Beam Dynamics and Beam Losses in Circular Accelerators

K. Fuchsberger, www.beampilots.com

Many Thanks to R. Schmidt and J. Wenninger for their ideas, slides and feedback!

In this lecture …

Timetable

- What can go wrong?
- What are the Consequences?
- Mitigation
- Controls & Operation
- Safety Engineering

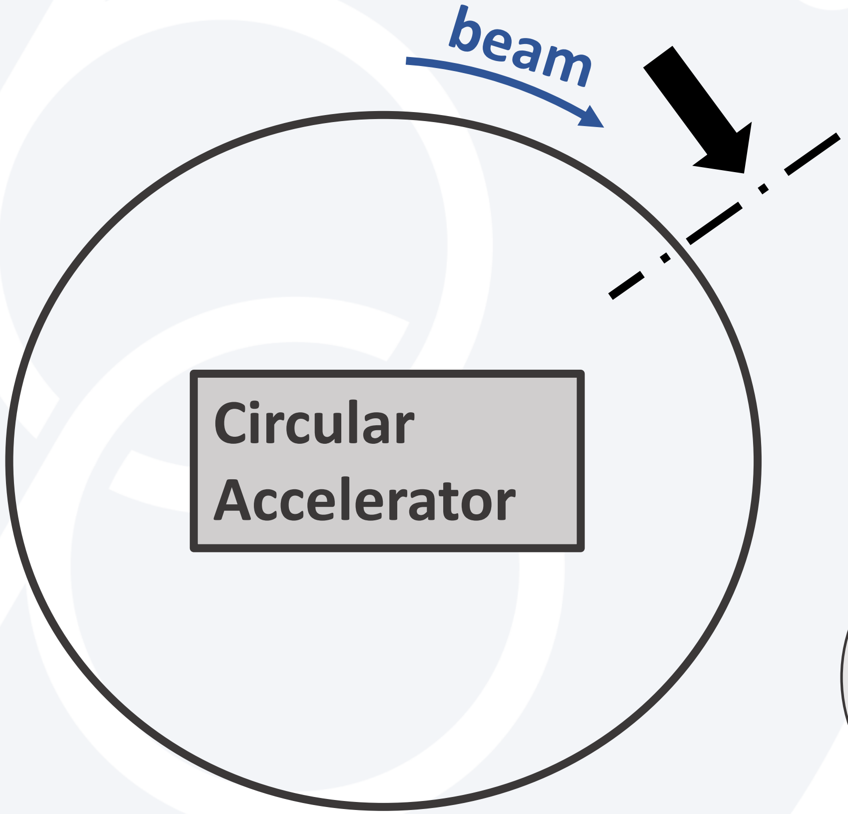
Over zoom

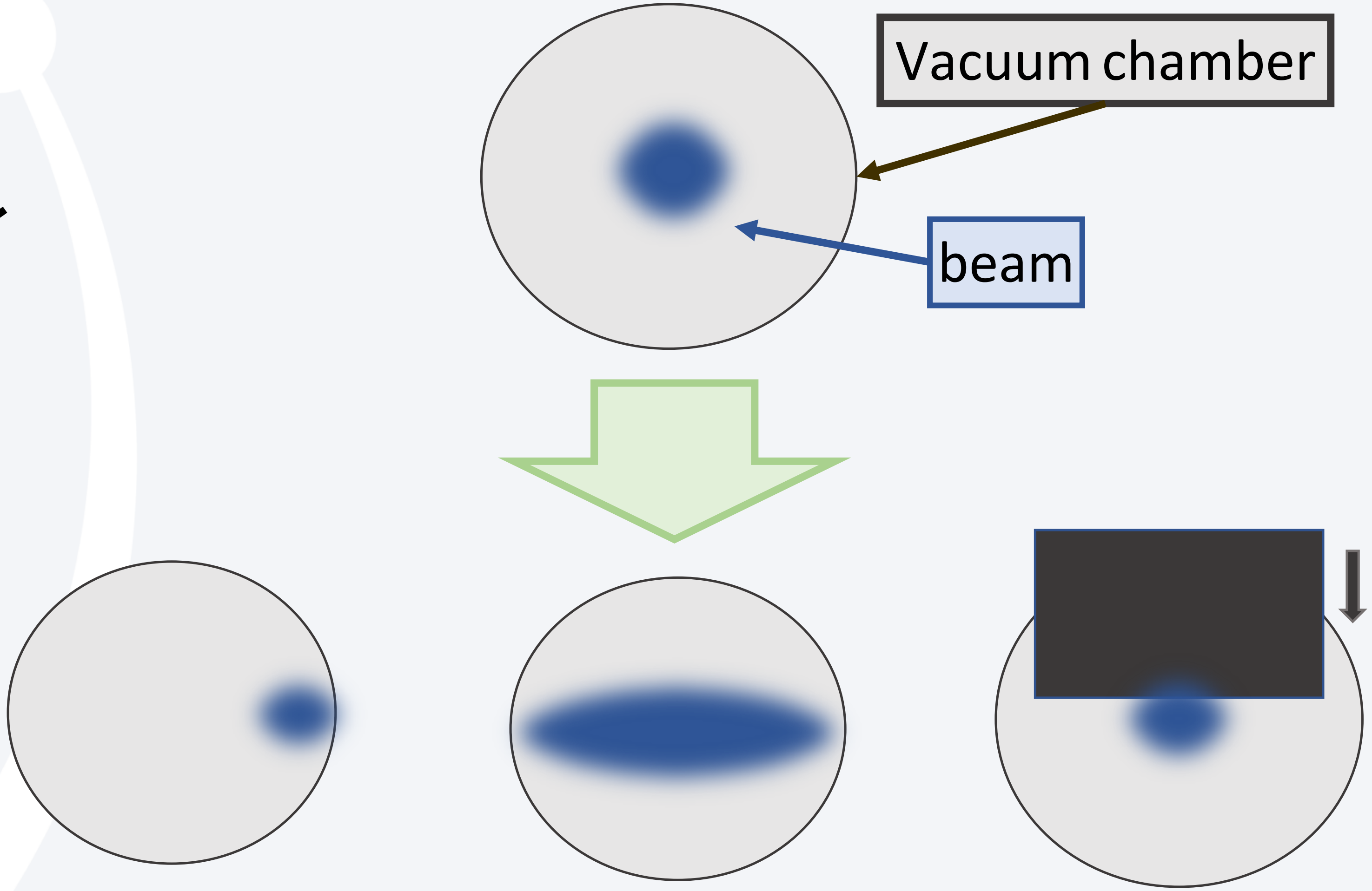
In person

	Monday	Tuesday	Wednesday	Thursday	Friday
08:30					
09:00	Welcome	Machine protection for	Beam loss induced damage	Controls and interlocks	Case studies
09:30	Introduction (R. Schmidt)	light sources (L. Emery)	(A. Bertarelli)	(D. Curry)	
10:00					
10:30					
11:00	Beam dynamics & beam losses	Beam material interaction	Reliability and availability	Machine protection	Case studies
11:30	circular (K.Fuchsberger)	(B. Barletta)	(A. Apollonio)	and operation (SNS)	
12:00	Beam dynamics & beam losses		(Ch. Peters)	(Ch. Peters)	
12:30	linear (Ch. Peters)				
13:00					
13:30					
14:00					
14:30	Ultra-fast failures	Beam cleaning and	Protection of SC circuits	Machine protection	
15:00	(J. Wenninger)	collimation	(M. Marchewsky)	in plasma accelerators	
15:30		(J. Wenninger)		(D. Curry)	
16:00	Detection of failures				
16:30	(Ch. Peters)	High power targets	Machine protection		
17:00		and their protection	and operation - LHC (J.W.)		
17:30		(Ch. Peters)			

30.01.2023 5 U.S. Particle Accelerator School 2023 - Beam Loss and Machine Protection

How can we loose particles in a circular accelerator?





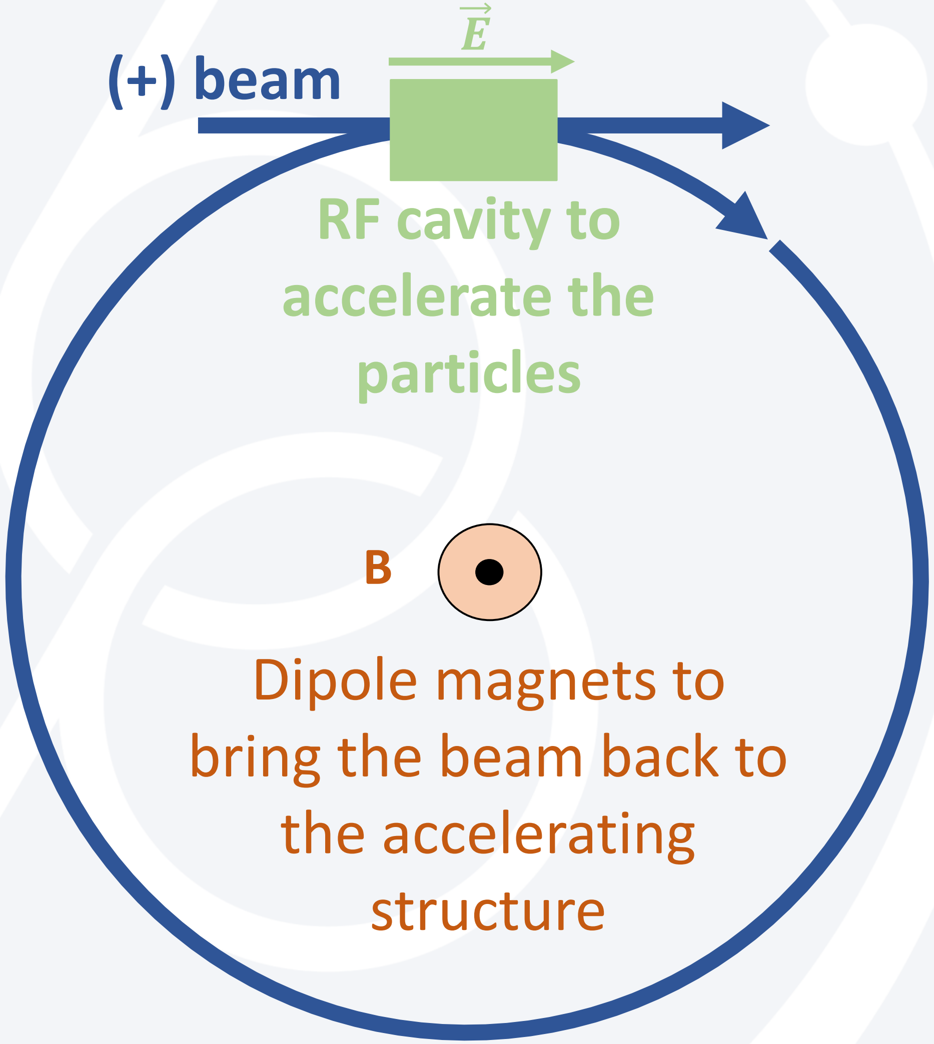
Transverse Offset

Blowup

Equipment movement

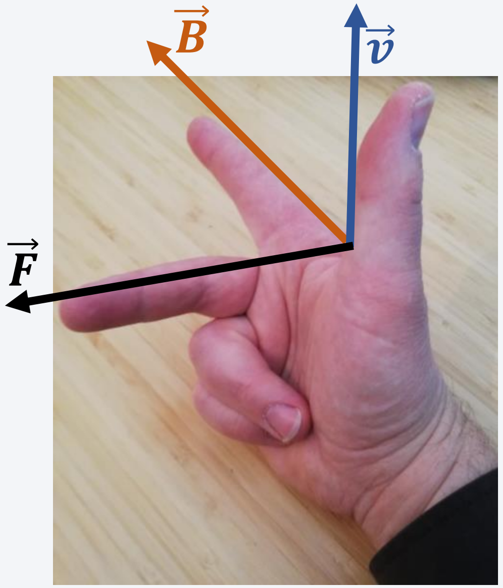
Circular Accelerators

Synchrotron – Basic Principle



Lorentz Force

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$



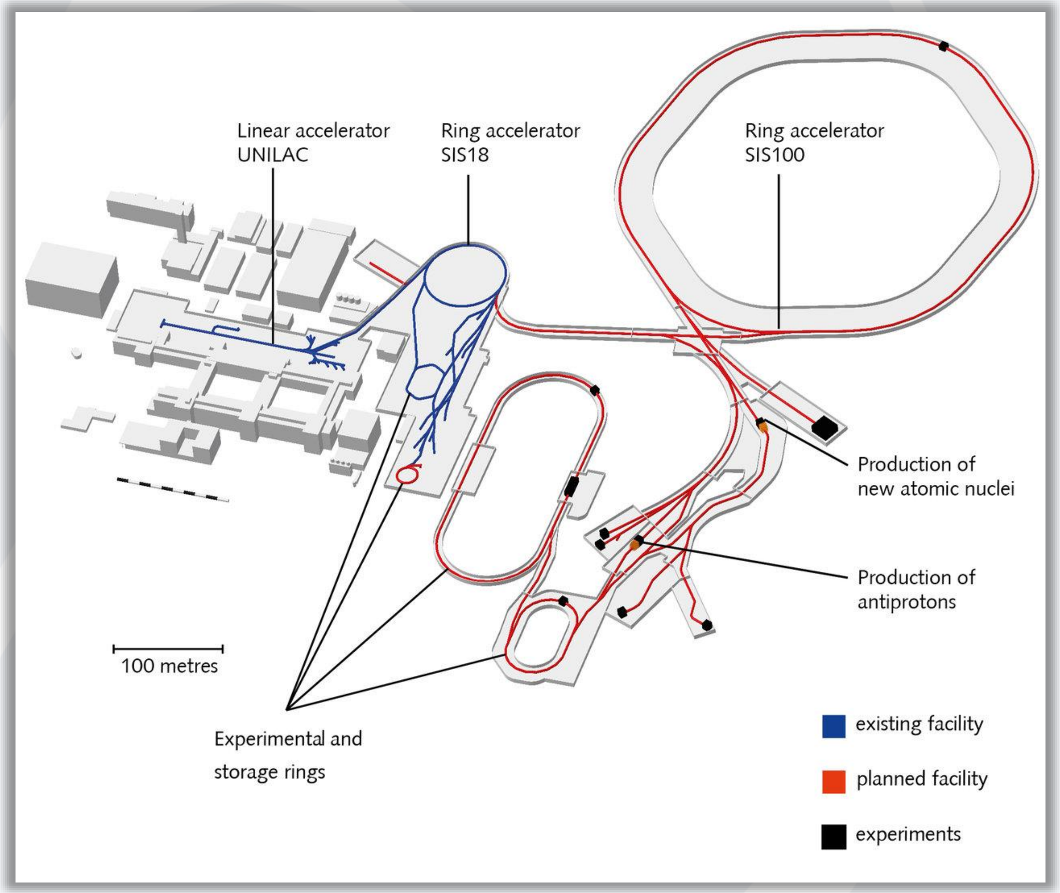
→ Reuse of accelerating Structure

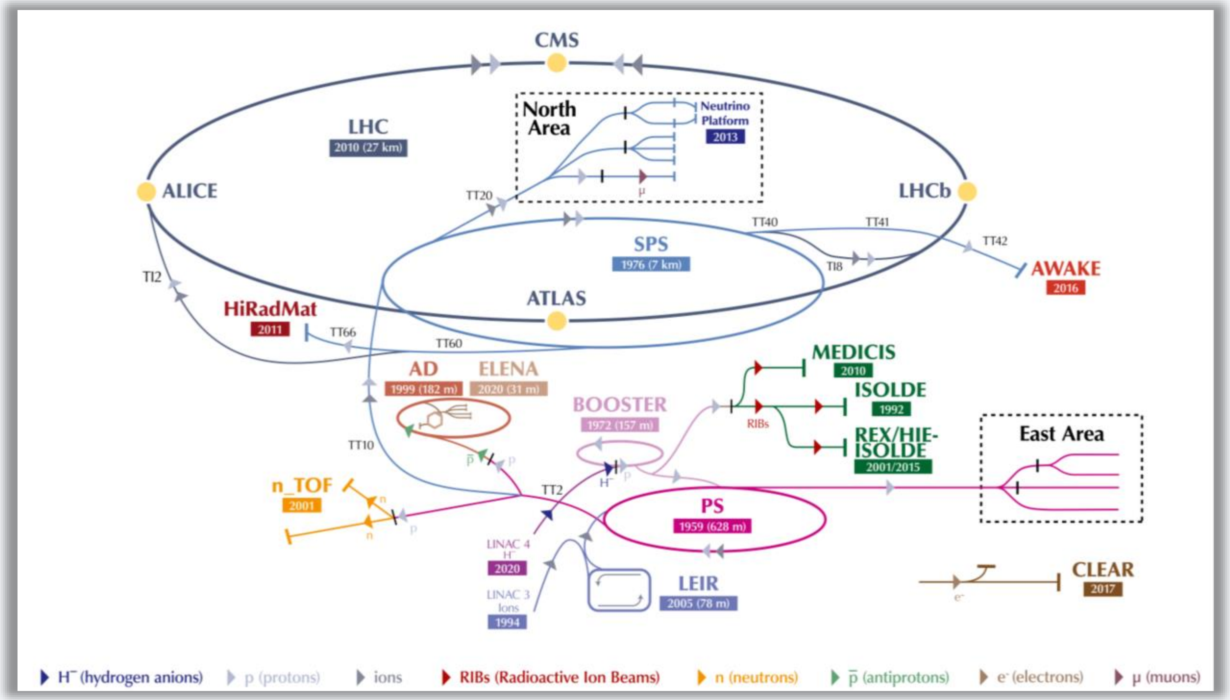
- magnetic field is increased; particles are accelerated;
- particle trajectory is (roughly) constant
- particles make many turns

B direction towards viewer

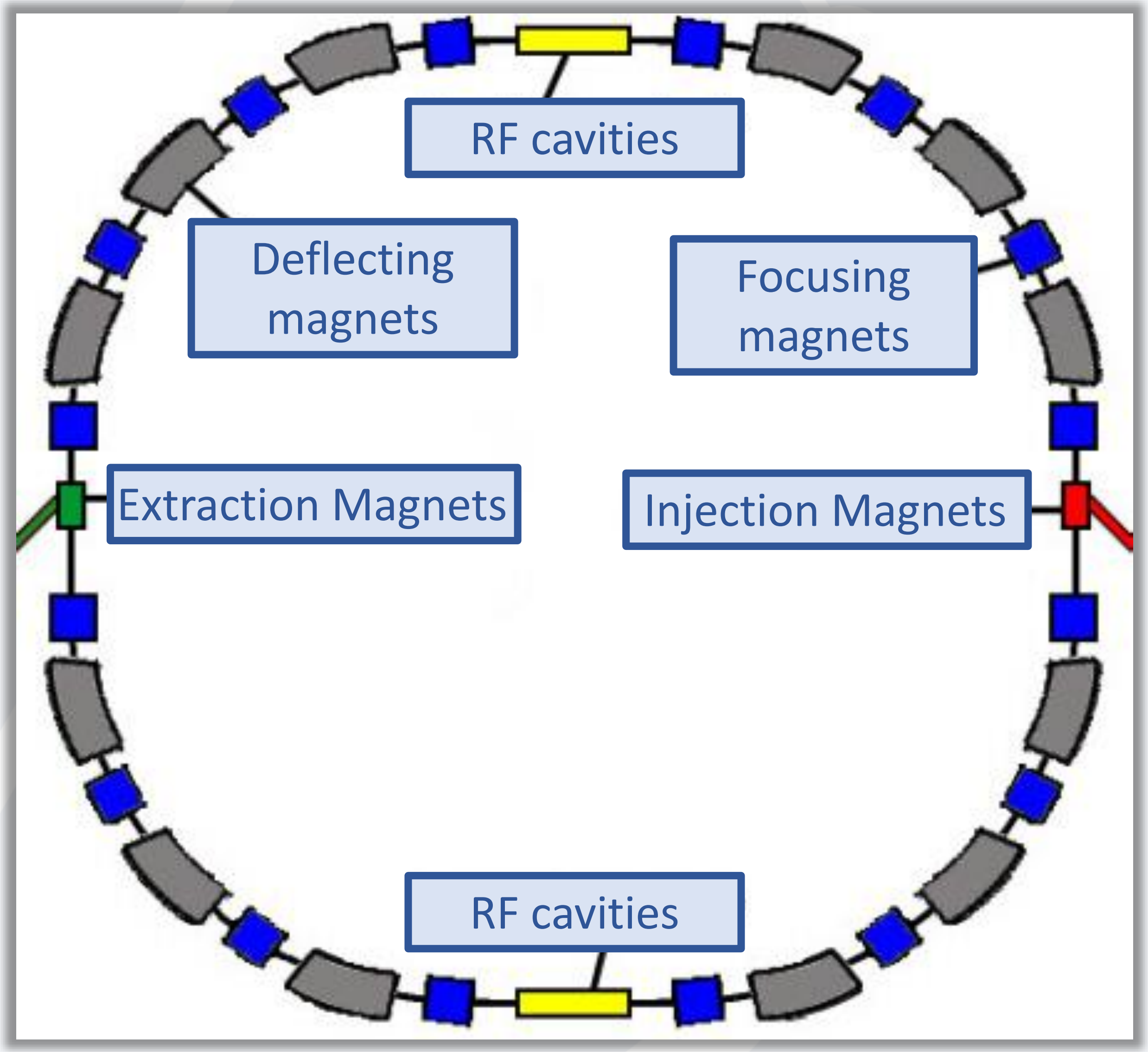
No way around Synchrotrons…

… if one wants to go to high energies.
(reuse of accelerating structure)





Key Components of a Synchrotron

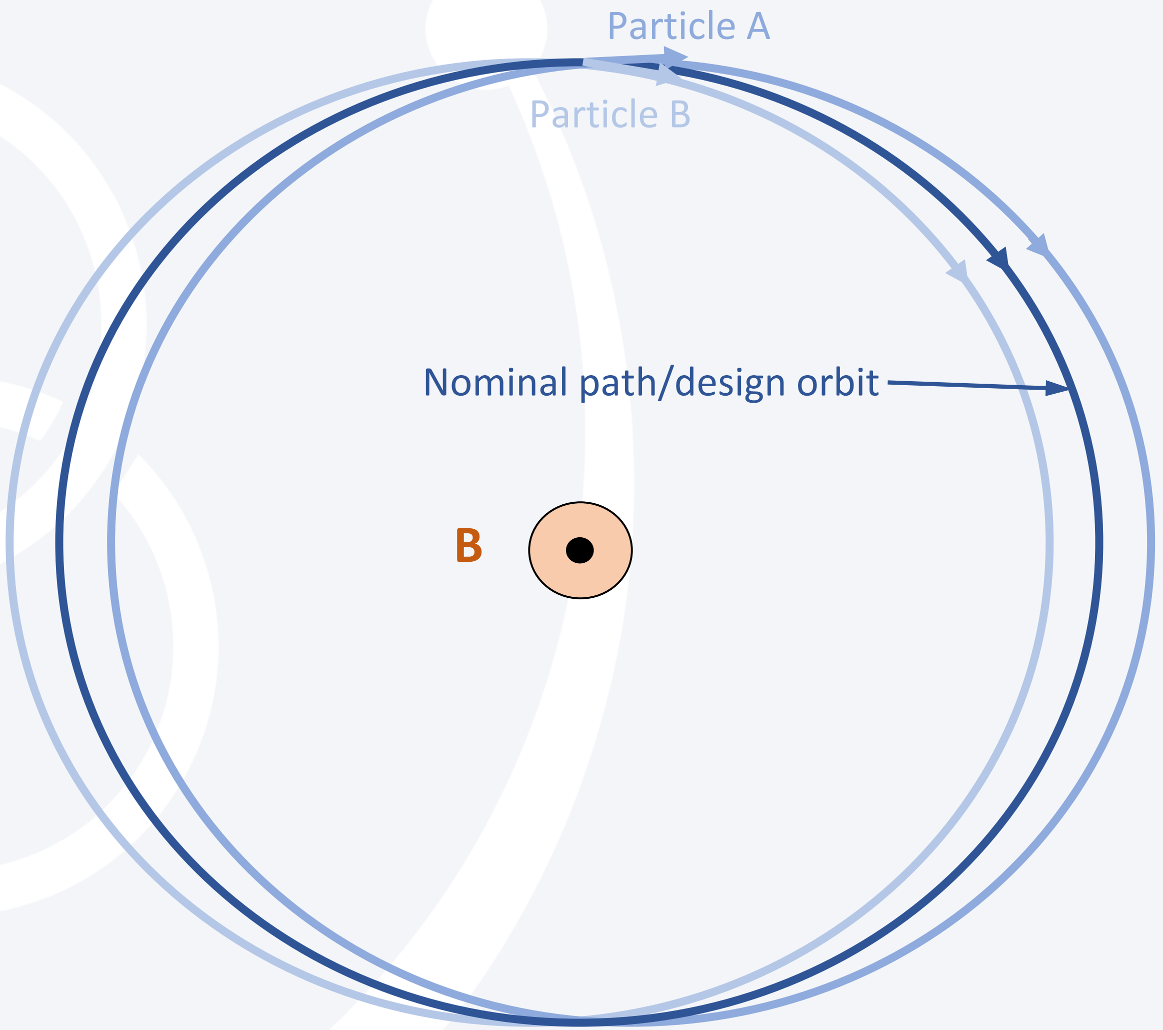


- RF cavities
- Magnets
  - Deflection
  - Focus,
  - Correction (dipole, quadrupole, sextupole …)
- Injection/Extraction magnets (pulsed)
- Power Converters (for each magnet)
- Vacuum Systems
- Diagnostics
- Control System

Particle Dynamics

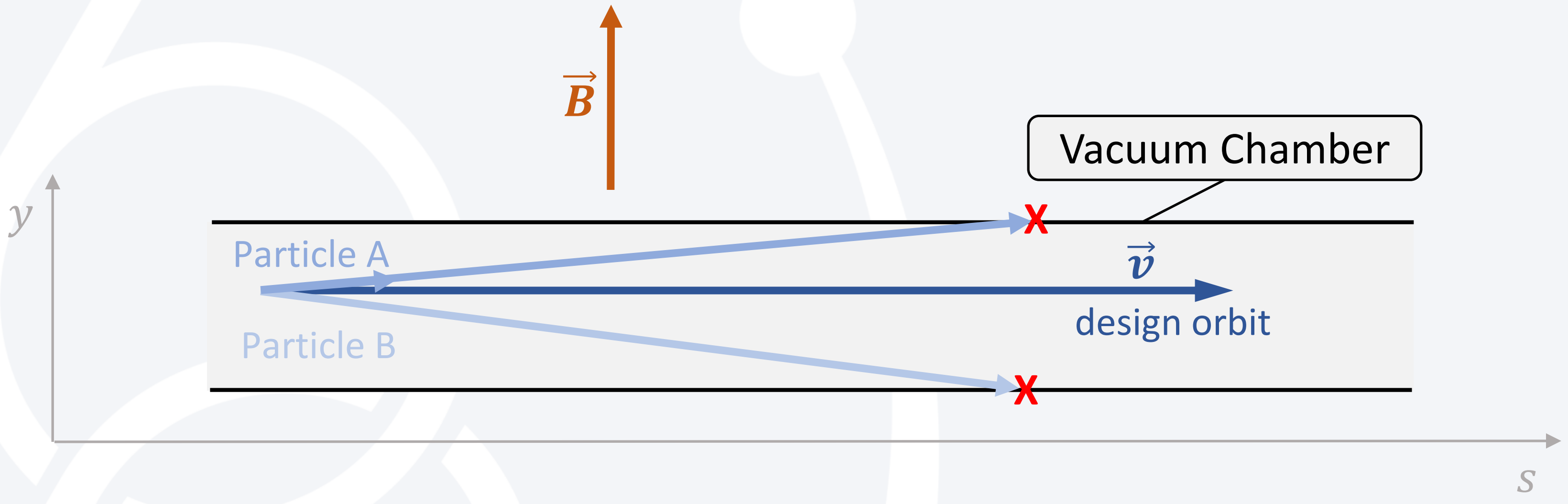
… The very basics

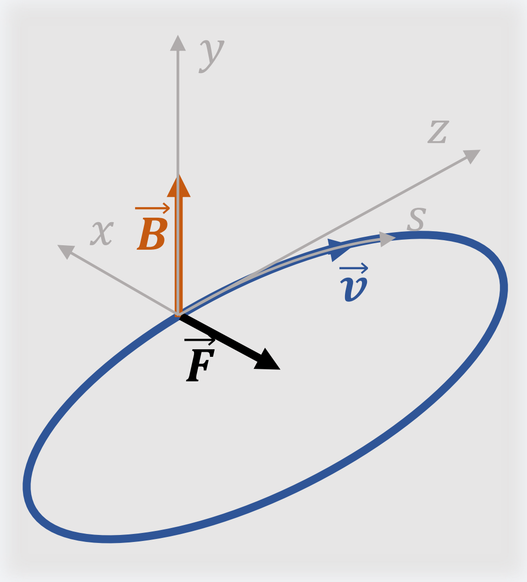
Particle movement in homogeneous dipole field (H)



Horizontal plane: Two particles with the same energy at the same position, but slightly different initial angles meet after each half-turn.

Particle movement in homogeneous dipole field (V)



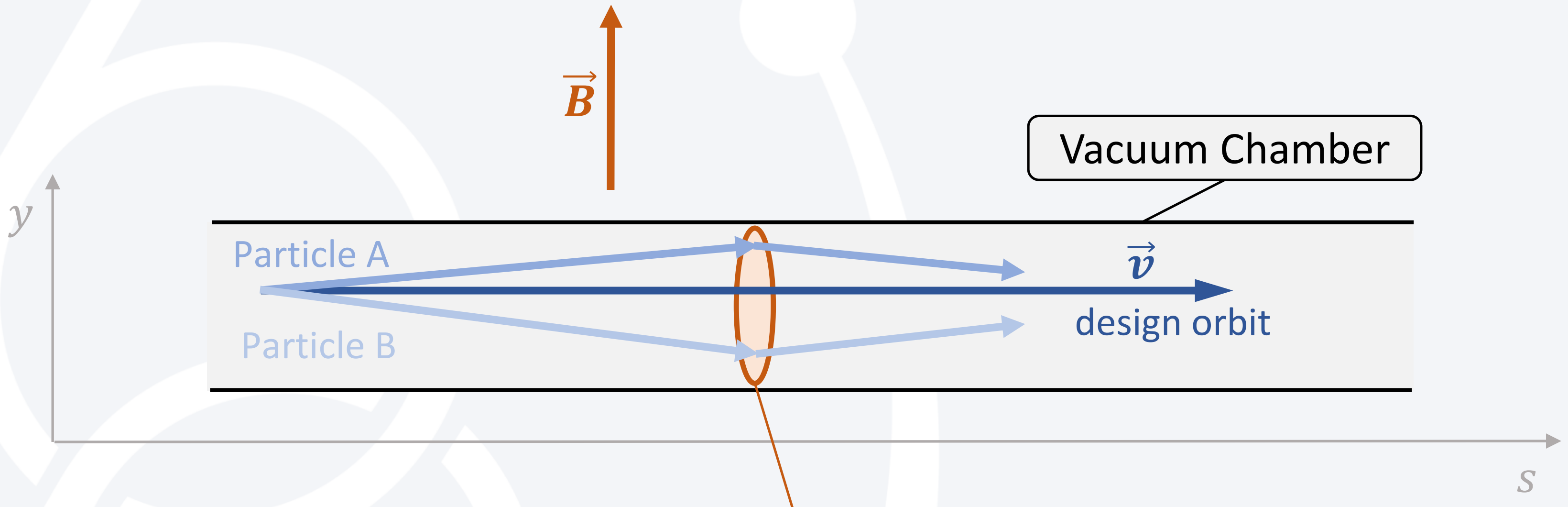


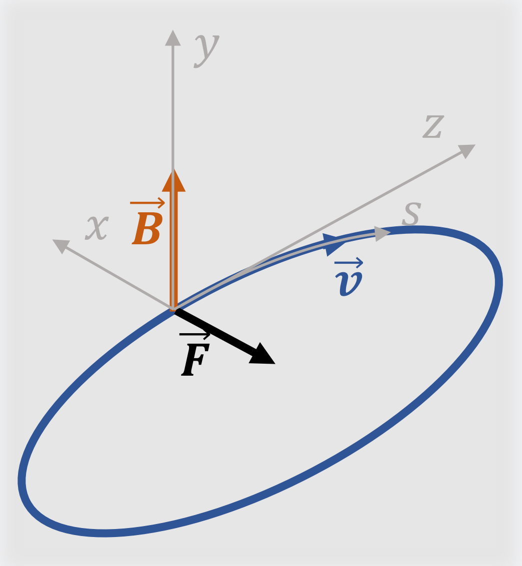
Vertical plane: Two particles with slightly different initial angles: the separation increases along the path

→ Mechanism to keep particles together in aperture is required

→ Co-rotating coordinate system

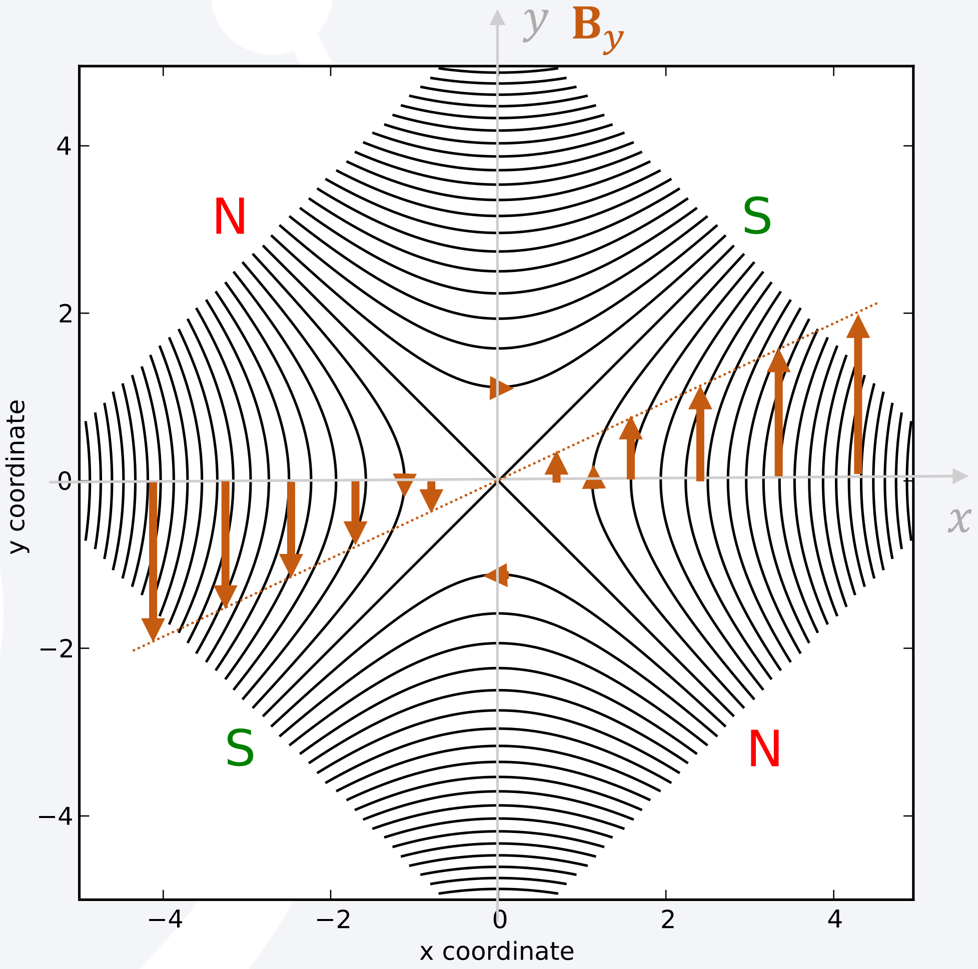
Particle movement in homogeneous dipole field (V)





Electromagnetic lens: quadrupole magnet

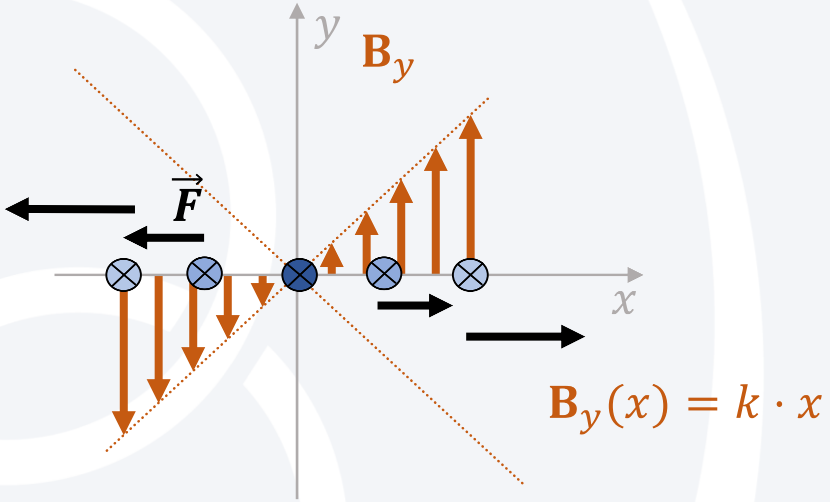
Quadrupole magnet

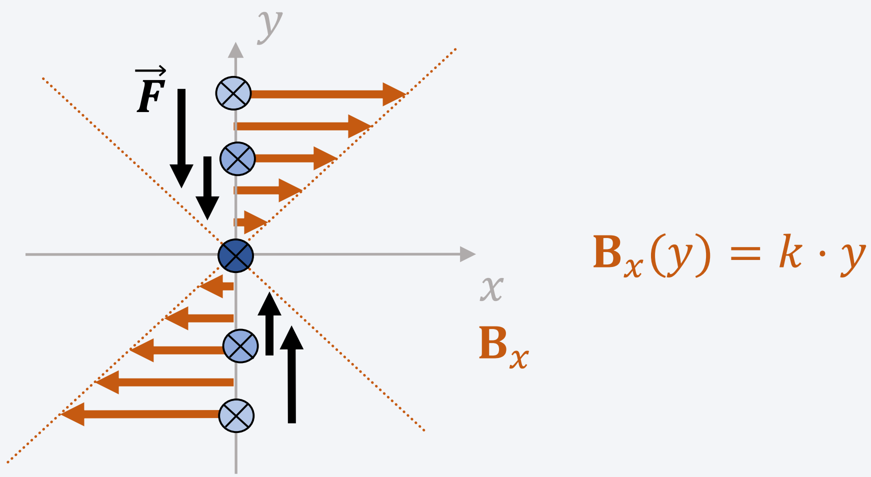


$$\mathbf{B}_y(x) = k \cdot x$$

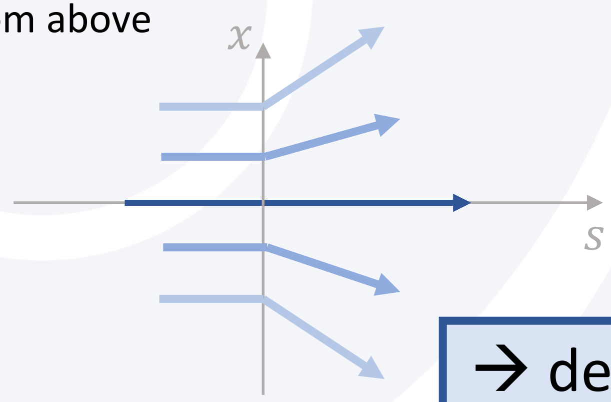
Deflection by quadrupole magnets

View along particle trajectory,
assume that a particle with positive charge is moving into the screen



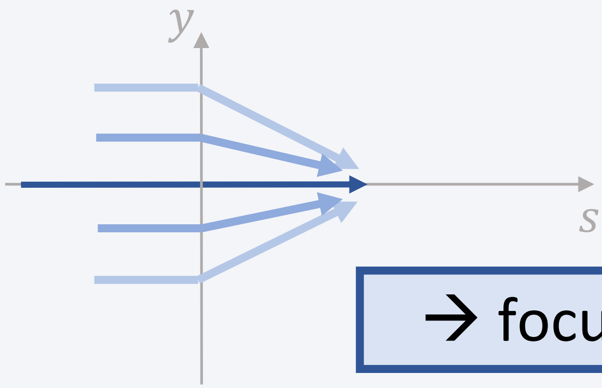


View from above



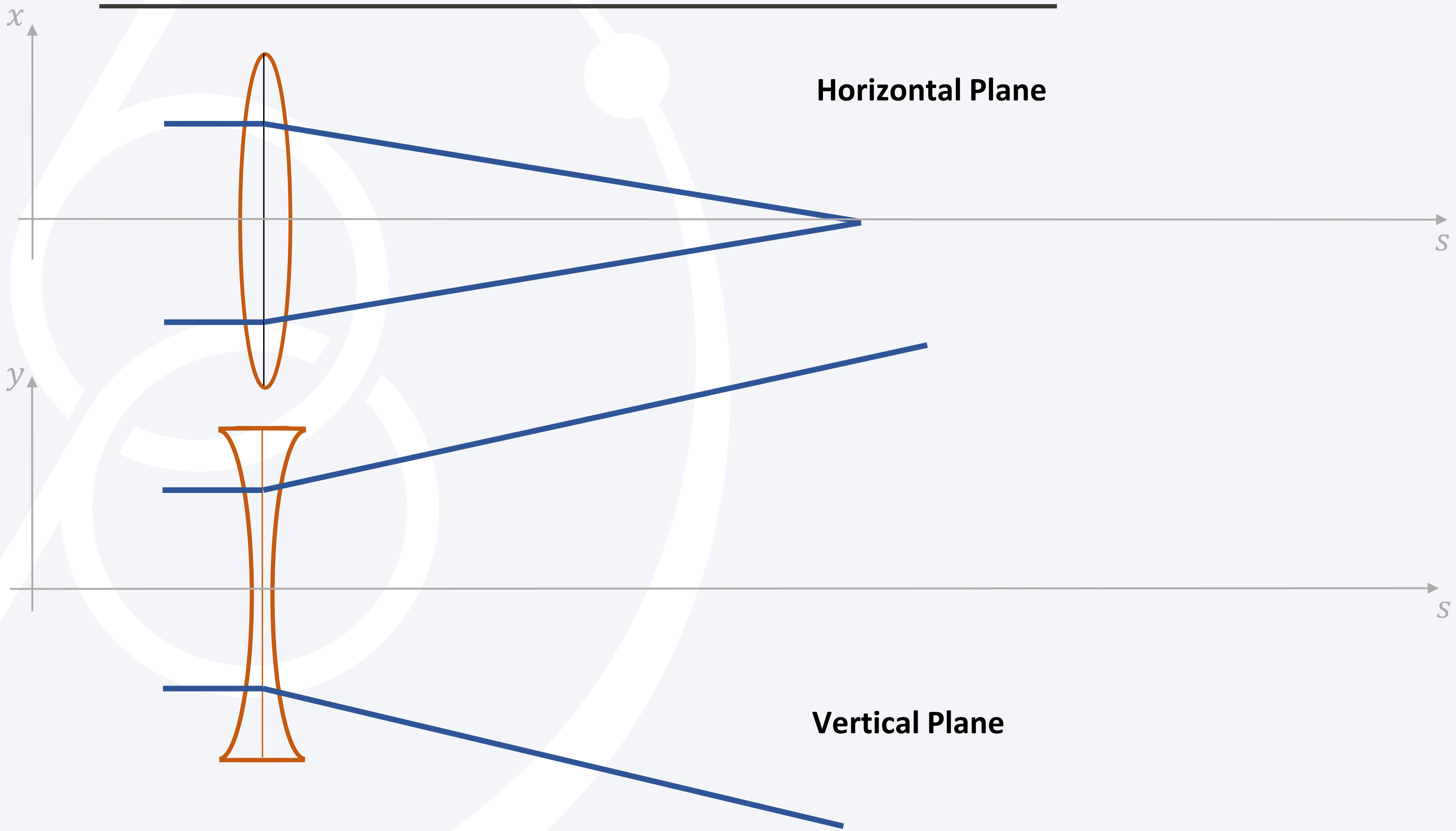
→ defocusing

Side view

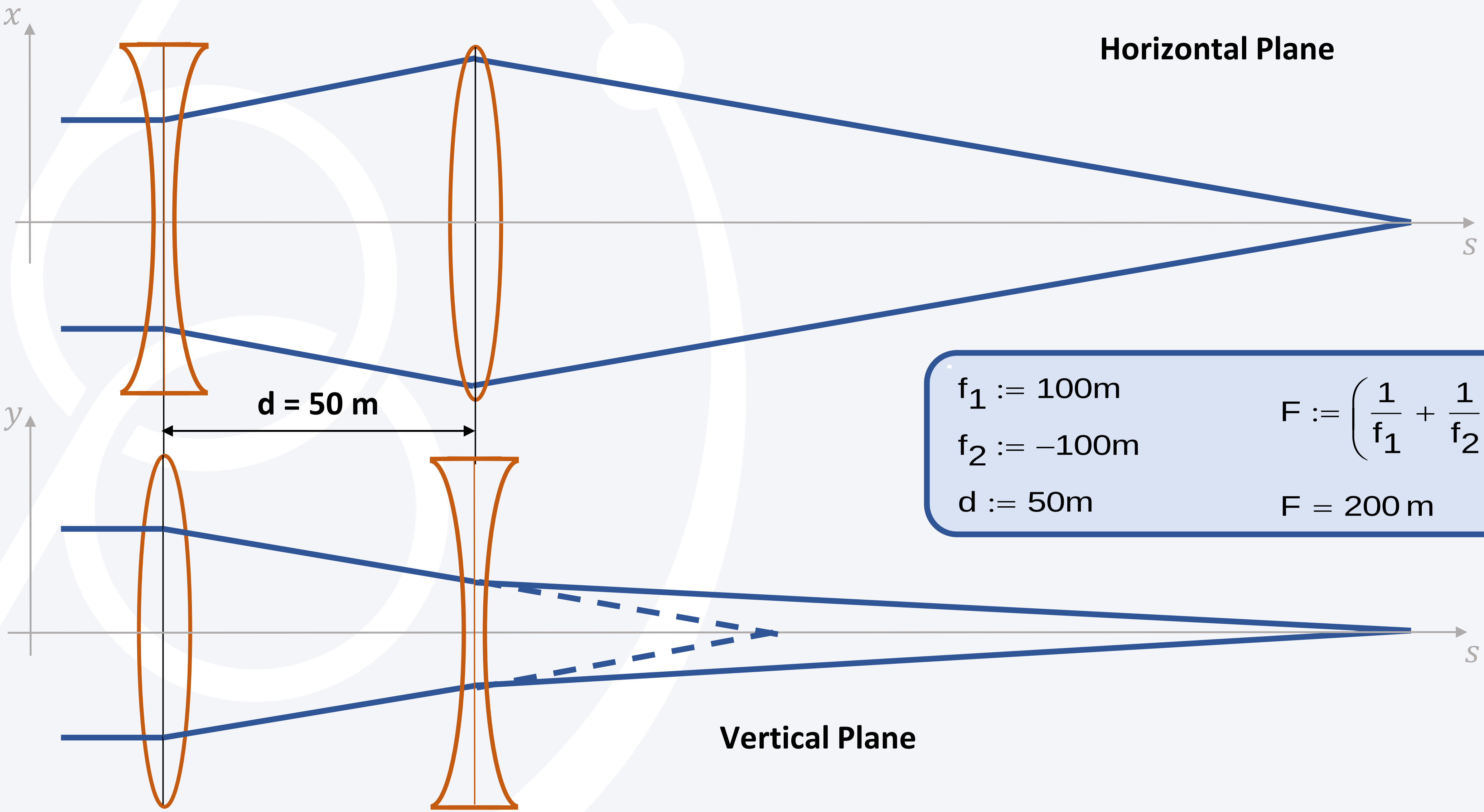


→ focusing

Quadrupole magnets and focusing



Quadrupole magnets and focusing



$f_1 := 100\mathrm{m}$

$f_2 := -100\mathrm{m}$

$d := 50\mathrm{m}$

$$F := \left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}\right)^{-1}$$

$$F = 200\,\mathrm{m}$$

How to understand beam dynamics in a synchrotron?

- "Perspectives":
  - Understanding of the movement of a single particle in the accelerator
  - Understanding of the movement of the entire beam in the accelerator
    - Introduction of parameters for particle ensemble – emittance
    - Derived quantities such as beam size
- Calculation
  - Option A: use transfer-matrices for each element in the accelerator, and calculate trajectory for a particle
  - Option B: use differential equation to calculate trajectory
- Common Parametrization:
  - Transverse Movement is commonly parametrized by:
    - x, y: Transverse position of particle
    - x', y': angles in respective directions
  - Parameter Vector per plane:

Horizontal: $\begin{pmatrix} x \\ x' \end{pmatrix}$ Vertical : $\begin{pmatrix} y \\ y' \end{pmatrix}$

$$u' = \frac{\partial u}{\partial s}$$ (u: x|y)

Option A: Transport matrices for particle coordinates

$$M_D = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Drift with length L

$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{k} \cdot s) & \frac{1}{\sqrt{k}} \cdot \sinh(\sqrt{k} \cdot s) \\ \sqrt{k} \cdot \sinh(\sqrt{k} \cdot s) & \cosh(\sqrt{k} \cdot s) \end{pmatrix}$$

Defocusing Quadrupol with strength k and length s

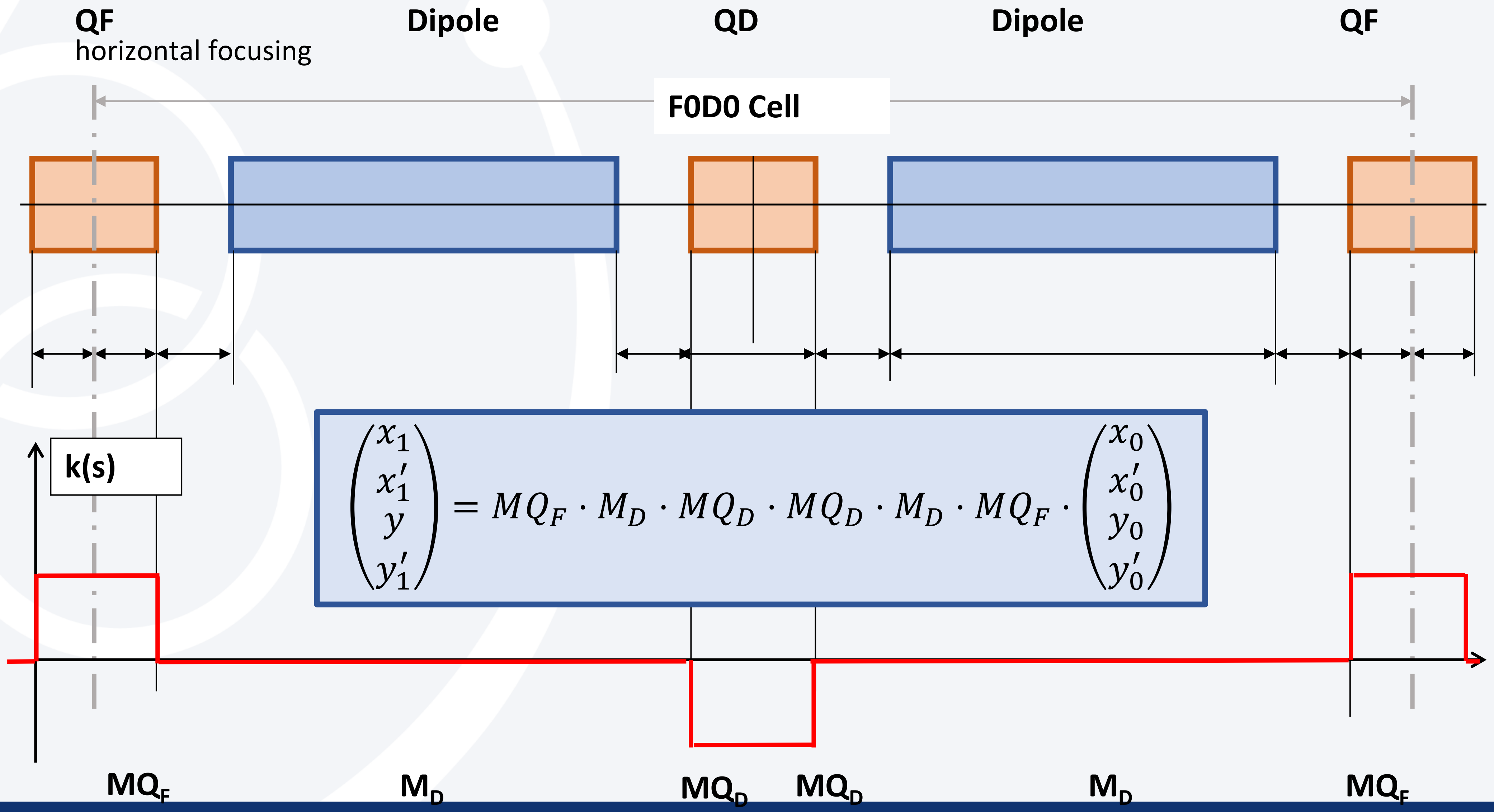
$$M_{QF} = \begin{pmatrix} \cos(\sqrt{k} \cdot s) & \frac{1}{\sqrt{k}} \cdot \sin(\sqrt{k} \cdot s) \\ -\sqrt{k} \cdot \sin(\sqrt{k} \cdot s) & \cos(\sqrt{k} \cdot s) \end{pmatrix}$$

Focusing Quadrupol with strength k and length s

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix} = M \times \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix}$$

$$k_x = \frac{e_0}{p} \cdot \frac{dB_y}{dx} \qquad k_y = \frac{e_0}{p} \cdot \frac{dB_x}{dy}$$

F0D0 cell



$$\begin{pmatrix} x_1 \\ x_1' \\ y \\ y_1' \end{pmatrix} = MQ_F \cdot M_D \cdot MQ_D \cdot MQ_D \cdot M_D \cdot MQ_F \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix}$$

Betatron function and betatron oscillations

– From the transfer matrices it is possible to derive equations for the particle movement around the accelerator

– The particle trajectory can be described as oscillation around the design orbit, with varying amplitude and phase: **betatron oscillations**

$$x(s) = \sqrt{\epsilon_x} \cdot \sqrt{\beta_x(s)} \cdot \cos(\mu_x(s) + \Phi_x)$$

– The beta function is a periodic function, always positive, determined by the focusing properties of the lattice: i.e. quadrupoles:

$$\beta_x(s + L) = \beta_x(s)$$

– The phase advance of the oscillation between the point 0 and point s in the lattice is given by:

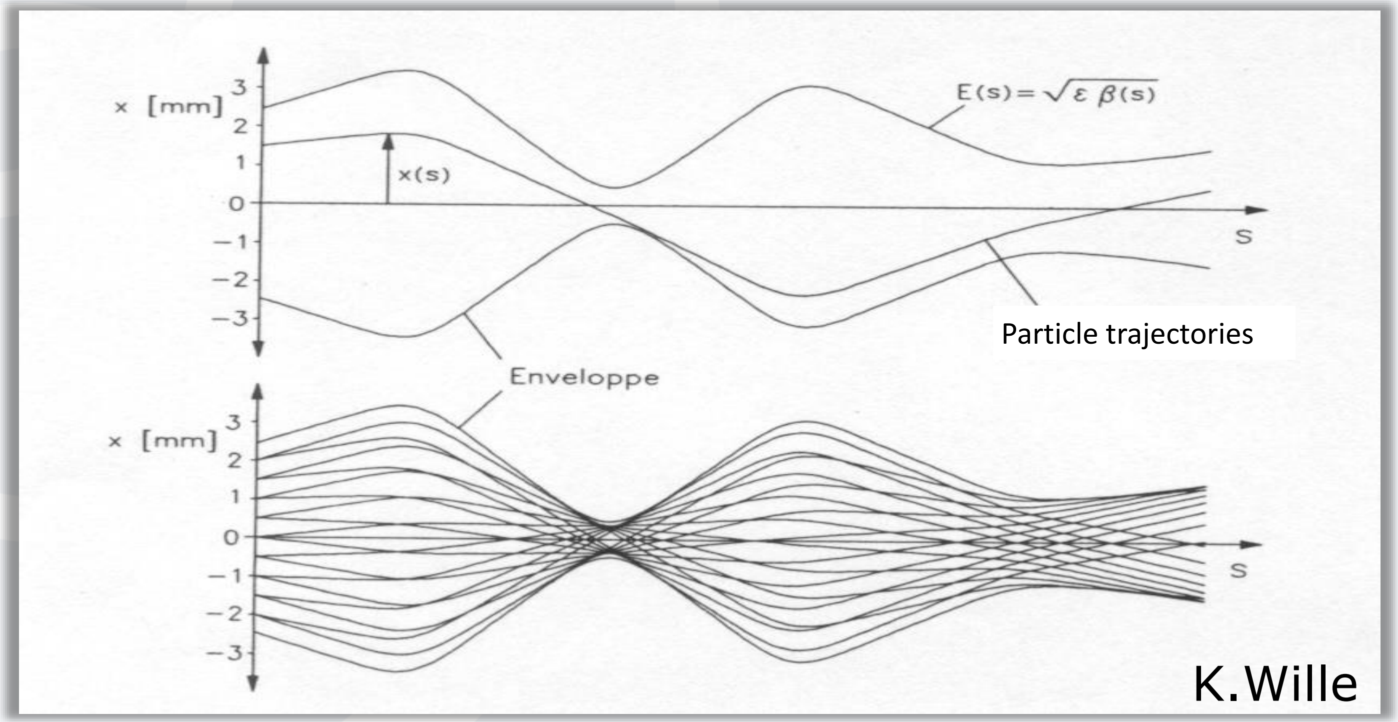
$$\mu_x(s) = \int_0^s ds/\beta_x(s)$$

Betatron trajectories and beam size

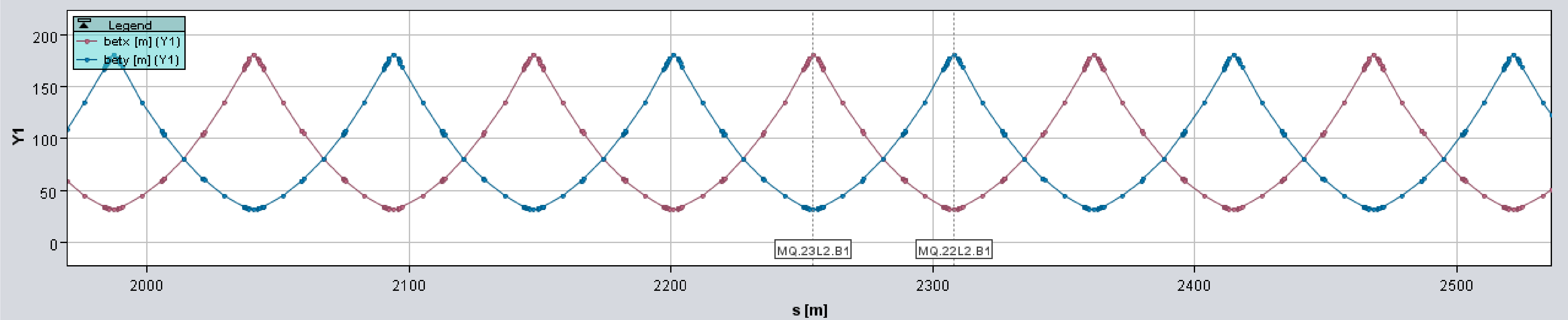
– The beam size σ (assuming Gaussian beams with rms value σ) in the accelerator is given by

$$\sigma_x(s) = \sqrt{\epsilon_x} \cdot \sqrt{\beta_x(s)} \quad \text{and} \quad \sigma_y(s) = \sqrt{\epsilon_y} \cdot \sqrt{\beta_y(s)}$$

– The beam emittance ϵ_x and ϵ_y are statistical quantities and are constant along the accelerator.

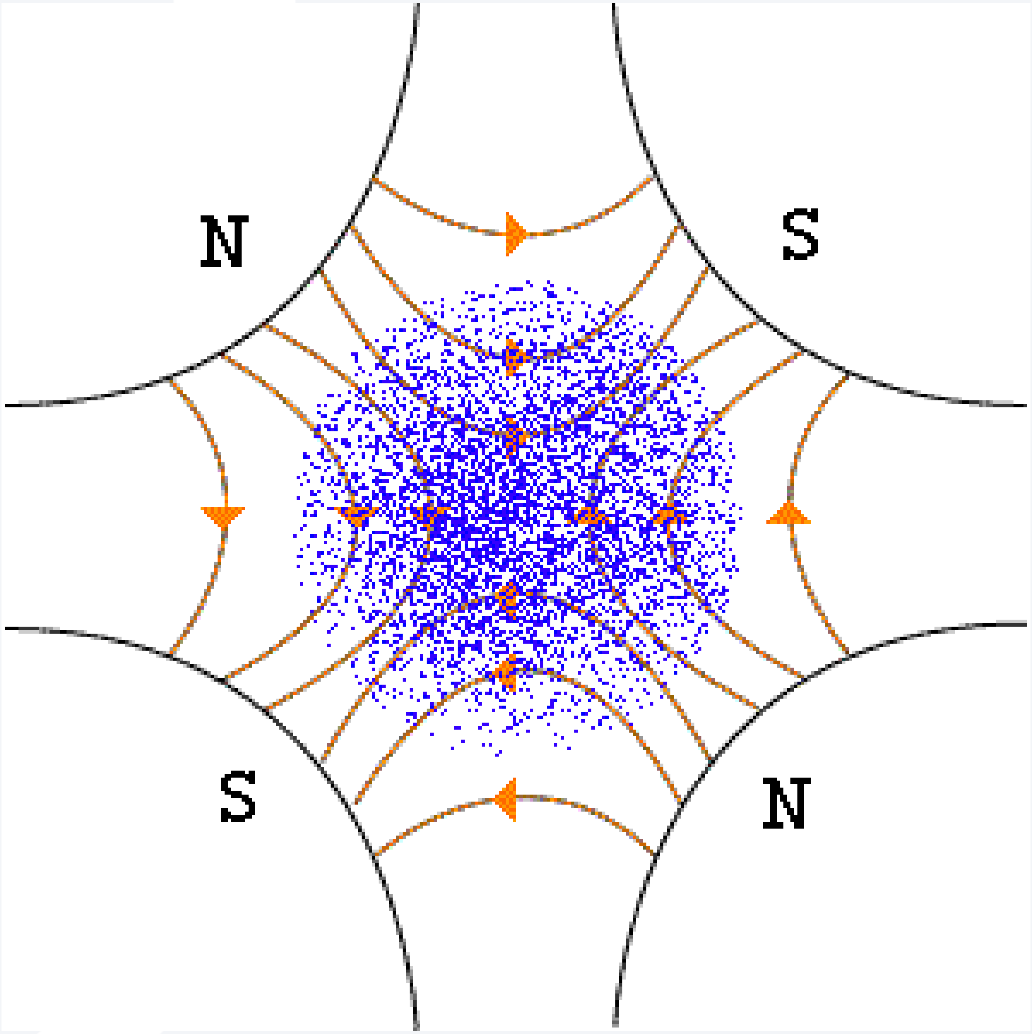


Beta function example

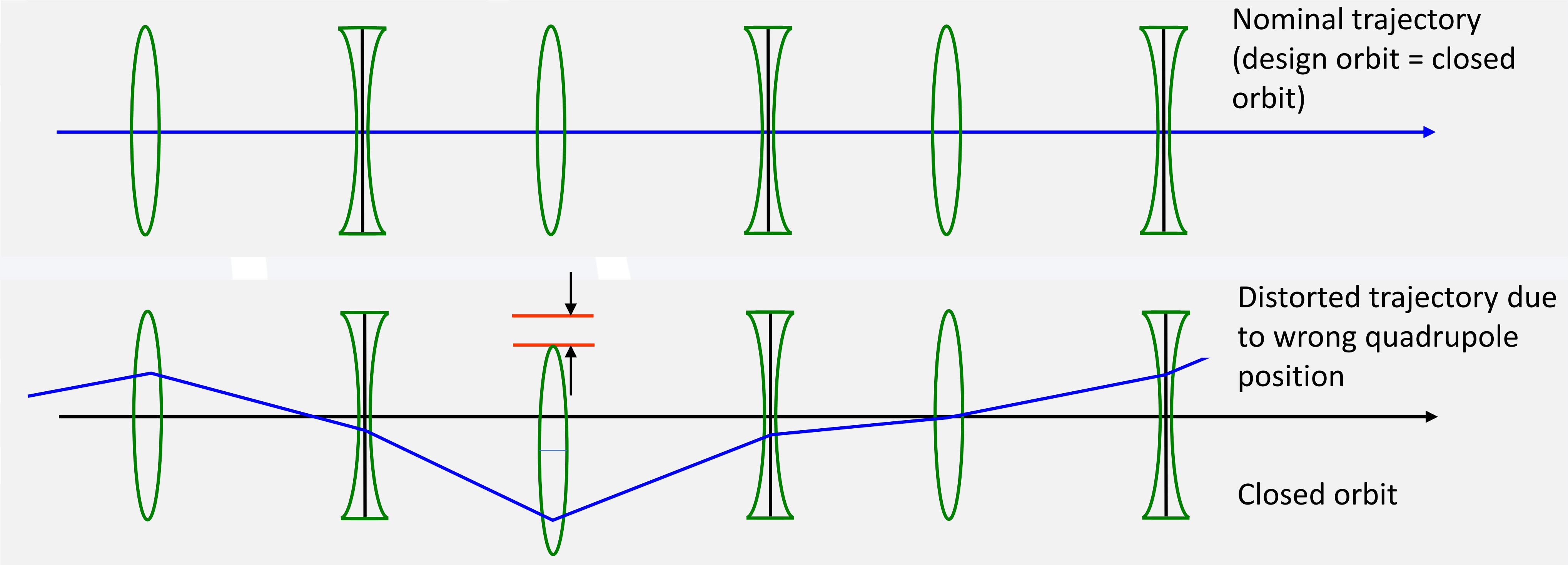


Big in focusing quadrupole, small in defocusing.

Visualisation



Closed orbit

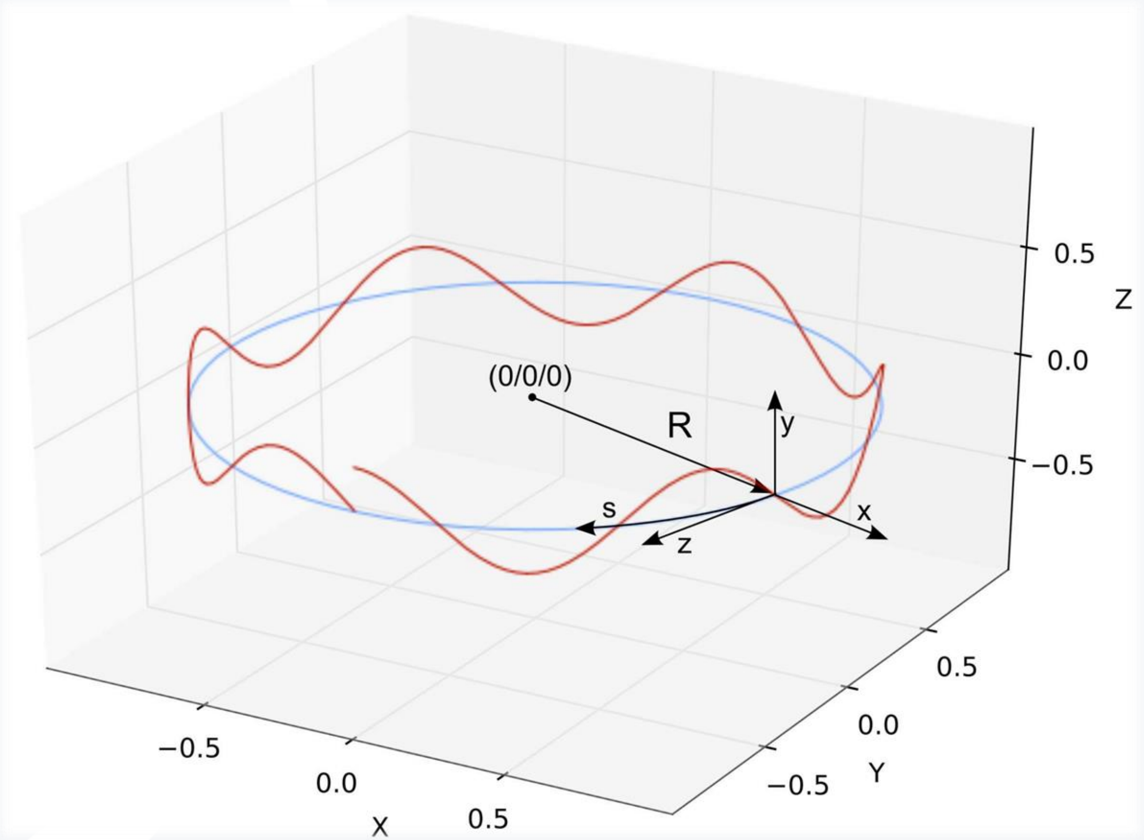


It is possible to show that there exists one closed particle trajectory around the accelerator → "Closed orbit"

Tune and Chromaticity

The Tune

Beam Oscillates around its ideal orbit.
Tune = Number of oscillations per turn

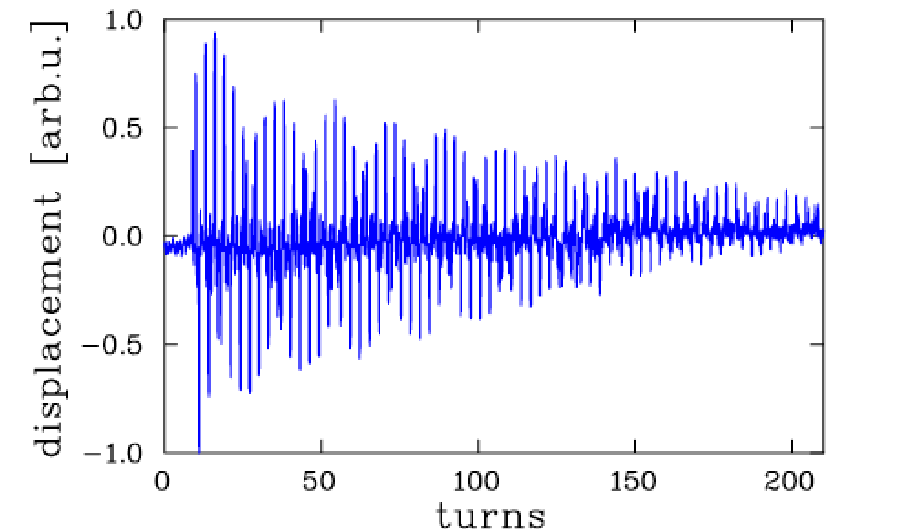


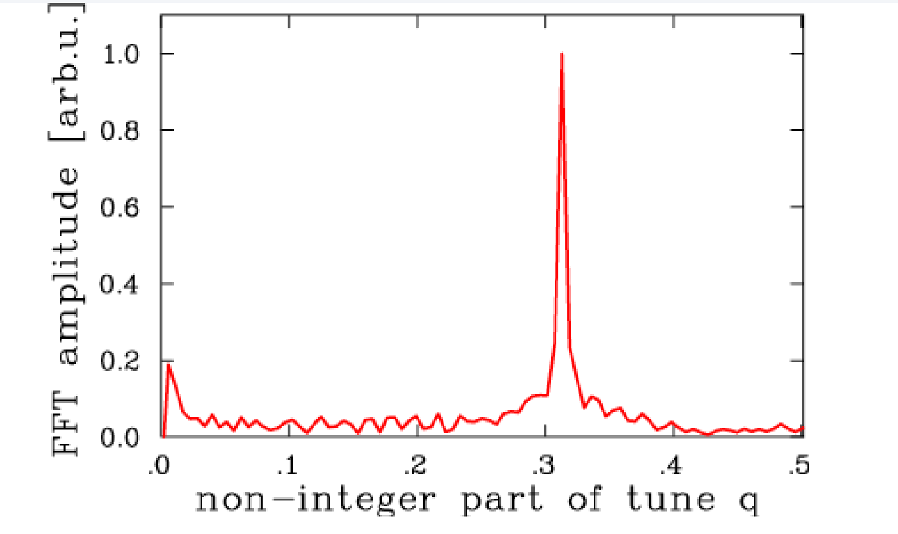
Betatron tune

– “phase advance for one turn” (for each plane):

$$Q = \frac{1}{2\pi}(\mu(s+C) - \mu(s)) = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$$

With a FFT of turn-by-turn data from a beam position monitor at one specific location in the accelerator we get the frequency of oscillation:

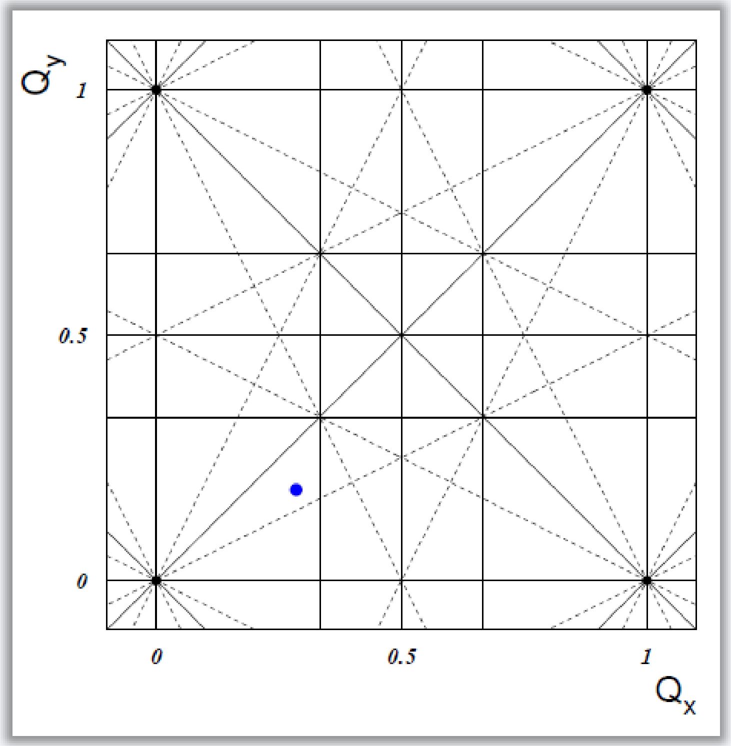




Why does it matter?

Motion can become unstable …

$$\Delta u_2 = \frac{\sqrt{\beta_1 \beta_2}}{2\boxed{\sin(\pi Q)}} \cdot \delta_1$$

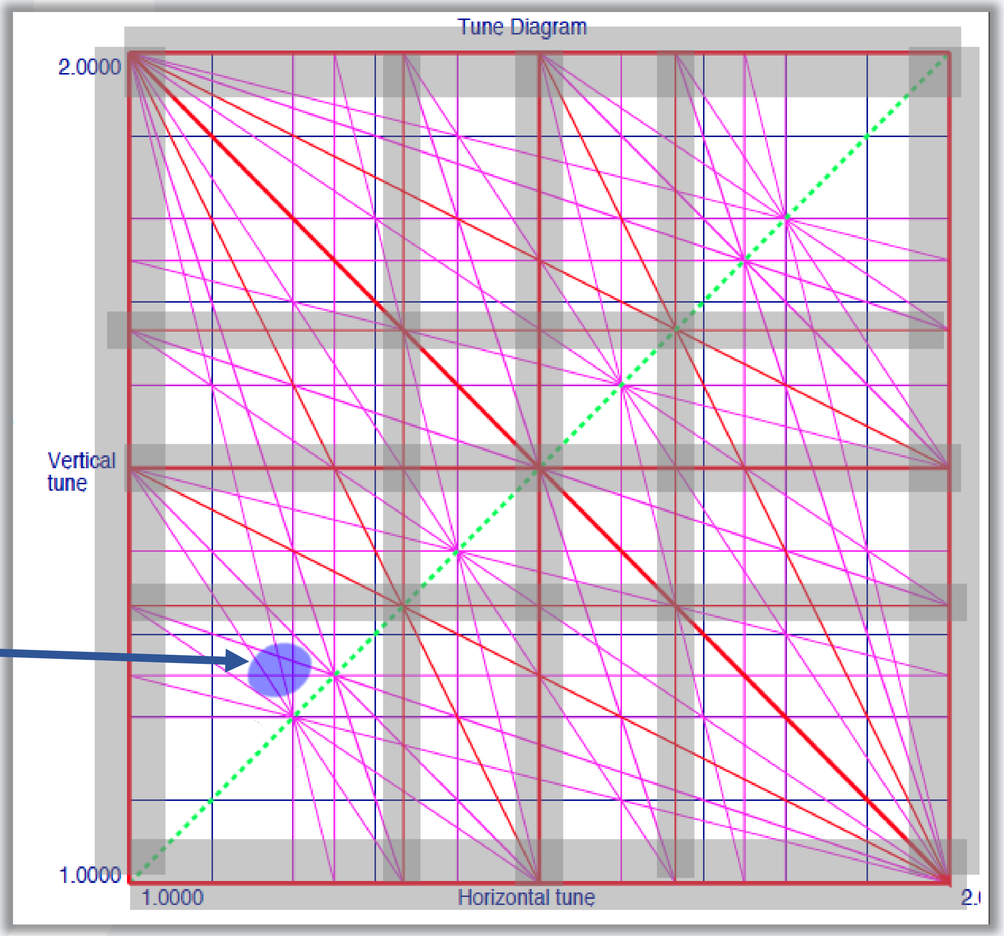


General case:

$mQ_x + nQ_y = p$, with $m, n, p \in Z$

→ Particles with integer, half-integer or third integer tunes risk to be lost

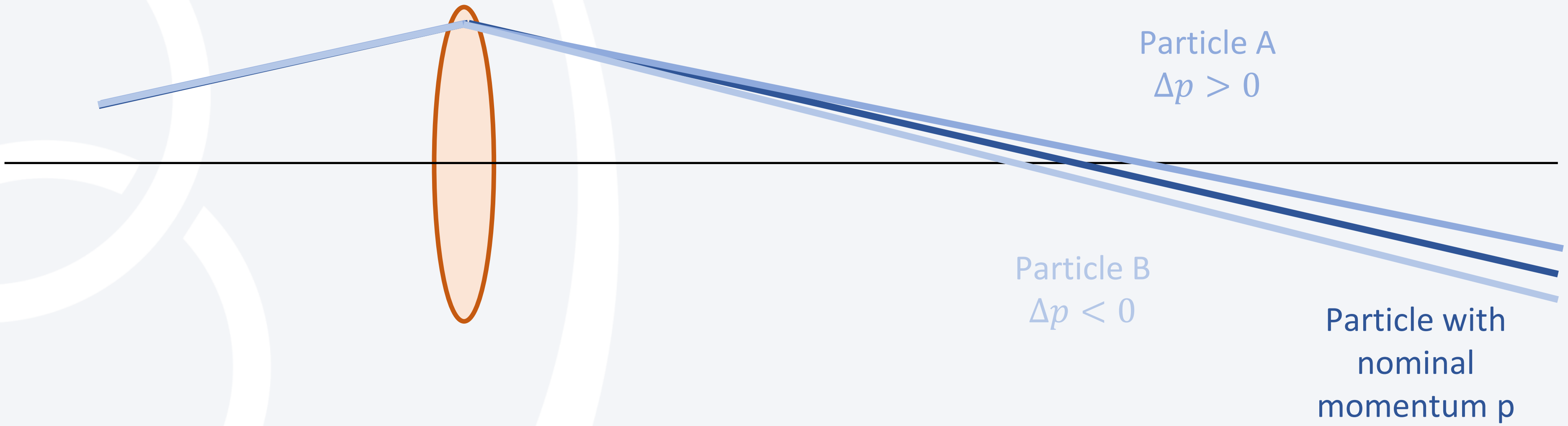
Betatron Tune diagram



„tune spread“
→Not a dot!

Tune Spread I – Chromaticity Q'

– Individual particles have slightly different momentum
– Particles with different momentum are deflected differently

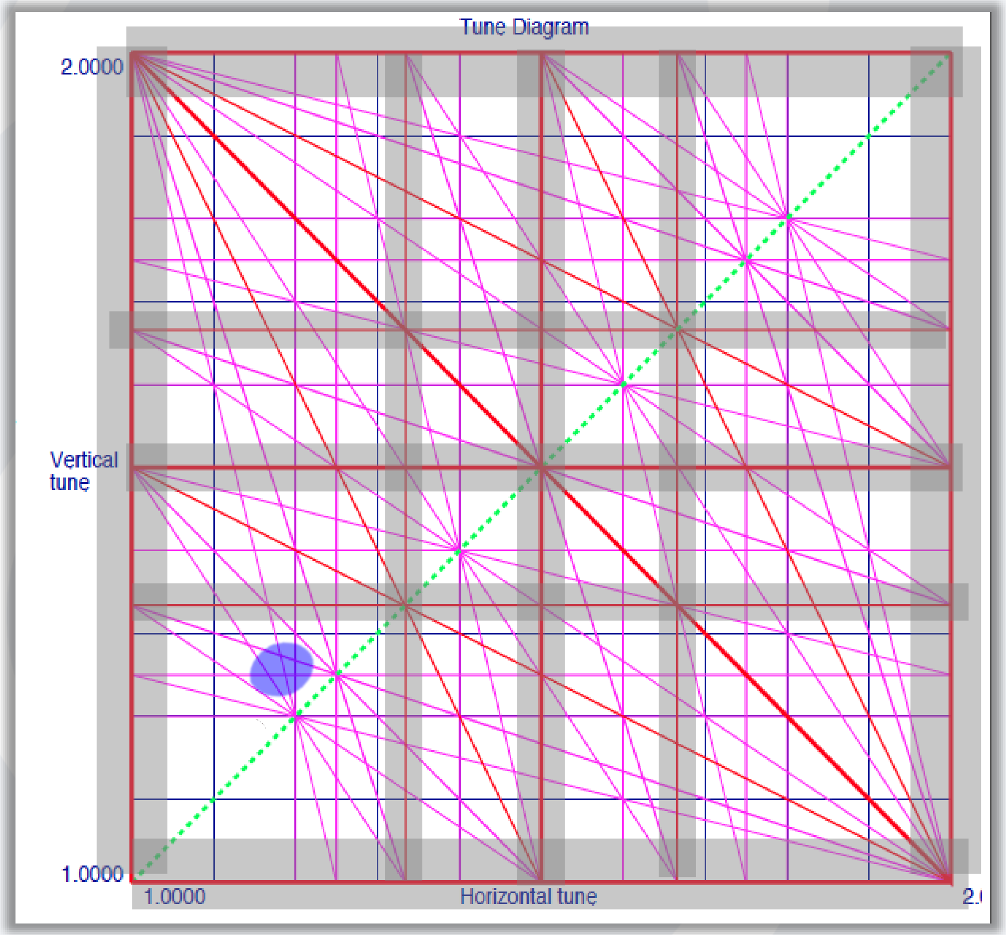


→ Particles with a momentum deviation have different betatron tunes

$$\Delta Q = Q' \frac{\Delta p}{p}$$

In general, trimmed slightly positive (head-tail)

Tune Spread II - Summary



- Chromaticity & energy spread
  - Partially corrected by sextupole magnets.
- Other sources
  (not discussed in detail)
  - beam-beam
  - nonlinear fields
  - effects due to high beam intensity

Dipole effects

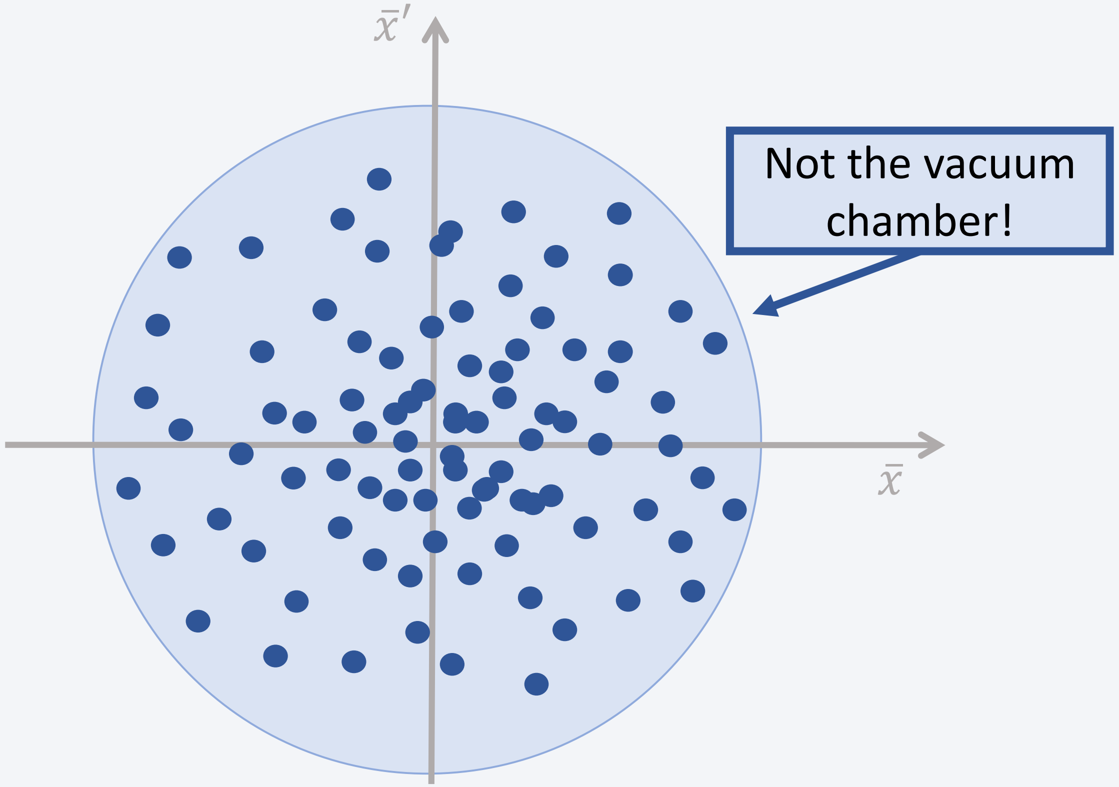
Deflection by dipole magnet

$$\delta = \frac{B \cdot L}{E} \cdot c \cdot e_0$$

With B the magnetic field, L the length of the magnet, and E the beam energy and $c \cdot e_0$ constants (speed of light, elementary charge)

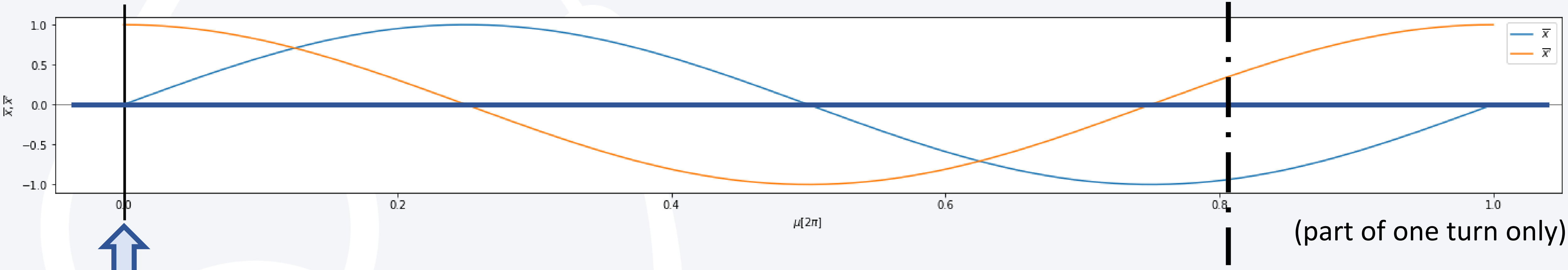
Phase space I

- Assume that position and angle of each particle at one position in the ring is measured and displayed - Phase Space
- In general, phase space is an ellipse and the area is conserved. Can also be parametrized as a circle ("normalized phase space" – see appendix)

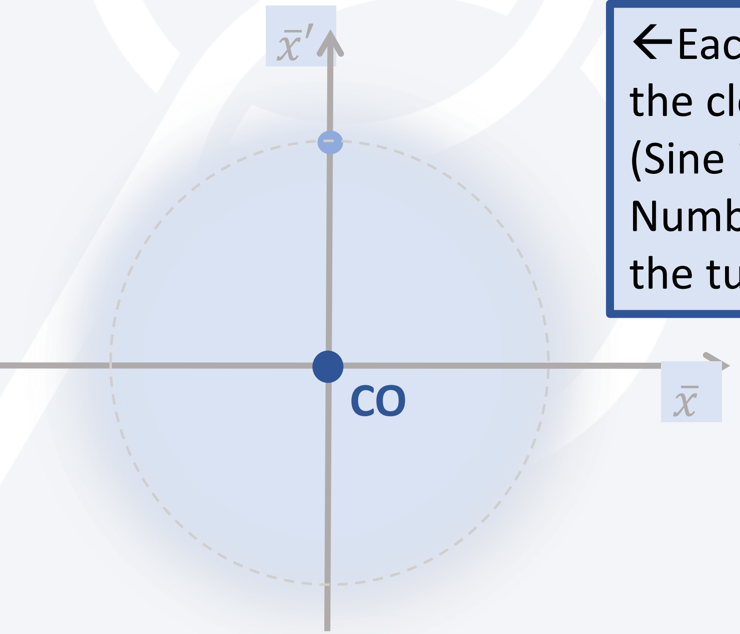


"normalized phase space"

Oscillation in Normalized Phase Space

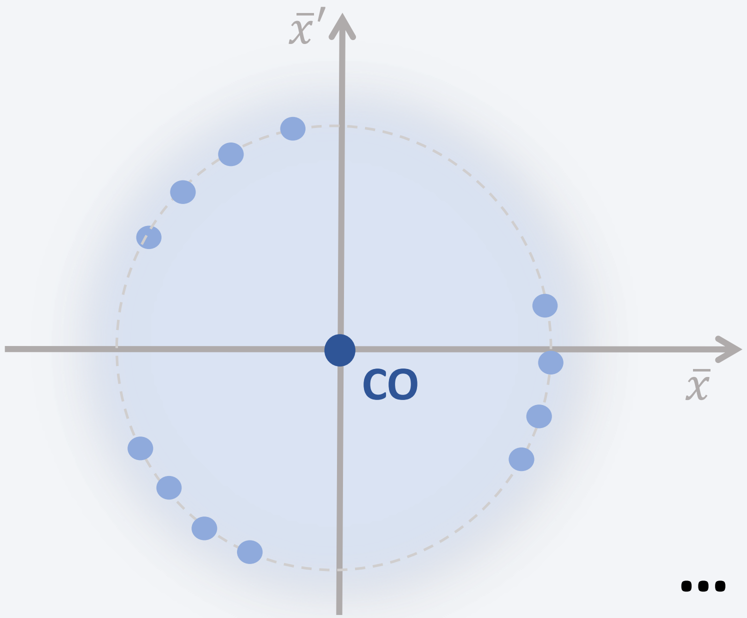


(part of one turn only)



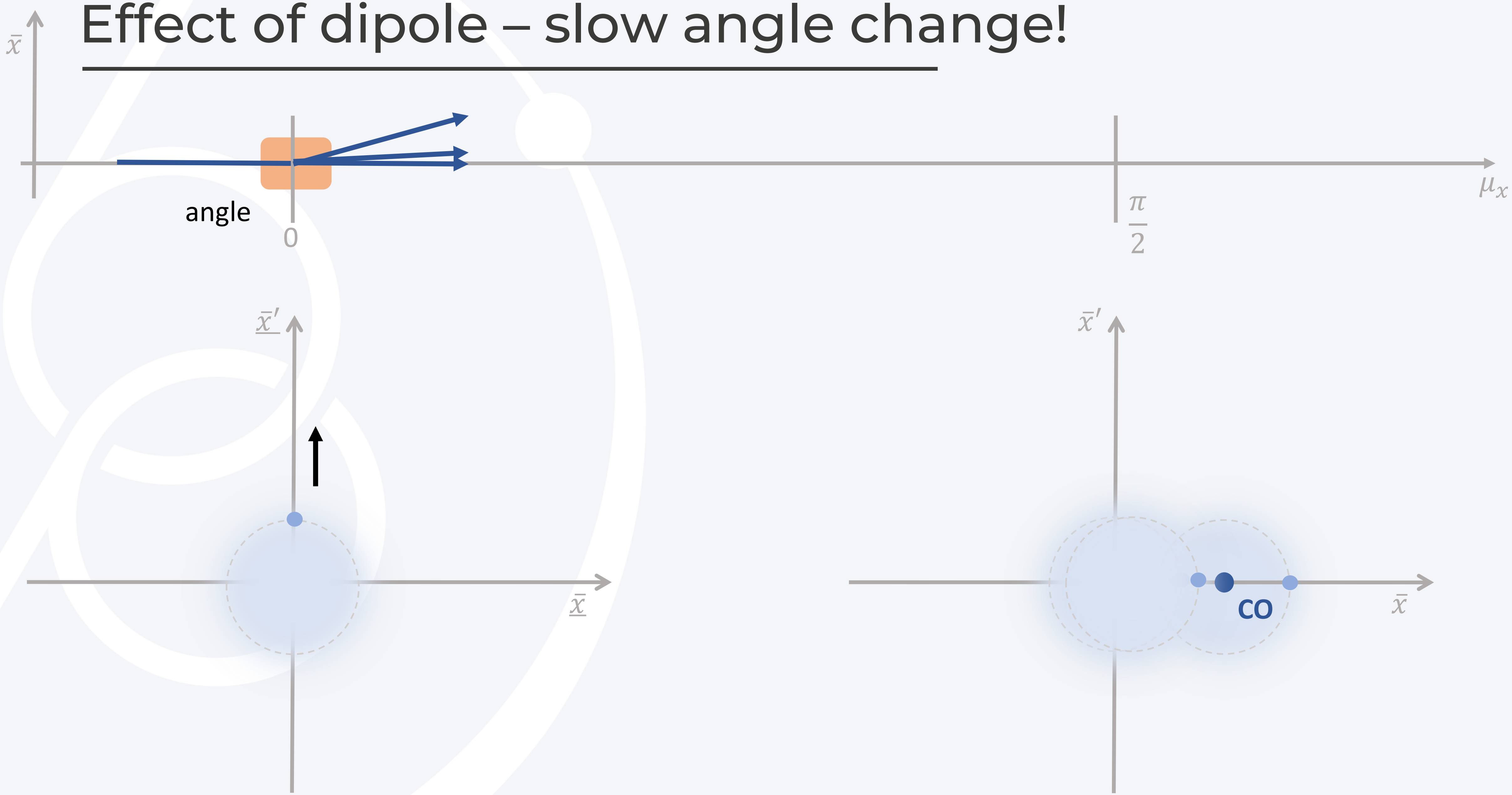
←Each particle oscillates around the closed orbit (CO).
(Sine in normalized phase space)
Number of oscillations per turn is the tune (of the particle)

Looking at one fixed position, the particle also moves in a circle in the phase space – each turn „jumping“ by the non-integer part of the tune ... →



...

Effect of dipole – slow angle change!



Dipole effect examples

Ring

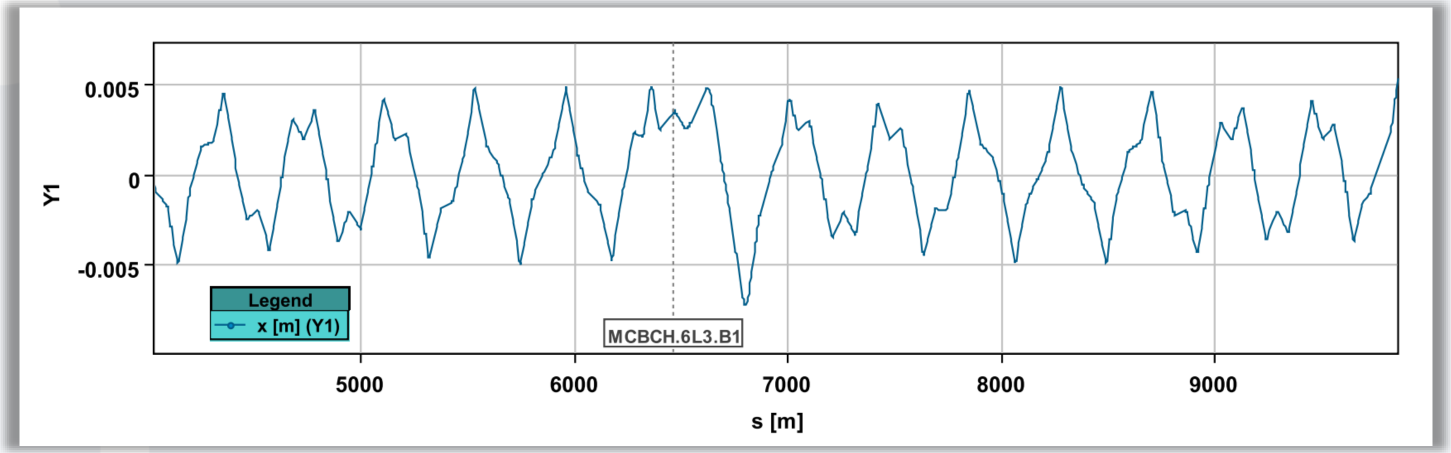
$$\Delta u_2 = \frac{\sqrt{\beta_1\beta_2}\cos\left(|\mu_1-\mu_2|-\pi Q\right)}{2\sin\left(\pi Q\right)}\cdot\delta_1$$

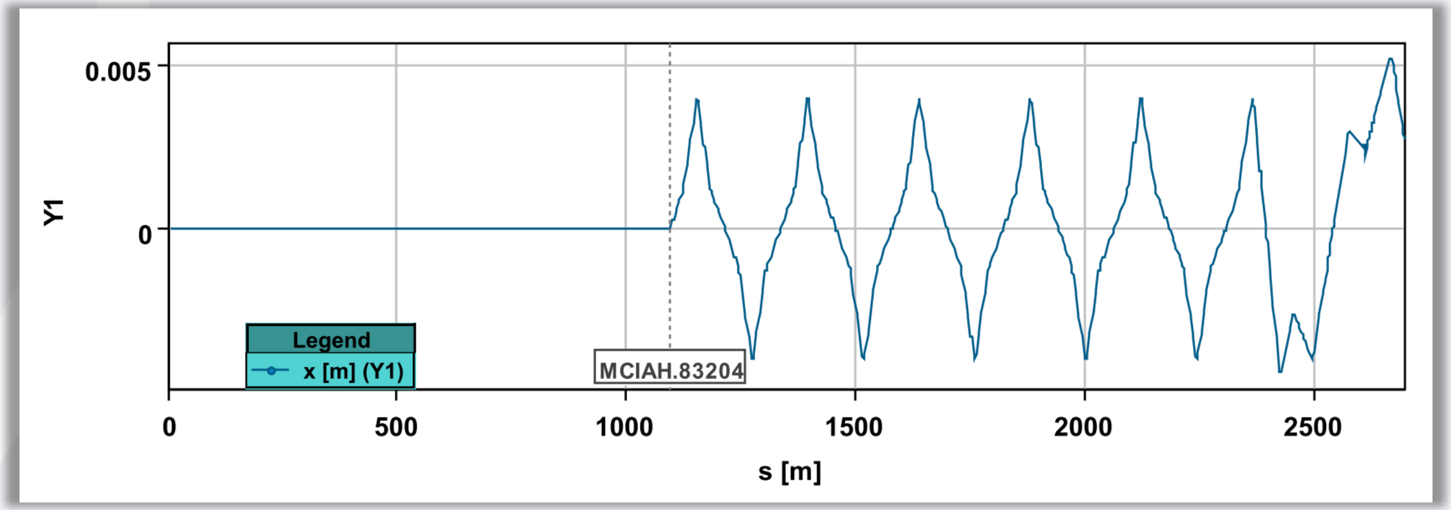
With $\beta_{1,2}$ the beta functions at the location of the magnet, and the location of the observation point, Q the betatron tune and δ the deflection angle in [rad]

Single pass:

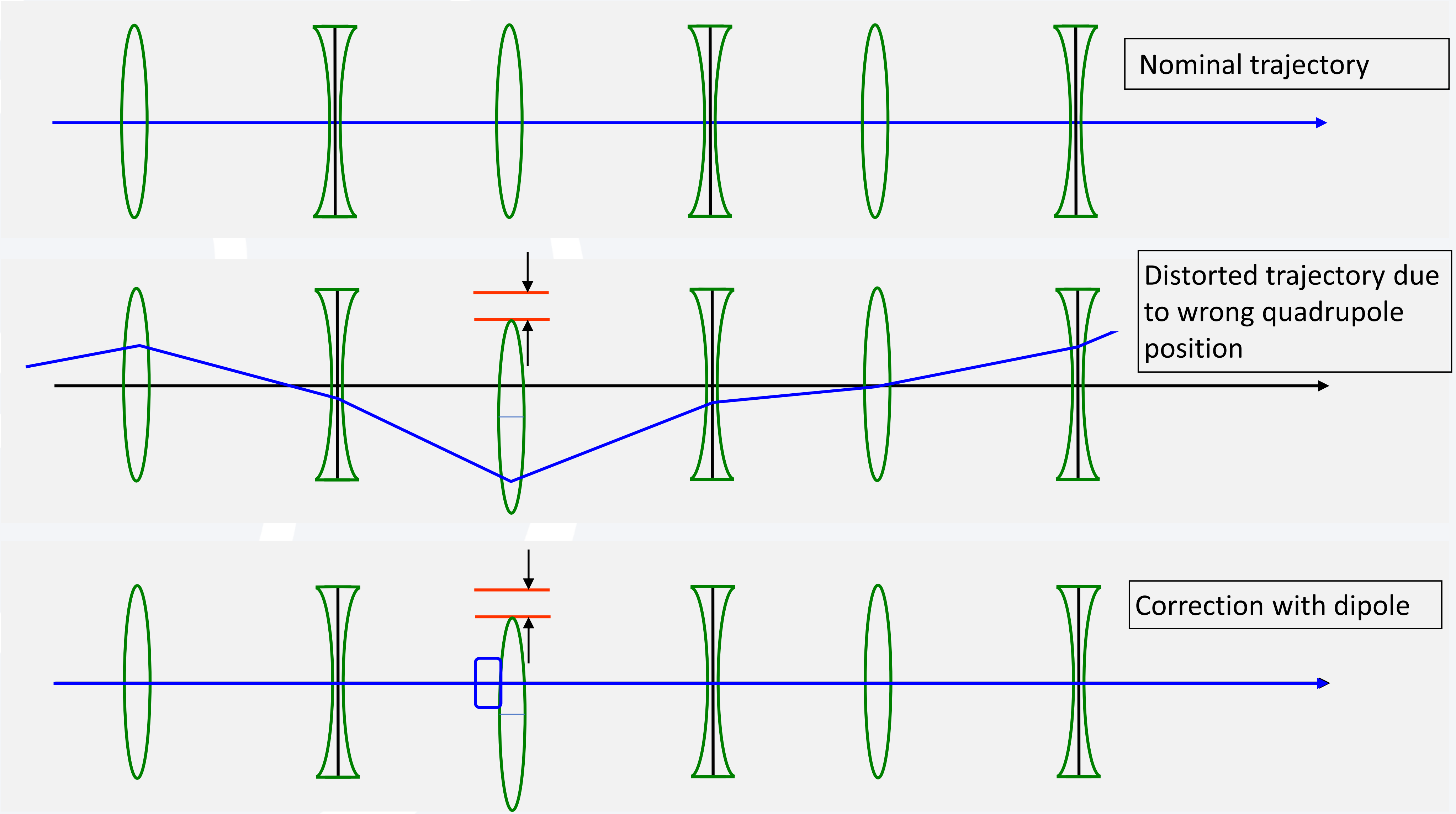
$$\Delta u_2 = \begin{cases}\sqrt{\beta_1\beta_2}\sin\left(\mu_1-\mu_2\right)\cdot\delta_1 & \text{for } \mu_2>\mu_1,\\ 0 & \text{otherwise.}\end{cases}$$

Note: when e.g. injecting, even a Circular Accelerator is a „Transfer Line“!

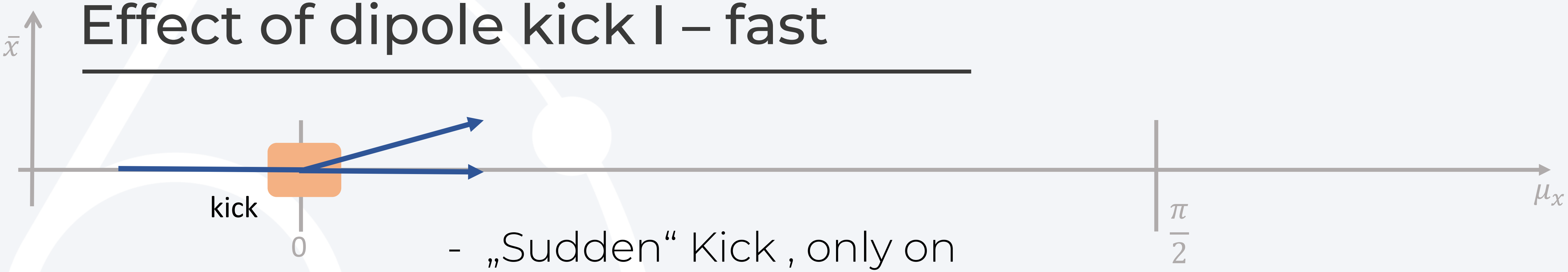




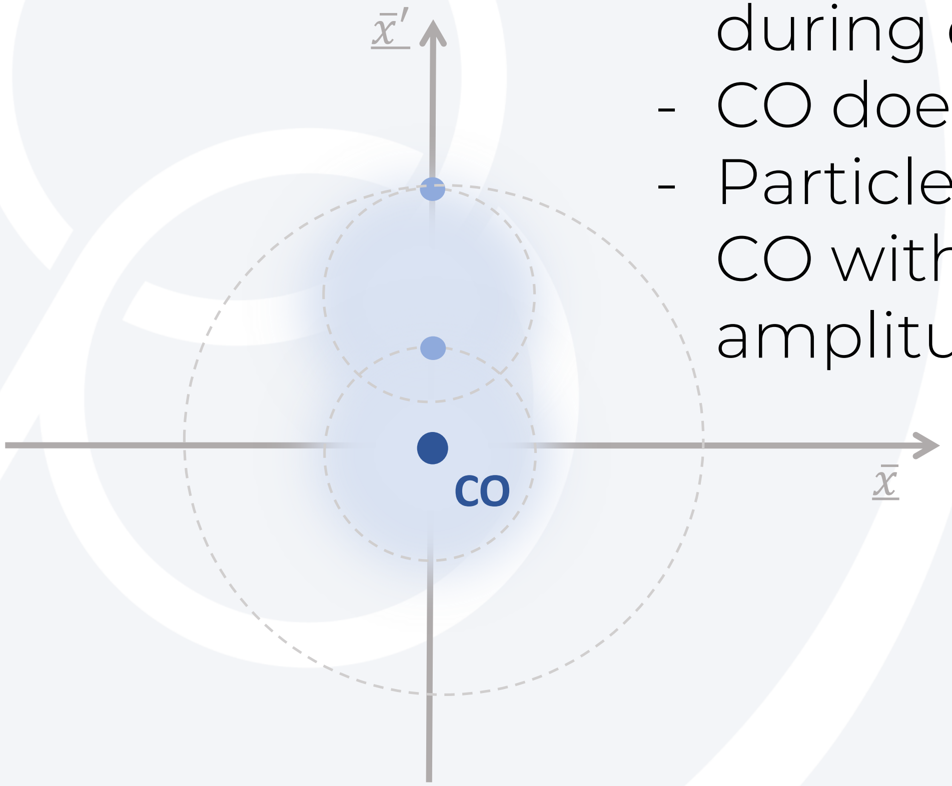
Quadrupole and Dipole kicks –orbit correction

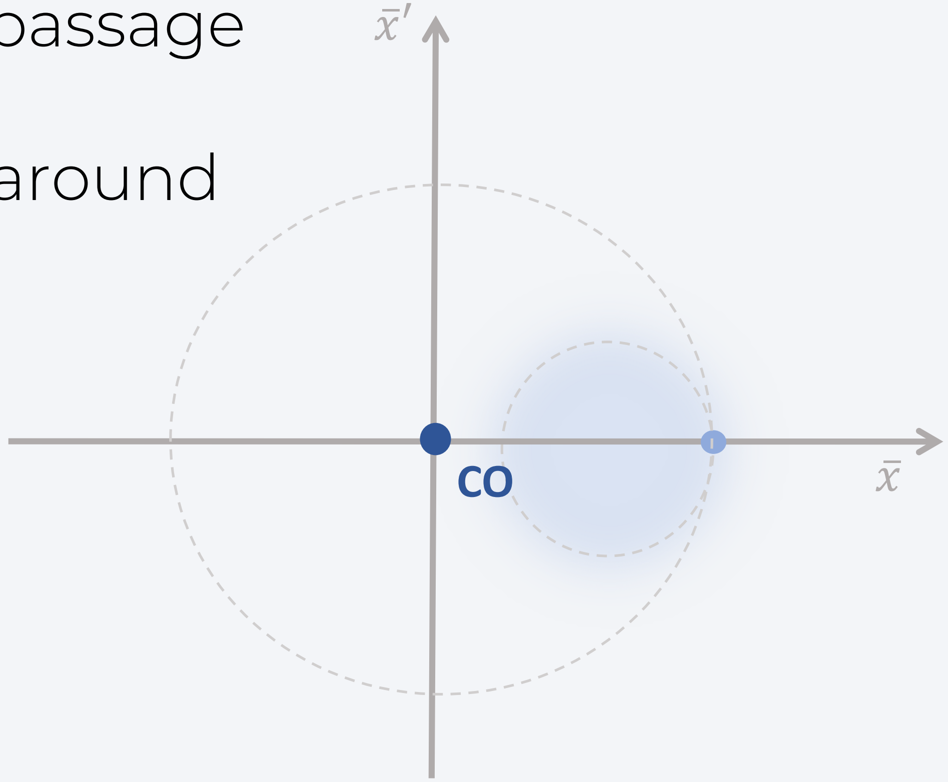


Effect of dipole kick I – fast

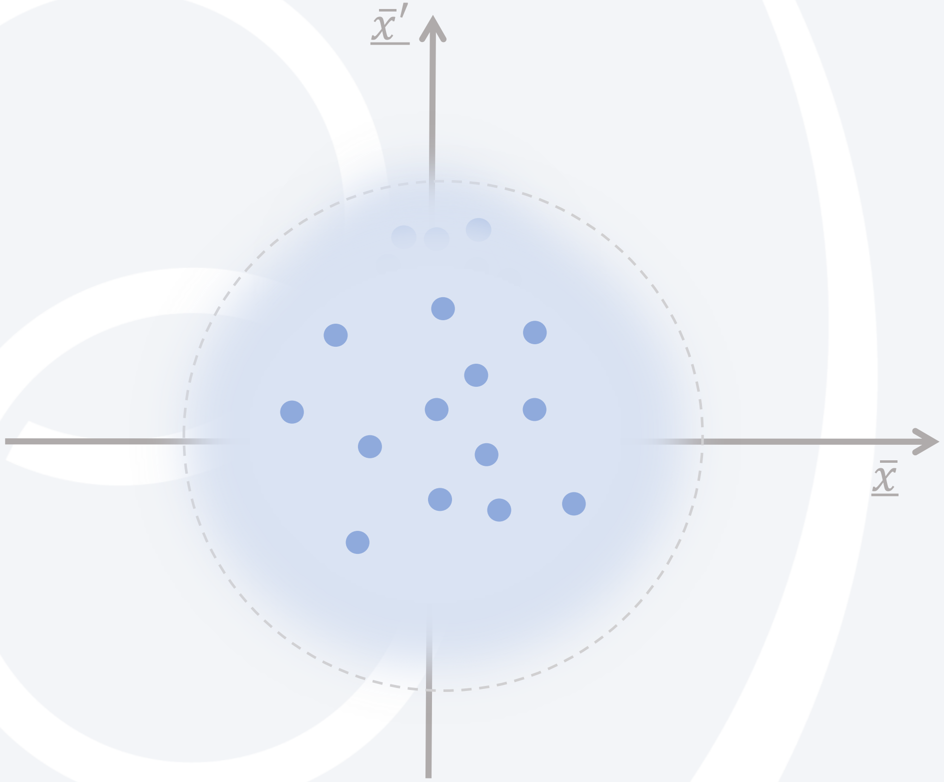


- „Sudden“ Kick , only on during one single passage
- CO does not move
- Particles circulate around CO with a higher amplitude





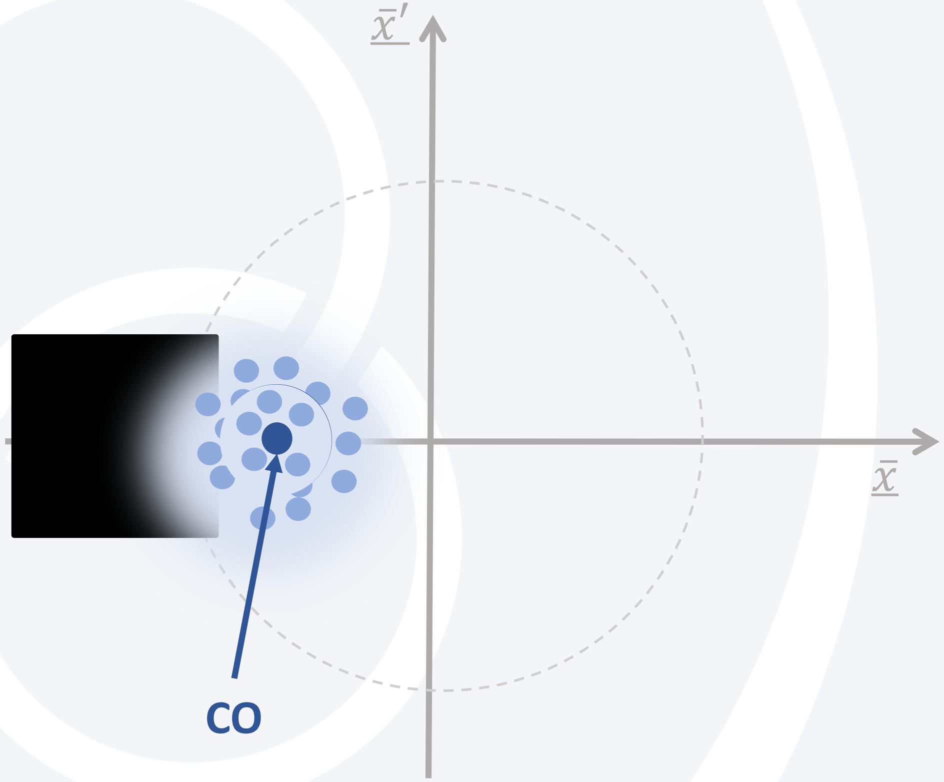
Effect of dipole kick I - fast!



- Individual particles have slightly different tune
- → After many turns, they fill up a bigger area of the phase-space.

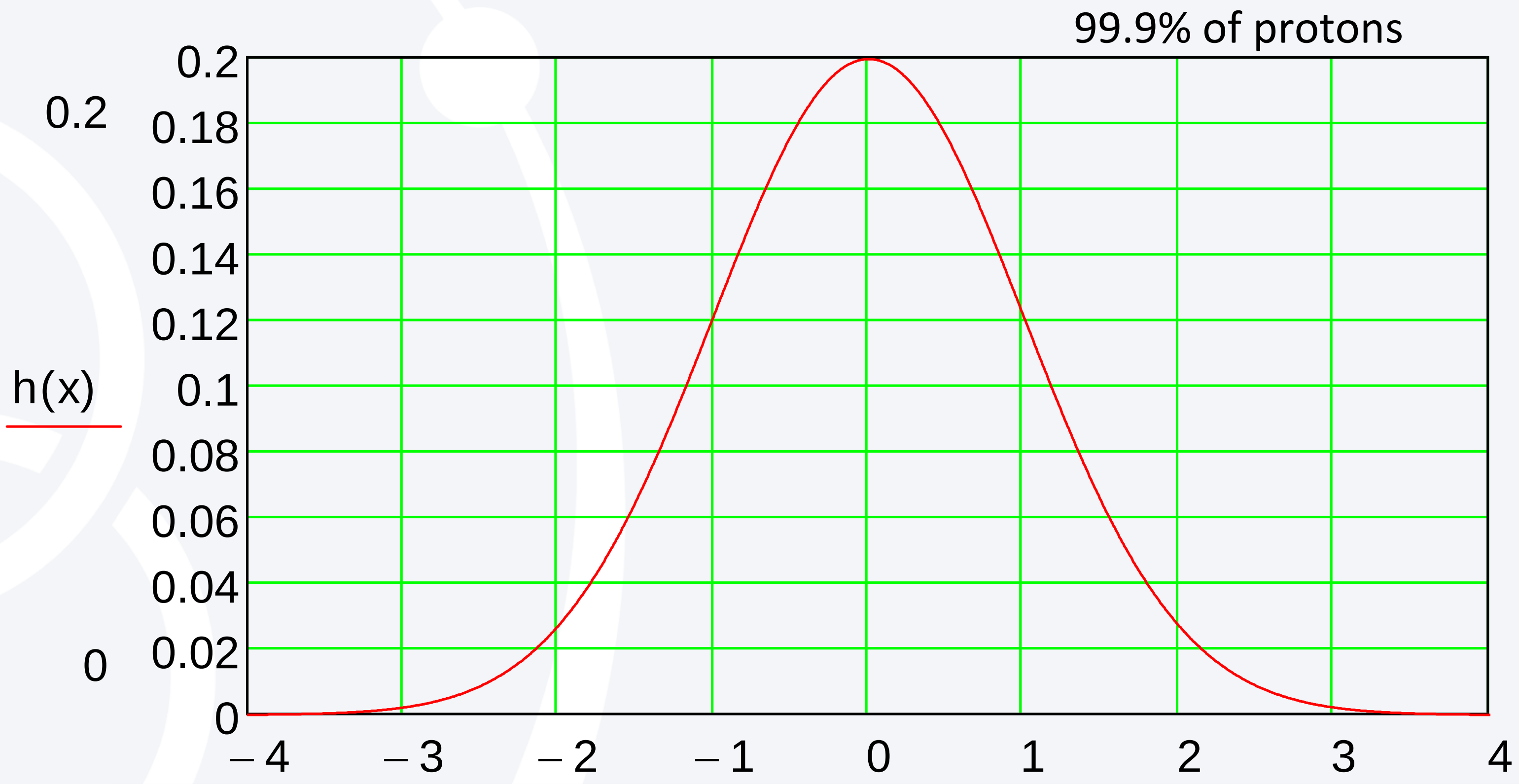
→ Blowup

Touching the aperture



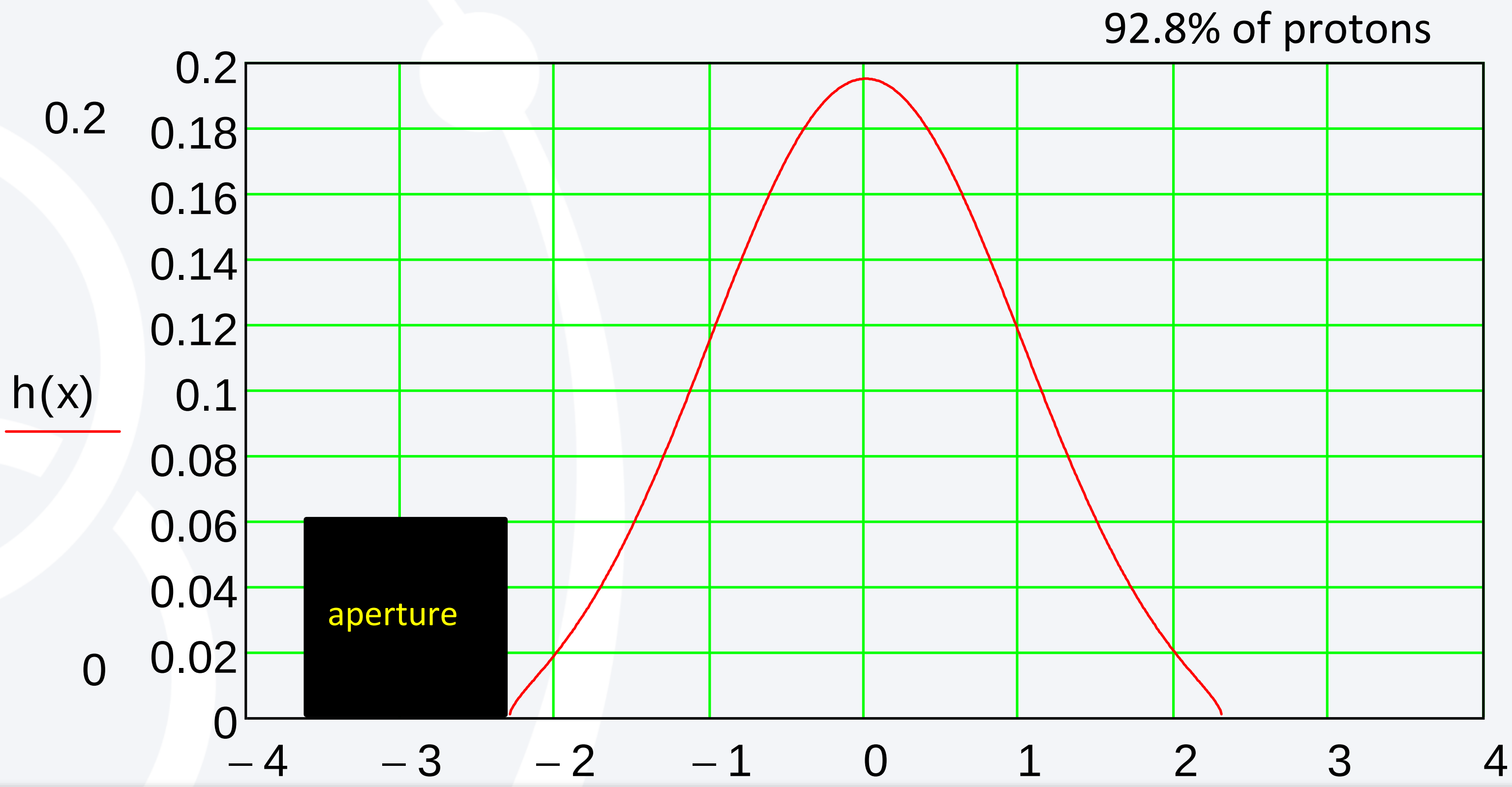
- E.g. beam at a certain position, collimator moved in …
- → Phase-space reduction (multiturn effect!)
- Major difference to e.g. transfer lines.

Gaussian beam and aperture



99.9% of all particles are inside an boundary of 4 σ
Depending on the accelerator and its operational parameters, the aperture can be much larger than 4 σ - but not smaller

Gaussian beam with an aperture at 2.3 σ



- Assume that the total energy stored in the beam is 500 MJ (HL-LHC)
- Assume a movement to a position with the aperture of 2.3 σ
- Assume that all particles above 2.3 σ are lost => corresponds to energy deposition of 35 MJ

RF failure

RF in circular accelerators

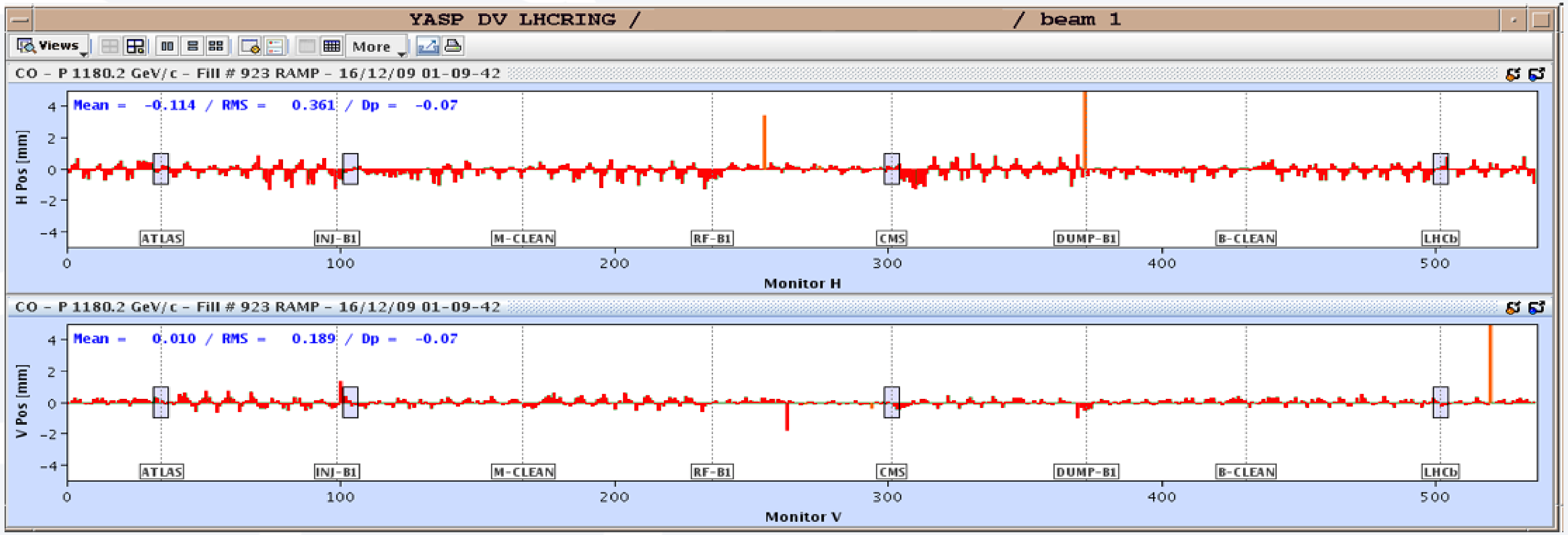
- Acceleration of the particles
- Compensation of energy loss at constant magnetic field
- Keeping the particles in a bunch

RF failure → particles are always lost in the transverse plane (vacuum chamber, collimator, …)

… mechanism depends on the operational phase

RF Failure… at constant magnetic field

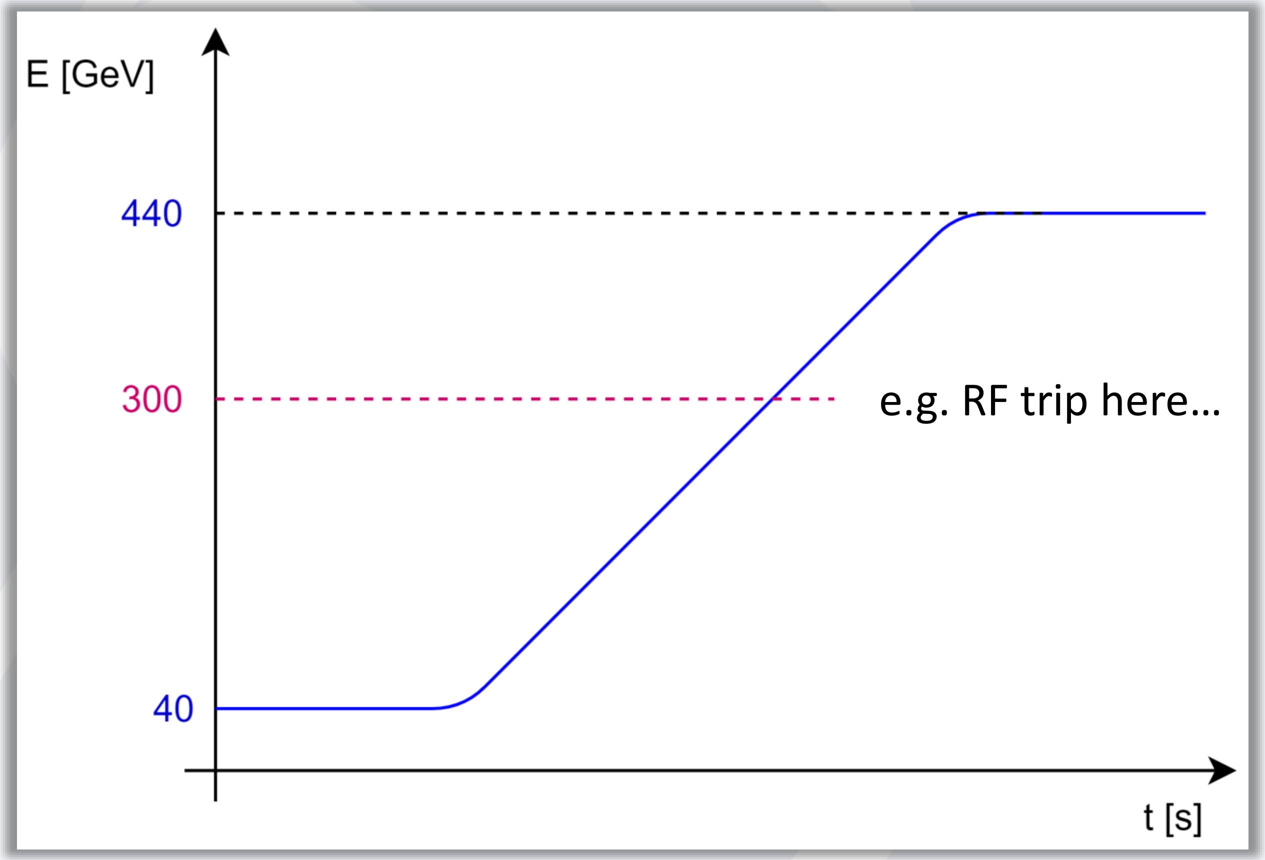
– Protons: beam de-bunches, very slow energy loss, but…



… diagnostics might be blind!

– Electrons: particle losses in short time (due to energy losses)

RF Failure … during energy ramp



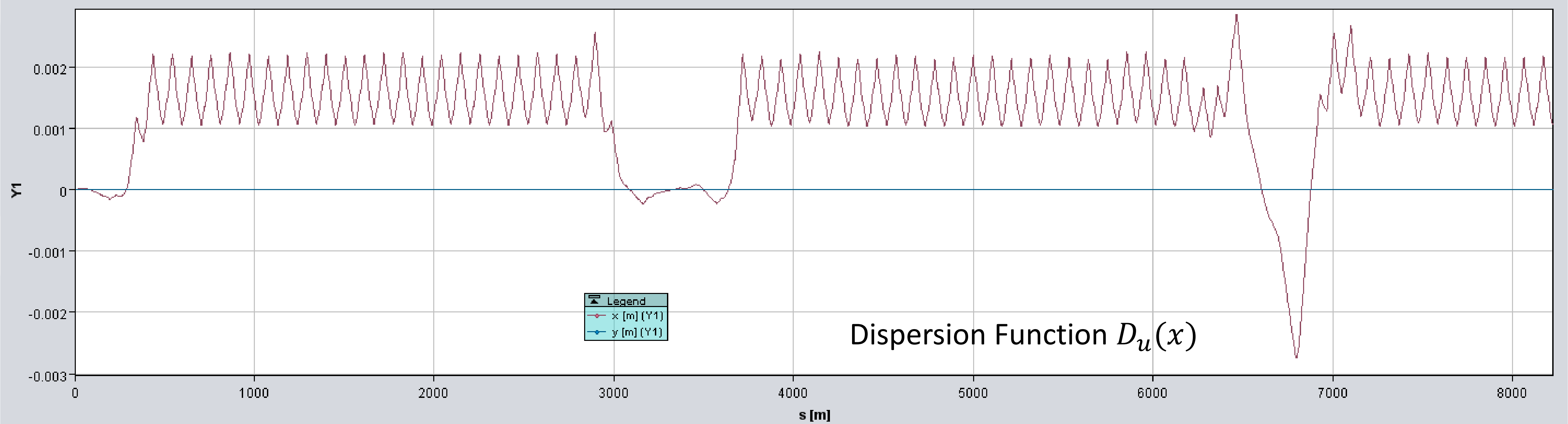
Magnetic field continues to increase, but particle momentum stays constant …

→ Momentum mismatch $\frac{\Delta p}{p}$

Dispersion

$$u = u_\beta + D_u \frac{\Delta p}{p}$$

… Orbit Change proportional to relative momentum error



Dispersion Function $D_u(x)$

Quadrupole Failure

Quadrupole Failure

– A quadrupole current error changes the betatron tune (and also the betatron functions):

$$\Delta Q = \frac{1}{4\pi} \beta_0 \cdot l \cdot \Delta k_1$$

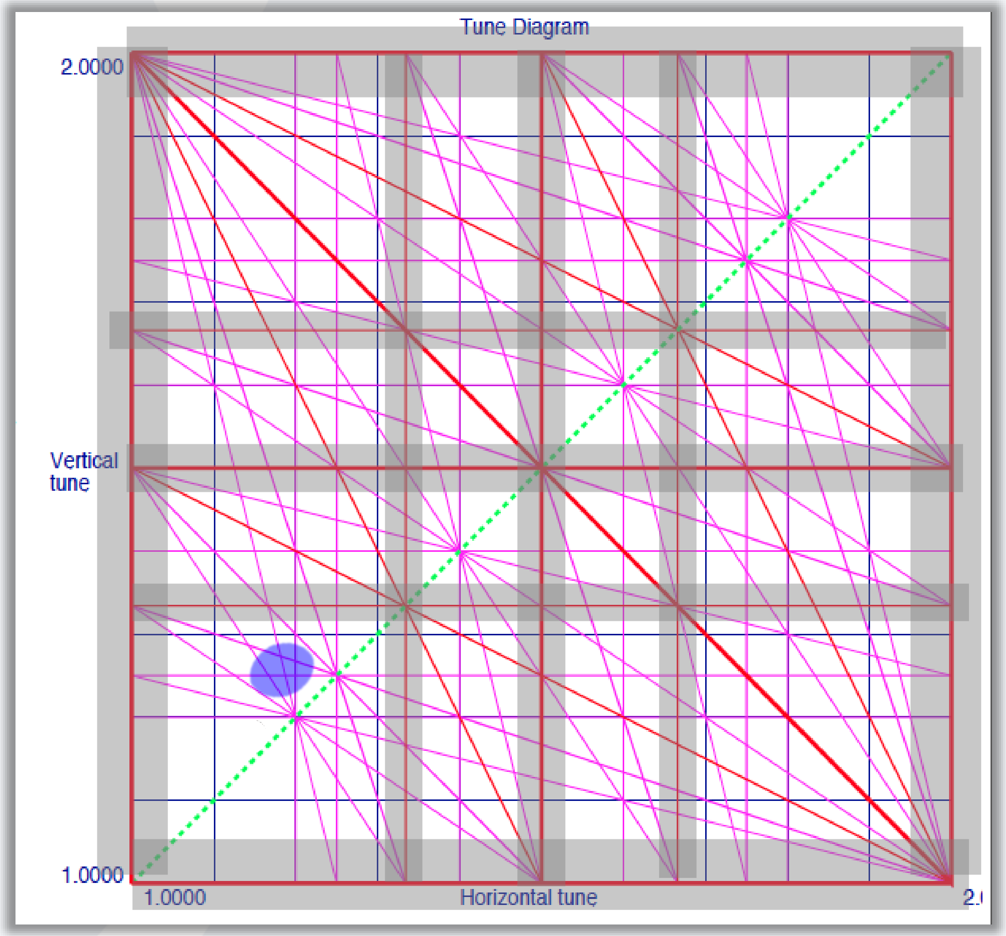
With l,the length of the quad, β_0 the beta function at the quad and Δk_1 the change of the quadrupole strength.

If the tune changes is large, the beam will cross resonances and get lost (in general, the beam size grows)

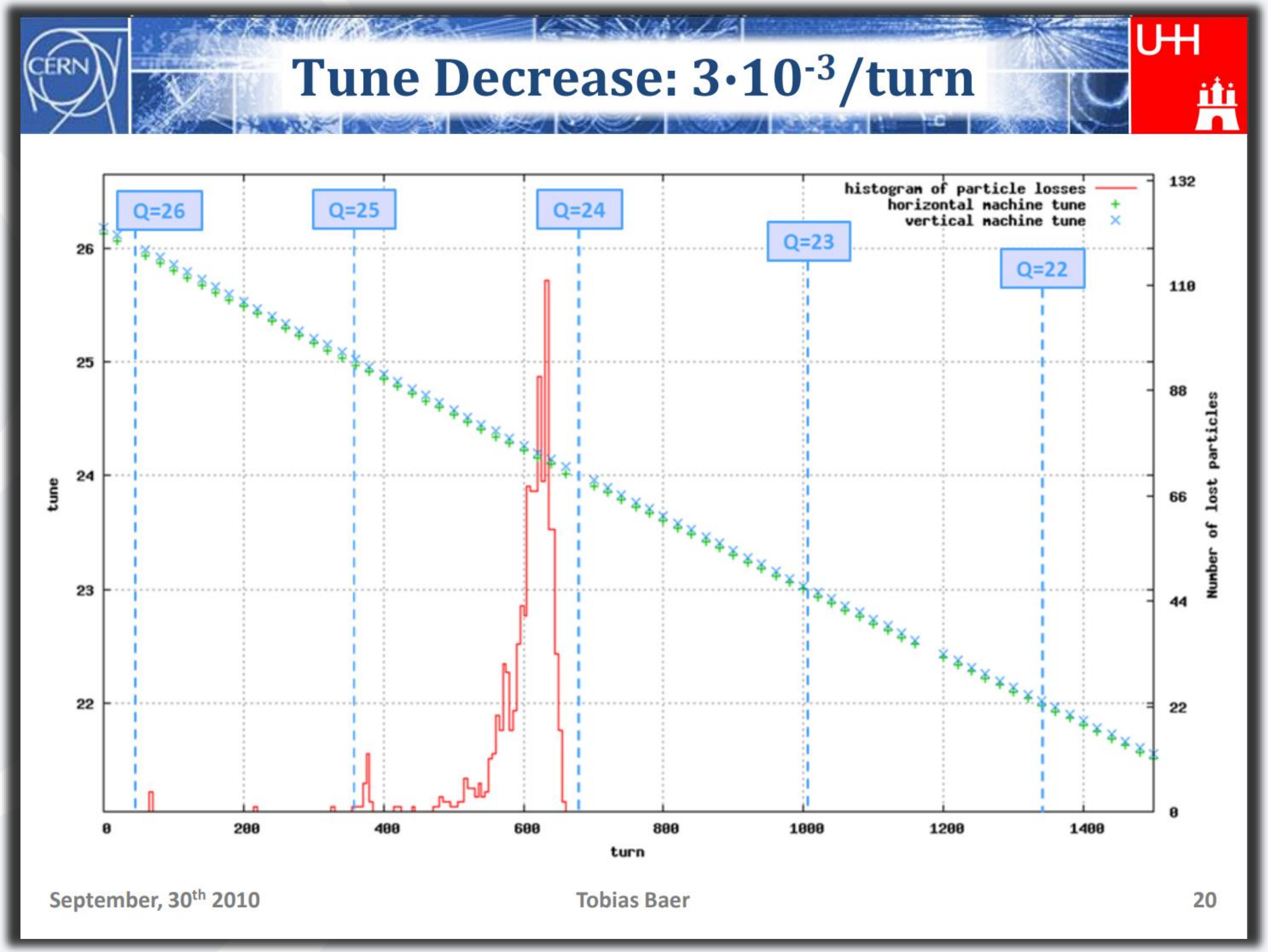
$$\Delta k_1 = \left(\frac{I(t)}{I_0} - 1\right) \cdot k_1$$

$$k_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$

Tune Change


Tune Diagram
2.0000
Vertical tune
1.0000
1.0000
Horizontal tune

No way to cross Resonances? …



… Yes we can ..
if we are fast enough ;-)

Source: Tune resonance phenomena in the SPS and related machine protection , T. Bär, 2010

Summarizing...

Beam loss mechanism

- What is required for beams not touching the aperture:
  - No mechanical elements in the beam pipe
  - Well corrected closed orbit
  - Correct betatron tunes
  - Correct chromaticity (in general, tune spread limited between resonances)
  - Beam intensity below threshold for instabilities

- What can go wrong:
  - Some mechanical element accidently moves into the vacuum pipe
  - Horizontal or vertical dipole magnet has wrong field
  - Quadrupole magnet has wrong field
  - Sextupole magnets have wrong field – losses due to single particle effects or instabilities
  - Too high beam current for the operational point – losses due to single particle effect or instabilities

Why does it go wrong?

For a cycle in an accelerator such as LHC, there are several million parameters used during the acceleration cycle (e.g. current versus time for 1700 power converters).

One single wrong parameter can cause beam losses

– Failure of some hardware (power converter)

– Single event upset in controller

– Thunderstorm (electrical system) affecting powering

– Software failure (wrong magnet current programmed)

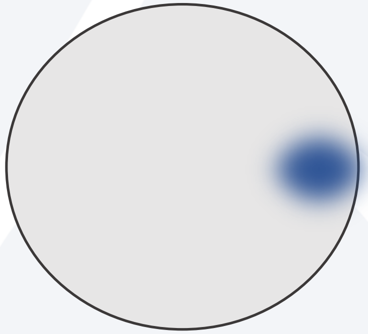
– Operator gives wrong command

– Too high beam intensity

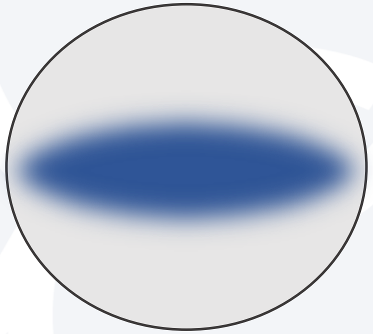
– Feedback system failure

– Wrong timing- functions not synchronised

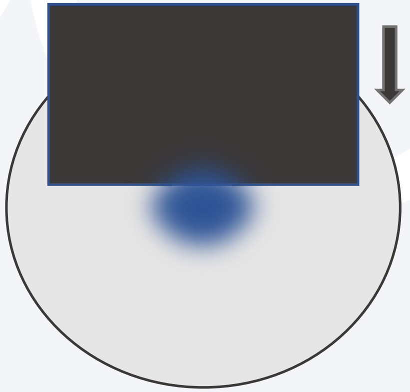
Beam losses summary



- Transverse
  - Dipole magnet - angle change
  - Quadrupole Offsets
  - RF
    - Energy mismatch → Dispersion
    - Debunching
  - Fast kicker magnets (see Jorg)



- Blowup
  - Tune and Chromaticity
  - Fast Kicks
  - Higher order magnets (not covered)
  - Instabilities (not covered)
    - Beam current
    - Impedance



- Equipment moves into vacuum chamber (without detail)
  - Vacuum valves (might close automatically)
  - Screens (might survive single pass; circular: who wins?)
  - Collimators (Effect on impedance!)
  - Dust Particles ("Ufos" → See Jorg)

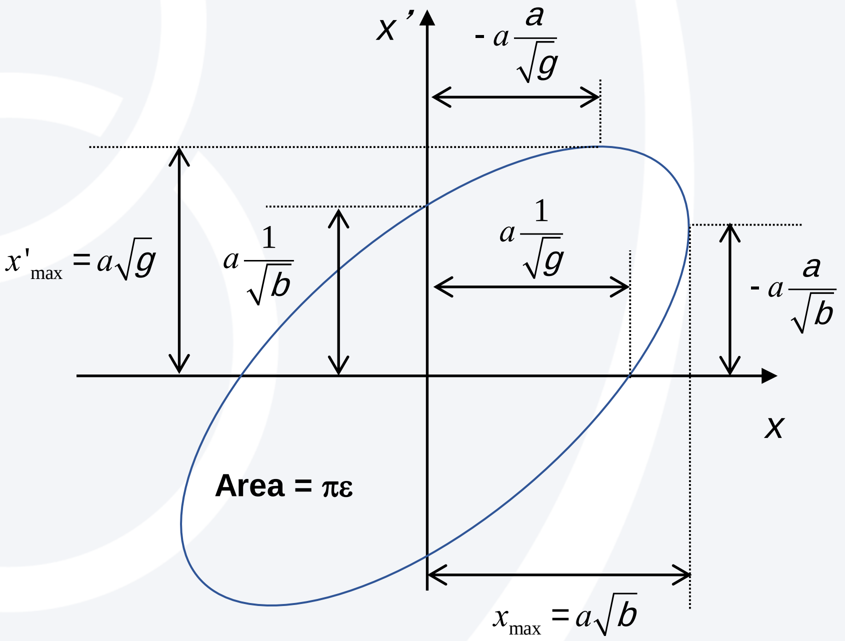
Thank you!

Appendix

Recap: Normalised phase space

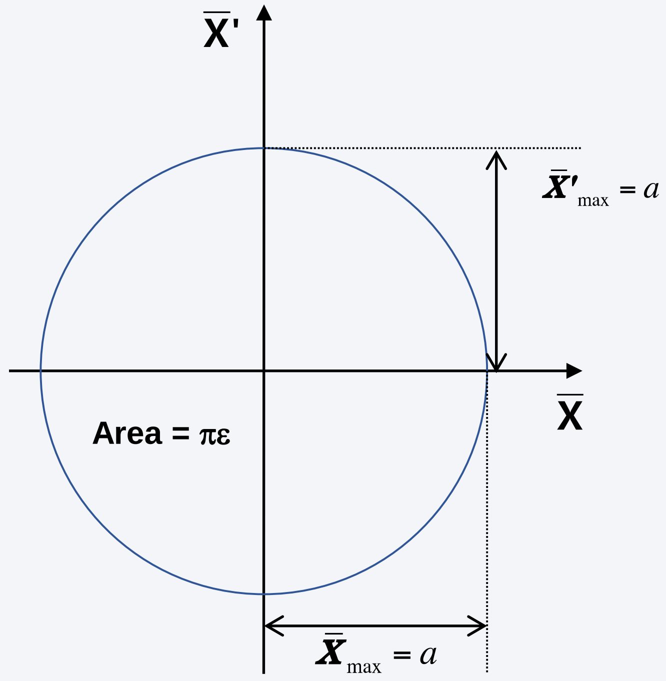
$$\overline{\mathbf{X}} = \sqrt{\frac{1}{\beta_S}} \cdot x \qquad \overline{\mathbf{X}}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S}\, x'$$

Real phase space



$\Rightarrow$

Normalised phase space (Circle)

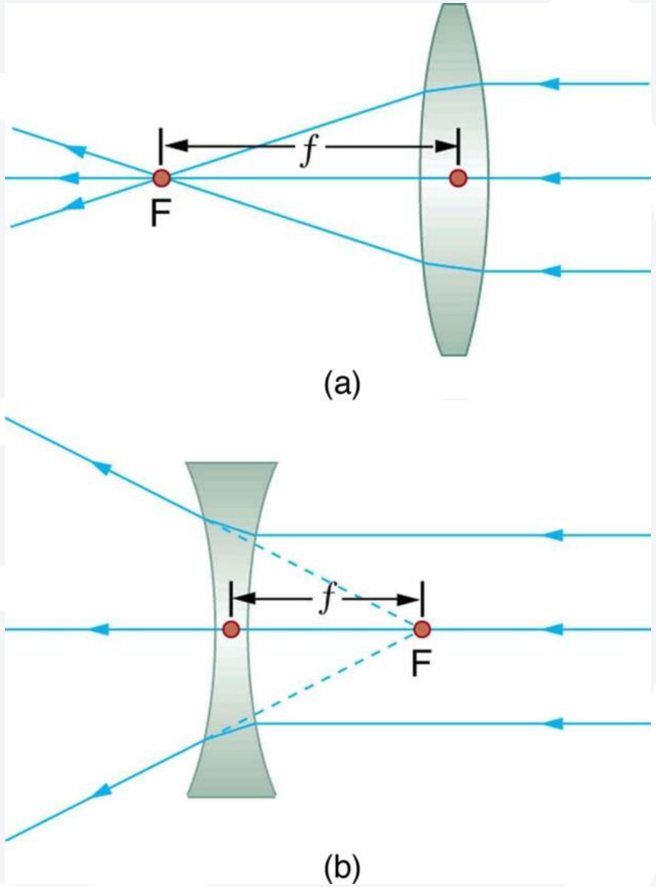


(Courant Snyder Invariant / invariant of motion)

$$a = g \times x^2 + 2a \times x \times x' + b \times x'^2$$

$$a = \overline{\boldsymbol{X}}^2 + \overline{\boldsymbol{X}}'^2$$

How to change the tune?



Tune Change ΔQ resulting from a change of quadrupole strength:

$$\Delta Q = \frac{1}{4\pi}\frac{\beta_0}{\Delta f}$$

β_0, beta function at the quadruple.

With Δf, the change in focussing length,

$$\Delta f = \frac{1}{l\,\Delta k_1}$$

l, length of the quadruple.

→ For N Quadrupoles (assuming same β_0):

$$\boxed{\Delta Q = \frac{1}{4\pi}\beta_0 \cdot l \cdot N \cdot \Delta k}$$

$k_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$, the quadrupole strength.

Momentum Error – RF freq

$$\frac{\Delta p}{p} = \frac{\frac{\Delta f}{f}}{\eta},$$

$$\eta = \frac{1}{\gamma_r^2} - \alpha_C.$$

f is the RF frequency, γ_r the relativistic gamma and α_C the momentum compaction factor of the ring.

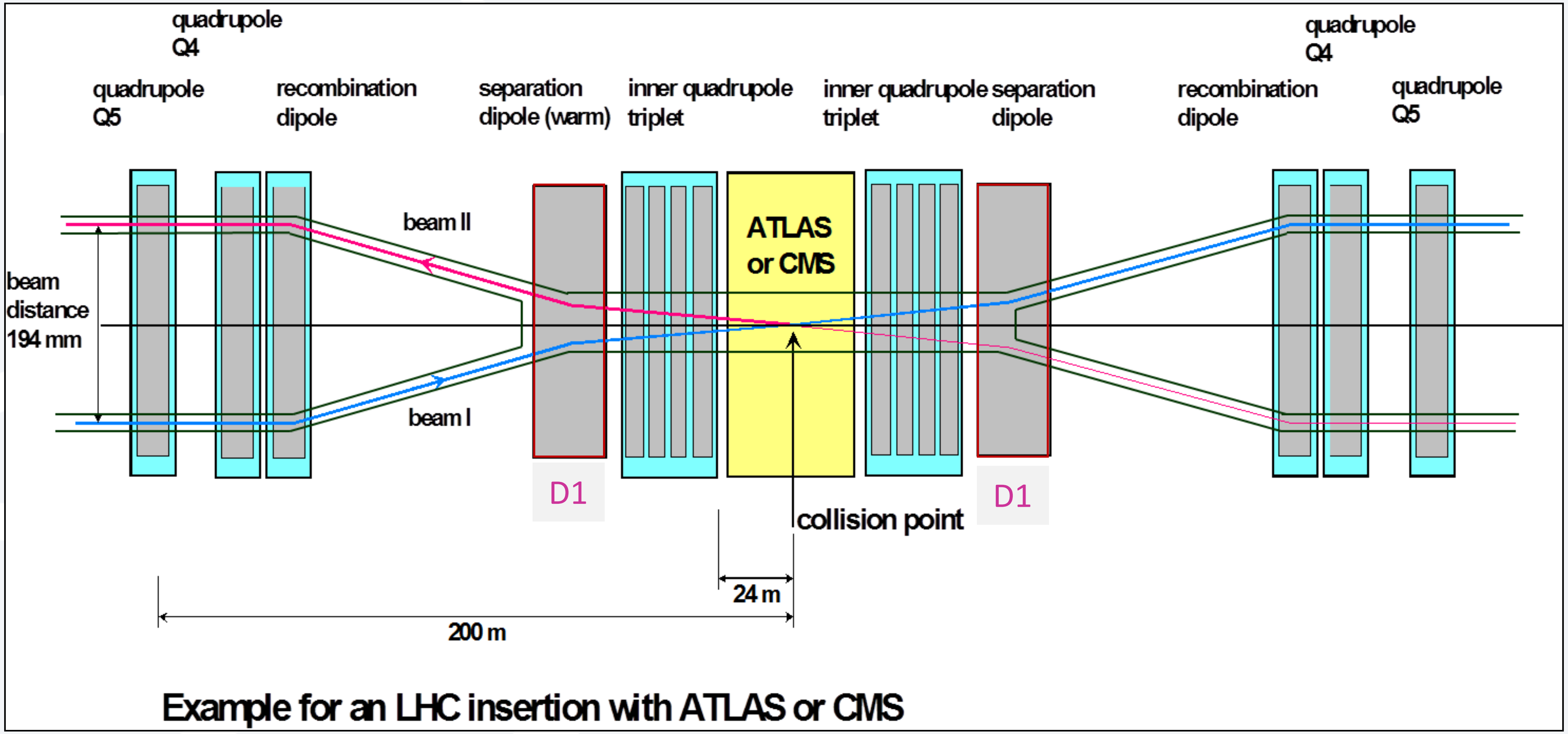
Notes

– Kick nur ein passage, closed orbit bleibt gleich, dadurch dass verschiedener tune – strahl platz auf

– Wenn Kick anbleibt, dann beide effekte

– Langsame Verschiebung

Example - LHC Separation dipole (D1) failure



- The 2 LHC beams are brought together to collide in a 'common' region
- Over ~260 m the beams circulate in one vacuum chamber with 'parasitic' encounters (when the spacing between bunches is small enough)
- D1 separates the two beams

Failure of a D1 magnet at LHC

Inductance of 12 magnets powered in series: $L_{D1} := 1.74H$

Resistance of 12 magnets powered in series: $R_{D1} := 0.78\Omega$

Energy of LHC protons at collision energy: $E_{lhc\_c} = 7.0000 \cdot TeV$ and Time for one revolution:

$T_0 := 89 \cdot 10^{-6} s$

Nominal field at 7 TeV: $B0_{D1} := 1.38T$

Length of one magnet: $L_{D1} := 3.4m$

Nominal deflection angle: $\alpha_{D1} := \dfrac{B0_{D1} \cdot 12 \cdot L_{D1}}{E_{lhc\_c}} \cdot c \cdot e_0$ => $\alpha_{D1} = 2.41 \times 10^{-3}$

Time constant in case of powering failure: $\tau_{D1} := \dfrac{L_{D1}}{R_{D1}}$ => $\tau_{D1} := 2.53s$

Error in deflection after $N := 10$ turns: $t_x := N \cdot 89 \cdot 10^{-6} \cdot s$

$$B_{D1} := B0_{D1} \cdot \left(1 - e^{\frac{-t_x}{\tau_{D1}}}\right)$$

$$\frac{B_{D1}}{B0_{D1}} = 3.52 \times 10^{-4}$$

Failure of a D1 magnet at LHC

Error in angle:

$$\alpha_{err} := \alpha_{D1} \cdot \frac{B_{D1}}{B0_{D1}}$$

$$\alpha_{err} = 8.4812 \times 10^{-7}$$

Change of the position orbit at a location of the LHC with $\beta_{D1} := 4000\text{m}$, $\beta_{test} := 100\text{m}$:

$$x_{D1} := \frac{\sqrt{\beta_{D1} \cdot \beta_{test}}}{2 \cdot \sin(\pi \cdot Q)} \cdot \alpha_{err}$$

$$x_{D1} = 0.3243 \cdot \text{mm}$$

Beam position change after 0.9 ms, about 1.4 σ

Beam size at a location with $\beta_{test} = 100.0\,\text{m}$ assuming a normalised emittance

$$\varepsilon_n = 3.75 \times 10^{-6}\,\text{m}$$

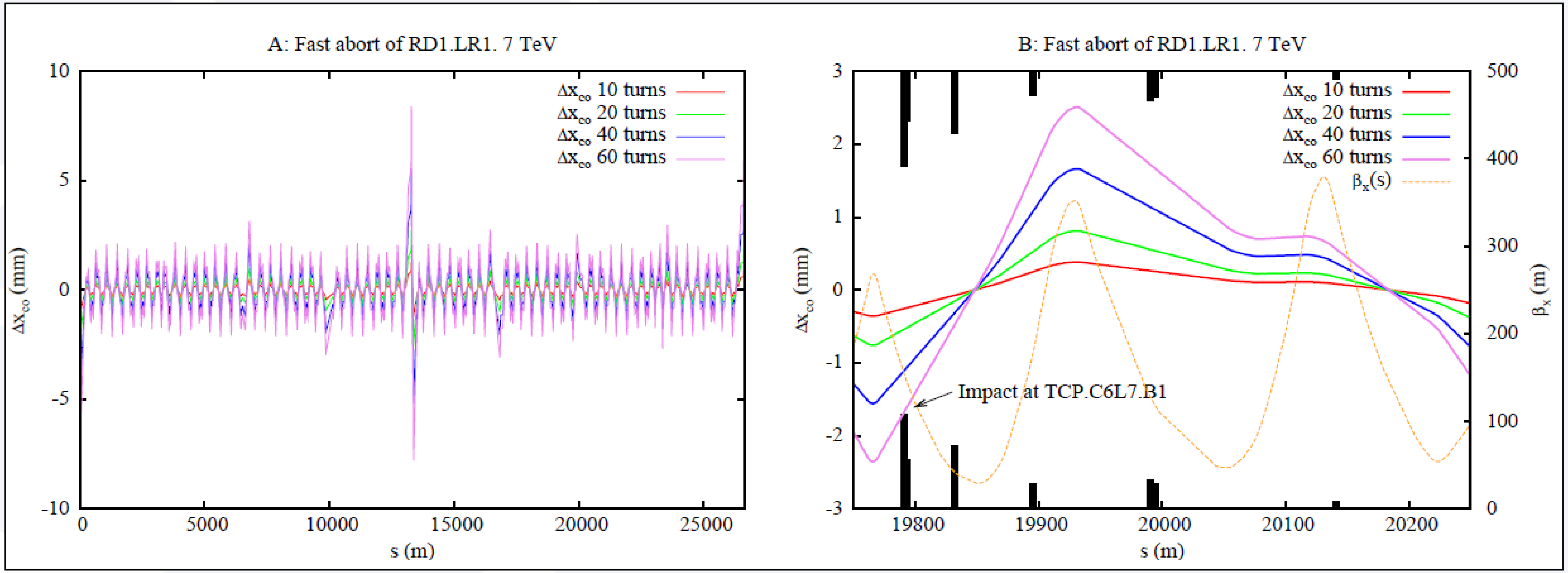
The gamma factor for an energy of $E_{lhc\_c} = 7.0000 \cdot \text{TeV}$ => $\gamma_c = 7459.9105$

$$\sigma_{100m} := \sqrt{\frac{\varepsilon_n}{\gamma_c} \cdot \beta_{test}}$$

$$\sigma_{100m} = 0.2242 \cdot \text{mm}$$

Simulation using MADX of this failure

- This failure (and many other failures) were simulated using MADX
- A failure of D1 is the most critical failure



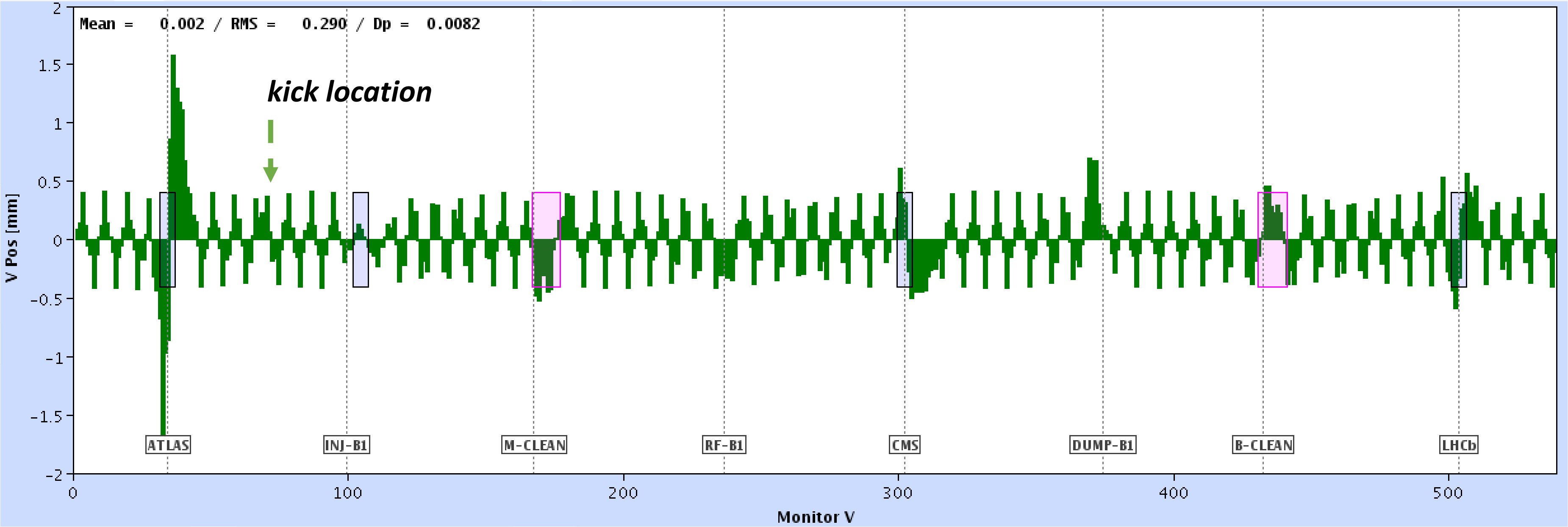
Andres Gomez Alonso

Consequences for machine protection

- In case of a trip of the D1 magnet the orbit starts to move rather rapidly (1 sigma in about 0.7 ms)
- In 10 ms the beam would move by 14 sigma, already outside of the aperture defined by the collimators
- For this failure, the beam has to be extracted in a very short time
- Probability that this will happen during the lifetime of LHC is high
- Detection of the failure by several different systems (diverse redundancy)
  - Detection of the failure of a wrong magnet current, challenging, since a fast detection on the level of 10^{-4} is required
  - Done with a specifically designed electronics (FMCM = Fast Magnet Current Monitor) – M.Werner (DESY) et al.
  - Beam loss monitors detect losses when the beam touches the aperture (e.g. collimator jaw, but also elsewhere)
- LHC MPS was designed for this type of failure => J.Wenninger

The Tune

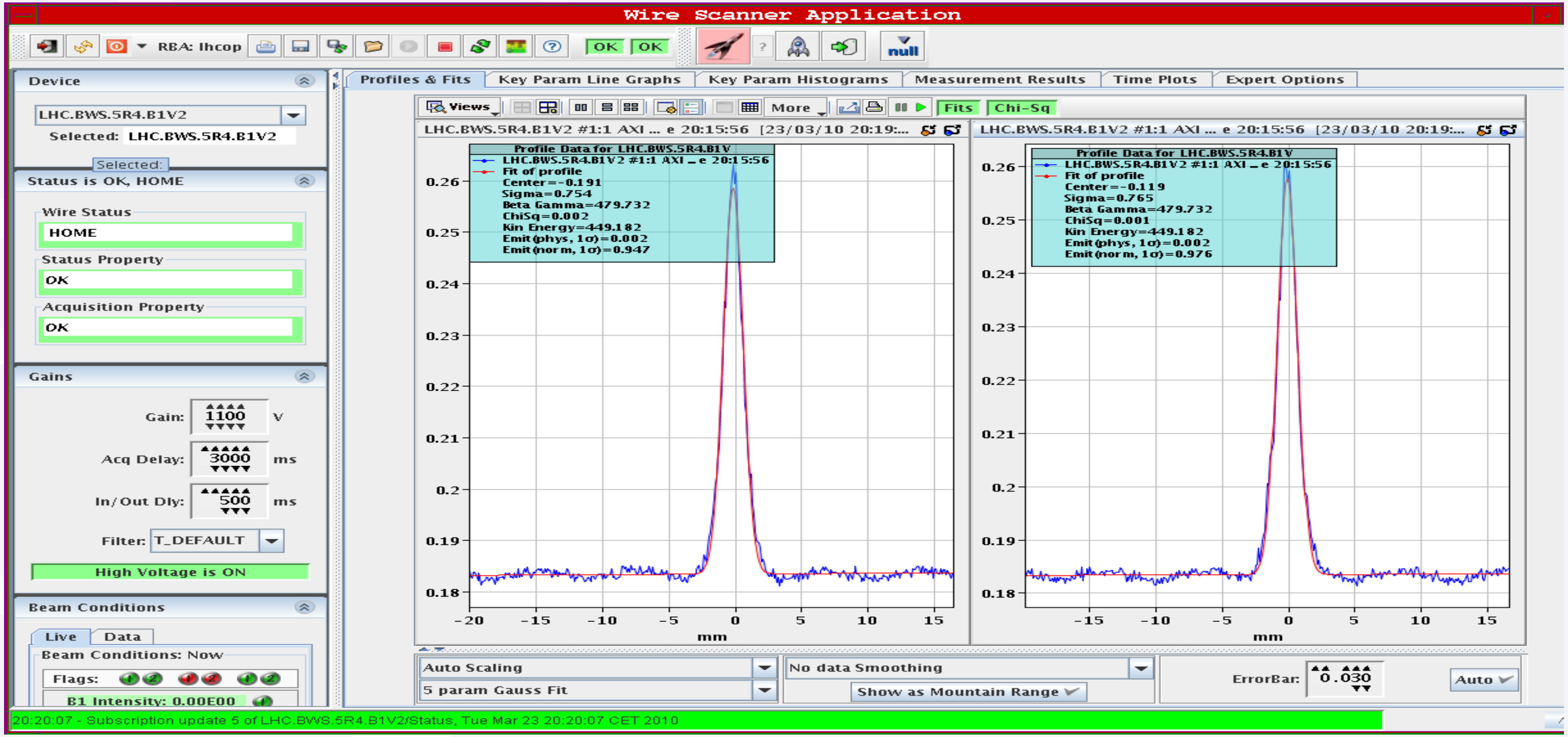
Beam Oscillates around its ideal orbit.
Tune = Number of oscillations per turn



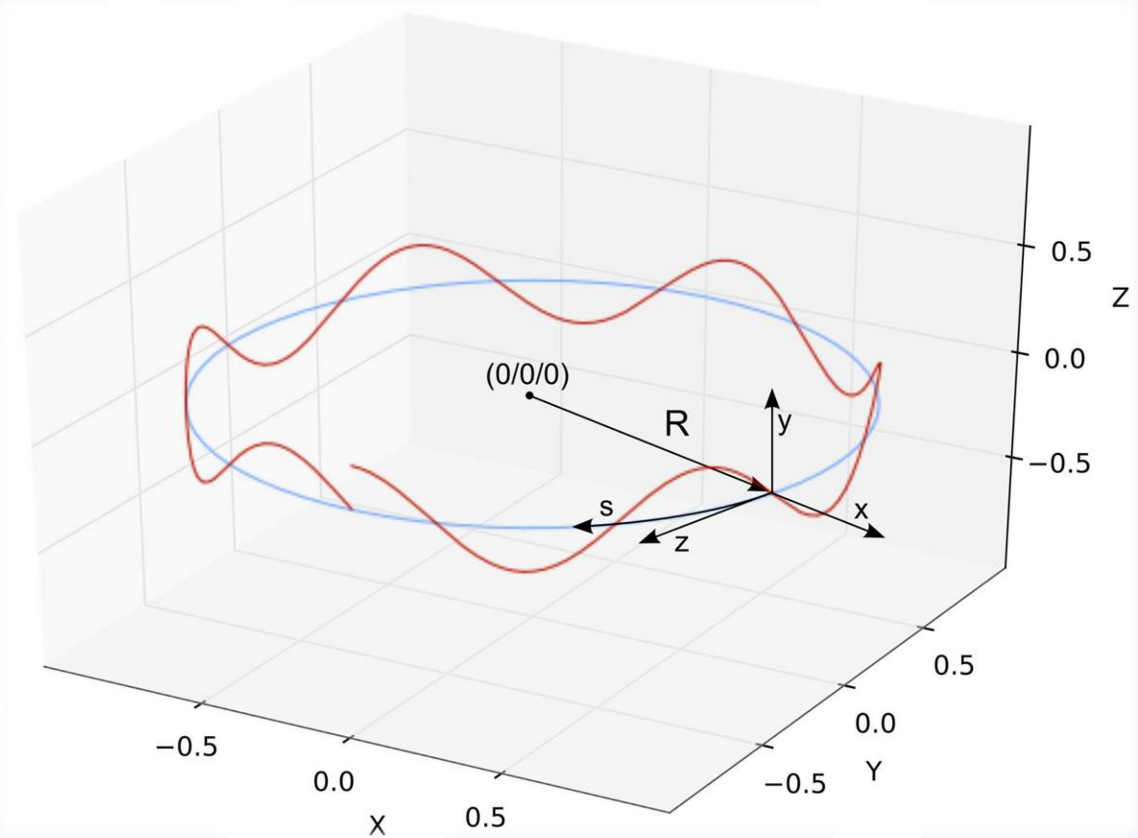
Example for the vertical plane of beam1 (at the BPMs)

Typical beam profile

– Typical beam profiles are close to Gaussian, here measured with a wire scanner (example for LHC)

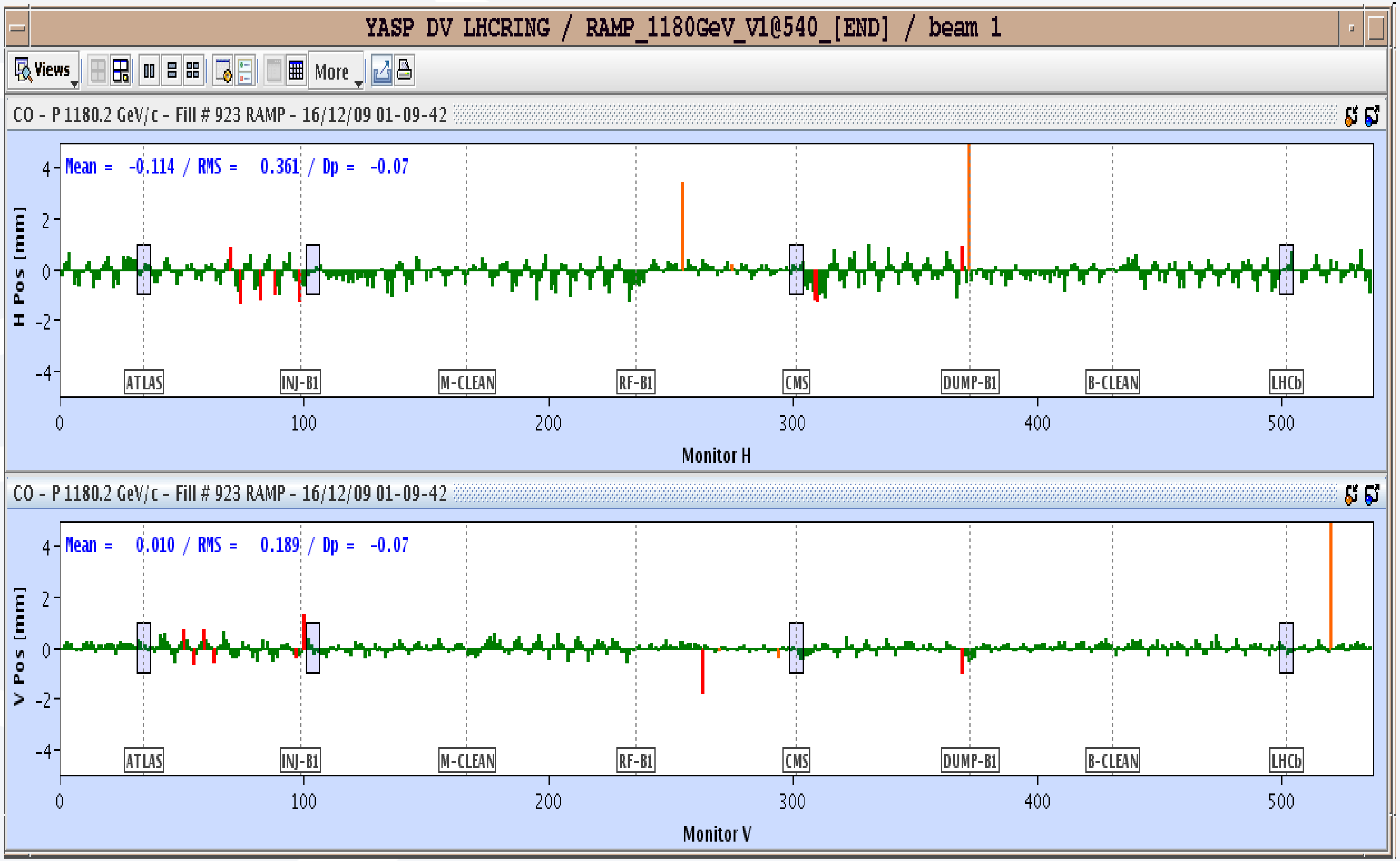


Co-rotating coordinate system



- blue: design orbit
- s: position along the design orbit
- y: upwards deviation from design
- x: outwards deviation from design

Closed orbit measurement at LHC



Orbit Change by dipole

Change of closed orbit as a function of the deflection angle of a magnet:

$$\Delta u_2 = \frac{\sqrt{\beta_1 \beta_2} \cos\left(|\mu_1 - \mu_2| - \pi Q\right)}{2 \sin\left(\pi Q\right)} \cdot \delta_1$$

Simplified:

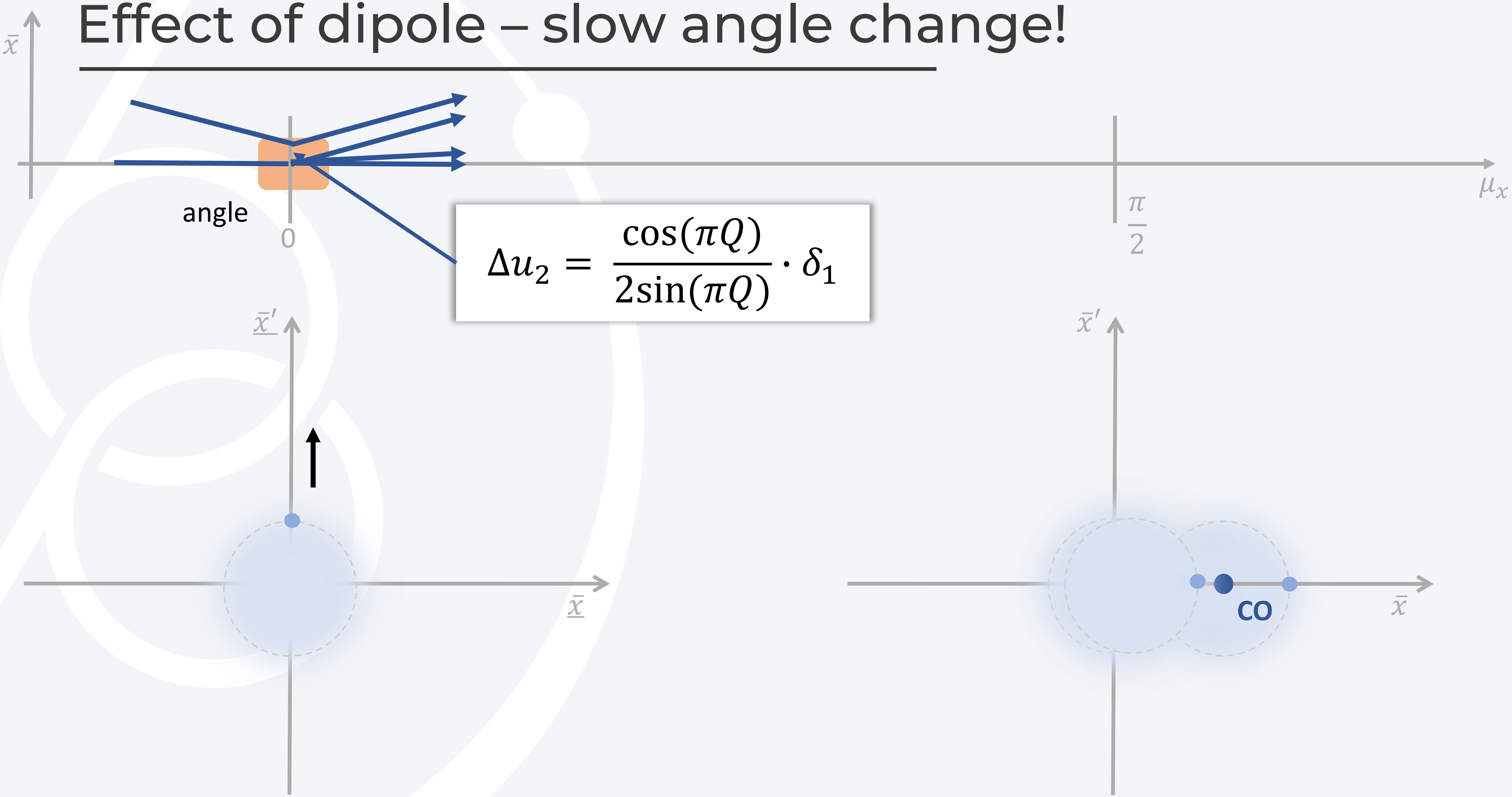
$$\Delta u_2 = \frac{\sqrt{\beta_1 \beta_2}}{2\sin(\pi Q)} \cdot \delta_1$$

With $\beta_{1,2}$ the beta functions at the location of the magnet, and the location of the observation point, Q the betatron tune and δ the deflection angle in [rad]

Single pass (for sake of completeness ;-):

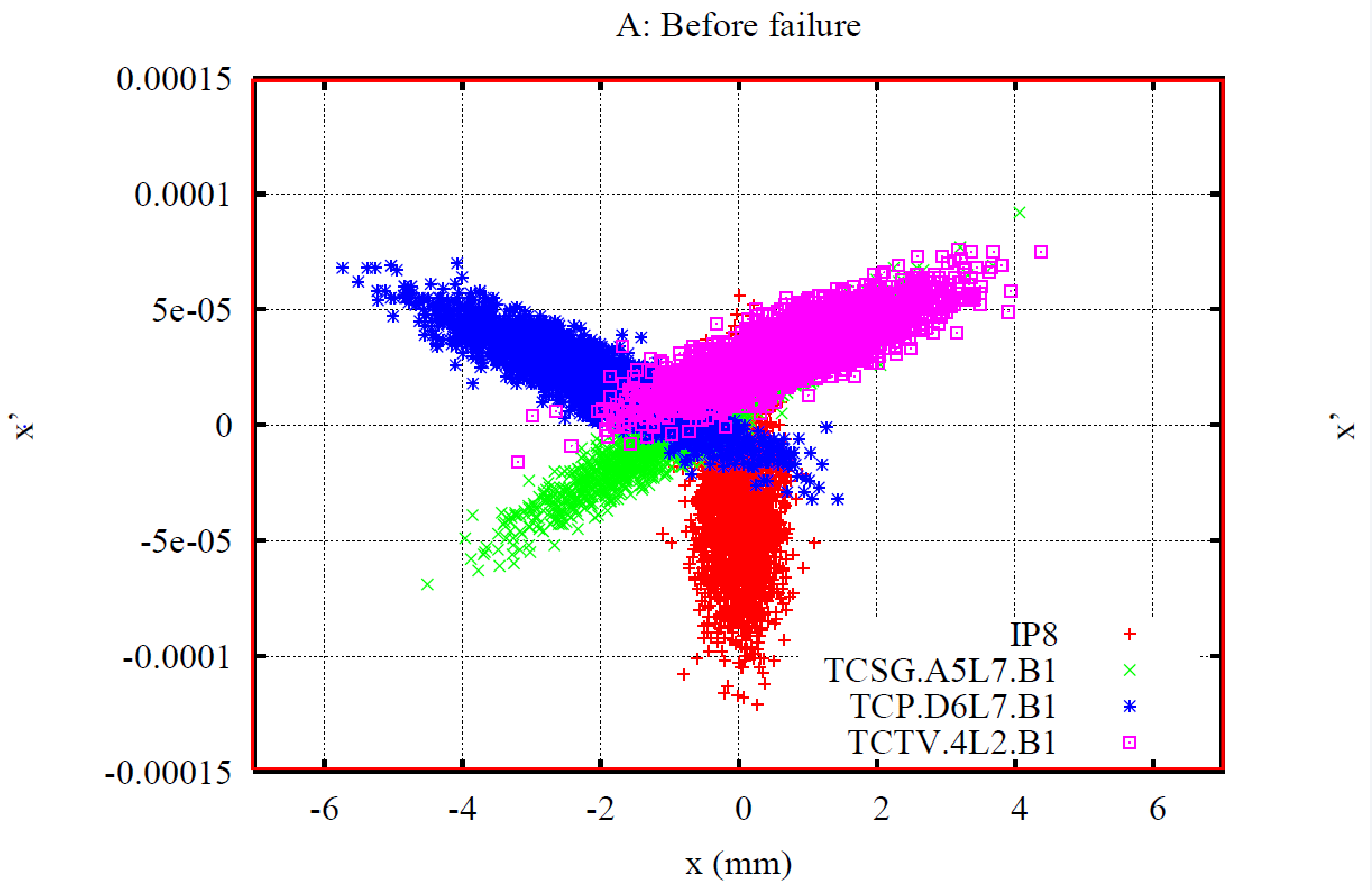
$$\Delta u_2 = \begin{cases} \sqrt{\beta_1 \beta_2} \sin\left(\mu_1 - \mu_2\right) \cdot \delta_1 & \text{for } \mu_2 > \mu_1, \\ 0 & \text{otherwise.} \end{cases}$$

Effect of dipole – slow angle change!

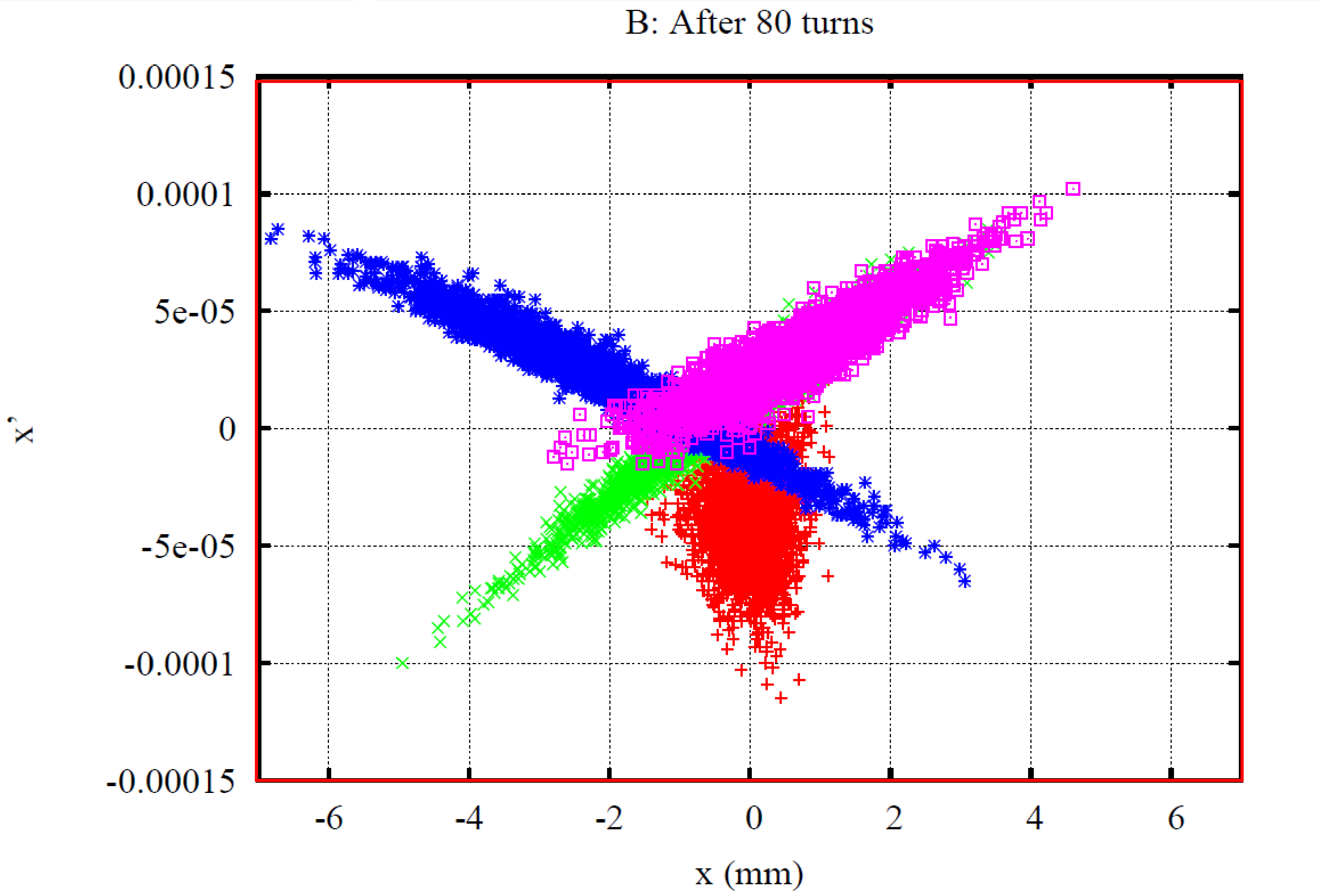


$$\Delta u_2 = \frac{\cos(\pi Q)}{2\sin(\pi Q)} \cdot \delta_1$$

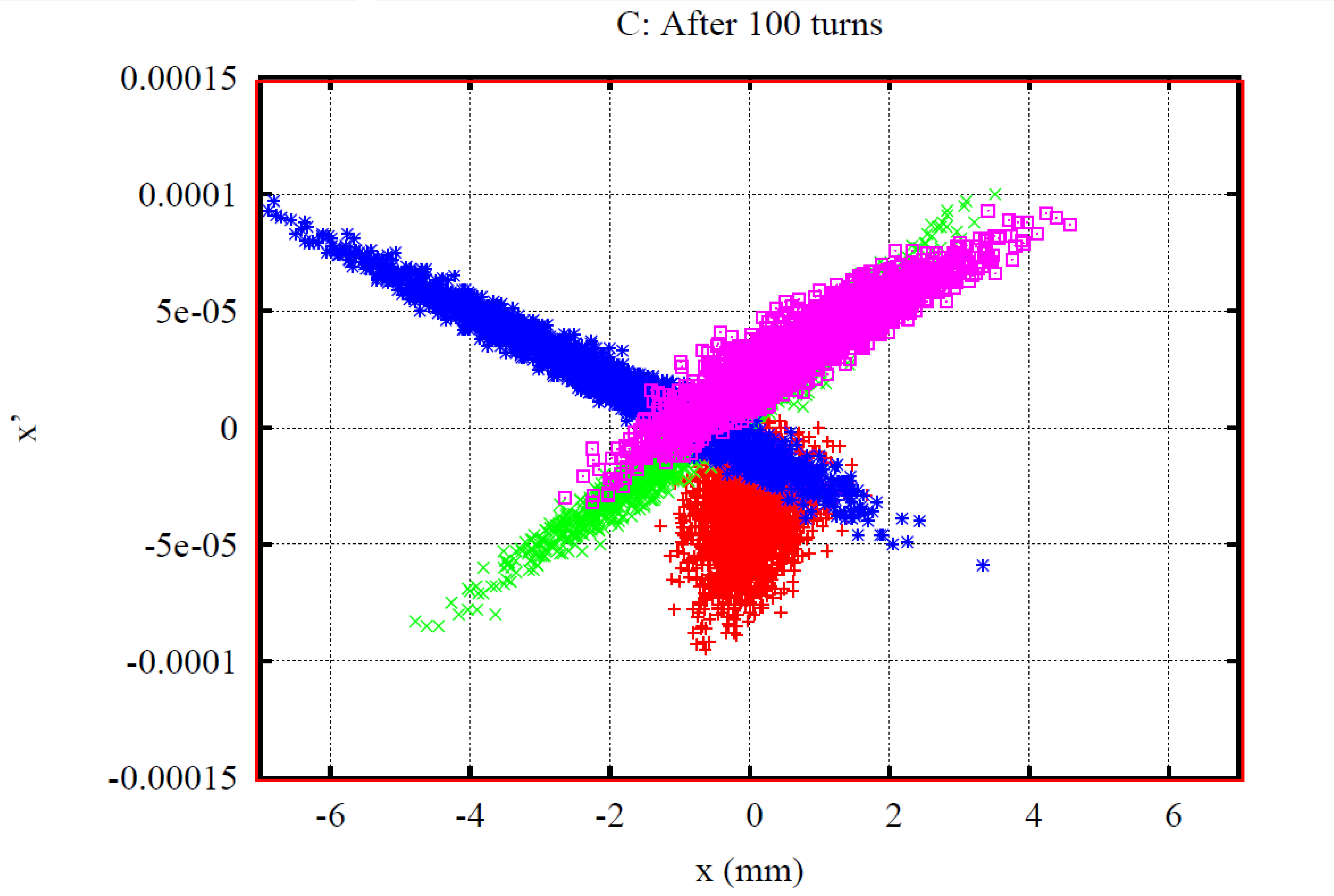
Beam phase space after quadrupole failure



Beam phase space after quadrupole failure



Beam phase space after quadrupole failure



Beam phase space after quadrupole failure