Calculating GPDs in Lattice QCD: Recent developments

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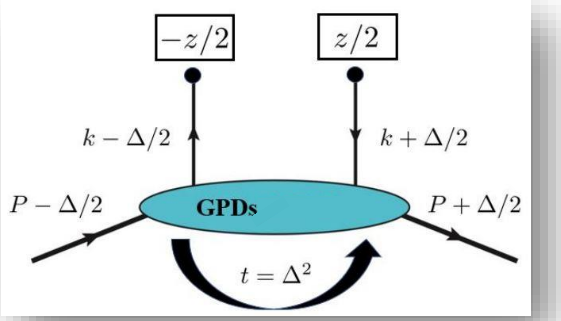
RIKEN BNL

27 June 2023

GPDs

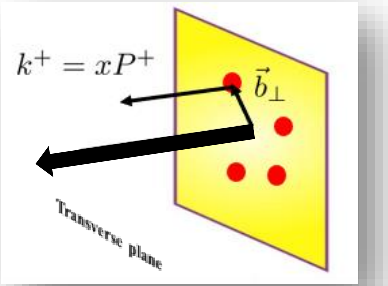


GPD correlator: Graphical representation

Definition:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Motivation for GPD studies



3D imaging (Burkardt, 0005108 ...)

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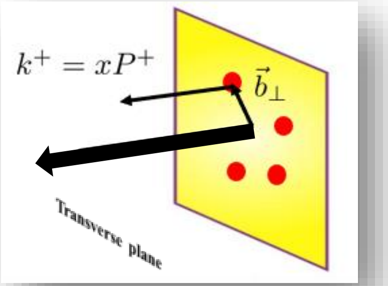

3D imaging (Burkardt, 0005108 ...)

Spin sum rule & orbital angular momentum (Ji, 9603249):

$$J^q = \int_{-1}^{1} dx\, x(H^q + E^q)|_{t=0}$$

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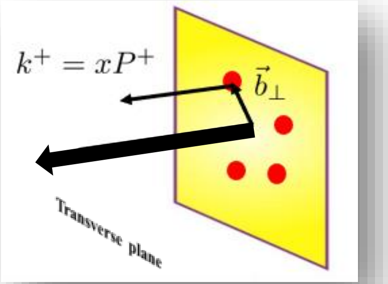

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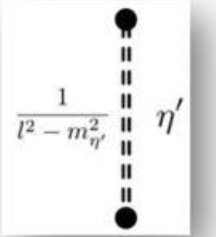
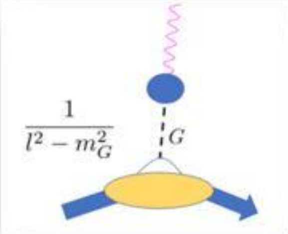
Imprints of chiral/trace anomalies in GPDs (SB, Hatta, Vogelsang, 2305.09431):



Eta-meson mass generation

$$g_P \sim \int dx \tilde{E}(x) \sim \frac{1}{l^2 - m_{\eta'}^2}$$



Glueball mass generation

$$A + \xi^2 D \sim \int dx\, xH(x) \sim \frac{1}{l^2 - m_G^2}$$

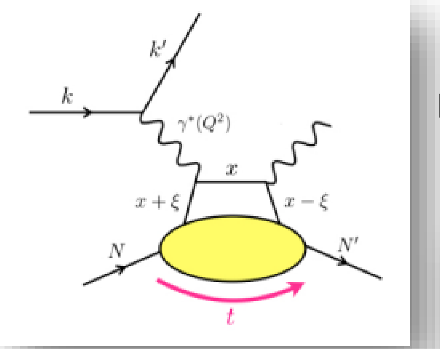
Novel avenue of GPD research

Profound physical implication of anomaly poles:

Touches questions on mass generations, Chiral symmetry breaking, ...

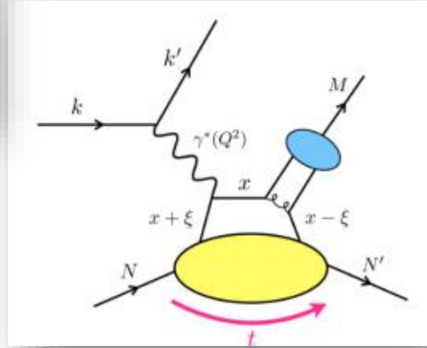
Motivation for GPD studies

Physical processes:



Deep Virtual Compton Scattering

See talks by Silvia, Charlotte, Karolina, Marija



Exclusive meson production

Amplitude: $\mathcal{M} \propto \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x \pm \xi + i\epsilon}$ **x-dependence lost!**

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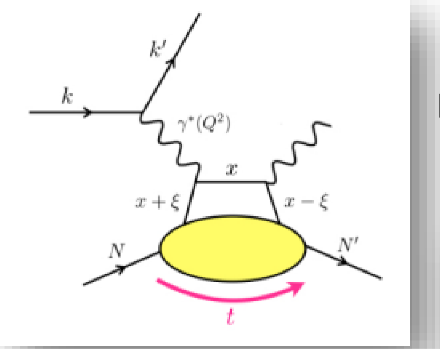

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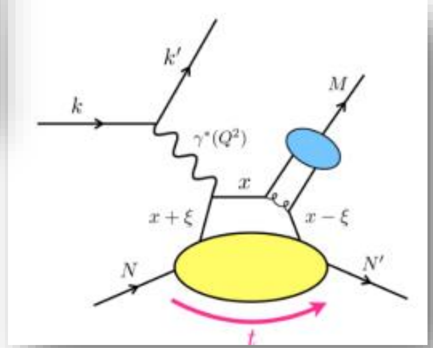
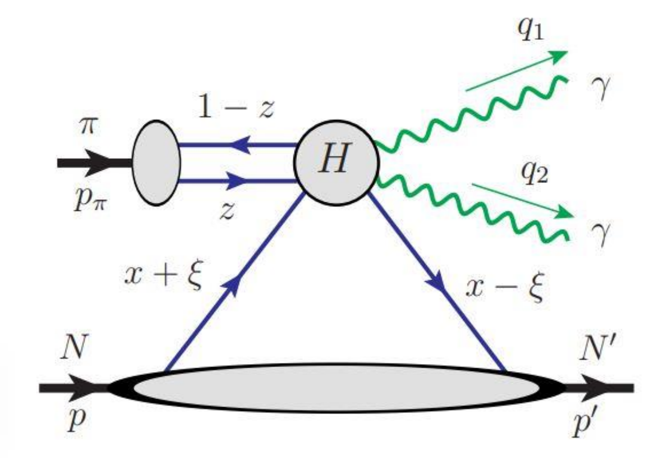


Exclusive meson production

Exclusive massive pair production

See Zhite Yu's talk

Access to x-dependence



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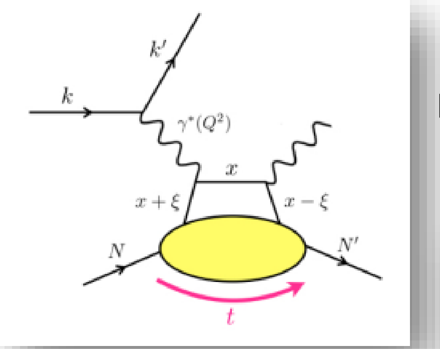

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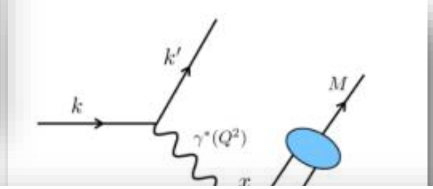

Exclusive meson production

We need GPD measurements from Lattice QCD

Exclusive massive pair production

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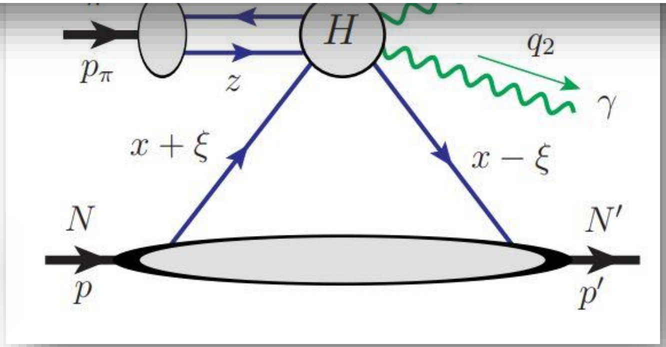


Can we extract these quantities from **lattice QCD**?

Physical processes:

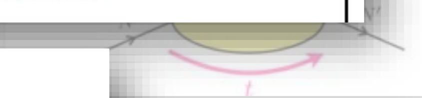
Light-cone (standard) correlator $-1 \leq x \leq 1$

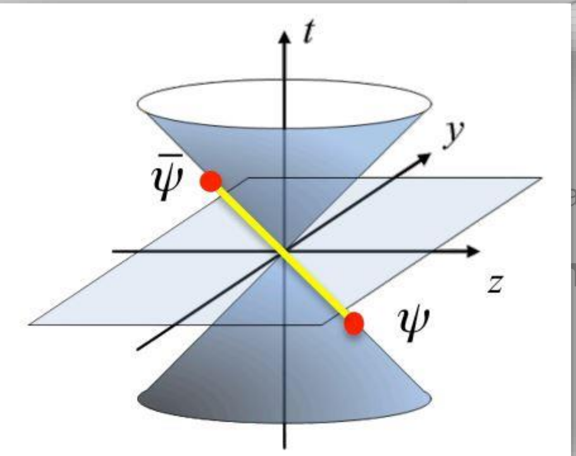
$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**

cattering

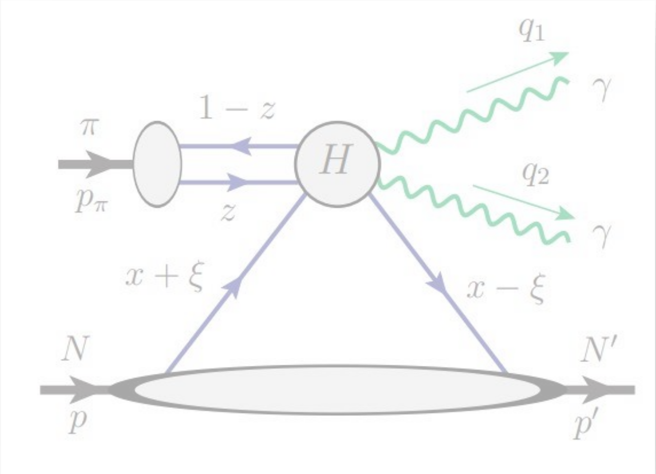
Exclusive meson production





assive pair production

nite Yu's talk



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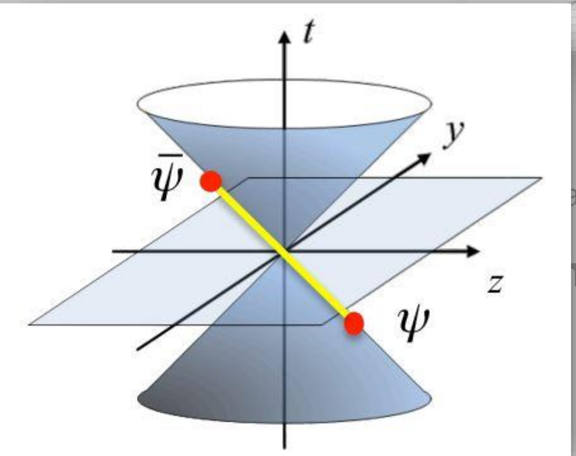

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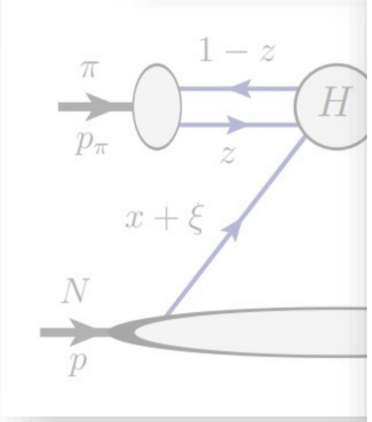
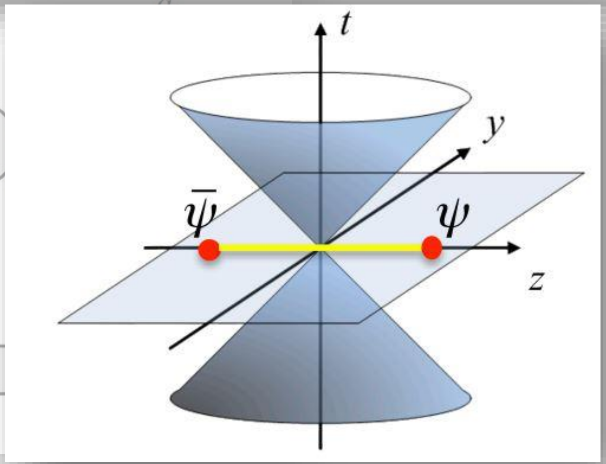


"Auxiliary" distributions

Correlator for quasi-GPDs (Ji, 2013) $-\infty \leq x \leq \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**



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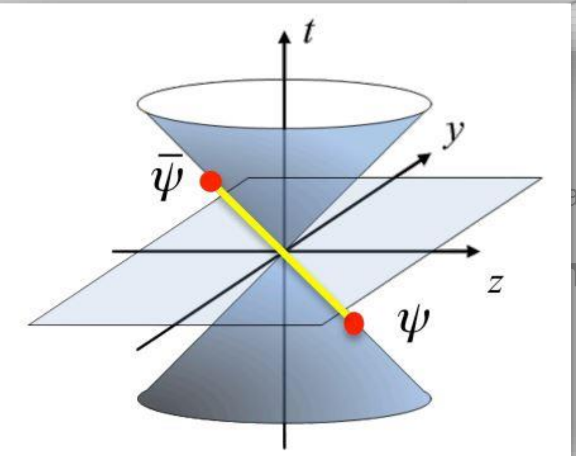

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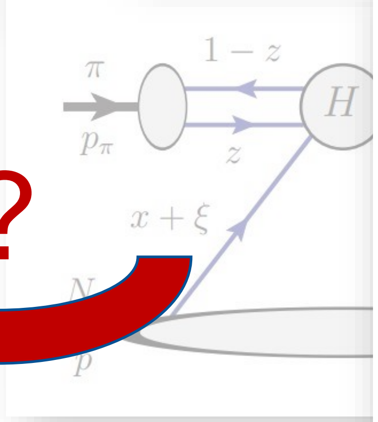

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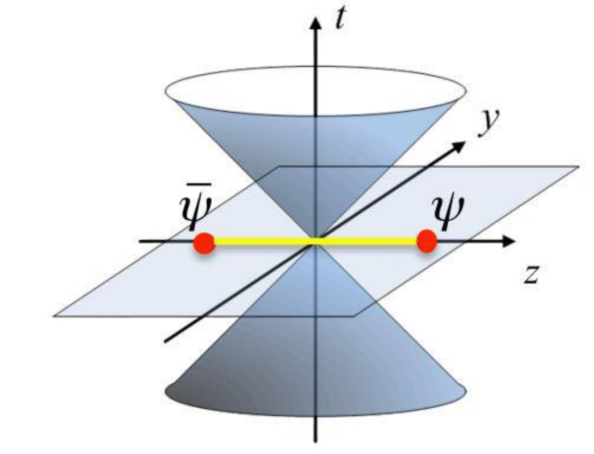

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$$F_{\mathrm{Q}}^{[\Gamma]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z}$$

Matching formula:

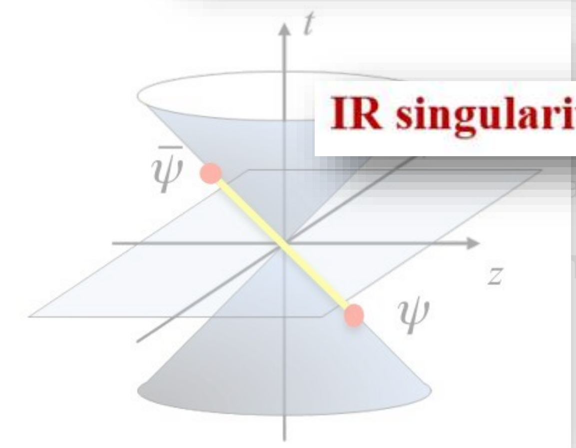
$$q_{\mathrm{Q}}(x; P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

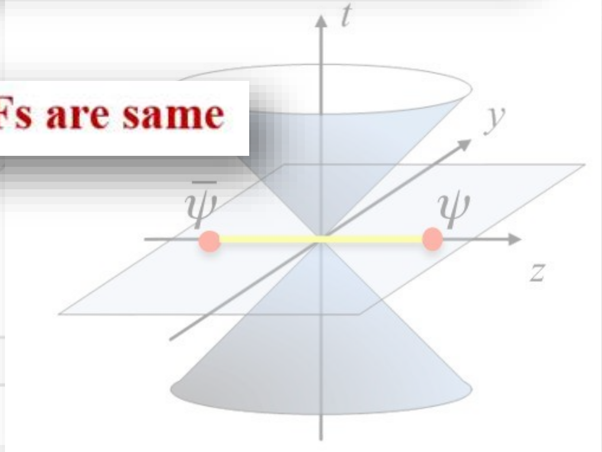
(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

Matching coefficient

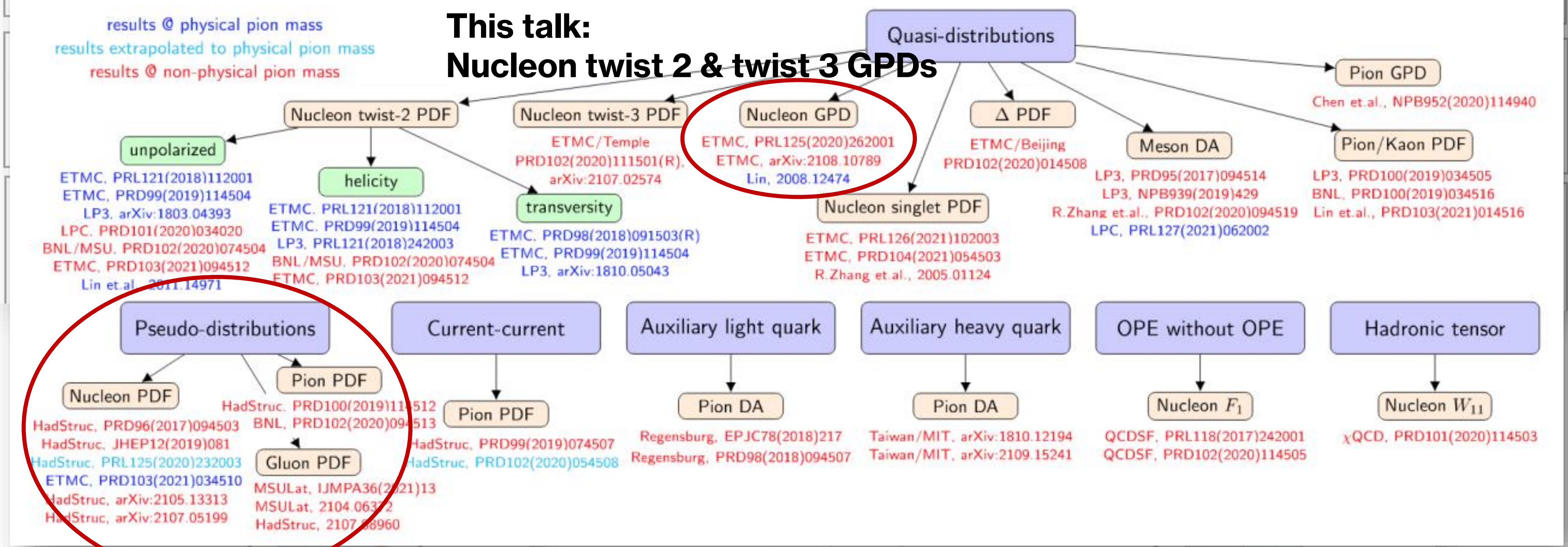
IR singularities of quasi-PDFs & light-cone PDFs are same





LQCD calculations of x-dependence of PDFs & related quantities, Using Euclidean correlators (compilation by K. Cichy, 2110.07440)

This talk:
Nucleon twist 2 & twist 3 GPDs



See talk by Joseph

First Lattice QCD results of the x-dependent GPDs

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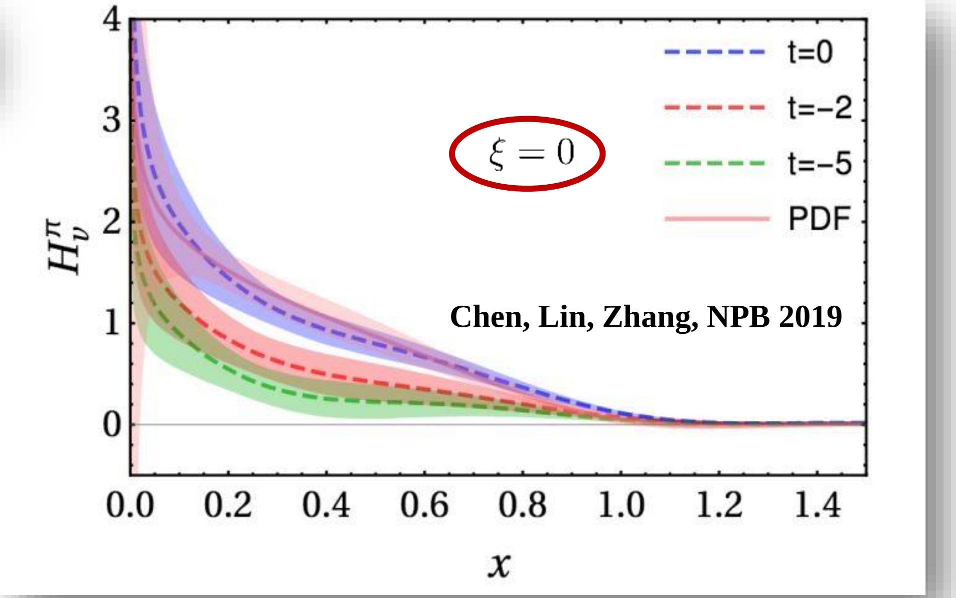
pion



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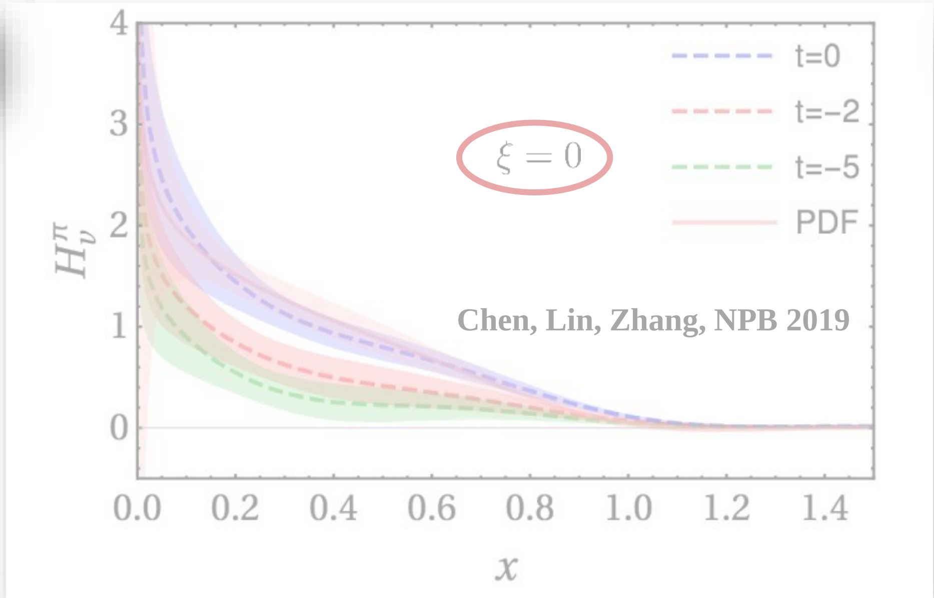
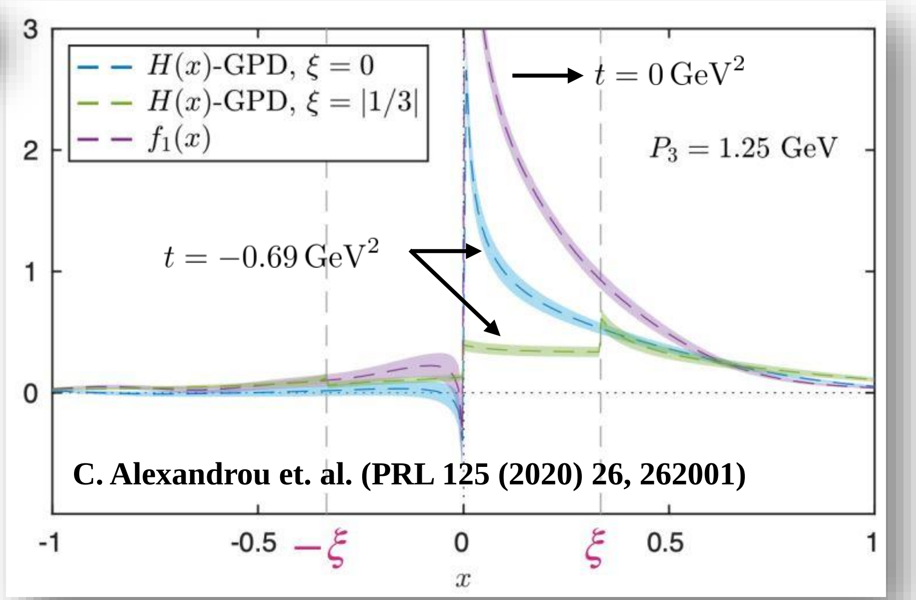


proton



ERBL/DGLAP: Qualitative differences

As $x \to 1$, qualitative behavior in agreement with power counting analysis

(F. Yuan, 0311288)

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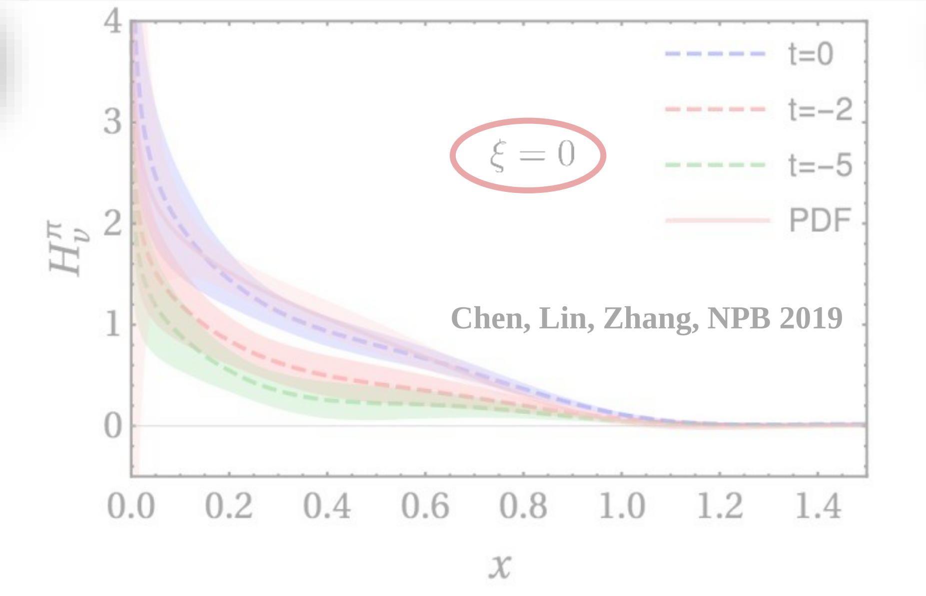

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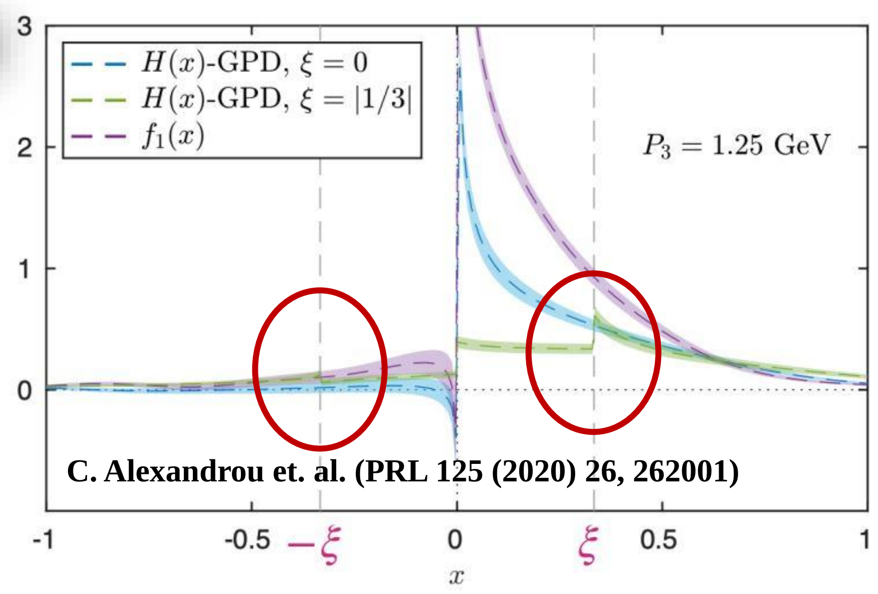

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At $x = \pm\xi$, matching formalism breaks down

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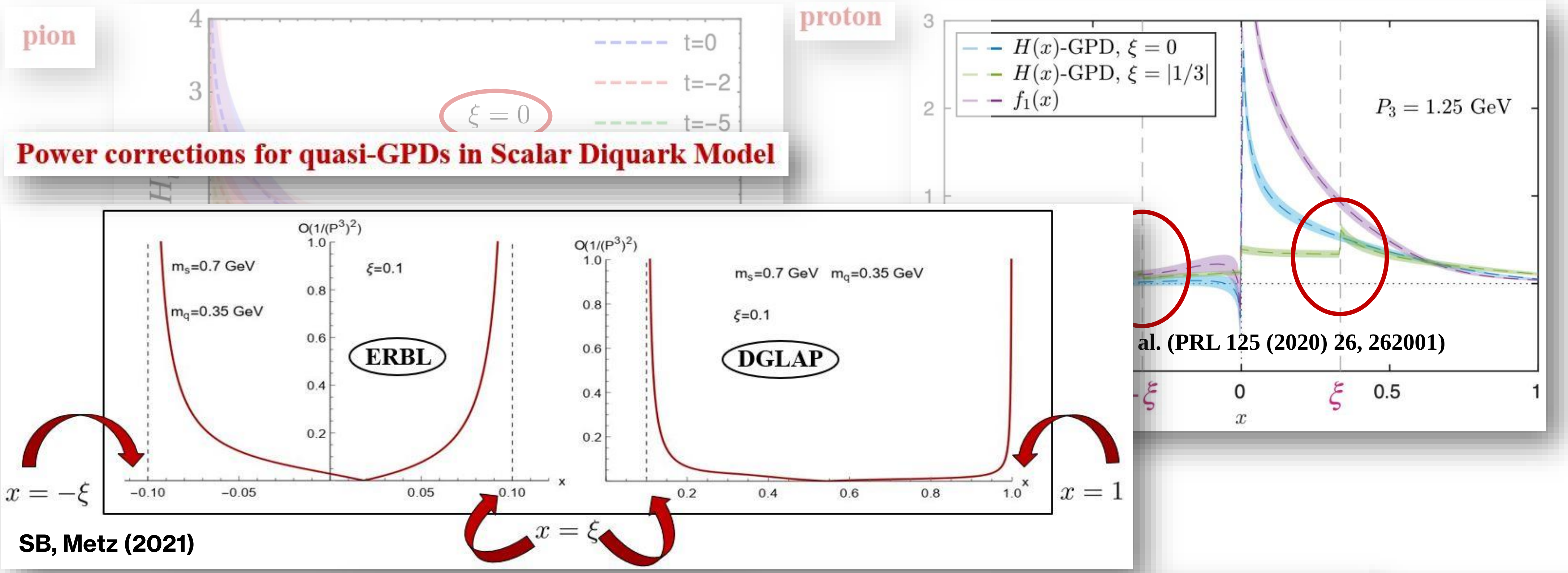

Our prediction regarding the structure of divergence: $q_Q(x) \approx \mathcal{O}\left(\frac{1}{(x+\xi)(x-\xi)(1-x)P_3^2}\right)$

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First Lattice QCD … he x-dependent GPDs

Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
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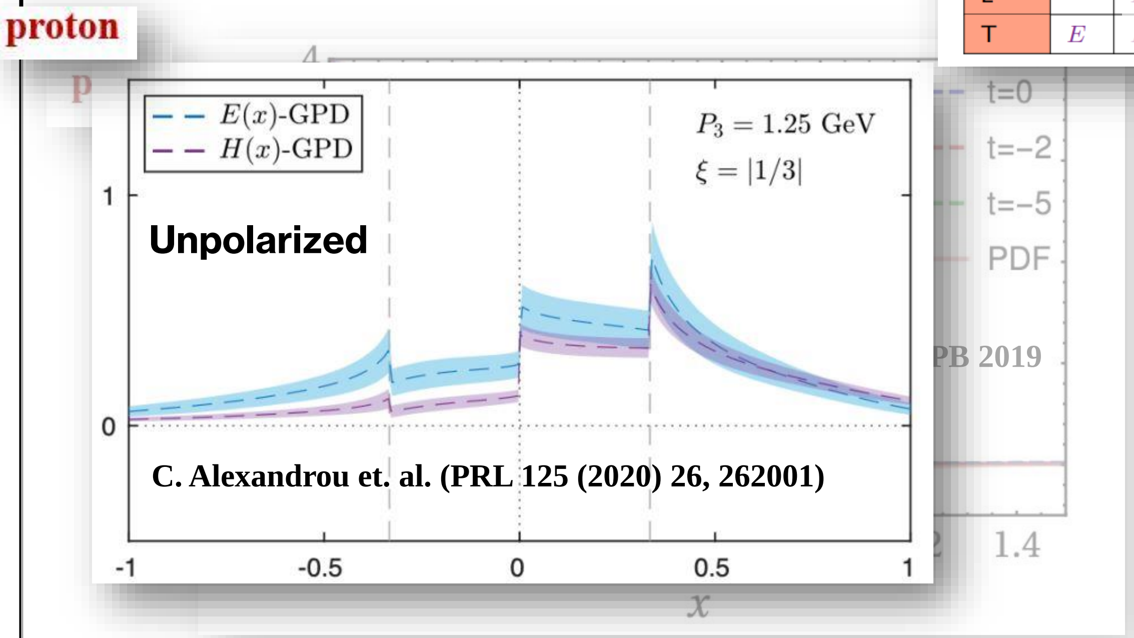

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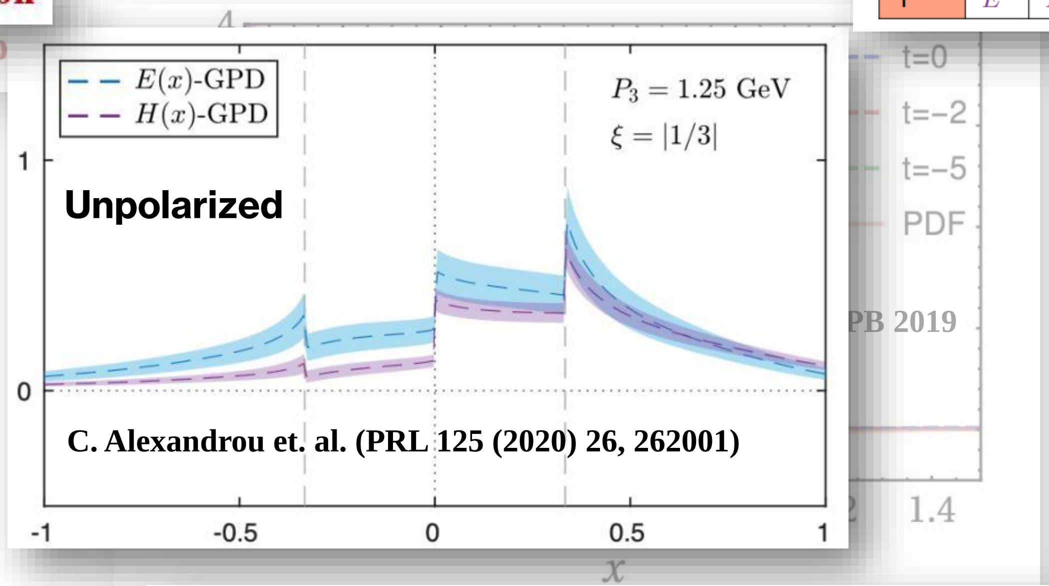

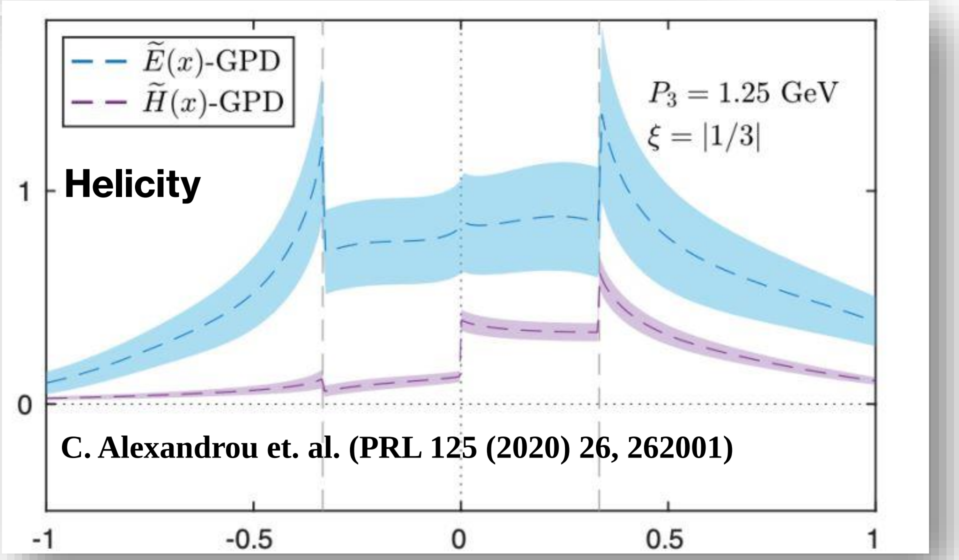

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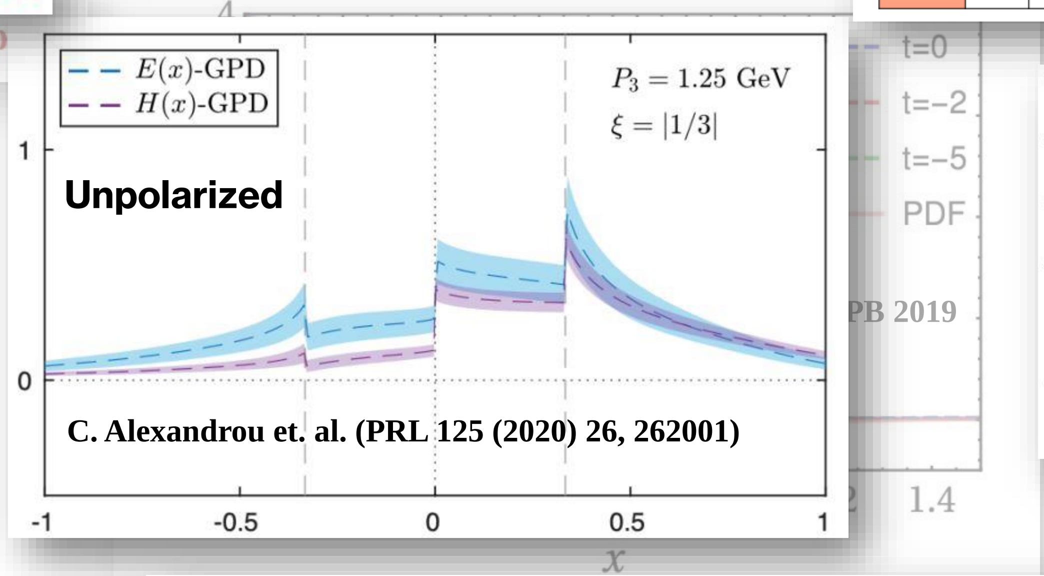
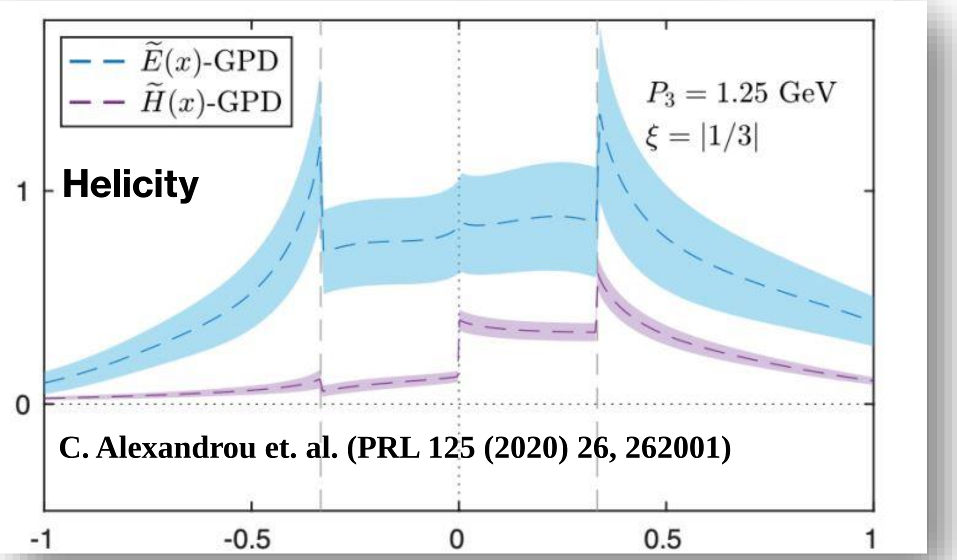


C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)



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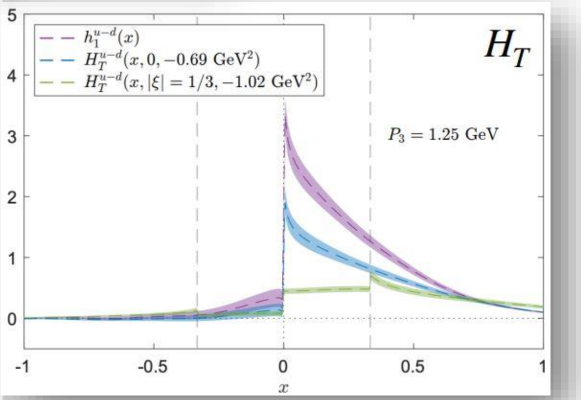
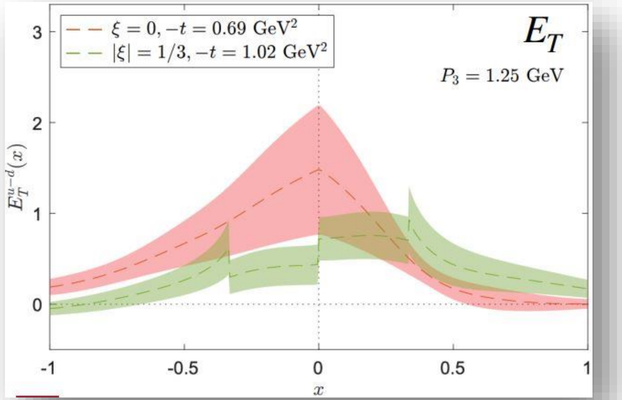
Transversity





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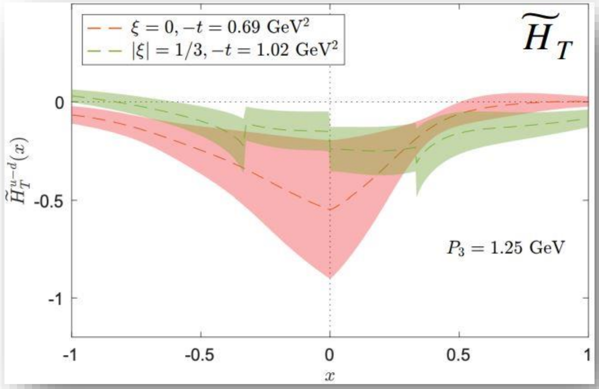
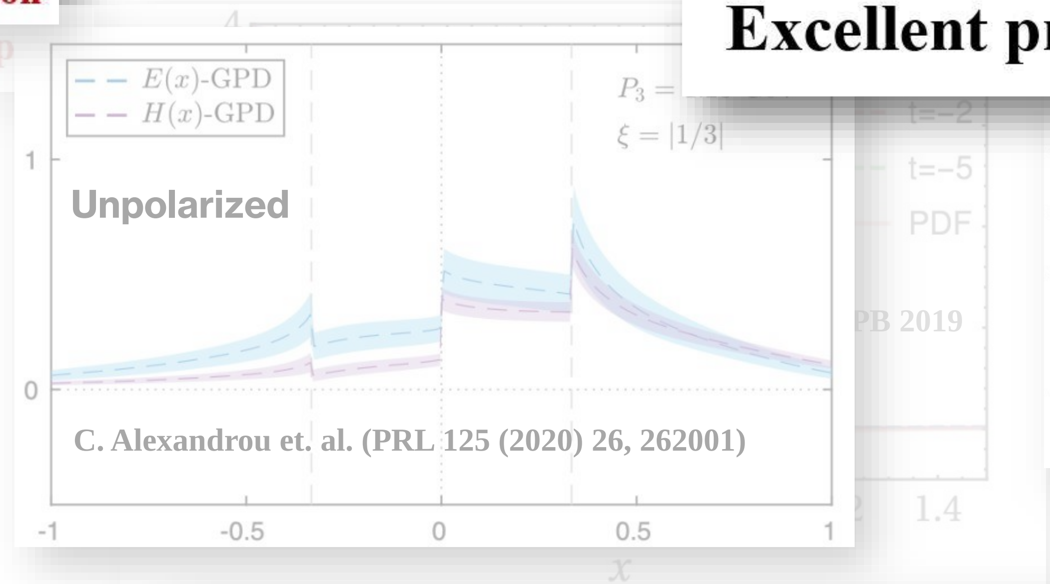


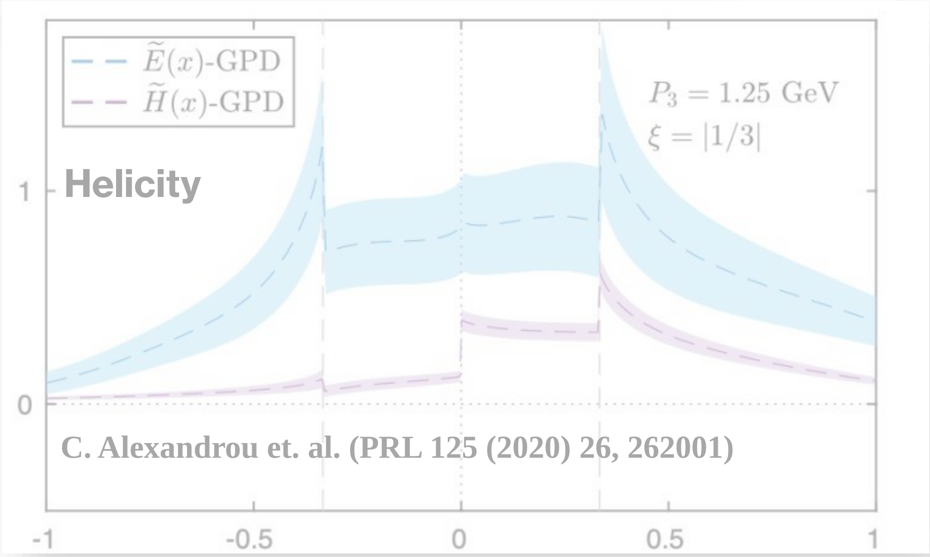
GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

First Lattice QCD results of the x-dependent GPDs

proton

Excellent progress!!!





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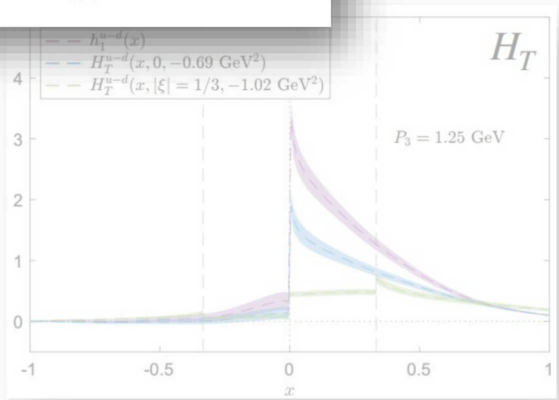

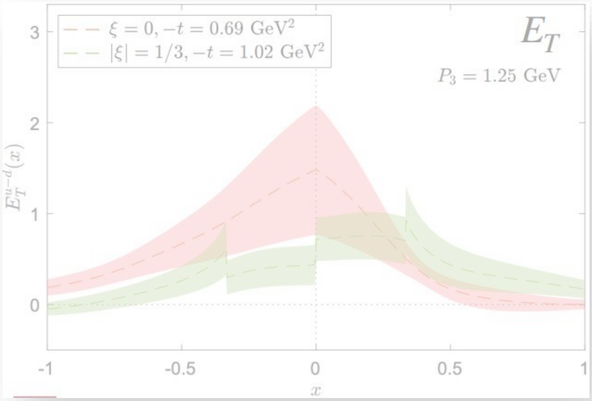

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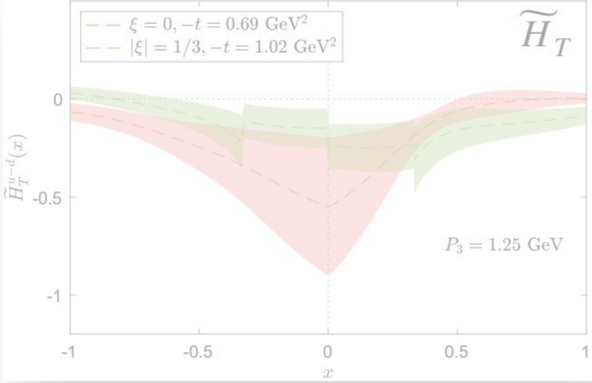

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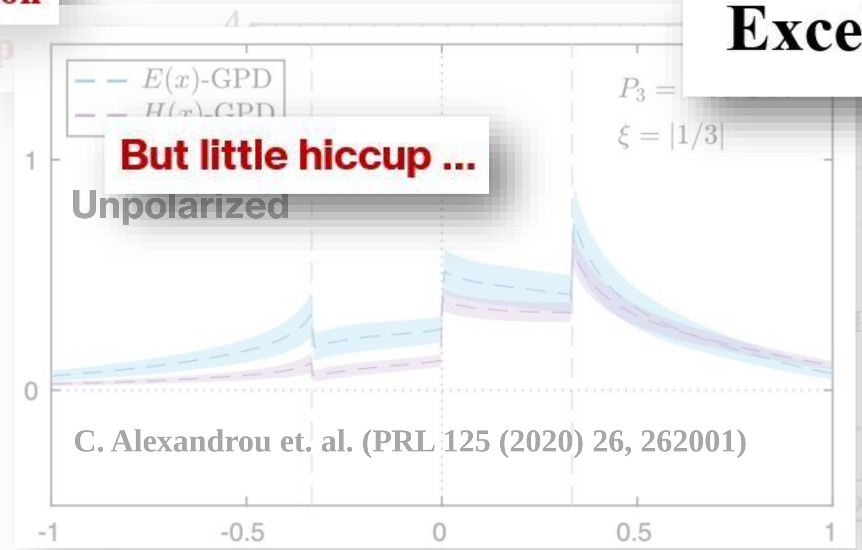
proton

Excellent progress!!!

But little hiccup ...

Unpolarized



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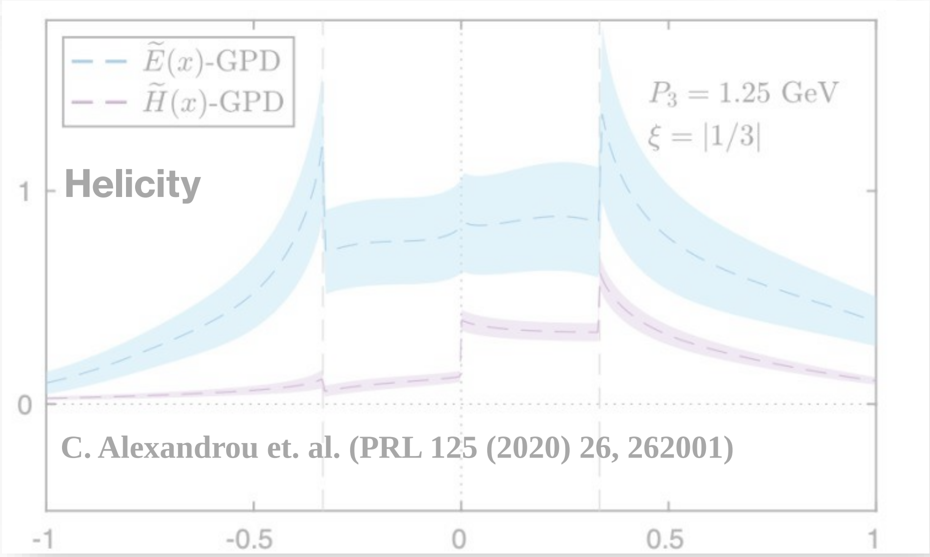
Helicity



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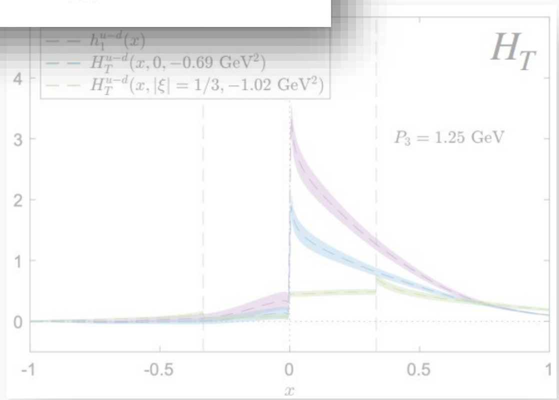

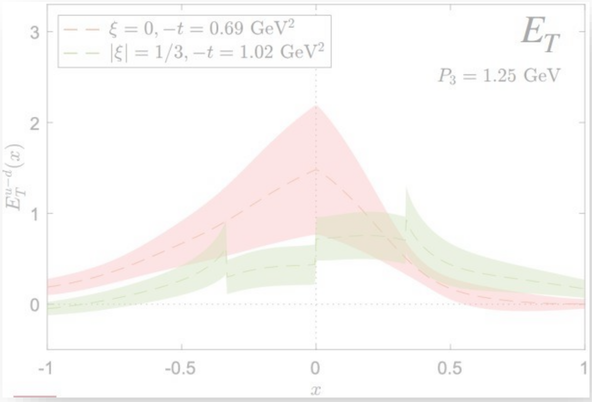

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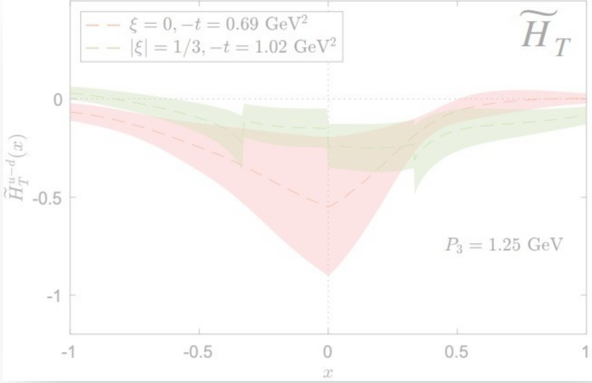
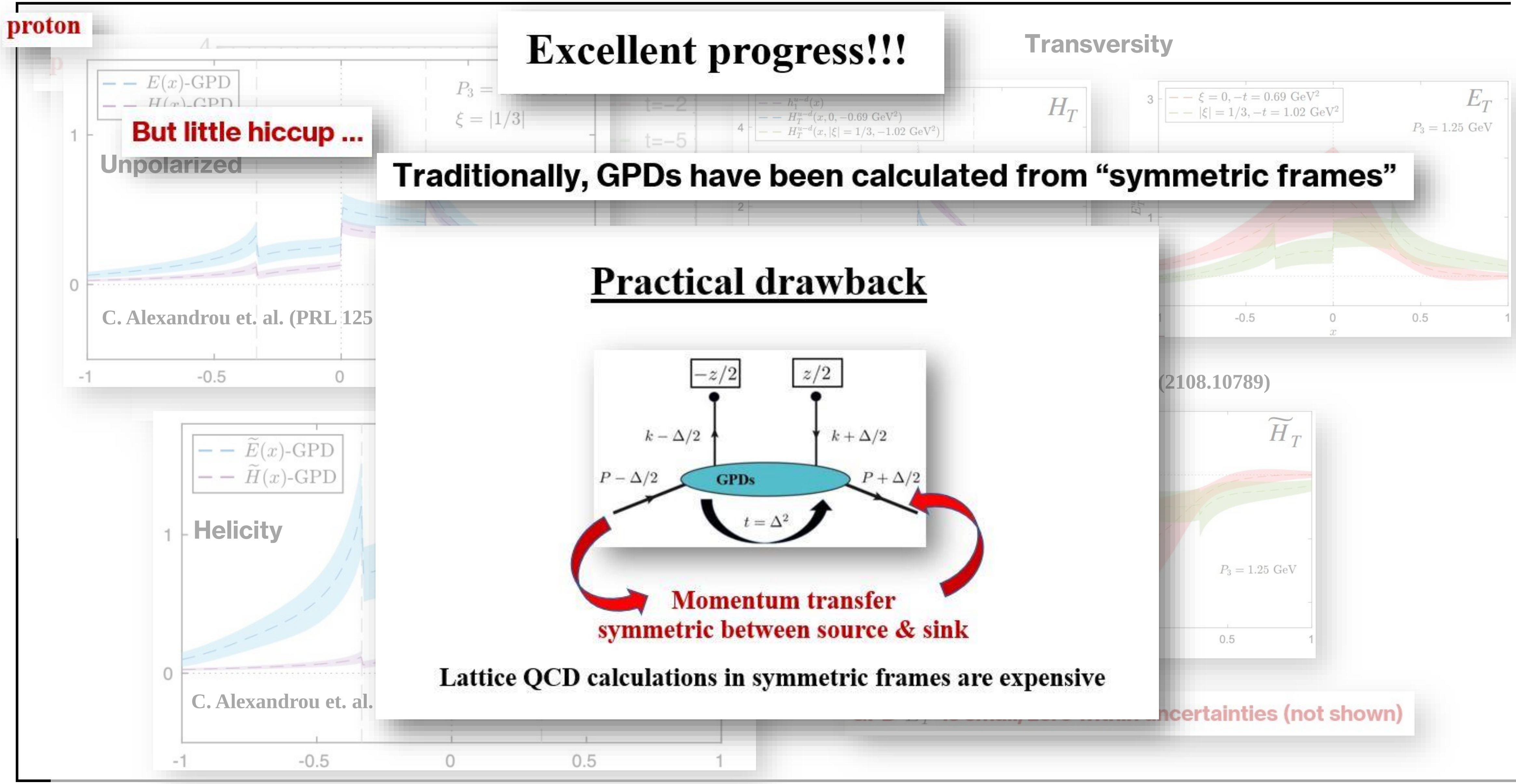


GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

First Lattice QCD results of the x-dependent GPDs



Excellent progress!!!

But little hiccup ...

Traditionally, GPDs have been calculated from "symmetric frames"

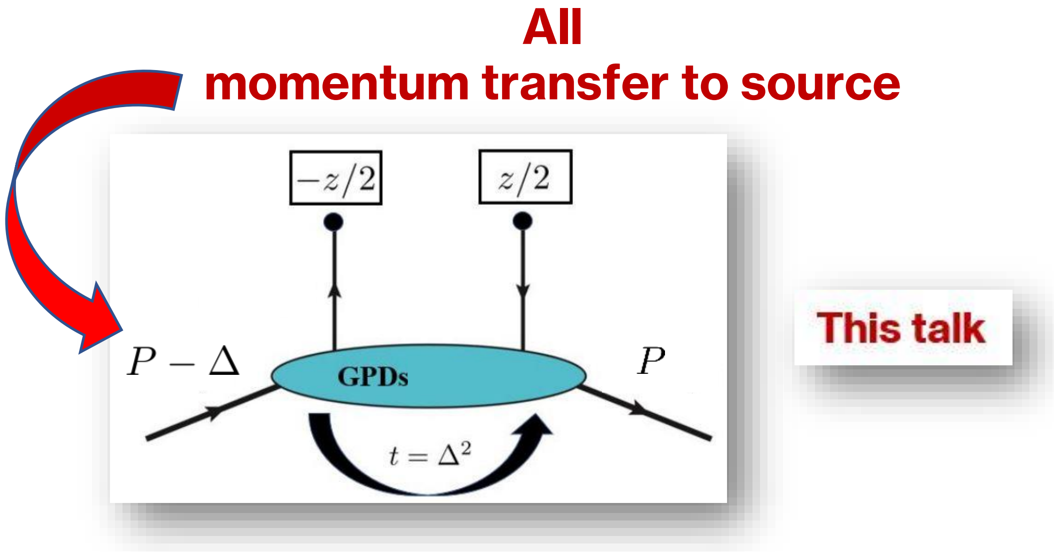
Practical drawback

Momentum transfer
symmetric between source & sink

Lattice QCD calculations in symmetric frames are expensive

Lattice QCD calculations of GPDs in asymmetric frames

Resolution:

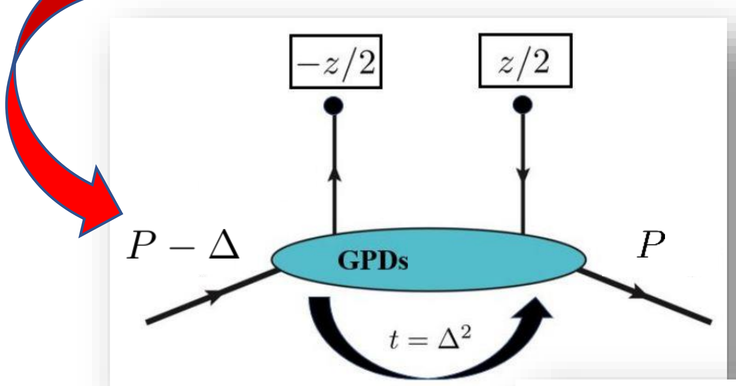


- **Perform Lattice QCD calculations of GPDs in asymmetric frames**

Lattice QCD calculations of GPDs in asymmetric frames

<u>Our contribution in a nutshell</u>:

All momentum transfer to source



This talk

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

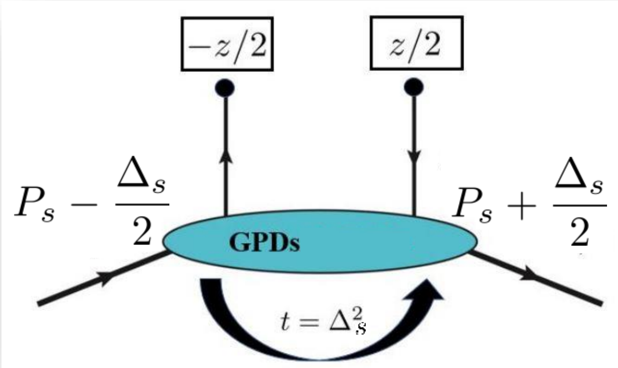
Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

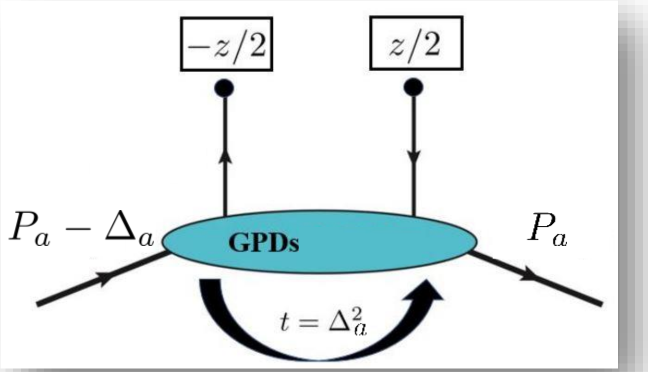
Key findings:

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames

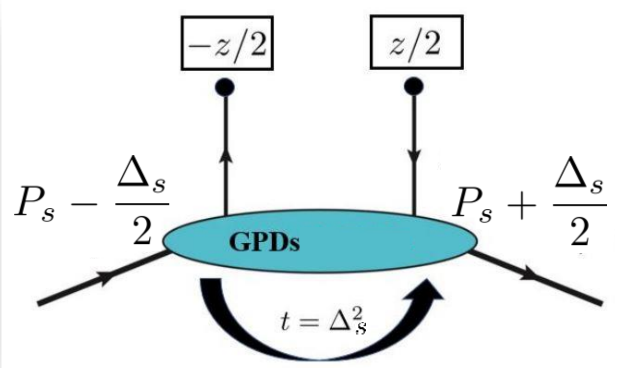




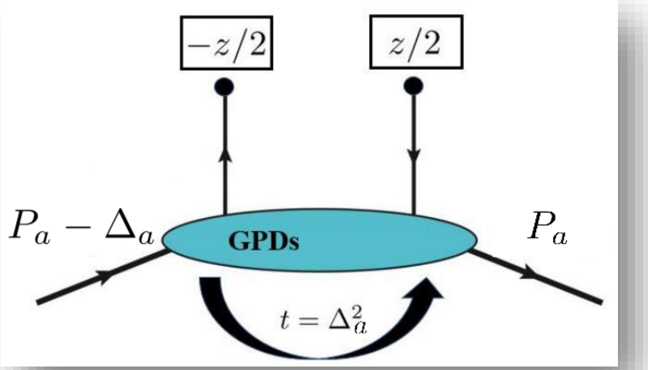
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



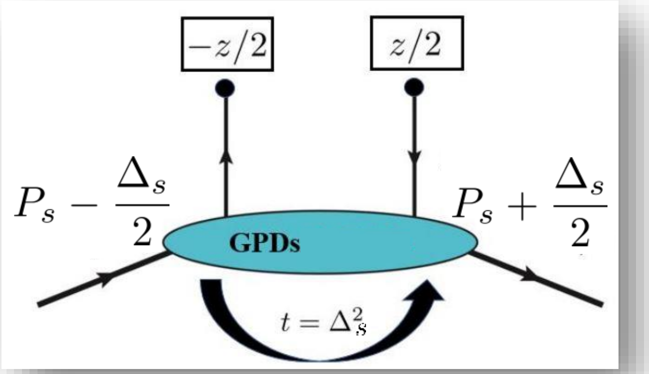
Related via
Lorentz transformation?



Yes, since symmetric & asymmetric frames are connected via Lorentz transformation

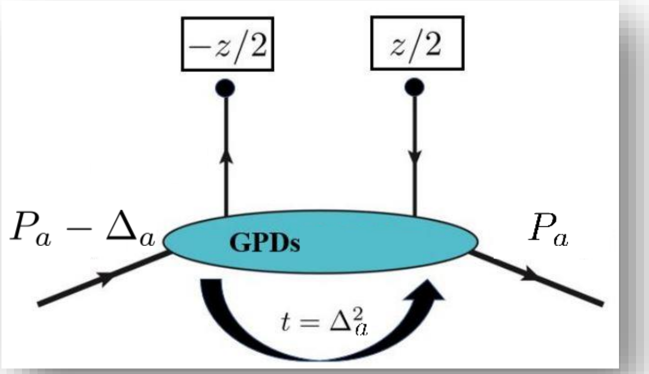
Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



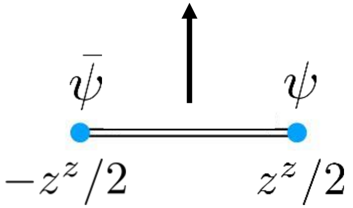
Related via
Lorentz transformation?

What kind?



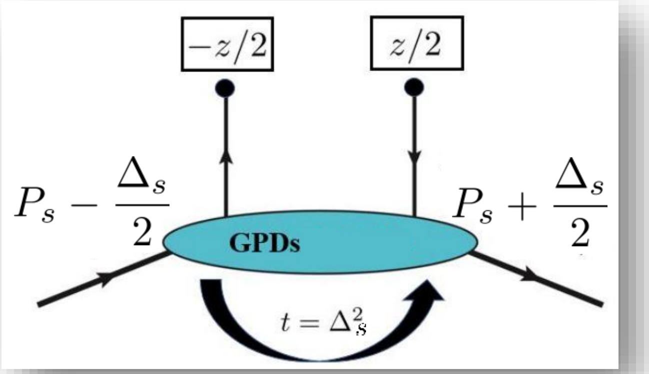
Case 1: Lorentz transformation in the z-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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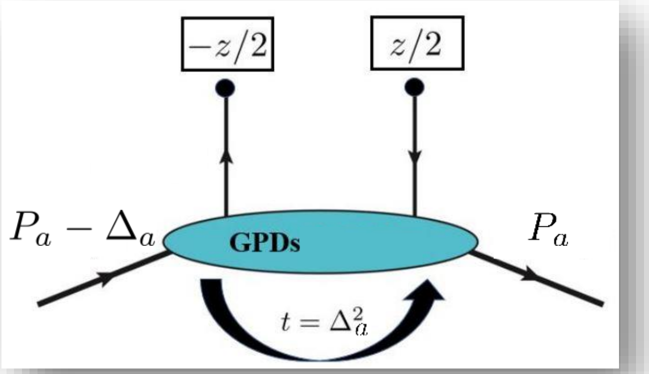

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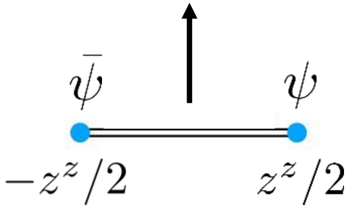


Results:

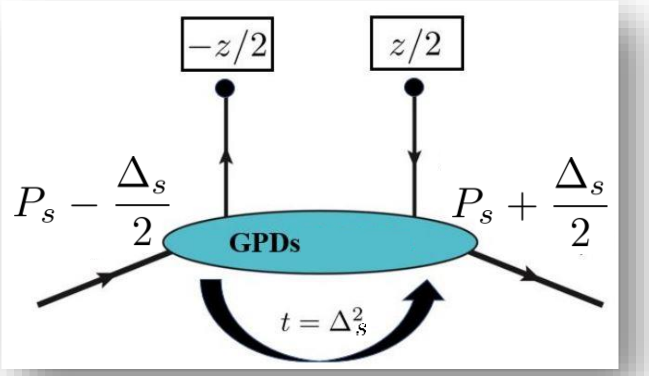
$$z_s^0 = -\gamma\beta z_a^z$$

$$z_s^z = \gamma z_a^z$$

Operator distance develops a non-zero temporal component

Lattice QCD calculations of GPDs in asymmetric frames

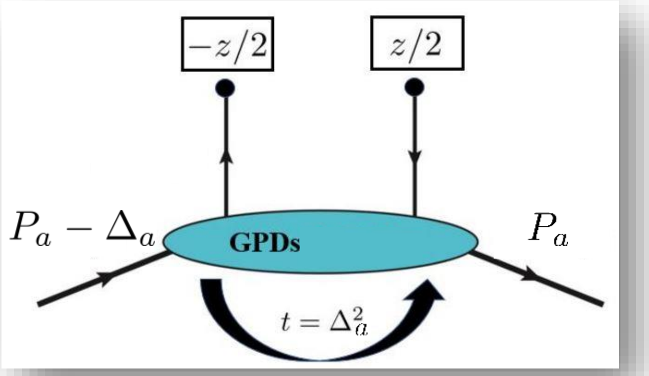
Symmetric & asymmetric frames



Related via
Lorentz transformation?

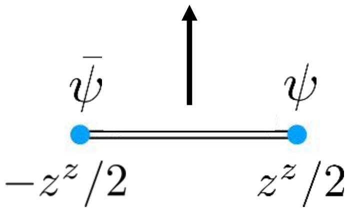
↔

What kind?



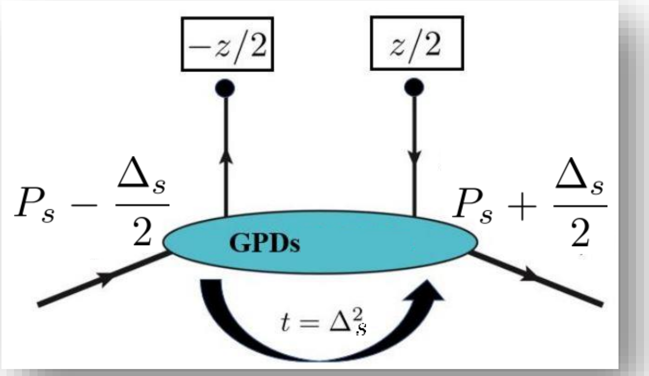
Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



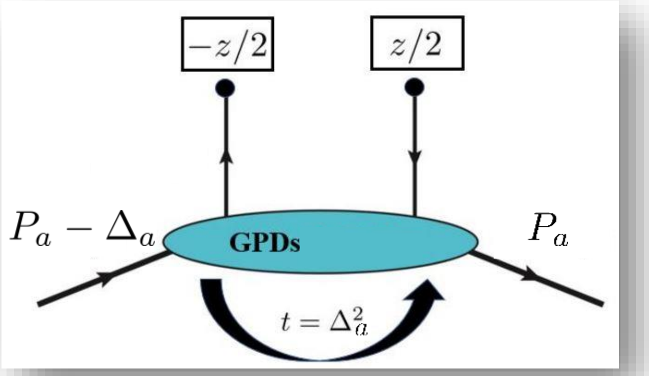
Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



Related via Lorentz transformation?

What kind?



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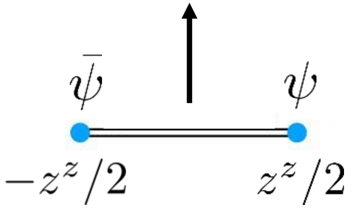


Results:

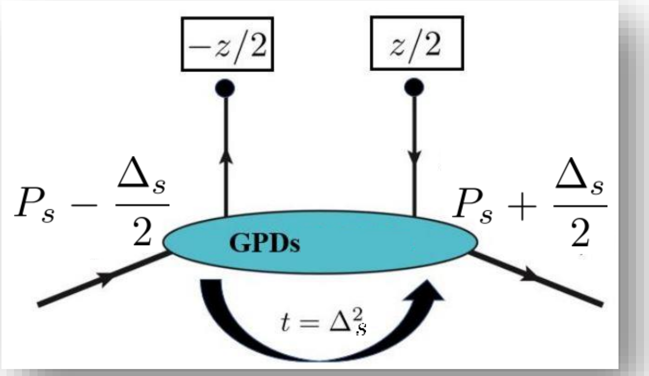
$$z_s^0 = 0$$

$$z_s^z = z_a^z$$

Operator distance remains spatial (& same)

Lattice QCD calculations of GPDs in asymmetric frames

Symmetric & asymmetric frames



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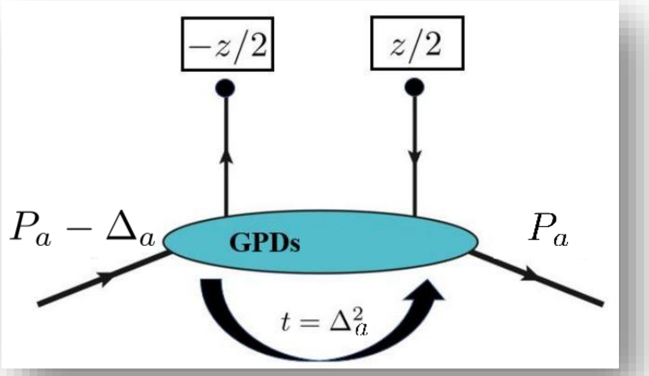


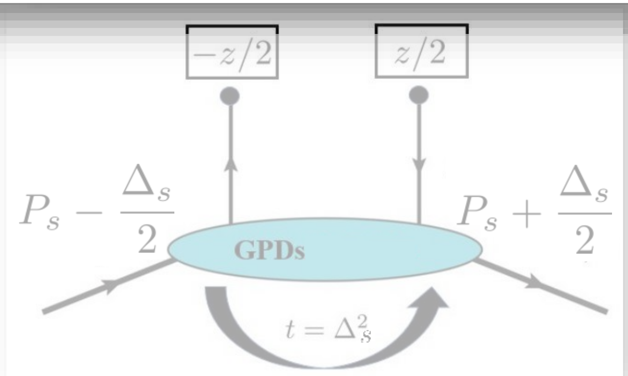
Case 2: Transv

Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

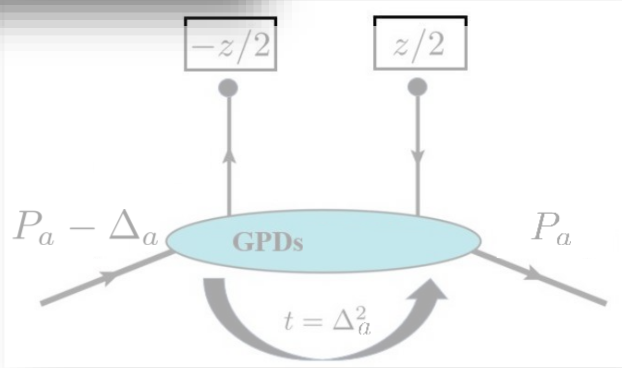
Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?



Related via
Lorentz transformation?

What kind?



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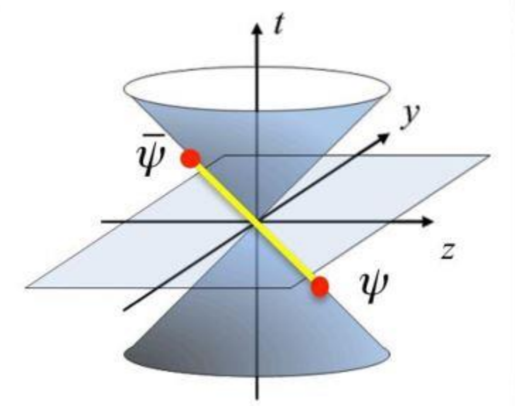
Lattice QCD calculations of GPDs in asymmetric frames

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

GPDs on the light-cone:



$$H(x,\xi,t) \to \int \frac{dz^-}{4\pi} e^{ixP\cdot z} \langle p'|\bar{q}\,\gamma^+ q|p\rangle \qquad z = (0, z^-, 0_\perp)$$

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\, q|p\rangle \qquad \text{Arbitrary light-like } z$$

GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Tra

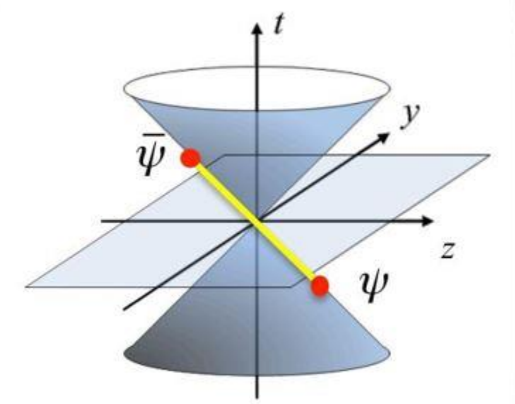
Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric **frame** through matrix elements of asymmetric frame

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

Key points:

P_s

GPDs on the light-cone:



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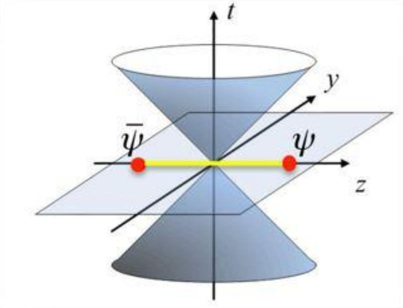
GPDs on the light-cone can be defined in a Lorentz-invariant way

Case 2: Tra

z_s^0

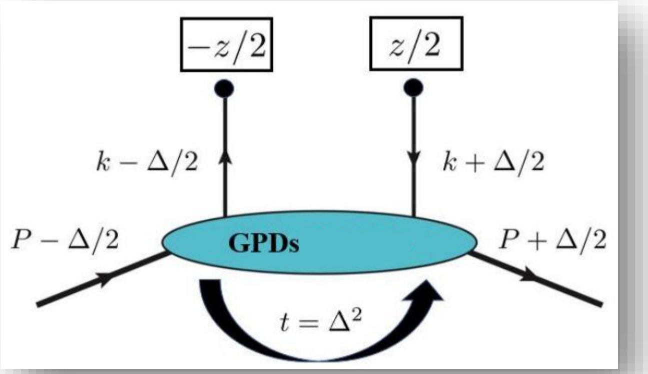
Transverse boost: This Lor … si-GPDs in symmetric **frame**



Are quasi-GPDs Lorentz-invariant?

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



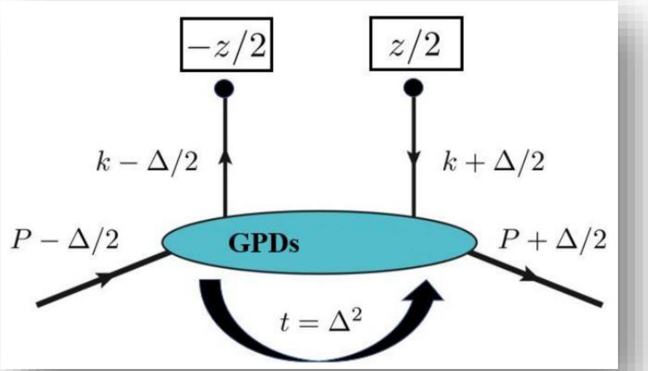
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



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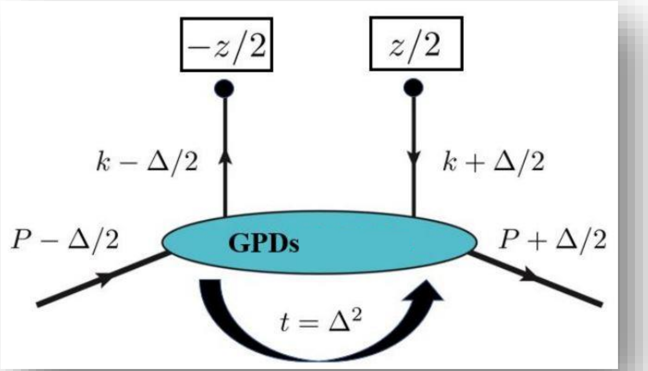
Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

Think about how γ^0 transforms under Lorentz transformation

Lattice QCD calculations of GPDs in asymmetric frames

Definitions of quasi-GPDs



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Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant

Can we come up with a

manifestly Lorentz invariant definition of quasi-GPDs for finite values of momentum?

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}\boldsymbol{A_1} + \frac{z^{\mu}}{M}\boldsymbol{A_2} + \frac{\Delta^{\mu}}{M}\boldsymbol{A_3} + \frac{i\sigma^{\mu z}}{M}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{M}\boldsymbol{A_5} + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_6} + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_7} + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}\boldsymbol{A_8}\right]u(p,\lambda)$$

Vector operator $F^{\mu}_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^{\mu}q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_{\perp}=\vec{0}_{\perp}}$

Lattice QCD calculations of GPDs in asymmetric frames

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

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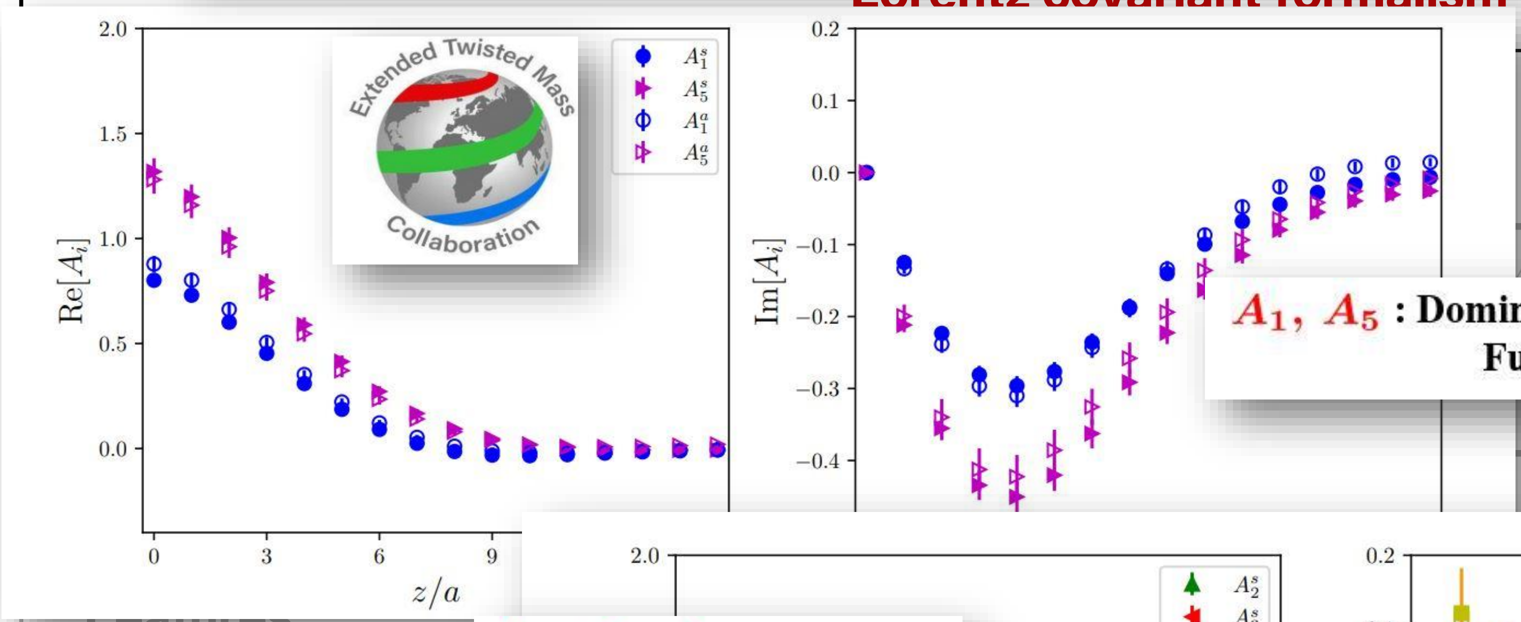
Features:

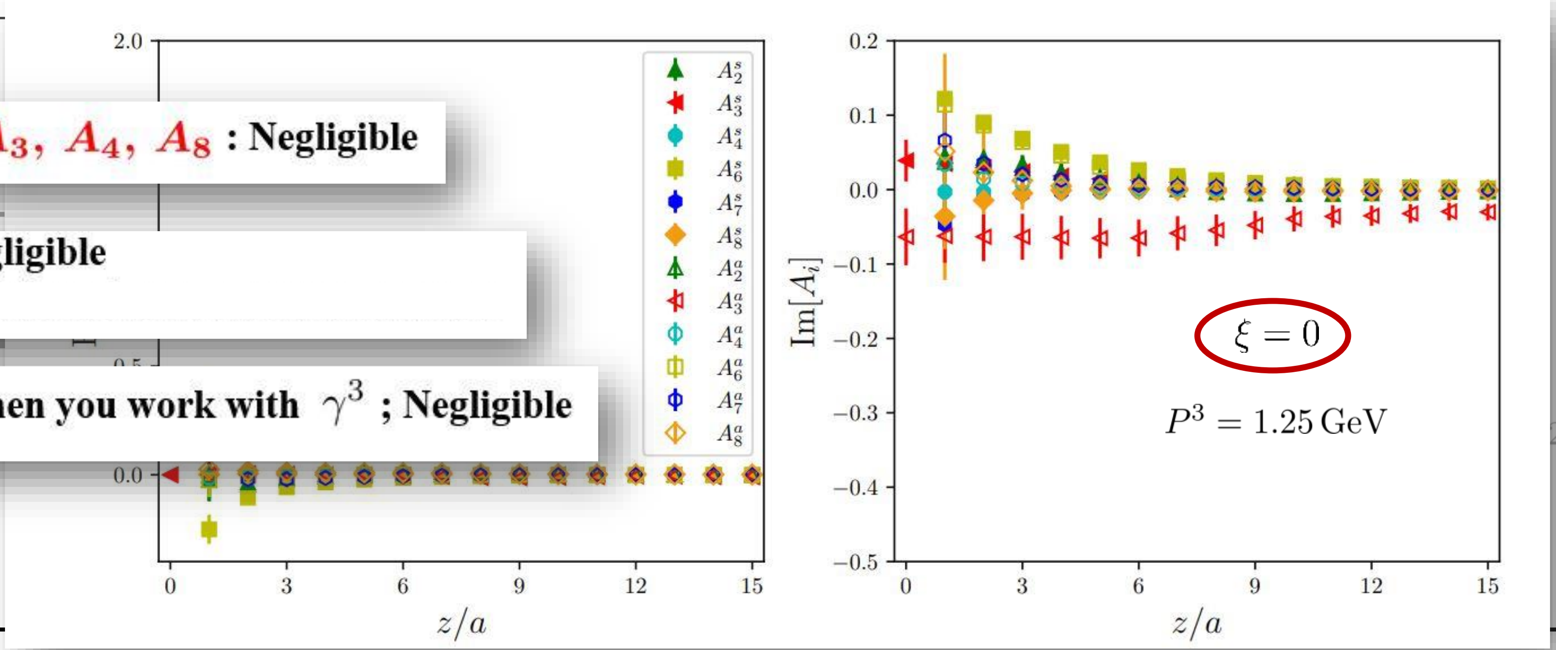
- **General structure of matrix element based on constraints from Parity**
- **8 linearly-independent Dirac structures**
- **8 Lorentz-invariant amplitudes (or Form Factors)** $A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

Validating the frame-independence of A's from Lattice QCD

Lorentz covariant formalism



$A_1,\ A_5$: Dominant contributions;
Full agreement in two frames for both Re & Im parts



$A_3,\ A_4,\ A_8$: Negligible

A_6 : Small but non-negligible

$A_2,\ A_7$: Appears only when you work with γ^3 ; Negligible

Lattice QCD calculations of GPDs in asymmetric frames

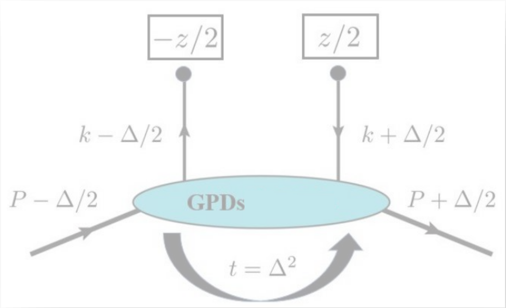
Re-exploring historical definitions of quasi-GPDs

Mapping amplitudes to the historical definitions of quasi-GPDs: (Sample results)

Lattice QCD calculations of GPDs in asymmetric frames

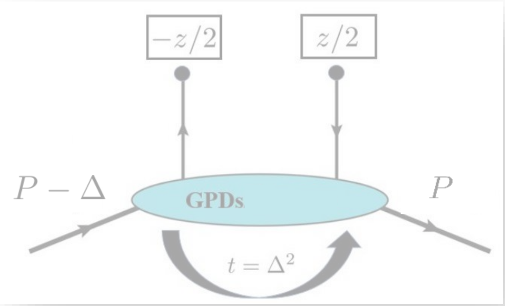
Re-exploring historical definitions of quasi-GPDs

Frame-dependent expressions: Explicit non-invariance from kinematics factors



Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = \mathbf{A_1} + \frac{\Delta_s^0}{P_s^0}\mathbf{A_3} - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}\mathbf{A_4} + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)\mathbf{A_6} + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)\mathbf{A_8}$$



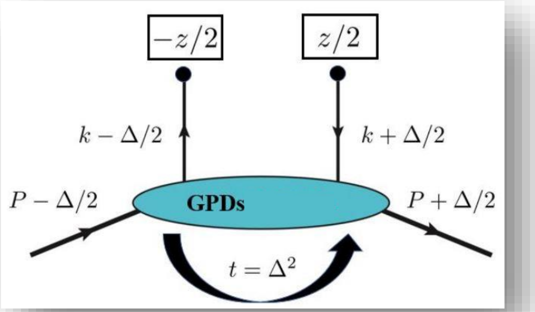
Asymmetric frame:

$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = \mathbf{A_1} + \frac{\Delta_a^0}{P_{avg,a}^0}\mathbf{A_3} - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)\mathbf{A_4}$$
$$+ \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)\mathbf{A_6}$$
$$+ \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)\mathbf{A_8}$$

Lattice QCD calculations of GPDs in asymmetric frames

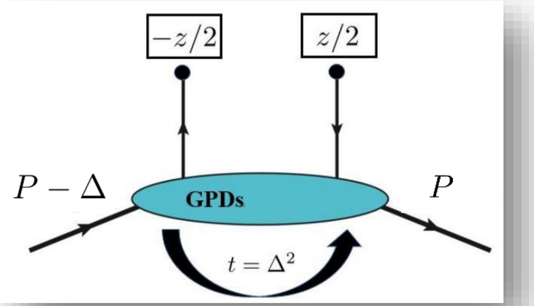
Light-cone GPDs

Mapping amplitudes to the light-cone GPDs: (Sample results)



Definition of light-cone GPD H:

$$H(x,\xi,t) \to \int \frac{d(P\cdot z)}{4\pi} e^{ixP\cdot z} \frac{1}{P\cdot z} \langle p'|\bar{q}\,\not{z}\,q|p\rangle$$



Relation between light-cone GPD H & amplitudes:

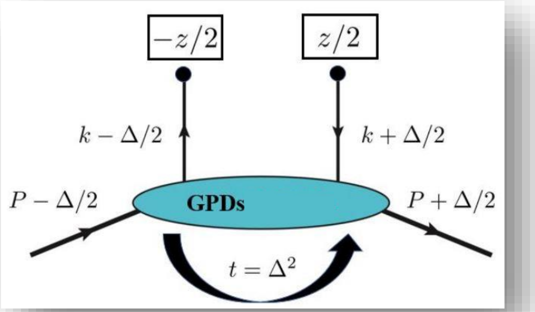
$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

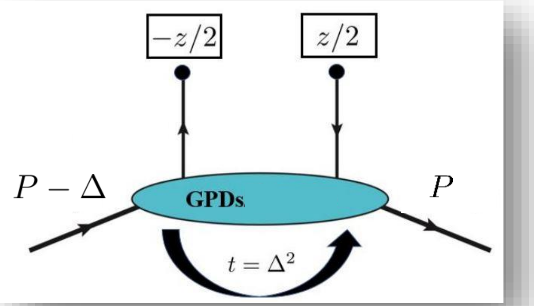
Lorentz-invariant expression

L…culations of GPDs in asymmetric frames

Sketch of the essence of a Lorentz-invariant definition of quasi-GPDs

Novel definition of quasi-GPDs





Lattice QCD calculations of GPDs in asymmetric frames

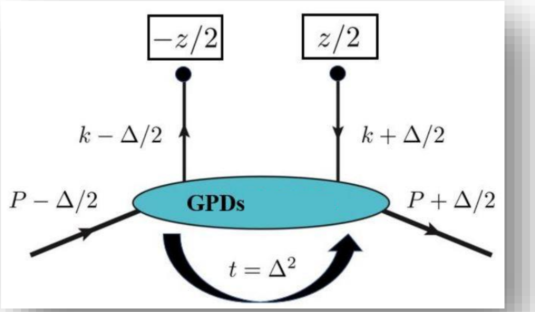
Relation between light-cone GPD H & amplitudes:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

Novel definition of quasi-GI

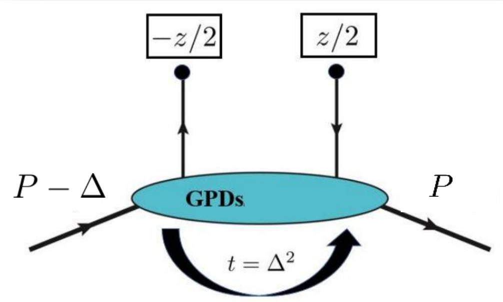
Mapping amplitudes to the historical definitions of quasi-GP

Symmetric frame:



$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Asymmetric frame:



$$H_{Q(0)}\big|_a(z,P_a,\Delta_a) = A_1 + \frac{\Delta_a^0}{P_{avg,a}^0}A_3 - \left(\frac{\Delta_a^0 z^3}{2P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{\Delta_a^0\Delta_a^3 z^3}{4P_{avg,a}^0(P_{avg,a}^3)^2}\right)A_4 + \left(\frac{(\Delta_a^0)^2 z^3}{2M^2 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{P_{avg,a}^0\Delta_a^0\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_{avg,a}^3}\right)A_6 + \left(\frac{(\Delta_a^0)^3 z^3}{2M^2 P_{avg,a}^0 P_{avg,a}^3} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2 P_{avg,a}^0(P_{avg,a}^3)^2} - \frac{1}{(1+\frac{\Delta_a^3}{2P_{avg,a}^3})}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P_{avg,a}^3)^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2 P_{avg,a}^0 P_{avg,a}^3}\right)A_8$$

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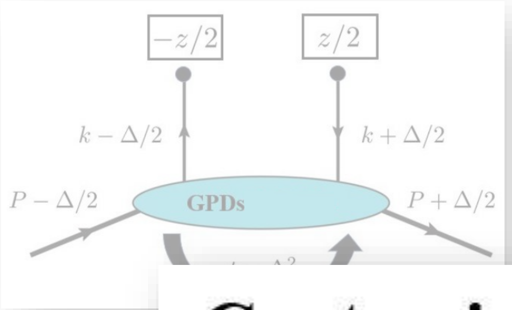

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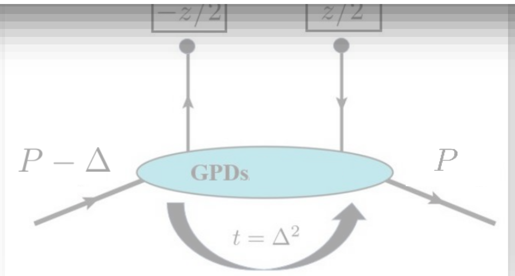
Contamination from additional amplitudes or power corrections

In the large-momentum limit, these expressions reduce to light-cone results



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Novel definition of quasi-GPDs

Interlude:

Mapping amplitudes to the historical definitions of quasi-GPDs

Let's go back to PDFs

frame:

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin

Old Dominion University, Norfolk, VA 23529, USA and

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

Contam... power corrections

In the ... these expressions reduce to light-cone results

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle \qquad (12)$$

type, where $\hat{E}(0,z;A)$ is the standard $0\to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)\,. \qquad (13)$$

2 Amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2\to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

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formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0\int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \qquad (14)$$

Therefore, γ^0 is better behaved than γ^3 with respect to power corrections

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Quasi-PDFs, momentum distributions and pseudo-PDFs

Old Domini
Thomas Jefferson Natio

Statement needs a qualifier: Situation more complicated for quasi-GPDs

(See next slide)

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Asymmetric frame:

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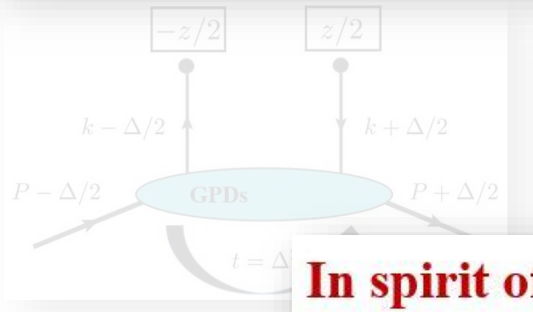

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In spirit of what's done for PDFs:

You can think of eliminating additional amplitudes by the addition of other operators:

$$(\gamma^1, \gamma^2)$$

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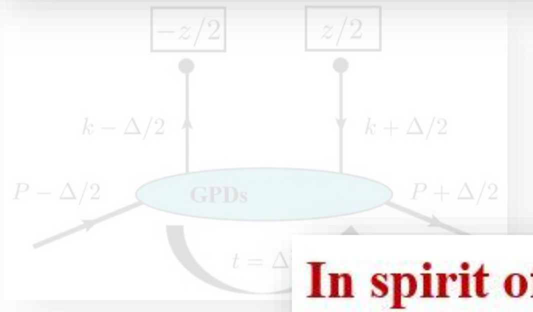

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Asymmetric frame:

Lorentz-invariant definition of quasi-GPDs:

Main finding:

Schematic structure: $H_Q \rightarrow c_0\langle\bar\psi\gamma^0\psi\rangle + c_1\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$

Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes

QCD calculations of GPDs in asymmetric frames

Same functional forms

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$A_i \equiv A_i(z^2 = 0)$

Lorentz-invariant generalization of LC definition to $z^2 \neq 0$:

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Note: Here c's are frame-dependent kinematic factors that cancel additional amplitudes to project quasi-GPD potentially faster (vs historic def.) onto light-cone GPD

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Key points:

In spirit of what's done for PDFs:

1) **Lorentz-invariant generalization of LC definition to $z^2 \neq 0$ might converge faster**

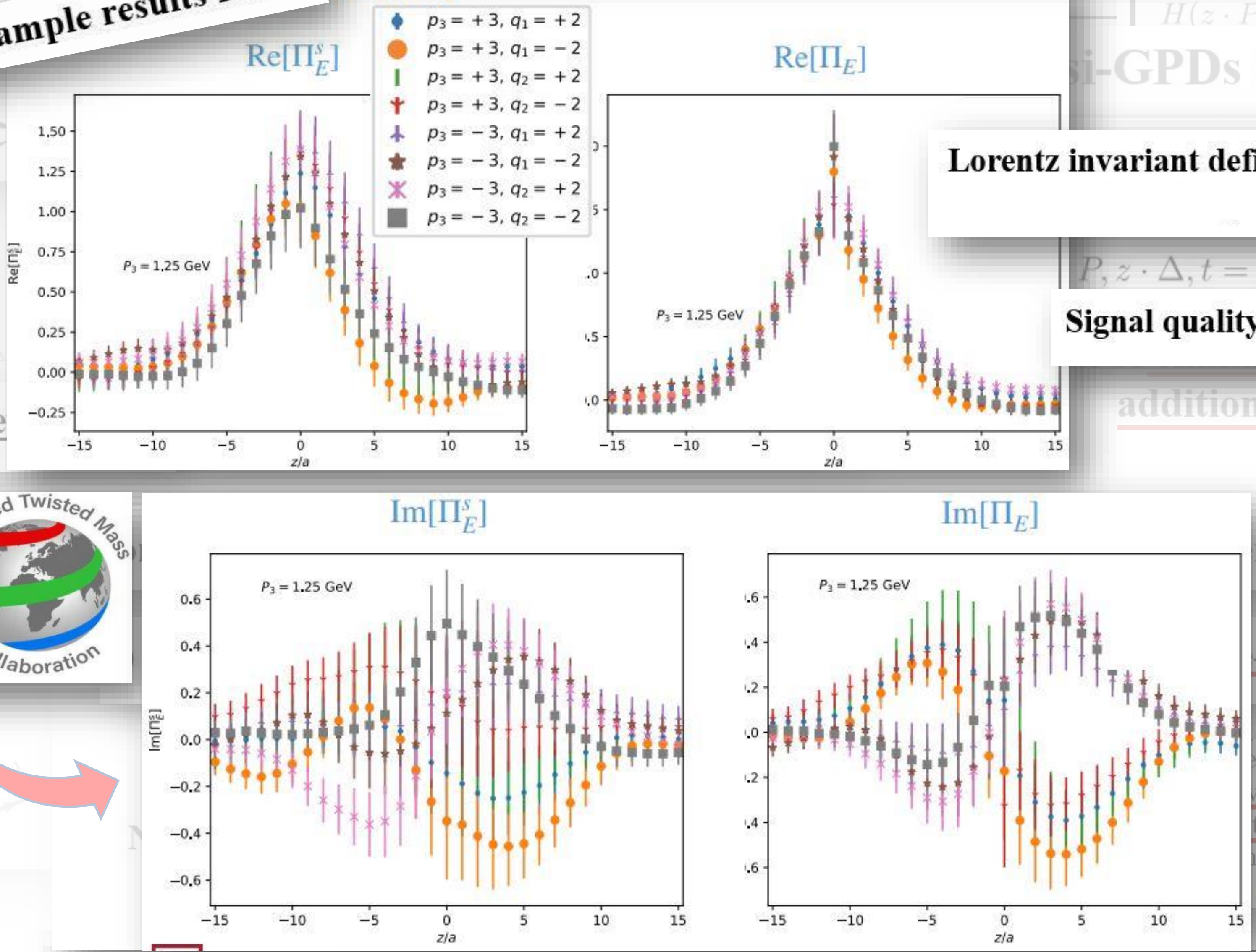
2) **Definition Lorentz invariant ⟶ differences suppressed by frame-independent power corrections**

Caveat:

However, it is essential to acknowledge that the amplitudes themselves also contain implicit power corrections. Moreover, it is worth noting that the presence of additional amplitudes in the first place could potentially serve to mitigate the implicit power corrections inherent in the amplitudes ... Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. Therefore, it is important to perform numerical comparisons

Numerical comparison between Lorentz invariant and historical definitions of quasi-GPDs:

Sample results for E



Lorentz invariant definition leads to more precise results for E

Signal quality for H same for all cases (not shown)

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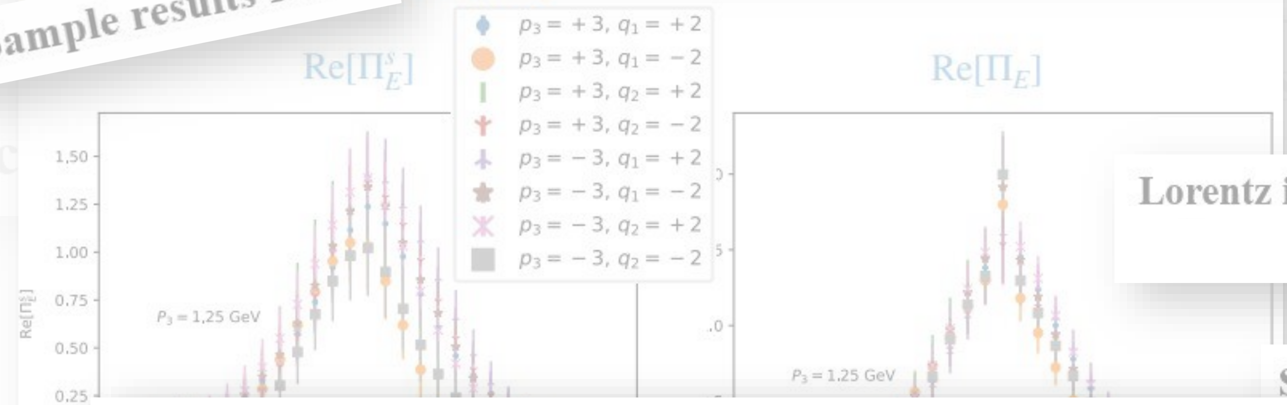

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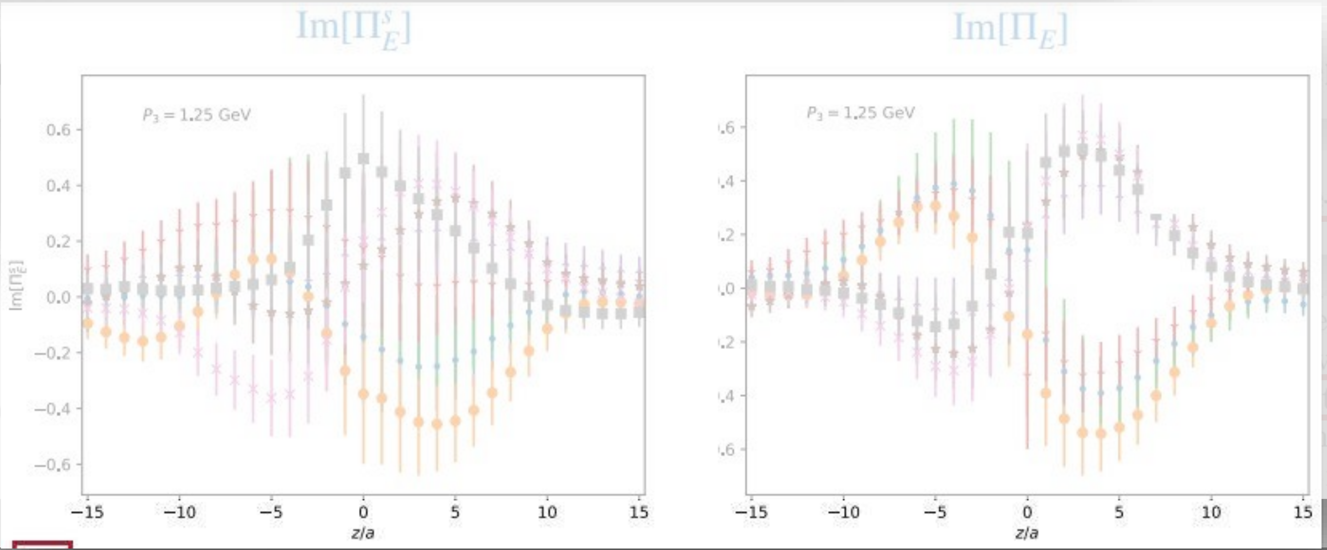
Revisiting GPD calculations for all cases from asymmetric frames



First exploration of twist-3 GPDs

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

(So far, from symmetric frames)

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Definition:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

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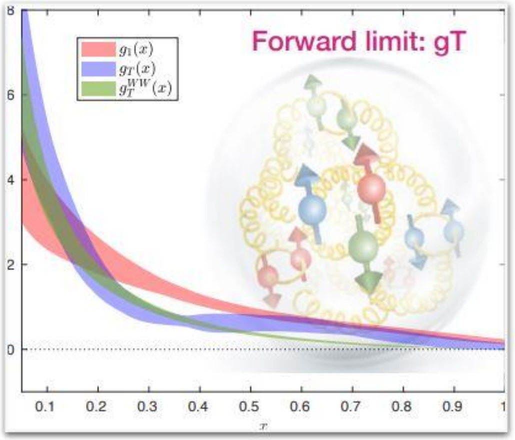
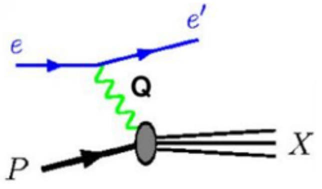


[S. Bhattacharya et al., PRD 102 (2020) 11]

PRD 102 (2020) 11, 111501 [Editor's suggestion]

New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

Twist-3 PDF	Processes	Data
$g_T(x)$		For instance: Hall A, 2016/ Hall C, 2018

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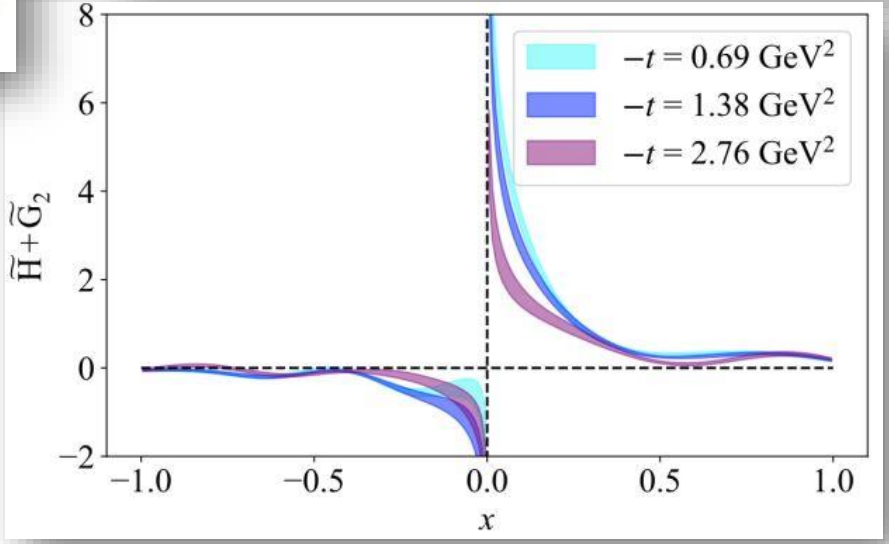

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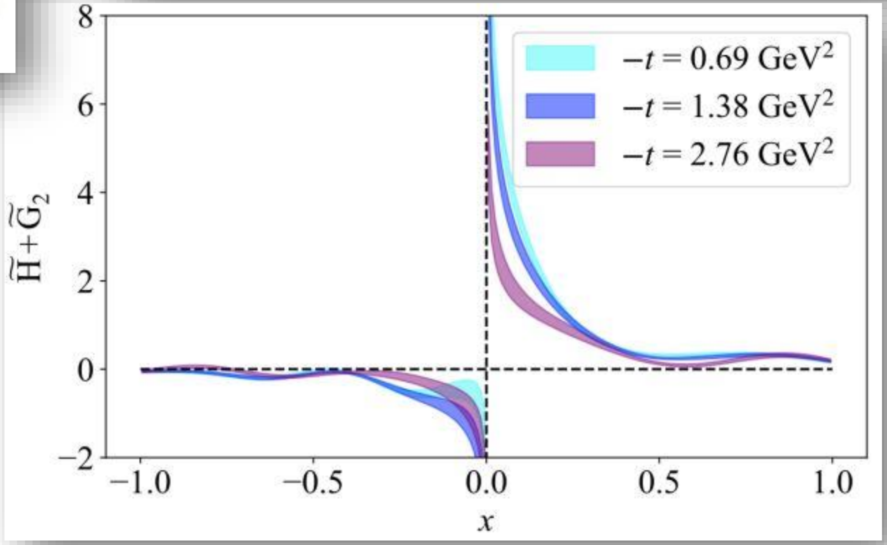
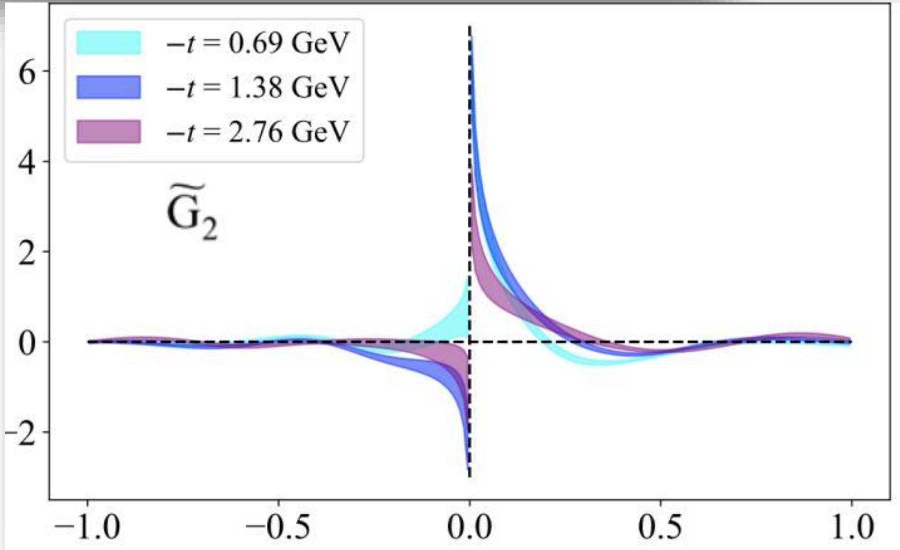


Isolating $\widetilde{G}_2$



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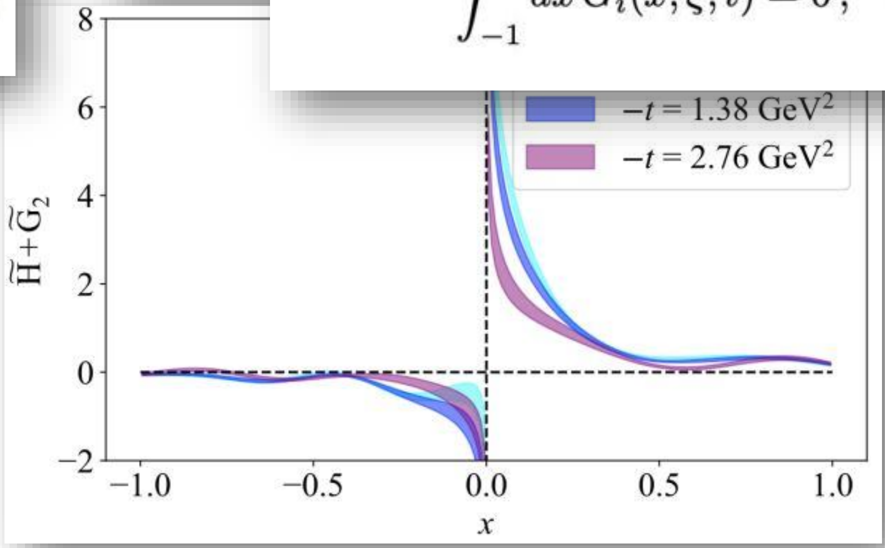
Negative regions in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$



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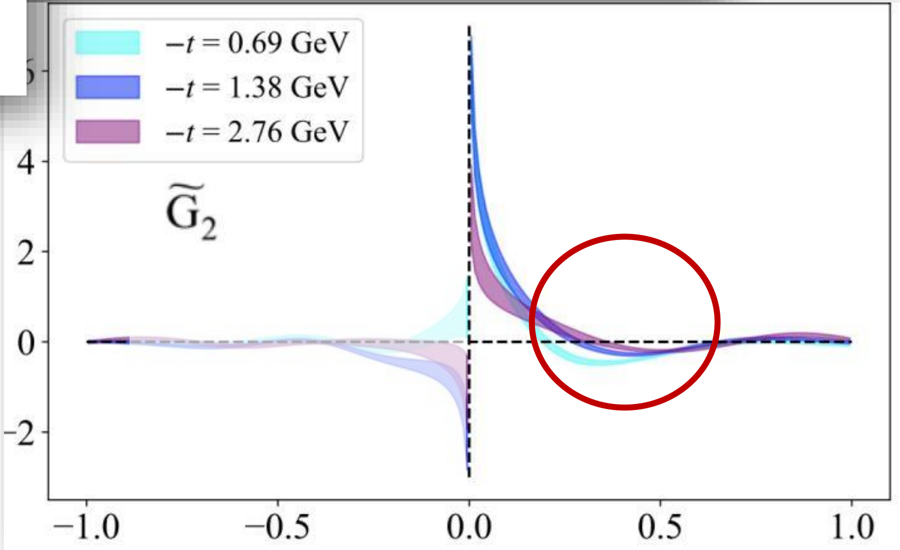

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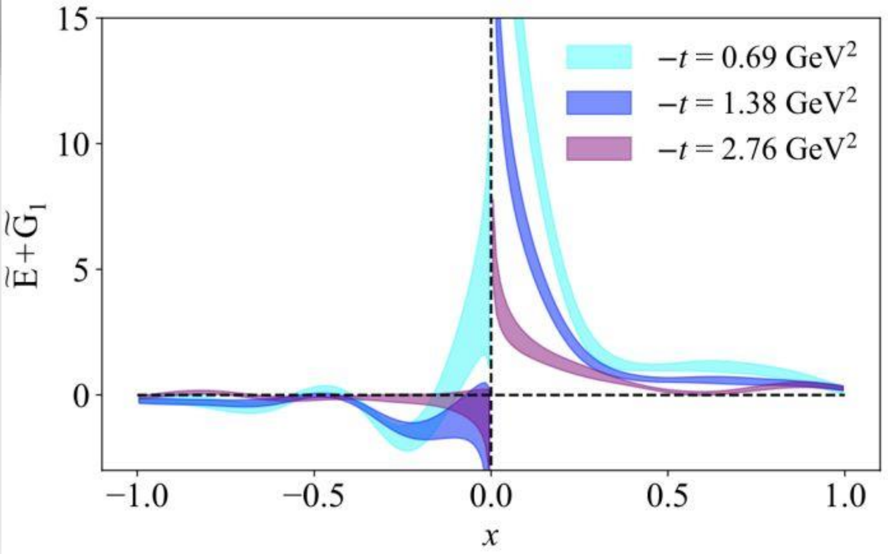

Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness

Sizeable contributed as expected:

$$\int_{-1}^{1} dx\,\widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\,\widetilde{G}_i(x,\xi,t) = 0\,, \quad i=1,2,3,4$$

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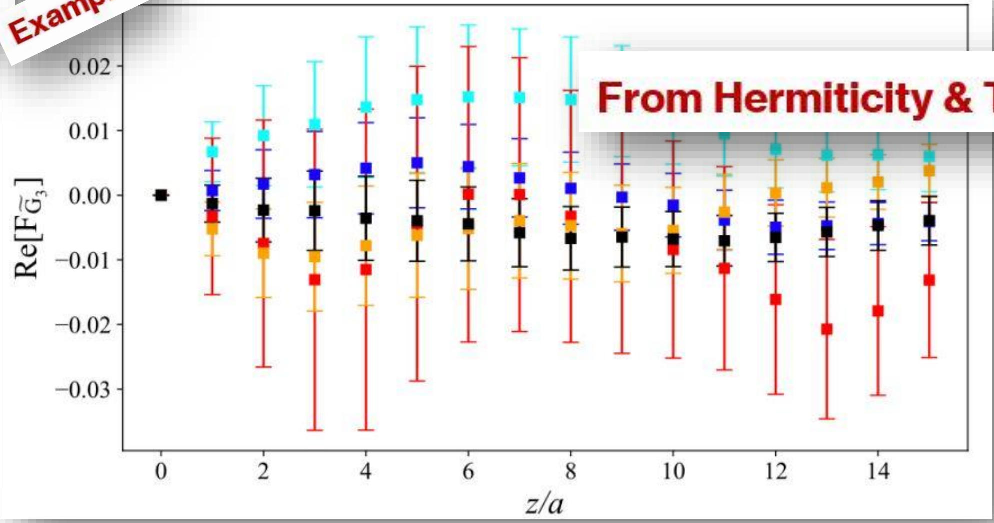
Example:



From Hermiticity & Time-reversal constraint, we find that $\widetilde{G}_3$ is odd under $\xi \to -\xi$

So $\widetilde{G}_3$ should exhibit at least linear scaling with respect to ξ

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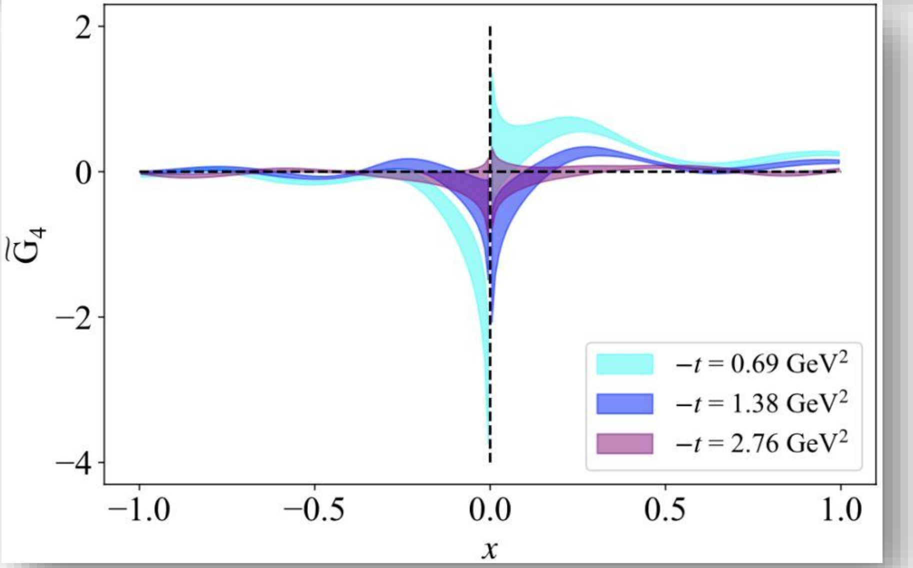


GPD $\widetilde{G}_4$ very small

No theoretical argument to be zero:

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4}G_E$$

First exploration of twist-3 GPDs

Matching formula:

$$q_{\mathrm{Q}}(x;P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

(Scale dependence omitted)

(Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

Matching coefficient

As discussed in the context of twist-2 GPDs, matching at $\xi = 0$ coincides with those for PDFs

We use the simplified matching coefficient for the twist-3 PDF $g_T(x)$

Matching coefficient

$$C^{(1)}_{\mathrm{M\overline{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} 0 \\ \delta(\xi) \\ 0 \end{cases} + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi}{\xi-1} + \frac{\xi}{1-\xi} + \frac{3}{2\xi}\right]_+ & \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2} + \frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{-\xi^2+2\xi+1}{1-\xi} \ln\frac{\xi-1}{\xi} - \frac{\xi}{1-\xi} + \frac{3}{2(1-\xi)}\right]_+ & \xi<0\,, \end{cases}$$

arXiv:2005.10939

One-loop matching for the twist-3 parton distribution $g_T(x)$

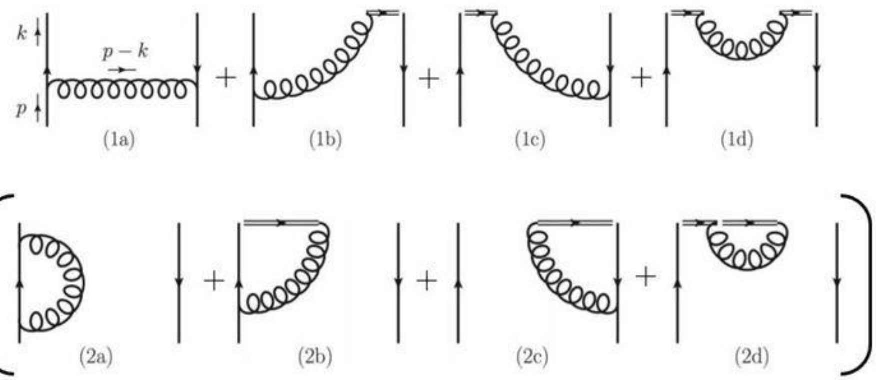
Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[3]

First exploration of twist-3 GPDs

Matching formu

1-loop corrections $= \delta(1-x) +$

(Feynman Gauge)



ao, 2013/

wart, Zhao, 2018/ ...)

2 parton correlator:

As dis sed in

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We use the simplified matching coefficient for the twist-3 PDF $g_T(x)$

GPD $\widetilde{G}_4$ very small

be zero:

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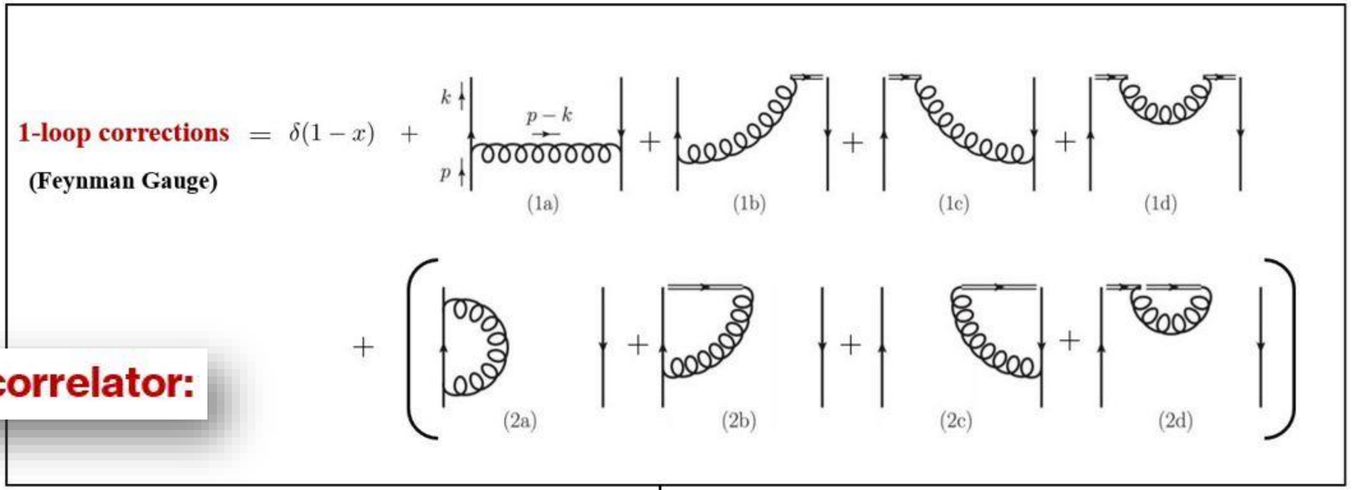


2 parton correlator:

For matching including 3 parton correlator, see:

QCD factorization for twist-three axial-vector parton quasidistributions

Vladimir M. Braun,[a] Yao Ji,[b] and Alexey Vladimirov[a]

Summary

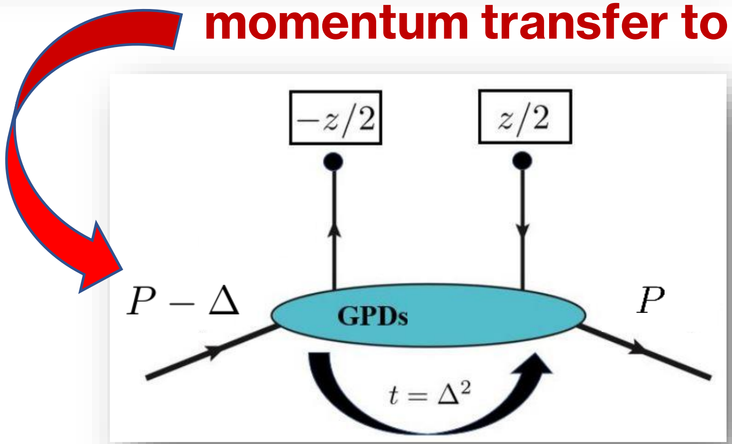
Connecting dots: Ending with what I started with

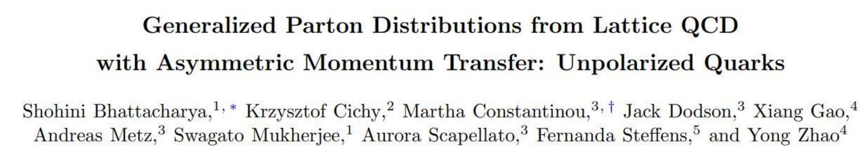
Summary

Goal: Connecting dots: Ending with what I started with

Perform Lattice QCD calculations of GPDs in asymmetric frames

All momentum transfer to source





Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Summary

Connecting dots: Ending with what I started with

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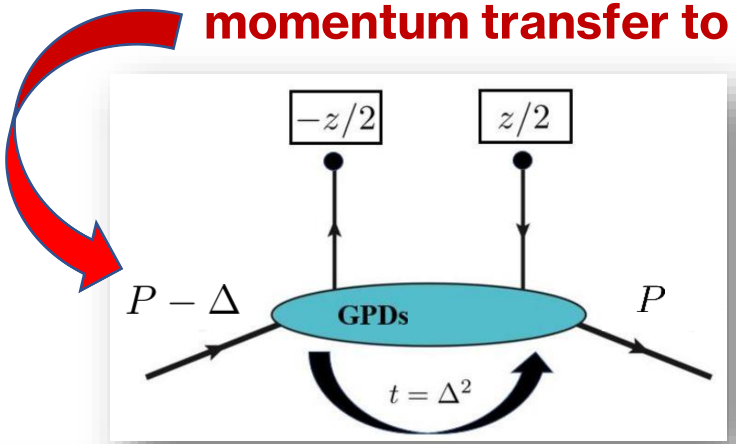


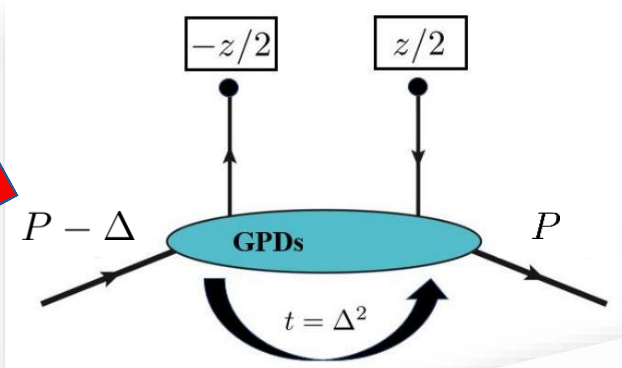
Approach 1: Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost: This Lorentz transformation allows for an exact calculation of quasi-GPDs in symmetric frame through matrix elements of asymmetric frame

Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



Key findings:

1) **Historic definitions of H & E quasi-GPDs are not manifestly Lorentz invariant**

$$H_Q \to c\langle\bar{\psi}\gamma^0\psi\rangle$$

Symmetric frame:

$$H_{Q(0)}(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0 P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2 P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2 P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2 P_s^0 P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2 P_s^0 P_s^3}\right)A_8$$

Contamination from additional amplitudes or power corrections

Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

arted with

All
momentum transfer to source

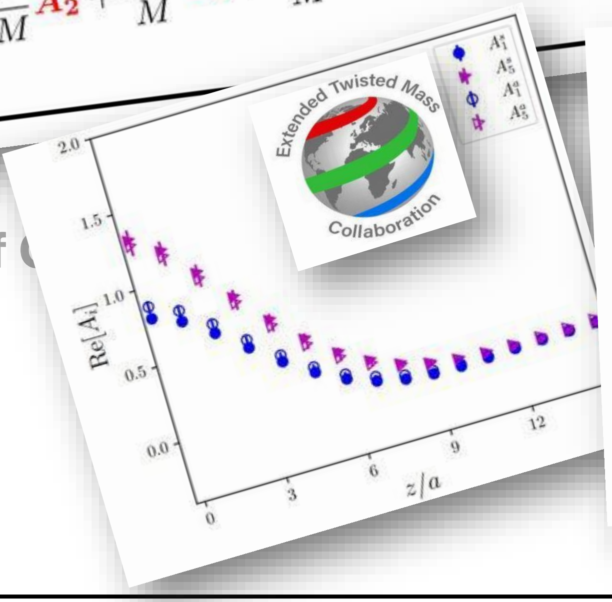
$-z/2$ $z/2$

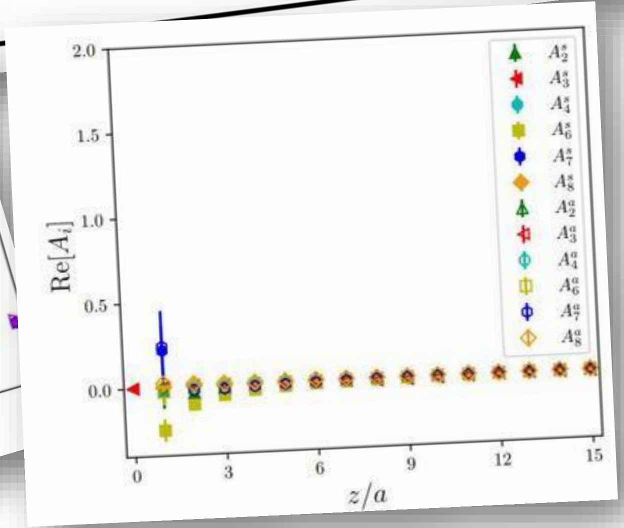
2) Novel parameterization of position-space matrix element: (Vector operator)

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + \frac{z^{\mu}}{M}A_2 + \frac{\Delta^{\mu}}{M}A_3 + \frac{i\sigma^{\mu z}}{M}A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M^3}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M^3}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M^3}A_8\right]u(p,\lambda)$$

Key findings:

ce QCD calculations of (





Summary

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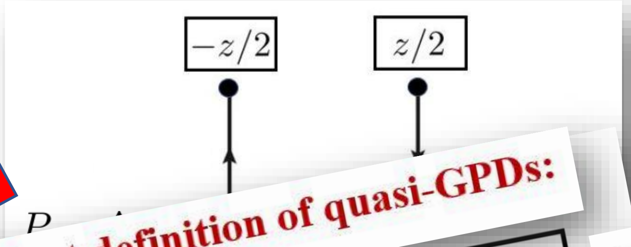


3)

Lorentz-invariant definition of quasi-GPDs:

$$H_Q(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

$$H_Q \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

Key findings: ce QC lculations of GPDs in asymmetric frames

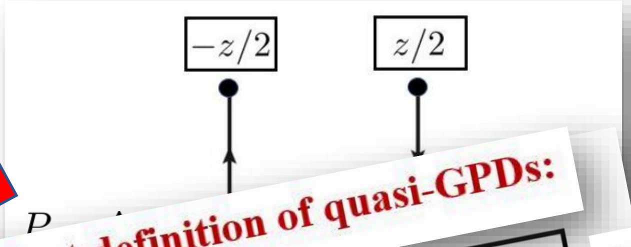
- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

Summary

arted with

Approach 2: Why does it matter in which frame quasi-GPDs are calculated?

All momentum transfer to source



3)

Lorentz-invariant definition of quasi-GPDs:

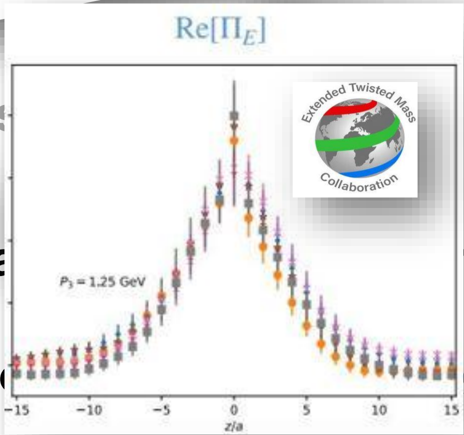
$$H_Q(z\cdot P, z\cdot \Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z} A_3$$

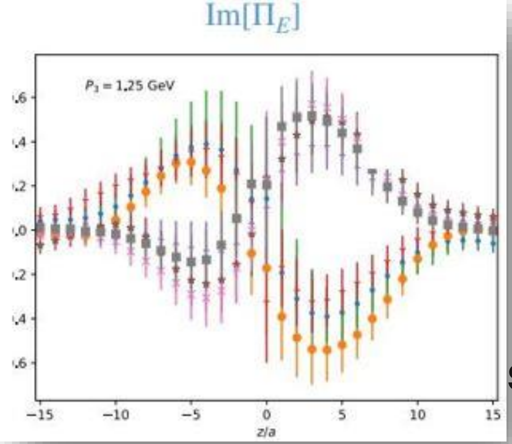
$$H_Q \rightarrow c_0 \langle \bar{\psi}\gamma^0\psi\rangle + c_1 \langle \bar{\psi}\gamma^1\psi\rangle + c_2 \langle \bar{\psi}\gamma^2\psi\rangle$$

Key findings:

Lorentz invariant definition leads to more precise results for E

ce QC ... lculations of GPDs in as





- **Lorentz cova... formalism for calculating qua... ne**
- **Elimination of power corrections potentially all... ge ... s**

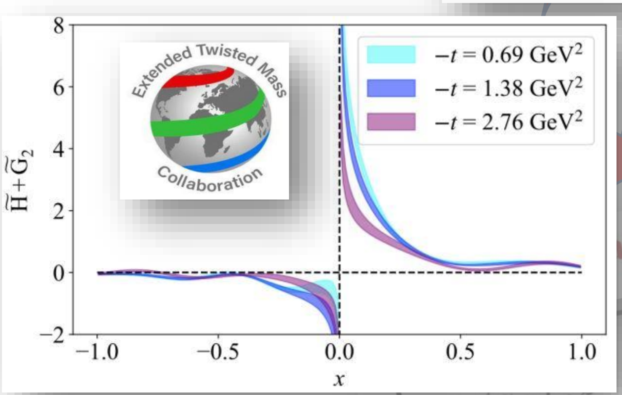
Summary

Approach 2: Why does it matter in which frame quasi-GPDs are calculated? …arted with

First exploration of twist-3 GPDs

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

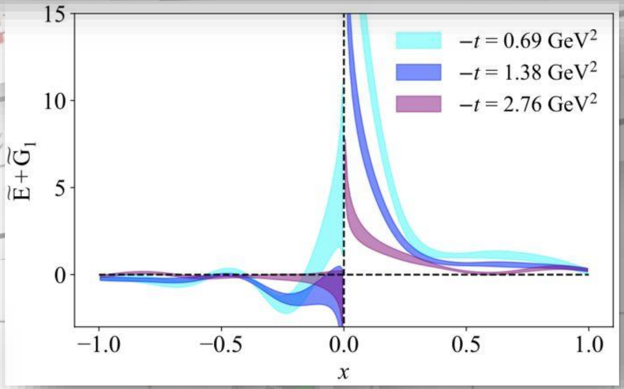
Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1],
Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]



Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:

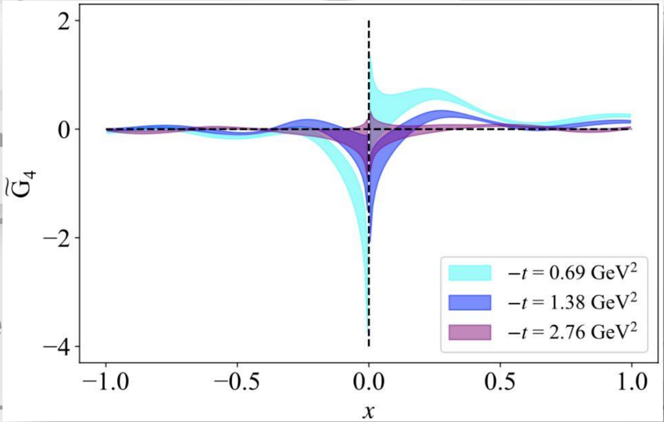
…riant definition of quasi-G…

$\langle\bar\psi\gamma^1\psi\rangle + c_2\langle\bar\psi\gamma^2\psi\rangle$



Key findings:

Lorentz invarian…



- Lorentz covariant fo… …ne
- Elimination of powe… …ge… …s