



International Workshop on Hadron Structure and Spectroscopy 2023

25-28 June 2023, Prague - Czechia

**MAPTMD22 :**  
**femtography of unpolarized**  
**protons and pions at N<sup>3</sup>LL accuracy**

**Marco Radici**

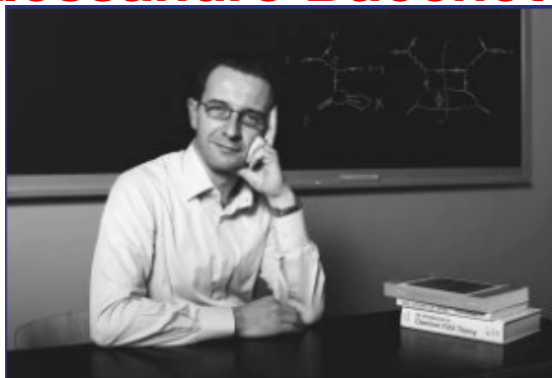


for the **MAP** Collaboration  
(**M**ulti-dimensional **A**nalysis of **P**artononic distributions)

<https://github.com/MapCollaboration/>

# Results with contributions from...

**Alessandro Bacchetta**



**Marco Radici**



**Matteo Cerutti**



**Andrea Signori**



**Valerio Bertone**



**Chiara Bissolotti**



**Giuseppe Bozzi**



**Fulvio Piacenza**



**Simone Venturini**





## Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data

### The MAP Collaboration<sup>1</sup>

Alessandro Bacchetta<sup>a,b</sup>, Valerio Bertone<sup>b,c</sup>, Chiara Biscolotti<sup>b,a,d</sup>,  
Giuseppe Bozzi<sup>b,e,f</sup>, Matteo Cerutti<sup>b,a,b</sup>, Fulvio Piacenza<sup>a</sup>, Marco Radici<sup>b</sup>  
and Andrea Signori<sup>b,a,b,2</sup>

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[marco.radici@pv.infn.it](mailto:marco.radici@pv.infn.it), [andrea.signori@unipv.it](mailto:andrea.signori@unipv.it)

arXiv:2206.07598

JHEP10(2022)127

arXiv:2210.01733

PHYSICAL REVIEW D **107**, 014014 (2023)

## Extraction of pion transverse momentum distributions from Drell-Yan data

Matteo Cerutti<sup>b,1,2,\*</sup>, Lorenzo Rossi<sup>b,1,2,†</sup>, Simone Venturini<sup>b,1,2,‡</sup>, Alessandro Bacchetta<sup>b,1,2,§</sup>,  
Valerio Bertone<sup>b,3,||</sup>, Chiara Biscolotti<sup>b,4,¶</sup> and Marco Radici<sup>b,2,\*\*</sup>

(MAP Collaboration)<sup>††</sup>

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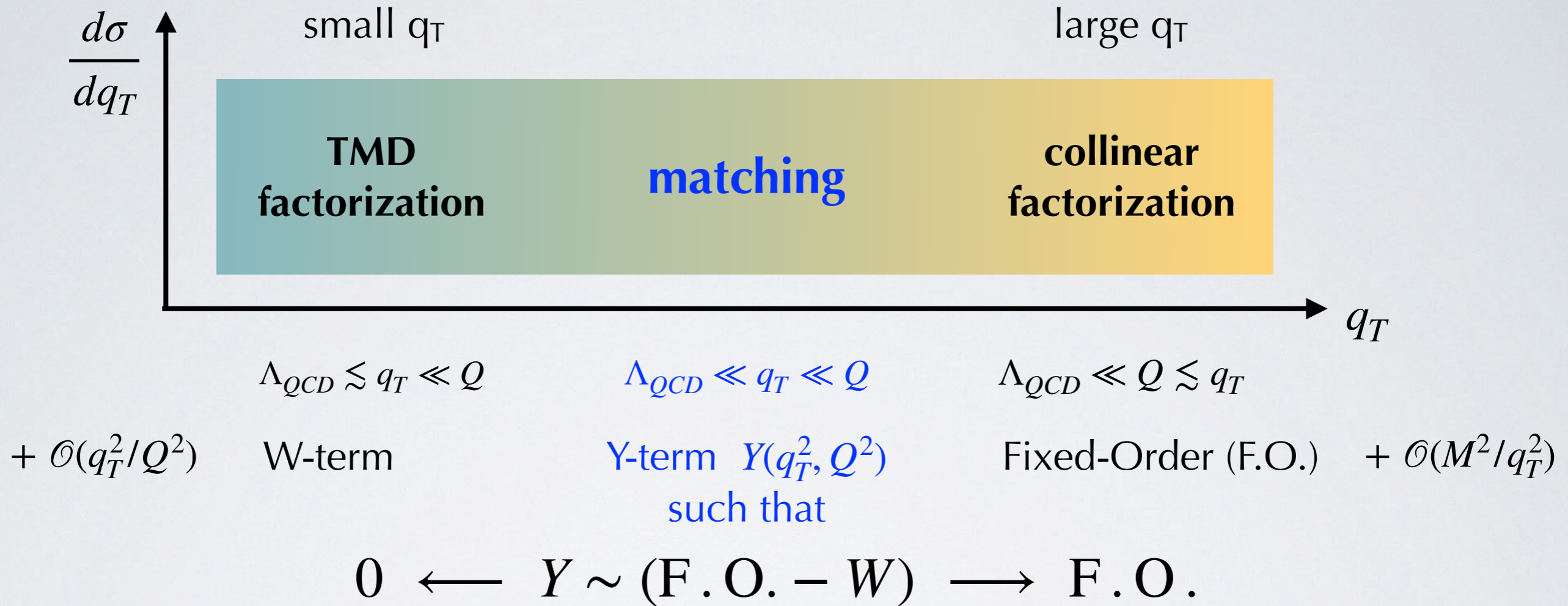
<sup>3</sup>IRFU, CEA, Université Paris-Saclay, F-91191 Gif-sur-Yvette, France

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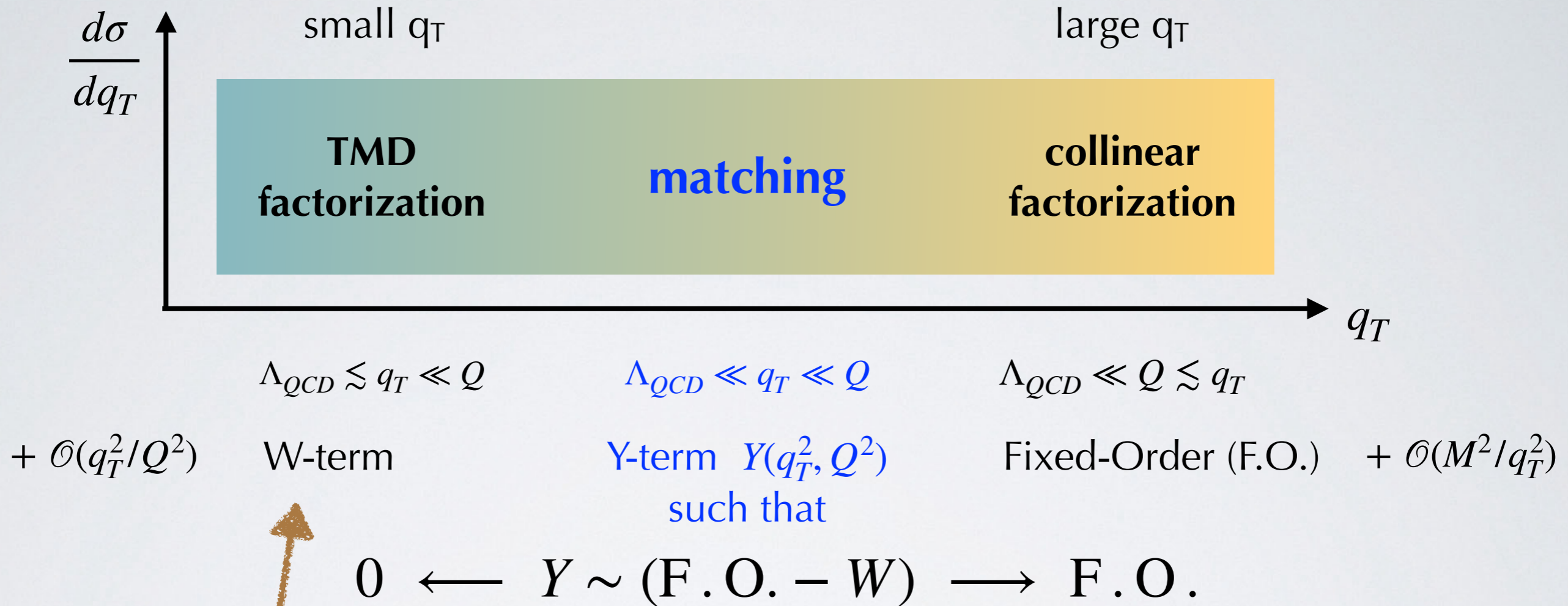
 (Received 11 October 2022; accepted 21 December 2022; published 11 January 2023)

We map the distribution of unpolarized quarks inside a unpolarized pion as a function of the quark's transverse momentum, encoded in unpolarized transverse momentum distributions (TMDs). We extract the pion TMDs from available data of unpolarized pion-nucleus Drell-Yan processes, where the cross section is differential in the lepton-pair transverse momentum. In the cross section, pion TMDs are convoluted with nucleon TMDs that we consistently take from our previous studies. We obtain a fairly good agreement with data. We present also predictions for pion-nucleus scattering that is being measured by the COMPASS Collaboration.

# TMD factorization

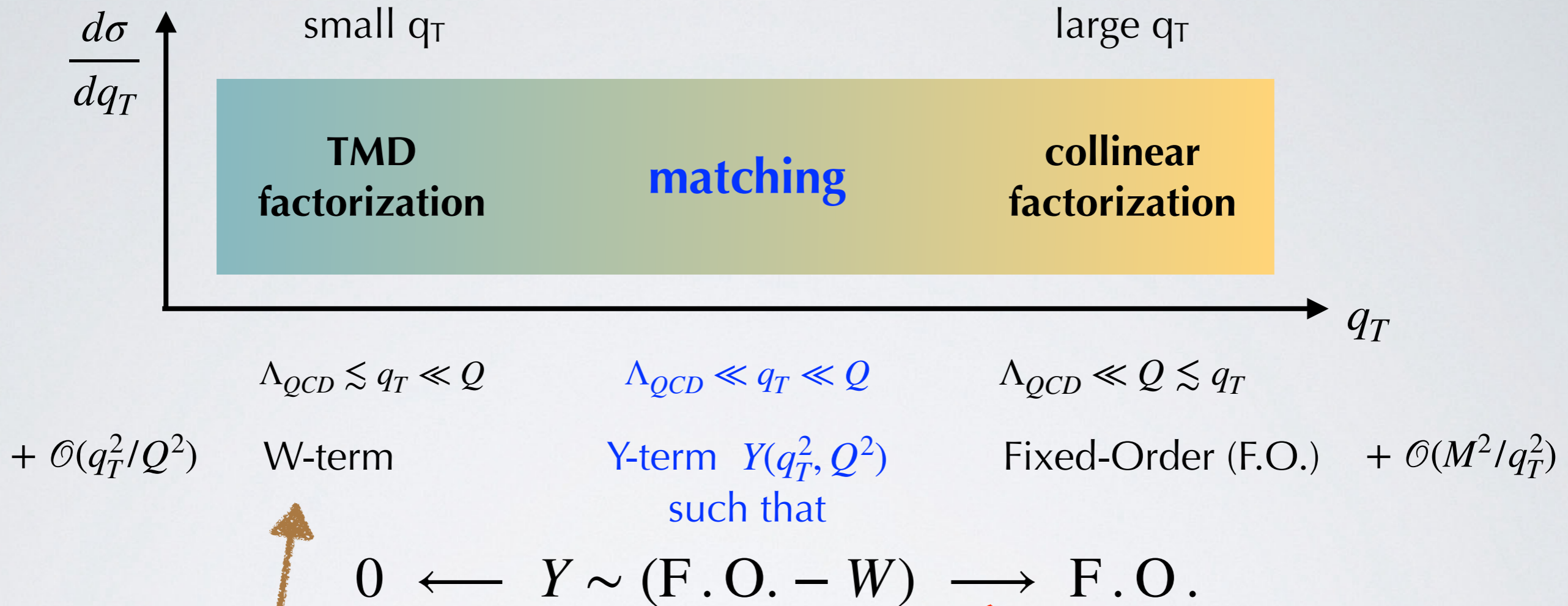


# TMD factorization



our analysis is in this regime where W-term dominates. Y-term is not included

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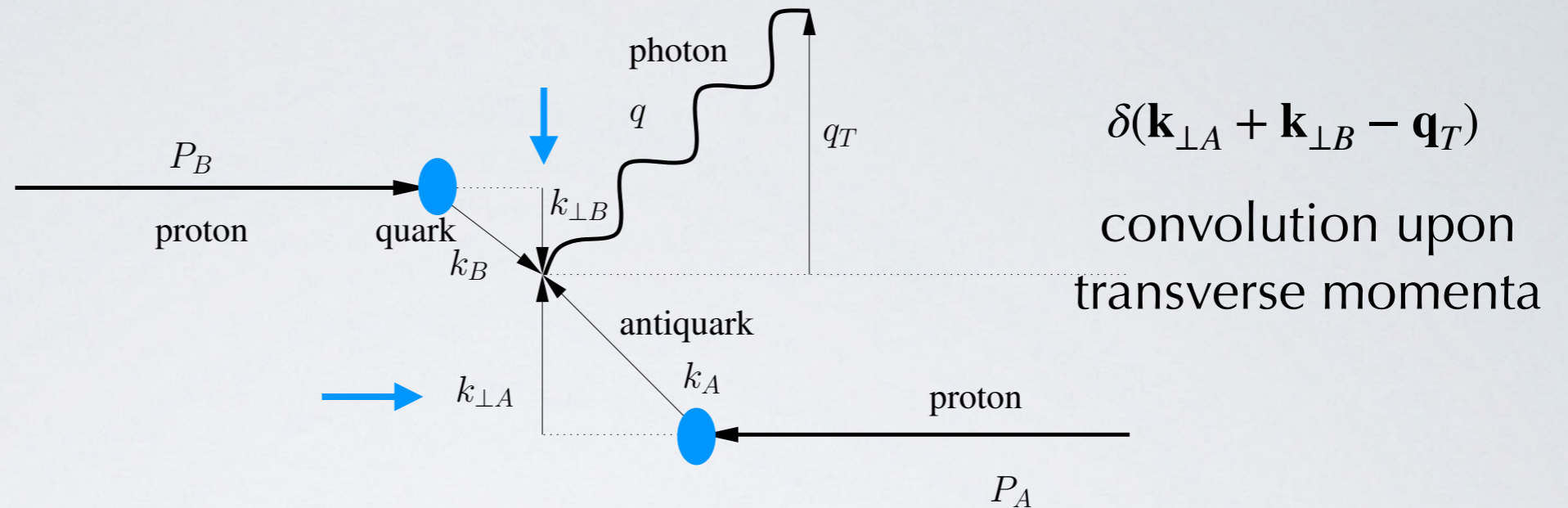
at low Q (SIDIS), these cancelations do not occur: where is the limit for TMD factorization (W) to be valid?

# TMD factorization: Drell-Yan

$$M^2 \ll Q^2$$

$$q_T^2 \ll Q^2$$

W-term dominant



$$\frac{d\sigma}{dq_T dy dQ} \sim W(x_A, x_B, q_T, Q) \sim \mathcal{H}^{\text{DY}}(Q^2) \left[ f_1^{\bar{q}}(x_A, \mathbf{k}_{\perp A}^2; Q^2) \otimes f_1^q(x_B, \mathbf{k}_{\perp B}^2; Q^2) \right]$$

hard factor

**TMD PDF anti-q**

**TMD PDF q**

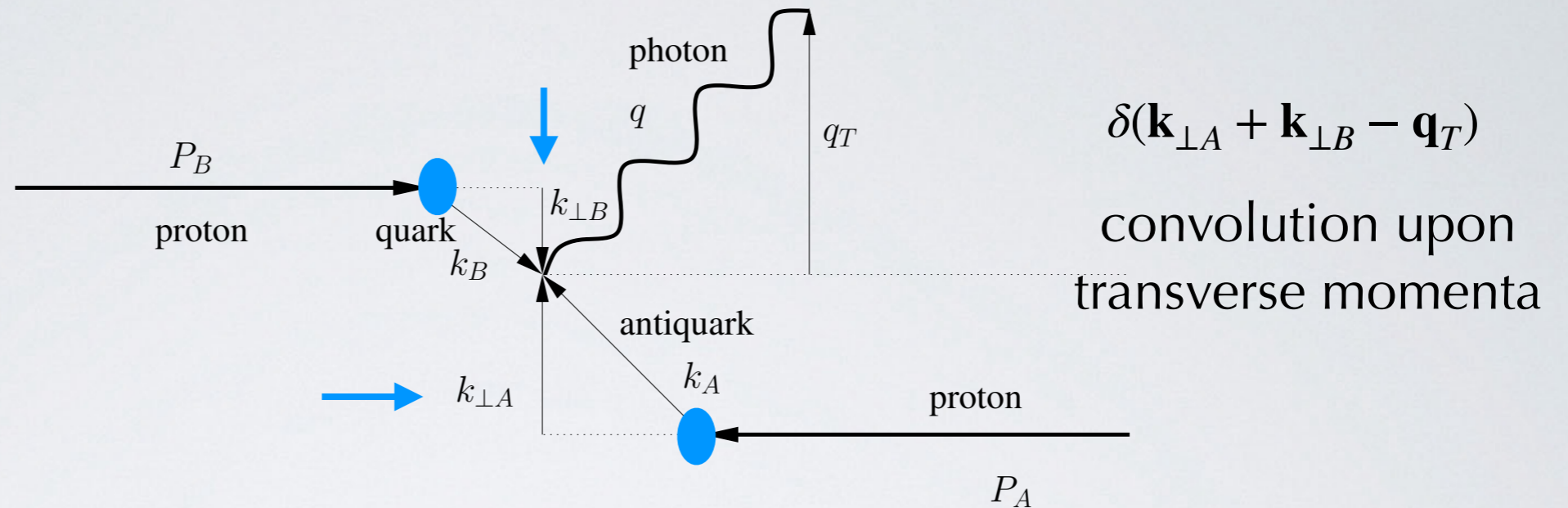
TMDs depend on two scales  $\mu, \zeta$ ; can be chosen as  $\zeta = \mu^2 = Q^2$

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**TMD PDF anti-q**

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Fourier Transform:  
from convolution to  
simple product

$$= \mathcal{H}^{\text{DY}}(Q^2) \frac{1}{2\pi} \int_0^\infty db_T b_T J_0(b_T, q_T) \tilde{f}_1^{\bar{q}}(x_A, b_T^2; Q^2) \tilde{f}_1^q(x_B, b_T^2; Q^2)$$

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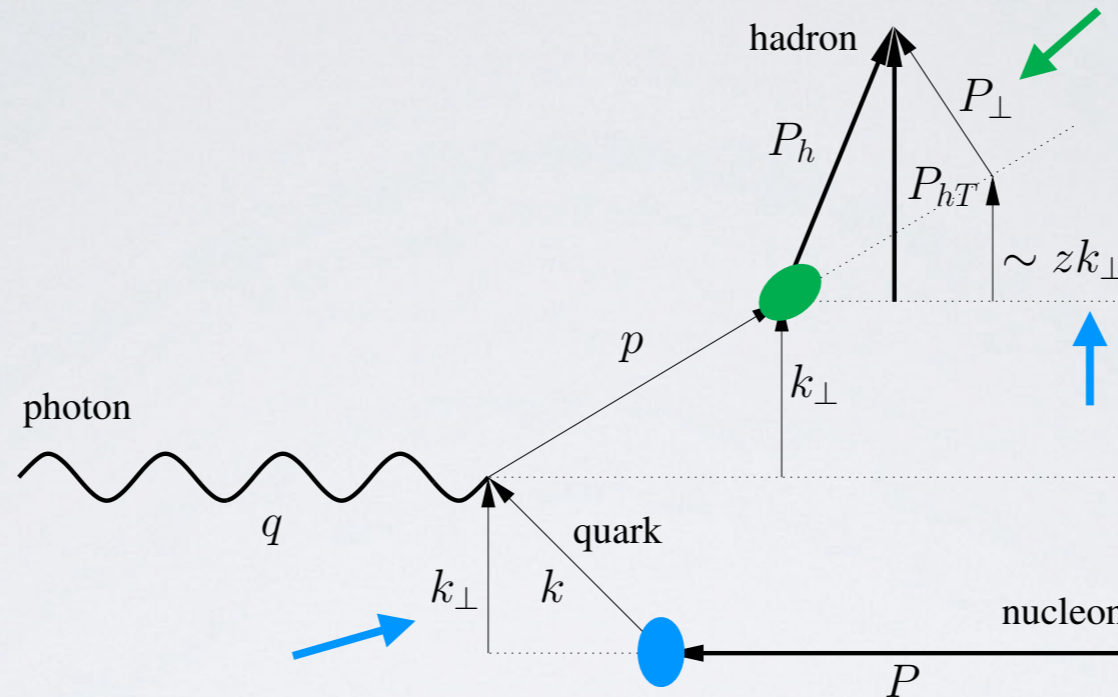


# TMD factorization: SIDIS

$$M^2 \ll Q^2$$

$$q_T^2 = \frac{P_{hT}^2}{z^2} \ll Q^2$$

W-term dominant



$$-z\mathbf{q}_T \approx \mathbf{P}_{hT} = \underbrace{z\mathbf{k}_\perp}_{\text{blue}} + \underbrace{\mathbf{P}_\perp}_{\text{green}}$$

$$\delta(\mathbf{k}_\perp + \mathbf{P}_\perp/z + \mathbf{q}_T)$$

convolution upon  
transverse momenta

$$\frac{d\sigma}{dx dz dq_T dQ} \sim W(x, z, q_T, Q) \sim \mathcal{H}^{\text{SIDIS}}(Q^2) \left[ f_1^q(x, \mathbf{k}_\perp^2; Q^2) \otimes D_1^{q \rightarrow h}(z, \mathbf{P}_\perp^2; Q^2) \right]$$

hard factor

**TMD PDF**

**TMD FF**

Fourier Transform:  
from convolution to  
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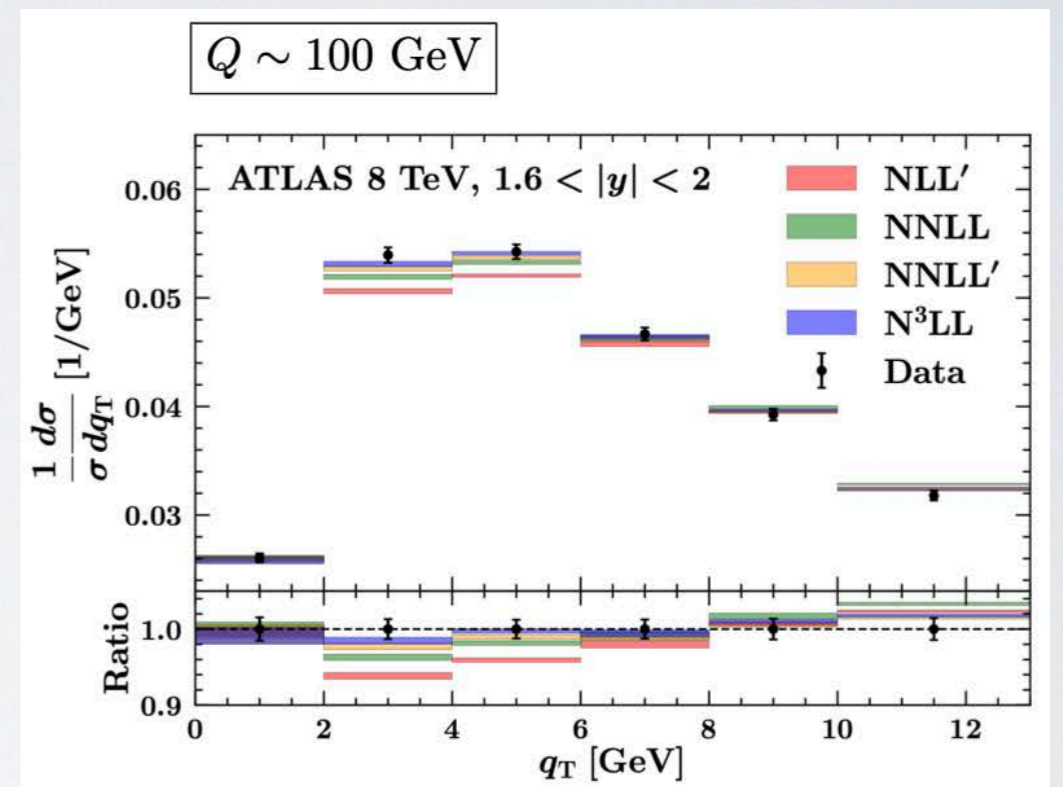
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# Normalization problem in SIDIS

increasing perturbative accuracy

increases agreement with Drell-Yan



# Normalization problem in SIDIS

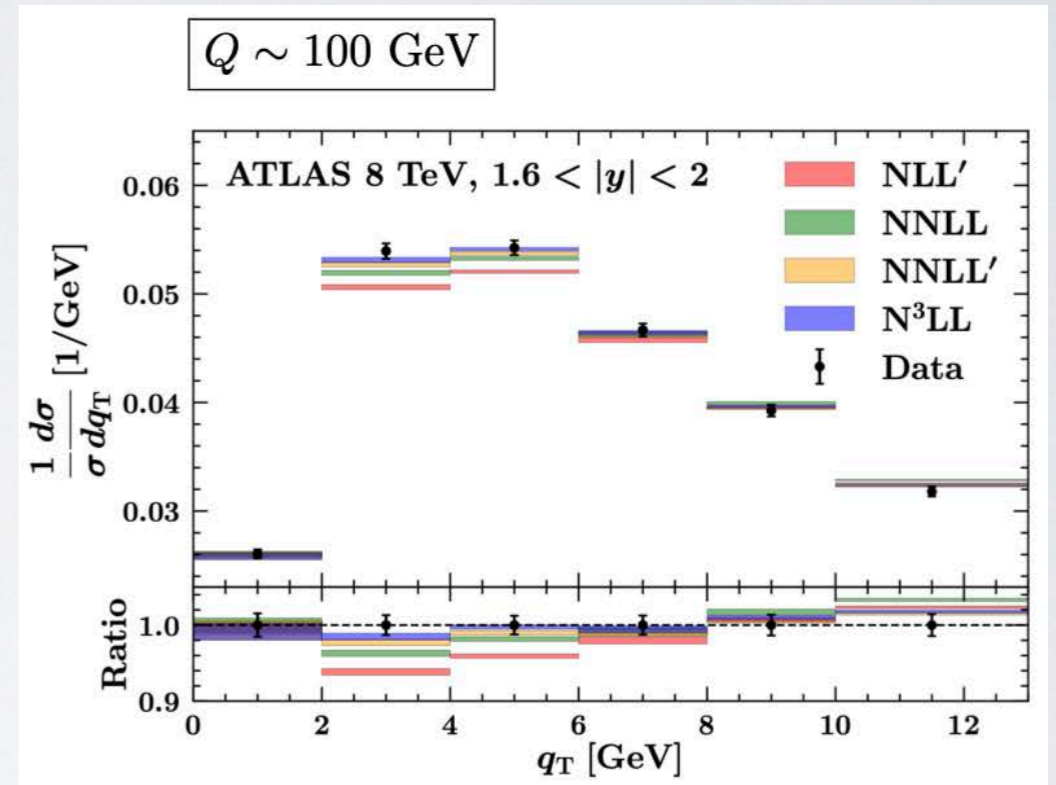
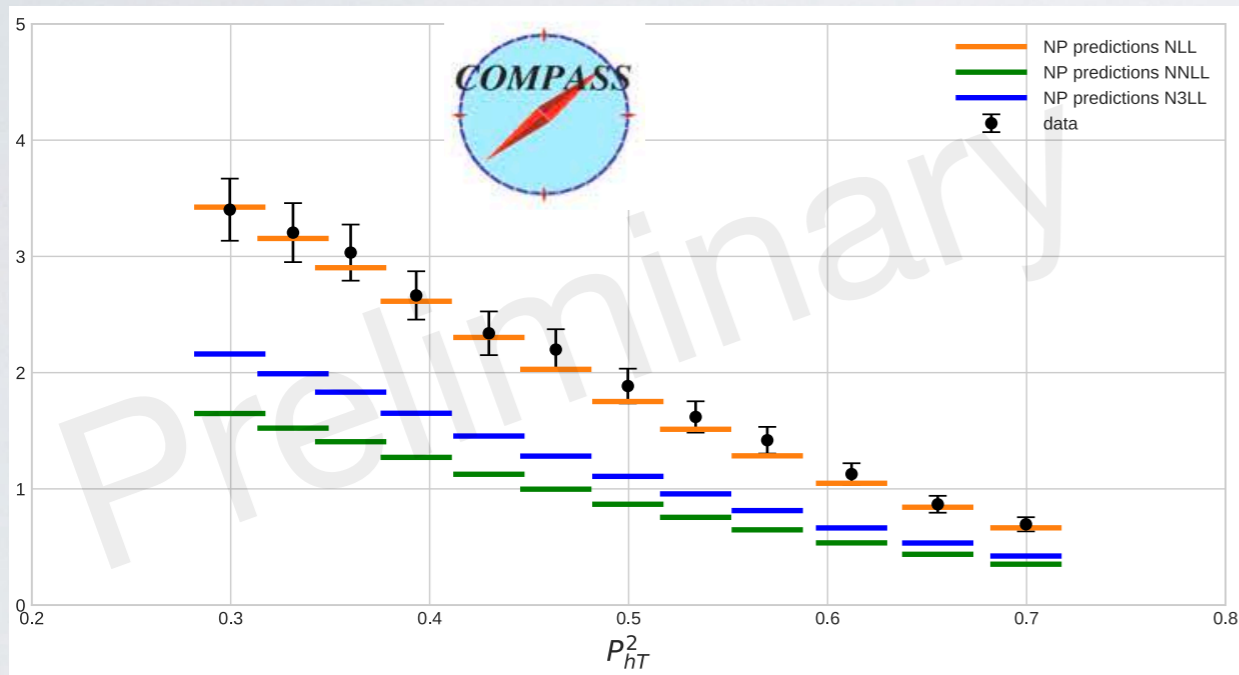
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$$M^h(\bar{x}, \bar{z}, \bar{Q}, P_{hT})$$

NLL    NNLL    N<sup>3</sup>LL



discrepancy is  $P_{hT}$ -independent:  $M_{\text{NLL}}/M_{\text{NNLL}} \sim 2$      $M_{\text{NLL}}/M_{\text{N}^3\text{LL}} \sim 1.5$

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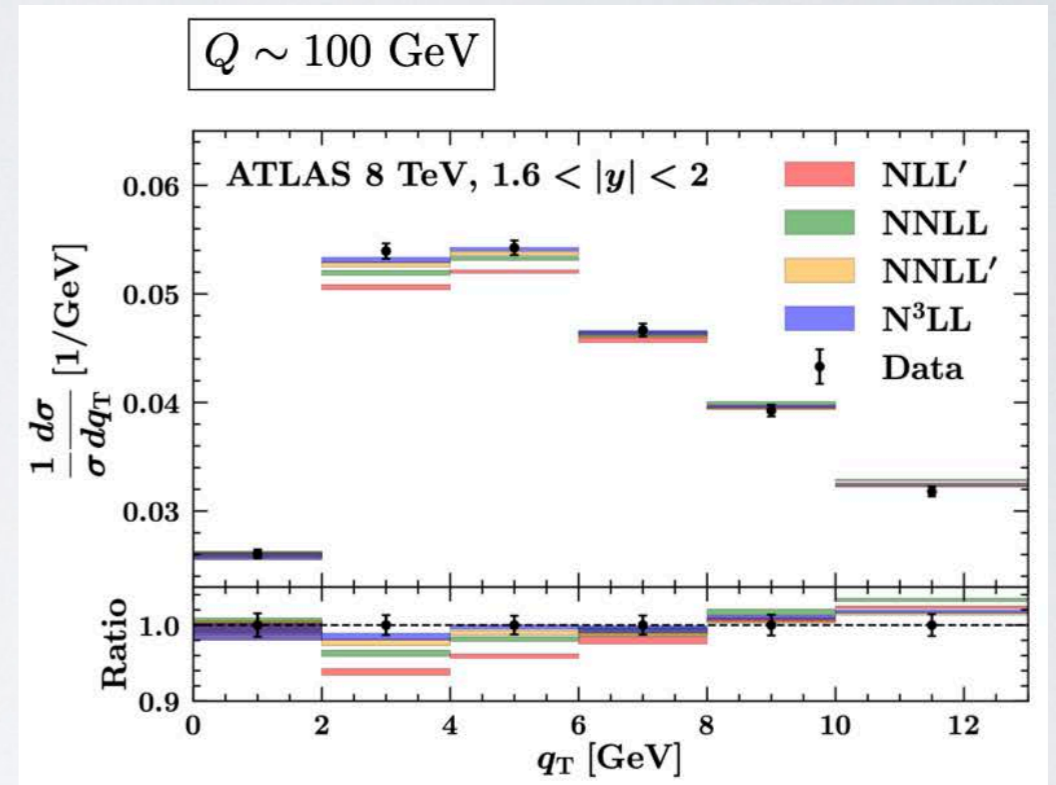
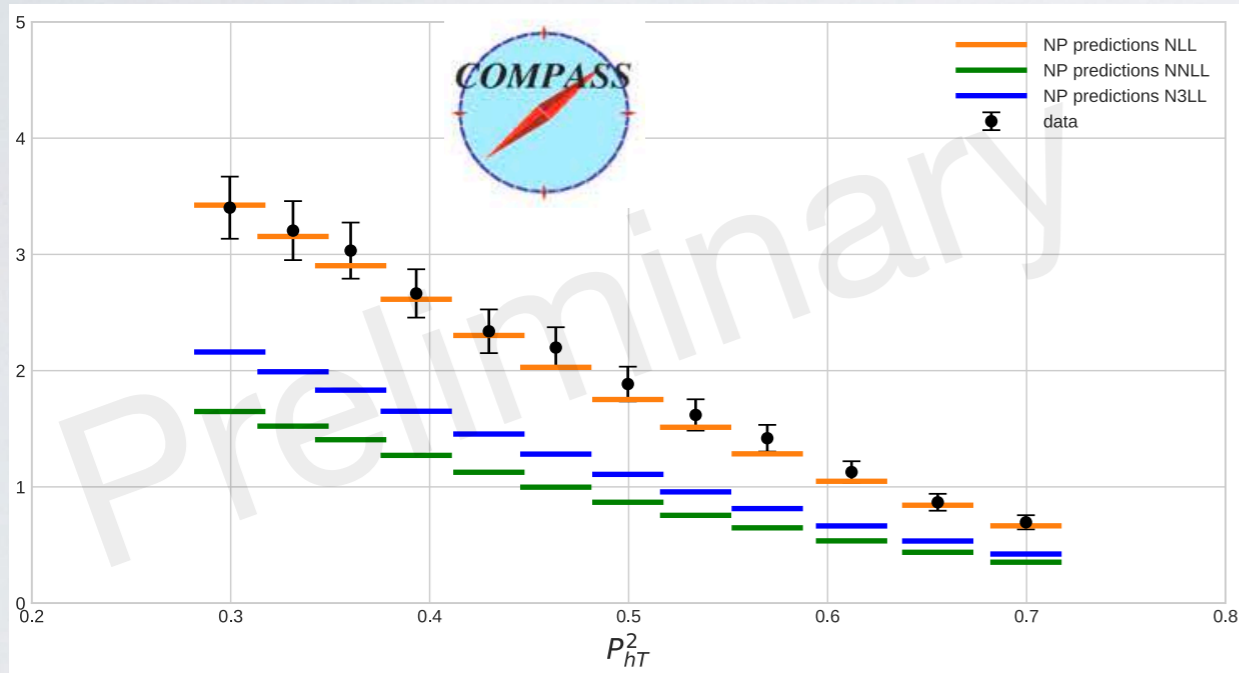
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tensions observed also at larger  $q_T$   
and also in Drell-Yan at low  $Q$   
and also in  $e^+e^-$  annihilations

Gonzalez et al., P.R. D**98** (18) 114005  
Bacchetta et al., P.R. D**100** (19) 014018  
Moffat et al., P.R. D**100** (19) 094014

but not in SV 2019 fit

Scimemi & Vladimirov,  
arXiv:1912.06532

No normalization problems for  
collinear SIDIS  $d\sigma/dx dz dQ$  :

MAPFF1.0 (Map Collaboration)  
Abdul Khalek et al., arXiv:2105.08725

# Normalization problem in SIDIS

at NLL,  $\mathcal{H}^{\text{SIDIS}} \approx 1$

the integrated W-term reproduces  
the SIDIS collinear  $d\sigma$  at LO,  
which reasonably describes data

De Florian et al., P.R. D75 (07) 114010

$$\begin{aligned} \int dq_T \frac{d\sigma}{dx dz dq_T dQ} &= \int dq_T W \Big|_{\text{NLL}} \propto f_1^q(x, Q) D_1^{q \rightarrow h}(z, Q) \\ &= \frac{d\sigma}{dx dz dQ} \Big|_{\text{LO}} \end{aligned}$$

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at higher orders,  $\mathcal{H}^{\text{SIDIS}} = 1 + c_1 \alpha_S + \dots$   $c_1 < 0$  spoils agreement

the integrated W-term contains only some terms of the SIDIS collinear  $d\sigma$

$$\int dq_T W \Big|_{\text{NNLL}} \neq \frac{d\sigma}{dx dz dQ} \Big|_{\text{NLO}}$$

Y-term  $\not\rightarrow 0$

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We define the normalization factor

$$\omega(x, z, Q) = \frac{d\sigma_{\text{nomix}}}{dx dz dQ} / \int d\mathbf{q}_T W = 1 \quad \text{at NLL}$$

includes all NLO terms where x- and z-dependence can be factorized  
 $\rightarrow$  includes W + part of the Y term...

Does not depend on fit parameters, precomputed

# Scale dependence of TMD

$$f_1^q(x, b_T^2; \mu_f, \zeta_f) =$$

$$\sum_i [C_{q \rightarrow i}(x, b_T^2; \mu_{b_*}) \otimes f_1^i(x, \mu_{b_*})]$$

matching collinear PDF  
at small  $b_T$

$$\times \exp[S(\mu_f, \mu_{b_*})]$$

Sudakov: evolution in  $\mu$  scale; contains  
anomalous dimensions  $\gamma_F, \gamma_K$

$$\times \left[ \frac{\zeta_f}{\mu_{b_*}^2} \right]^{K(b_*, \mu_{b_*})/2}$$

evolution in  $\zeta$  scale; contains  
Collins-Soper kernel  $K$

perturbative

perturbative  $\alpha_S^n$

accuracy	$\mathcal{H}$ and $C$	$K$ and $\gamma_F$	$\gamma_K$	PDF and $\alpha_S$ evol.	FF
LL	0	-	1	-	-
NLL	0	1	2	LO	LO
NLL'	1	1	2	NLO	NLO
NNLL	1	2	3	NLO	NLO
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N <sup>3</sup> LL	2	3	4	NNLO	NNLO
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NLO

FF at NNLO  
not yet  
implemented

Borsa et al.,  
P.R.L. **129** (22) 012002  
arXiv:2202.05060

Abdul Khalek et al.,  
P.L. **B834** (22) 137456  
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nonperturbative Collins-Soper kernel (arbitrary  $Q_0=1$  GeV)

$$\times f_{NP}(x, b_T; Q_0)$$

nonperturbative TMD at initial (arbitrary) scale  $Q_0$

perturbative

non perturbative

$$\mu_f \geq \mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*(b_T)} \geq 1$$

prescription to smoothly connect to large  $b_T$ , avoiding Landau pole. Introduces **nonperturbative** part

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# Most recent extractions of unpolarized TMD $f_1$




## SIDIS

	Accuracy	HERMES	COMPASS	DY	Z production	N of points	$\chi^2/N_{\text{points}}$
PV 2017 <a href="#">arXiv:1703.10157</a>	NLL	✓	✓	✓	✓	8059	1.5
SV 2017 <a href="#">arXiv:1706.01473</a>	NNLL'	✗	✗	✓	✓	309	1.23
BSV 2019 <a href="#">arXiv:1902.08474</a>	NNLL'	✗	✗	✓	✓	457	1.17
SV 2019 <a href="#">arXiv:1912.06532</a>	N <sup>3</sup> LL	✓	✓	✓	✓	1039	1.06
PV 2019 <a href="#">arXiv:1912.07550</a>	N <sup>3</sup> LL	✗	✗	✓	✓	353	1.07
SV19 + flavor dep. <a href="#">arXiv:2201.07114</a>	N <sup>3</sup> LL	✗	✗	✓	✓	309	<1.08>
MAPTMD 2022 <a href="#">arXiv:2206.07598</a>	N <sup>3</sup> LL	✓	✓	✓	✓	2031	1.06
ART23 <a href="#">arXiv:2305.07473</a>	N <sup>4</sup> LL	✗	✗	✓	✓	627	0.96

increasing precision & accuracy

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increasing accuracy & precision

only three global fits

# Main features of MAPTMD22 fit

- global fit of Drell-Yan and SIDIS data with
  - **largest** number of **data** points: **2031**
  - **highest** perturbative **accuracy**: **N<sup>3</sup>LL**  
(with FF currently only at NLO)
- prescription to fix SIDIS normalization problem beyond NLL (effectively introducing part of Y-term)
- number of fitted **parameters**: **21**
- extremely good description:  **$\chi^2 / N_{\text{data}} = 1.06$**
- account of **correlated** (exp. and th.) **errors** with nuisance parameters, including PDF & FF uncertainties
  - MMHT2014
  - DSS14 ( $\pi$ ), DSS17 (K)

# The MAPTMD22 data sets

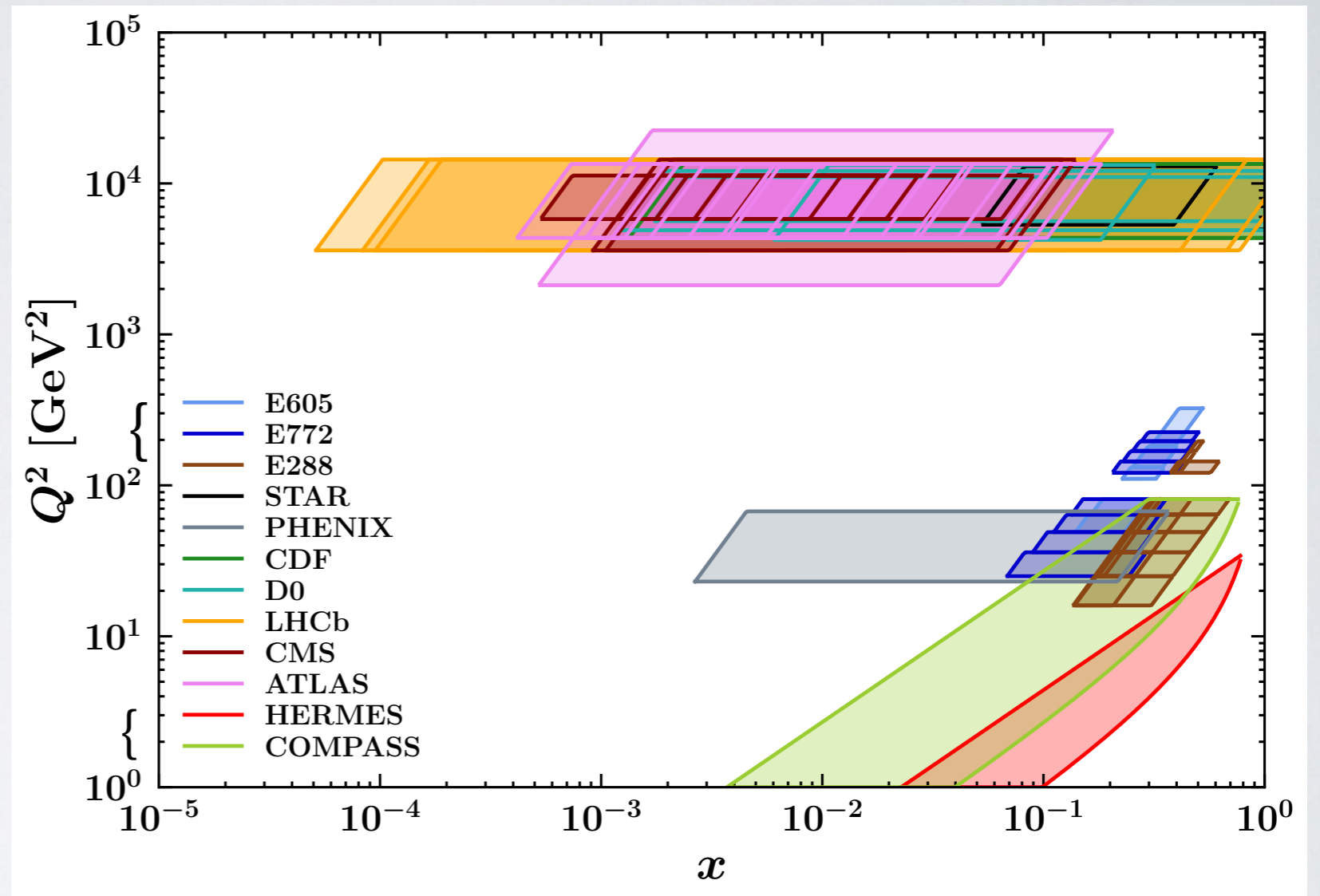
$N_{\text{data}}$  after cuts

Drell Yan { 233 fixed target  
251 collider

1547 SIDIS

---

**2031 data points**



kinematic cuts

$$\langle Q \rangle > 1.4 \text{ GeV}$$

$$0.2 < z < 0.7$$

Drell-Yan

$$q_T < 0.2 Q$$

SIDIS

$$P_{hT} < \min \left[ \min [0.2 Q, 0.5 Qz] + 0.3 \text{ GeV}, zQ \right]$$

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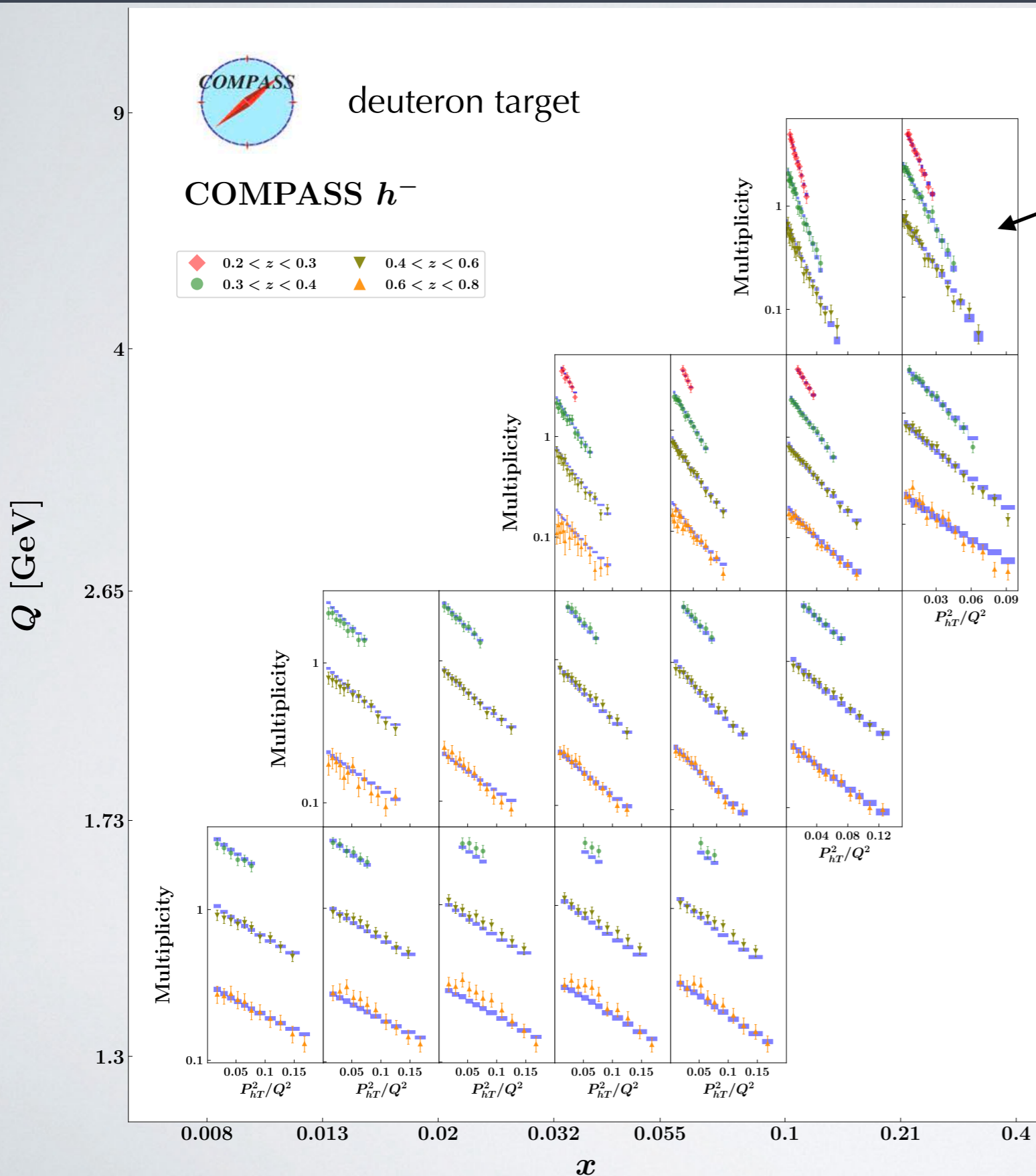
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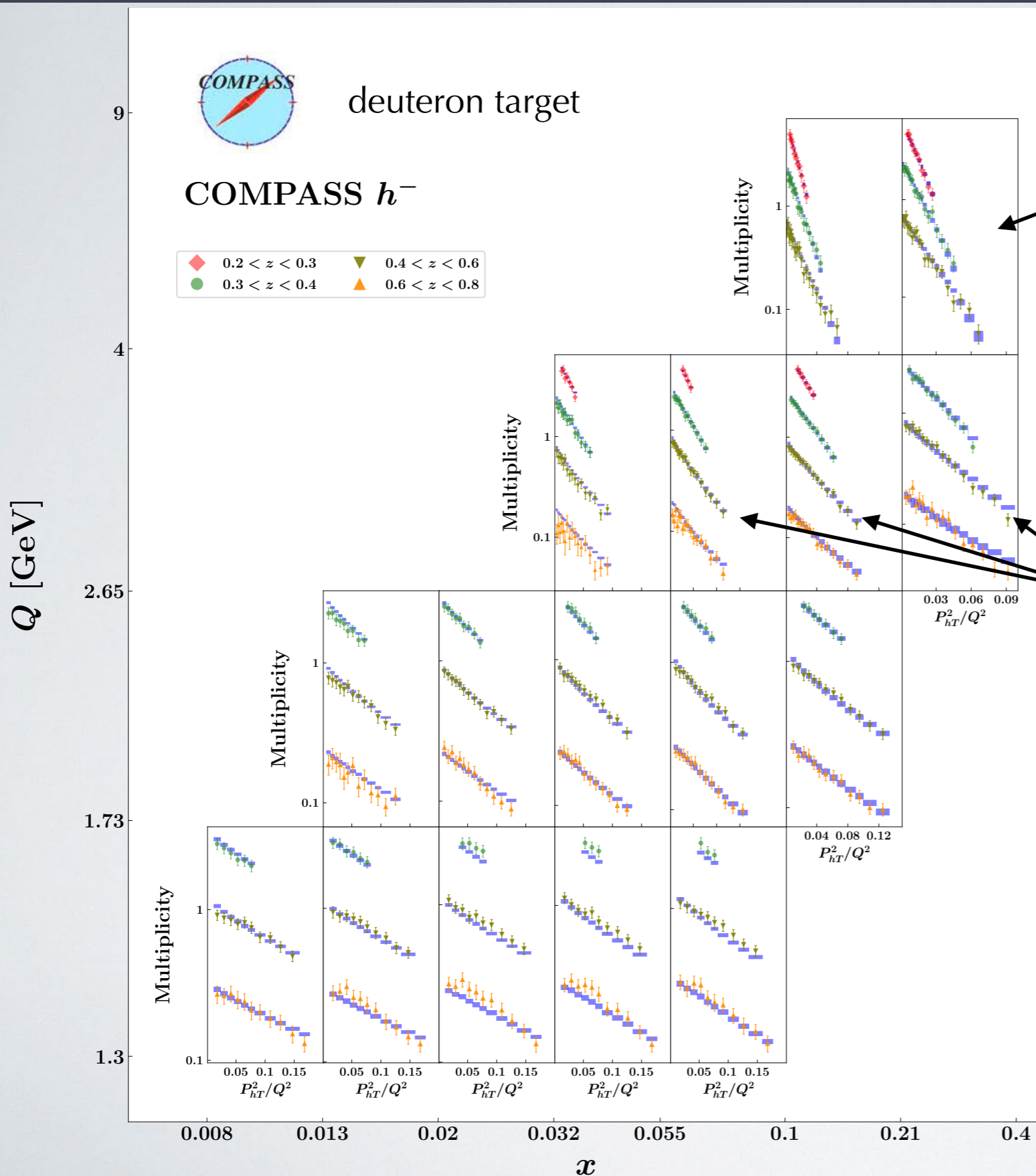
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# Data-driven nonperturbative TMD



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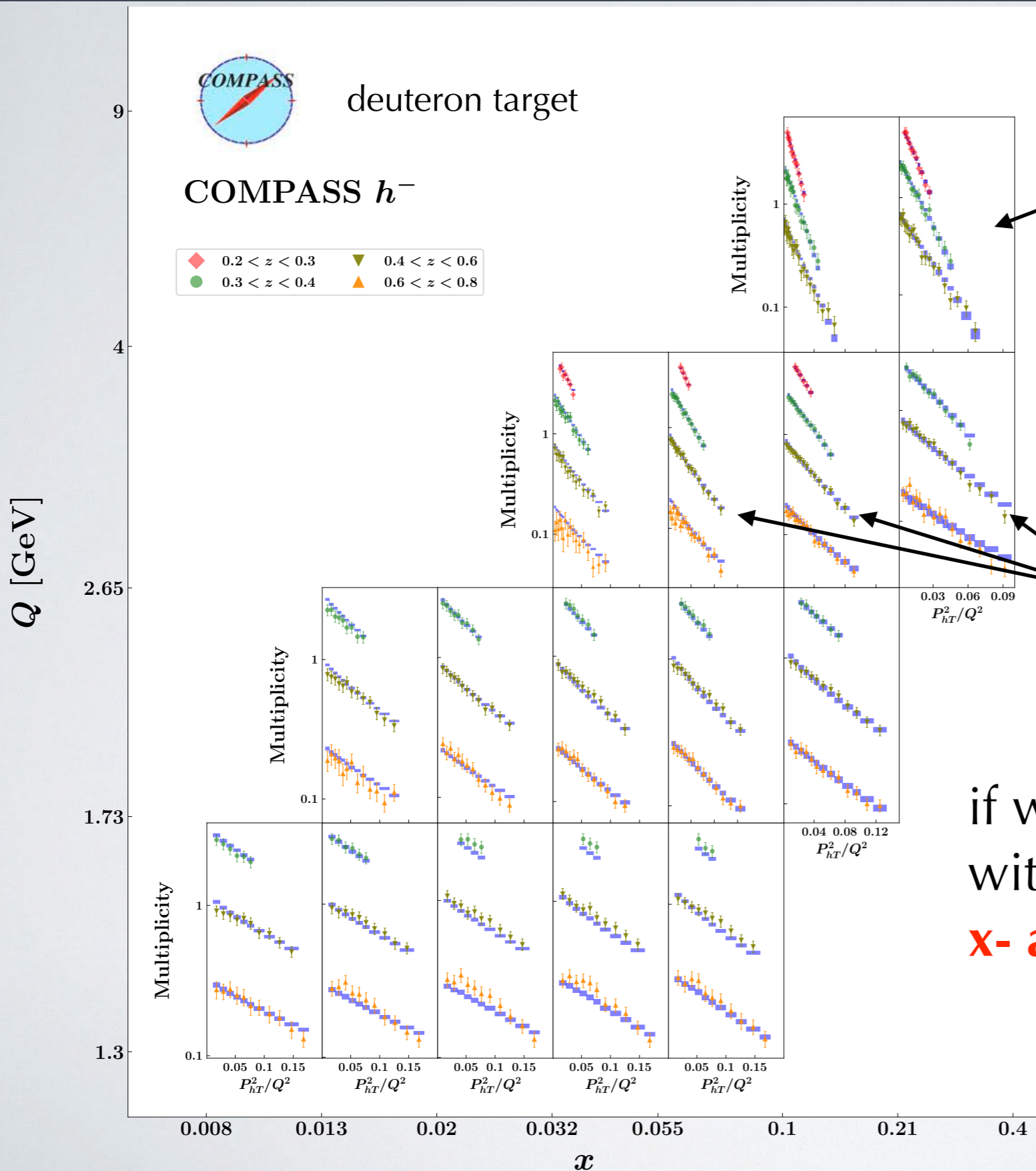


at given  $(x, Q^2)$ ,  
different slopes for different  $z$

at given  $z$ ,  
different slopes for different  $x$



# Data-driven nonperturbative TMD



at given  $(x, Q^2)$ ,  
different slopes for different  $z$

at given  $z$ ,  
different slopes for different  $x$

if we model nonperturbative TMDs  
with Gaussians, we need  
 **$x$ - and  $z$ -dependent widths!**

# Parametrization of non-perturbative TMD

nonperturbative TMD PDF  
Fourier Transform of sum  
of 3 Gaussians with  
x-dependent widths

$$f_{\text{NP}}(x, b_T; Q_0) = \text{F.T.} \left( e^{-k_{\perp}^2 / g_{1A}(x)} + \lambda_B k_{\perp}^2 e^{-k_{\perp}^2 / g_{1B}(x)} + \lambda_C e^{-k_{\perp}^2 / g_{1C}(x)} \right)$$

with  $g_{1X}(x) = N_{1X} \frac{(1-x)^{\alpha_X^2} x^{\sigma_X}}{(1-\hat{x})^{\alpha_X^2} \hat{x}^{\sigma_X}} \quad \hat{x} = 0.1$  **11 param.**

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nonperturbative TMD FF  
Fourier Transform of sum  
of 2 Gaussians with  
z-dependent widths

$$D_{\text{NP}}(z, b_T; Q_0) = \text{F.T.} \left( e^{-P_{\perp}^2 / g_{3A}(z)} + \lambda_F P_{\perp}^2 e^{-P_{\perp}^2 / g_{3B}(z)} \right)$$

with  $g_{3X}(z) = N_{3X} \frac{(1-z)^{\gamma_X^2} (z^{\beta_X} + \delta_X^2)}{(1-\hat{z})^{\gamma_X^2} (\hat{z}^{\beta_X} + \delta_X^2)} \quad \hat{z} = 0.5$  **9 param.**

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nonperturbative TMD PDF  
Fourier Transform of sum  
of 3 Gaussians with  
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nonperturbative TMD FF  
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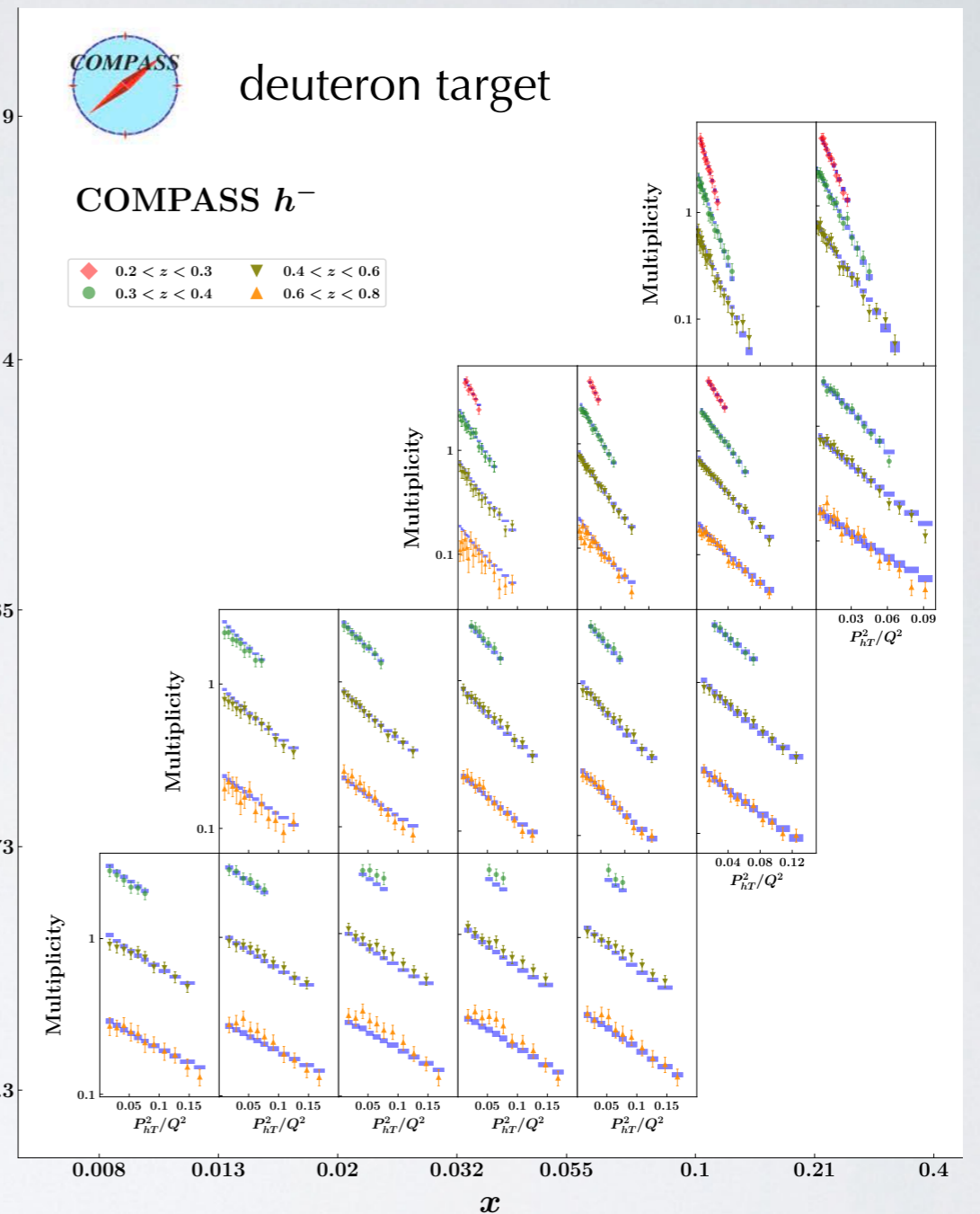
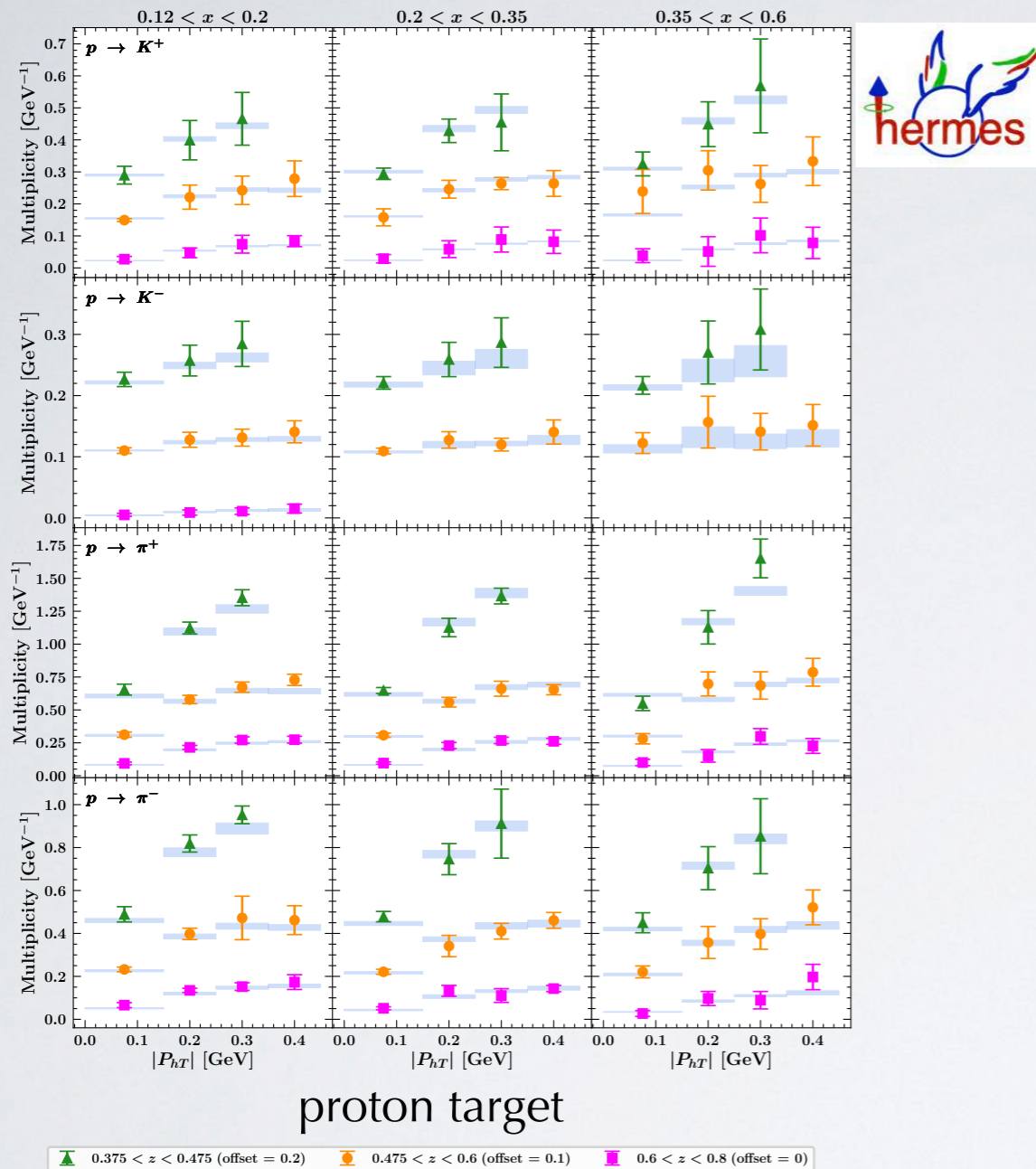
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nonperturbative part of  
Collins-Soper kernel

$$\left[ \frac{\zeta_f}{Q_0^2} \right]^{g_K(b_T)/2} \quad g_K(b_T) = -g_2^2 \frac{b_T^2}{4} \quad \mathbf{1 \text{ param.}}$$

**Total 21 param.**

# Fit results for SIDIS



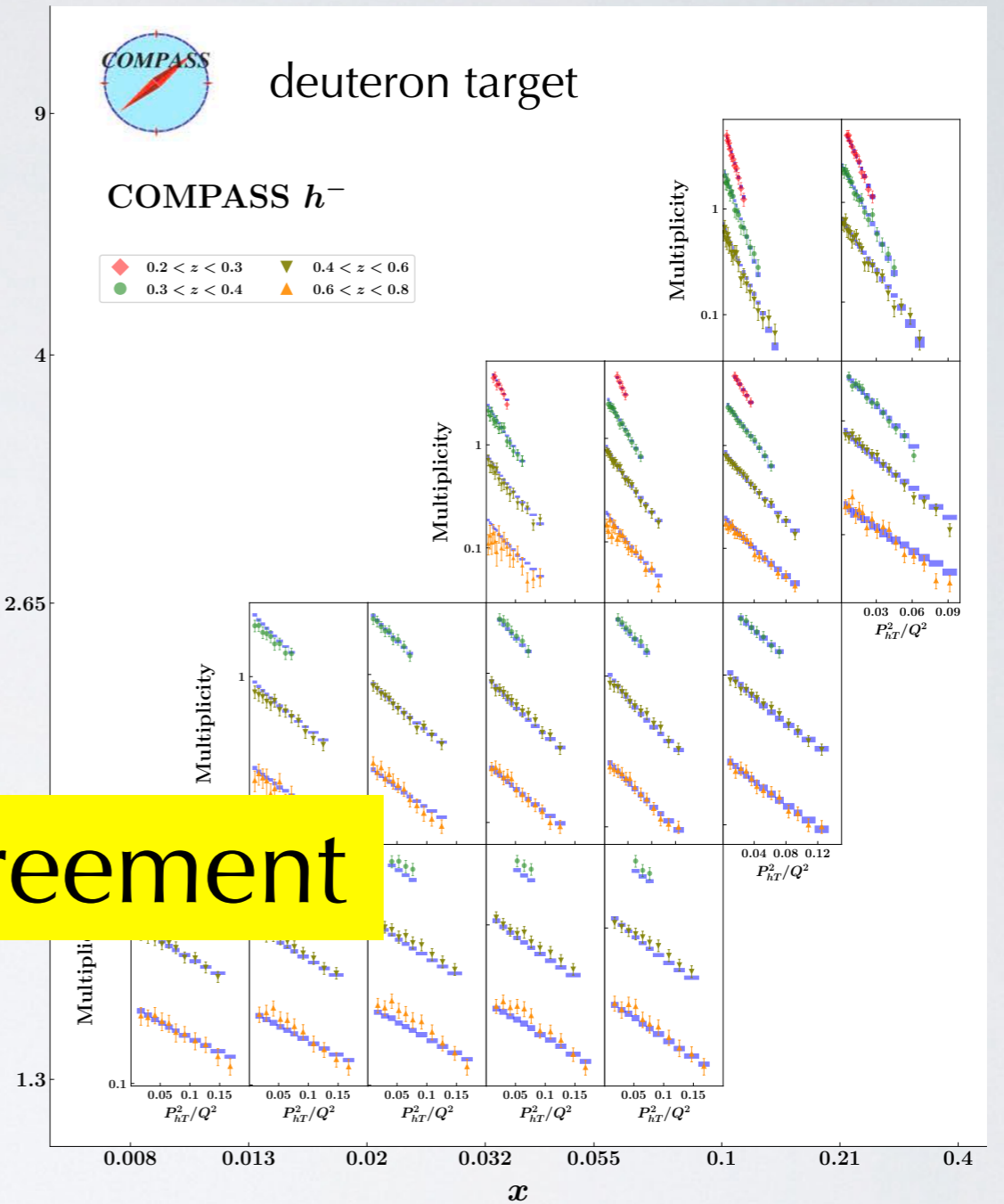
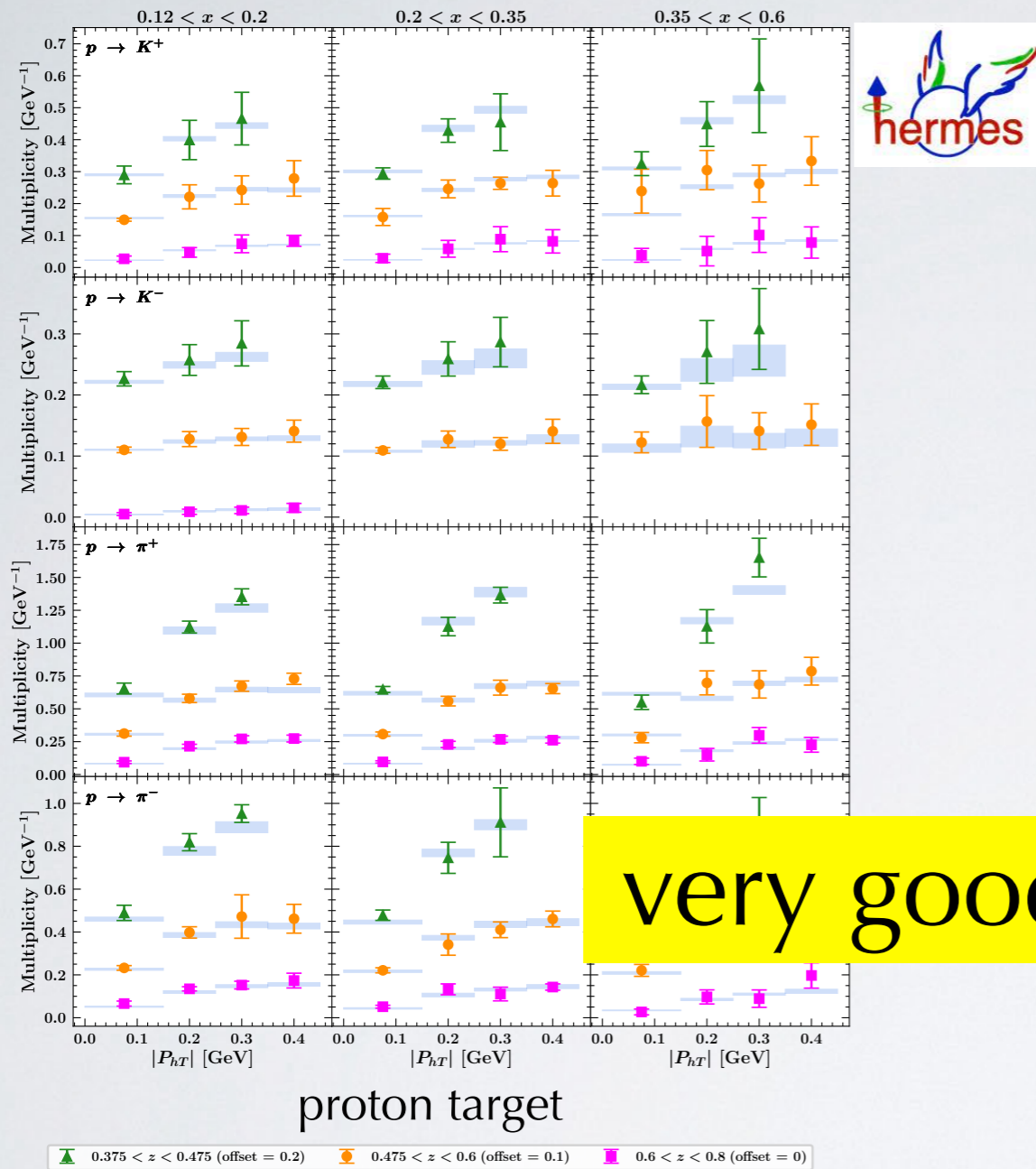
data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi^2$
HERMES	344	0.48	0.23	0.71
COMPASS	1203	0.62	0.3	0.92
<b>SIDIS total</b>	<b>1547</b>	<b>0.59</b>	<b>0.28</b>	<b>0.87</b>

$\chi_D^2$  = uncorrelated error

$\chi_\lambda^2$  = correlated error

$$\chi^2 = \chi_D^2 + \chi_\lambda^2$$

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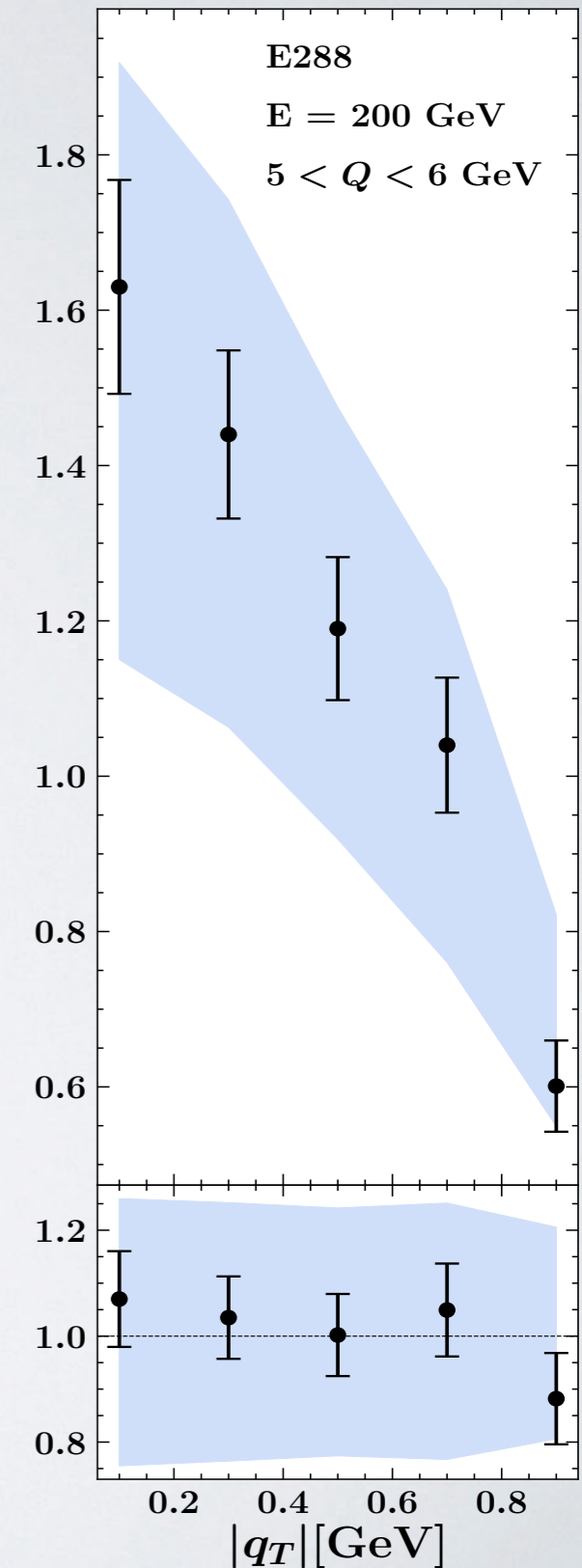
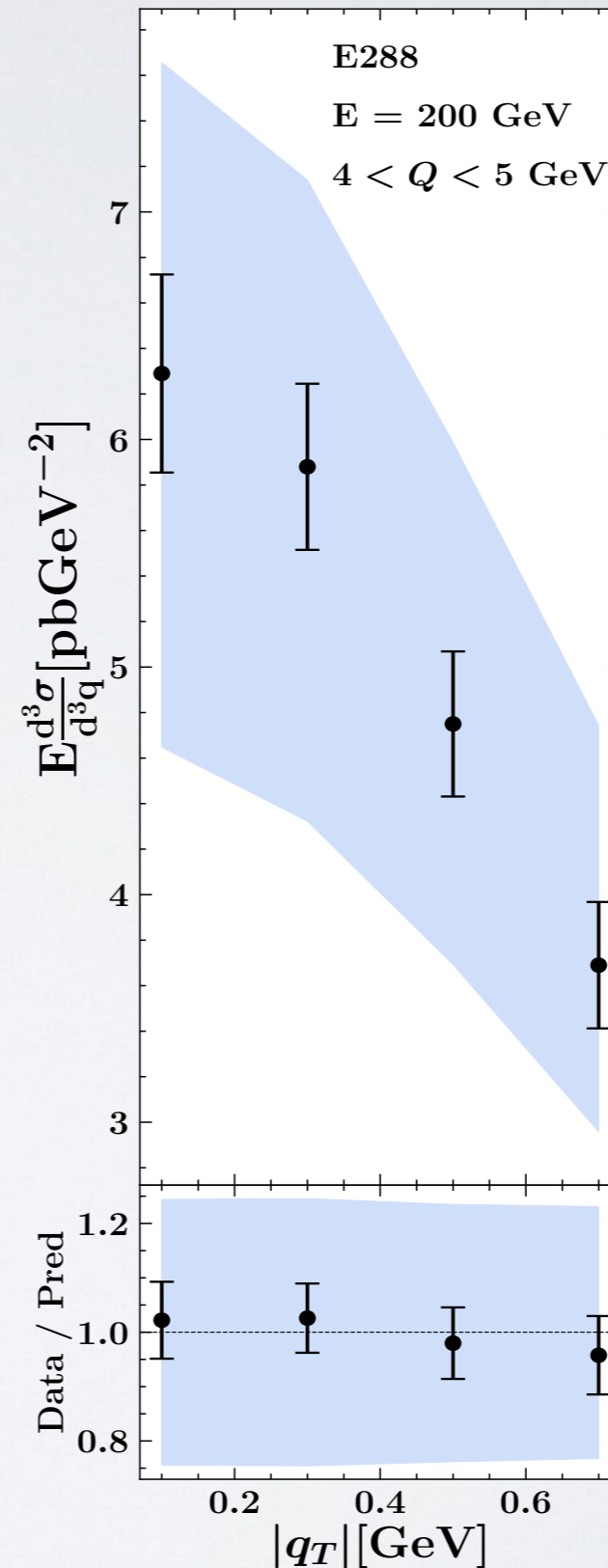
# Fit results for fixed-target Drell-Yan

data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi^2$
SIDIS total	1547	0.59	0.28	0.87
Drell-Yan fixed target tot	233	0.84	0.4	1.24
Drell-Yan collider total	251	1.86	0.2	2.06

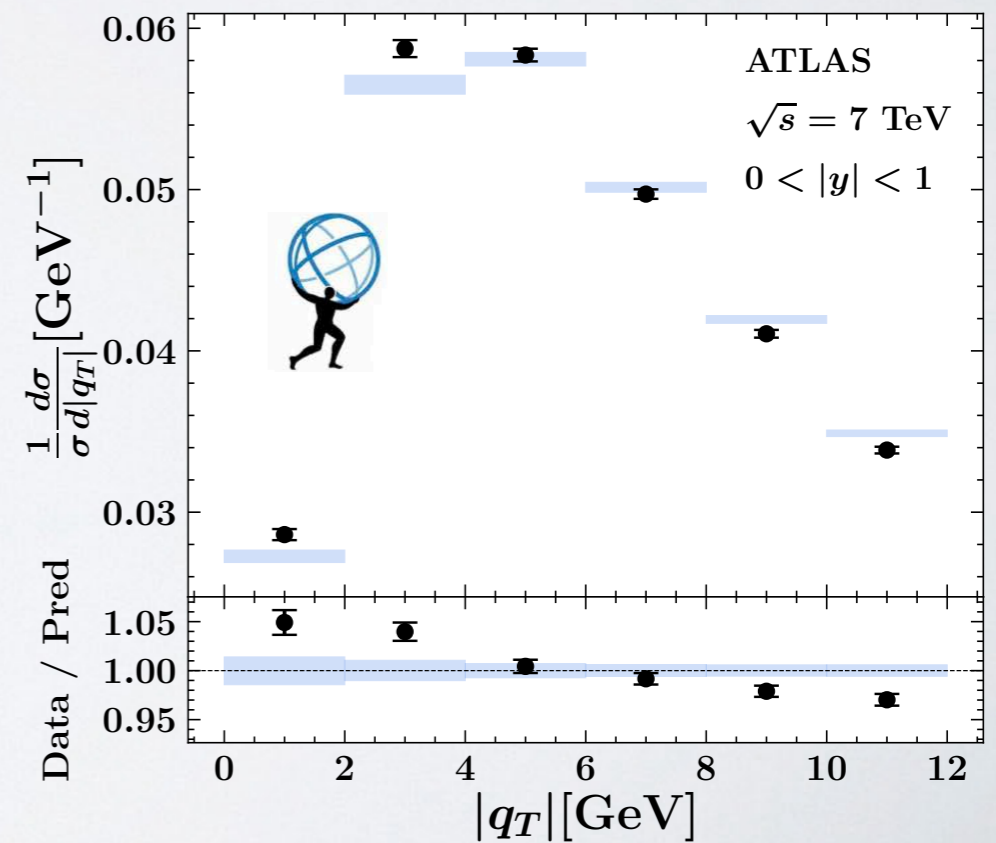
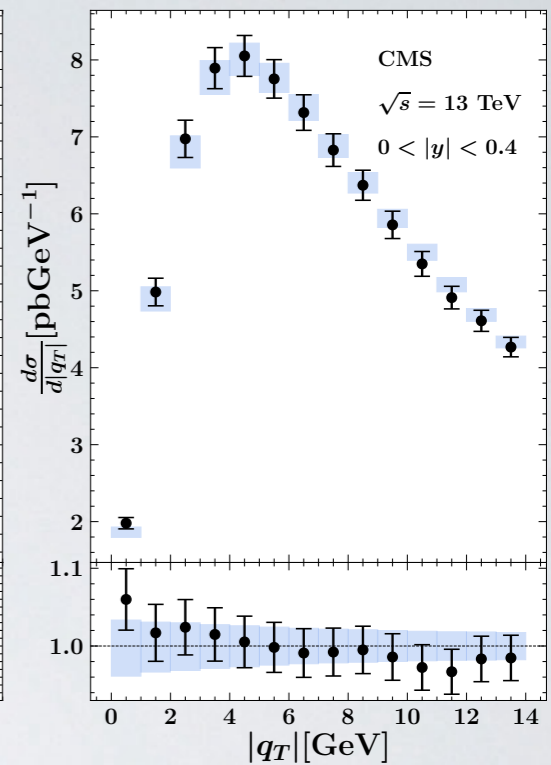
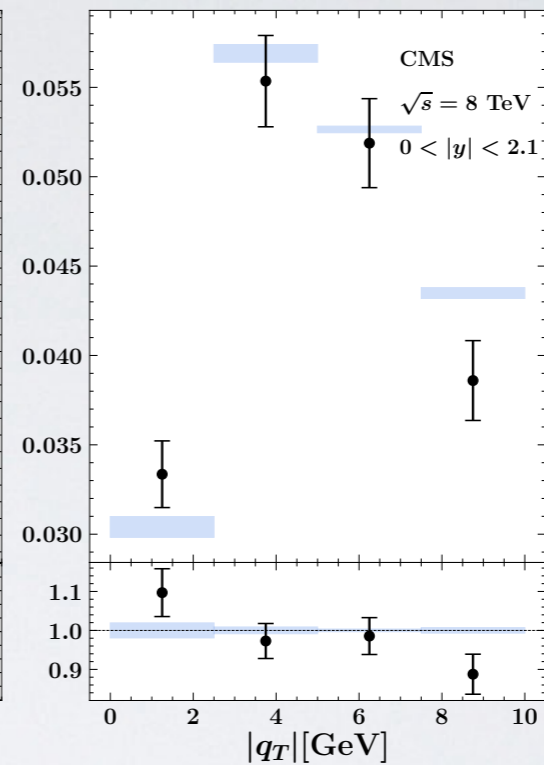
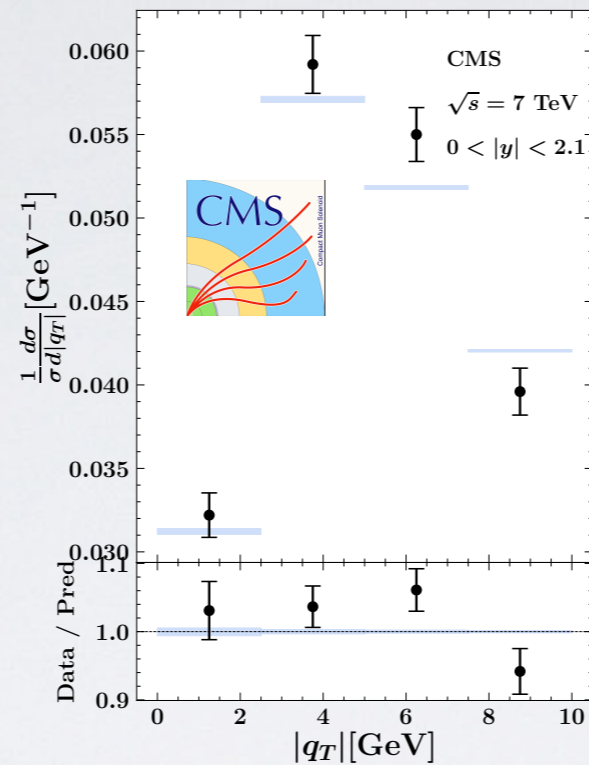
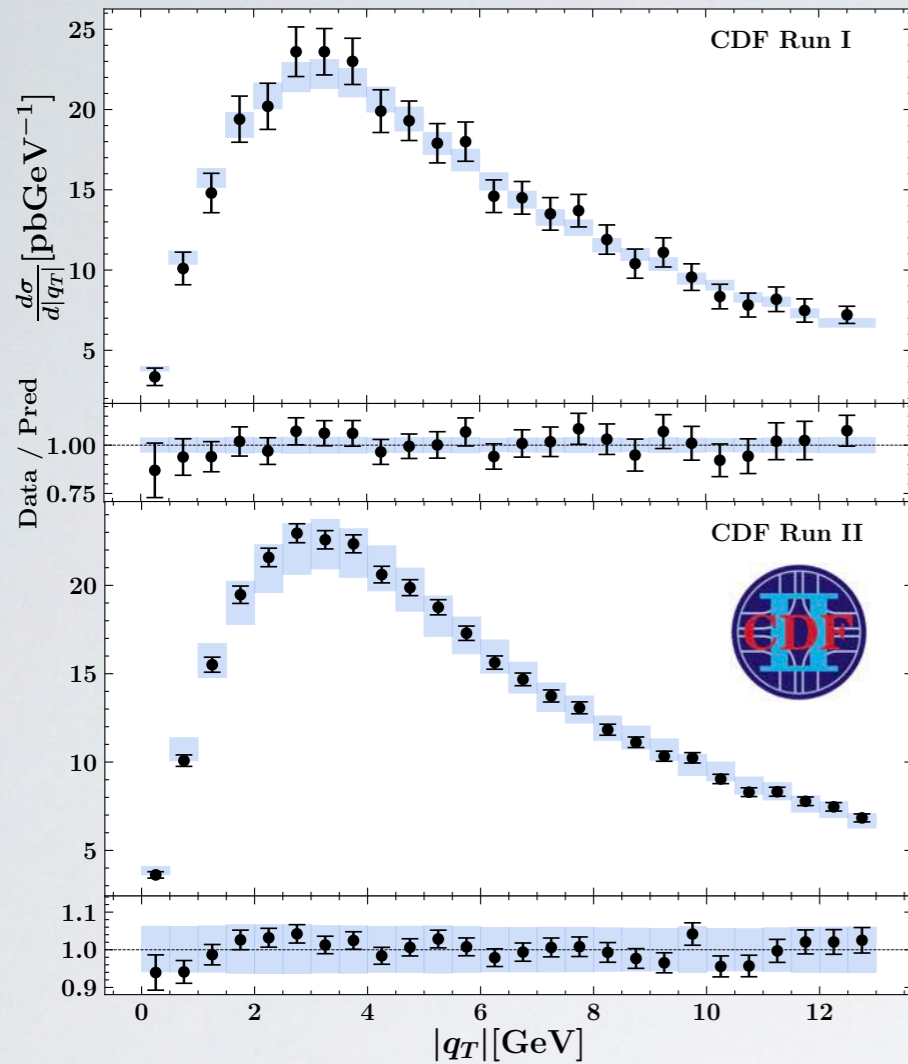
$\chi_D^2$  = uncorrelated error  
 $\chi_\lambda^2$  = correlated error  
 $\chi^2 = \chi_D^2 + \chi_\lambda^2$

good agreement

th. error band =  
68% of all replicas



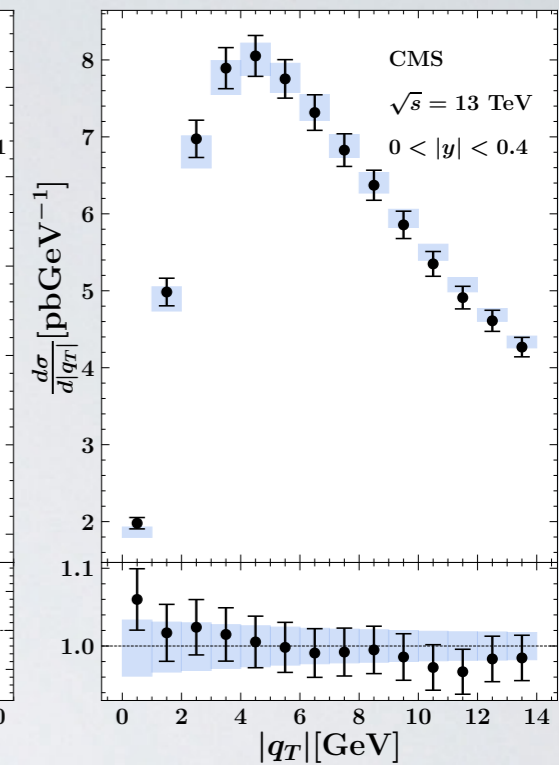
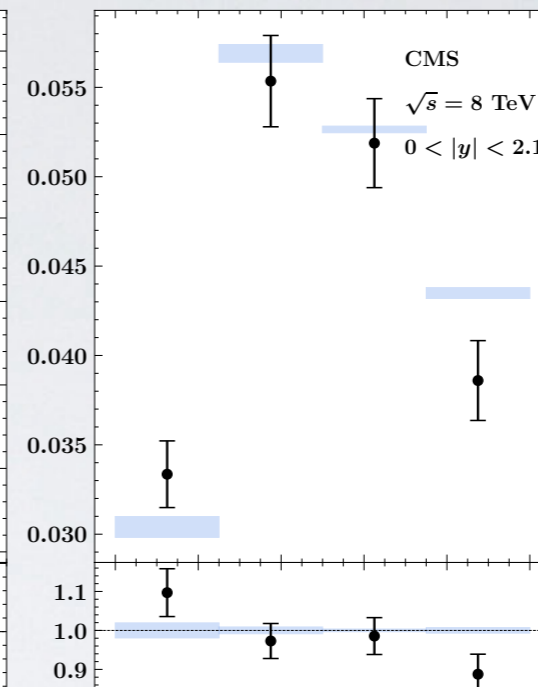
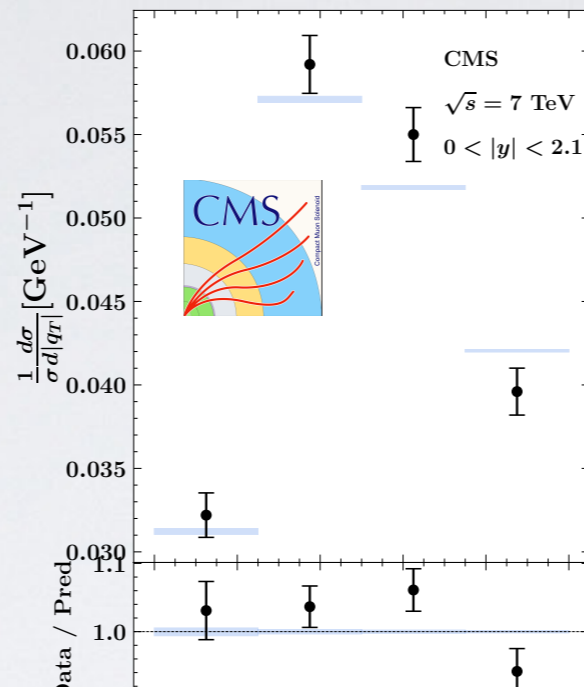
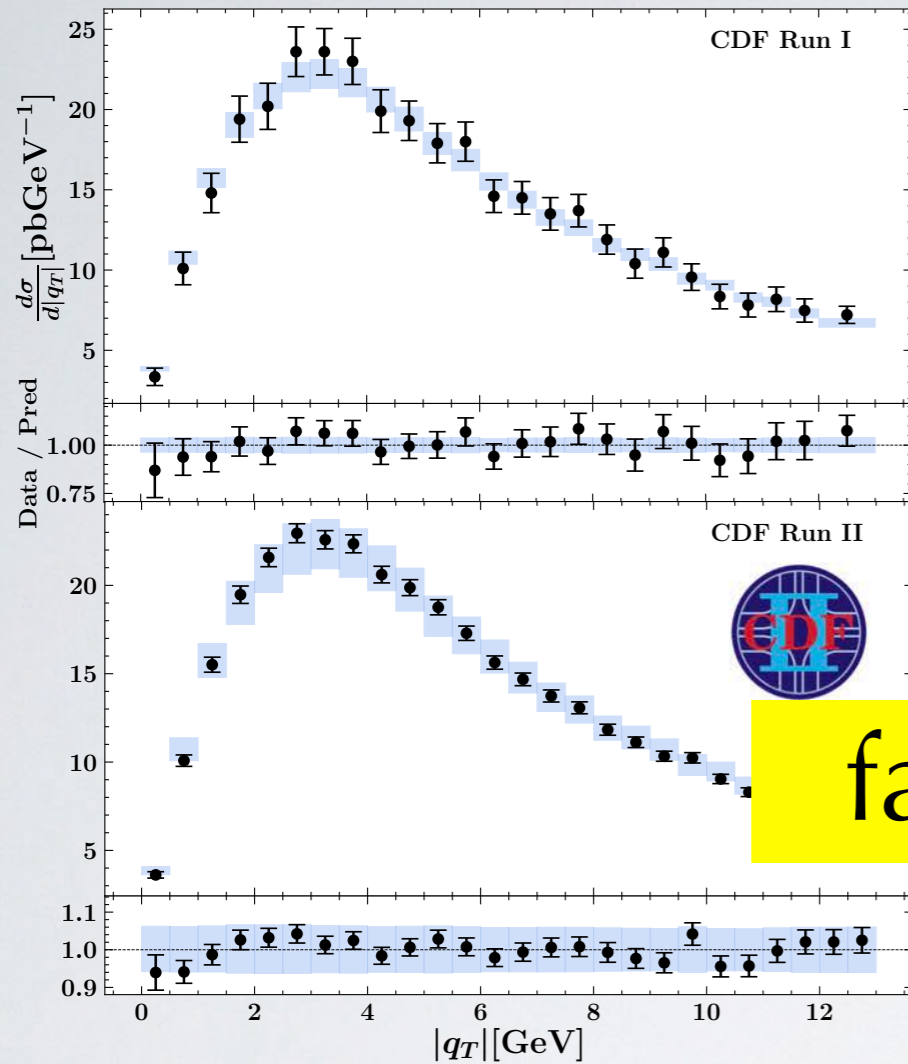
# Fit results for collider Drell-Yan



data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi^2$
SIDIS total	1547	0.59	0.28	0.87
Drell-Yan fixed target tot	233	0.85	0.4	1.24
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ATLAS total	72	4.56	0.48	5.05

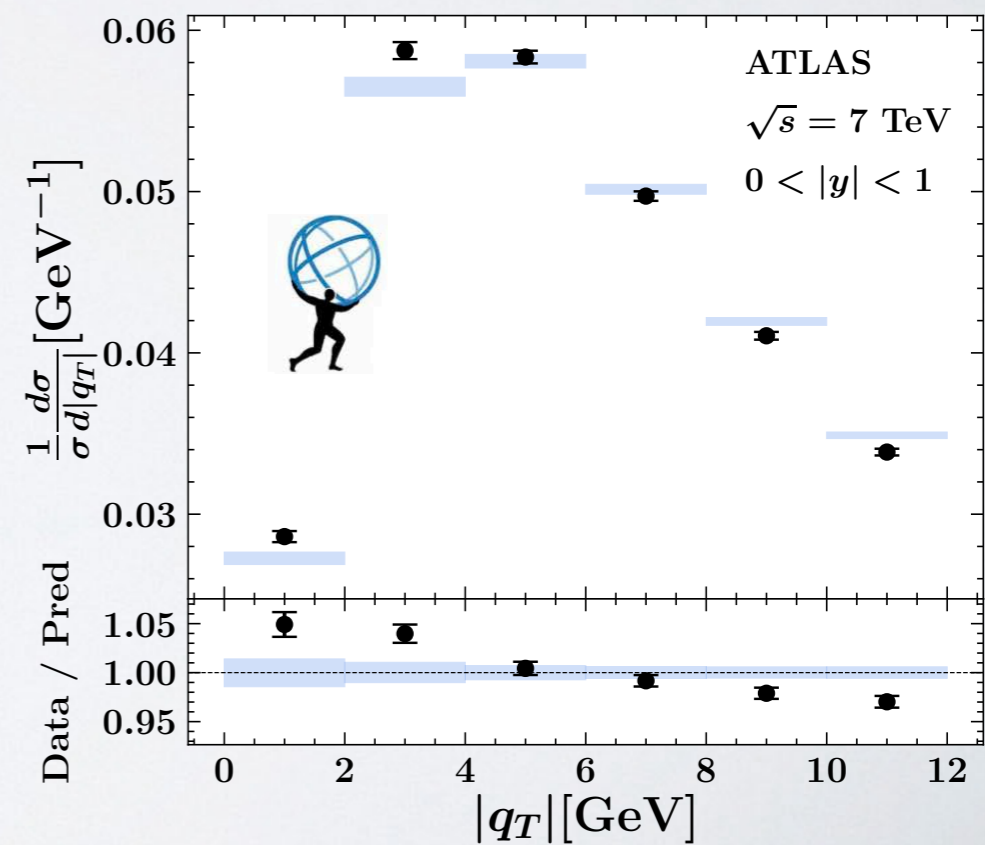


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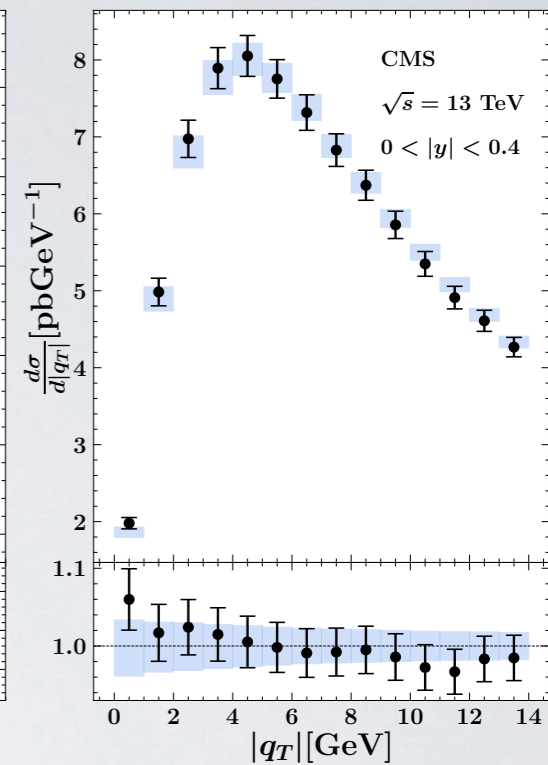
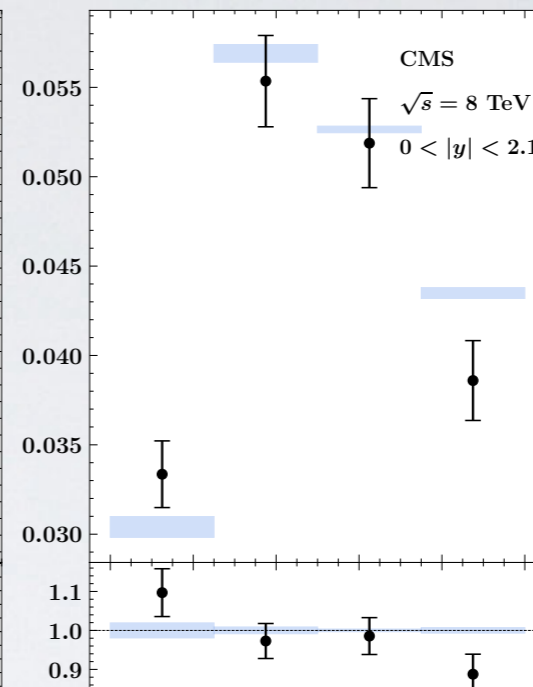
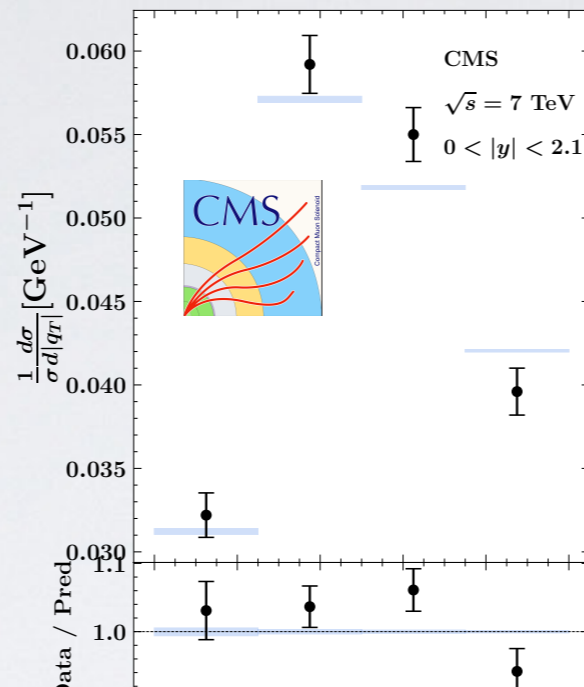
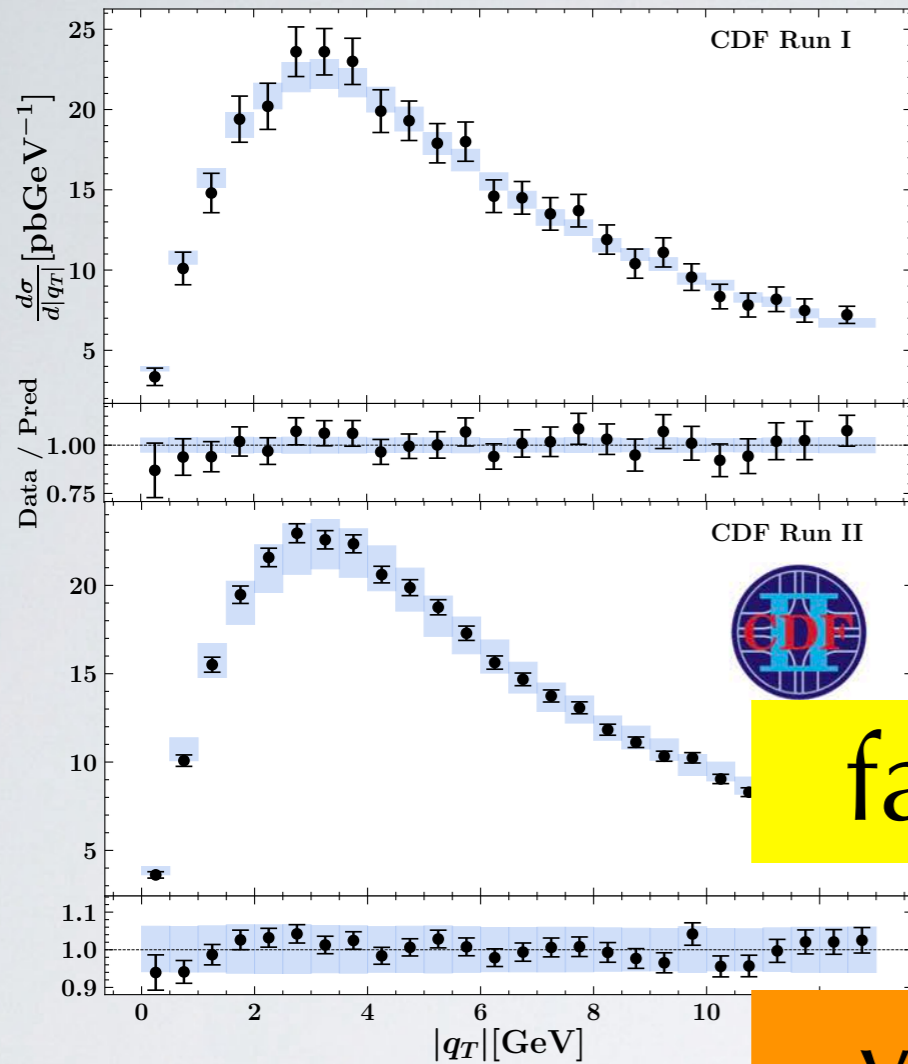


fairly good agreement

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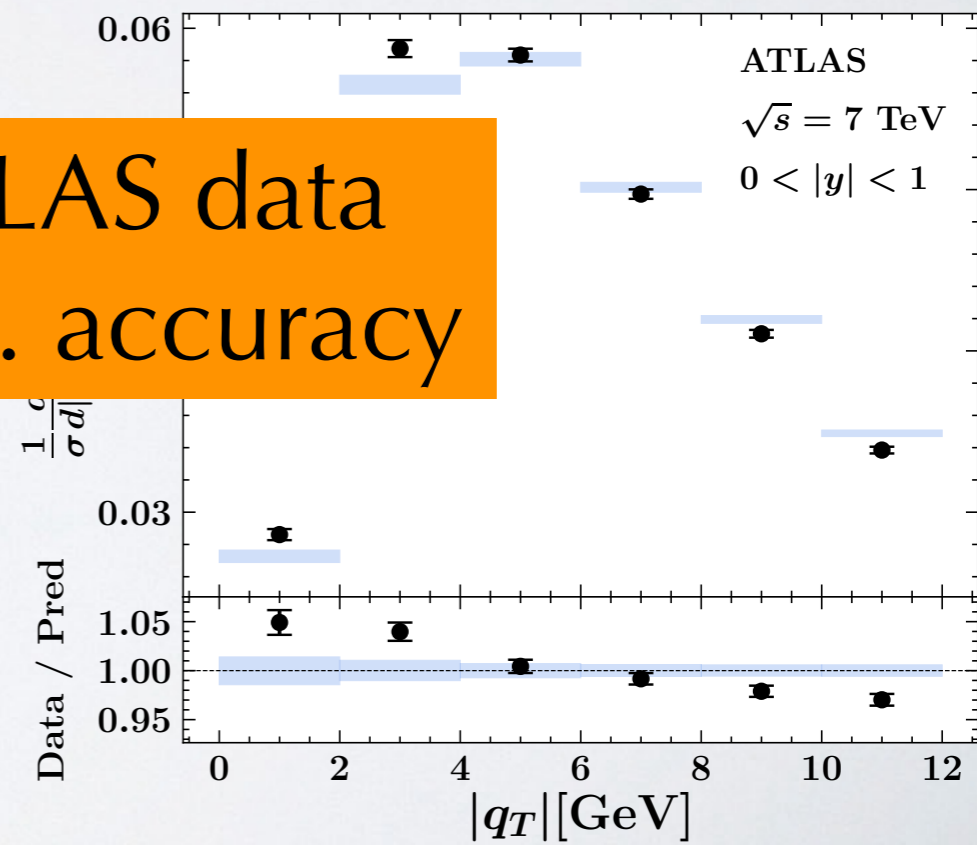
# Fit results for collider Drell-Yan



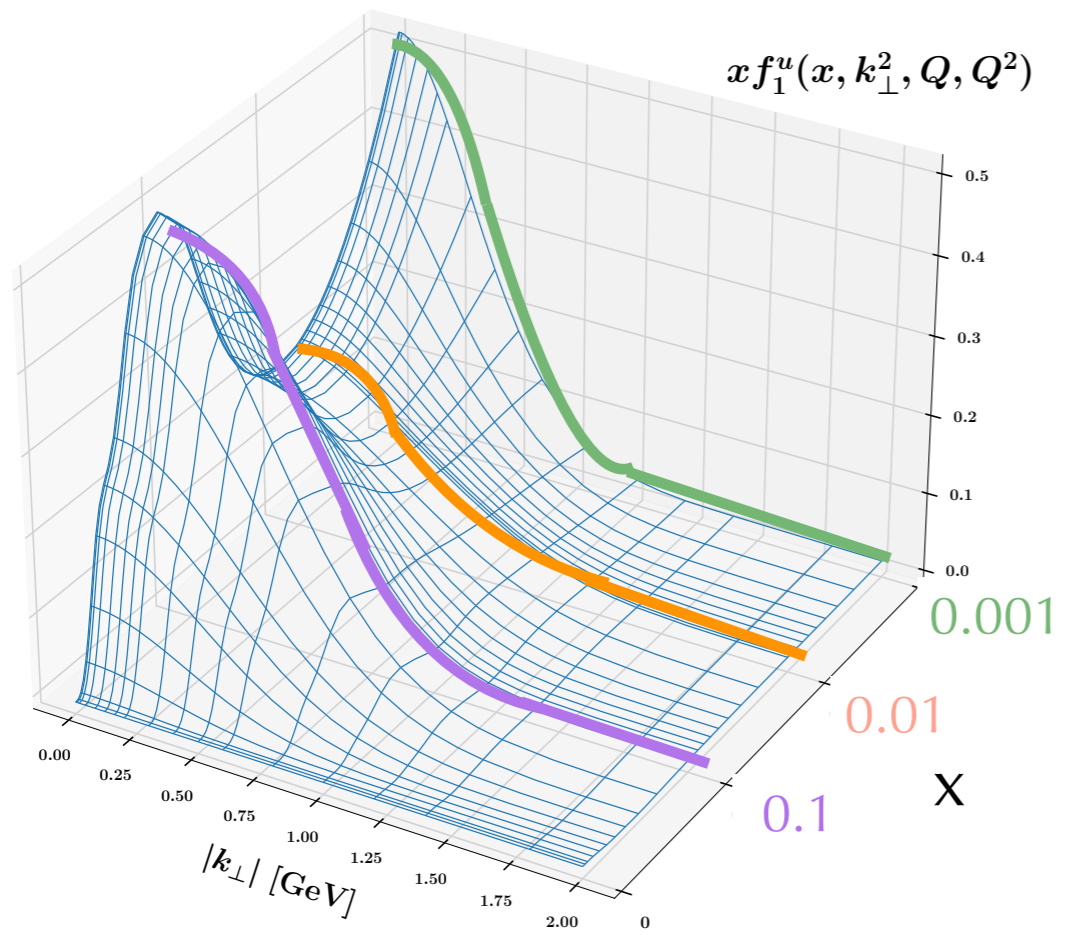
fairly good agreement

very precise ATLAS data  
require higher Th. accuracy

data set	$N_{\text{points}}$	$\chi^2/N$	$\chi^2/N_{\text{th}}$	$\chi^2/N_{\text{stat}}$
SIDIS total	1547	0.59	0.28	0.87
Drell-Yan fixed target tot	233	0.85	0.4	1.24
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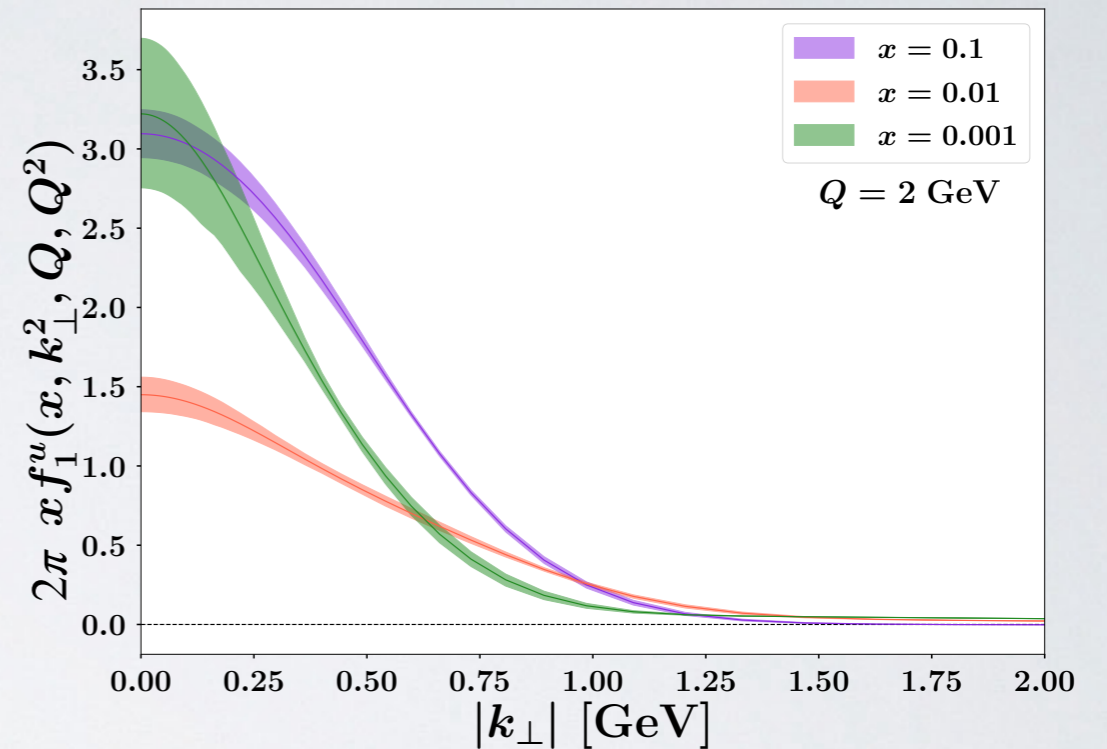


# Visualizing MAPTMD22 TMD PDF



unpolarized up quark in the proton

$Q=2$  GeV

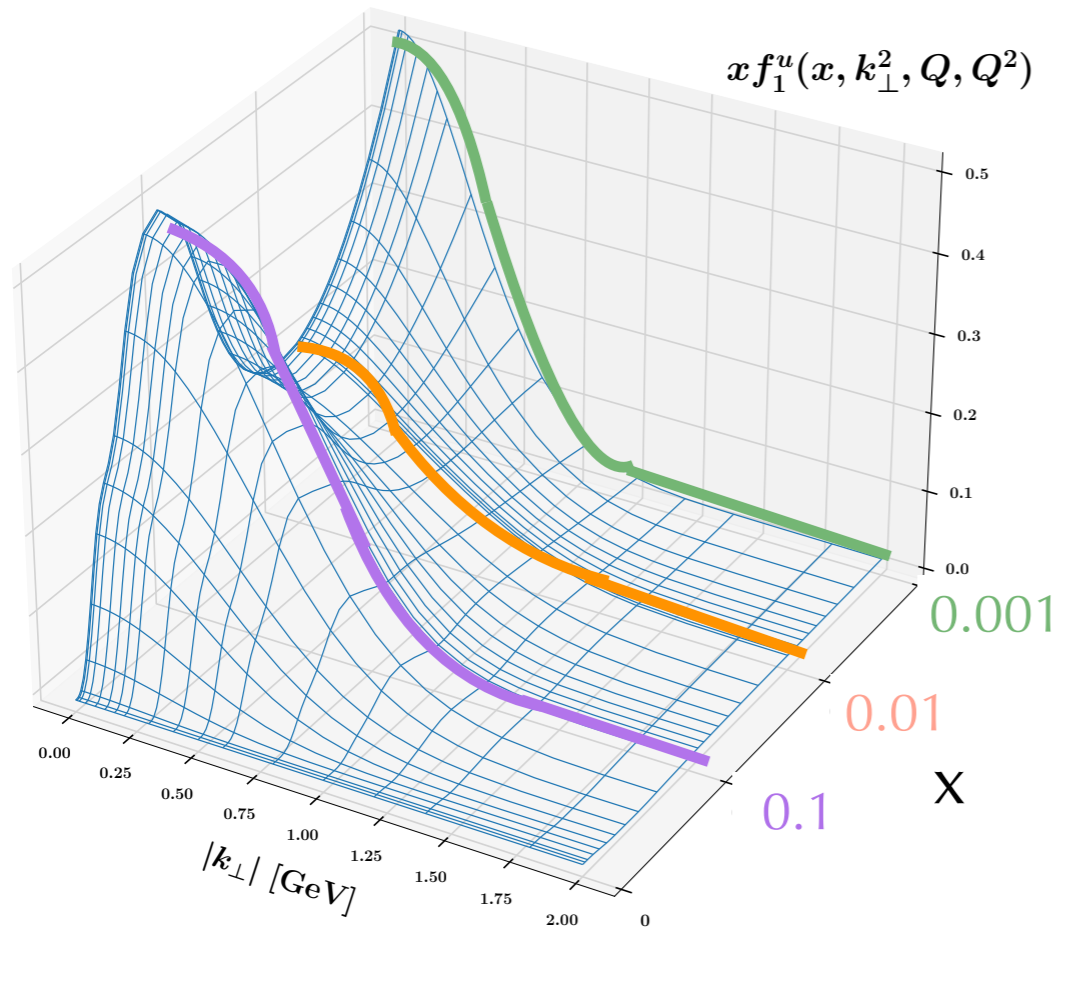


$$f_{\text{NP}}(x, k_\perp; Q_0) \sim \text{Gauss} + \lambda_B k_\perp^2 \text{ Gauss} + \lambda_C \text{ Gauss}$$

effect of  $x$ -dependent widths

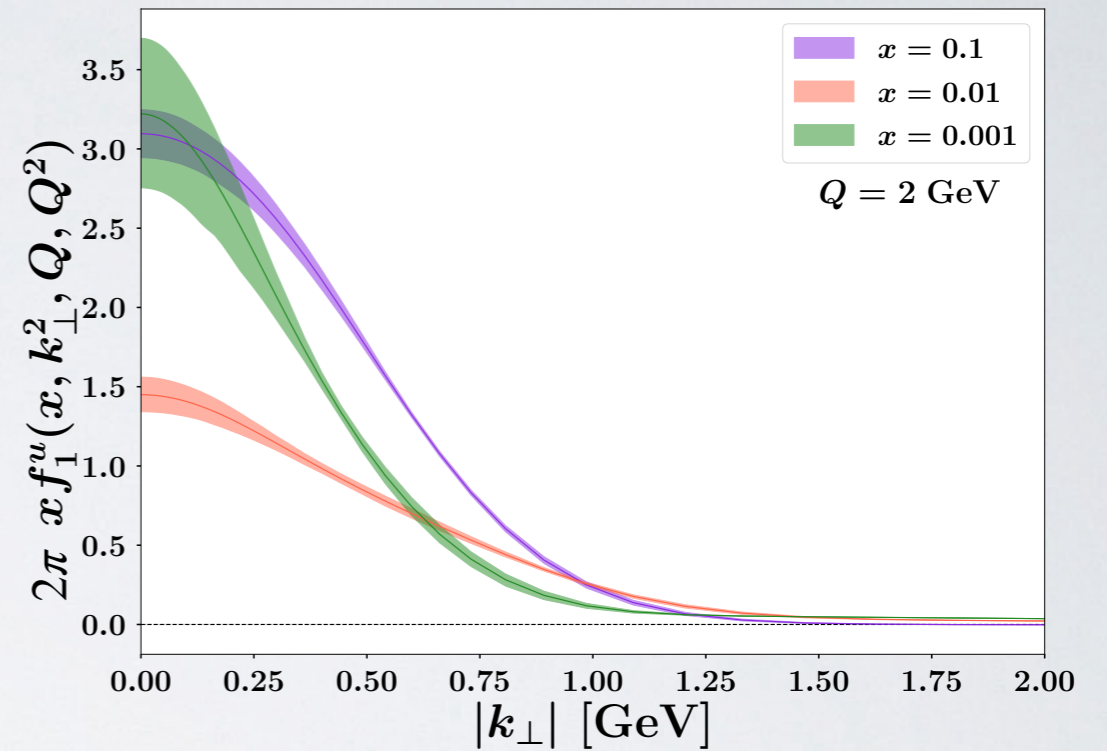
$$\langle k_\perp^2 \rangle(x = 0.1, Q = 1) = 0.3 \pm 0.05 \text{ GeV}^2$$

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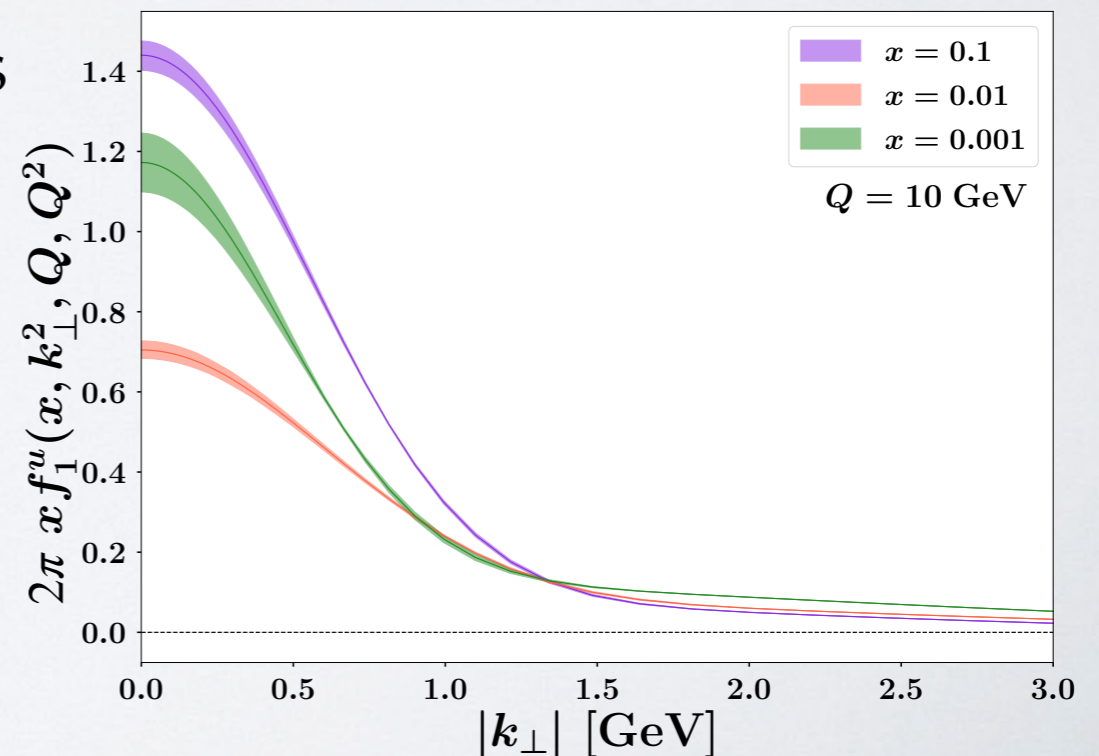
$$f_{\text{NP}}(x, k_{\perp}; Q_0) \sim \text{Gauss} + \lambda_B k_{\perp}^2 \text{ Gauss} + \lambda_C \text{ Gauss}$$

effect of x-dependent widths

$$\langle k_{\perp}^2 \rangle(x = 0.1, Q = 1) = 0.3 \pm 0.05 \text{ GeV}^2$$

$$e^{-g_2^2 b_T^2/4} \quad g_2 = 0.248 \pm 0.008 \text{ GeV}$$

effect of nonperturbative evolution



# Visualizing the Collins-Soper evolution kernel

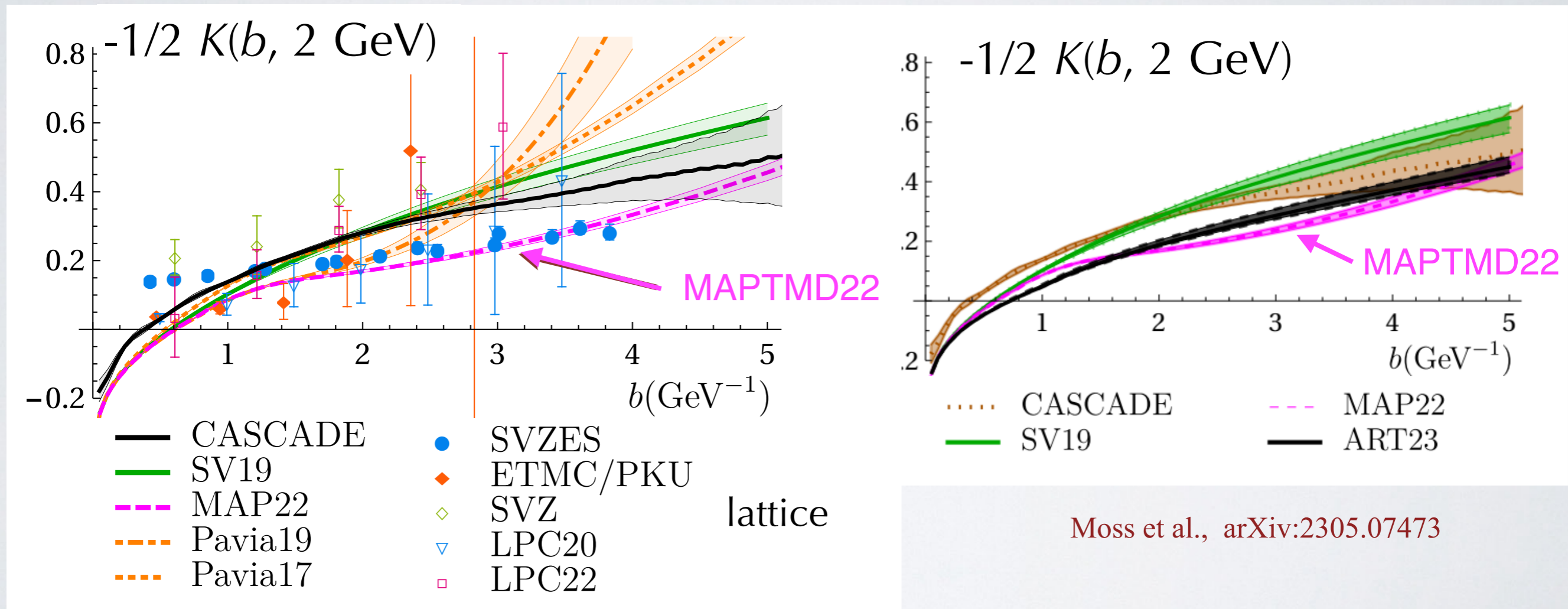
Collins-Soper kernel

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

drives evolution in rapidity  $\zeta$

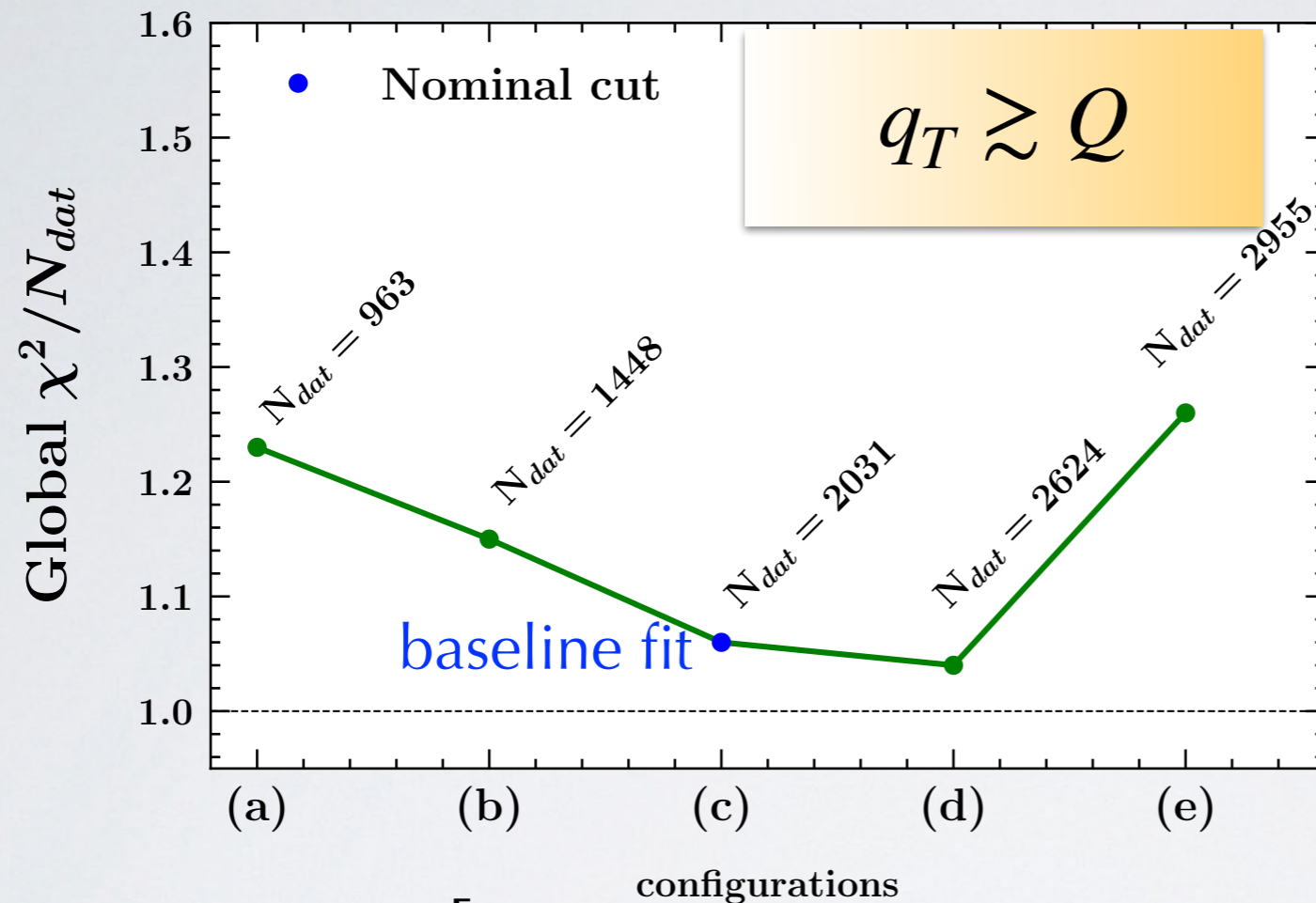
perturbative

non-perturbative  
(fitted)



Moss et al., arXiv:2305.07473

# Kinematic cuts and validity of TMD region



$$P_{hT} < \min \left[ \min [c_1 Q, c_2 Qz] + c_3 \text{ GeV}, zQ \right]$$

a)  $c_1=c_2=0.4, c_3=0$   $q_T < 0.4 Q$

b)  $c_1=0.15, c_2=0.4, c_3=0.2$

c)  $c_1=0.2, c_2=0.5, c_3=0.3$  baseline fit

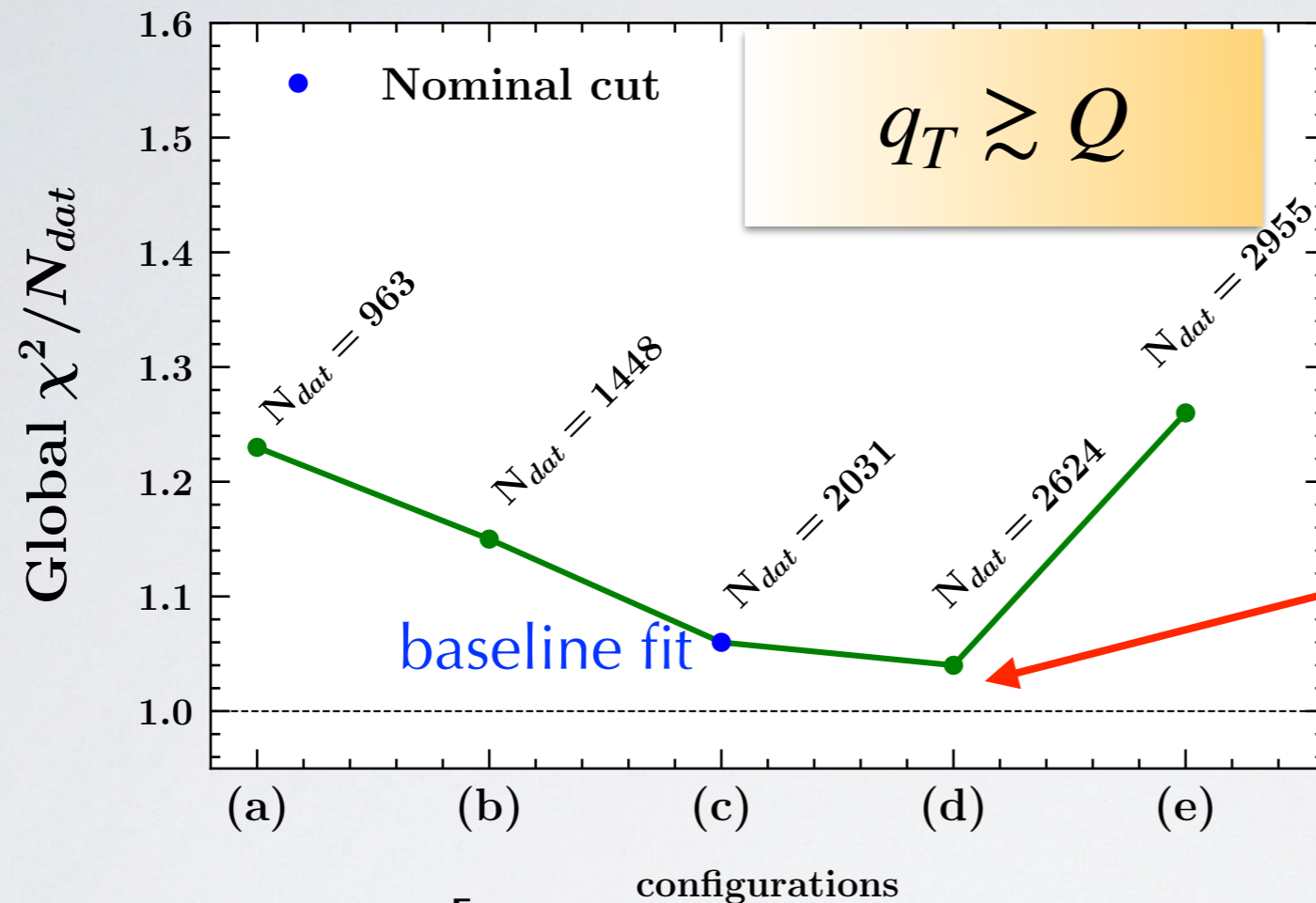
d)  $c_1=0.2, c_2=0.6, c_3=0.4$  can be  $q_T > Q$

e)  $c_1=0.2, c_2=0.7, c_3=0.5$  can be  $q_T > Q$

more conservative

less conservative

# Kinematic cuts and validity of TMD region



better  $\chi^2$  with less conservative cuts allowing for  $q_T > Q$

Where is the limit for TMD factorization??

$$P_{hT} < \min \left[ \min [c_1 Q, c_2 Qz] + c_3 \text{ GeV}, zQ \right]$$

- a)  $c_1=c_2=0.4, c_3=0$   $q_T < 0.4 Q$
- b)  $c_1=0.15, c_2=0.4, c_3=0.2$
- c)  $c_1=0.2, c_2=0.5, c_3=0.3$  baseline fit
- d)  $c_1=0.2, c_2=0.6, c_3=0.4$  can be  $q_T > Q$
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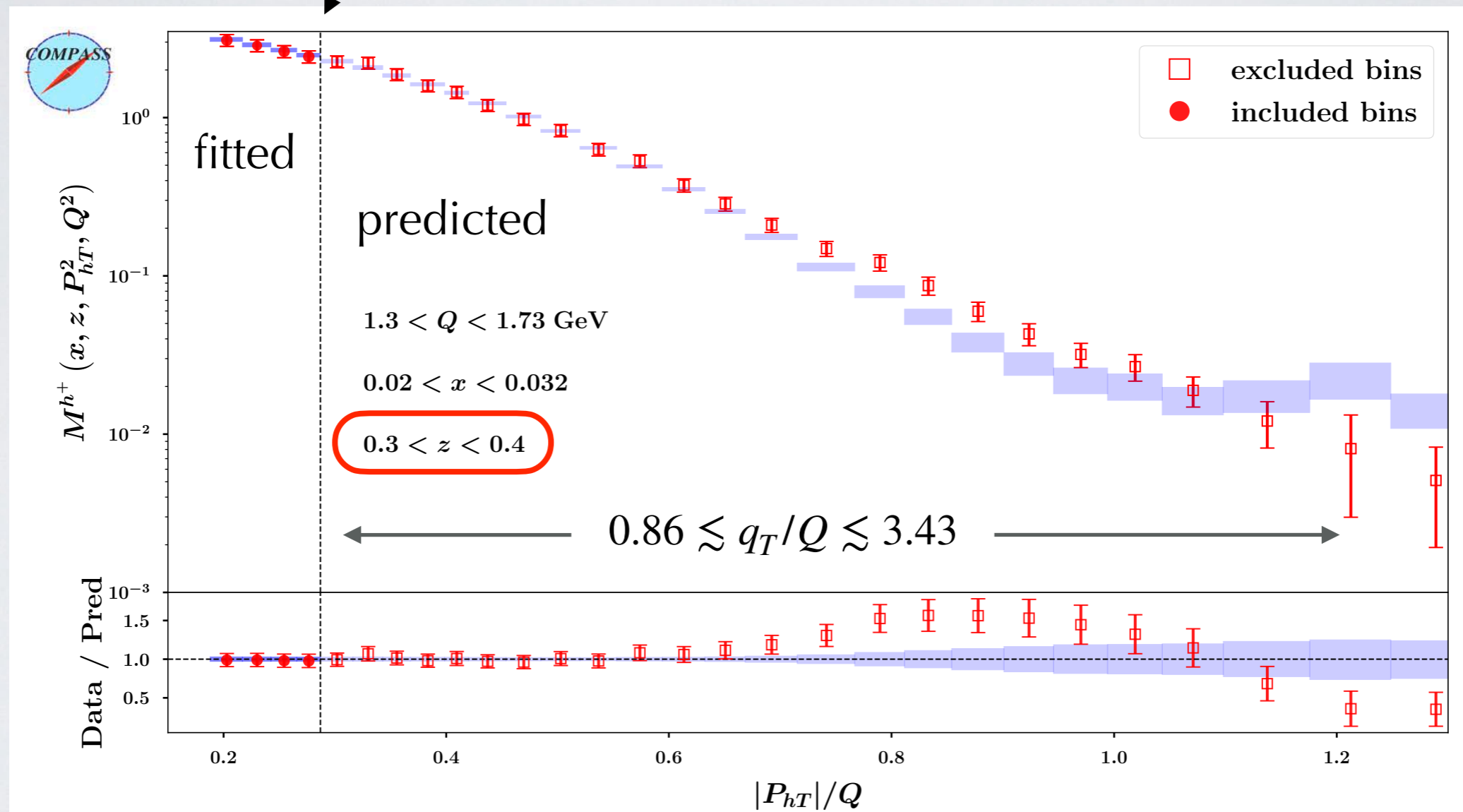
more conservative

less conservative



# Kinematic cuts and validity of TMD region

cut of  
baseline fit

$$P_{hT} < \min \left[ \min [0.2 Q, 0.5 Qz] + 0.3 \text{ GeV}, zQ \right]$$


validity of TMD factorization seems to extend well beyond  $P_{hT}/z \ll Q$  !



# The MAPTMDPion22 fit

## Extracting TMDs for the pion

	Framework	DY	N of points	$\chi^2/N_{\text{points}}$
Wang et al., 2017 <a href="#">arXiv:1707.05207</a>	NLL	✓	96	1.61
Vladimirov, 2019 <a href="#">arXiv:1907.10356</a>	NNLL'	✓	80	1.44
MAPTMDPion22, 2022 <a href="#">arXiv:2210.01733</a>	N <sup>3</sup> LL	✓	138	1.54
JAM 2023 <a href="#">arXiv:2302.01192</a>	NNLL	✓	93	1.37

- MAPTMDPion22: - best theoretical accuracy: **N<sup>3</sup>LL**  
[arXiv:2210.01733](#) - largest number of included data: **138**  
- same consistent framework of MAPTMD22

$$f_{\text{NP}}^\pi(x, b_T; Q_0) = e^{-g_{1\pi}(x) b_T^2/4} \quad g_{1\pi}(x) = N_{1\pi} \frac{(1-x)^{\alpha_\pi^2} x^{\sigma_\pi}}{(1-\hat{x})^{\alpha_\pi^2} \hat{x}^{\sigma_\pi}} \quad 3 \text{ param.}$$

$\hat{x} = 0.1$

# The MAPTMDPion22 data sets

$$\pi^- W \rightarrow \mu^+ \mu^- X$$

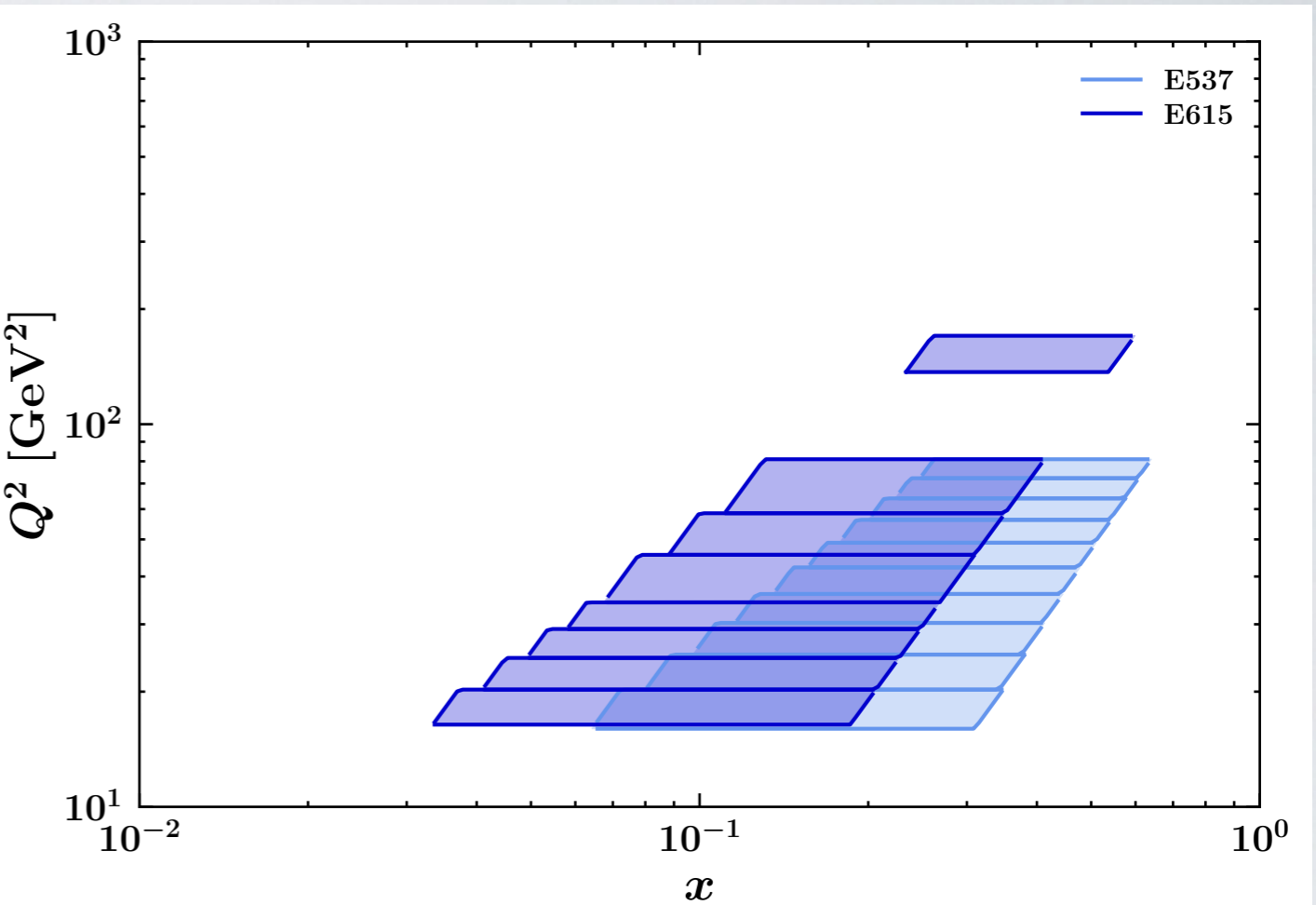
**E615:**  $\sqrt{s} = 21.8 \text{ GeV}$   $0 < x_F < 1$   
 $4.05 < Q < 13.05 \text{ GeV}$   
*Stirling et al., 1993*

**E537:**  $\sqrt{s} = 15.3 \text{ GeV}$   $-0.1 < x_F < 1$   
 $4 < Q < 9 \text{ GeV}$   
*Anassontzis et al., 1988*

Kinematical cut  $q_T < 0.3 Q + 0.6$

$N_{\text{data}}$  after cut: **74** + **64**

Many sources of (large) errors:



**E615:** stat. 5%      syst. 16% (normalization)

**E537:** stat. 15-20%    syst. 8%

**Th.:** 5-8% on PDF from xFitter20

# The MAPTMDPion22 data sets

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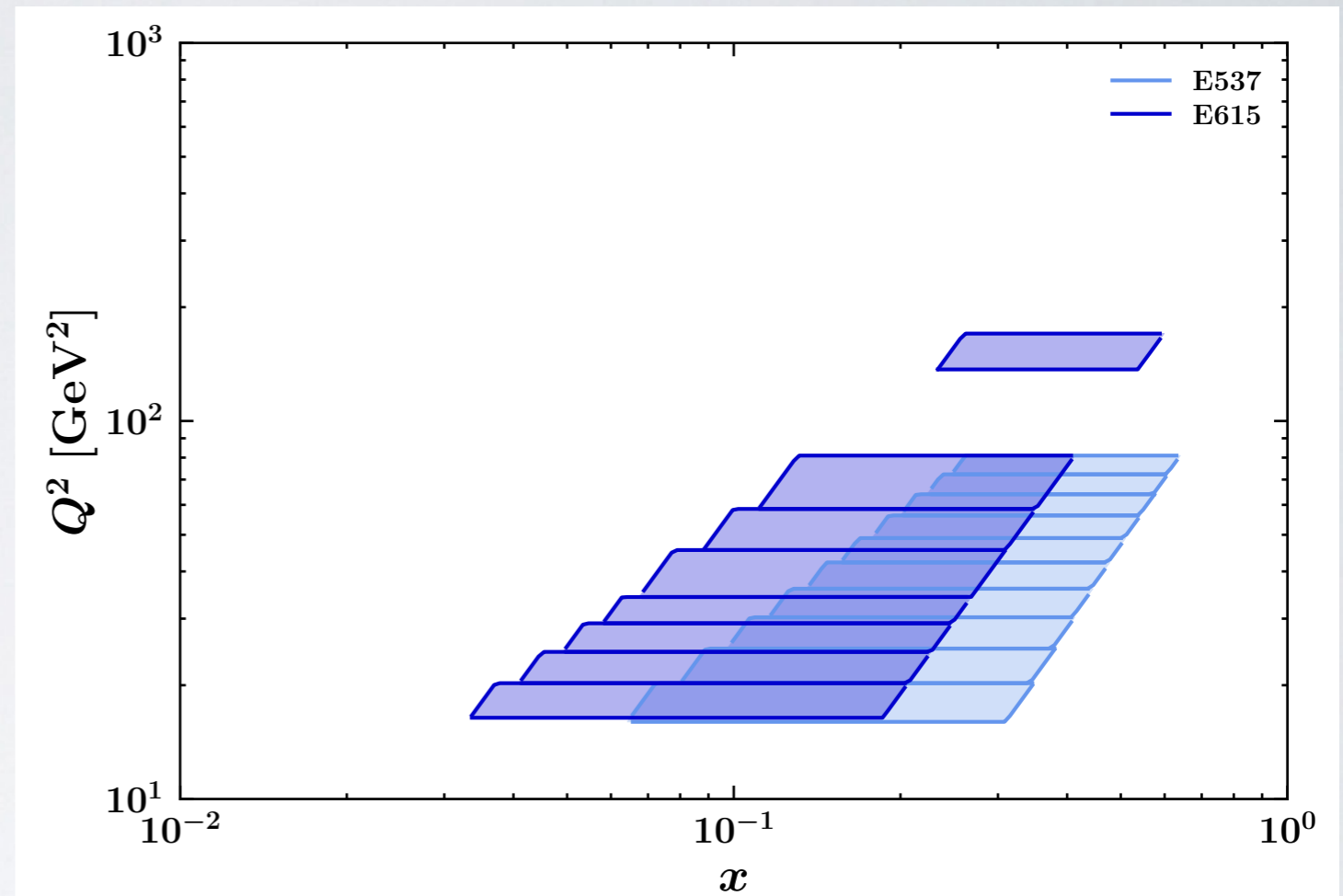
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**E537:**  $\sqrt{s} = 15.3 \text{ GeV}$   $-0.1 < x_F < 1$   
 $4 < Q < 9 \text{ GeV}$   
*Anassontzis et al., 1988*

Kinematical cut  $q_T < 0.2 Q$

$N_{\text{data}}$  after cut: **74** + **64**

Many sources of (large) errors:



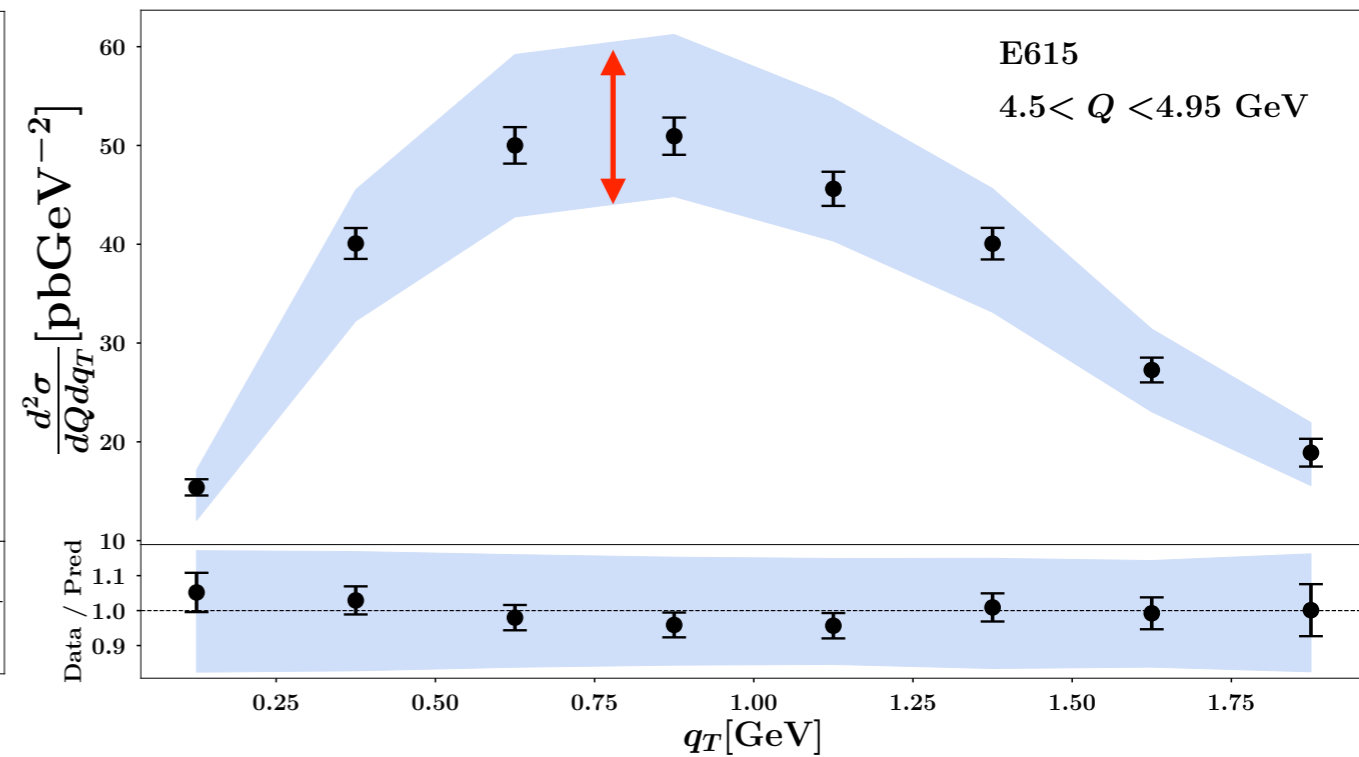
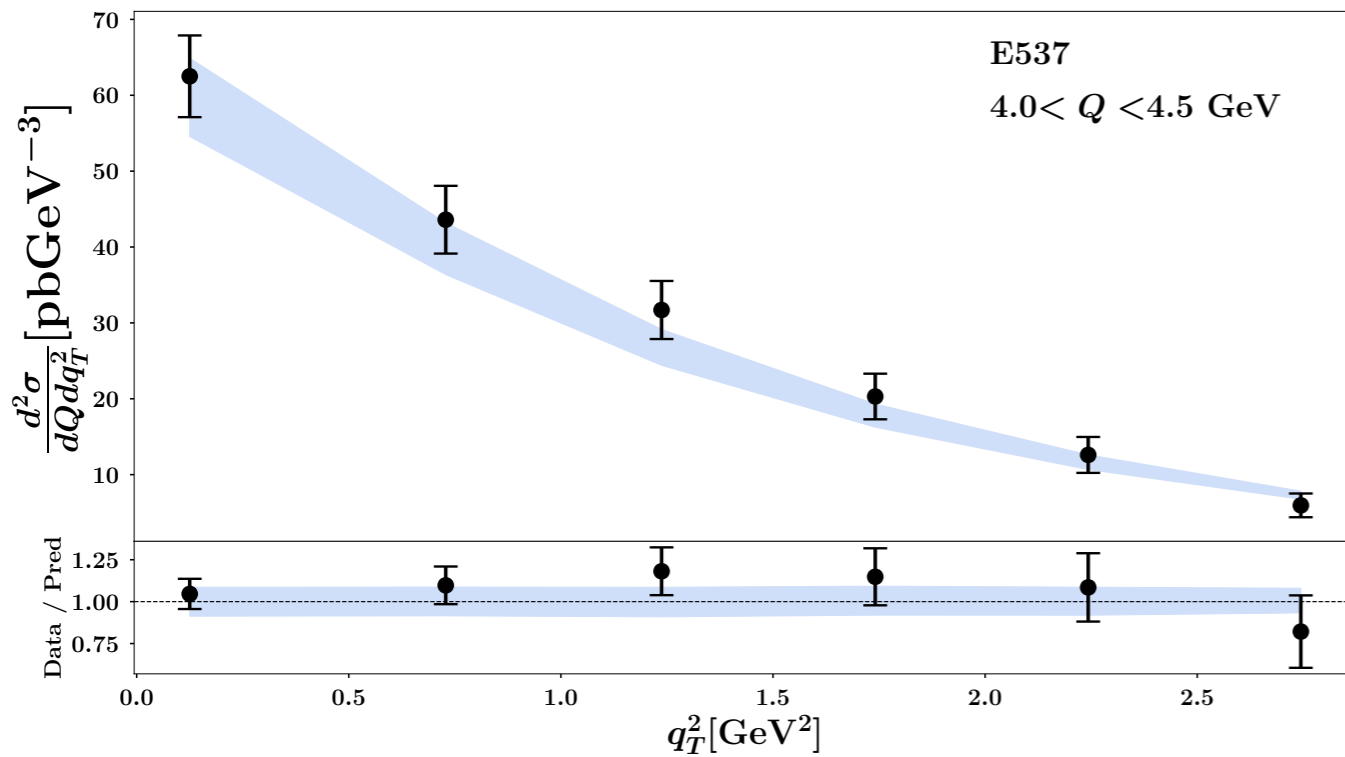
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# Fit results

arXiv:2210.01733

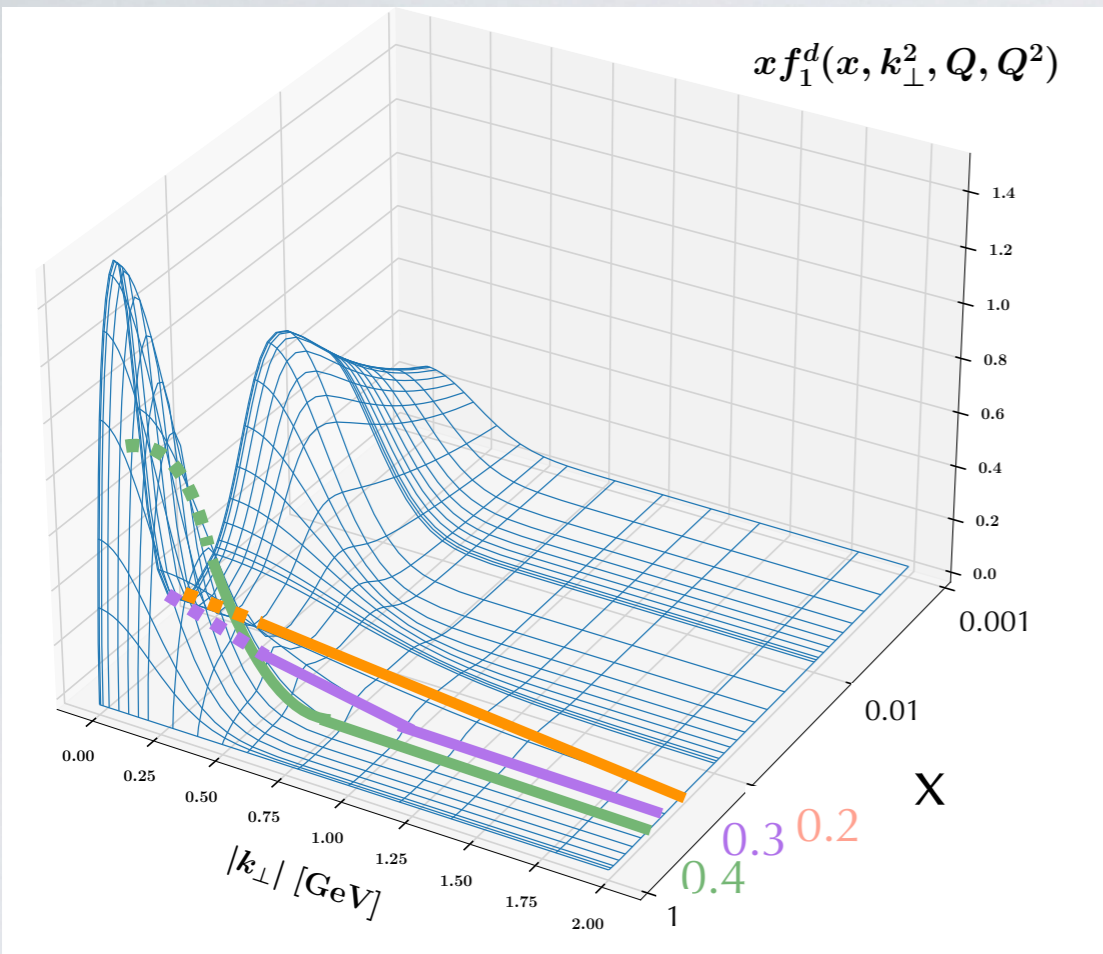


MAPTMDPion22: - good agreement in shape ↑  
 - **large normalization errors** (for E615)

data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi^2$
E537	64	1.0	0.57	1.57
E615	74	0.31	1.22	1.53
<b>total</b>	<b>138</b>	<b>0.63</b>	<b>0.92</b>	<b>1.55</b>

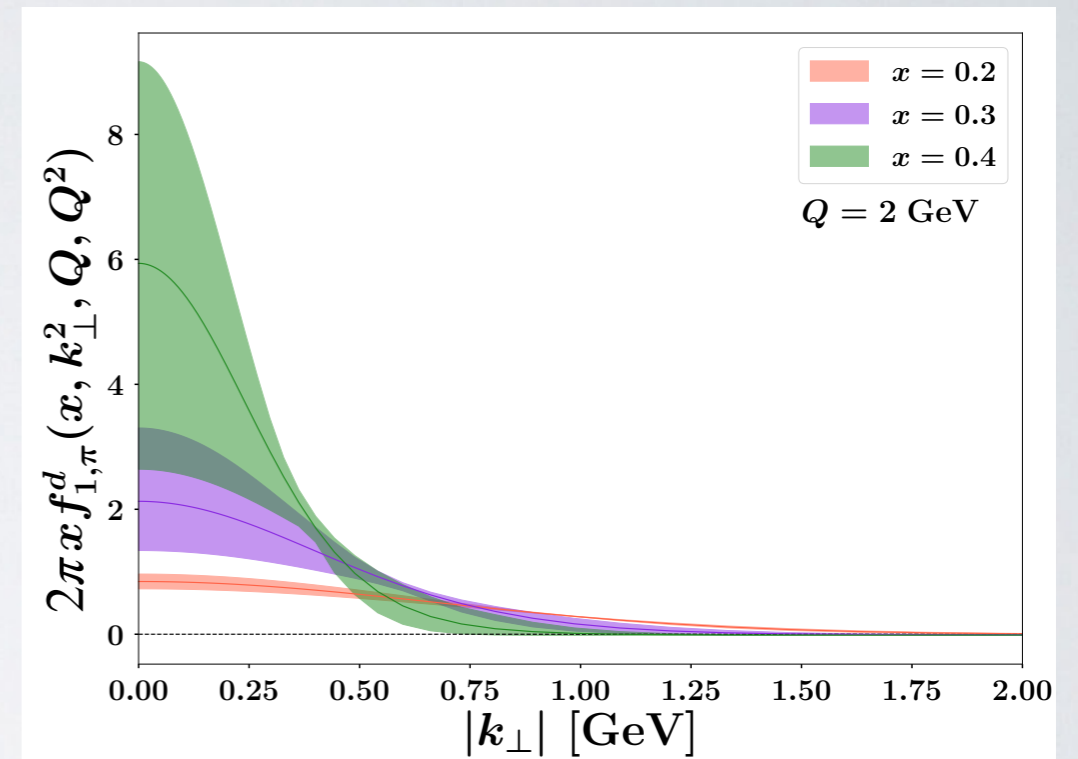
$\chi_D^2$  = uncorrelated error  
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# Visualizing MAPTMDPion22 TMD PDF

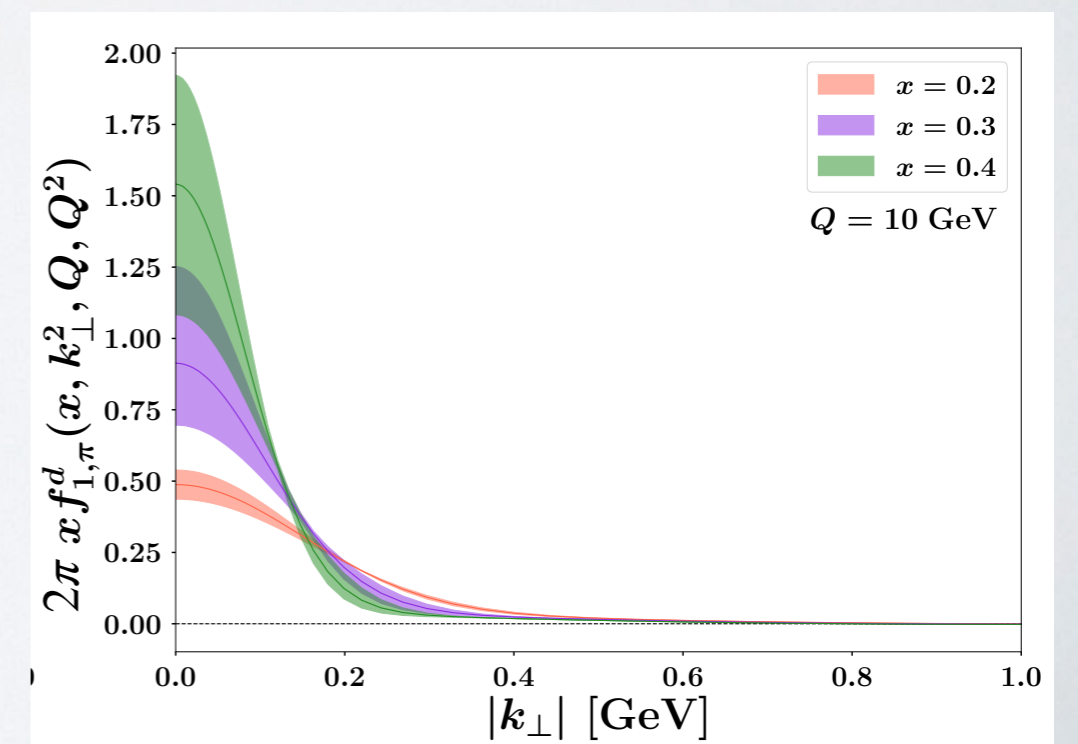


unpolarized down quark in  $\pi^-$

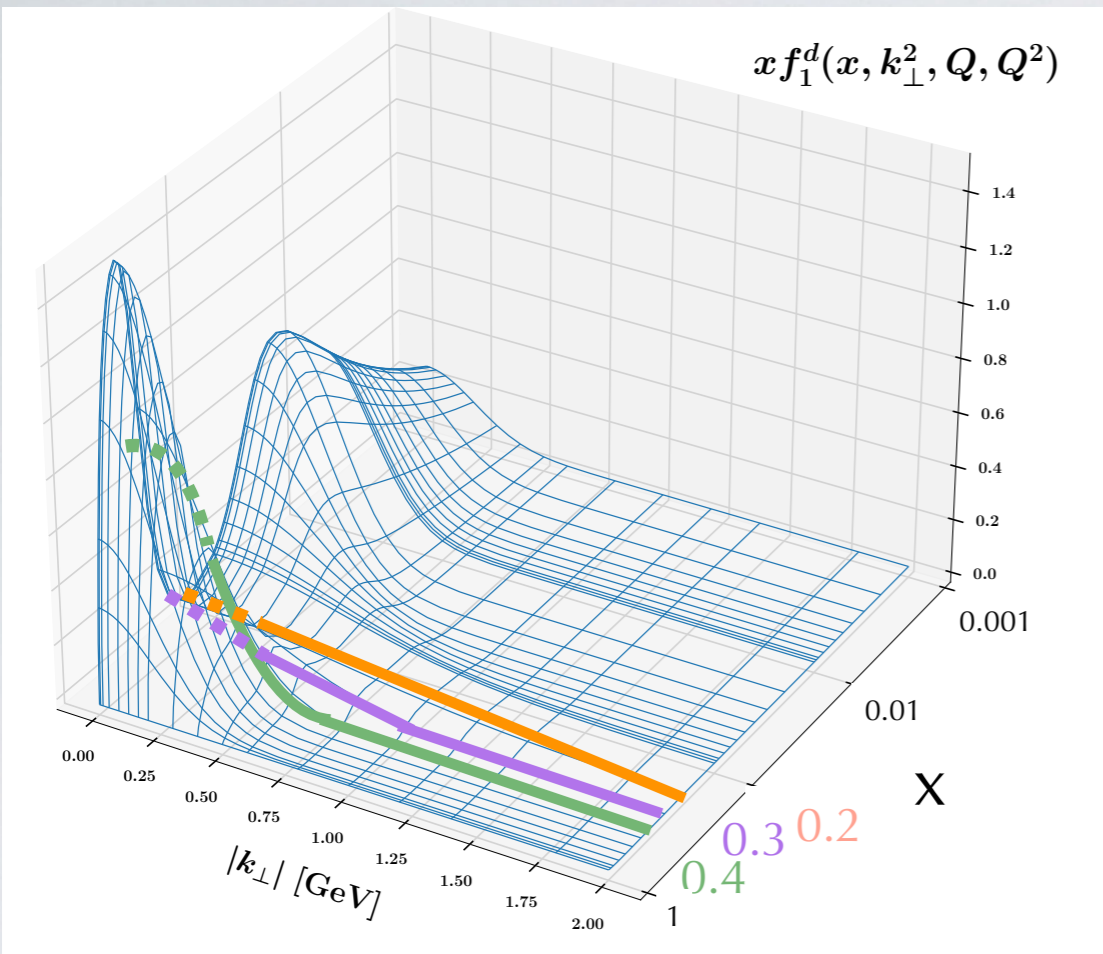
$Q=2$  GeV



$e^{-g_2^2 b_T^2/4}$  same nonperturbative evolution as for proton

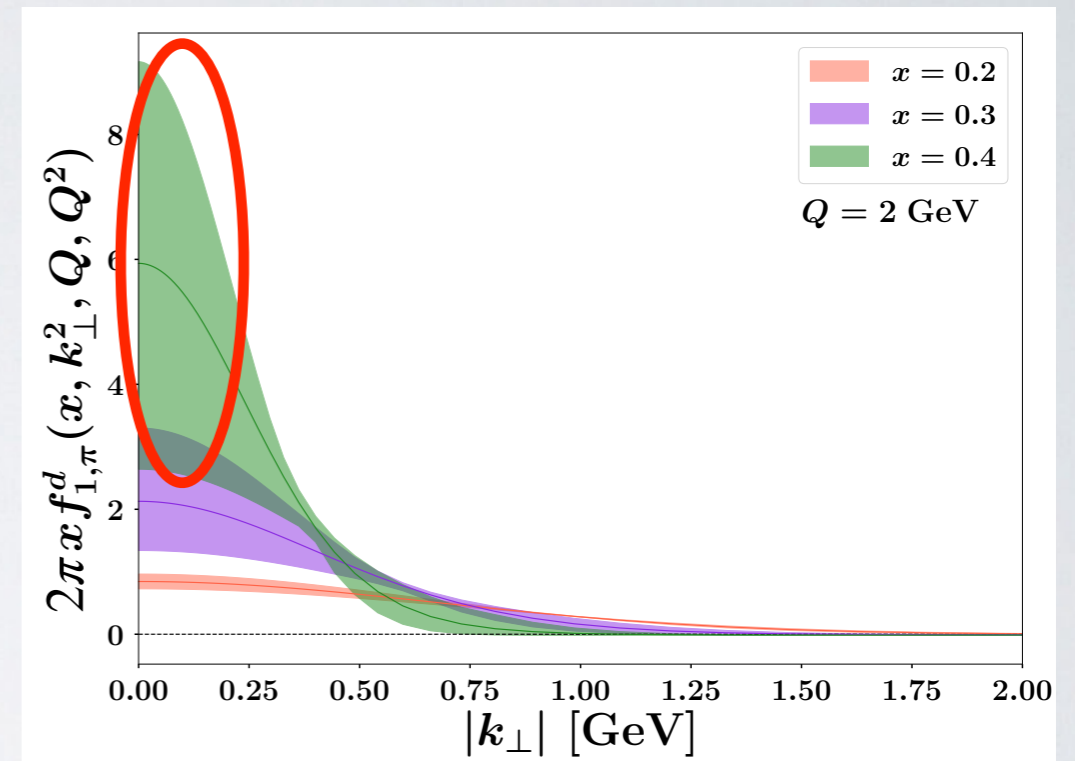


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unpolarized down quark in  $\pi^-$

$Q=2$  GeV

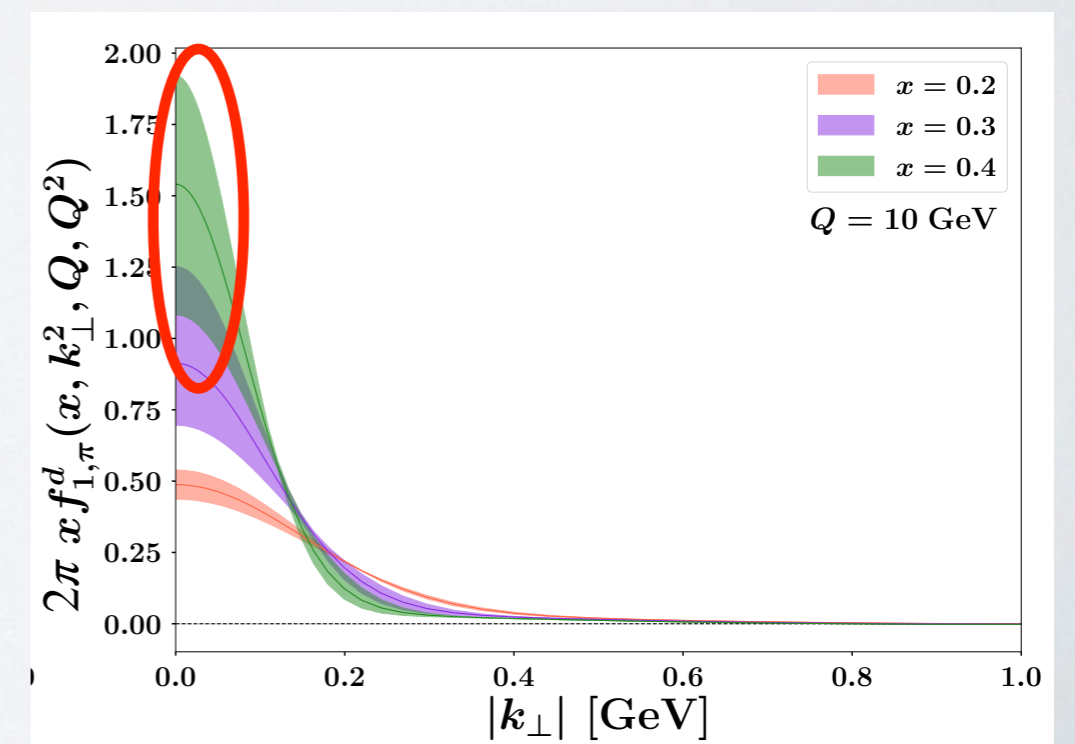


$e^{-g_2^2 b_T^2/4}$  same nonperturbative evolution as for proton

$$N_{1\pi} = 0.47 \pm 0.12 \quad \sigma_{1\pi} = 4.50 \pm 2.25 \quad \alpha_{1\pi} = 4.40 \pm 1.34$$

$\downarrow$   
 $\langle k_{\perp}^2 \rangle(x = 0.1, Q = 1)$  larger than in the proton

data not very sensitive to nonperturbative parameters => **need new data!**



# Summary and Outlook

- **MAPTMD22**: a **global fit** at **N<sup>3</sup>LL** of SIDIS and Drell-Yan data from LHC, Tevatron, RHIC; **2031 data** pts., **21 parameters**,  $\chi^2/N_{\text{data}} = 1.06$  : the current most robust and precise extraction of unpolarized quark TMDs in the proton, accuracy comparable to modern PDF fits
- surprisingly, MAPTMD22 performs **well** in SIDIS well **outside** TMD factorization limit  $q_T \ll Q$

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  - surprisingly, MAPTMD22 performs **well** in SIDIS well **outside** TMD factorization limit  $q_T \ll Q$
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-



# Summary and Outlook

- **MAPTMD22**: a **global fit** at **N<sup>3</sup>LL** of SIDIS and Drell-Yan data from LHC, Tevatron, RHIC; **2031 data** pts., **21 parameters**,  $\chi^2/N_{\text{data}} = 1.06$  : the current most robust and precise extraction of unpolarized quark TMDs in the proton, accuracy comparable to modern PDF fits
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- 
- introduce flavor-dependent  $k_{\perp}$  distributions (ongoing...)
  - include upcoming Compass data for pion-induced Drell-Yan
  - improve perturbative accuracy (very precise ATLAS data...)



**THANK YOU**  
for your  
**ATTENTION!**

# Backup

# Drell-Yan observables

collider

**Exp:** normalized cross section differential in  $q_T$  in each bin

**Th:** for each bin [i,f]  $\frac{1}{\sigma_{\text{fiducial}}} \frac{1}{(\Delta q_T)_{if}} \int_{q_{Ti}}^{q_{Tf}} dq_T \int_{y_i}^{y_f} dy \int_{Q_i}^{Q_f} dQ \frac{d\sigma}{dq_T dy dQ}$

DYNNLO  
with MMHT14 PDFs

**E288:** cross section differential in average  $\bar{q}_T$  and  $\bar{y}$

**Th:** for each bin [i,f]  $\frac{1}{2\pi\bar{q}_T} \int_{Q_i}^{Q_f} dQ \frac{d\sigma}{dq_T dy dQ} \Big|_{y=\bar{y}, q_T=\bar{q}_T}$

fixed  
target

**E772:** cross section differential in average  $\bar{q}_T$  and  $x_F = x_A - x_B$  bins

**Th:** for each bin [i,f]  $\frac{1}{(\Delta x_F)_{if}} \int_{x_{Fi}}^{x_{Ff}} dx_F \int_{Q_i}^{Q_f} dQ \frac{2E}{\pi\sqrt{s}} \frac{d\sigma}{dq_T^2 dx_F dQ} \Big|_{q_T=\bar{q}_T}$

**E605:** cross section differential in average  $\bar{q}_T$  and  $\bar{x}_F$

**Th:** for each bin [i,f]  $\int_{Q_i}^{Q_f} dQ \frac{2E}{\pi\sqrt{s}} \frac{d\sigma}{dq_T^2 dx_F dQ} \Big|_{x_F=\bar{x}_F, q_T=\bar{q}_T}$

# SIDIS observable

**Exp:** differential SIDIS cross section divided by DIS one

$$M(x, z, P_{hT}, Q) = \frac{d\sigma^{\text{SIDIS}}}{dx dz dP_{hT} dQ} \bigg/ \frac{d\sigma^{\text{DIS}}}{dx dQ}$$

Multiplicity

**Th:** for each bin [i,f]

$$\mathcal{O}^{\text{SIDIS}} = \frac{1}{(\Delta Q)_{if}} \int_{Q_i}^{Q_f} dQ \frac{1}{(\Delta x)_{if}} \int_{x_i}^{x_f} dx \frac{1}{(\Delta z)_{if}} \int_{z_i}^{z_f} dz \frac{1}{(\Delta P_{hT})_{if}} \int_{P_{hTi}}^{P_{hTf}} dP_{hT} \frac{d\sigma^{\text{SIDIS}}}{dx dz dP_{hT} dQ}$$

$$\mathcal{O}^{\text{DIS}} = \frac{1}{(\Delta Q)_{if}} \int_{Q_i}^{Q_f} dQ \frac{1}{(\Delta x)_{if}} \int_{x_i}^{x_f} dx \frac{d\sigma^{\text{DIS}}}{dx dQ}$$

$$M^{\text{th}}(x_{if}, z_{if}, P_{hTif}, Q_{if}) = \frac{\mathcal{O}^{\text{SIDIS}}}{\mathcal{O}^{\text{DIS}}}$$

# Error treatment

bootstrap method: fitting 250 replicas of fluctuated exp. data  
quality indicator:  $\chi_0^2$  of *central* replica (fitting not " " " )

**MAPTMD22** at N<sup>3</sup>LL<sup>(-)</sup>:  $N_{\text{data}}=2031$ , **21** parameters,  $\chi_0^2/N_{\text{data}} = 1.06$

$$\chi_0^2 \sim \langle \chi^2 \rangle_{\text{replicas}}$$

(exp. / th.) errors can be **uncorrelated** or **correlated**

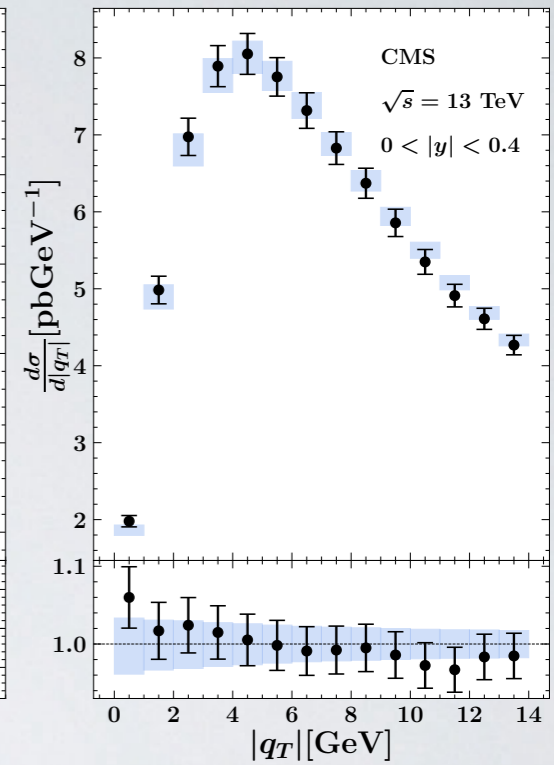
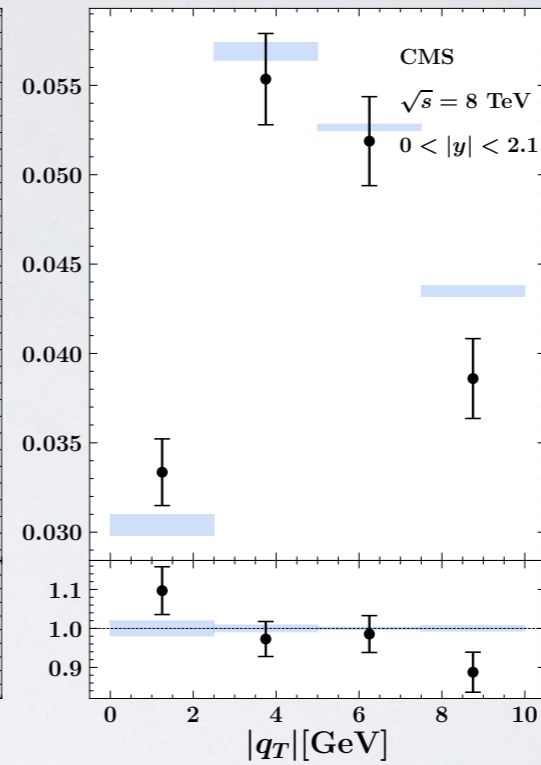
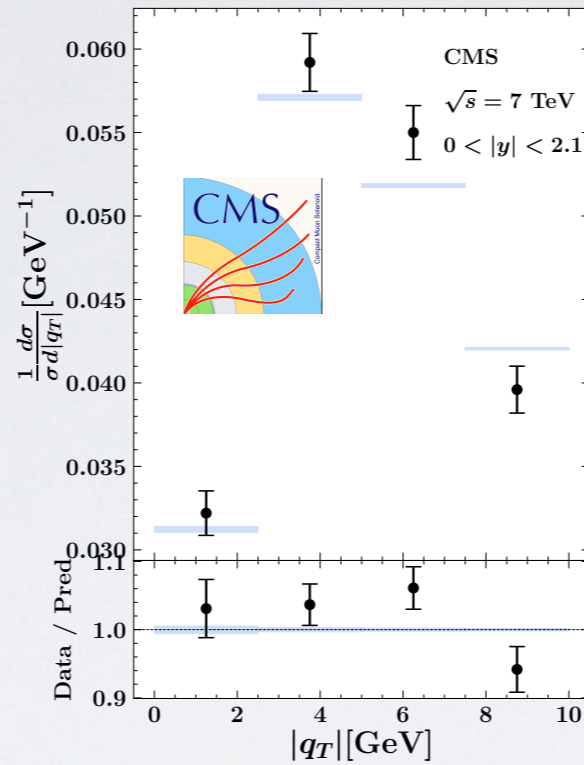
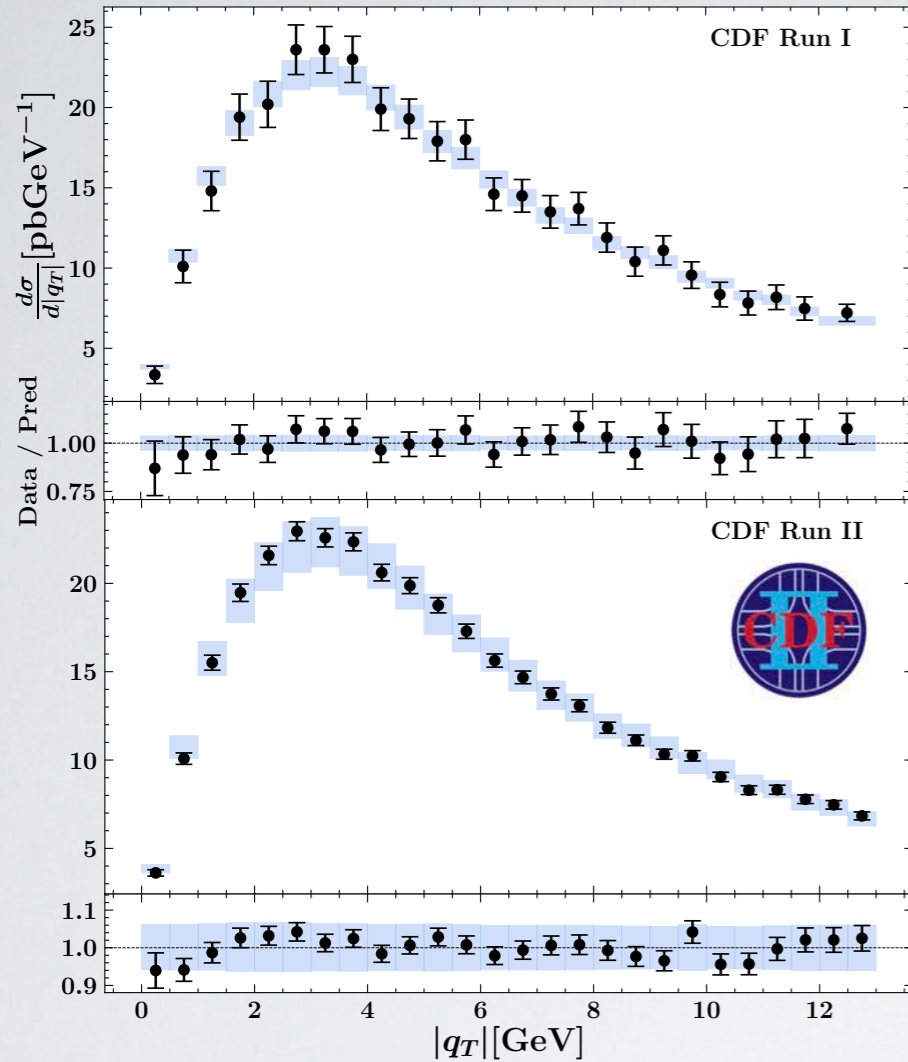
$$\chi^2 = \underbrace{\sum_{\text{bins}} \left( \frac{\text{exp} - \bar{\text{th}}}{\sigma} \right)^2}_{\sigma^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{uncorr}}^2}^{\chi_D^2} + \underbrace{\chi_\lambda^2}_{\text{penalty for correlated errors}}$$
$$\bar{\text{th}} = \text{th} + \sum_{\alpha} \lambda_{\alpha} \sigma_{\text{corr}}^{(\alpha)}$$
$$\chi_\lambda^2 = \sum_{\alpha} \lambda_{\alpha}^2$$

nuisance params.

Examples of **(partly) correlated errors** :

- exp.: some normalization systematic errors
- th. : uncertainties of PDFs
  - MMHT2014
  - FFs
  - DSS14 for  $\pi^{\pm}$
  - DSS17 for  $K^{\pm}$

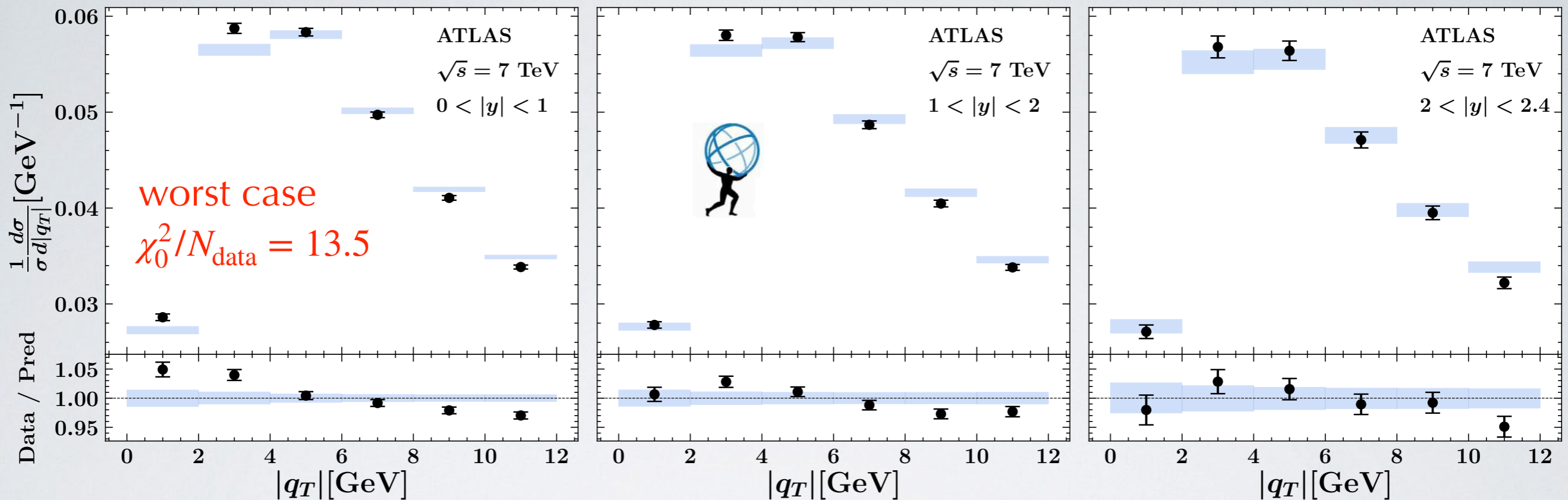
# MAPTMD22 N<sup>3</sup>LL<sup>(-)</sup> global fit $\chi_0^2/N_{\text{data}} = 1.06$



th. error band =  
68% of all replicas

data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi_0^2$
Tevatron total	71	0.87	0.06	0.93
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
LHCb total	21	1.15	0.3	1.45
<b>ATLAS total</b>	<b>72</b>	<b>4.56</b>	<b>0.48</b>	<b>5.05</b>
CMS total	78	0.53	0.02	0.55
collider total	251	1.86	0.2	2.06
fixed target tot	233	0.85	0.4	1.24

# MAPTMD22 N<sup>3</sup>LL<sup>(-)</sup> global fit $\chi_0^2/N_{\text{data}} = 1.06$



extremely small exp. errors  
 → very sensitive to small th. corrections

Examples:

- numerical implementation of lepton cuts
- power corrections
- effects of matching  $Y$  term

Chen et al.,  
 arXiv:2203.01565  
 Camarda et al.,  
 arXiv:2111.14509  
 Buonocore et al.,  
 arXiv:2111.13661

data set	$N_{\text{data}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi_0^2$
ATLAS 7 TeV	18	6.43	0.92	7.35
ATLAS 8 TeV	48	3.7	0.32	4.02
ATLAS 13 TeV	6	5.09	0.5	6.4
ATLAS total	72	4.56	0.48	5.05
collider total	251	1.86	0.2	2.06

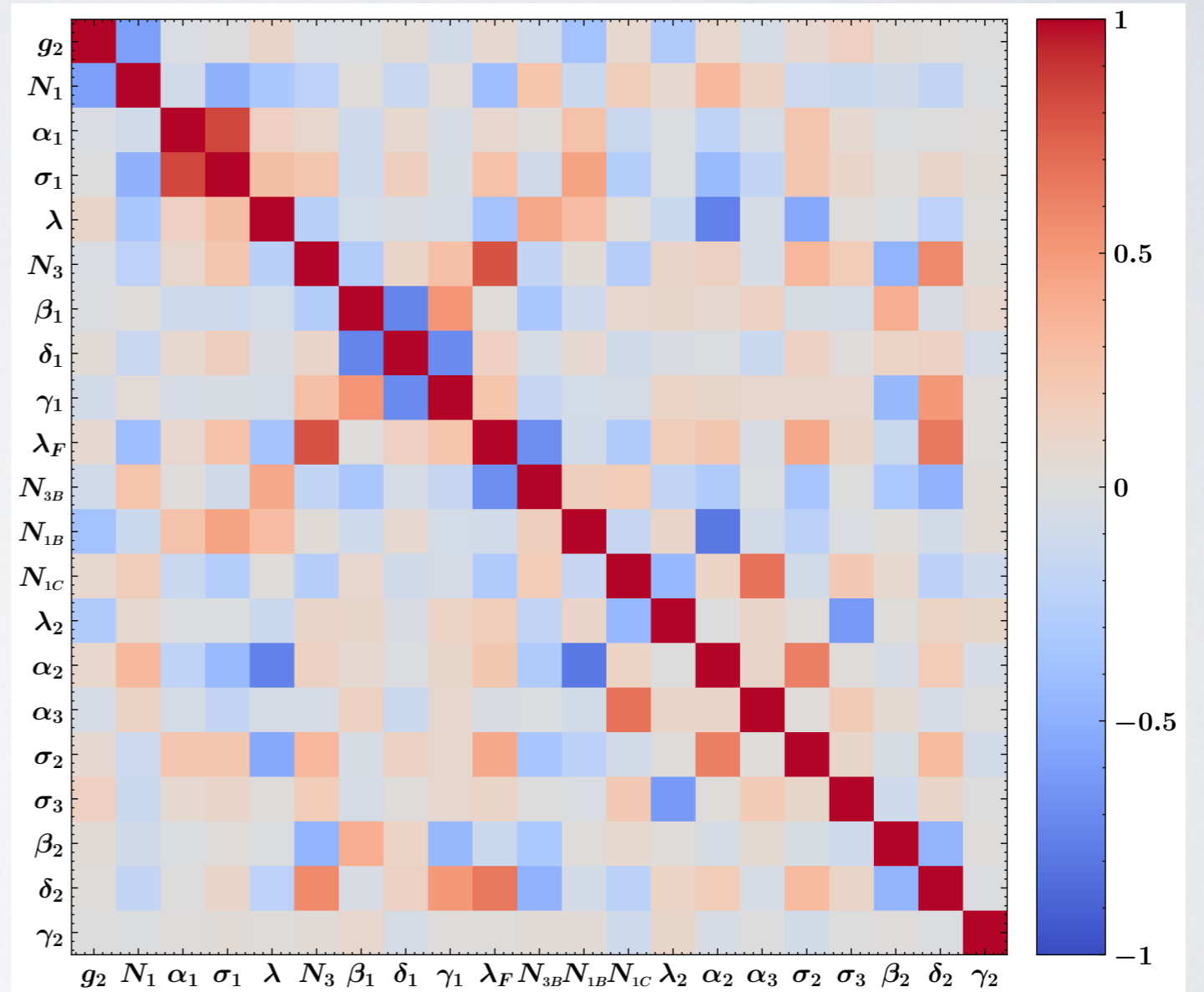


# Backup: chi2 breakout

Data set	$N^3LL^-$			
	$N_{\text{dat}}$	$\chi_D^2$	$\chi_\lambda^2$	$\chi_0^2$
CDF Run I	25	0.45	0.09	0.54
CDF Run II	26	0.995	0.004	1.0
D0 Run I	12	0.67	0.01	0.68
D0 Run II	5	0.89	0.21	1.10
D0 Run II ( $\mu$ )	3	3.96	0.28	4.2
<i>Tevatron total</i>	71	0.87	0.06	0.93
LHCb 7 TeV	7	1.24	0.49	1.73
LHCb 8 TeV	7	0.78	0.36	1.14
LHCb 13 TeV	7	1.42	0.06	1.48
<i>LHCb total</i>	21	1.15	0.3	1.45
ATLAS 7 TeV	18	6.43	0.92	7.35
ATLAS 8 TeV	48	3.7	0.32	4.02
ATLAS 13 TeV	6	5.9	0.5	6.4
<i>ATLAS total</i>	72	4.56	0.48	5.05
CMS 7 TeV	4	2.21	0.10	2.31
CMS 8 TeV	4	1.938	0.001	1.94
CMS 13 TeV	70	0.36	0.02	0.37
<i>CMS total</i>	78	0.53	0.02	0.55
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
<i>DY collider total</i>	251	1.86	0.2	2.06
E288 200 GeV	30	0.35	0.19	0.54
E288 300 GeV	39	0.33	0.09	0.42
E288 400 GeV	61	0.5	0.11	0.61
E772	53	1.52	1.03	2.56
E605	50	1.26	0.44	1.7
<i>DY fixed-target total</i>	233	0.85	0.4	1.24
HERMES ( $p \rightarrow \pi^+$ )	45	0.86	0.42	1.28
HERMES ( $p \rightarrow \pi^-$ )	45	0.61	0.31	0.92
HERMES ( $p \rightarrow K^+$ )	45	0.49	0.04	0.53
HERMES ( $p \rightarrow K^-$ )	37	0.18	0.13	0.31
HERMES ( $d \rightarrow \pi^+$ )	41	0.68	0.45	1.13
HERMES ( $d \rightarrow \pi^-$ )	45	0.63	0.35	0.97
HERMES ( $d \rightarrow K^+$ )	45	0.2	0.02	0.22
HERMES ( $d \rightarrow K^-$ )	41	0.14	0.08	0.22
<i>HERMES total</i>	344	0.48	0.23	0.71
COMPASS ( $d \rightarrow h^+$ )	602	0.55	0.31	0.86
COMPASS ( $d \rightarrow h^-$ )	601	0.68	0.3	0.98
<i>COMPASS total</i>	1203	0.62	0.3	0.92
<i>SIDIS total</i>	1547	0.59	0.28	0.87
<b>Total</b>	<b>2031</b>	<b>0.77</b>	<b>0.29</b>	<b>1.06</b>

# Backup: N3LL(-) fit parameters

Parameter	Average over replicas
$g_2$ [GeV]	$0.248 \pm 0.008$
$N_1$ [GeV <sup>2</sup> ]	$0.316 \pm 0.025$
$\alpha_1$	$1.29 \pm 0.19$
$\sigma_1$	$0.68 \pm 0.13$
$\lambda$ [GeV <sup>-1</sup> ]	$1.82 \pm 0.29$
$N_3$ [GeV <sup>2</sup> ]	$0.0055 \pm 0.0006$
$\beta_1$	$10.23 \pm 0.29$
$\delta_1$	$0.0094 \pm 0.0012$
$\gamma_1$	$1.406 \pm 0.084$
$\lambda_F$ [GeV <sup>-2</sup> ]	$0.078 \pm 0.011$
$N_{3B}$ [GeV <sup>2</sup> ]	$0.2167 \pm 0.0055$
$N_{1B}$ [GeV <sup>2</sup> ]	$0.134 \pm 0.017$
$N_{1C}$ [GeV <sup>2</sup> ]	$0.0130 \pm 0.0069$
$\lambda_2$ [GeV <sup>-1</sup> ]	$0.0215 \pm 0.0058$
$\alpha_2$	$4.27 \pm 0.31$
$\alpha_3$	$4.27 \pm 0.13$
$\sigma_2$	$0.455 \pm 0.050$
$\sigma_3$	$12.71 \pm 0.21$
$\beta_2$	$4.17 \pm 0.13$
$\delta_2$	$0.167 \pm 0.006$
$\gamma_2$	$0.0007 \pm 0.0110$



# Backup: NNLL and NLL fits

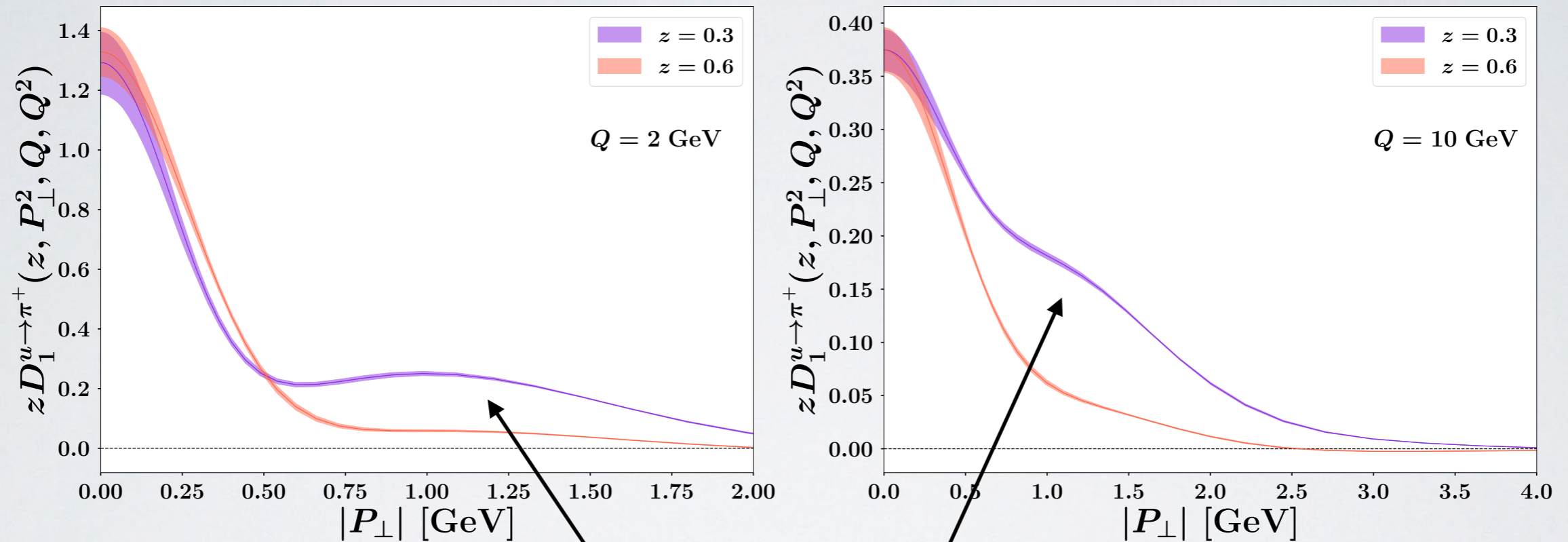
Data set	N <sup>3</sup> LL <sup>-</sup>		NNLL		NLL	
	$N_{\text{dat}}$	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$	$N_{\text{dat}}$	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$	$N_{\text{dat}}$	$\langle\chi^2\rangle \pm \delta\langle\chi^2\rangle$
ATLAS	72	$5.01 \pm 0.26$	/	/	/	/
PHENIX 200	2	$3.26 \pm 0.31$	2	$0.81 \pm 0.11$	/	/
STAR 510	7	$1.16 \pm 0.04$	7	$0.99 \pm 0.03$	/	/
Other sets	170	$0.83 \pm 0.01$	170	$2.37 \pm 0.11$	/	/
DY collider	251	$2.06 \pm 0.07$	179	$2.3 \pm 0.1$	/	/
E772	53	$2.48 \pm 0.12$	53	$2.05 \pm 0.22$	/	/
Other sets	180	$0.87 \pm 0.04$	180	$0.71 \pm 0.04$	180	$0.81 \pm 0.04$
DY fixed-target	233	$1.24 \pm 0.04$	233	$1.01 \pm 0.05$	180	$0.81 \pm 0.04$
HERMES	344	$0.71 \pm 0.04$	344	$1.1 \pm 0.06$	344	$0.51 \pm 0.02$
COMPASS	1203	$0.95 \pm 0.02$	1203	$0.6 \pm 0.06$	1203	$0.41 \pm 0.01$
SIDIS	1547	$0.89 \pm 0.02$	1547	$0.71 \pm 0.05$	1547	$0.43 \pm 0.01$
Total	2031	$1.08 \pm 0.01$	1959	$0.89 \pm 0.01$	1727	$0.47 \pm 0.01$

data sets requiring higher pert. accuracy need to be excluded.

Still, these fits useful for polarized situations with less available accuracy

# Visualizing MAPTMD22 TMD FF

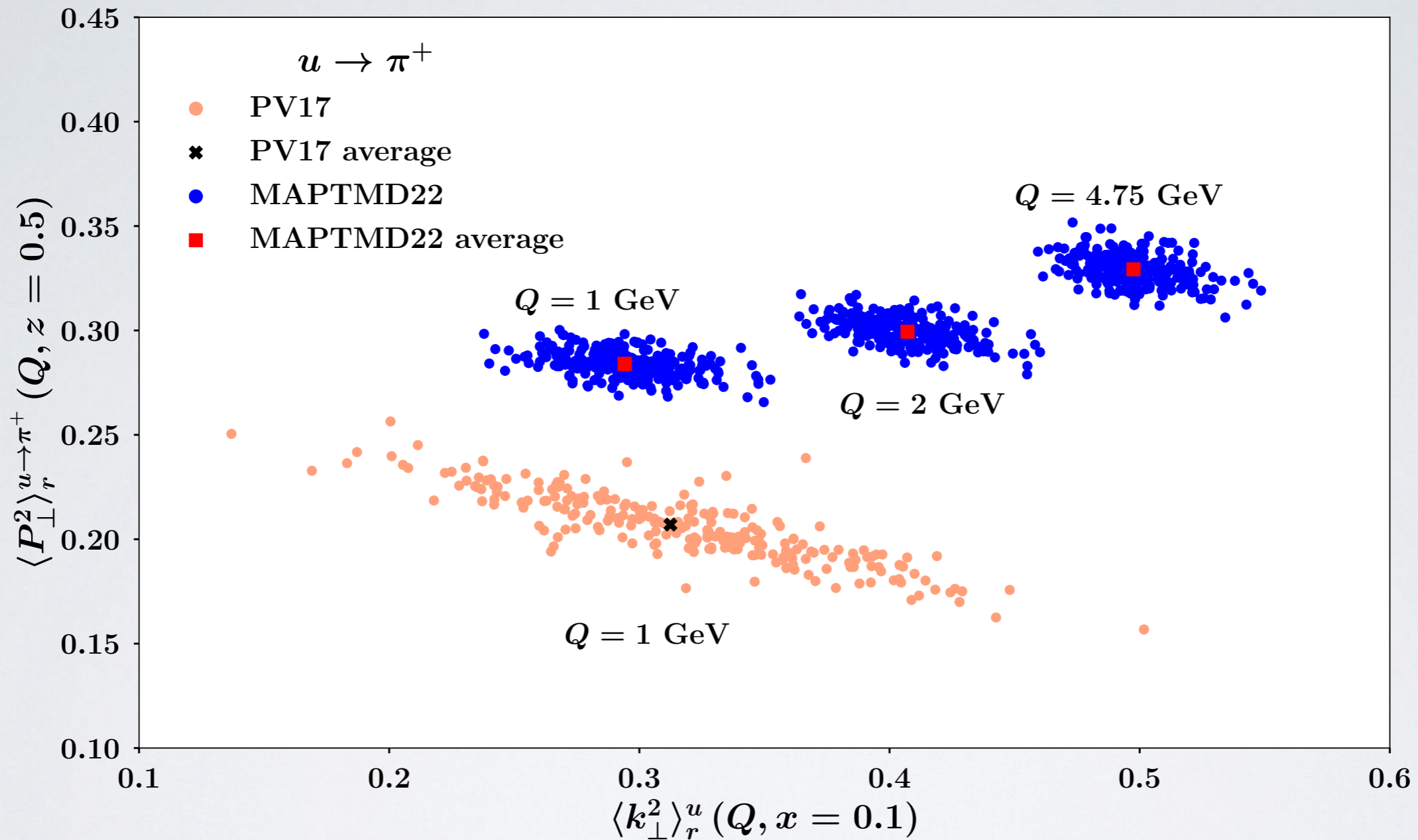
$u\bar{p} \rightarrow \pi^+$



$$D_{\text{NP}}(z, P_\perp; Q_0) \sim \text{Gauss} + \lambda_F P_\perp^2 \text{ Gauss}$$

$$\lambda_F = 0.078 \pm 0.011 \text{ GeV}^{-2}$$

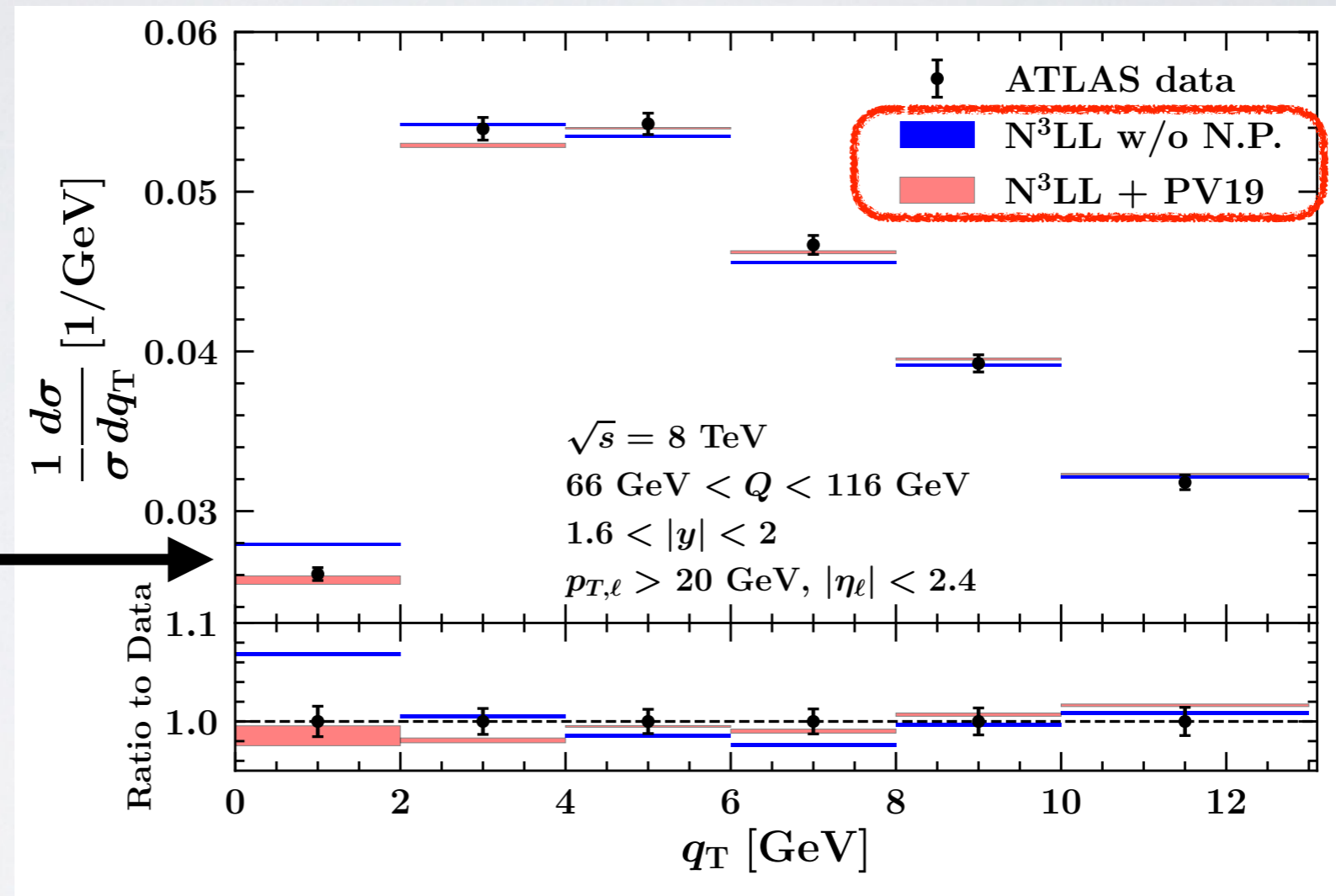
# Backup: avg. transverse momenta



PV17 ( NLL )  $\leftrightarrow$  MAPTMD22 ( N3LL(-) )  $\rightarrow$  less anticorrelation

# TMD impact at the LHC

$q_T$  distribution of Z in ATLAS kin.



effect of intrinsic  
parton  $k_\perp$   
from PV 2019 fit

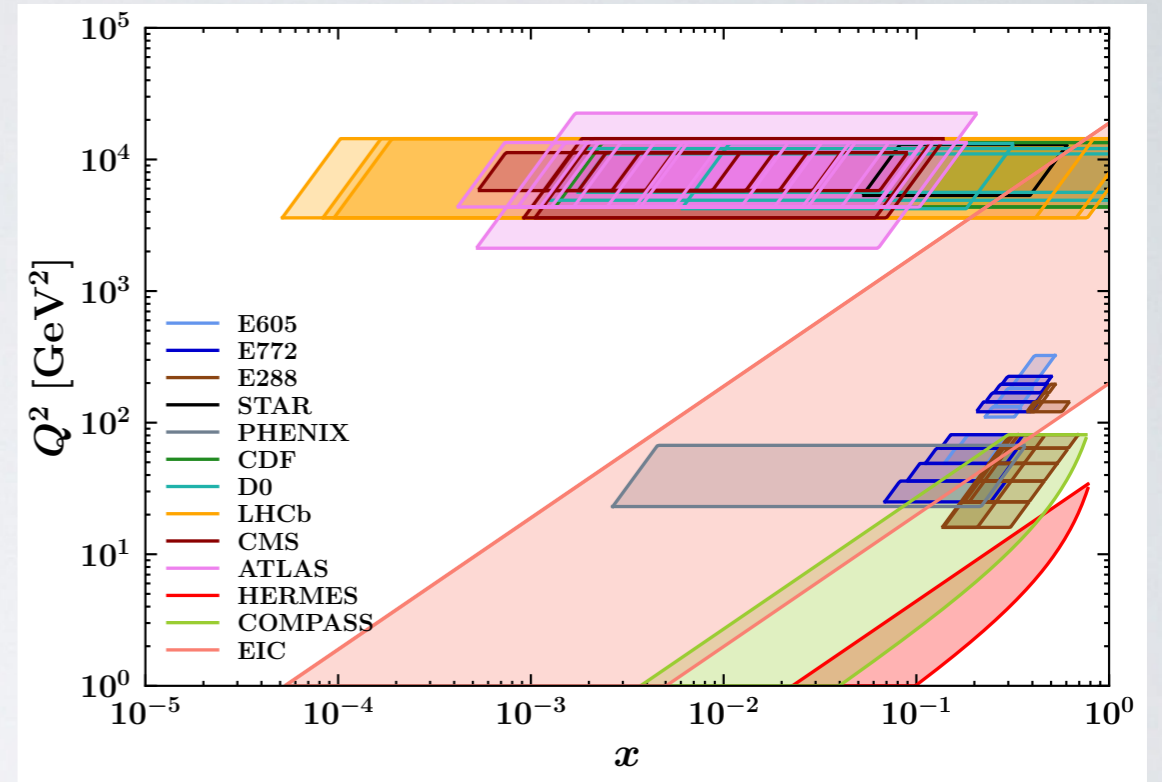
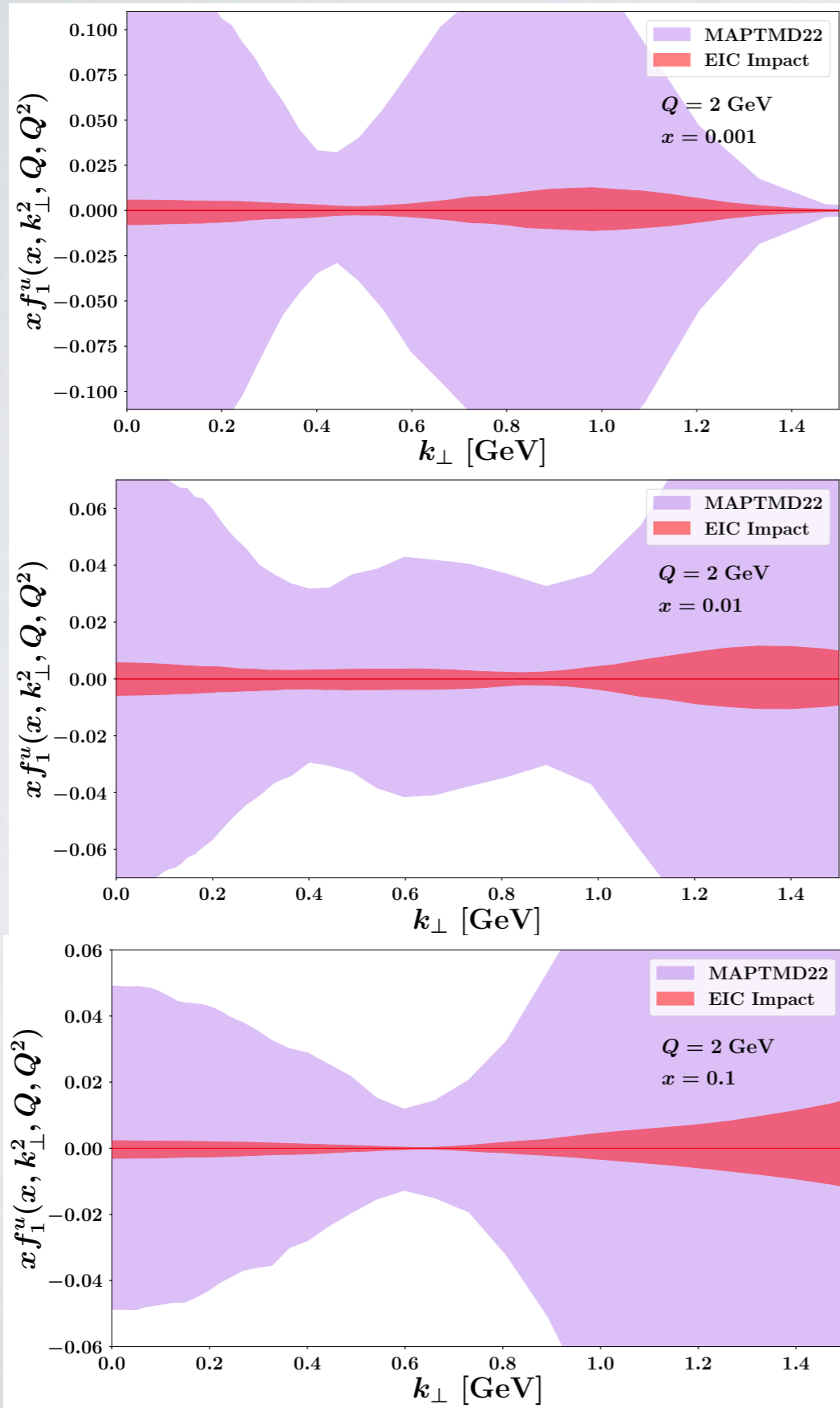
see also later for potential impact on W mass extraction

G. Bozzi, I. Scimemi (eds.) et al.,

*Resummed predictions of the transverse momentum distribution of Drell-Yan lepton pairs in p-p collisions at LHC*

Yellow Report of CERN EW Working Group, in preparation

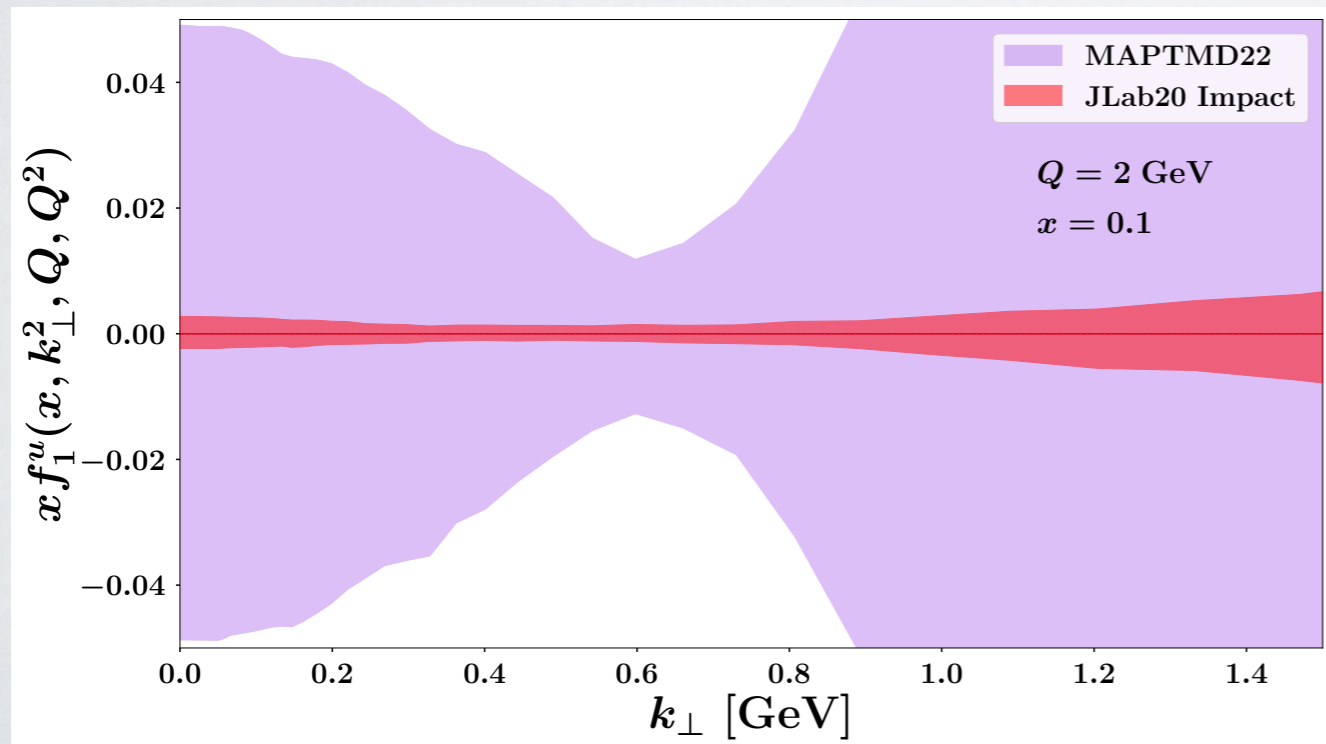
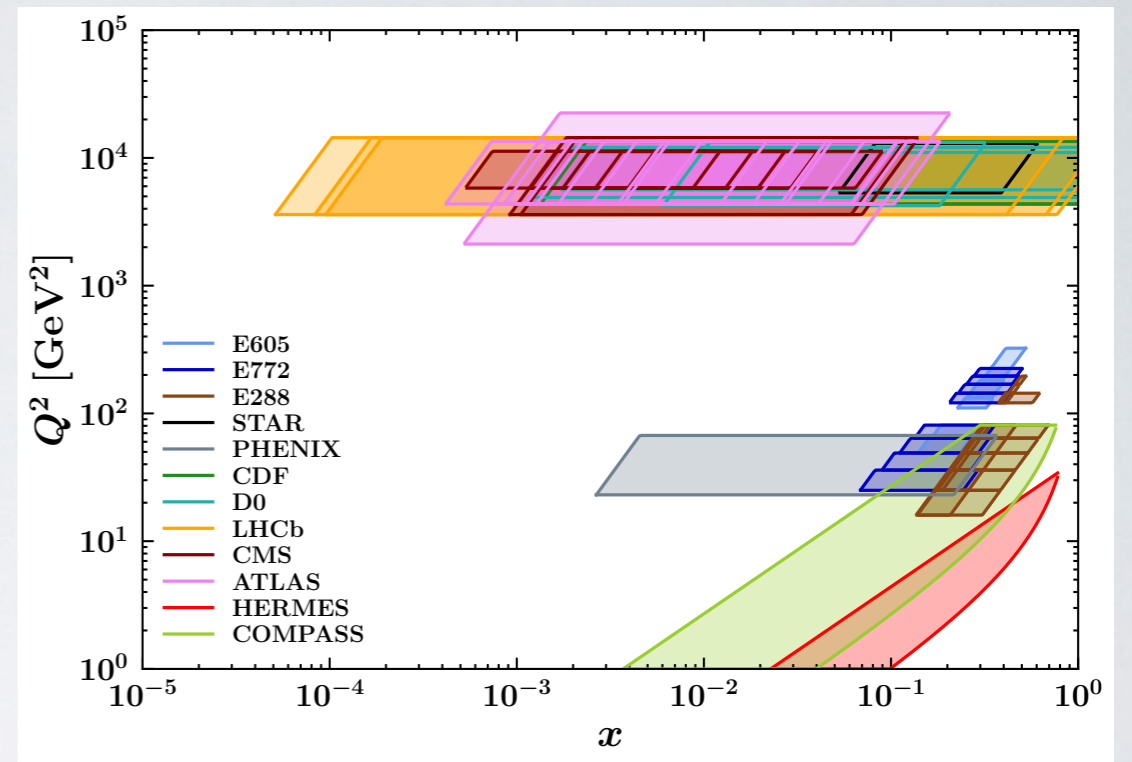
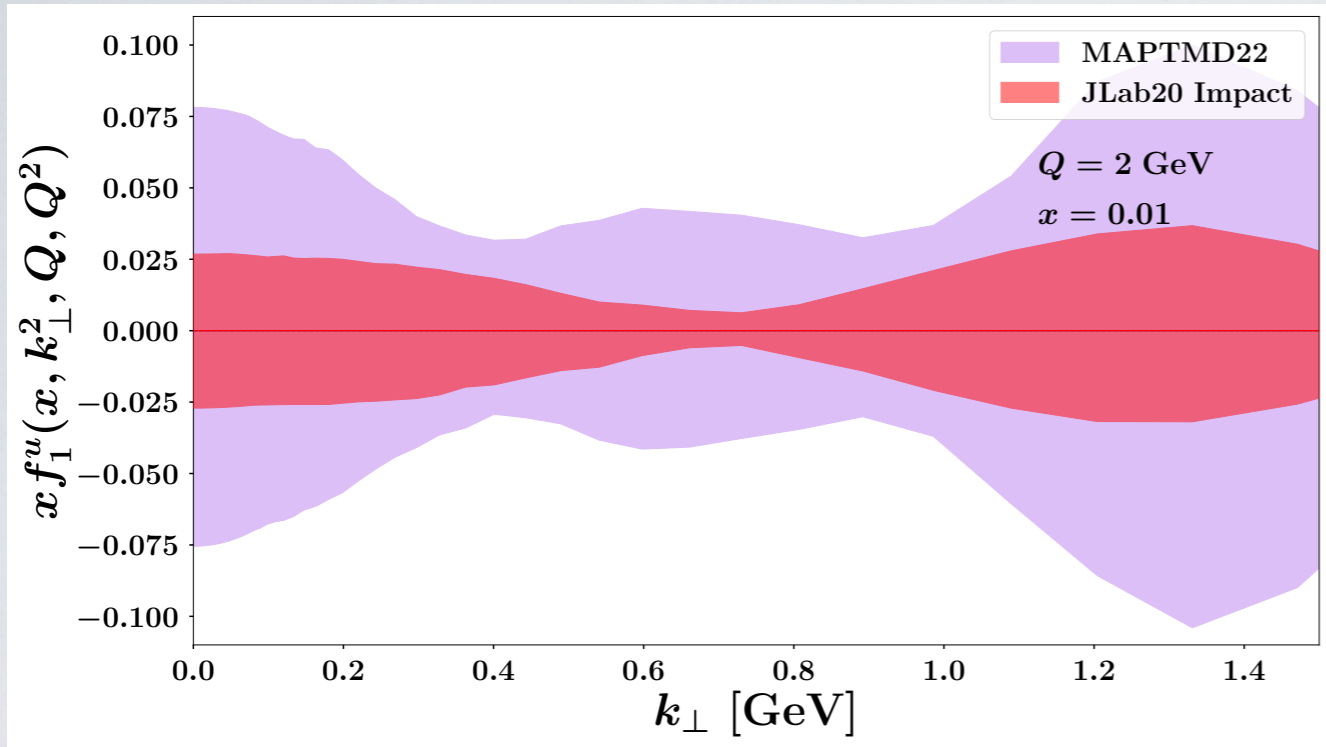
# The EIC impact



kinematics 10x100

major improvements at smaller  $x$

# The JLab20+ impact



kinematics JLab20

major improvements at valence  $x$



# The Nanga Parbat fitting framework

All material available at the Nanga Parbat github site



## Nanga Parbat: a TMD fitting framework

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Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

### Download

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You can obtain NangaParbat directly from the github repository:

<https://github.com/MapCollaboration/NangaParbat>

