

June 27th, 2023
Prague

International Workshop on Hadron Structure and Spectroscopy 2023

Modeling spin effects in electron-positron annihilation to hadrons

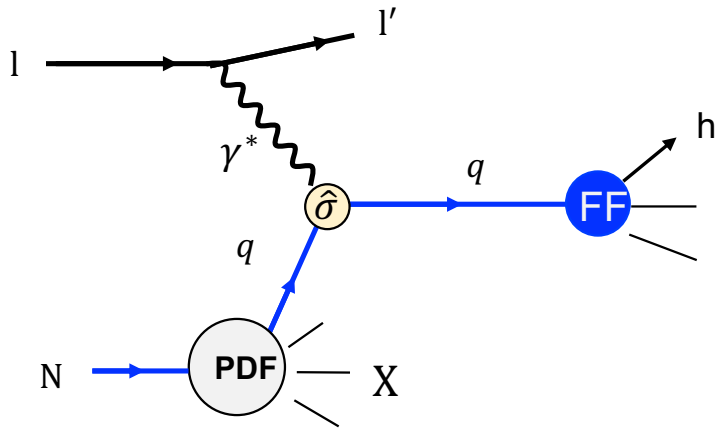
Albi Kerbizi

University of Trieste and INFN Trieste

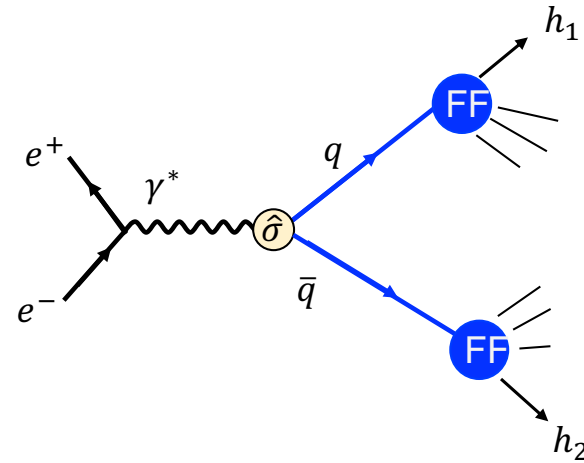
work done in the context of the POLFRAG project



Introduction: nucleon structure and hadronization

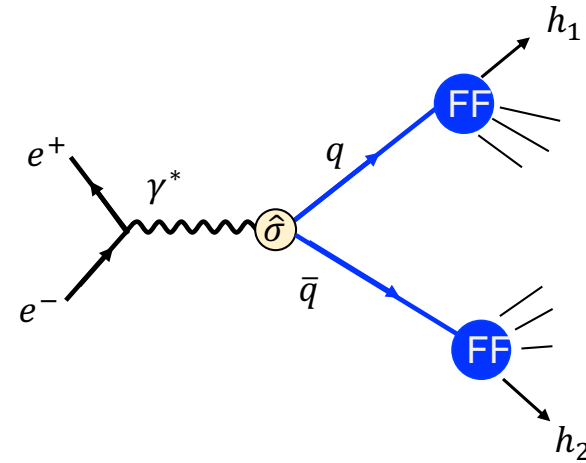
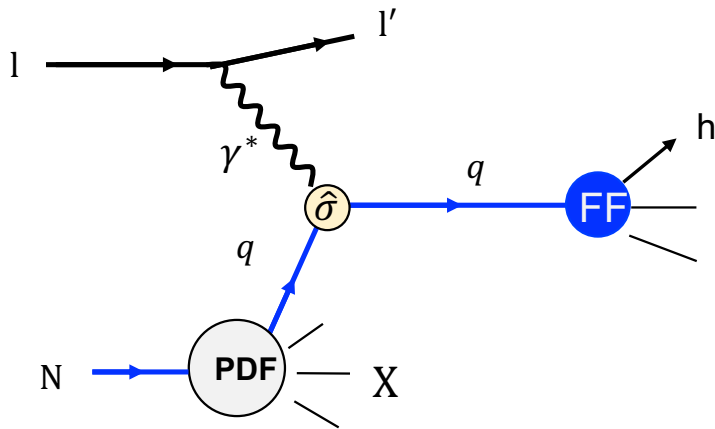


SIDIS \rightarrow access to nucleon structure
via convolutions of PDFs and FFs



e^+e^- annihilation to hadrons \rightarrow access to FFs

Introduction: nucleon structure and hadronization



SIDIS → access to nucleon structure via convolutions of PDFs and FFs

e^+e^- annihilation to hadrons → access to FFs

Combined to extract information on the transverse spin structure of nucleons e.g., transversity (but also Boer-Mulders TMD,..)

2h channel → talk of A. Metz

Example: Collins asymmetry in SIDIS

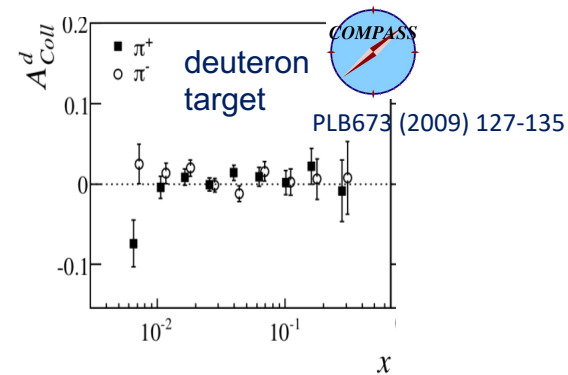
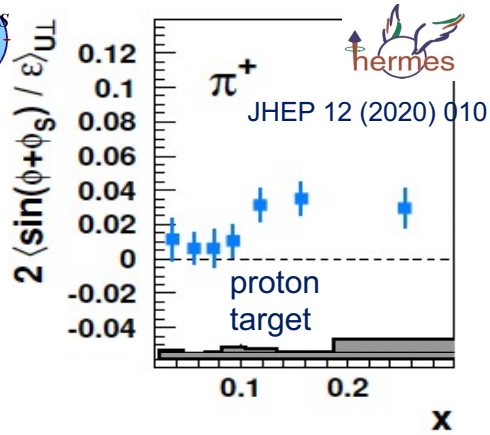
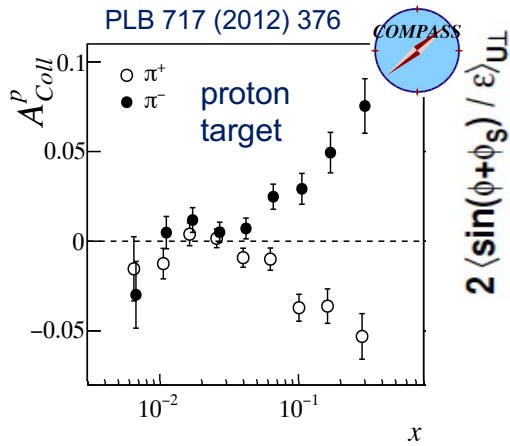
$$A_{UT}^{\sin \phi_h + \phi_S - \pi} = \frac{\sum_q e_q^2 \overset{\text{transversity}}{h_1^q} \otimes \overset{\text{Collins FF}}{H_{1q}^{\perp h}}}{\sum_q e_q^2 f_1^q \otimes D_{1q}^h} \quad \text{Collins, NPB 396, 161 (1993).}$$

Collins asymmetry in e^+e^-

$$A_{12}^{UL} = \frac{\sum_q e_q^2 H_{1q}^{\perp h_1} H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 D_{1q}^{h_1} D_{1\bar{q}}^{h_2}}$$

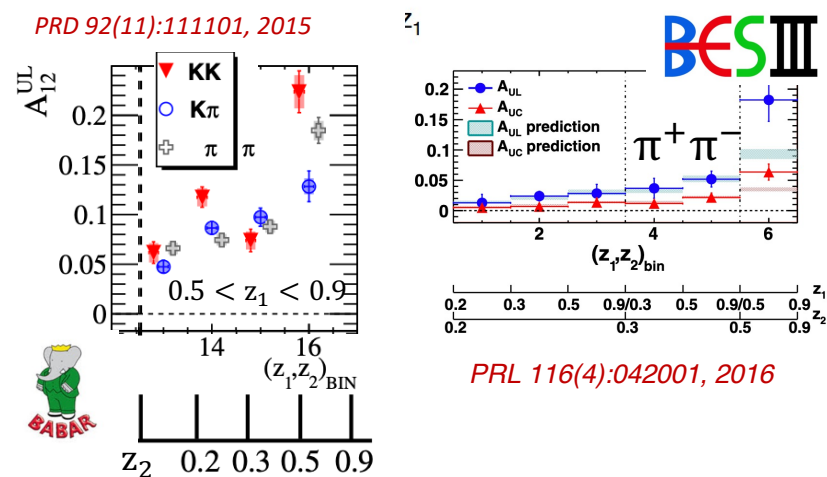
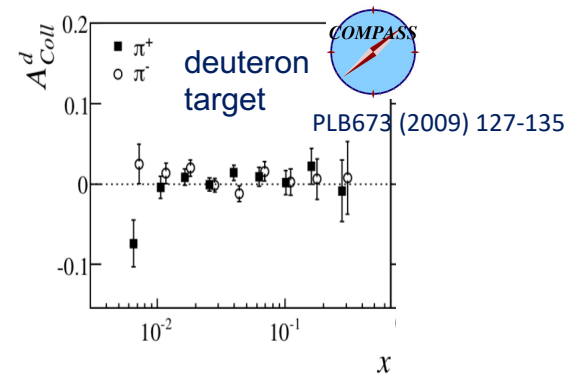
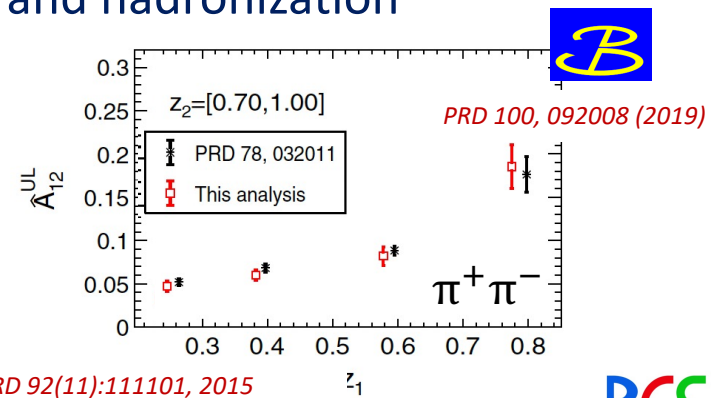
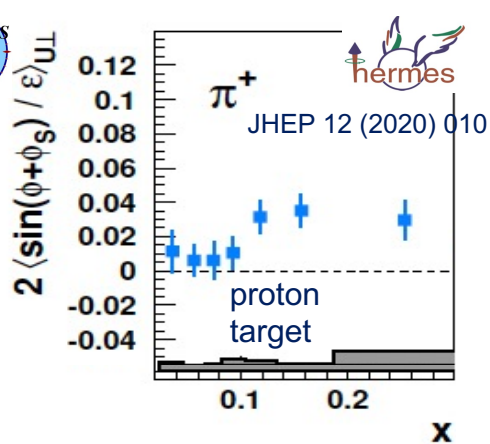
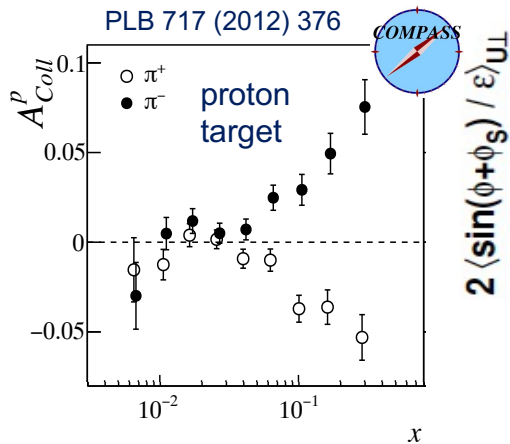
Boer, NPB, 806:23–67, 2009
D'Alesio et al., JHEP 10 (2021) 078 3

Introduction: nucleon structure and hadronization



Many measurements in SIDIS
HERMES (p), COMPASS (p,d), Jlab (n)

Introduction: nucleon structure and hadronization



Many measurements in SIDIS
HERMES (p), COMPASS (p,d), Jlab (n)

Different measurements in e^+e^-
Belle, BaBar, BESIII

Used to extract transversity (and Collins FF) by different groups
Anselmino et al, PRD 92 (11) (2015) 114023
Martin et al., PRD 91(1):014034, 2015
Kang et al., PRD 93 (1) (2016) 014009
...
But also a benchmark for hadronization models!

Modeling hadronization: the string+ 3P_0 model

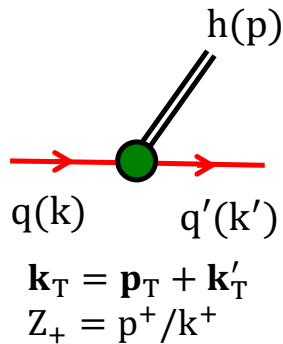
We have developed a model for the simulation of the fragmentation polarized quarks

→ string+ 3P_0 model

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018) PS

AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019) PS

AK, Artru, Martin, PRD 104, 114038 (2021) PS + VM



Quark splitting described by a 2x2 splitting amplitude

$$T_{q',h,q} \propto \left[F_{q',h,q}^{\text{Lund}}(Z_+, \mathbf{p}_T; \mathbf{k}_T) \right]^{1/2} [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \Gamma_{h,S_h}$$

3P_0 mechanism
 μ complex mass parameter
 Coupling
 e.g. $\Gamma_{h=PS} = \sigma_z$

$\text{Im}(\mu) \rightarrow$ T spin effects (Collins, dihadron)

$\text{Im}(\mu) \rightarrow$ L spin effects (jet handedness)

Modeling hadronization: the string+ 3P_0 model

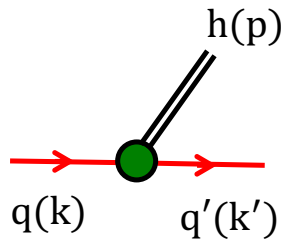
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$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$

$$Z_+ = p^+ / k^+$$

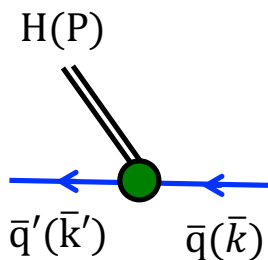
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3P_0 mechanism Coupling
 μ complex mass parameter e.g.
 $\Gamma_{h=PS} = \sigma_z$

$\text{Im}(\mu) \rightarrow$ T spin effects (Collins, dihadron)

$\text{Im}(\mu) \rightarrow$ L spin effects (jet handedness)



$$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$$

$$Z_- = P^- / \bar{k}^-$$

For anti-quark splitting

$$\{q, h, q'\} \rightarrow \{\bar{q}, H, \bar{q}'\}, \quad Z_+ \rightarrow Z_-, \quad \{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \rightarrow \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

Modeling hadronization: the string+ 3P_0 model

We have developed a model for the simulation of the fragmentation polarized quarks

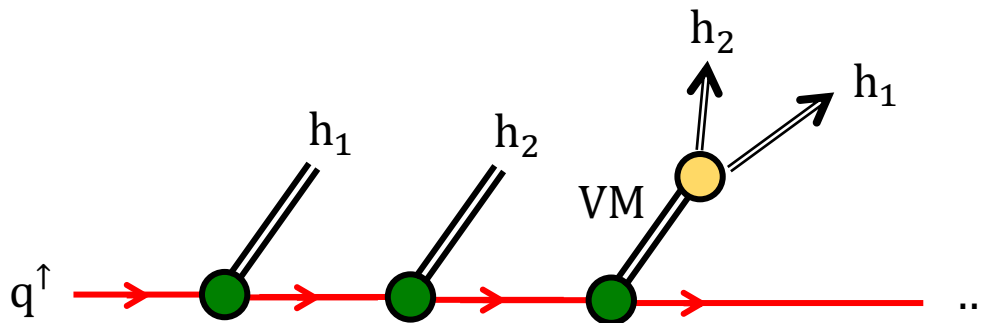
→ string+ 3P_0 model

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Used to describe the fragmentation of (transversely) polarized quarks in SIDIS



Modeling hadronization: the string+ 3P_0 model

We have developed a model for the simulation of the fragmentation polarized quarks

→ string+ 3P_0 model

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018) PS

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AK, Artru, Martin, PRD 104, 114038 (2021) PS + VM

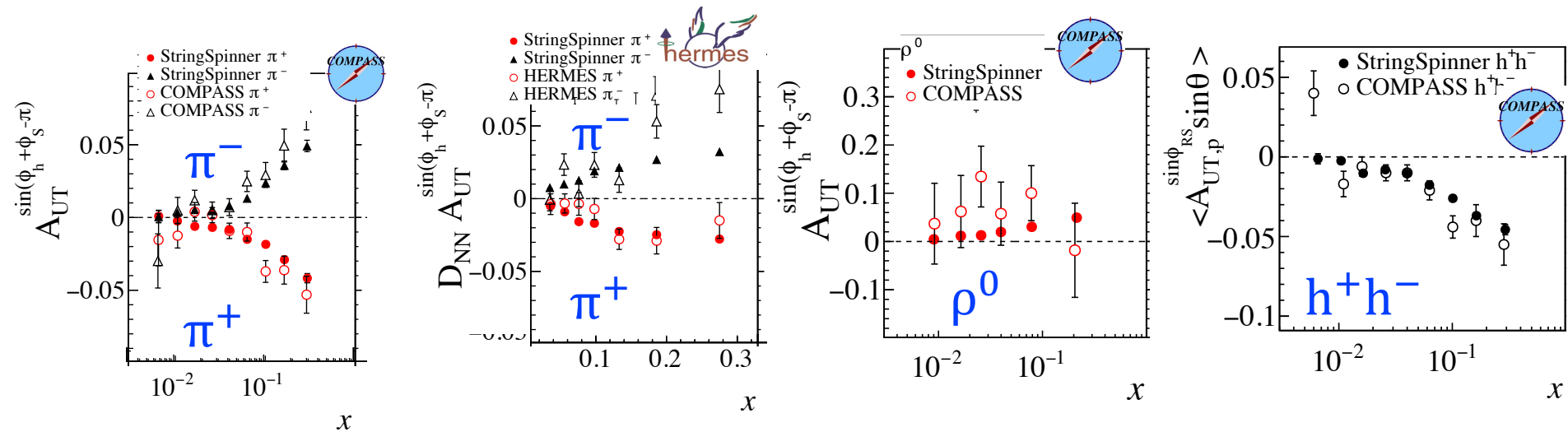
Implemented in Pythia for SIDIS → StringSpinner

AK, L. Lönnblad, CPC **272** (2022) 108234

PS, Pythia 8.2

AK, L. Lönnblad, arXiv: 2305.05058

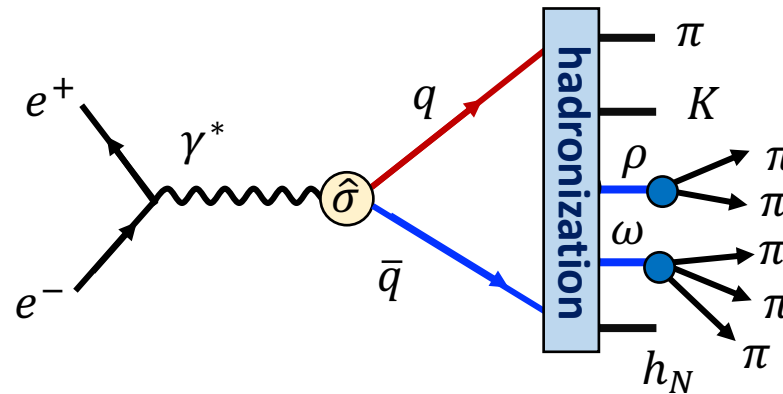
PS + VM, Pythia 8.3



Promising results for SIDIS! (more in AK, L. Lönnblad, arXiv: 2305.05058)

Can we use the model for e^+e^- ?

The recursive recipe for simulating e^+e^- annihilation



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from \bar{q}
6. Exit condition

The goal is to hadronize the $q\bar{q}$ system by using the string+ 3P_0 model and accounting for

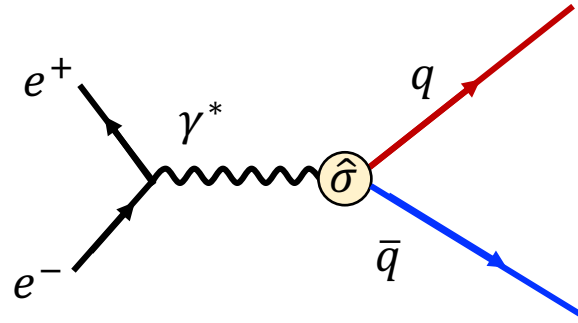
- i) correlated spin states of q and \bar{q}
- ii) quantum mechanical spin-correlations in the fragmentation chain

in collaboration with X. Artru

The recursive recipe for simulating e^+e^- annihilation

Steps:

1. **Hard scattering**
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from \bar{q}
6. Exit condition

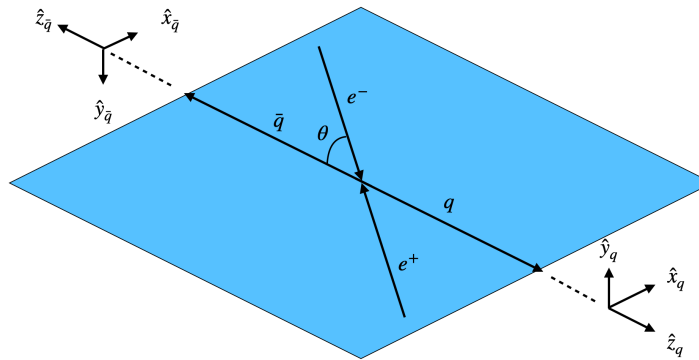


Set up the scattering $e^+e^- \rightarrow q\bar{q}$ in the c.m.s
generate the quark flavors and kinematics using

$$d\hat{\sigma}(q\bar{q})/d\cos\theta \propto \langle |\hat{M}|^2 \rangle$$

antiquark helicity frame
(AHF)

$$\begin{aligned}\hat{\mathbf{z}}_q &\propto \mathbf{k}_q \\ \hat{\mathbf{y}}_q &\propto \mathbf{p}_- \times \hat{\mathbf{z}}_q \\ \hat{\mathbf{x}}_q &= \hat{\mathbf{z}}_q \times \hat{\mathbf{y}}_q\end{aligned}$$



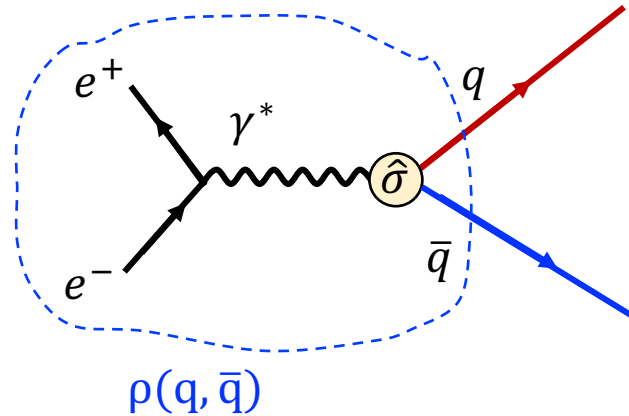
quark helicity frame
(QHF)

$$\begin{aligned}\hat{\mathbf{z}}_q &\propto \mathbf{k}_q \\ \hat{\mathbf{y}}_q &\propto \mathbf{p}_- \times \hat{\mathbf{z}}_q \\ \hat{\mathbf{x}}_q &= \hat{\mathbf{z}}_q \times \hat{\mathbf{y}}_q\end{aligned}$$

The recursive recipe for simulating e^+e^- annihilation

Steps:

1. Hard scattering
- 2. Joint spin density matrix**
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from \bar{q}
6. Exit condition



Set up the joint spin density matrix of the $q\bar{q}$ pair

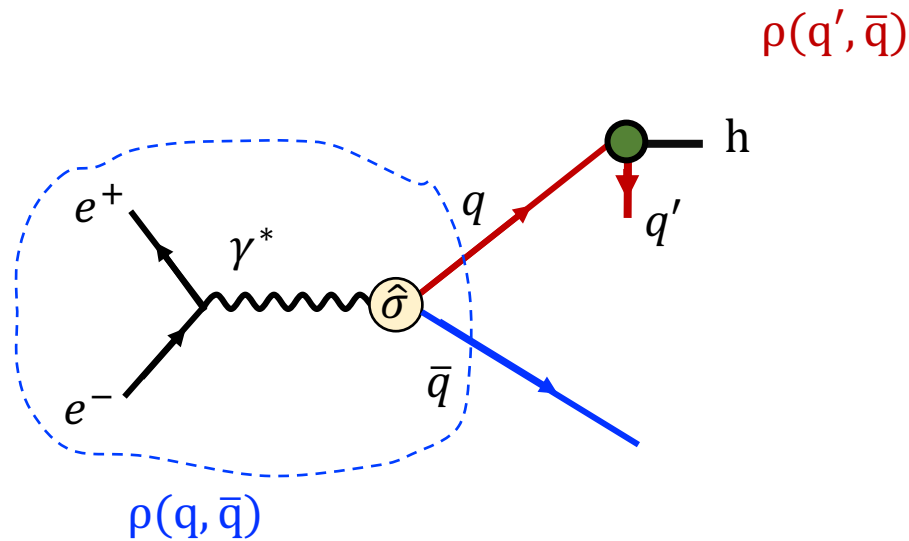
$$\rho(q, \bar{q}) = C_{\alpha\beta}^{q\bar{q}} \sigma_q^\alpha \otimes \sigma_{\bar{q}}^\beta$$

correlation coefficients
 Pauli matrices along QHF and AHF
 $\alpha = 0, x_q, y_q, z_q$
 $\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q, \bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma_q^z \otimes \sigma_{\bar{q}}^z + \frac{\sin^2\theta}{1+\cos^2\theta} [\sigma_q^x \otimes \sigma_{\bar{q}}^x + \sigma_q^y \otimes \sigma_{\bar{q}}^y]$$

The recursive recipe for simulating e^+e^- annihilation



Steps:

1. Hard scattering
2. Joint spin density matrix
- 3. Hadron emission from q**
4. Update density matrix
5. Hadron emission from \bar{q}
6. Exit condition

Emit the first hadron using the splitting matrix of the string+ 3P_0 model

splitting function (emission probability density)

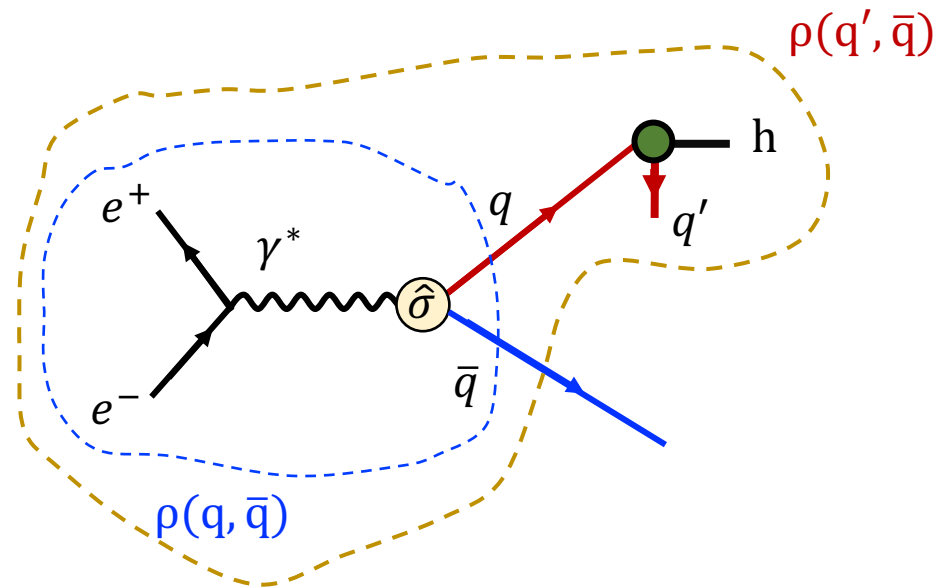
$$\frac{dP(q \rightarrow h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2p_T} = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q} \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^\dagger = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv \mathbf{T}_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

For VM emission see backup

The recursive recipe for simulating e^+e^- annihilation



Steps:

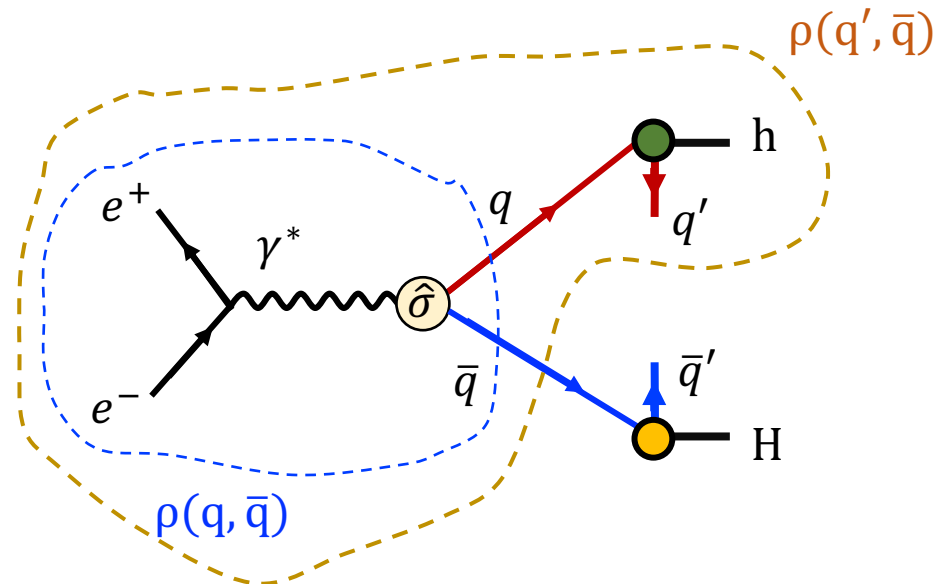
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
- 4. Update density matrix**
5. Hadron emission from \bar{q}
6. Exit condition

Evaluate the spin density matrix $\rho(q', \bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^\dagger$$

includes the information on the emission of h

The recursive recipe for simulating e^+e^- annihilation



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
- 5. Hadron emission from \bar{q}**
6. Exit condition

Emit a hadron from the \bar{q} side using the **splitting function** (emission probability density)

$$\frac{dP(\bar{q} \rightarrow H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2P_T} = \text{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}', H, \bar{q}} \rho(q', \bar{q}) \mathbf{T}_{\bar{q}', H, \bar{q}}^\dagger = F_{\bar{q}', H, \bar{q}}(Z_-, P_T; \bar{\mathbf{k}}_T, \mathbf{C}^{q'\bar{q}})$$

conditional probability of emitting H, having emitted h
 → correlations between their transverse momenta

Depend on the azimuthal angle h

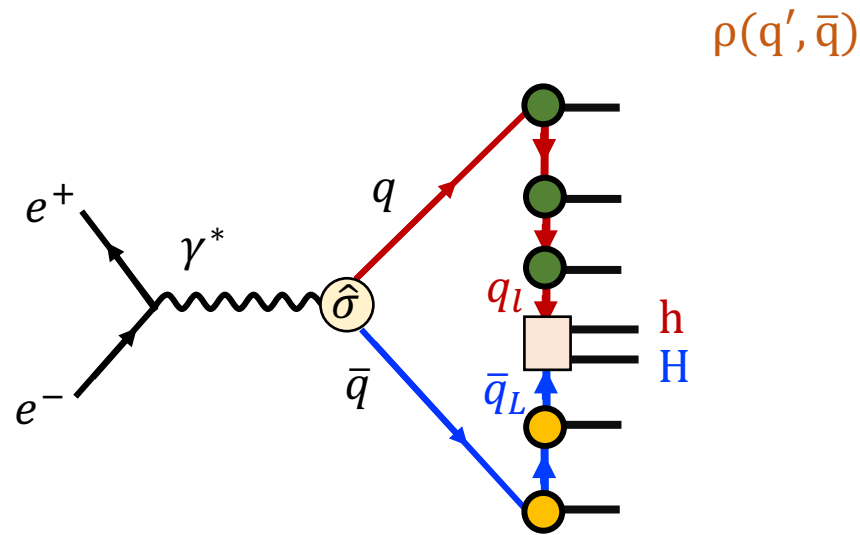


Expressed in the AHF



[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

The recursive recipe for simulating e^+e^- annihilation



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from \bar{q}
6. **Exit condition**

After several emissions, hadronize the last pair $q_l \bar{q}_L$
 emit the hadron $h = q_l \bar{q}'$ from q_l and project $\bar{q}_L q'$ to the state H

$$dP(q_l \rightarrow h + q'; q_l \bar{q}_L) = \text{Tr}_{q' \bar{q}_L} [T_{q',h,q_l} \otimes \Gamma_{H,S_H}] \quad \rho(q_l, \bar{q}_L) \quad [T_{q',h,q_l}^\dagger \otimes \Gamma_{H,S_H}^\dagger]$$

or emit the hadron $H = q' \bar{q}_L$ from \bar{q}_L and project $q_l q'$ to the state h

The model can be shown analytically to reproduce the expected form of the $e^+e^- \rightarrow h_1h_2X$ cross section see backup

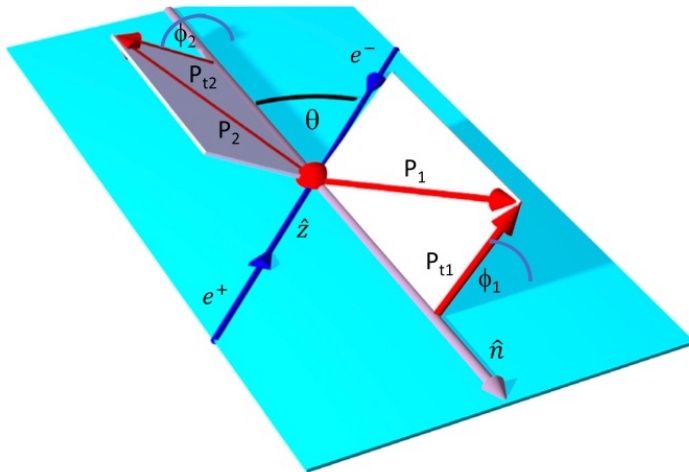
For quantitative results and phenomenology → [implementation in Pythia 8.3](#) for e^+e^-

ongoing work, in collaboration with A. Martin and L. Lönnblad

Simulations of e^+e^- with Pythia 8.3

We are extending the StringSpinner package to e^+e^-

Next slides → preliminary results on Collins asymmetries for back-to-back hadrons



Simulations (caveats)

$$e^+e^- \rightarrow q\bar{q} \rightarrow h_1 h_2 X @ \sqrt{s} = 10 \text{ GeV}, \theta = \frac{\pi}{2}$$

only PS meson production

$$q\bar{q} = u\bar{u}$$

$q\bar{q}$ axis instead of the thrust axis

complex mass as in CPC **272** (2022) 108234

(to account for the absence of VMs)

Steps for the extraction of Collins asymmetries

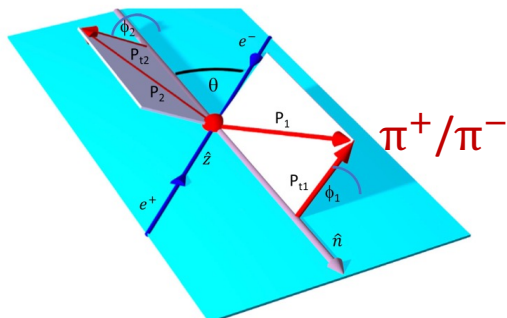
Example of $e^+e^- \rightarrow \pi^+\pi^-X$

i) Evaluate normalized yields for

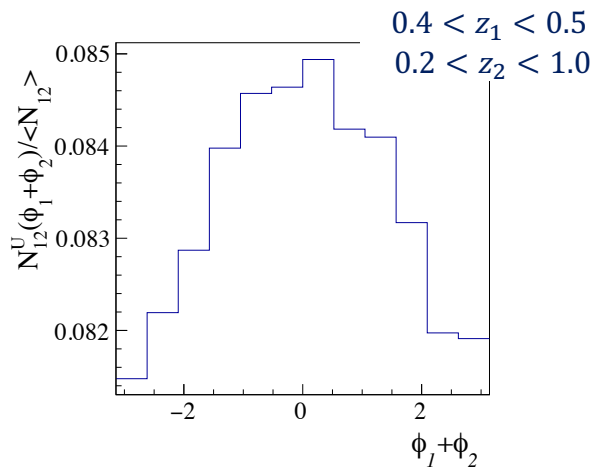
$\pi^\pm - \pi^\mp$ “Unlike pairs”

$$R_{12}^U = \frac{N_{12}^U(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

π^-/π^+



π^+/π^-

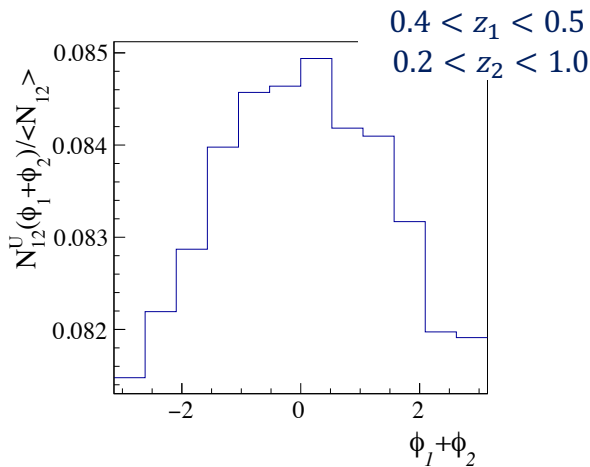
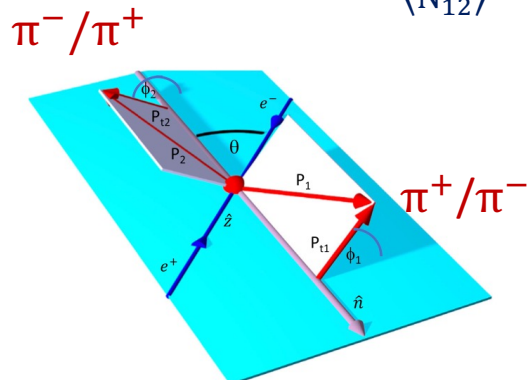


Steps for the extraction of Collins asymmetries

Example of $e^+e^- \rightarrow \pi^+\pi^-X$

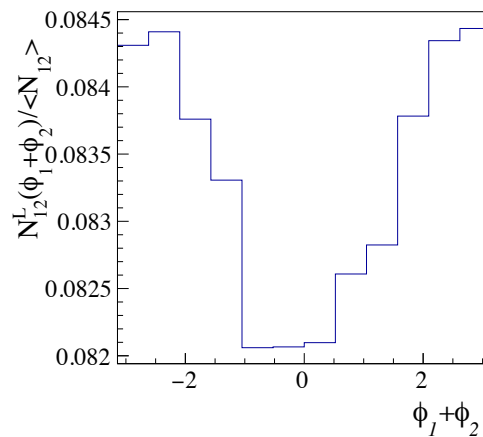
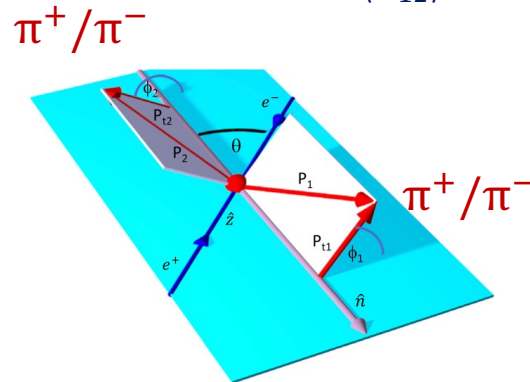
i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ “Unlike pairs”

$$R_{12}^U = \frac{N_{12}^U(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$



ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ “Like pairs”

$$R_{12}^L = \frac{N_{12}^L(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

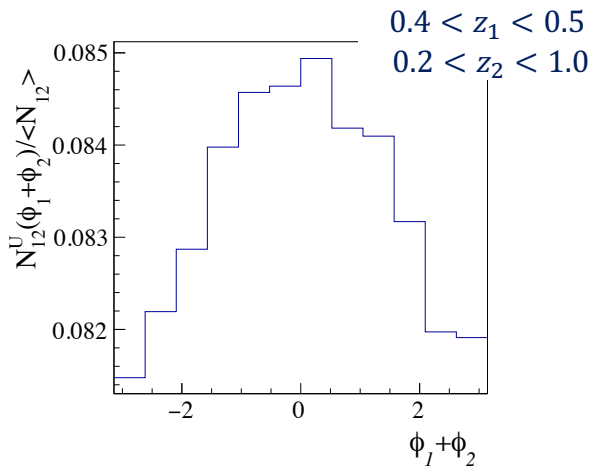
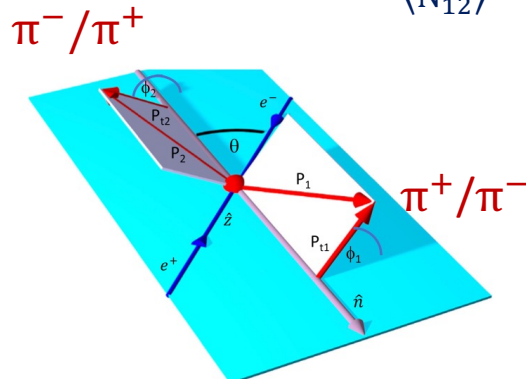


Steps for the extraction of Collins asymmetries

Example of $e^+e^- \rightarrow \pi^+\pi^-X$

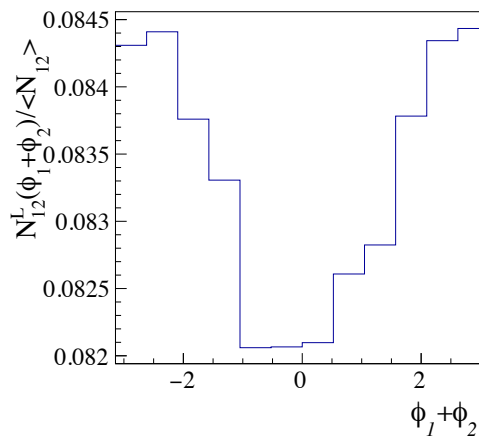
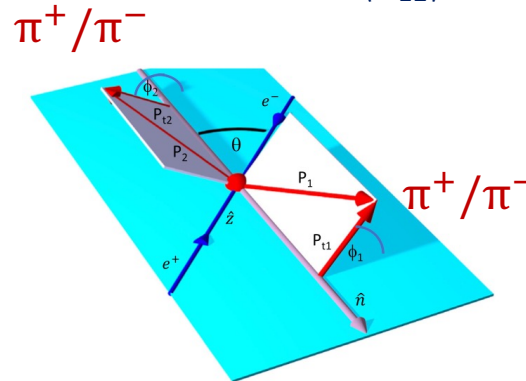
i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ “Unlike pairs”

$$R_{12}^U = \frac{N_{12}^U(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$



ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ “Like pairs”

$$R_{12}^L = \frac{N_{12}^L(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

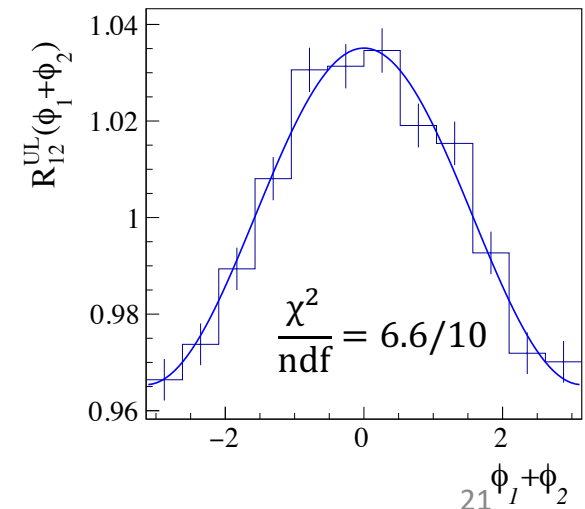


ii) Evaluate the ratio $\frac{R_{12}^U}{R_{12}^L}$ and fit the asymmetry

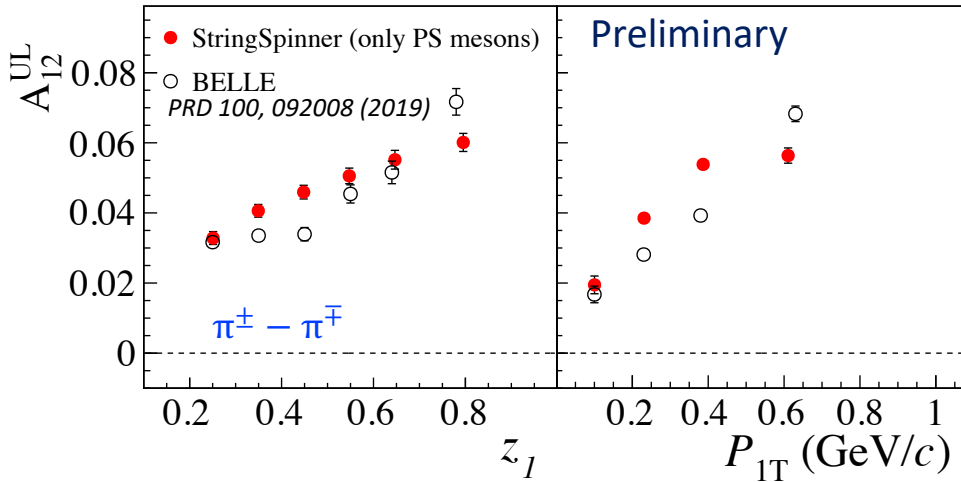
$$R_{12}^{UL} = \frac{R_{12}^U}{R_{12}^L} \approx 1 + A_{12}^{UL} \cos(\phi_1 + \phi_2)$$

Fit function

$$f(\phi_1 + \phi_2) = p_0 + p_1 \cos(\phi_1 + \phi_2)$$



Preliminary results from simulations with Pythia 8.3



The asymmetry reproduces the qualitative features of the data

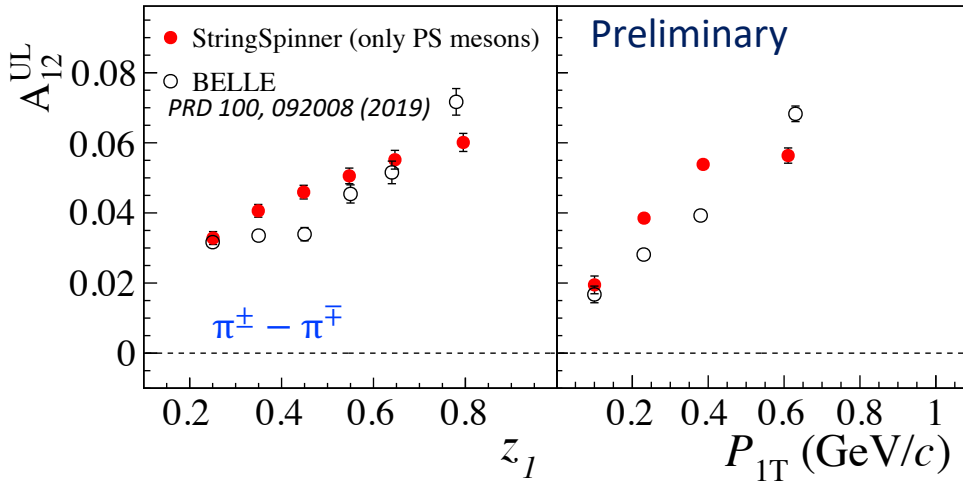
- positive sign

- rising trend with z and PT

- comparable size

Introduction of VMs and decays is expected to change trends at small z and in PT

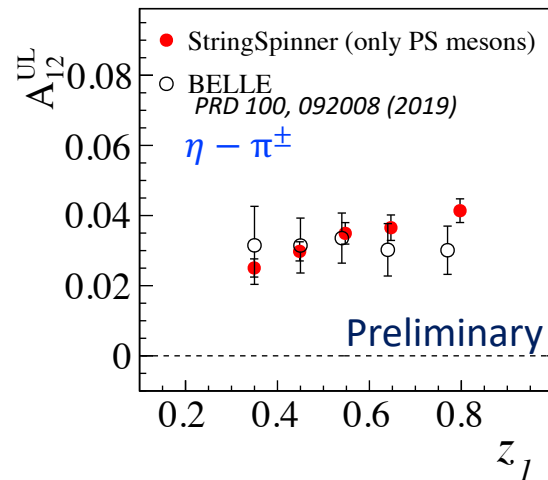
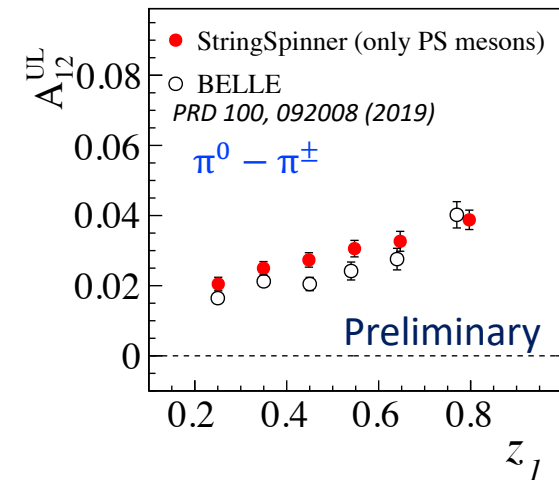
Preliminary results from simulations with Pythia 8.3



The asymmetry reproduces the qualitative features of the data

- positive sign
- rising trend with z and PT
- comparable size

Introduction of VMs and decays is expected to change trends at small z and in PT



Conclusions

We generalized the $\text{string}+{}^3P_0$ model of hadronization to
 $e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$
recursive quantum mechanical recipe

The recipe is general
can be applied to other production channels of the $q\bar{q}$ pair

The implementation in Pythia 8.3 is ongoing
preliminary Collins asymmetry for back-to-back pions promising

(More) phenomenological studies ongoing
the goal is to publish the results in few months..

Backup

Relevant free parameters for string fragmentation used in simulations

(see AK, L. Lönnblad, arXiv: 2305.05058)

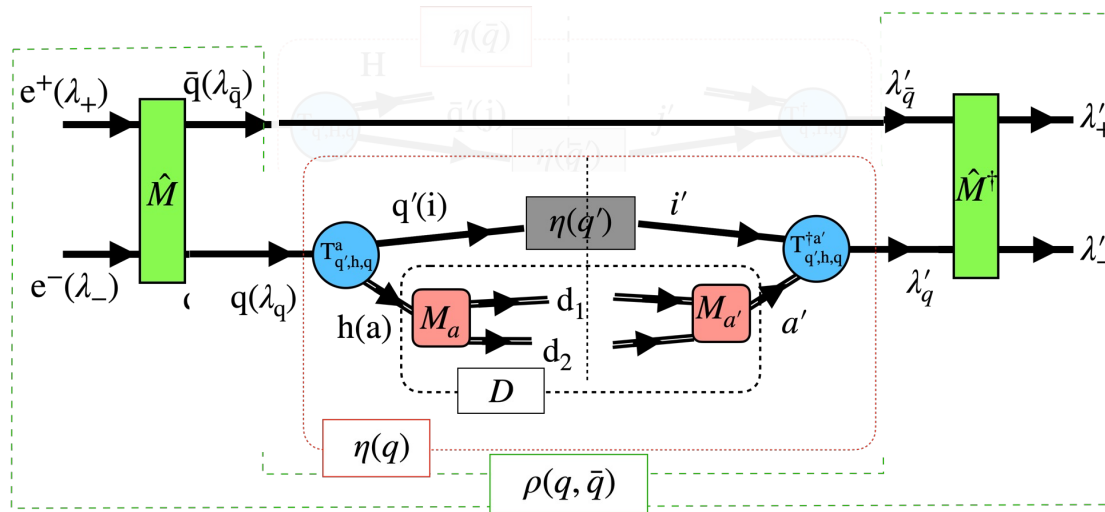
Pythia parameters

StringZ:aLund	default
StringZ:bLund	default
StringPT:sigma	default
StringPT:enhancedFraction	0.0
StringPT:enhancedWidth	0.0 GeV/c

String+³P₀ parameters

Re(μ)	0.42 GeV/c ²
Im(μ)	0.76 GeV/c ²
f_L	0.93
θ_{LT}	0

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson $h=VM$

$$\rightarrow \eta(q) = \mathbf{T}_{q',h=VM,q}^{a'} \eta(q') \mathbf{T}_{q',h=VM,q}^a D_{a'a}, \quad \eta(q') = 1_{q'}, \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2 p_T} = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}^a \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^{a'} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of $h=VM$, and decay the meson

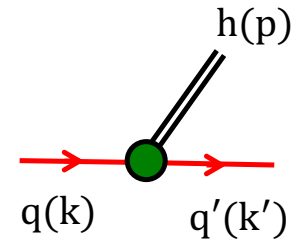
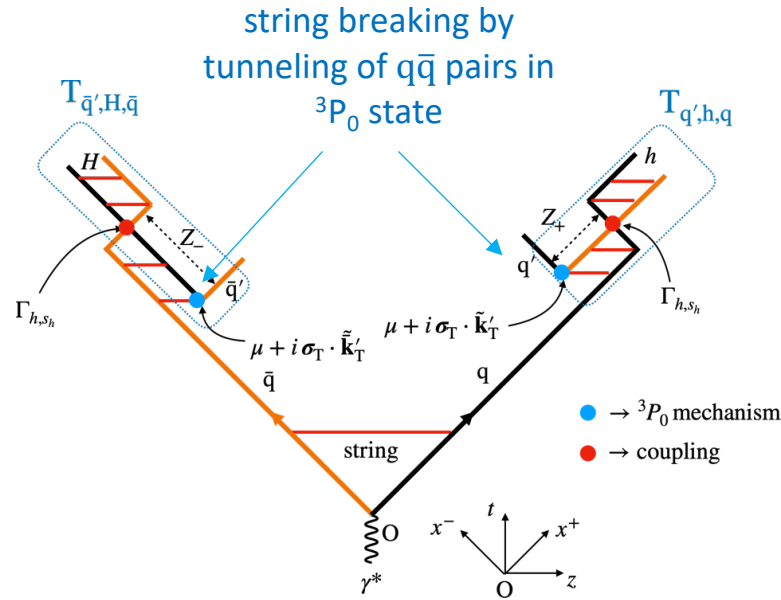
$$\rho_{aa'}(h) = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}^a \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^{a'}$$

iii) Decay the meson $p \rightarrow p_1 p_2 \dots$

$$dN(p_1, p_2 \dots) / d\Omega \propto M_{\text{dec}}^a(p \rightarrow p_1 p_2 \dots) \rho_{aa'}(h) M_{\text{dec}}^{a'}(p \rightarrow p_1 p_2 \dots)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, \dots) = M_{\text{dec}}^{a'}(p \rightarrow p_1 p_2 \dots) M_{\text{dec}}^a(p \rightarrow p_1 p_2 \dots)$

The hadronization model: string+ 3P_0



quark splitting $q \rightarrow h + q'$

Relevant variables:

$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$

$$Z_+ = p^+ / k^+$$

$$\varepsilon_h^2 = M^2 + p_T^2$$

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+ 3P_0 model

$$T_{q',h,q} \propto C_{q',h,q} D_h(M^2) \underbrace{\left(\frac{1 - Z_+}{\varepsilon_h^2} \right)^{\frac{a}{2}} \exp \left[-\frac{\mathbf{b}_L \varepsilon_h^2}{2Z_+} \right]}_{\text{longitudinal momentum}} \underbrace{N_a^{-\frac{1}{2}} (\varepsilon_h^2) e^{-\frac{\mathbf{b}_T k'^2_T}{2}}}_{\text{transverse momentum (w.r.t string axis)}}$$

$[\mu + \sigma_z \sigma_T \cdot \mathbf{k}'_T]$
 3P_0 mechanism
 $[\mu$ complex mass parameter]

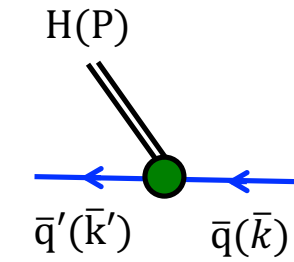
Γ_{h,s_h}
Coupling
e.g.
 $\Gamma_{h=PS} = \sigma_z$

Free param. Lund

Free param. string+ 3P_0

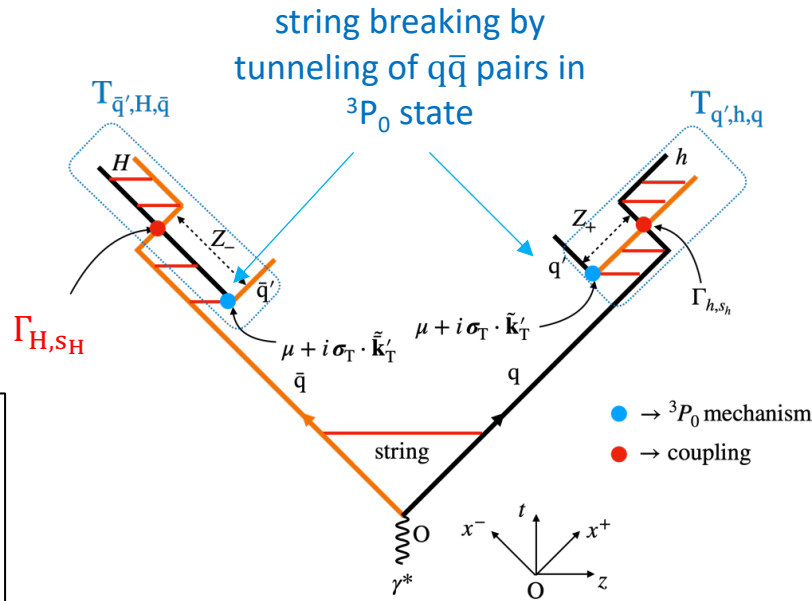
AK, Artru, Martin, PRD 104, 114038 (2021)

The hadronization model: $\text{string} + {}^3P_0$



antiquark splitting
 $\bar{q} \rightarrow H + \bar{q}'$

Relevant variables:
 $\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$
 $Z_- = P^- / \bar{k}^-$
 $\epsilon_H^2 = M^2 + P_T^2$



Antiquark splitting amplitude in the $\text{string} + {}^3P_0$ model obtained by the quark one by

$$\{q, h, q'\} \rightarrow \{\bar{q}, H, \bar{q}'\}, \quad Z_+ \rightarrow Z_-, \quad \{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \rightarrow \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

Application of the recipe to the first two hadrons produced

Application of the recipe to $e^+e^- \rightarrow h H X$

$h = \text{PS}$ and $H = \text{PS}$ being the first two hadrons produced

$$dP(e^+e^- \rightarrow h H X) = \hat{\sigma}^{-1} \frac{d\hat{\sigma}}{d \cos \theta} \times \underset{\text{Prob}(e^+e^- \rightarrow q\bar{q})}{F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})} \times \underset{\text{Prob}(q \rightarrow h + q')}{F_{\bar{q}',H,\bar{q}}(Z_-, \bar{\mathbf{p}}_T; \bar{\mathbf{k}}_T, C^{q'\bar{q}})} \times \underset{\text{Prob}(\bar{q} \rightarrow H + \bar{q}'; q \rightarrow h + q')}{F_{\bar{q}',H,\bar{q}}(Z_-, \bar{\mathbf{p}}_T; \bar{\mathbf{k}}_T, C^{q'\bar{q}})}$$

$$\propto (1 + \cos^2 \theta) \times (\dots) \times \left[1 + \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{2\text{Im}(\mu)p_T}{|\mu|^2 + p_T^2} \frac{2\text{Im}(\mu)P_T}{|\mu|^2 + P_T^2} \cos(\phi_h + \phi_H) \right]$$

expected form for the azimuthal distribution of back-to-back hadrons!

For quantitative results and phenomenology

→ implementation of the model in Pythia 8.3 for of $e^+e^- \rightarrow \text{hadrons}$