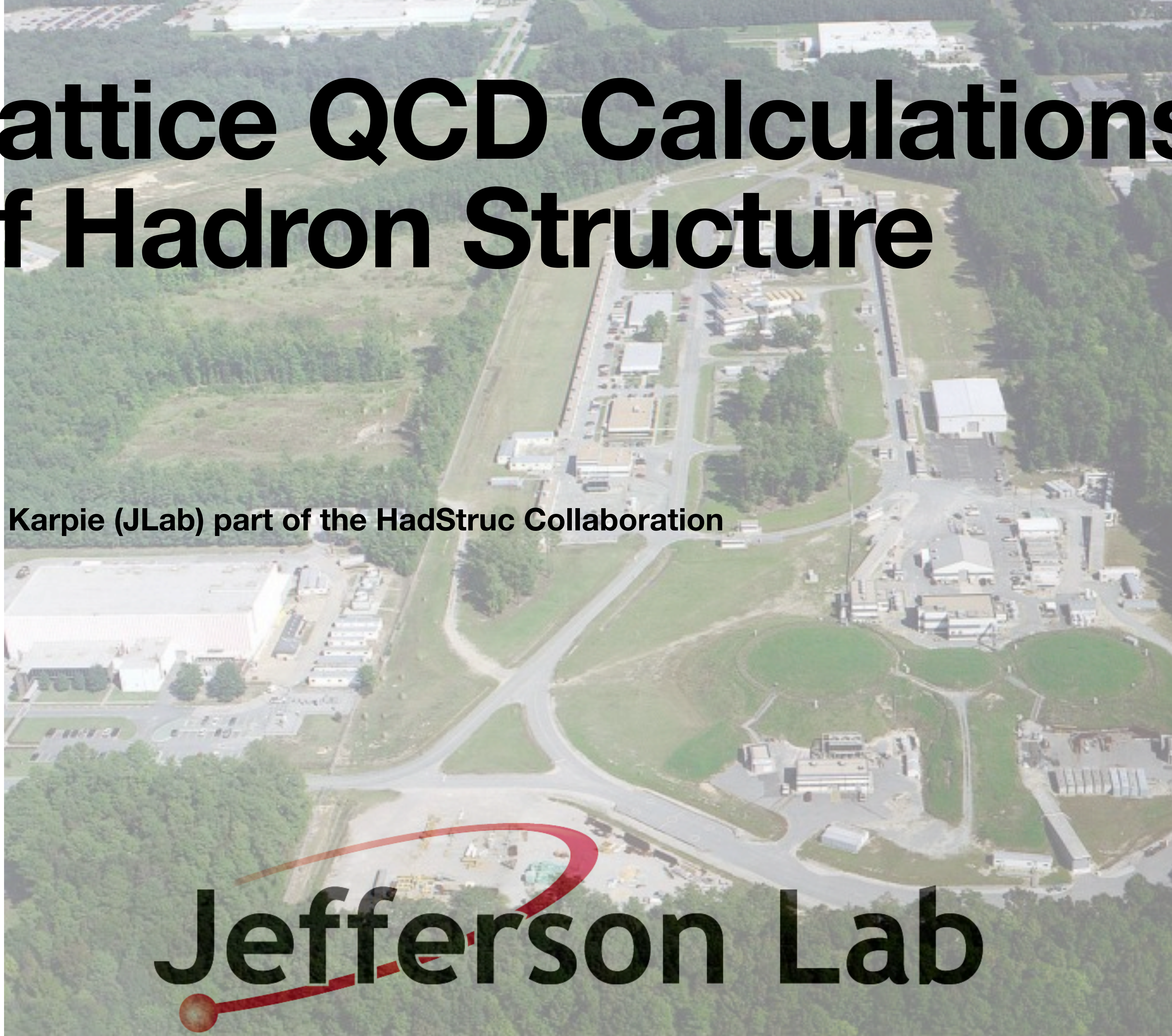
Lattice QCD Calculations of Hadron Structure

Joe Karpie (JLab) part of the HadStruc Collaboration



Digging Deeper and Deeper

- Typical humans in Highline Park (NYC)

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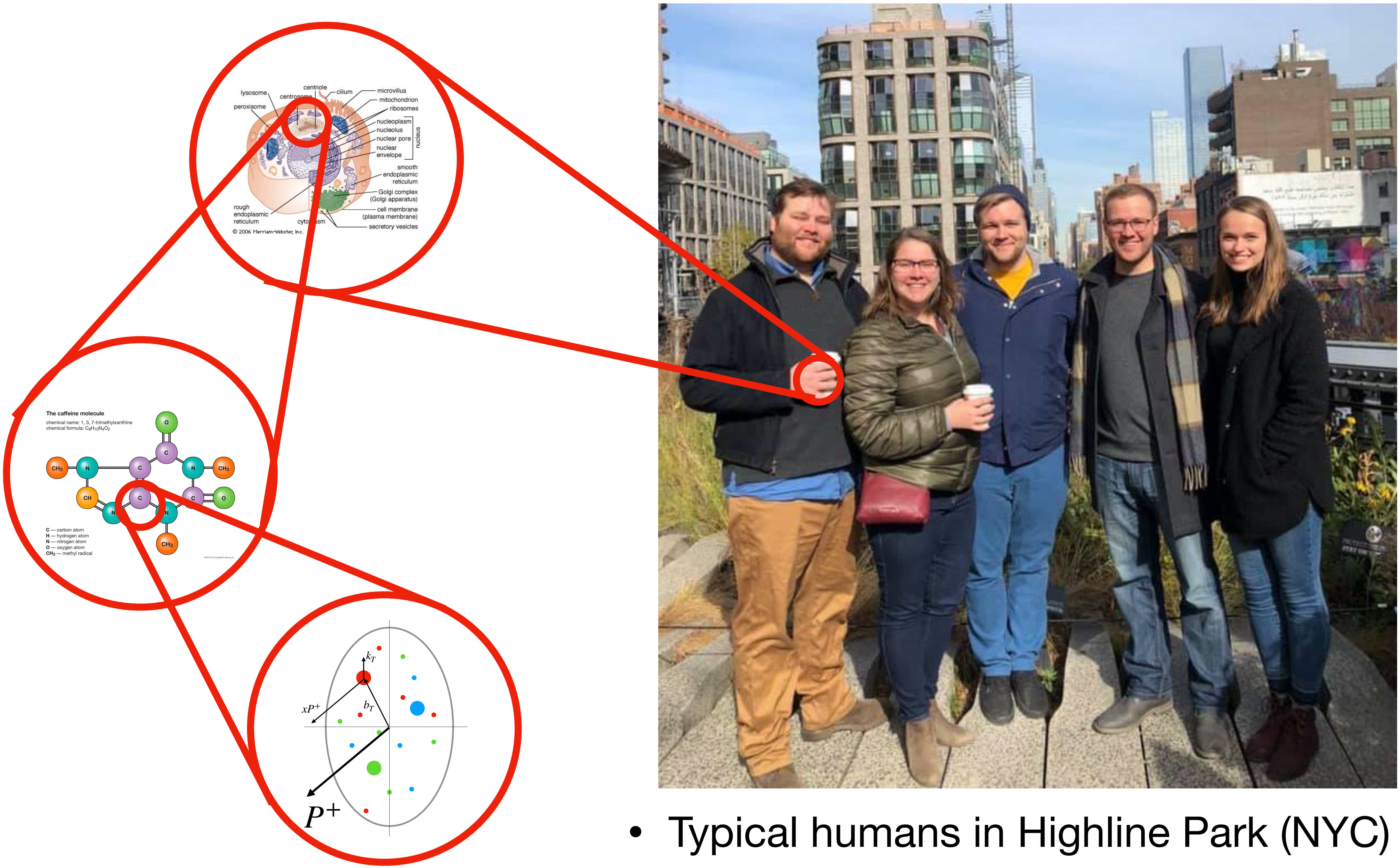
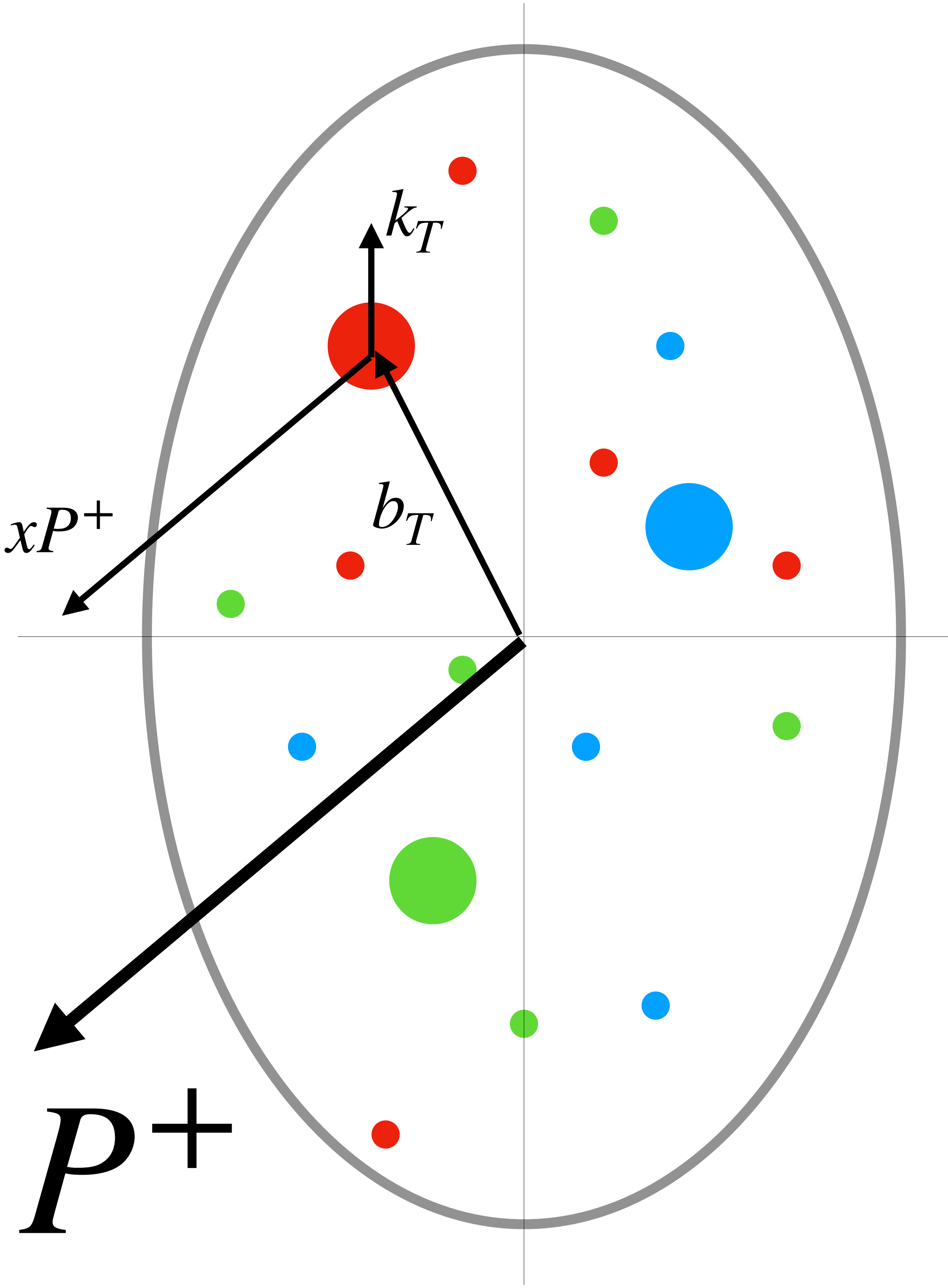
Digging Deeper and Deeper



- Typical humans in Highline Park (NYC)

Parton Structure

- Internal structure of hadrons
- How are quarks and gluons distributed?
- How do they contribute to cross sections, hadron mass, spin, …..?
  - **Longitudinal** structure from momentum fraction x
  - **Transverse** structure from
    - Impact parameter b_T
    - Transverse momentum k_T



Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum
Distribution (GTMD)

$F(x, b_T, k_T)$

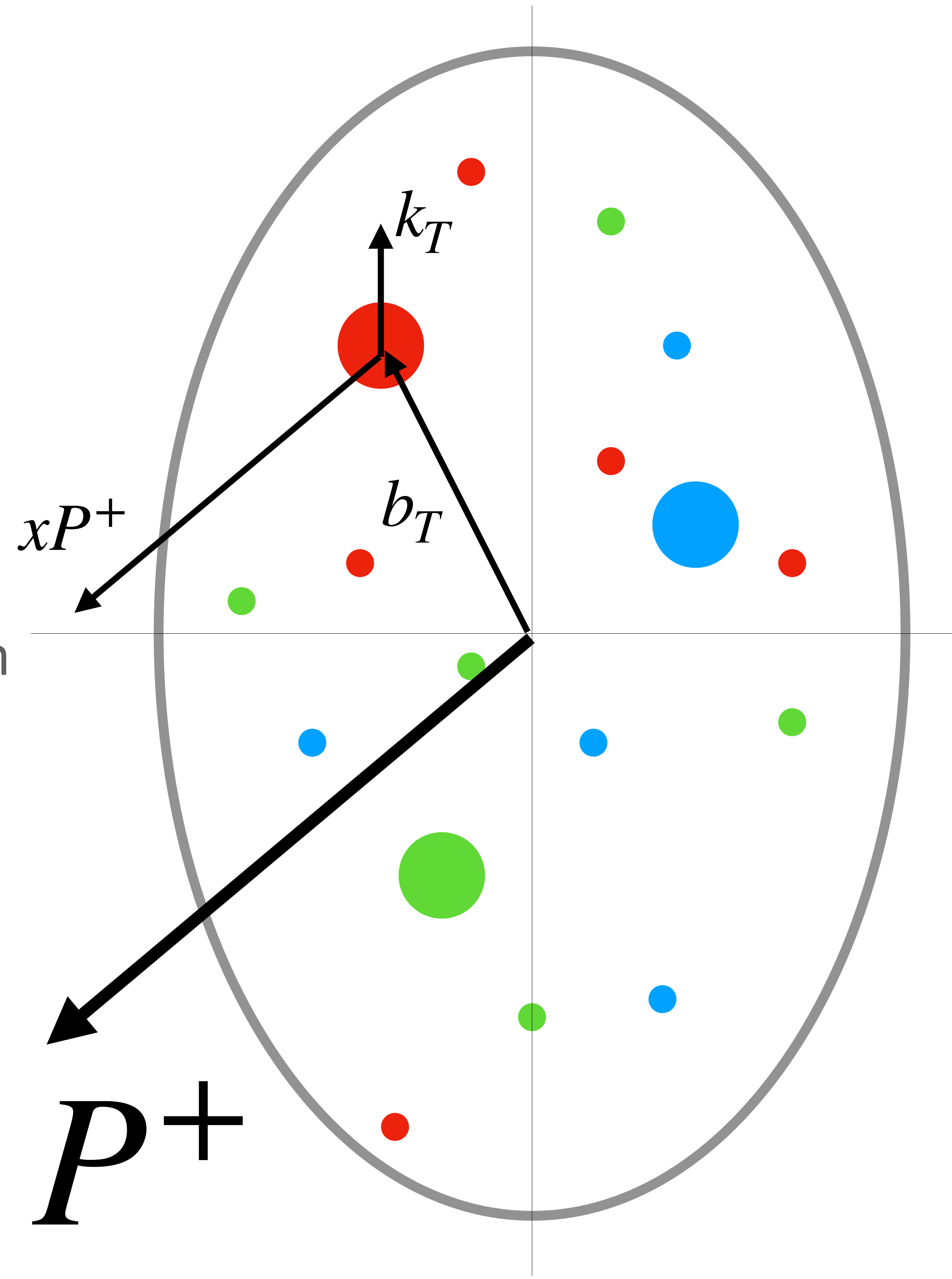
$\int d^2 b_t$ $\int d^2 k_t$

Transverse Momentum
Distribution (TMD)
$f(x, k_T)$

Generalized Parton
Distribution (GPD)
$f(x, b_T)$

$\int d^2 k_t$ $\int d^2 b_t$

Parton Distribution Function (PDF)
$f(x)$



Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions

Ioffe time: $\nu = p \cdot z$

V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

  - $I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-)\gamma^+ W(z^-; 0)\psi_q(0) \,|\, p \rangle_\mu$

$z^2 = 0$

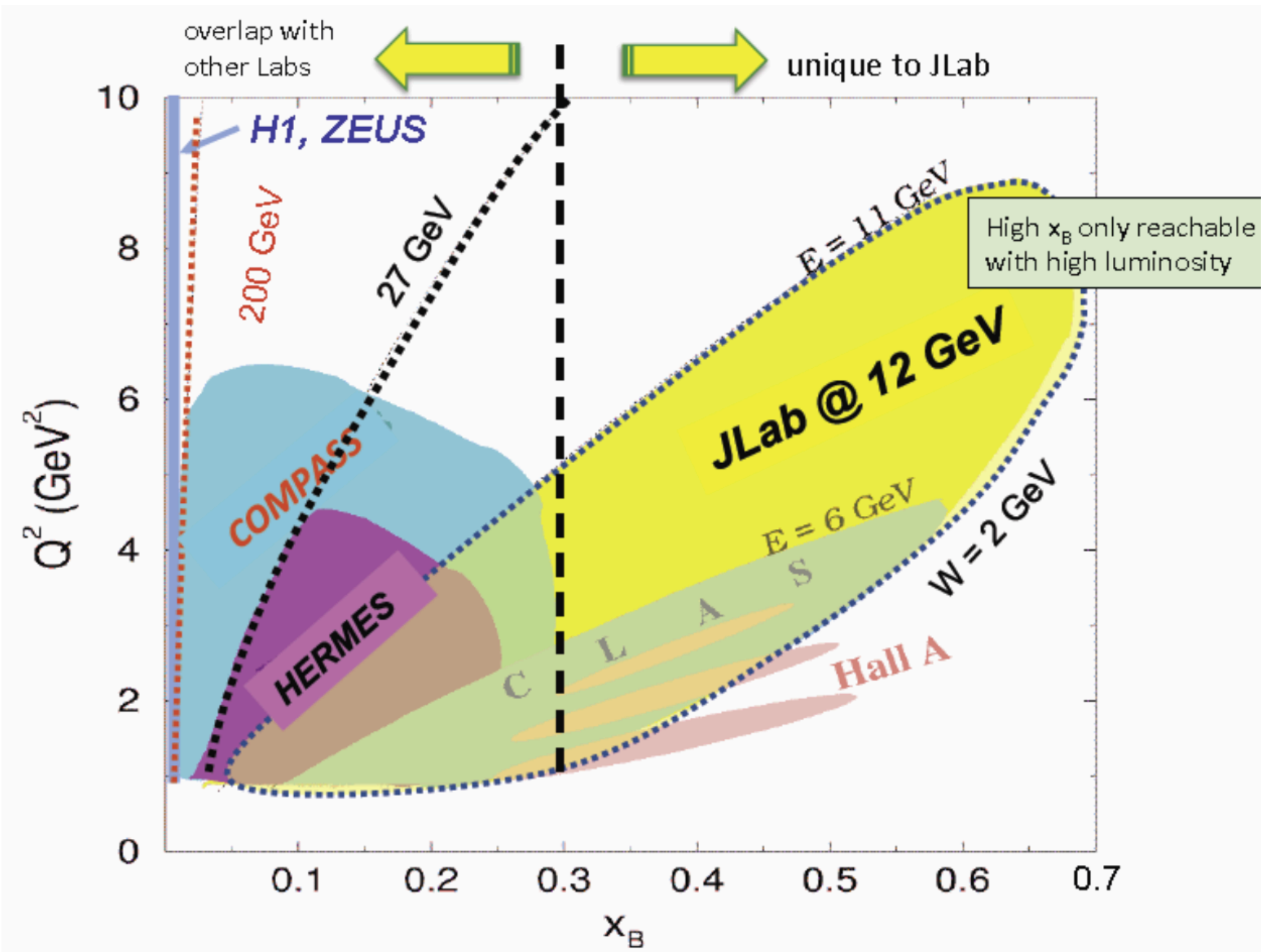
  - $I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-; 0) F^i_+(0) \,|\, p \rangle_\mu$

$i = x, y$

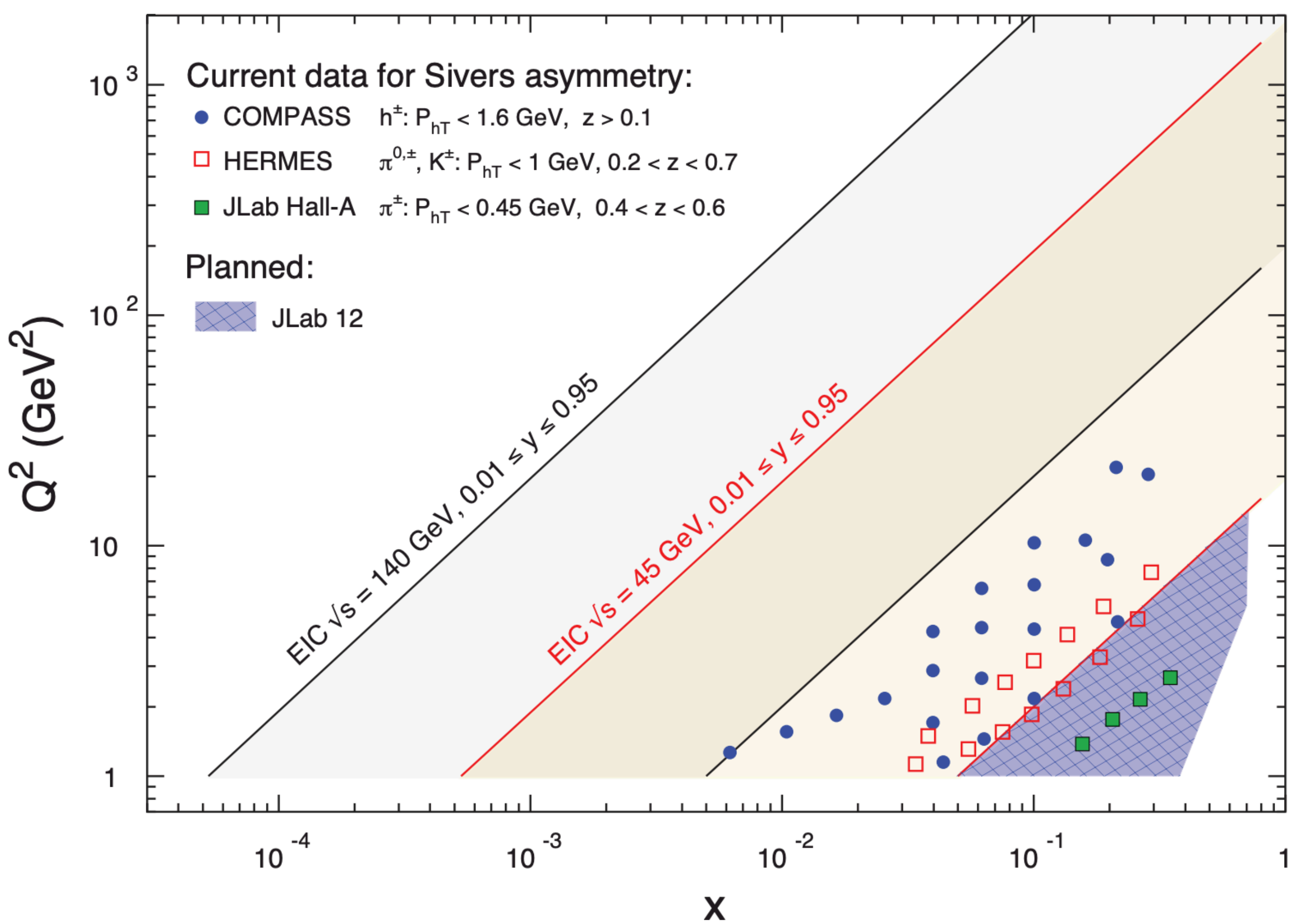
- Parton Distribution Functions
  - $I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$
  - $I_g(\nu, \mu^2) = \int_{0}^{1} dx\, \cos(x\nu)\, x f_g(x, \mu^2)$

JLab 12 GeV and the EIC

- Expand Kinematic Coverage
  - JLab extends to large x (also where lattice is most sensitive)
  - Unpolarized gluon PDF dominates low x and is goal of EIC
- Expand to polarized and 3D distributions (lattice goal as well)

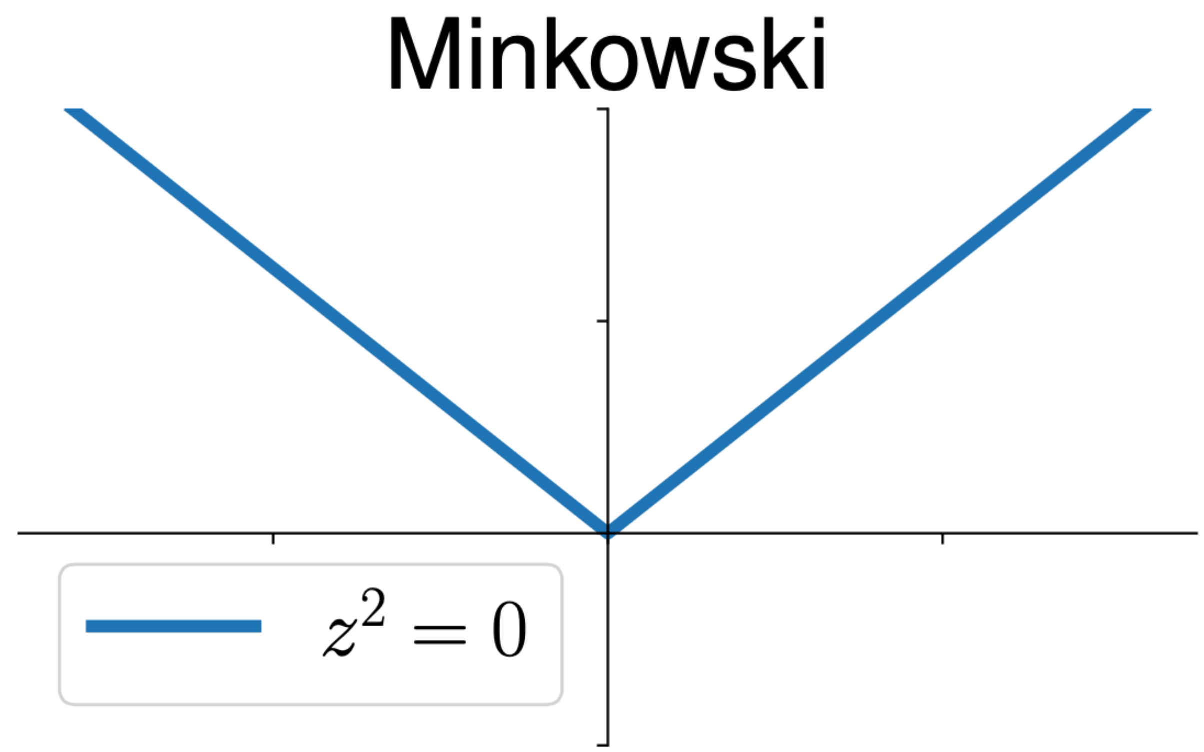


JLab 12 GeV White Paper:
Eur. Phys. J. A 48 (2012) 187



EIC White Paper:
Eur. Phys. J. A 52 (2016) 9, 268

Parton Distributions and the Lattice



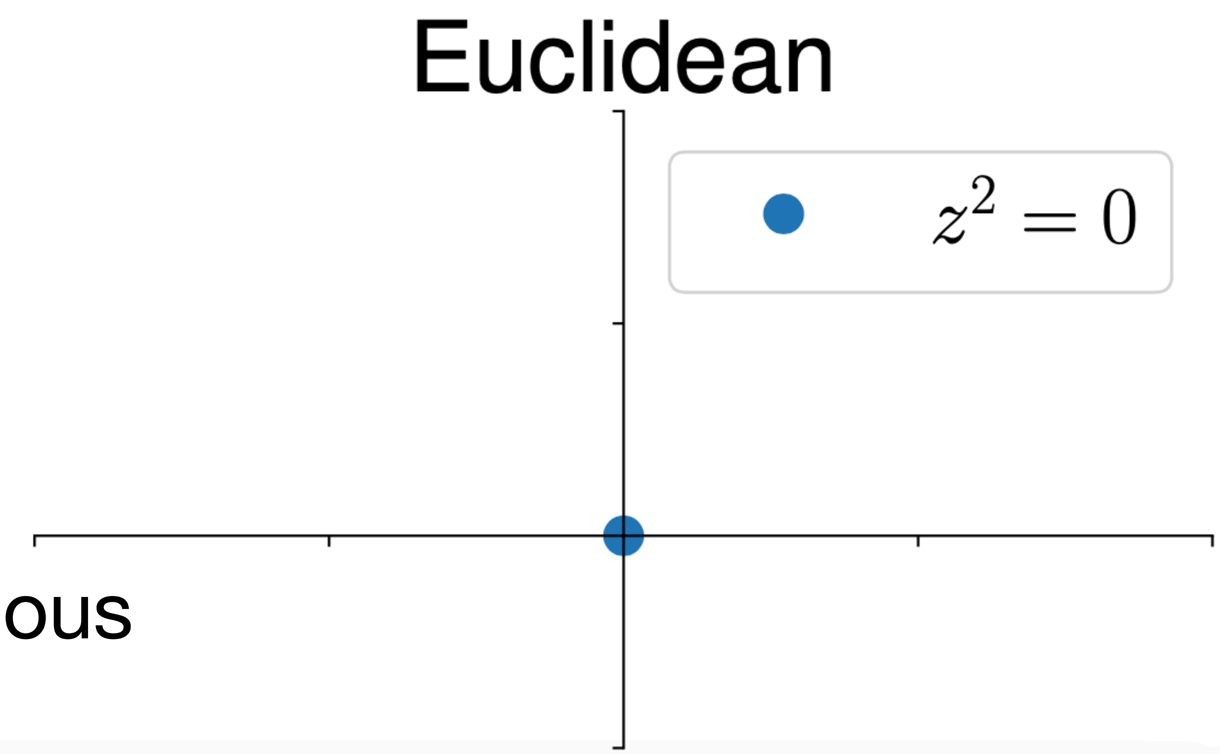
- Parton Distributions are defined by operators with light-like separations

$$M(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^{\alpha} W(z;0)\psi(0) \,|\, p \rangle$$

- Use space-like separations $z^2 \neq 0$

  X. Ji *Phys Rev Lett* 110 (2013) 262002

  - Wilson line operators or pairs of space-like separated currents



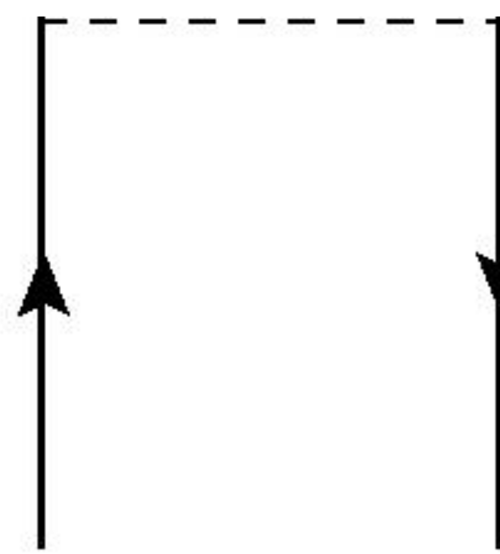
- Fourier transformations of matrix elements give PDF in certain limits

- Factorizable matrix elements analogous to cross sections

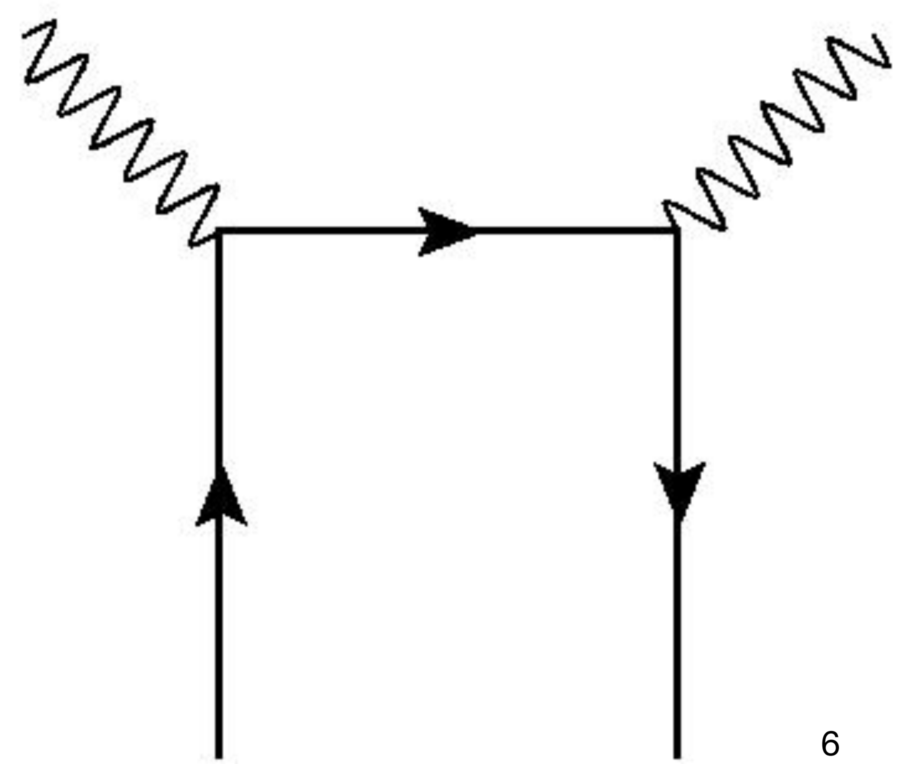
Many approaches

- **Wilson line operators**
  - LaMET X. Ji *Phys. Rev. Lett.* 110 (2013) 262002
  - Pseudo-PDF A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025
- Two current correlators
  - Hadronic Tensor
    K.-F. Liu et al *Phys. Rev. Lett.* 72 1790 (1994)
    Phys. Rev. D 62 (2000) 074501
  - HOPE
    W. Detmold and C.-J. D. Lin, *Phys. Rev. D* 73 (2006) 014501
  - Short distance OPE
    V. Braun and D. Muller *Eur. Phys. J. C* 55 (2008) 349
  - OPE-without-OPE
    A. Chambers et al, *Phys. Rev. Lett.* 118 (2017) 242001
  - Good Lattice Cross Sections
    Y.-Q. Ma and J.-W. Qiu *Phys. Rev. Lett.* 120 (2018) 2, 022003

$$O_{WL}(x;z) = \bar{\psi}(x+z)\Gamma W(x+z;x)\psi(x)$$



$$O_{CC}(x,y) = J_{\Gamma}(x)J_{\Gamma'}(y)$$



The Role of Separation and Momentum

- The separation and momenta will play the role x_B and Q^2 is DIS
- In quasi-PDF and pseudo-PDF, separation and momentum swap roles

Scale:

Q^2 / p_z^2 / z^2

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ_{QCD}^2
- Technically only requires single value

Dynamical variable:

x_B / z / p_z , or $\nu = p \cdot z$

- Variable for inverse problem
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

What do we do with this data?

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$q(x) = \int_0^\infty d\nu \, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$q(x) = \int_0^\infty d\nu \, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

- Forward integral to an ill posed matrix equation

$$\mathfrak{M}(\nu) = \int_0^1 dx \, C(x\nu) \, q(x) \to [\mathbf{C}][\mathbf{q}]$$

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

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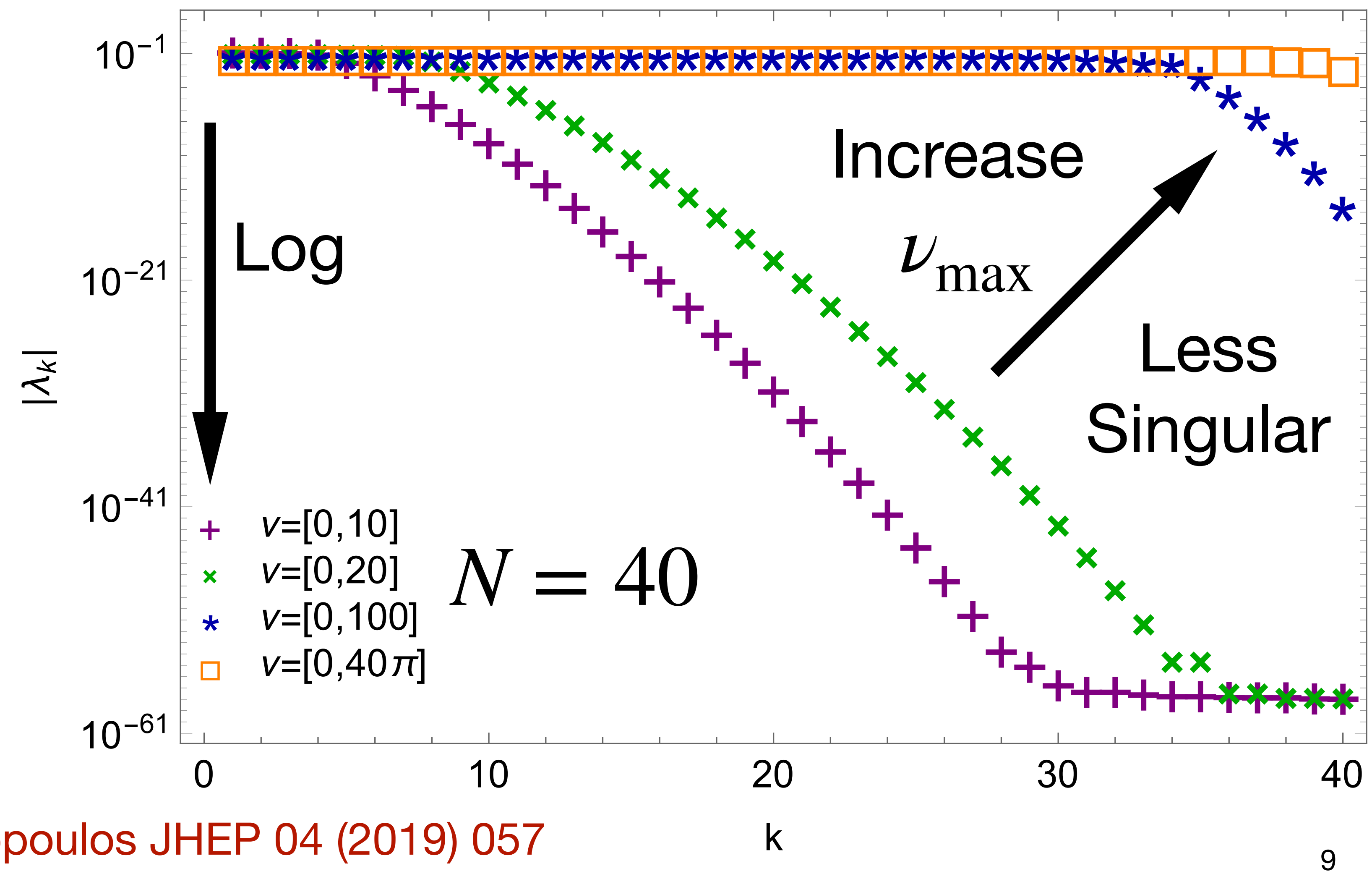

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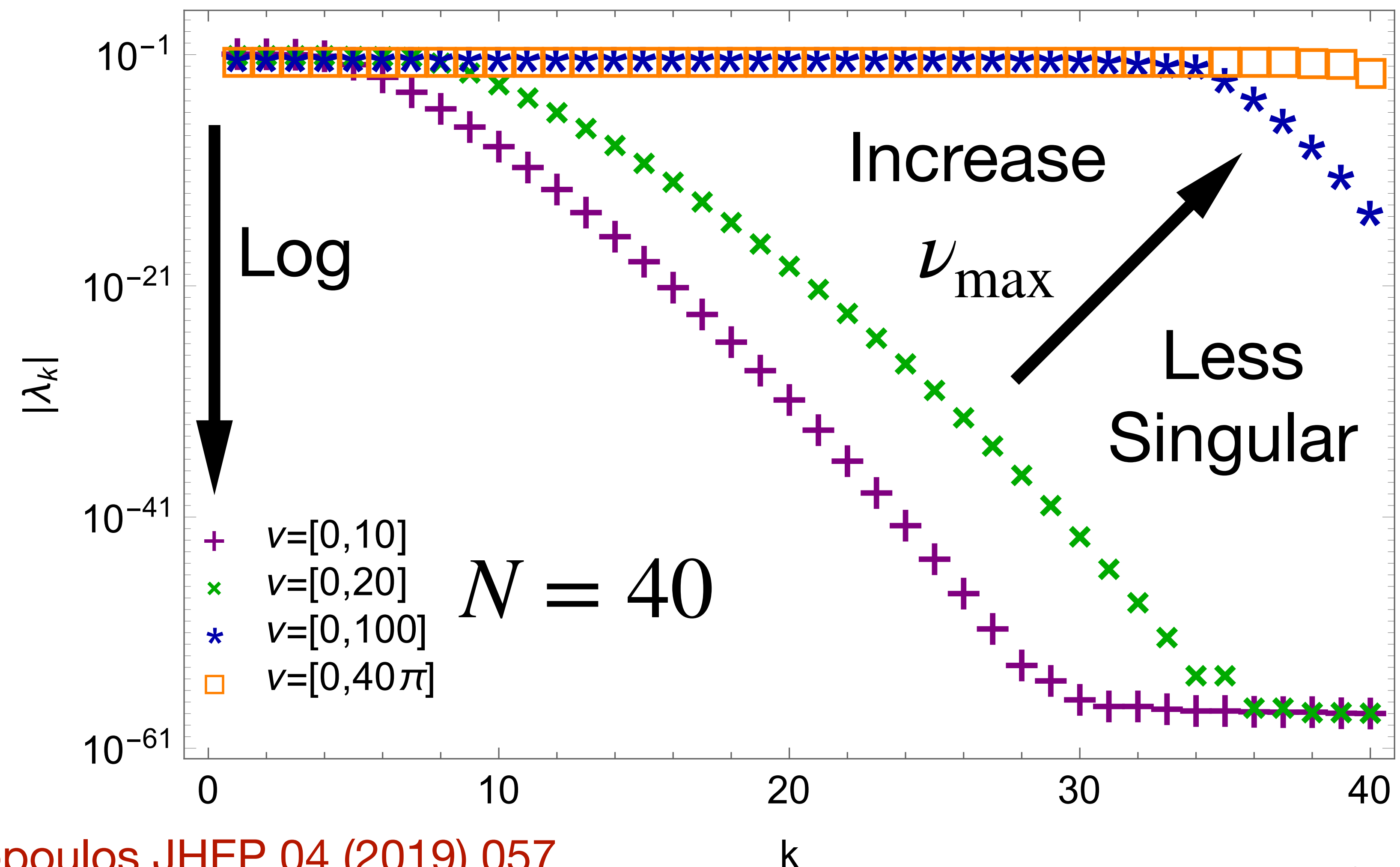
- Must be regulated by additional information
  - Restricted functional form
  - Constraints on the PDF or parameters
  - Assumptions of smoothness, continuity, ….



JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for Parton Physics

- **Structure Functions**

$$F_2(x, Q^2) = \sum_i \int_x^1 d\xi\, C(\xi, \frac{\mu^2}{Q^2})\, q(\frac{x}{\xi}, \mu^2)$$

- **LaMET (on the lattice)**

$$M(p_z, z) = \int_{-\infty}^{\infty} dy\, e^{iyp_z z}\, \tilde{q}(y, p_z^2)$$

- **pseudo-Distributions / Good Lattice Cross Sections**

$$\mathfrak{M}(\nu, z^2) = \int_{-1}^{1} dx\, C(x\nu, \mu^2 z^2)\, q(x, \mu^2)$$

- **Local Matrix elements / HOPE / OPE-without-OPE**

$$a_n(\mu^2) = \int_{-1}^{1} dx\, x^{n-1}\, q(x, \mu^2)$$

- **Hadronic Tensor**

$$\tilde{W}_{\mu\nu}(\tau) = \int d\nu\, e^{-\nu\tau}\, W_{\mu\nu}(\nu)$$

Unknown Functions

- **Lattice systematic errors**

$$\mathfrak{M}^{\text{latt}}(p,z,a) = \mathfrak{M}^{\text{cont}}(\nu,z^2) + \sum_n \left(\frac{a}{|z|}\right)^n P_n(\nu) + \left(a\Lambda_{\text{QCD}}\right)^n R_n(\nu) + \dots$$

- **Power Corrections**

$$\mathfrak{M}^{\text{cont}}(p,z,a) = \mathfrak{M}^{\text{lt}}(\nu,z^2) + \sum_n \left(z^2\Lambda^2_{\text{QCD}}\right)^n B_n(\nu)$$

- **Factorization**

$$[\text{Re}/\text{Im}]\mathfrak{M}_{\text{lt}}(\nu,z^2) = \int_0^1 dx\, K_{\text{R/I}}(x\nu,\mu^2 z^2) q_{\mp}(x,\mu^2)$$

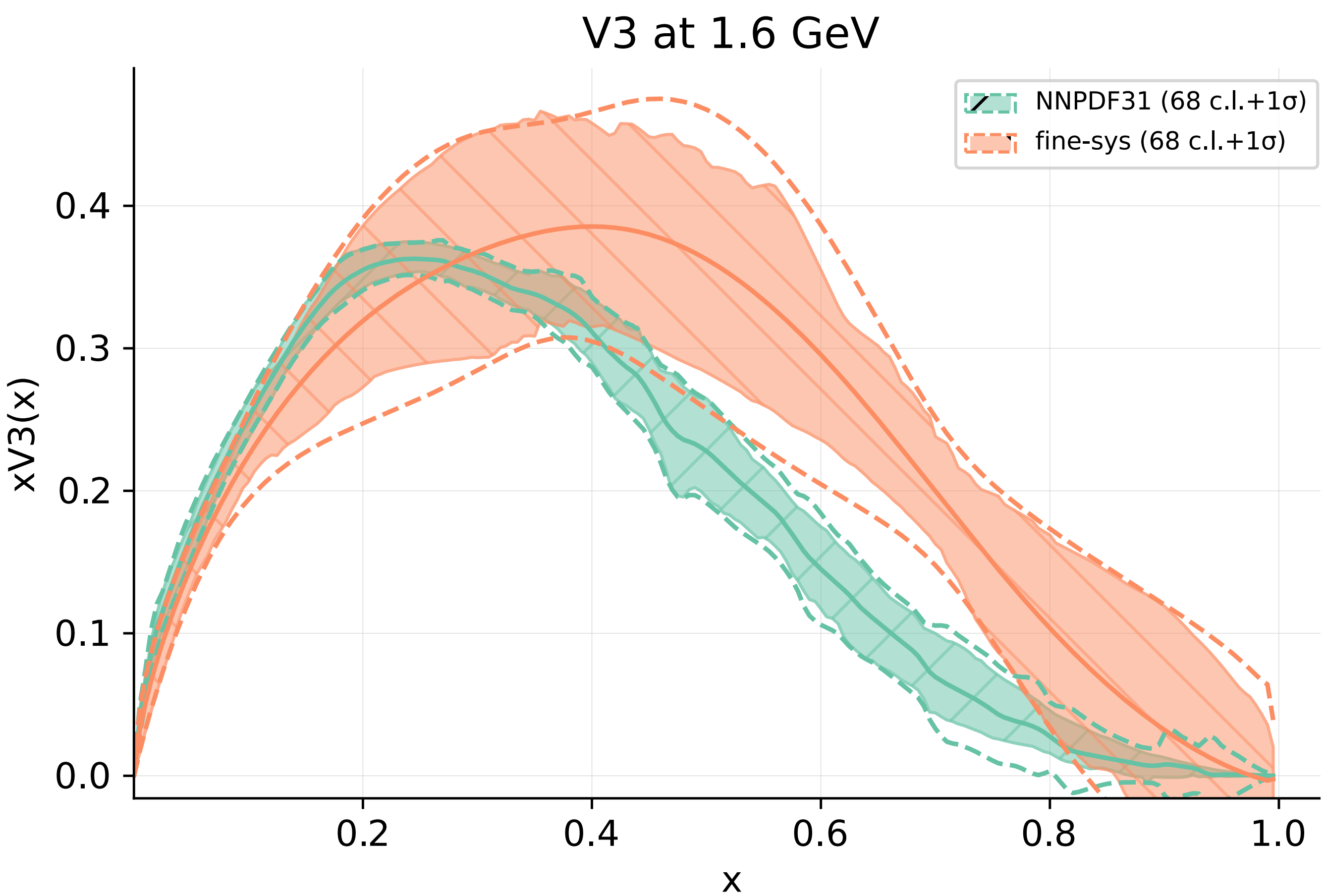
$$K_R(x\nu,\mu^2z^2) = \cos(x\nu) + O(\alpha) \qquad K_I(x\nu,\mu^2z^2) = \sin(x\nu) + O(\alpha)$$

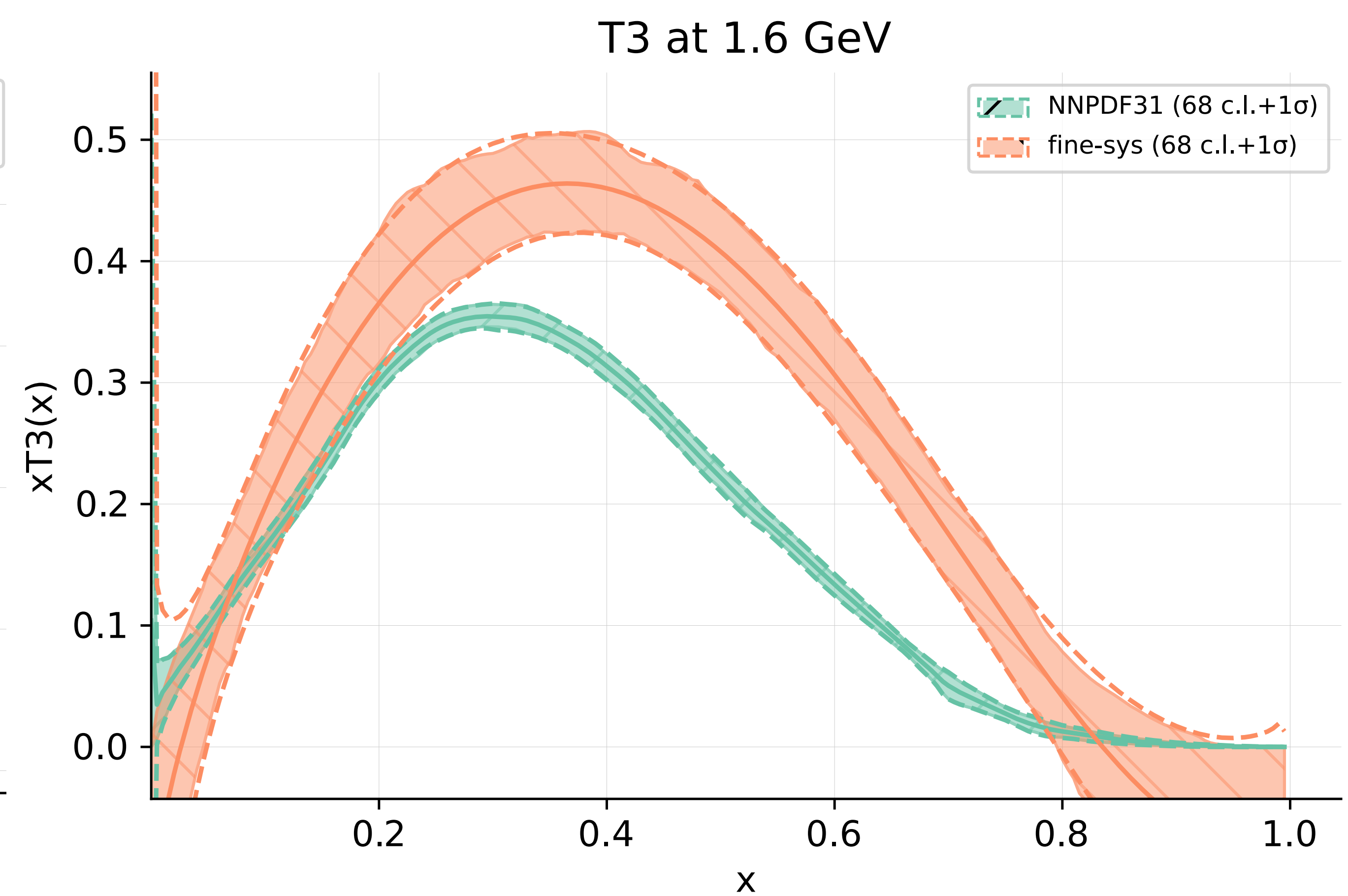
JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos JHEP 11 (2021) 024

Neural Networks for Inverse Problems

- NNPDF Analysis with NN geom 2-5-3-1

L. Del Debbio, T. Giani, JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos JHEP 02 (2021) 138





$V_3 = q_- = q_v \quad T_3 = q_+$

Jacobi Polynomials

- Change of variables from true Jacobi polynomials

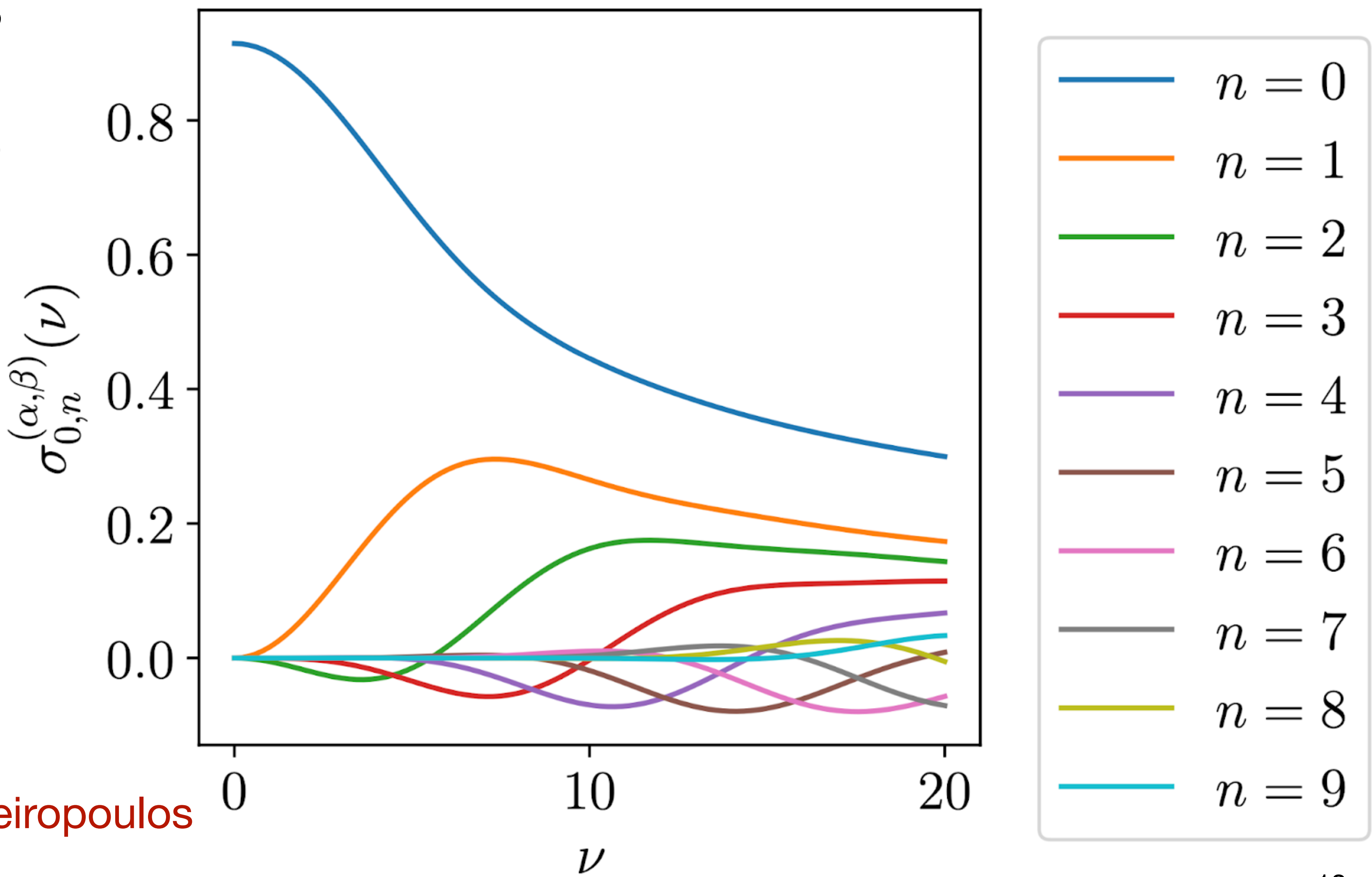
$$\int_{-1}^{1} dz(1+z)^\alpha(1-z)^\beta P_n(z)P_m(z) = \delta_{nm}\tilde{N}_n \rightarrow \int_0^1 dx x^\alpha(1-x)^\beta J_n(x)J_m(x) = \delta_{nm}N_n$$

- PDF Parameterized as

$$q_- = x^\alpha(1-x)^\beta \sum_n {}_- d_n P_n(x)$$

- ITD becomes

$$\mathrm{Re}\, I_q(\nu) = \sum_n {}_- d_n \sigma_n(\nu)$$



JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos
JHEP 11 (2021) 024

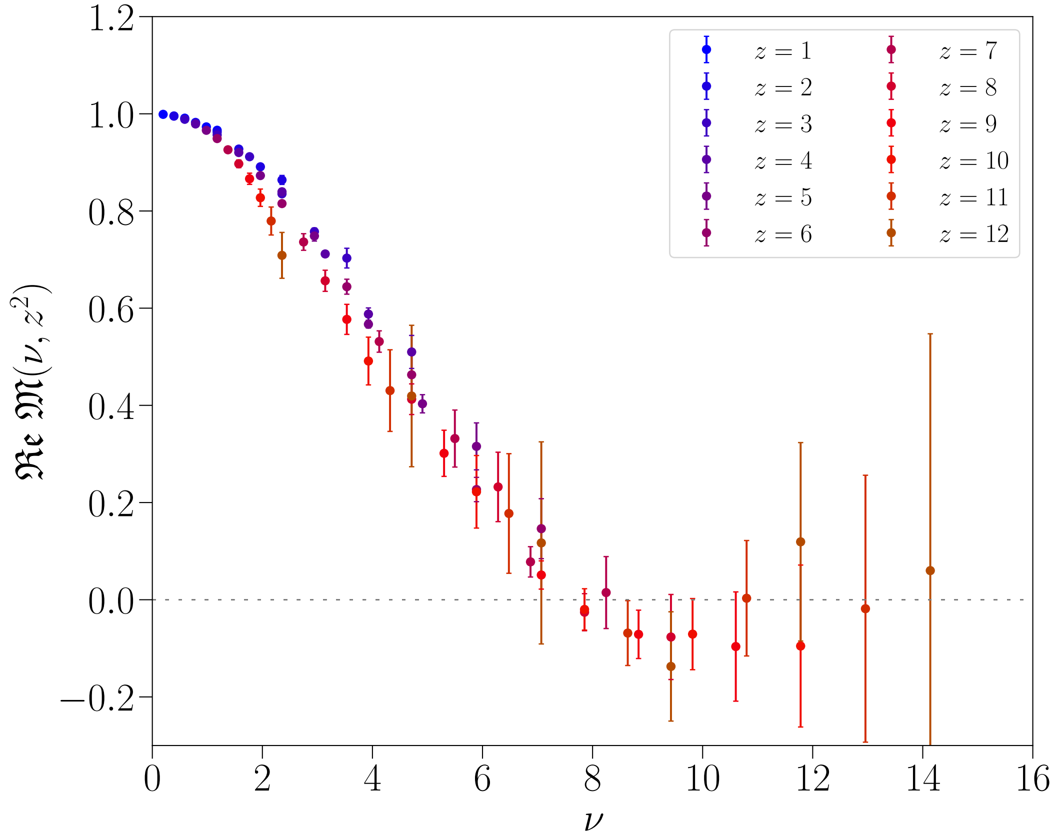
Quark pseudo-PDFs

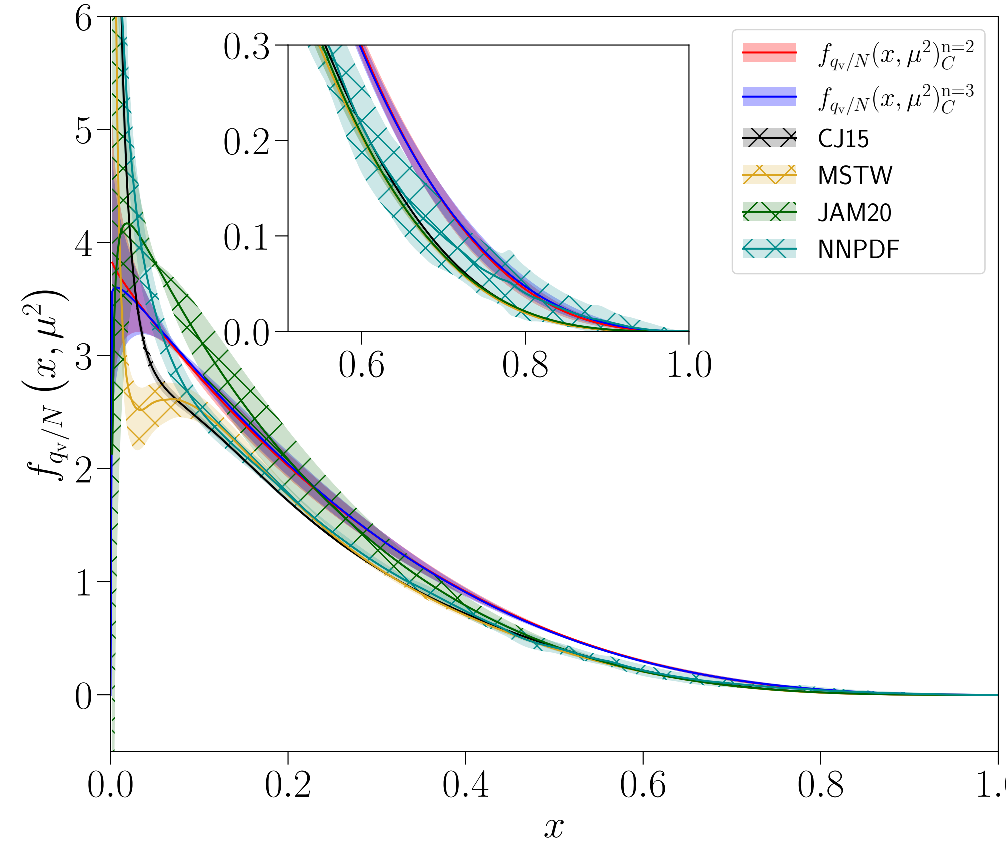
- Bare Matrix element: $M = \langle p \,|\, \bar{\psi}(z)\Gamma W(z;0)\psi(0)p\rangle$
- Choose multiplicatively renormalizable operators
  - Unpolarized PDF : $\Gamma = \gamma^t$
  - Helicity PDF : $\Gamma = \gamma^z\gamma^5$
  - Transversity PDF : $\Gamma = \gamma^5\gamma^t\gamma^i \quad ; \quad i = x, y$
- First study with distillation with unphysically heavy quarks and coarse ensemble

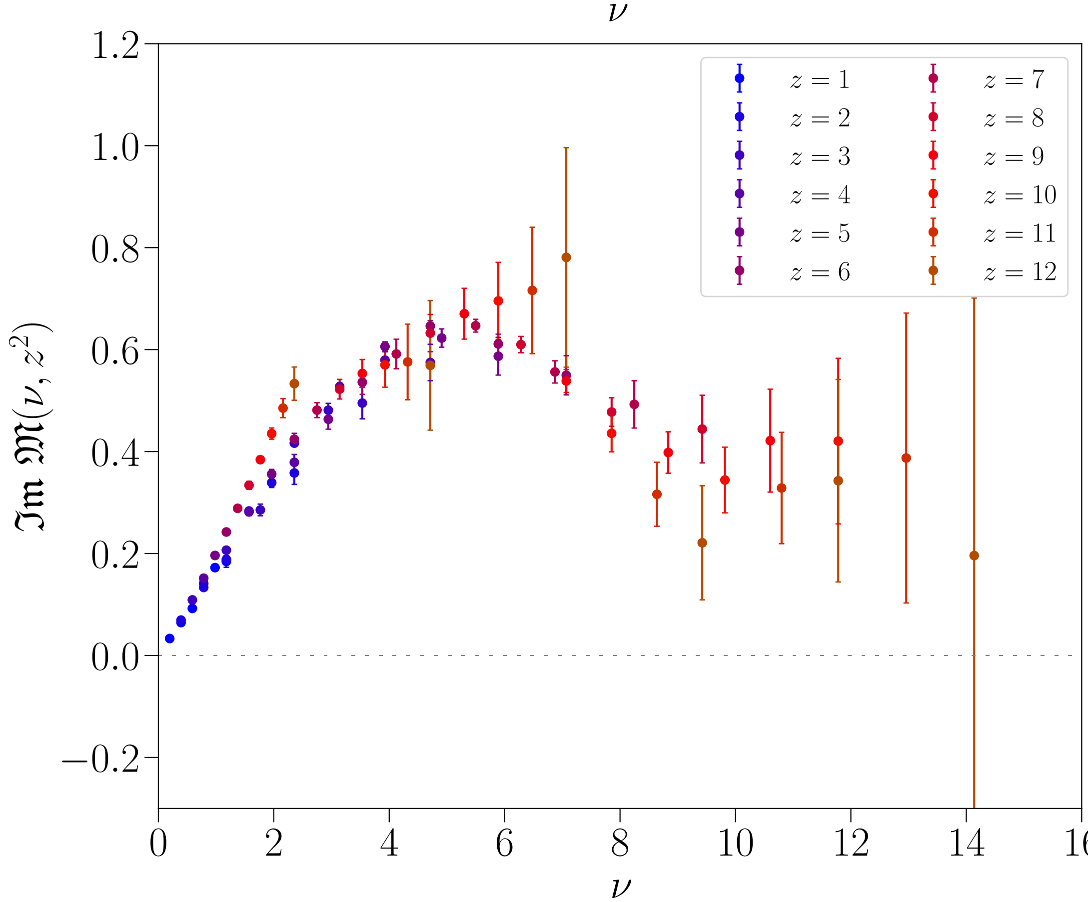
$a = 0.094$ fm
$m_\pi = 358$ MeV

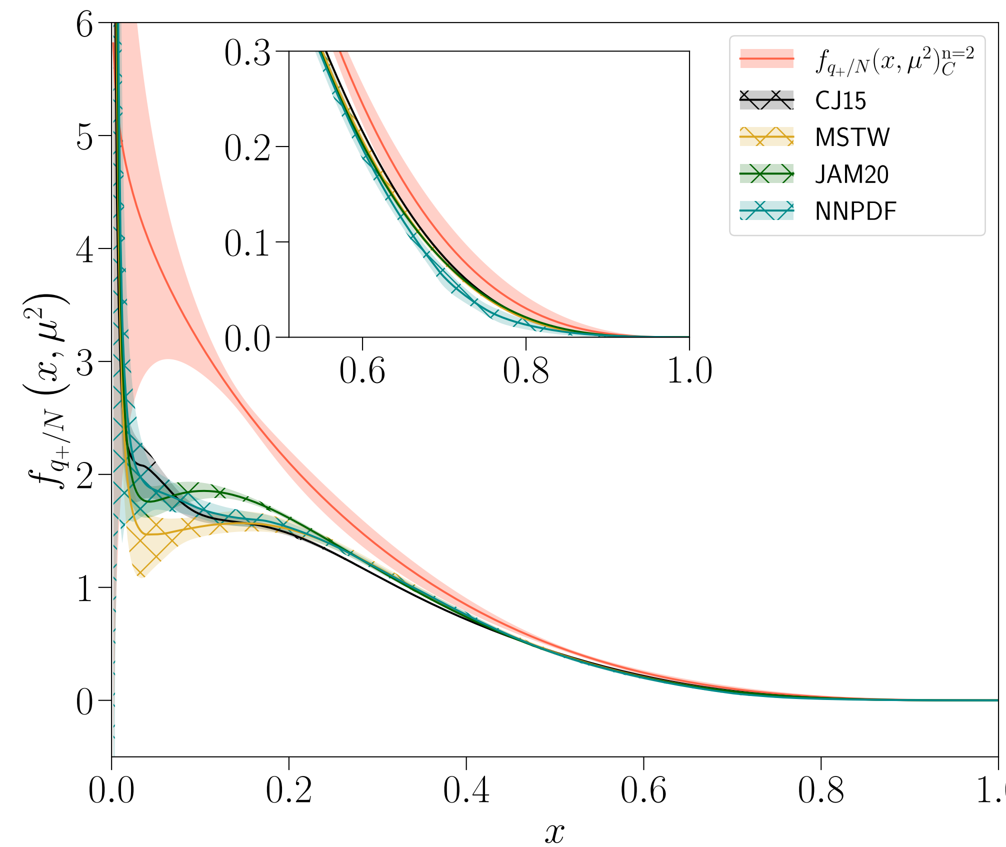
Unpolarized Quark PDF

C. Egerer et al (HadStruc)
JHEP 11 (2021) 148



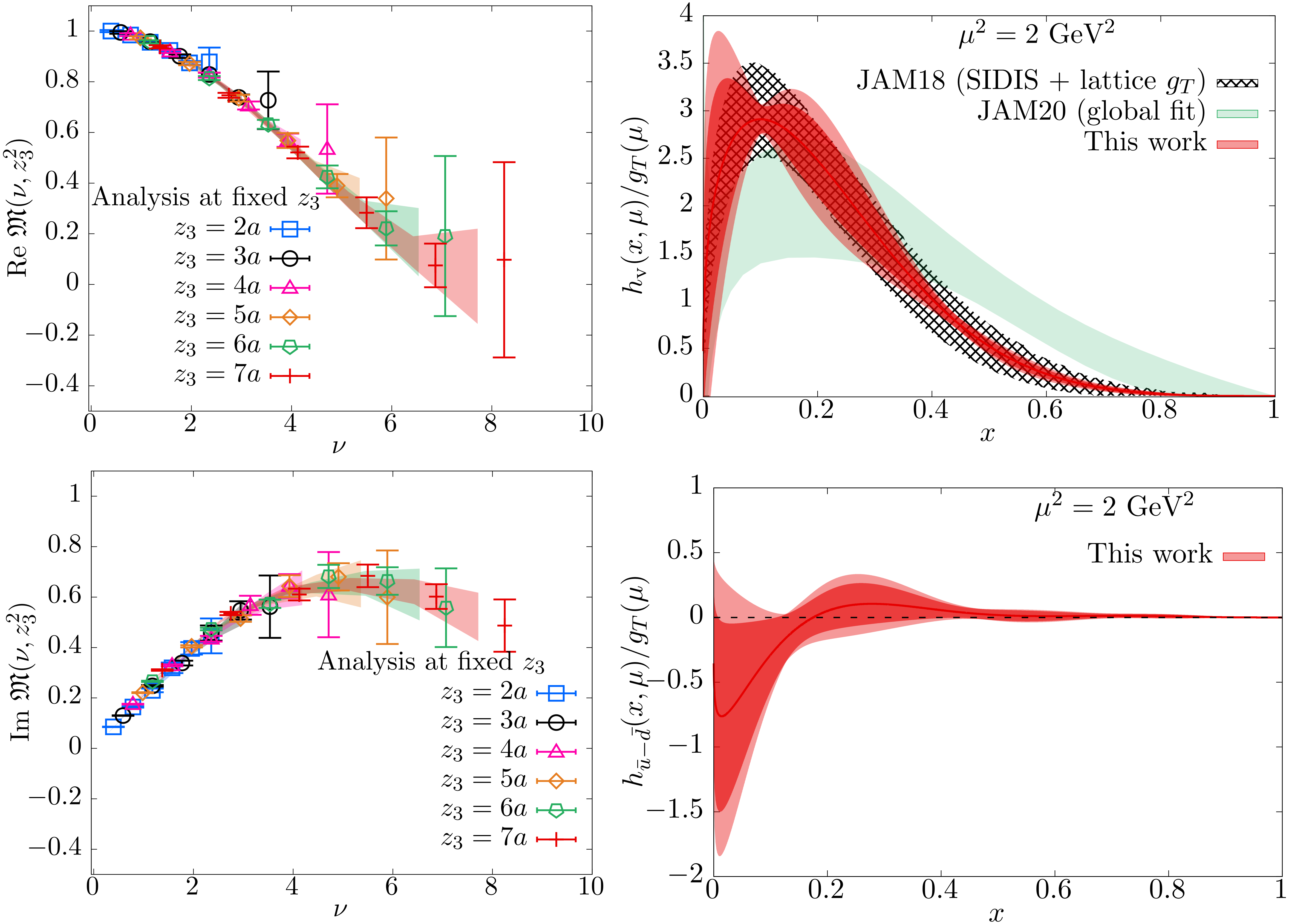






Transversity Quark PDF

C. Egerer et al (HadStruc)
Phys Rev D 105 (2022) 3, 034507



Gluon Matrix Elements

- **General Matrix Element**

$$M^{\mu\alpha;\nu\beta}(z,p,s) = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)F^{\nu\beta}(0)\Big]\,|\,p,s\rangle$$

  - Assume z is along cardinal direction (eventually lattice axis)

- **Renormalization** Z-Y. Li, Y-Q. Ma, J-W. Qiu. *Phys. Rev. Lett.* 122 (2019) 6, 062002

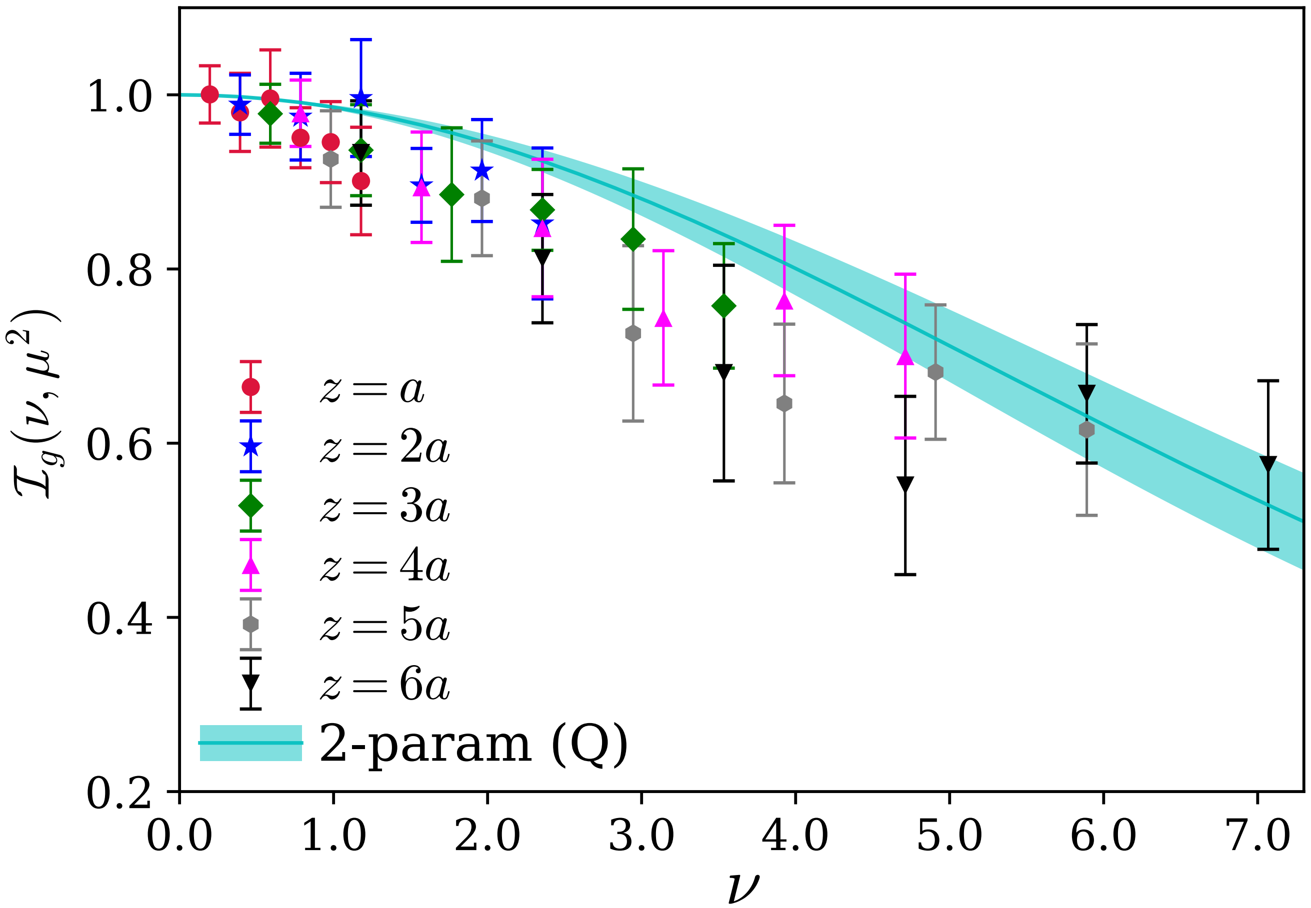
  - Multiplicatively renormalizable

  - Depends on how many of μ, ν, ρ, σ are in z direction.

- Matrix element has complicated Lorentz decomposition in terms of p^μ, z^μ, s^μ

  - Need to isolate amplitudes with leading twist contributions

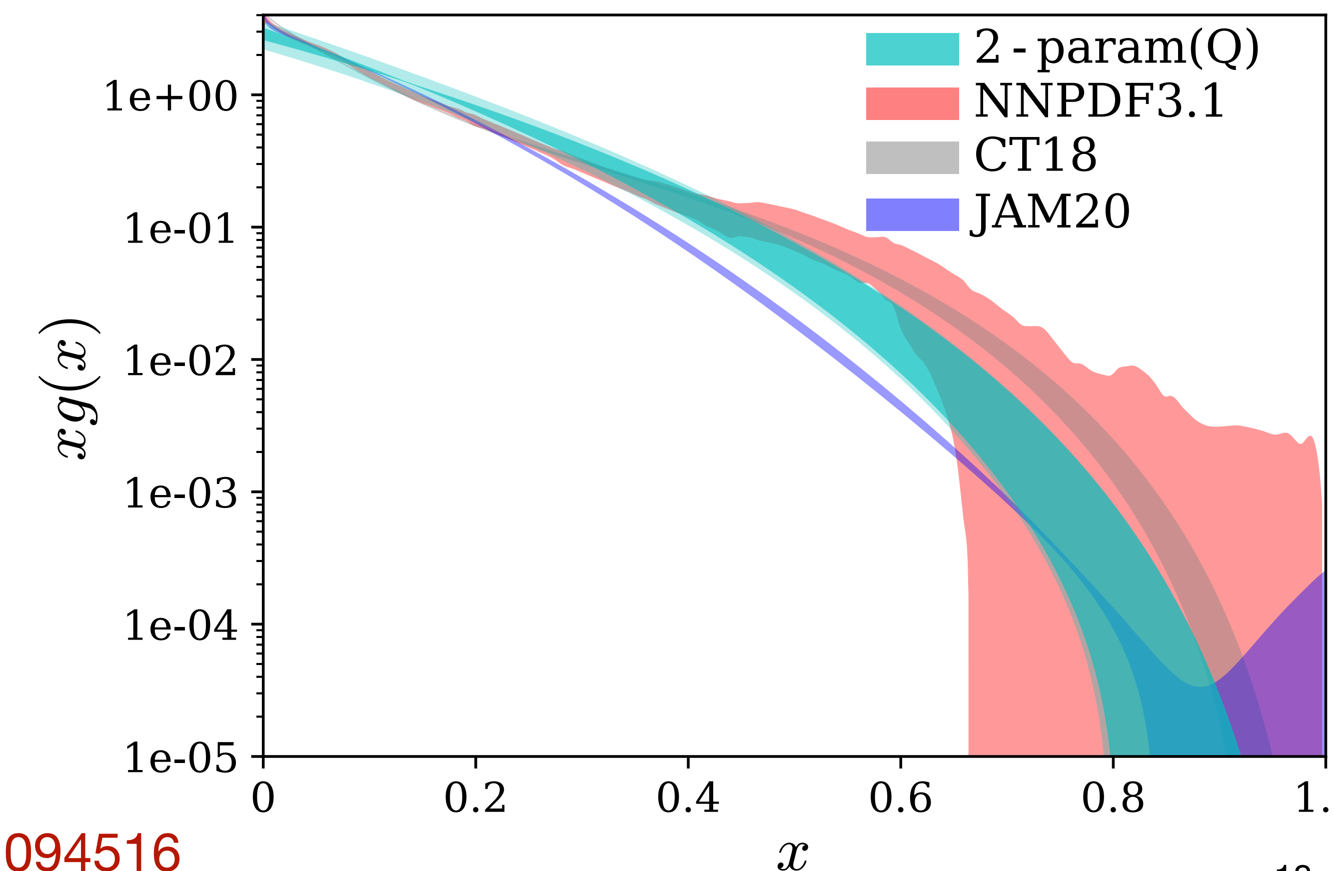
Unpolarized Gluon PDF



- Data modified by NLO formula ignoring quark mixing
- Data normalized by $\langle x \rangle_g$

$a = 0.094$ fm
$m_\pi = 358$ MeV

- ITD fit to cosine transform of $xg(x) = x^a(1-x)^b/B(a+1,b+1)$
- Qualitative agreement with global analysis



T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\left(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\left(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{zz}$$
$$+(sz)\left(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{zp}$$
$$+(sz)\left(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\left(p_\mu z_\alpha - p_\alpha z_\mu\right)\left(p_\lambda z_\beta - p_\beta z_\lambda\right)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\left(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\right)\widetilde{\mathcal{M}}_{gg}$$

$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \left(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\right)\widetilde{\mathcal{M}}_{sp}$$
$$+\left(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\right)\widetilde{\mathcal{M}}_{ps}$$
$$+\left(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\right)\widetilde{\mathcal{M}}_{sz}$$
$$+\left(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\right)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

Want: $M_{\Delta g}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\right]$

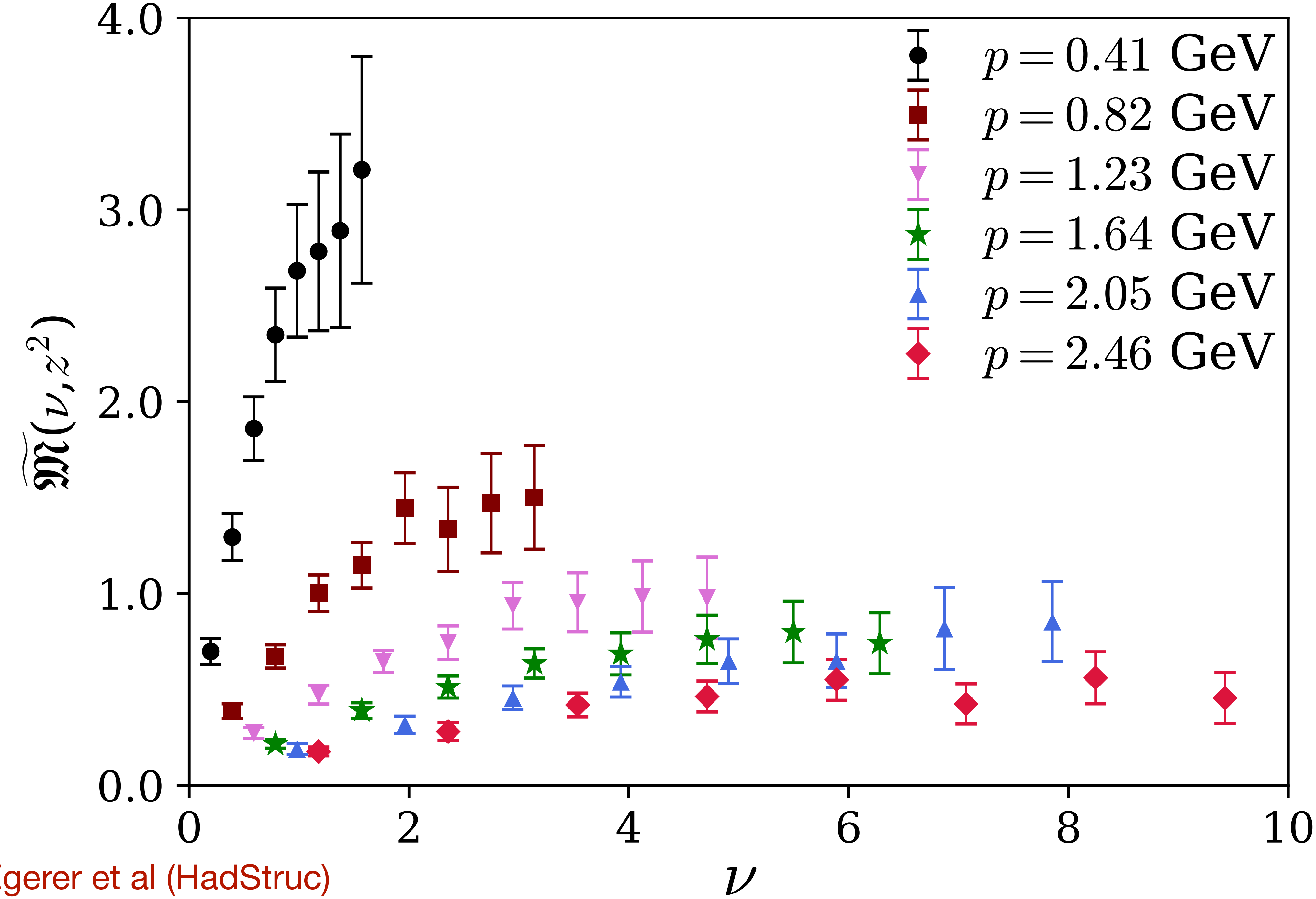
Can get: $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$

$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$

$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon Matrix Element

- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$ will need to be removed

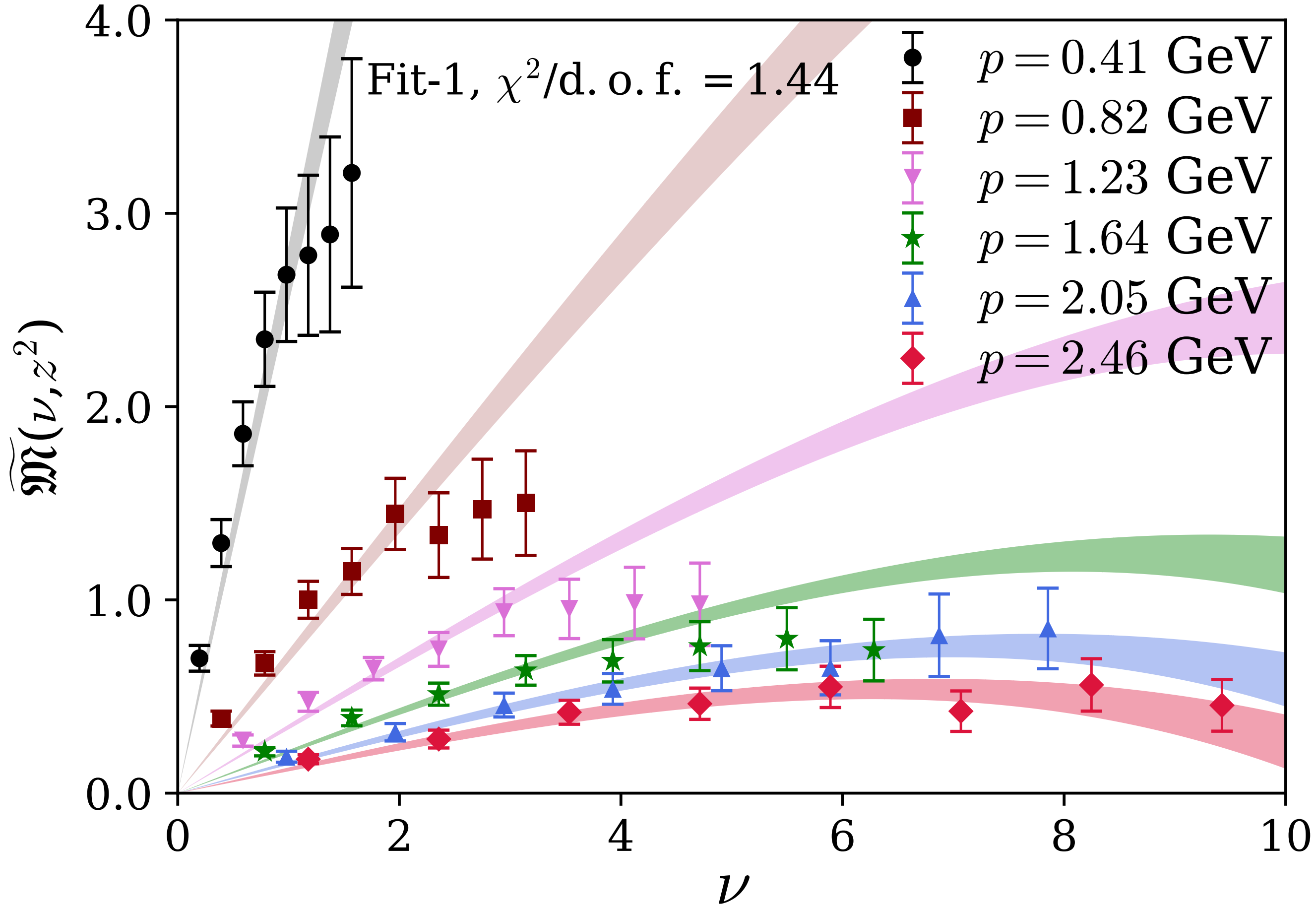


$a = 0.094$ fm
$m_\pi = 358$ MeV

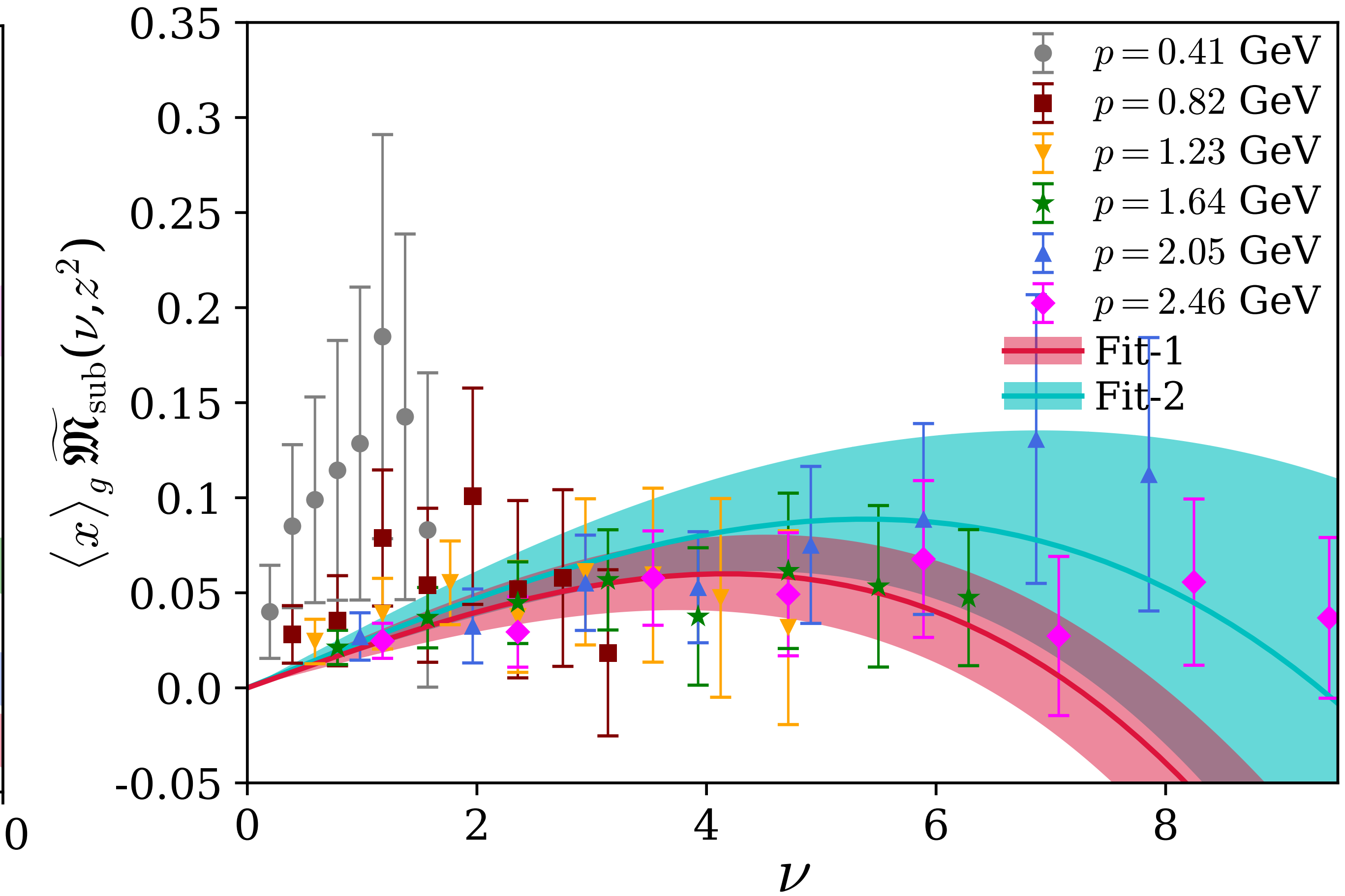
C. Egerer et al (HadStruc)
arXiv: 2207.08733

Helicity Gluon PDF

- Model both terms



- Subtract rest frame



$a = 0.094$ fm

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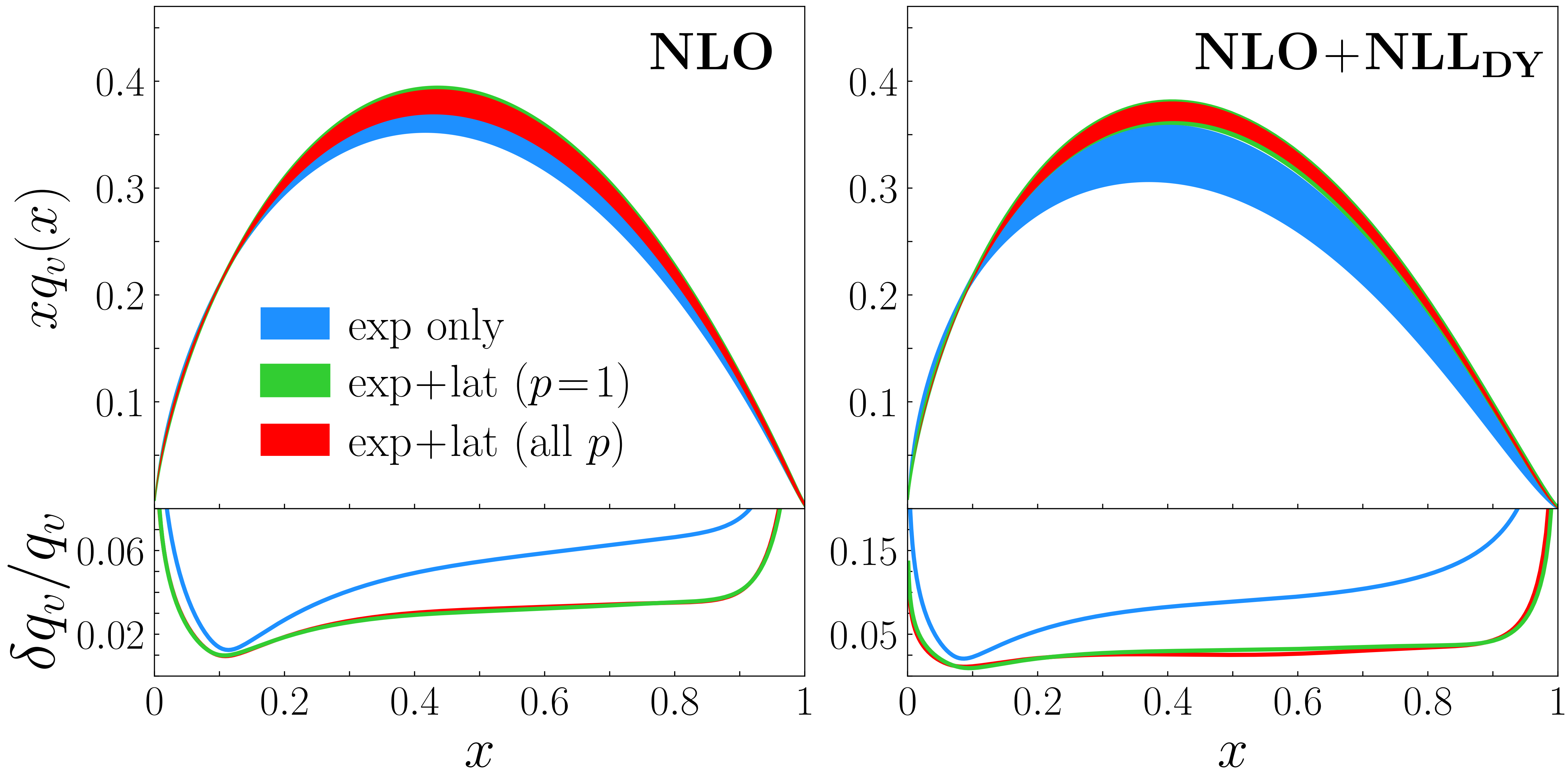
C. Egerer et al (HadStruc) arXiv: 2207.08733

Lattice data aren't that different from experimental data

Combining Lattice and Experiment

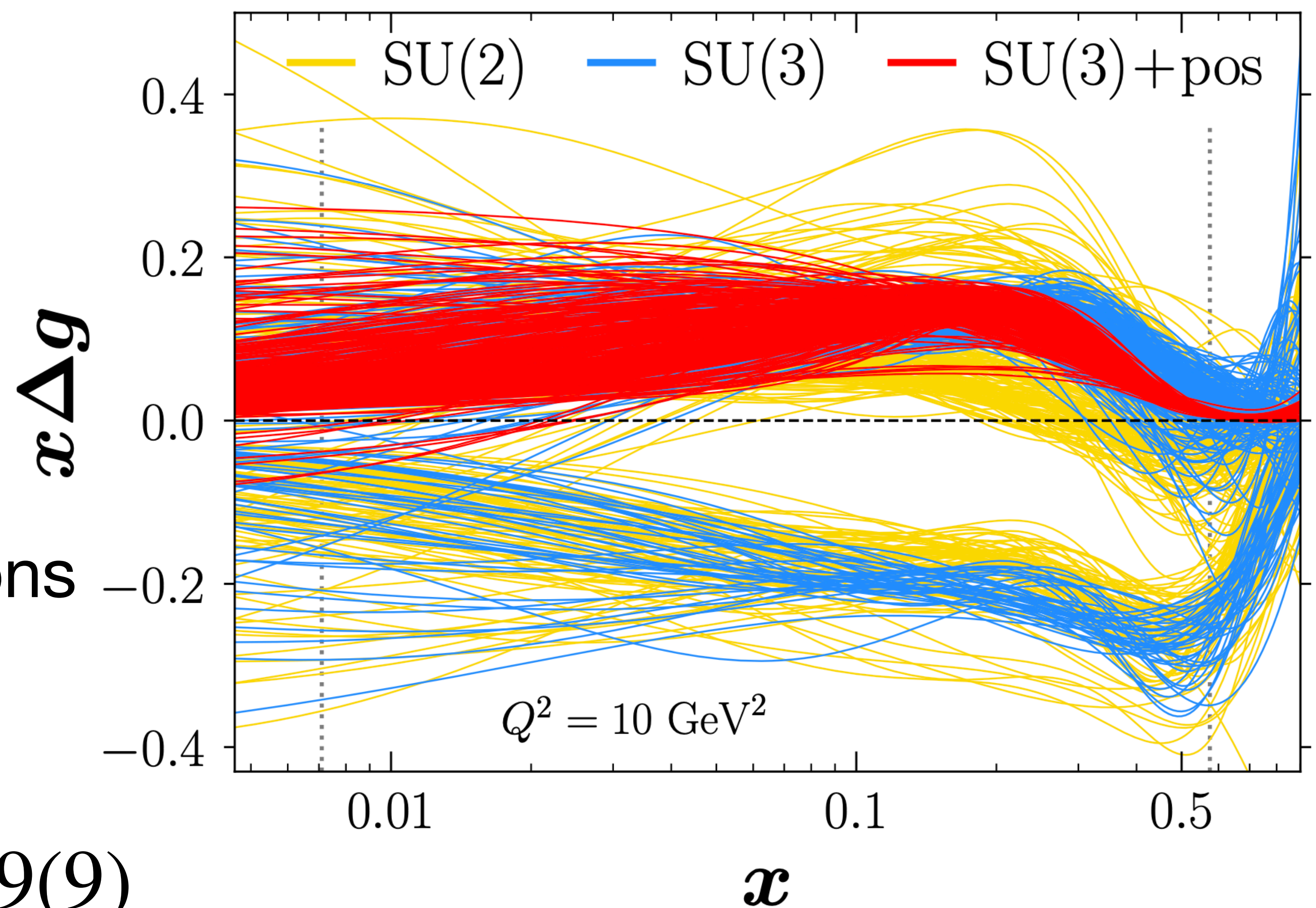
- Simultaneously fit Lattice and Experimental pion PDF data
- Each gives unique information complementing each other



P. Barry et al (HadStruc and JAM), *Phys. Rev. D* 105 (2022) 11, 114051

Spinning gluons (revisited)

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g(x)$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

- Without constraint: $\Delta G = 0.3(5)$

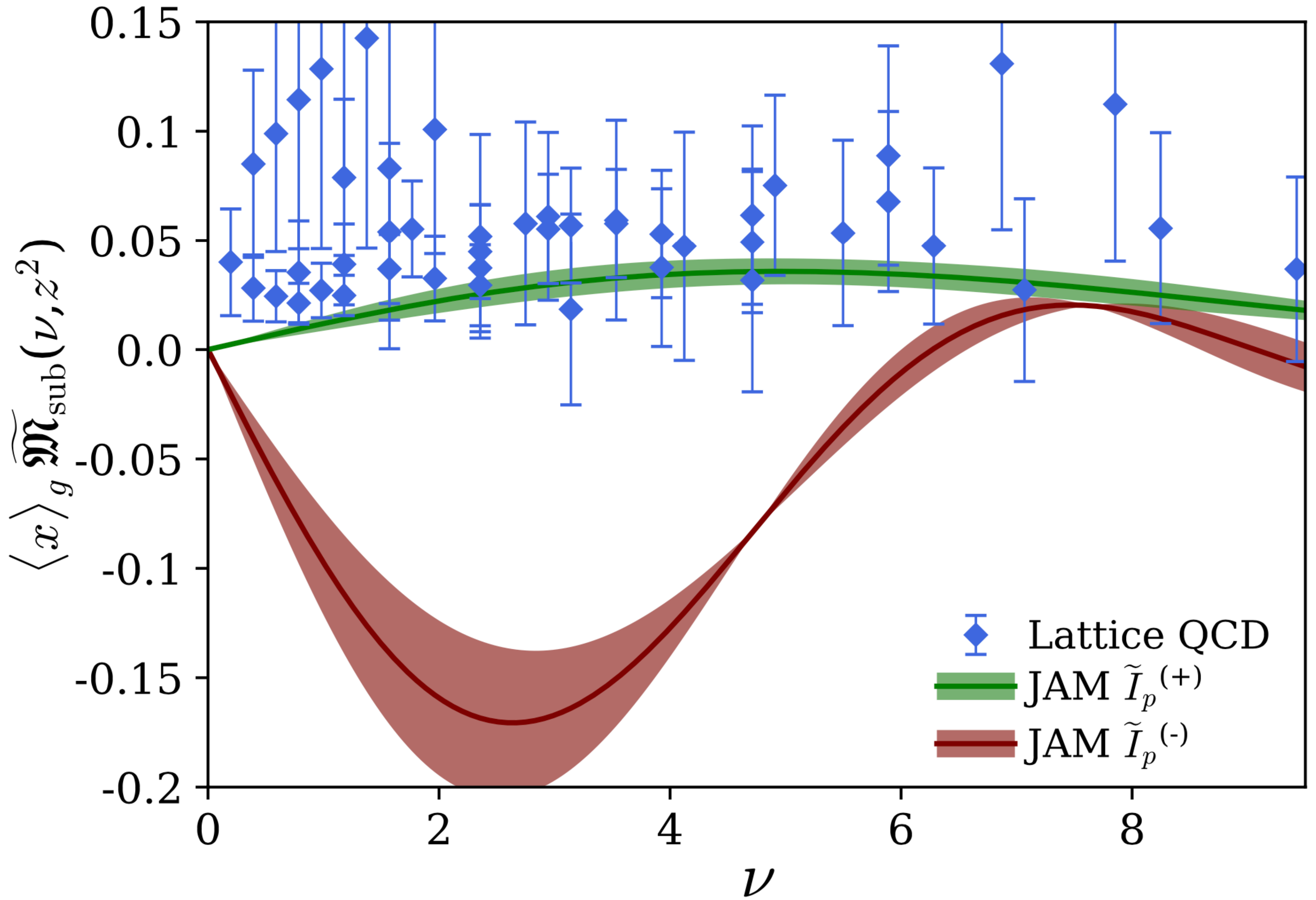
- Lattice: $\Delta G = 0.251(47)(16)$

$$\Delta G = \int dx\, \Delta g(x)$$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

Spinning gluons (revisited)

Can lattice data affect phenomenological polarized gluon analysis?



$$\Delta G = \int d\nu \, I_g(\nu)$$

- The positive and negative solutions without positivity constraints plotted in ν space

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733

- Only positive band consistent with lattice data

Conclusions

- Modern Lattice QCD can find parton x or ν dependent structure
  - High precision results require complex techniques
  - Precision needed to impact phenomenology is possible
  - Need to improve systematic errors at this precision
- Possibly impact phenomenological PDF analyses
  - **Today:** with pion PDFs and polarized gluon PDF
  - **Future:** JLab 12GeV and EIC data on PDF and on new distributions

Thank you and the organizers!