Extraction of $f_{1T}^{\perp}$ and $g_{1T}^{\perp}$ from SIDIS and Drell-Yan data

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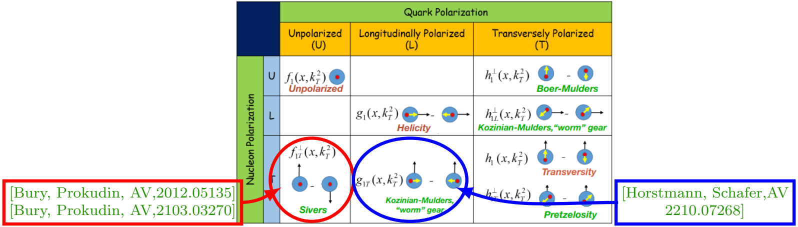
IPARCOS

Comunidad de Madrid

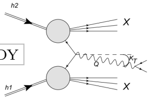


Outline

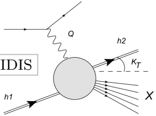
- Extraction of $f_{1T}^\perp$ and $g_{1T}^\perp$
- Problems of these extractions

Factorization valid at:

$Q \gg \Lambda \quad Q \gg q_T \quad Q \gg k_T \quad Q \gg M$

DY 

$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(-\frac{Q^2}{\mu^2}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

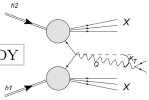
SIDIS 

$$\frac{d\sigma}{dp_\perp} \propto \int \frac{d^2b}{(2\pi)^2} e^{i\frac{(bp_\perp)}{z}} C\left(\frac{Q^2}{\mu^2}\right) F(x, b; \mu, \zeta) D(z, b; \mu, \bar{\zeta})$$

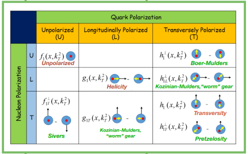
Factorization valid at:

$$Q \gg \Lambda \quad Q \gg q_T \quad Q \gg k_T \quad Q \gg M$$

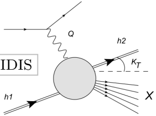
DY



$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(-\frac{Q^2}{\mu^2}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$



SIDIS



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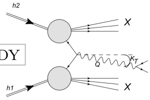
From the theory point of view
all LP asymetries are alike –
just substitute different function!

Factorization valid at:

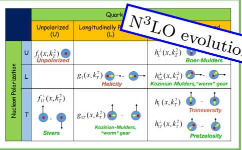
$$Q \gg \Lambda \qquad Q \gg q_T \qquad Q \gg k_T \qquad Q \gg M$$

DY



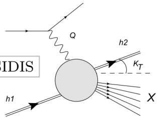
$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(-\frac{Q^2}{\mu^2}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

N^{3}LO coef.function



N^{3}LO evolution

SIDIS



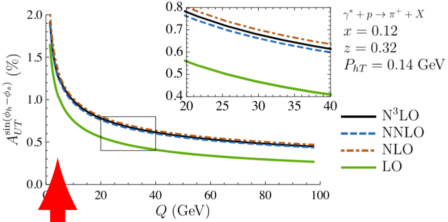
$$\frac{d\sigma}{dp_\perp} \propto \int \frac{d^2b}{(2\pi)^2} e^{i\frac{(bp_\perp)}{z}} C\left(\frac{Q^2}{\mu^2}\right) F(x, b; \mu, \zeta) D(z, b; \mu, \bar{\zeta})$$

Pert.input is same for all cases!

From the theory point of view
all LP asymetries are alike –
just substitute different function!

N^{4}LO is avalible!

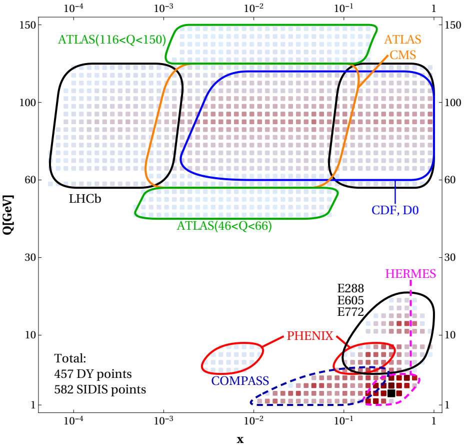
TMD evolution is very important



Very large effect
at small-Q

$$A_{UT}^{\sin(\phi_h-\phi_S)} = -M \frac{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi}\, b\, J_1\left(\frac{b|P_{hT}|}{z}\right) R(b,Q) f_{1T,q\leftarrow h_1}^{\perp}(x,b) D_{1,q\to h_2}(z,b)}{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi}\, J_0\left(\frac{b|P_{hT}|}{z}\right) R(b,Q) f_{1,q\leftarrow h_1}(x,b) D_{1,q\to h_2}(z,b)}$$

Unpolarized part is critical!



$f_1(\text{proton}),\ D_1(\pi, K)$

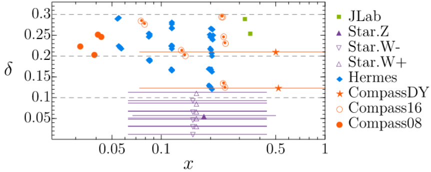
- SV19 [Scimemi, AV,1912.06532]
  - DY+SIDIS ($\sim 10^3$pt)
  - N^3LO - TMD factorization
  - N^2LO - small-b matching
  - NNPDF + DSS

$f_1(\text{pion})$

- Vpion19 [AV,1907.10356]
  - πDY (80 pt)
  - Same as SV19

$\Rightarrow$ ART23

Data for the Sivers function



Dataset name	Ref.	Reaction	# Points
Compass08	[36]	$d^\uparrow + \gamma^* \to \pi^+$	1 / 9
		$d^\uparrow + \gamma^* \to \pi^-$	1 / 9
		$d^\uparrow + \gamma^* \to K^+$	1 / 9
		$d^\uparrow + \gamma^* \to K^-$	1 / 9
Compass16	[39]	$p^\uparrow + \gamma^* \to h^+$	5 / 40
		$p^\uparrow + \gamma^* \to h^-$	5 / 40
Hermes	[35]	$p^\uparrow + \gamma^* \to \pi^+$	11 / 64
		$p^\uparrow + \gamma^* \to \pi^-$	11 / 64
		$p^\uparrow + \gamma^* \to K^+$	12 / 64
		$p^\uparrow + \gamma^* \to K^-$	12 / 64
JLab	[41, 42]	$p^\uparrow + \gamma^* \to \pi^+$	1 / 4
		$p^\uparrow + \gamma^* \to \pi^-$	1 / 4
		$p^\uparrow + \gamma^* \to K^+$	1 / 4
		$p^\uparrow + \gamma^* \to K^-$	0 / 4
SIDIS total			63
CompassDY	[40]	$\pi^- + d^\uparrow \to \gamma^*$	2 / 3
Star.W+	[43]	$p^\uparrow + p \to W^+$	5 / 5
Star.W-		$p^\uparrow + p \to W^-$	5 / 5
Star.Z		$p^\uparrow + p \to \gamma^*/Z$	1 / 1
DY total			13
Total			76

TMD factorization is valid **ONLY** at

$$q_T \ll Q, M \ll Q$$

DY: $q_T = q_T$

SIDIS: $q_T = \frac{p_{h\perp}}{z}$

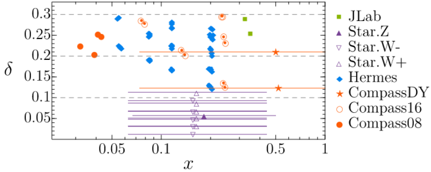
estimation

at $\delta = \frac{q_T}{Q} = 0.3 \sim 10\%$ power-correction

at $Q = 2\text{GeV} \quad \sim 25\%$ power-correction

Too optimistic!

Data for the Sivers function



TMD factorization is valid
ONLY at
$q_T \ll Q, M \ll Q$

we use
$\delta < 0.3$ $Q >$2GeV

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Ansatz for Sivers function

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$

$u,\ d,\ s,\ sea(=\bar{u},\bar{d},\bar{s})$

Common for all flavors
Similar to unpol.

12 free parameters

- $\{r_0, r_1, r_2\}$ – TMD part
- $\{N_{u,d}, \beta_{u,d}, \epsilon_{u,d}\}$ – valence quarks
- $\{N_{s,sea}, \beta_s = \beta_{sea}\}$ – rest

No restrictions for parameters

- Data driven extraction
- Positivity (almost satisfied)
- No "$T \sim f_1$" assumption and fake Q-dependence

Ansatz for Sivers function

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)^{\alpha} x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$

$u,\ d,\ s,\ sea(=\bar{u},\bar{d},\bar{s})$

Sea is not constrained
$\beta_q = \beta_{sea}, \quad \epsilon_s = \epsilon_{sea} = 0$

No resctrictions
typical fit $\alpha \in [0,5]$
fix $\alpha = 1$

Common for all flavors
Similar to unpol.

12 free parameters

- $\{r_0, r_1, r_2\}$ – TMD part
- $\{N_{u,d}, \beta_{u,d}, \epsilon_{u,d}\}$ – valence quarks
- $\{N_{s,sea}, \beta_s = \beta_{sea}\}$ – rest

No restrictions for parameters

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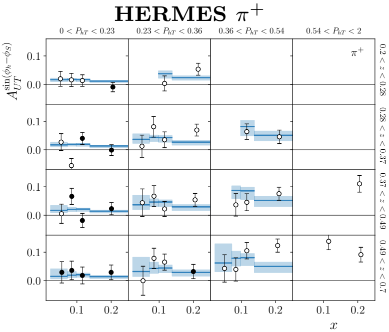
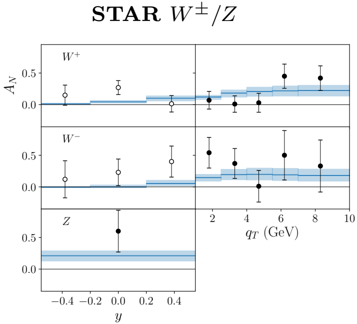
Example of data description



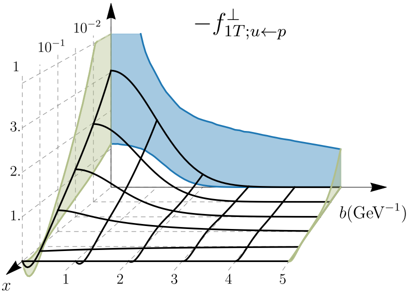


Filled points = in fit, Open point = prediction

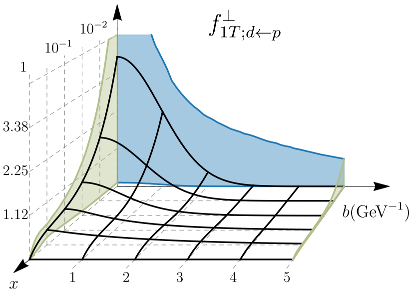
Actually, we can explain more data (up to $q_T < 0.4Q$ in SIDIS)

Sivers function

u-quark

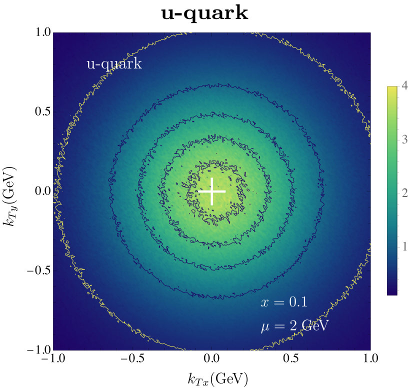


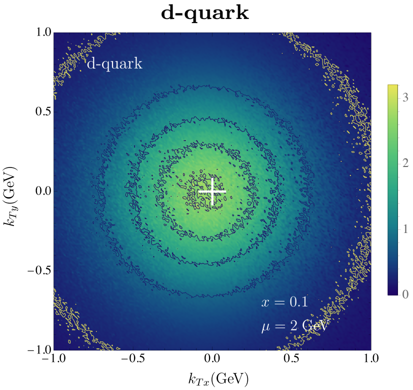
d-quark



- Notably huge uncertainties
- Not sign definite

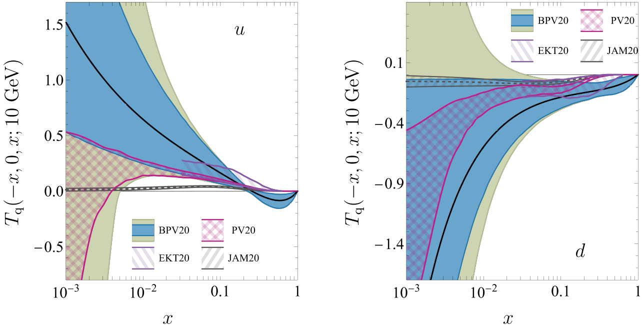
Sivers function



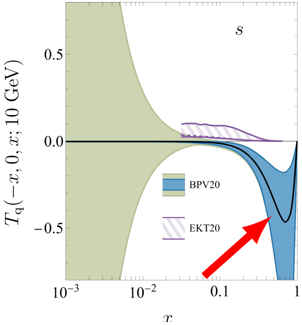


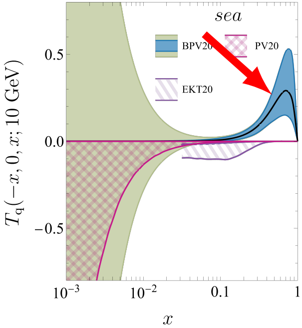
- Notably huge uncertainties
- Not sign definite

Qiu-Sterman function



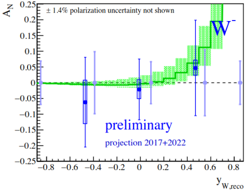
Un-biased extraction of Qiu-Sterman function. Using the NLO small-b matching [Tarasov, Scimemi, AV; 1901.04519]

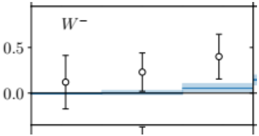




Un-biased extraction of Qiu-Sterman function. Using the NLO small-b matching [Tarasov, Scimemi, AV; 1901.04519]

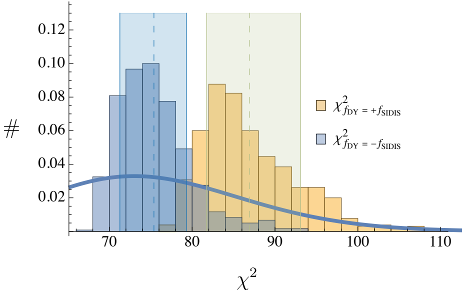
Large-x anomaly: due to VERY large A_N at STAR





Check sign-change

$$f_{1T}^{\perp}(SIDIS) = -f_{1T}^{\perp}(DY)$$



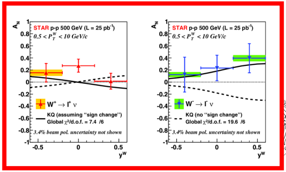
$$f_{1T}^{\perp}(sea) \to -f_{1T}^{\perp}(sea)$$

$\chi^2/N_{pt} = 0.88^{+0.16}_{-0.06}$ **vs.** $\chi^2/N_{pt} = 1.00^{+0.22}_{-0.08}$

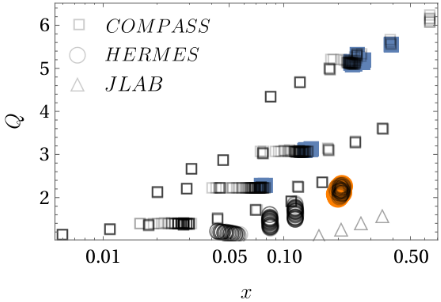
Current data does not check sign-change!

If there would be DY at anti-proton...

Naive picture



Data for the worm-gear-T function



we use

$\delta < 0.35 \qquad Q > 2\text{GeV}$

	Hermes				Compass	
	π^+	π^-	K^+	K^-	h^+	h^-
N_{pt}	11	11	11	11	14	12

most part of data
at very large δ
$N_{pt} = 70$
49 pt at $x \in [0.1, 0.2]$

Also uncertanties larger ...

Ansatz for worm-gear-T function

$$g_{1T,q}^{\perp}(x,b) = \sum_f x \int_x^1 \frac{dy}{y} C_{q\leftarrow f}^{\text{tw2}}\left(\frac{x}{y}, \mu_{\text{OPE}}\right) g_{1f}(y,\mu_{\text{OPE}}) + \sum_f [C_{q\leftarrow f}^{\text{tw3}} \otimes T_f](x) + \mathcal{O}(b^2),$$

Twist-2 part (WW)
helicity PDF

Twist-3 part
$\langle \bar{q} G q \rangle$

$$C_{q\leftarrow q}^{\text{tw2}}(x,\mu) = 1 + a_s(\mu) C_F \left[\mathbf{L}_b(2\ln x - 4\ln(1-x) - 1 - 2x) - 2(1-x) - 2\ln x - \frac{\pi^2}{6}\right] + \mathcal{O}(a_s^2),$$
$$C_{q\leftarrow g}^{\text{tw2}}(x,\mu) = a_s \left[-\mathbf{L}_b(\ln x + 2 - 2x) + 1 - x + \frac{1}{2}\ln x\right] + \mathcal{O}(a_s^2),$$

Complete NLO computation
[Rein,Rodini,AV:2209.00962]
(all distributions)

Ansatz for worm-gear-T function

$$g_{1T,q}^{\perp}(x,b)=\sum_f x\int_x^1 \frac{dy}{y}C^{\text{tw2}}_{q\leftarrow f}\left(\frac{x}{y},\mu_{\text{OPE}}\right)g_{1f}(y,\mu_{\text{OPE}})+\sum_f[C^{\text{tw3}}_{q\leftarrow f}\otimes T_f](x)+\mathcal{O}(b^2),$$

Twist-2 part (WW)
helicity PDF

Twist-3 part
$\langle \bar{q}Gq\rangle$

$$C^{\text{tw2}}_{q\leftarrow q}(x,\mu)=1+a_s(\mu)C_F\left[\mathbf{L}_b(2\ln x-4\ln(1-x)-1-2x)-2(1-x)-2\ln x-\frac{\pi^2}{6}\right]+\mathcal{O}(a_s^2),$$

$$C^{\text{tw2}}_{q\leftarrow g}(x,\mu)=a_s\left[-\mathbf{L}_b(\ln x+2-2x)+1-x+\frac{1}{2}\ln x\right]+\mathcal{O}(a_s^2),$$

Complete NLO computation
[Rein,Rodini,AV:2209.00962]
(all distributions)

Ansatz:

$$g_{1T}^{\perp}(x,b)=\underbrace{\sum_f x\int_x^1 \frac{dy}{y}C^{\text{tw2}}_{q\leftarrow f}\left(\frac{x}{y},\mu\right)g_{1f}(y,\mu)}_{\text{WW}}\frac{\lambda_2}{\cosh(\lambda_1 b)}$$

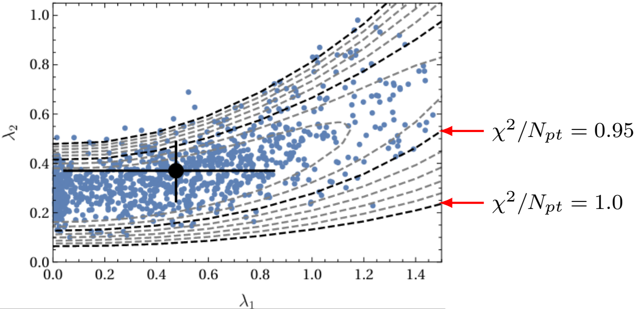
$\sim$tw-3 effects
$(\lambda_2=1)$ = pure WW

b-profile

Data barely constraints parameters

$\lambda_1 = 0.47^{+0.38}_{-0.43}$

$\lambda_2 = 0.37^{+0.12}_{-0.12}$

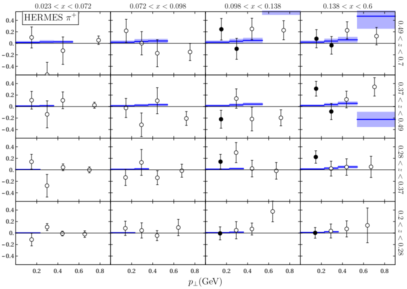


null-hypothesis: $\lambda_2 = 0 \quad \chi^2/N_{pt} = 1.06$

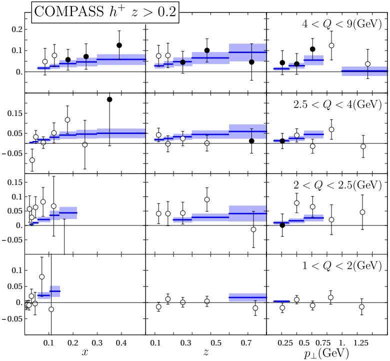
tw3-null-hypothesis: $\lambda_2 = 1 \quad \chi^2/N_{pt} = 0.95$

Example of data description

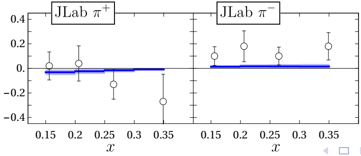
HERMES π^+



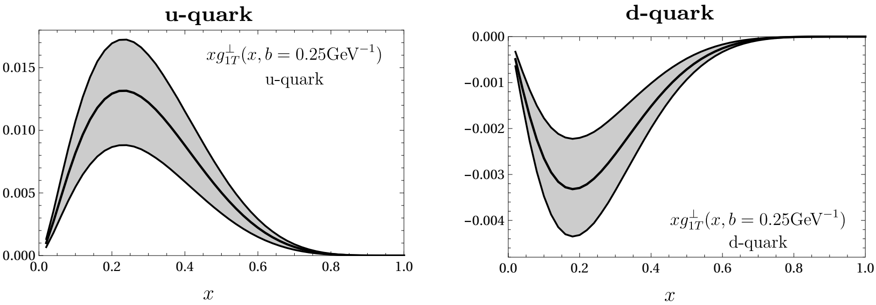
COMPASS π^+



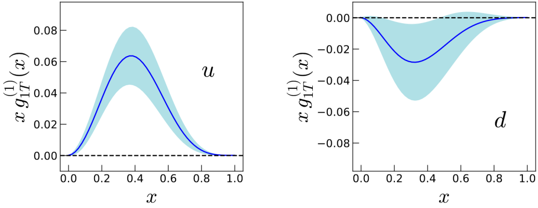
Filled points = in fit, Open point = prediction



Worm-gear-T function (at $b = 0.25\text{GeV}^{-1}$)



[Bhattacharya, Kang, Metz, Penn, Pitonyak: 2110.10253]



ΔF_{UT}^1 in W/Z production

$$\frac{d\sigma}{dQ^2 dy d\varphi dq_T^2} = \frac{\alpha_{\text{em}}^2(Q)}{9sQ^2}\Big\{F_{UU}^1$$
$$+|s_T|\sin(\varphi-\phi_S)F_{TU}^1 + |s_T|\cos(\varphi-\phi_S)\Delta F_{TU}^1$$

ΔF_{TU}^1 is only due to P-violation by W/Z alike Sivers asymetry turned by 90^o

$$F_{TU}^1 = -M|C_V(-Q^2,\mu)|^2 \sum_{f,\text{ch.}} z_{ll'}^{\text{ch.}} z_{ff'}^{\text{ch.}} \Delta^{\text{ch.}} \int_0^\infty \frac{b^2 db}{2\pi} J_1(b|q_T|)\left[f_{1T;f\leftarrow h_1}^\perp f_{1;\bar f'\leftarrow h_2} + f_{1T;\bar f\leftarrow h_1}^\perp f_{1;f'\leftarrow h_2}\right]$$

$$\Delta F_{TU}^1 = M|C_V(-Q^2,\mu)|^2 \sum_{q,\text{ch.}} \bar z_{ll'}^{\text{ch.}} z_{ff'}^{\text{ch.}} \Delta^{\text{ch.}} \int_0^\infty \frac{b^2 db}{2\pi} J_1(b|q_T|)\left[g_{1T;f\leftarrow h_1} f_{1;\bar f'\leftarrow h_2} - g_{1T;\bar f\leftarrow h_1} f_{1;f'\leftarrow h_2}\right],$$

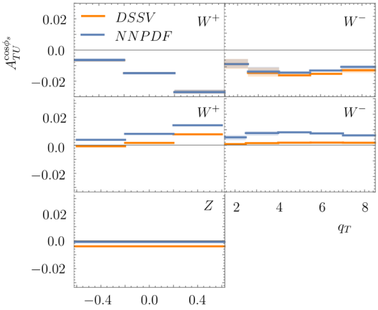
ΔF^1_{UT} in W/Z production

$$\frac{d\sigma}{dQ^2 dy d\varphi dq_T^2} = \frac{\alpha_{\rm em}^2(Q)}{9sQ^2}\Big\{F^1_{UU} + |s_T|\sin(\varphi-\phi_S)F^1_{TU} + |s_T|\cos(\varphi-\phi_S)\Delta F^1_{TU}$$

ΔF^1_{TU} is only due to P-violation by W/Z alike Sivers asymetry turned by 90^o

$$F^1_{TU} = -M|C_V(-Q^2,\mu)|^2 \sum_{f,\mathrm{ch.}} z^{\mathrm{ch.}}_{ll'} z^{\mathrm{ch.}}_{ff'} \Delta^{\mathrm{ch.}} \int_0^\infty \frac{b^2 db}{2\pi} J_1(b|q_T|)\left[f^\perp_{1T;f\leftarrow h_1} f_{1;\bar f'\leftarrow h_2} + f^\perp_{1T;\bar f\leftarrow h_1} f_{1;f'\leftarrow h_2}\right]$$

$$\Delta F^1_{TU} = M|C_V(-Q^2,\mu)|^2 \sum_{q,\mathrm{ch.}} \bar z^{\mathrm{ch.}}_{ll'} z^{\mathrm{ch.}}_{ff'} \Delta^{\mathrm{ch.}} \int_0^\infty \frac{b^2 db}{2\pi} J_1(b|q_T|)\left[g_{1T;f\leftarrow h_1} f_{1;\bar f'\leftarrow h_2} - g_{1T;\bar f\leftarrow h_1} f_{1;f'\leftarrow h_2}\right],$$



STAR kinematic up to $\sim 2\%$

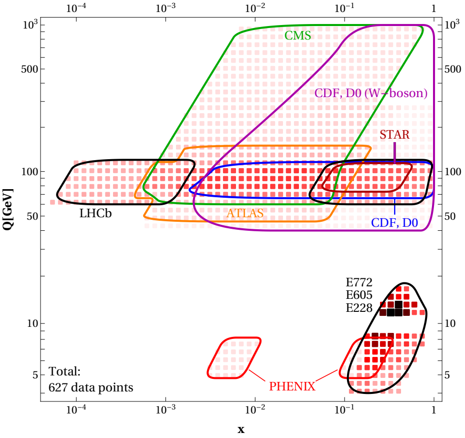
Conclusions

Currently, the extractions of "off-diagonal" TMDs
are "questionable"
Issues both in the data (large uncertainties) and theory (problems with low-Q) sides

How we can overcome this problem

- New data: STAR W/Z, COMPASS(?)
- New unpolarized sector (MAP22, ART23) & more accurate uncertainty estimation
- New theory (corrections at low-energy)
- New generation of global fits (twist-3 collinear + TMDs)

Backup slides

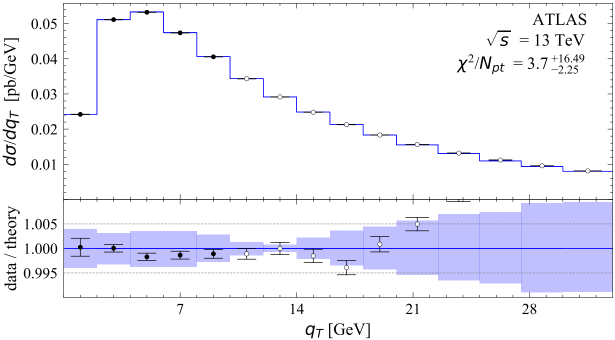


*** data included for the first time**

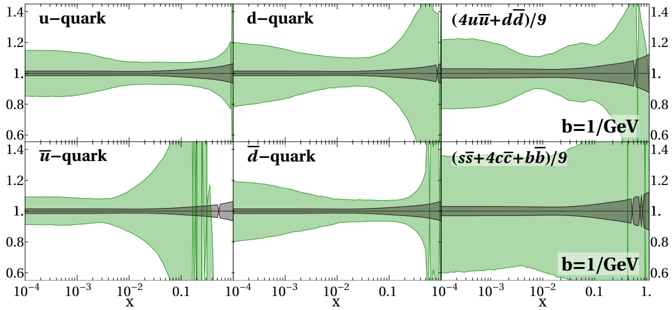
- ATLAS
  - Z-boson at 8 (y-diff.)
  - **Z-boson at 13 TeV (0.1% prec.!)**
- CMS
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
  - **Z/γ up to $Q = 1000$GeV**
- LHCb
  - Z-boson at 7 and 8 TeV
  - **Z-boson at 13 TeV (y-diff.)**
- **Further more:**
  - Z-boson at Tevatron
  - **W-boson at Tevatron**
  - **Z-boson at RHIC**
  - DY at PHENIX
  - DY at FERMILAB (fix target)

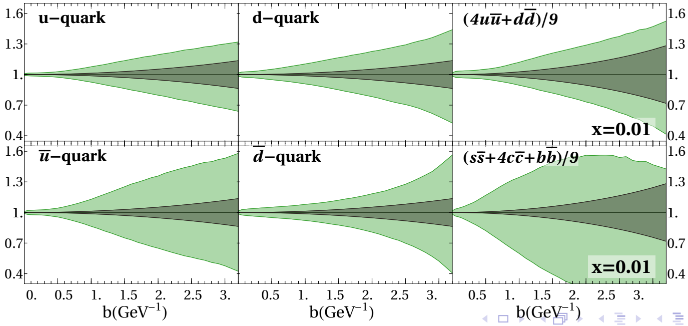
627 data points

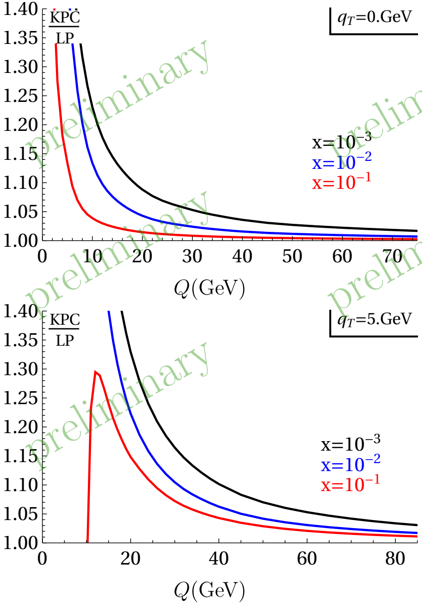
vs. 457 in SV19
vs. 484 in MAP22



$$\text{TOTAL } (N_{\text{pt}} = 627): \quad \chi^2/N_{\text{pt}} = 0.96^{+0.09}_{-0.01}$$


u−quark
d−quark
(4uū+dd̄)/9
ū−quark
d̄−quark
(ss̄+4cc̄+bb̄)/9
b=1/GeV
x


u−quark
d−quark
(4uū+dd̄)/9
ū−quark
d̄−quark
(ss̄+4cc̄+bb̄)/9
x=0.01
b(GeV^−1)



- Mostly change the normalization
- $\sim 1\%$ at LHC energy
- $\sim 50 - 100\%$ at HERMES
- Estimation for SV19 ($q_T = 0$)

$$\sim \frac{2\text{GeV}^2}{\sqrt{x}Q^2}$$