Pion GPDs: Theory, modeling and experimental access

Jose Manuel Morgado Chávez

International Workshop on Hadron Structure and Spectroscopy 2023

Prague, Czech Republic

June 26-28th 2023

Email: jose-manuel.morgadochavez@cea.fr

Generalized parton distributions

Generalized parton distributions (GPDs)

(GPD) – Generalized parton distributions:
Non-local quark and gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light front.

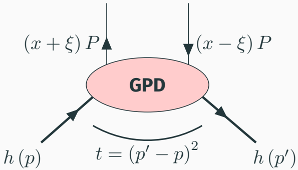
[Fortsch.Phys.:42(1994)101]
[Phys.Lett.B:380(1996)417]
[Phys.Rev.D:55(1197)7114]

Example: Leading twist chiral-even GPDs of a spinless hadron.

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle\pi(p')\right|\bar{\psi}^q(-\lambda n/2)\,\gamma^\mu\psi^q(\lambda n/2)\left|\pi(p)\right\rangle n_\mu\,,$$

$$H_\pi^g(x,\xi,t) = \int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle\pi(p')\right|G^{\mu\alpha}(-\lambda n/2)\,G^\nu_\alpha(\lambda n/2)\left|\pi(p)\right\rangle n_\mu n_\nu$$

GPDs: Definition and properties



x: Momentum fraction of P.

ξ: Fraction of momentum longitudinally transferred.

t: Momentum transfer.

Kinematics:
[Phys.Rept:388(2003)41]

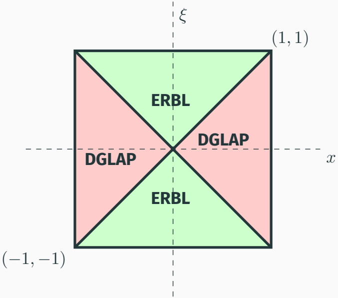
DGLAP: $(|x| > |\xi|)$:

$x > 0$: Emits quark.

$x < 0$: Takes antiquark.

ERBL: $(|x| < |\xi|)$:

Emits pair quark-antiquark.



GPDs: Definition and properties

- **Support:** [Phys.Lett.B:428(1998)359] — **Analyticity/Causality**

$$(x,\xi) \in [-1,1] \otimes [-1,1]$$

- **Polynomiality:** [X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999] — **Lorentz invariance**

$$\int_{-1}^{1} dx x^m H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\,\xi^k$$

- **Positivity:** [P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999] — **Positivity of Hilbert space norm**

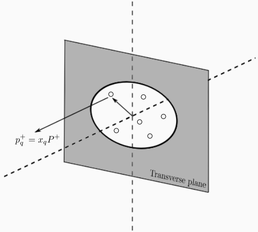
$$|H^q(x,\xi,t=0)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)} \quad , \qquad |x| \geq \xi$$

- **Low energy soft-pion theorem** [M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015] — **PCAC/Axial-Vector WTI**

GPDs: Definition and properties

Probabilistic interpretation:

Probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction "x" of the hadron's averaged light-cone momentum.
[Phys.Rev.D:66(2002)119903]



"3D picture" of hadrons

Striking properties:

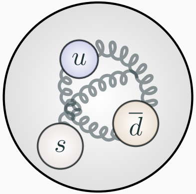
- PDFs as forward limit.
- Electromagnetic and gravitational FFs as Mellin moments.
- Connection with energy-momentum tensor: Angular momentun, pressure...
[Phys.Lett.B:555(2003)57, Phys.Rev.Lett.:28(1997)610]
- Parametrize DVCS amplitudes through CFFs.
[e.g. Phys.Rev.D:59(1999)07009, Phys.Rev.D:56(1997)2982]

Hadron structure

How do quarks and gluons combine to make hadrons up?

PIONS

1. "Simpler" than baryons
2. Connected with DCSB



GPDs

1. Connection with elementary degrees of freedom
2. "3D" picture of hadrons
3. Experimentally accessible

1. *Can we obtain "theoretically complete" pion GPDs?*
2. *What can we learn about actual pions?*

GPD modeling

GPD modeling

Question: *Can we build pion GPDs fulfilling all these constraints?*

1. **Overlap representation**
   [Nucl.Phys.B596(2001)33]
   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$

   Polynomiality ?
   Positivity ✓

---

2. **Double Distribution representation**
   [Fortsch.Phys.:42(1994)101, hep-ph/0101225]
   Relying on Radon transform, $\mathcal{R}$

   Polynomiality ✓
   Positivity ?

Problem: Different modeling strategies and different issues.

GPD modeling

Question: *Can we build pion GPDs fulfilling all these constraints?*

1. **Overlap representation**
   [Nucl.Phys.B596(2001)33]

   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$ } Polynomiality ? Positivity ✓

---

2. **Double Distribution representation**
   [Fortsch.Phys.:42(1994)101, hep-ph/0101225]

   Relying on Radon transform, $\mathcal{R}$ } Polynomiality ✓ Positivity ?

Problem: Different modeling strategies and different issues.

Solution

Covariant extension: Given a DGLAP GPD, the covariant extension allows computing the ERBL GPD such that polynomiality is satisfied. [Eur.Phys.J.C:77(2017)12,906]

GPD modeling: DSE

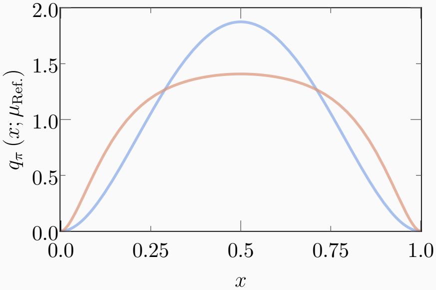
GPD modeling: DSE *Ansätze*

1. "Factorization" of LFWFs: DSE-PTIRs.
[Phys. Lett. B 815 (2021) 136158, Chin. Phys. C 46 (2022) 1, 013105]

$$\Psi_{\pi}^{\lambda_1\lambda_2}\left(x,k_\perp^2\right)=\sqrt{q_\pi\left(x\right)}\frac{i^{\lambda_1\lambda_2}M^2}{\left(k_\perp^2+M^2\right)^2}$$

2. Overlap representation: **Positive** pion GPD [Phys. Rev. D 105 (2022) 9, 094012]

$$H_\pi^q\left(x,\xi,t\right)\big|_{\mathrm{DGLAP}}=\frac{\sqrt{q_\pi\left(x_-\right)q_\pi\left(x_+\right)}}{\left(1+z^2\right)^2}\left[3+\frac{1-2z}{1+z}\frac{\operatorname{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}}\right],\ z=\frac{-t\left(1-x\right)^2}{4M^2\left(1-\xi^2\right)}$$



Two models:

- Algebraic model

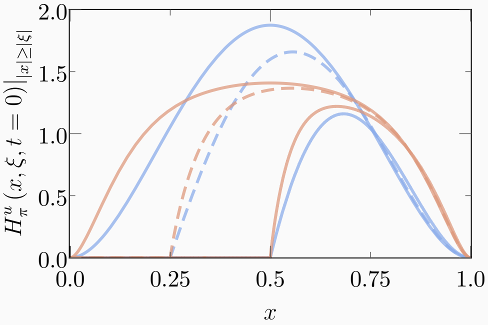
$$q_\pi^{\mathrm{Alg.}}\left(x\right)=\mathcal{N}_q x^2\left(1-x\right)^2$$

- Realistic model (DSE)
[Phys.Rev.D:101(2020)5,054014]

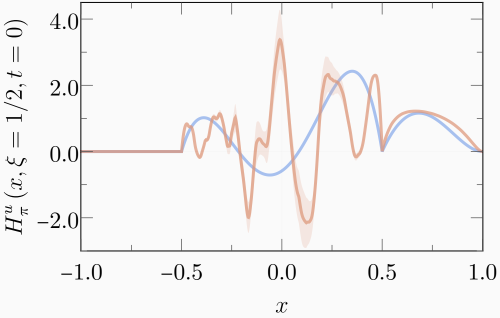
$$q_\pi\left(x\right)=q_\pi^{\mathrm{Alg.}}\left(x\right)\left[1+\gamma x\left(1-x\right)+\rho\sqrt{x\left(1-x\right)}\right]$$

GPD modeling: DSE *Ansätze*

DGLAP GPD



DGLAP+ERBL GPD



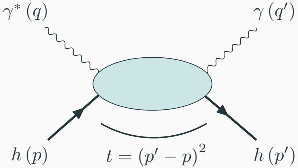
Continuity (but non-differentiability) along the $x = \xi$ line, as expected in DVCS amplitudes

Phenomenology of pion GPDs

Phenomenology of pion GPDs

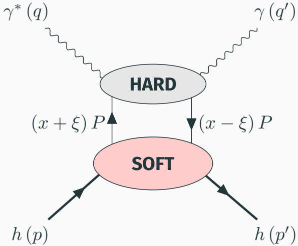
Hadron structure

How do quarks and gluons combine to make hadrons up?



Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

Factorization
[Phys.Rev.D59(1999)074009]



DVCS: $$\mathcal{H}\left(\xi,t;Q^2\right)=\sum_{p=q,g}\int_{-1}^{1}\frac{dx}{\xi}\mathcal{K}^p\left(\frac{x}{\xi},\frac{Q^2}{\mu_F^2},\alpha_s\left(\mu_F^2\right)\right)H^p\left(x,\xi,t;\mu_F^2\right)$$

Hard kernel $\mathcal{K}^p$: perturbative information

Generalized Parton distributions H^p: non perturbative QCD

Phenomenology of pion GPDs: Sullivan process

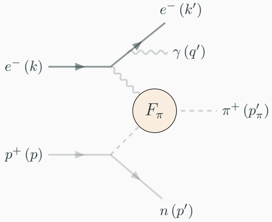
Sullivan process [Phys.Rev.D:5(1972)1732

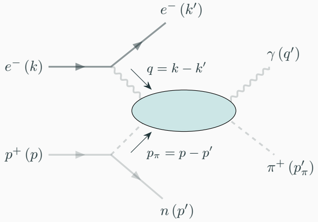
Deep inelastic electron-proton scattering with πn fixed final states.

One-pion-exchange approximation: [Eur.Phys.J.C:58(2008)179]

- $|t|^{\text{Max.}} = 0.6\,\text{GeV}^2$
- Factorization: $\sigma_L^{\gamma^*} >> \sigma_\perp^{\gamma^*}$

} Met at EIC [arXiv[phys.ins-det]2103.05419]





Employed for EFFs
[Phys.Rev.C:78(2008)045203]

Provide access to pion GPDs

Phenomenology of pion GPDs: The path towards DVCS

1. Start from state-of-the-art **models for pion GPDs**.
2. Leading order **scale evolution** for GPDs.
3. Convolution with coefficient functions: **CFFs**.

PARTONS

APFEL++

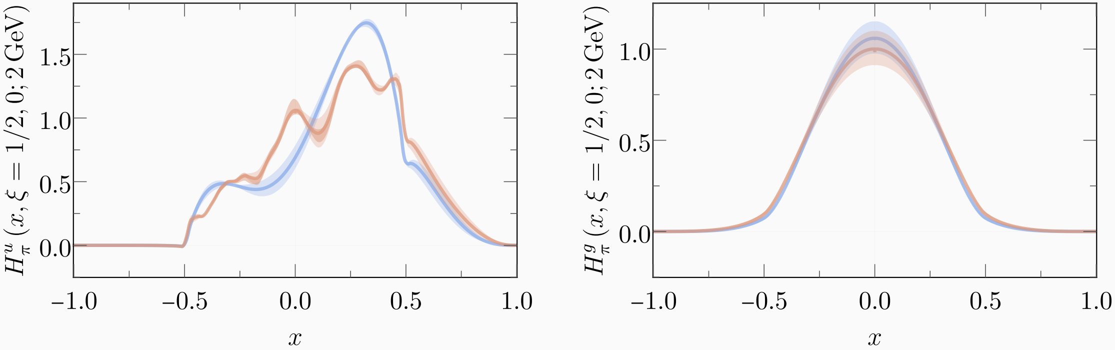
GPD Modeling		QCD evolution		LO/NLO CFFs
[Phys.Lett.B 815 (2021) 136158] [Chin. Phys. C 46 (2022) 1, 013105] [Phys. Rev. D 105 (2022) 9, 094012]	+	[Eur. Phys. J. A 58 (2022) 1, 10] [Phys. Rev. D 105 (2022)9, L091502] [Eur. Phys. J. C 82 (2022) 10, 888]	+	[Phys. Rev. D 105 (2022) 9, 094012] [Phys. Rev. Lett. 128 (2022) 20, 202501]

Phenomenological study of pion's structure

Phys. Rev. Lett. 128 (2022) 20, 202501

Phenomenology of pion GPDs: Scale evolution

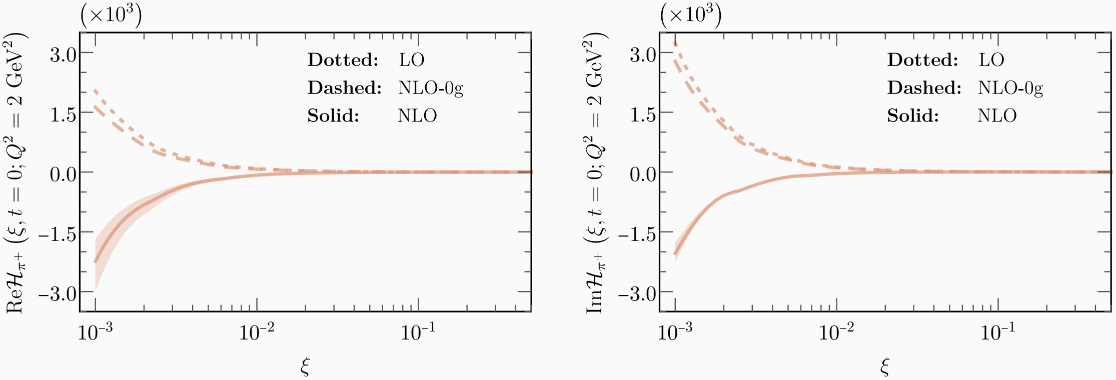
[Phys. Rev. D 105 (2022) 9, 094012]



- Non-zero gluon distribution generated by scale evolution.
- Uncertainty band narrowed.
- Continuity along $x = \xi$ lines.

Phenomenology of pion GPDs: Compton Form Factors

[Phys. Rev. D 105 (2022) 9, 094012]



- Small effect of NLO corrections to quark amplitudes.
- Dominant effect of gluons.

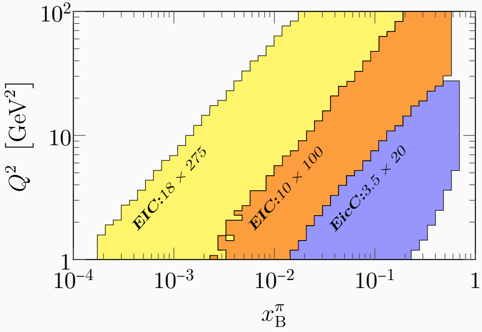
Gluon dominance makes essential at least NLO accuracy in any phenomenlogical analysis of DVCS at an EIC.

Phenomenology of pion GPDs: Generating events

[Phys. Rev. Lett. 128 (2022) 20, 202501]

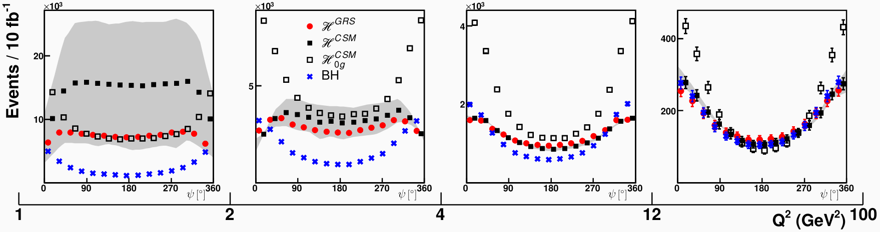
Monte Carlo event generation

- Pick events compatible with detector geometry and performance: EIC and EicC.
  [arXiv[phys.ins-det]2103.05419, arXiv[nucl-ex]2102.09222]
- Add kinematical cuts: [Eur.Phys.J.C:58(2008)179]
  - DVCS kinematics and one pion exchange approximation:
    - ⋆ $s_\pi^{\mathrm{Min}} = 4\,\mathrm{GeV}^2$
    - ⋆ $|t_\pi|^{\mathrm{Max.}} = 0.6\,\mathrm{GeV}^2$
  - Reduce contamination of resonances (Δ): $M_{n\pi}^2 \gtrsim 4\,\mathrm{GeV}^2$
- Integrated one-year luminosity.



Phenomenology of pion GPDs: EIC event-rates

[Phys. Rev. Lett. 128 (2022) 20, 202501]

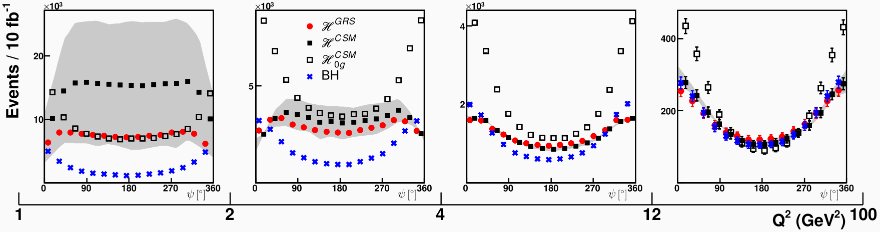


The cross-section can be written as: [Phys.Rev.D:79(2009)014017s]

$$\left|\mathcal{M}_{e\pi\to e\gamma\pi}\right|^2 = C_{\text{BH}}F_\pi^2\left(t_\pi\right) \pm \frac{F_\pi}{Q}\left(C_{\text{int}}\left(\psi\right)\text{Re}\mathcal{H}_\pi + \lambda S_{\text{Int}}\left(\psi\right)\text{Im}\mathcal{H}_\pi\right) + \frac{1}{Q^2}C_{\text{DVCS}}\left|\mathcal{H}_\pi\right|^2$$

Phenomenology of pion GPDs: EIC event-rates

[Phys. Rev. Lett. 128 (2022) 20, 202501]



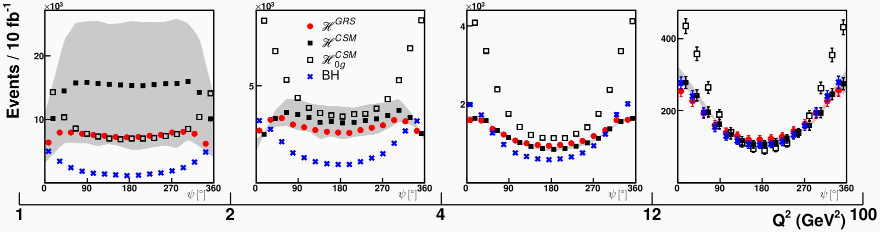
The cross-section can be written as: [Phys.Rev.D:79(2009)014017s]

$$|\mathcal{M}_{e\pi\to e\gamma\pi}|^2 = C_{\rm BH} F_\pi^2(t_\pi) \pm \frac{F_\pi}{Q}\left(C_{\rm int}(\psi)\,{\rm Re}\mathcal{H}_\pi + \lambda S_{\rm Int}(\psi)\,{\rm Im}\mathcal{H}_\pi\right) + \frac{1}{Q^2} C_{\rm DVCS}|\mathcal{H}_\pi|^2$$

$$|\mathcal{H}_\pi|^2 = {\rm Re}^2(\mathcal{H}_\pi) + {\rm Im}^2(\mathcal{H}_\pi) \sim {\rm Re}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - |\mathcal{H}_\pi^g|\right) + {\rm Im}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - |\mathcal{H}_\pi^g|\right)$$

Phenomenology of pion GPDs: EIC event-rates

[Phys. Rev. Lett. 128 (2022) 20, 202501]



The cross-section can be written as: [Phys.Rev.D:79(2009)014017s]

$$\left|\mathcal{M}_{e\pi\to e\gamma\pi}\right|^2 = C_{\rm BH} F_\pi^2\left(t_\pi\right) \pm \frac{F_\pi}{Q}\left(C_{\rm int}\left(\psi\right){\rm Re}\mathcal{H}_\pi + \lambda S_{\rm Int}\left(\psi\right){\rm Im}\mathcal{H}_\pi\right) + \frac{1}{Q^2} C_{\rm DVCS}\left|\mathcal{H}_\pi\right|^2$$

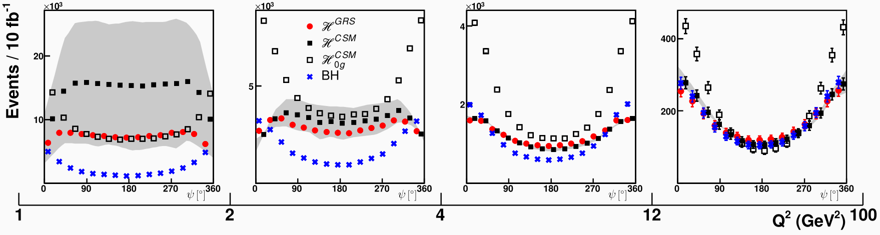
$$\left|\mathcal{H}_\pi\right|^2 = {\rm Re}^2\left(\mathcal{H}_\pi\right) + {\rm Im}^2\left(\mathcal{H}_\pi\right) \sim {\rm Re}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - \left|\mathcal{H}_\pi^{g}\right|\right) + {\rm Im}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - \left|\mathcal{H}_\pi^{g}\right|\right)$$

Low energy, $Q^2 < 2$ GeV^2 — Intermediate energies

$$\left|\mathcal{H}_\pi\right|^2_{Q^2<2\ {\rm GeV}^2} \sim \left|\mathcal{H}_\pi^g\right|^2_{Q^2<2\ {\rm GeV}^2} > \left|\mathcal{H}_\pi^{q,{\rm LO}} - \mathcal{H}_\pi^g\right|^2_{2<Q^2<12\ {\rm GeV}^2} \sim \left|\mathcal{H}_\pi\right|^2_{2<Q^2<12\ {\rm GeV}^2}$$

Phenomenology of pion GPDs: EIC event-rates

[Phys. Rev. Lett. 128 (2022) 20, 202501]



The cross-section can be written as: [Phys.Rev.D:79(2009)014017s]

$$\left|\mathcal{M}_{e\pi\to e\gamma\pi}\right|^2 = C_{\rm BH}F_\pi^2\left(t_\pi\right) \pm \frac{F_\pi}{Q}\left(C_{\rm int}\left(\psi\right){\rm Re}\mathcal{H}_\pi + \lambda S_{\rm Int}\left(\psi\right){\rm Im}\mathcal{H}_\pi\right) + \frac{1}{Q^2}C_{\rm DVCS}\left|\mathcal{H}_\pi\right|^2$$

$$\left|\mathcal{H}_\pi\right|^2 = {\rm Re}^2\left(\mathcal{H}_\pi\right) + {\rm Im}^2\left(\mathcal{H}_\pi\right) \sim {\rm Re}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - \left|\mathcal{H}_\pi^{g}\right|\right) + {\rm Im}^2\left(\left|\mathcal{H}_\pi^{q,{\rm LO}}\right| - \left|\mathcal{H}_\pi^{g}\right|\right)$$

Low energy, $Q^2 < 2$ GeV^2 — Intermediate energies

Quark-gluon "interference": Modulates expected number of events

Summary and perspectives

Summary and perspectives

1. *Can we obtain "theoretically complete" pion GPDs?* **YES!**
    - Positivity: "Factorization hypothesis"
    - Polynomiality: Covariant extension
    - Support and soft-pion theorem
    - Continuity along $x = \xi$
2. *What can we learn from them about actual pions?*
    - GPDs might be accessible in future experiments
    - Crucial role of gluons within pions
        - Modulate expected statistics
        - Manifest themselves through beam-spin asymmetries

Summary and perspectives

1. ***Can we obtain "theoretically complete" pion GPDs?*** **YES!**
   - Positivity: "Factorization hypothesis"
   - Polynomiality: Covariant extension
   - Support and soft-pion theorem
   - Continuity along $x = \xi$
2. ***What can we learn from them about actual pions?***
   - GPDs might be accessible in future experiments
   - Crucial role of gluons within pions
     - Modulate expected statistics
     - Manifest themselves through beam-spin asymmetries

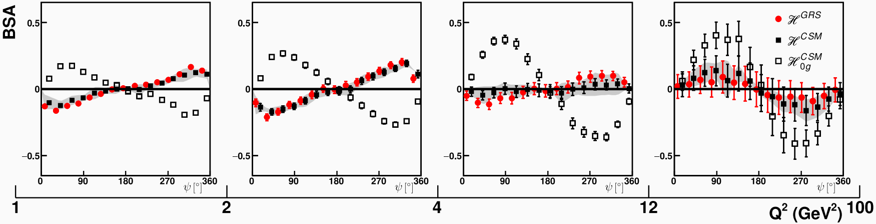
Future work

- "Incomplete" covariant extension: Trigger Lattice efforts (**In preparation**)
- Refine evolution
- Neutral pions: Direct window to GPDs?

Thank you!

Phenomenology of pion GPDs: EIC beam-spin asymmetries

[Phys. Rev. Lett. 128 (2022) 20, 202501]

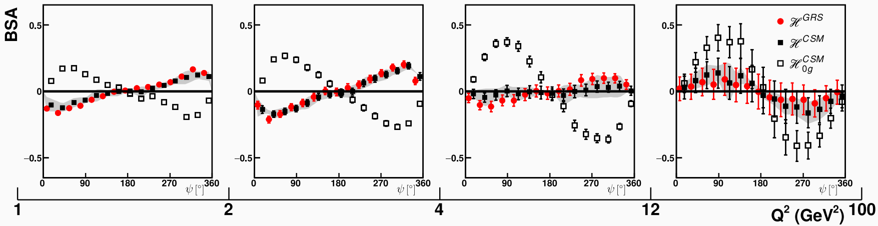


$$\mathcal{A}(\psi) = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \sim s^{1}_{\mathrm{Int}} \mathrm{Im}\mathcal{H}_{\pi} \sin\psi$$

$$\mathrm{Im}\left(\mathcal{H}_{\pi}\right) = \mathrm{Im}\left(\mathcal{H}_{\pi}^{q,\mathrm{LO}}\right) + \mathrm{Im}\left(\mathcal{H}_{\pi}^{q,\mathrm{NLO}}\right) + \mathrm{Im}\left(\mathcal{H}_{\pi}^{g}\right) \sim \mathrm{Im}\left|\mathcal{H}_{\pi}^{q,\mathrm{LO}}\right| - \mathrm{Im}\left|\mathcal{H}_{\pi}^{g}\right|$$

Phenomenology of pion GPDs: EIC beam-spin asymmetries

[Phys. Rev. Lett. 128 (2022) 20, 202501]



$$\mathcal{A}(\psi) = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \sim s^{1}_{\mathrm{Int}} \mathrm{Im}\mathcal{H}_{\pi} \sin\psi$$

$$\mathrm{Im}\left(\mathcal{H}_{\pi}\right) = \mathrm{Im}\left(\mathcal{H}_{\pi}^{q,\mathrm{LO}}\right) + \mathrm{Im}\left(\mathcal{H}_{\pi}^{q,\mathrm{NLO}}\right) + \mathrm{Im}\left(\mathcal{H}_{\pi}^{g}\right) \sim \mathrm{Im}\left|\mathcal{H}_{\pi}^{q,\mathrm{LO}}\right| - \mathrm{Im}\left|\mathcal{H}_{\pi}^{g}\right|$$

Sign inversion: **Smoking gun** for gluon dominance

GPD modeling: Covariant extension

$$H\left(x,\xi,t\right)=\int_{\Omega}d\beta d\alpha\delta\left(x-\beta-\alpha\xi\right)h\left(\beta,\alpha,t\right)+\frac{1}{|\xi|}D^{+}\left(\frac{x}{\xi},t\right)+sign\left(\xi\right)D^{-}\left(\frac{x}{\xi},t\right)$$

- DGLAP GPDs receive **NO** D-term contributions.
- The inverse Radon transform exists, is unique and can be evaluated.
- Soft-pion theorem tames D-term contributions to the ERBL region.

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

GPD modeling: Covariant extension

$$H(x,\xi,t) = \int_\Omega d\beta d\alpha \delta(x-\beta-\alpha\xi)\, h(\beta,\alpha,t) + \frac{1}{|\xi|} D^+\left(\frac{x}{\xi},t\right) + sign(\xi)\, D^-\left(\frac{x}{\xi},t\right)$$

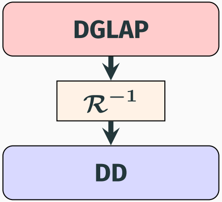
DGLAP

- DGLAP GPDs receive **NO** D-term contributions.
- The inverse Radon transform exists, is unique and can be evaluated.
- Soft-pion theorem tames D-term contributions to the ERBL region.

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

GPD modeling: Covariant extension

$$H(x,\xi,t) = \int_\Omega d\beta d\alpha \delta(x-\beta-\alpha\xi)\, h(\beta,\alpha,t) + \frac{1}{|\xi|} D^+\left(\frac{x}{\xi},t\right) + sign(\xi)\, D^-\left(\frac{x}{\xi},t\right)$$

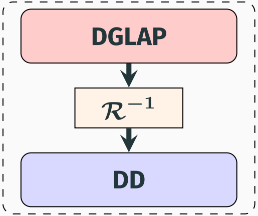


- DGLAP GPDs receive **NO** D-term contributions.
- The inverse Radon transform exists, is unique and can be evaluated.
- Soft-pion theorem tames D-term contributions to the ERBL region.

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

GPD modeling: Covariant extension

$$H\left(x,\xi,t\right)=\int_{\Omega}d\beta d\alpha\delta\left(x-\beta-\alpha\xi\right)h\left(\beta,\alpha,t\right)+\frac{1}{|\xi|}D^{+}\left(\frac{x}{\xi},t\right)+sign\left(\xi\right)D^{-}\left(\frac{x}{\xi},t\right)$$

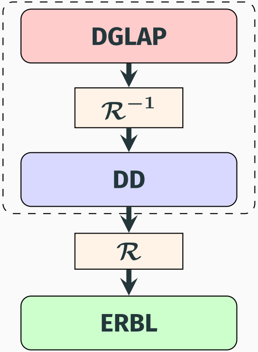


- DGLAP GPDs receive **NO** D-term contributions.
- The inverse Radon transform exists, is unique and can be evaluated.
- Soft-pion theorem tames D-term contributions to the ERBL region.

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

GPD modeling: Covariant extension

$$H\left(x,\xi,t\right)=\int_{\Omega}d\beta d\alpha\delta\left(x-\beta-\alpha\xi\right)h\left(\beta,\alpha,t\right)+\frac{1}{|\xi|}D^{+}\left(\frac{x}{\xi},t\right)+sign\left(\xi\right)D^{-}\left(\frac{x}{\xi},t\right)$$



- DGLAP GPDs receive **NO** D-term contributions.
- The inverse Radon transform exists, is unique and can be evaluated.
- Soft-pion theorem tames D-term contributions to the ERBL region.

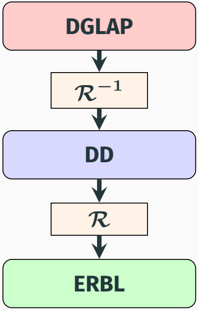
GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

Covariant extension: Allows to overcome challenges of known modeling strategies.
Input: (part of) **DGLAP GPD**

Covariant extension

Covariant extension: Given a DGLAP GPD, the covariant extension allows for computing the corresponding ERBL GPD such that polynomiality is satisfied.[Eur.Phys.J.C:77(2017)12,906]

$$H(x,\xi) = \mathcal{R}\left[h(\beta,\alpha)\right] + \frac{1}{|\xi|} D^{+}\left(\frac{x}{\xi}\right) + sign(\xi)\, D^{-}\left(\frac{x}{\xi}\right)$$



1. Build positive DGLAP GPD

2. Covariant extension: ERBL GPD

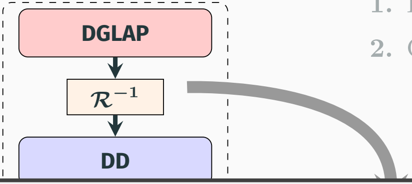
GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

3. Soft pion theorem: fix $D^{\pm}(\alpha, 0)$

Covariant extension

Covariant extension: Given a DGLAP GPD, the covariant extension allows for computing the corresponding ERBL GPD such that polynomiality is satisfied. [Eur.Phys.J.C:77(2017)12,906]

$$H(x,\xi) = \mathcal{R}\left[h(\beta,\alpha)\right] + \frac{1}{|\xi|} D^{+}\left(\frac{x}{\xi}\right) + sign(\xi)\, D^{-}\left(\frac{x}{\xi}\right)$$



1. Build positive DGLAP GPD
2. Covariant extension: ERBL GPD

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓

$$H(x,\xi)\big|_{|x|\geq\xi} = \mathcal{R}\left[h(\beta,\alpha)\right] \Rightarrow h(\beta,\alpha) = \mathcal{R}^{-1}\left[H(x,\xi)\right]$$

Can we evaluate the inverse Radon transform?

Covariant extension

Question: *Can we evaluate the inverse Radon transform?*

$$H\left(x,\xi\right)\big|_{|x|\geq|\xi|} = \int_\Omega d\beta d\alpha \delta\left(x-\beta-\alpha\xi\right) h\left(\beta,\alpha\right)$$

Uniqueness theorem

(Boman and Todd-Quinto [D.Math.J.:943(55)1987, Eur.Phys.J.C:77(2017)12,906])

If $H\left(x,\xi\right) = 0 \; \forall \left(x,\xi\right) \in [-1,1] \otimes [-1,1] \,/\, |x| \geq |\xi|$

Then, $h\left(\beta,\alpha\right) = 0 \; \forall \left(\beta \neq 0, \alpha\right) \in \Omega$

Covariant extension

Question: *Can we evaluate the inverse Radon transform?*

$$H\left(x,\xi\right)\big|_{|x|\geq|\xi|}=\int_\Omega d\beta d\alpha\delta\left(x-\beta-\alpha\xi\right)h\left(\beta,\alpha\right)$$

Uniqueness theorem

(Boman and Todd-Quinto [D.Math.J.:943(55)1987, Eur.Phys.J.C:77(2017)12,906])

If $H\left(x,\xi\right)=0\ \forall\left(x,\xi\right)\in\left[-1,1\right]\otimes\left[-1,1\right]/\left|x\right|\geq\left|\xi\right|$

Then, $h\left(\beta,\alpha\right)=0\ \forall\left(\beta\neq0,\alpha\right)\in\Omega$

Although it exists, $\mathcal{R}^{-1}$ is a non-continuous operator:

ILL-POSED PROBLEM (HADAMARD)

Numerical solution of the inverse Radon transform

[Phys. Rev. D 105 (2022) 9, 094012]

GPD modeling: Factorized LFWFs

1. Overlap representation [Nucl.Phys.B596(2001)33]

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \int \frac{d^2k_\perp}{16\pi^3}\Psi^{q*}\left(x_-,k^2_{\perp,-}\right)\Psi^q\left(x_+,k^2_{\perp,+}\right)$$

2. Assume factorization of the LFWF

$$\Psi^q\left(x,k_\perp^2\right) \propto \varphi(x)\,\phi\left(k_\perp^2\right)$$

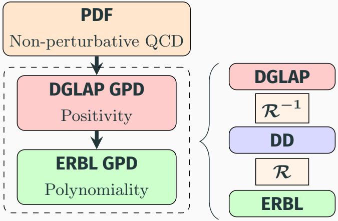
↓ (Overalp rep.)

$$H^q(x,\xi,t)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}\,\Phi(x,\xi,t)$$

↓ $(t=0)$

$$H^q(x,\xi,0)\big|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right)q\left(\frac{x+\xi}{1+\xi}\right)}$$

[Phys. Rev. D 105 (2022) 9, 094012]



PDFs are the sole ingredient needed to build positive DGLAP GPD

Scale evolution

[Eur. Phys. J. C 82 (2022) 10, 888]

$$H\left(x,\xi,t;\mu^2\right) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle p' \right| \mathcal{O}\left(-\lambda n, \lambda n\right) \left| p \right\rangle$$

Light-cone operator product expansion:

$$\mathcal{O}\left(-\lambda n, \lambda n\right) \xrightarrow{n^2=0} \sum_i c_i n^{2i} \mathcal{O}_i$$

Renormalization group equations for the operators $\mathcal{O}_i$ yield scale evolution for GPDs:

[Phys.Rev.D:55(1997)7114, Eur.Phys.J.C:82(2022)10,888]

$$\frac{dH^{(\pm)}\left(x,\xi,t;\mu^2\right)}{d\log\mu^2} = \frac{\alpha_s\left(\mu^2\right)}{4\pi} \int_x^\infty \frac{dy}{y} \mathcal{P}^{(\pm)}\left(y,\kappa,\alpha_S\left(\mu^2\right)\right) H^{(\pm)}\left(\frac{x}{y},\xi,t;\mu^2\right)$$

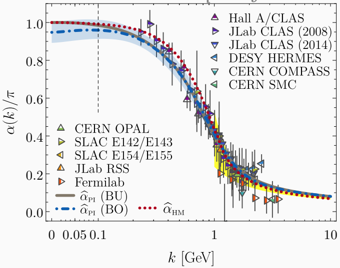
“Splitting functions”: $\mathcal{P}^{(\pm)}\left(y,\kappa;\alpha_S\left(\mu^2\right)\right) = \sum_{n=1}^{\infty} \frac{\alpha_S\left(\mu^2\right)}{4\pi} \mathcal{P}_n^{(\pm)}\left(y,\kappa\right) \;\; ; \;\; \kappa = \frac{\xi}{x}$

Scale evolution: Hadron scale

Reference scale: $\mu_{\text{Ref.}} = 331$ MeV

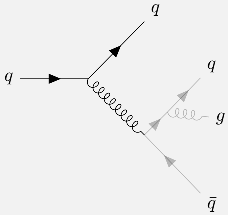
No gluons nor sea quarks

[J.Phys.Conf.Ser.1643(2020)1,0



Parton splitting (DGLAP)

[Nucl.Phys.B:126(1977)298]



Renormalization group equations for GPD operators yield:

[Phys.Rev.D:55(1997)7114, Eur.Phys.J.C:82(2022)10,888]

$$\frac{dH^{(\pm)}\left(x,\xi,t;\mu^2\right)}{d\log\mu^2} = \frac{\alpha_s\left(\mu^2\right)}{4\pi}\int_x^\infty \frac{dy}{y}\mathcal{P}^{(\pm)}\left(y,\kappa,\alpha_S\left(\mu^2\right)\right)H^{(\pm)}\left(\frac{x}{y},\xi,t;\mu^2\right)$$

Scale evolution: PDFs