MadLoop

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@ FermiLab

Contents

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- Basics of NLO computations
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Motivations

NLO is important because

- NLO corrections are large in QCD
- NLO corrections significantly affect the shape of distributions
- It reduces the scale dependence inherent to tree-level cross-sections
- New production channels open at NLO

LesHouches '06 Wish list

Wish list for one-loop high multiplicity processes

Process	background to
1. $pp \to VV$ jet	$t\bar{t}H$, new physics
2. $pp \to H+$ 2 jet	H production by boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t}+$ 2 jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV+$ 2 jets	VBF $\to H \to VV$
7. $pp \to V+$ 3 jets	various new physics signatures
8. $pp \to VVV$	SUSY trilepton

Automation would help!

- Accurate theoretical prediction are necessary for the search of signals events in large background samples.

Automation advantages

- Save time

  Trade time spent on computing a process with time on studying the physics behind it.

- Avoid bugs

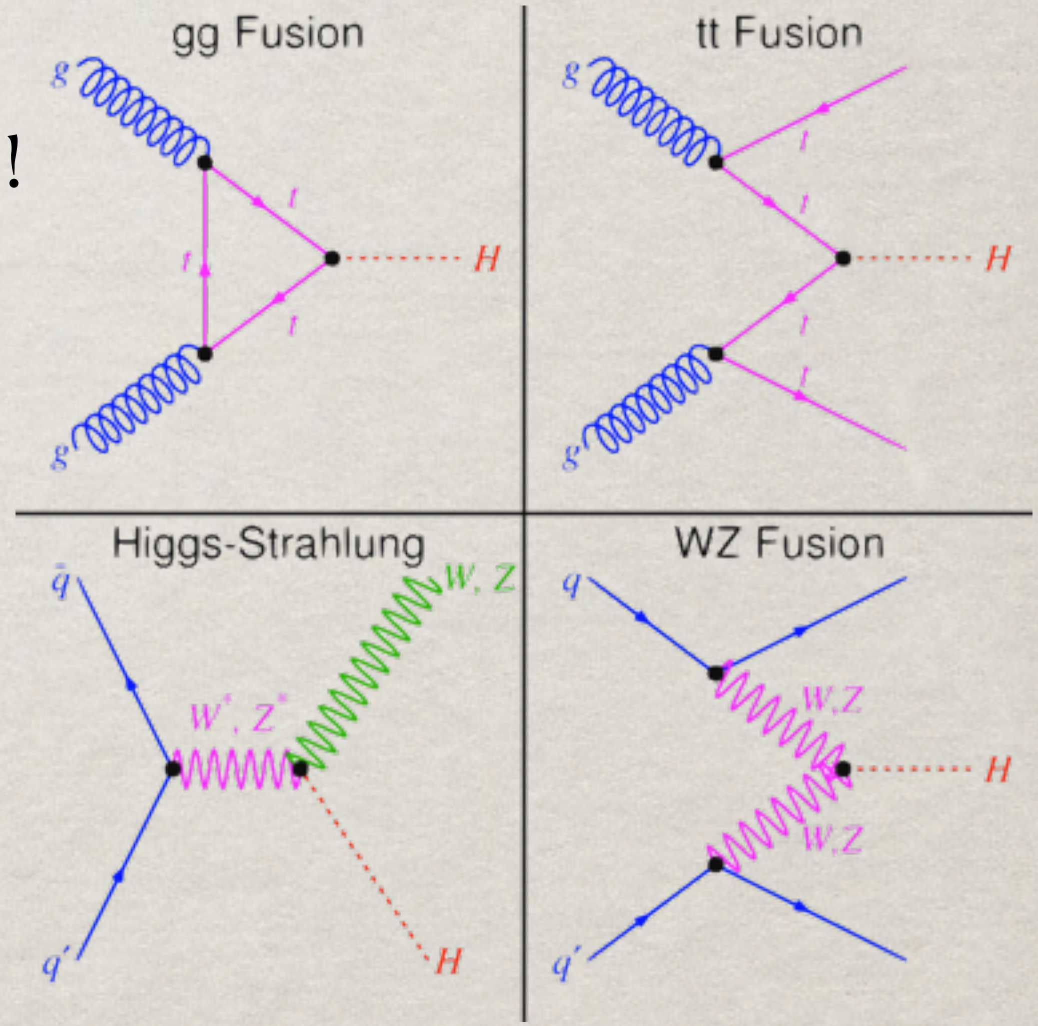
  Having a trusted program extensively checked once and for all, eliminates obvious bugs when running different processes.

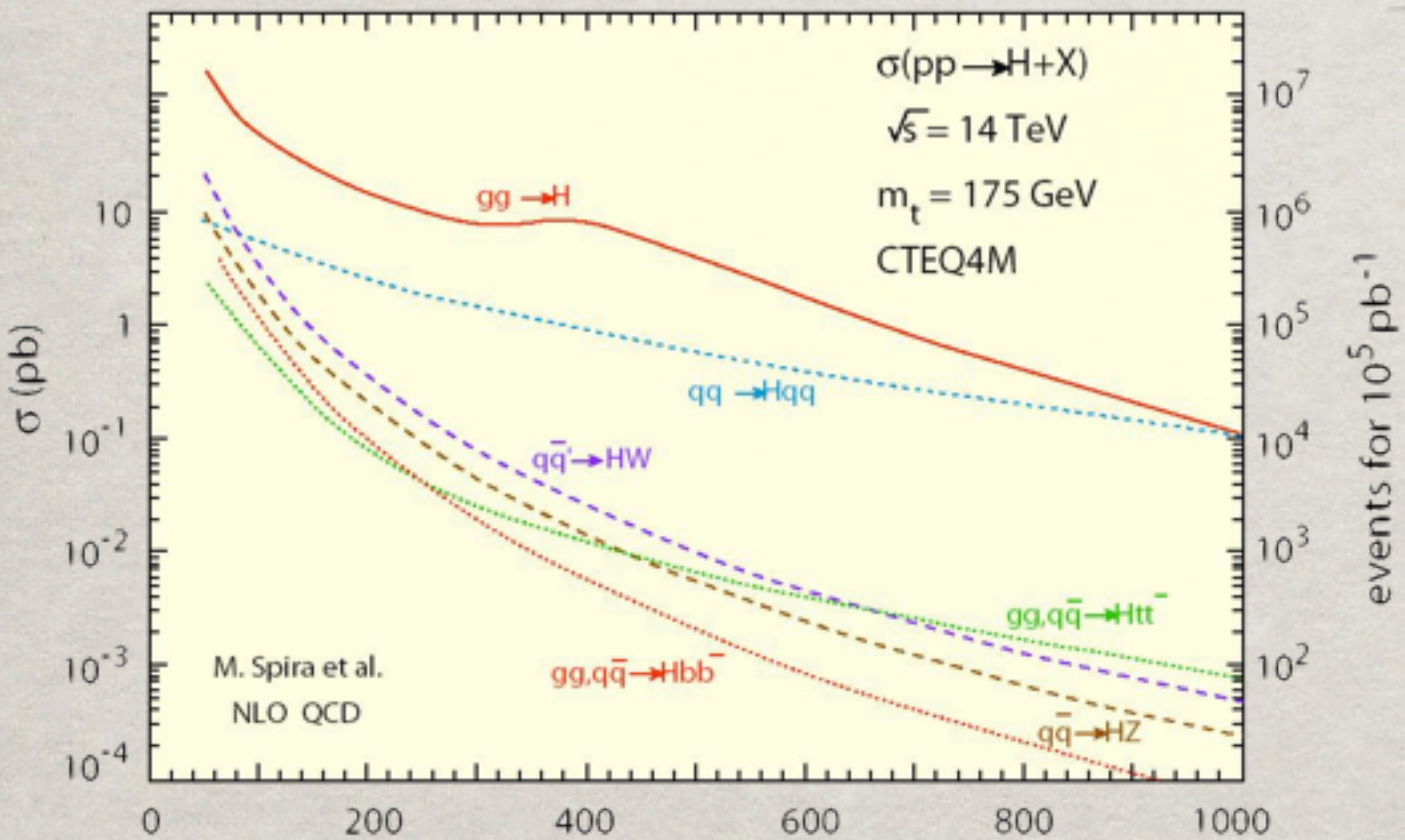
- Use of the same framework for all processes

  It only requires to know how to efficiently use one single program to do all NLO phenomenology.

Higgs Production

- Gluon fusion exclusively loop induced!
- Still very relevant compared to other production channels.



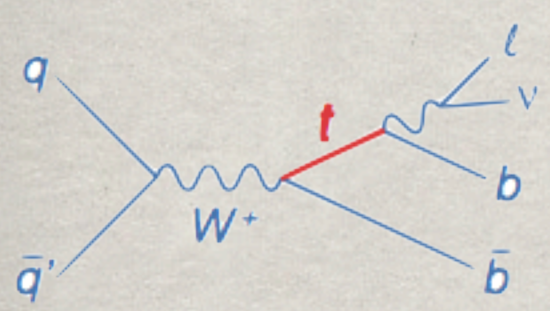


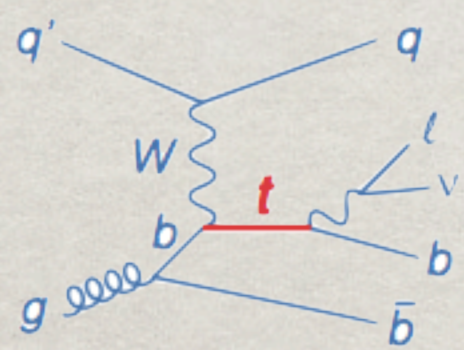
From CMS collaboration

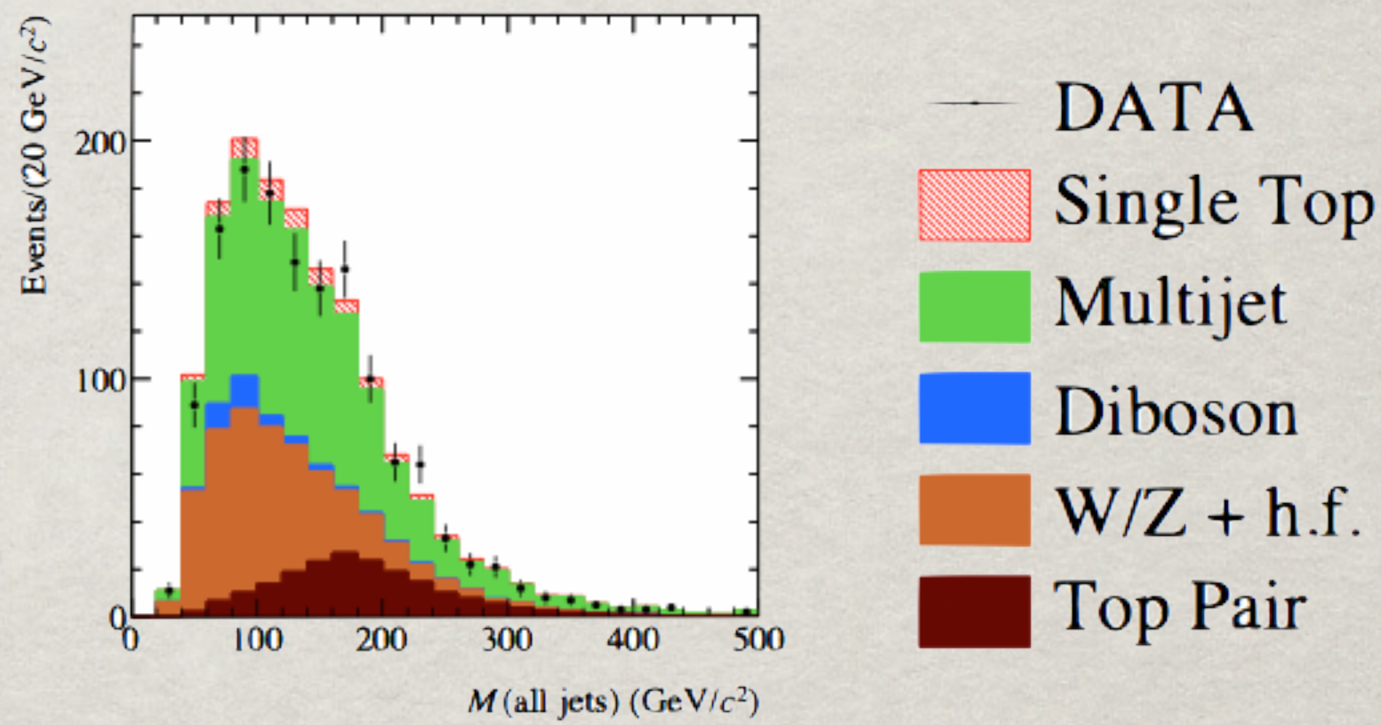
- But consider $H \to b\bar{b}$ and compare to $\sigma_{LHC}(b\bar{b}) = \mathcal{O}(10^8\ [pb])$!
- So accurate estimation of background process is crucial.

Single-top searches

This process closely resemble to many background events which have to be carefully evaluated



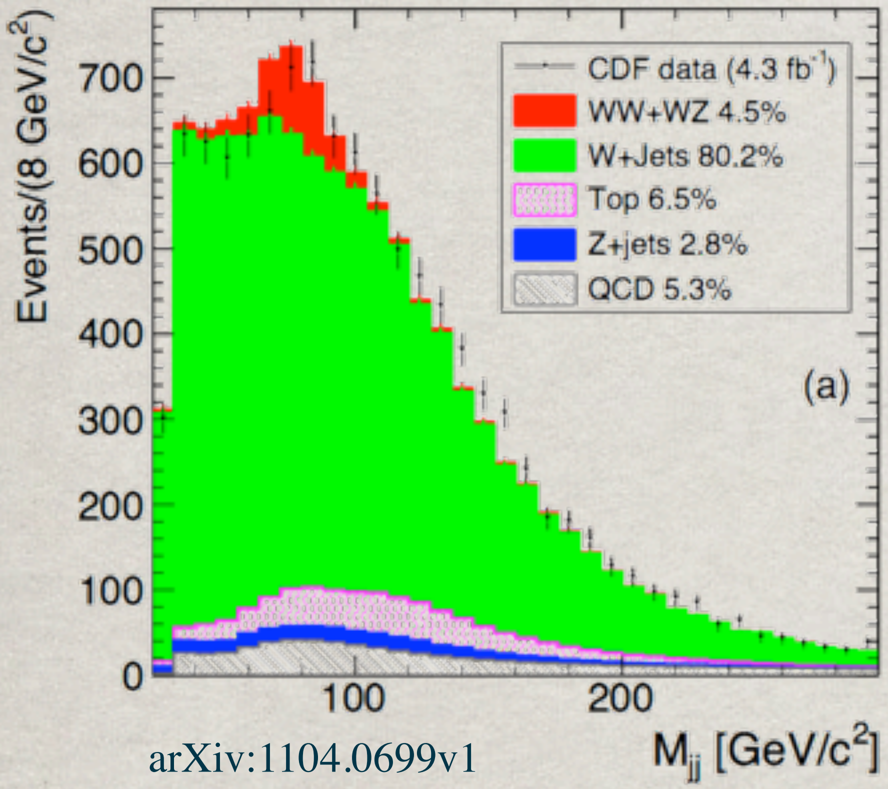




arXiv: 1001.4577

Single top events have been observed at Tevatron, giving the first and only direct estimation of V_{tb} !

Wjj at Tevatron

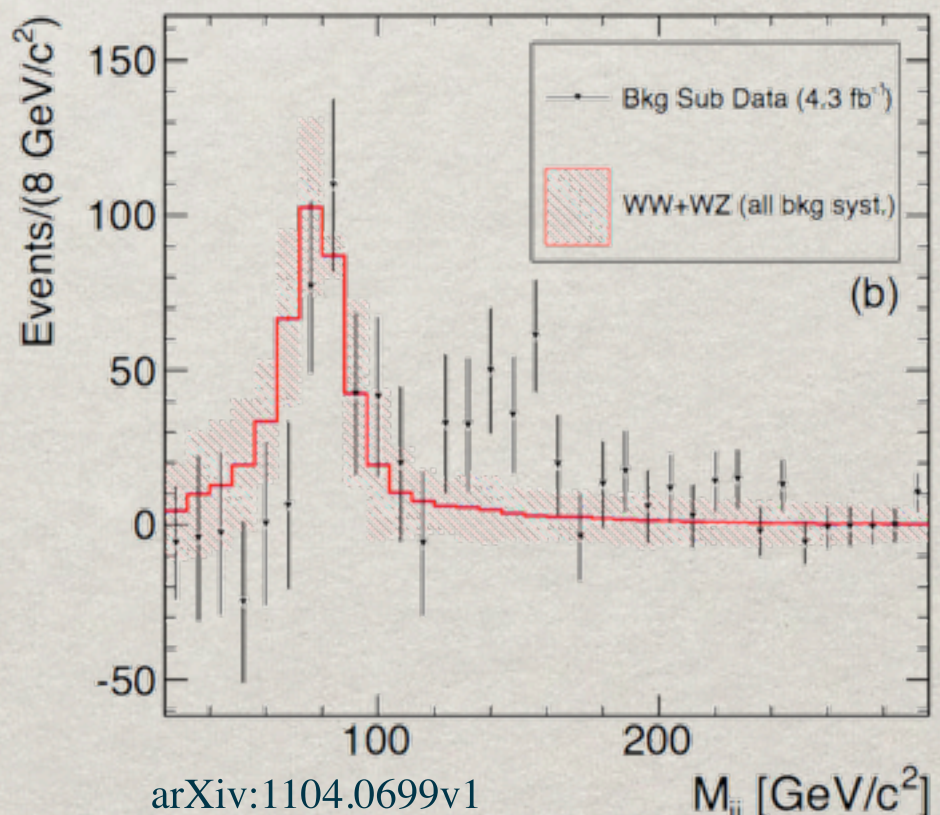


arXiv:1104.0699v1

CDF observes 3-σ deviation to the SM signal.

- New Physics, stat. fluctuations?
- Unreliable prediction?
  - ➡ W+jets treated at LO !
  - ➡ Mistreatment of background?

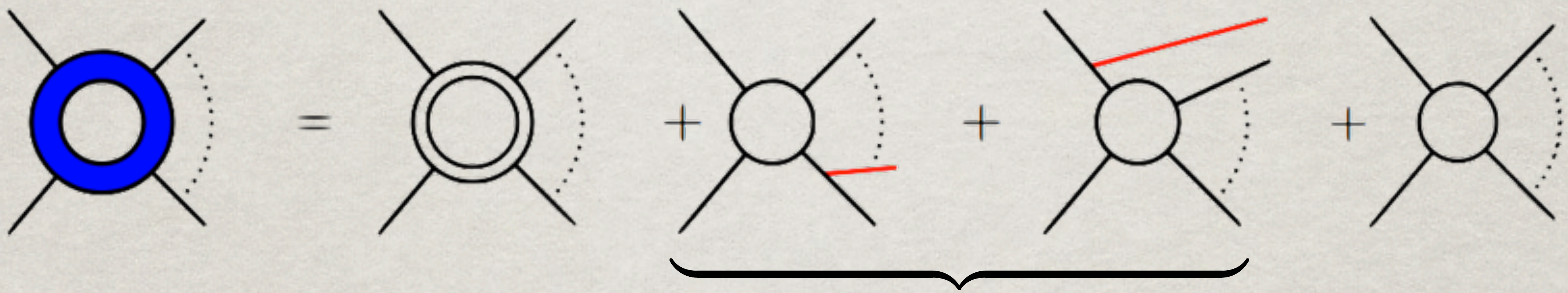
Having NLO computations by default lead to more conclusive observations.



arXiv:1104.0699v1

NLO Basics

NLO contributions have two parts



$$\sigma^{\text{NLO}} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(4)}\sigma^B$$

Virtual part

- Current bottleneck of NLO computations
- Algorithms for automation known in principle but not yet efficiently implemented
- This work brings automation using MadGraph and CutTools interfaced through MadLoop.

Real emission part

- Automated for different methods
- Challenge is the systematic extraction of singularities
- MadFKS using the FKS subtraction method successfully implemented on MGv4

Subtraction terms

IR divergences are dealt with using subtraction terms

$$\sigma^{\mathrm{NLO}} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(4)}\sigma^B$$

$$\sigma^{\mathrm{NLO}} = \int_m \left[d^{(4)}\sigma^B + \int_l d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A\right] + \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A\right]$$

- Each of the two integrals are separately finite.
- The only missing input required from MadFKS is the *finite part* of the virtual amplitude.
- This is the part MadLoop provides!

Towards automation

- MadGraph is a tree-level matrix element generator
- MadFKS is built on MG and computes everything but the finite part of the amplitude.
- CutTools is a code which computes very general loop integrals using the OPP method

↳ Idea: To create a third-party program, MadLoop which interfaces these three tools to automatically compute some loop x-sections.

MadFKS

Phase-space: Divide and Conquer

- Real emission part : $d\sigma^R = |M^{n+1}|^2 d\phi_{n+1}$
- $|M^{n+1}|^2$ diverges as $\frac{1}{\chi_i^2}\frac{1}{1-y_{ij}}$ with $\begin{aligned}\chi_i &= \frac{E_i}{\sqrt{\hat{s}}} \\ y_{ij} &= \cos\theta_{ij}\end{aligned}$
- Divide phase-space so that each partition has at most one soft and one collinear singularity

$$d\sigma^R = \sum_{ij} S_{ij} |M^{n+1}|^2 d\phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

- Use plus distribution to regulate the singularities $\int d\chi \left(\frac{1}{\chi}\right)_+ f(\chi) = \int d\chi \frac{f(\chi)-f(0)}{\chi}$

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\chi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \chi_i^2 (1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

CutTools

Or how to compute loops without doing so

- CutTools uses the OPP method for loop reduction at the integrand level

CutTools

Or how to compute loops without doing so

- CutTools uses the OPP method for loop reduction at the integrand level

$$\bar{q}^2 = q^2 + \tilde{q}^2 \qquad (q \cdot \tilde{q}) = 0$$

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \,, \quad p_0 \neq 0 \,.$$

$$\int d^{(d)}\sigma^V = \int d^{(4+\epsilon)} \left(A(\bar{q}) + \tilde{A}(\bar{q})\right)$$

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \left(\tilde{A}(\bar{q}) \rightarrow \mathrm{R2}\right)$$

$$\begin{aligned} N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\ + & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ + & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\ + & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i \\ + & \tilde{P}(q) \prod_{i}^{m-1} D_i \end{aligned}$$

- R2 can be obtained with a tree-level-like computation with special Feynman-Rules.
- Evaluation of N(q) for different specific q's allows to algebraically obtain the coefficients a, b, c and d
- Reconstruction of the $\tilde{q}$ dependance of the numerator gives the cut-constructible part R1 of the finite part of the virtual amplitude

Finite part = R1 + R2

MadLoop

Helping MadGraph and CutTools to be friends

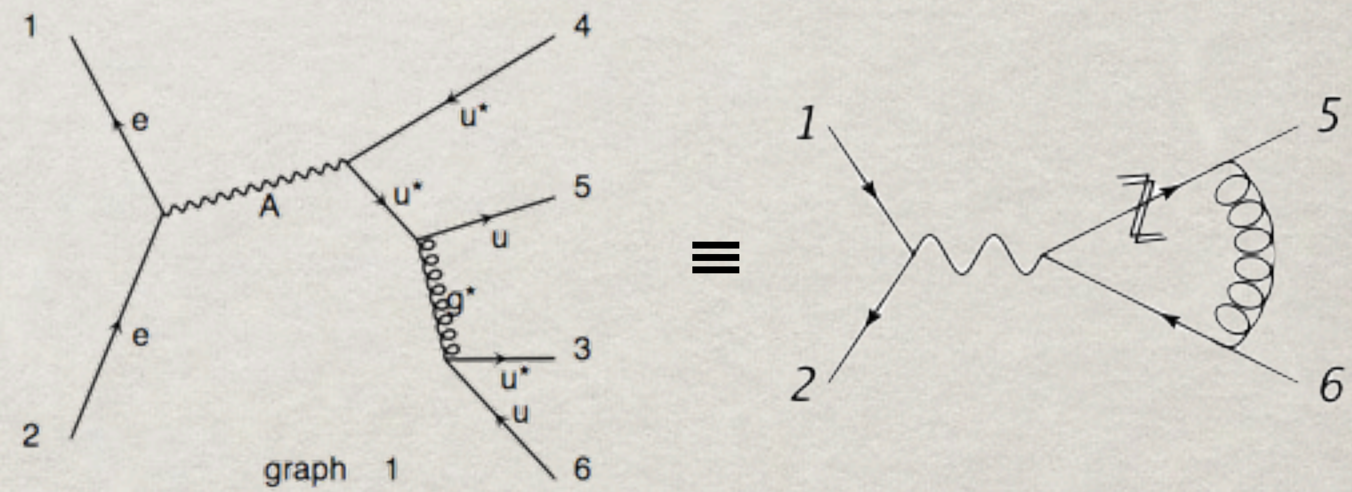
- Tree-like operations : Generate and obtain Born and R2 amplitudes
- Loop diagrams :
  - Generate cut-loop diagrams
  - Select a non-redundant basis
  - Compute the color factors
  - Provide numerator of the loop integrant
  - Handle exceptional PS points
- Compute R2 contribution
- Square born against virtual amplitude
- Carry UV renormalization.
- Perform Sanity checks (Double pole, Ward Identity, ...)

Cut-Loop diagrams

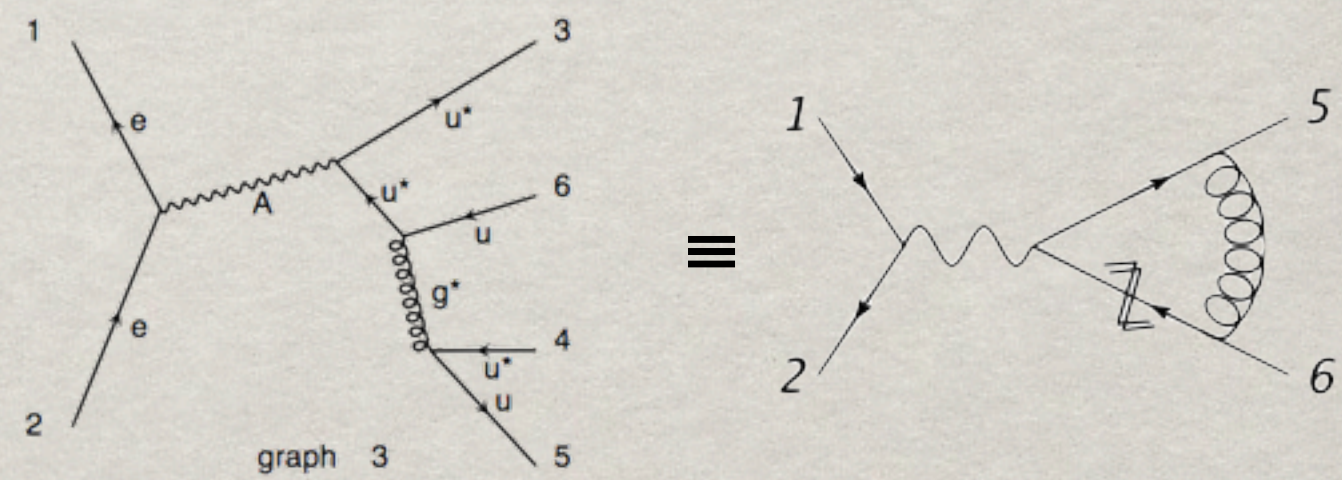
With a specific example

Consider $e^+e^- \to \gamma \to u\bar{u}$:

- Loop particles are denoted with a star. When MG is asked for $e^+e^- \to u^*\bar{u}^*u\bar{u}$ it gives back eight diagrams. Two of them are:
- Selection is performed to keep only one cut-diagram per loop <u>contributing</u> in the process
- Tags are associated to each cut-diagram. Those whose tags are **mirror and/or cyclic permutations** of tags of diagram already in the loop-basis are taken out.
- Additional custom filter to eliminate tadpoles and bubbles attached to external legs.



Diag $_1 = [u^*(6)g^*(5)u^*(A)]$



Diag $_3 = [u^*(A)u^*(6)g^*(5)]$

MadLoop

New Features implemented

- Recognition of the loop topologies in order to filter L-cut diagrams.
- Structure to deal with two MG process simultaneously (the L-cut and the born-like).
- Treat color to obtain the squaring of the loop color structure against the born one.
- Right form of integrant to CutTools: no denominators, complex momenta and reconstruction of missing propagator for the sewed particles.

$$\sum_{i=-1,1} \epsilon_i^{\mu}(p)\epsilon_i^{\nu *}(p) \rightarrow -g^{\mu\nu}$$

- Implementation of QCD ghosts.
- Implementation of the special R2 vertices and automatic UV renormalization.

MadLoop

Fighting Exceptional Phase Space points

- CutTools can asses the numerical stability of the computation of a loop by

  ➥ By sending $m_i^2 \to m_i^2 + M^2$, CT has an independent reconstruction of the numerator and can check if both match.
  ➥ CT ask MadLoop to evaluate the integrand at a given loop momentum and check if the result is close enough to the one from the reconstructed integrand.

- When an EPS occurs, MadLoop tries to cure it:

  ➥ Check if Ward Identities hold at a satisfactory level

  ➥ Shift the PS point by rescaling momenta : $k_i^3 = (1+\lambda_\pm)k_i^3$

  ➥ Provide an estimate of the virtual for the original PS point with uncertainty:

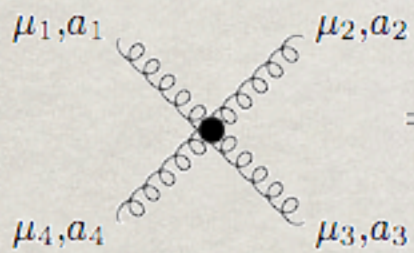
$$v_{\lambda_\pm}^{FIN} = \frac{V_{\lambda_\pm}^{FIN}}{|\mathcal{A}_{\lambda=0}^{born}|^2} \quad c = \frac{1}{2}\left(v_{\lambda_+}^{FIN} + v_{\lambda_-}^{FIN}\right) \quad \Delta = \left|v_{\lambda_+}^{FIN} - v_{\lambda_-}^{FIN}\right| \quad V_{\lambda=0}^{FIN} = \left|\mathcal{A}_{\lambda=0}^{born}\right|^2 (c \pm \Delta)$$

  ➥ If nothing works, then use the median of the results of the last 100 stable points

MadLoop

What it cannot do yet

- No four-gluon vertex at born level :



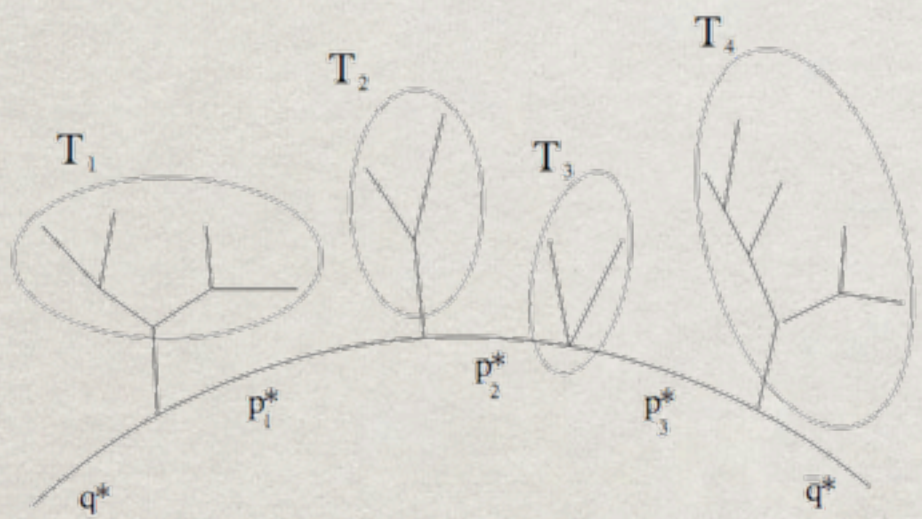
$$
= -\frac{i g^4 N_{col}}{96\pi^2} \sum_{P(234)} \Big\{ \Big[\frac{\delta_{a_1a_2}\delta_{a_3a_4} + \delta_{a_1a_3}\delta_{a_4a_2} + \delta_{a_1a_4}\delta_{a_2a_3}}{N_{col}}
+ 4\,Tr(t^{a_1}t^{a_3}t^{a_2}t^{a_4} + t^{a_1}t^{a_4}t^{a_2}t^{a_3})\,(3+\lambda_{HV})
- Tr(\{t^{a_1}t^{a_2}\}\{t^{a_3}t^{a_4}\})\,(5+2\lambda_{HV}) \Big]\, g_{\mu_1\mu_2}g_{\mu_3\mu_4}
+ 12\frac{N_f}{N_{col}} Tr(t^{a_1}t^{a_2}t^{a_3}t^{a_4}) \left(\frac{5}{3} g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_2\mu_3}g_{\mu_1\mu_4}\right)\Big\}
$$

- If EW bosons in the loop, it might be that CutTools cannot handle certain loops.
- All born contribution must factorize the same power of all coupling orders.
- No finite-width effects of unstable massive particles also appearing in the loop.

MadLoop

How to make it faster

- Recycling of the tree-structures attached to the loop.



- Identify identical contributions (i.e. massless fermion loops of diff. flavors)

- Call CT not per diagram, but per set of diagrams with the same loop kinematics.

- Use of recursion relations: Big gain in reals, not so much in the virtual.

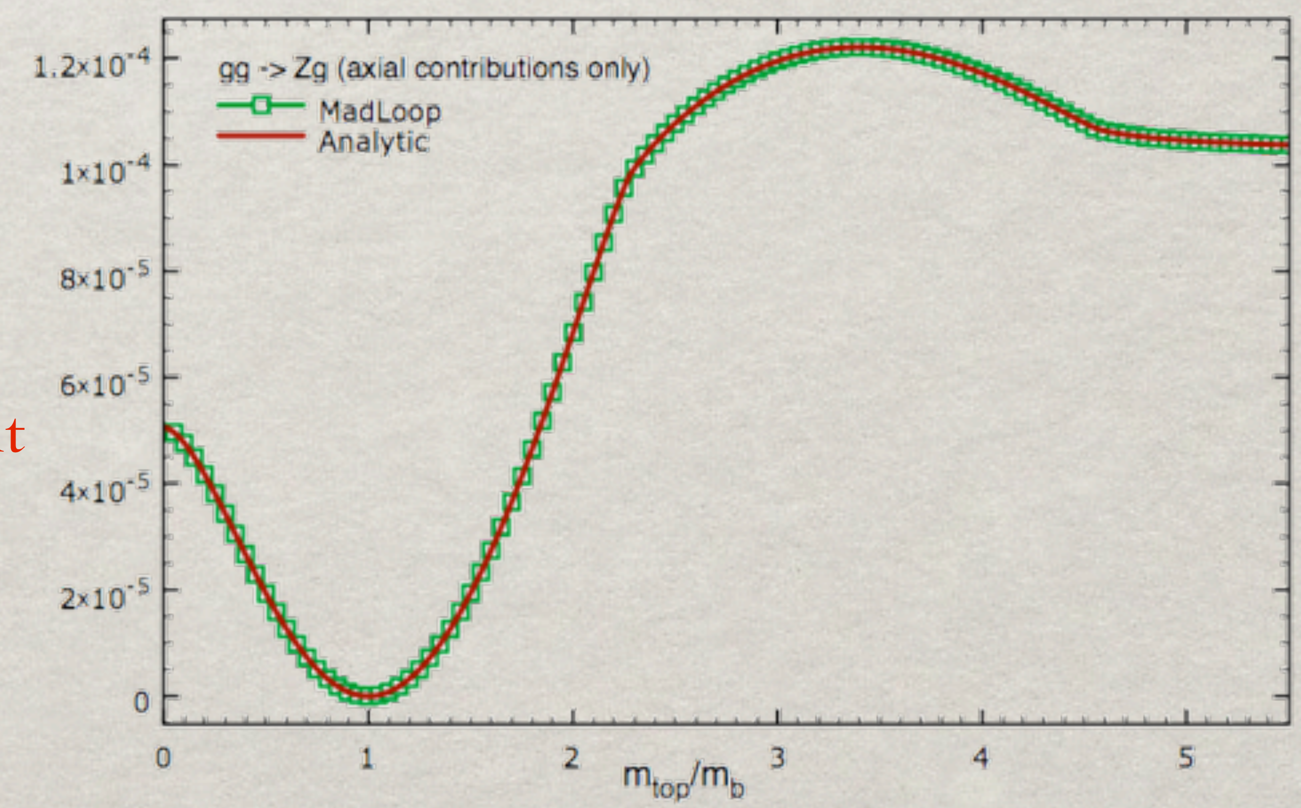
Local checks

You don't want the exhaustive list...

$u\bar{u} \to W^+W^-b\bar{b}$	MadLoop	Ref. [33]
a_0	2.338047209268890E-008	2.338047130649064E-008
c_{-2}	-2.493920703542680E-007	-2.493916939359002E-007
c_{-1}	-4.885901939046758E-007	-4.885901774740355E-007
c_0	-2.775800623041098E-007	-2.775787767591390E-007
$gg \to W^+W^-b\bar{b}$		
a_0	1.549795815702494E-008	1.549794572435312E-008
c_{-2}	-2.686312747217639E-007	-2.686310592221201E-007
c_{-1}	-6.078687041491385E-007	-6.078682316434646E-007
c_0	-5.519004042667462E-007	-5.519004727276688E-007

Ref. [33] : A. van Hameren *et al.*

- We believe the code is very robust - e.g., MadLoop helped spot mistakes in published loop computations (Zjj , W^+W^+jj)



- The numerics are pin-point on analytical data, even with several mass scales.

- Analytic computations from an independent implementation of the helicity amplitudes by J.J van der Bij *et al.*

Integrated Results

- Running time: Two weeks on a 150+ node cluster
- Proof of efficient EPS handling with $Zt\bar{t}$
- Successful cross-check against known results
- Large K-factors sometimes
- No cuts on b, robust numerics with small P_T

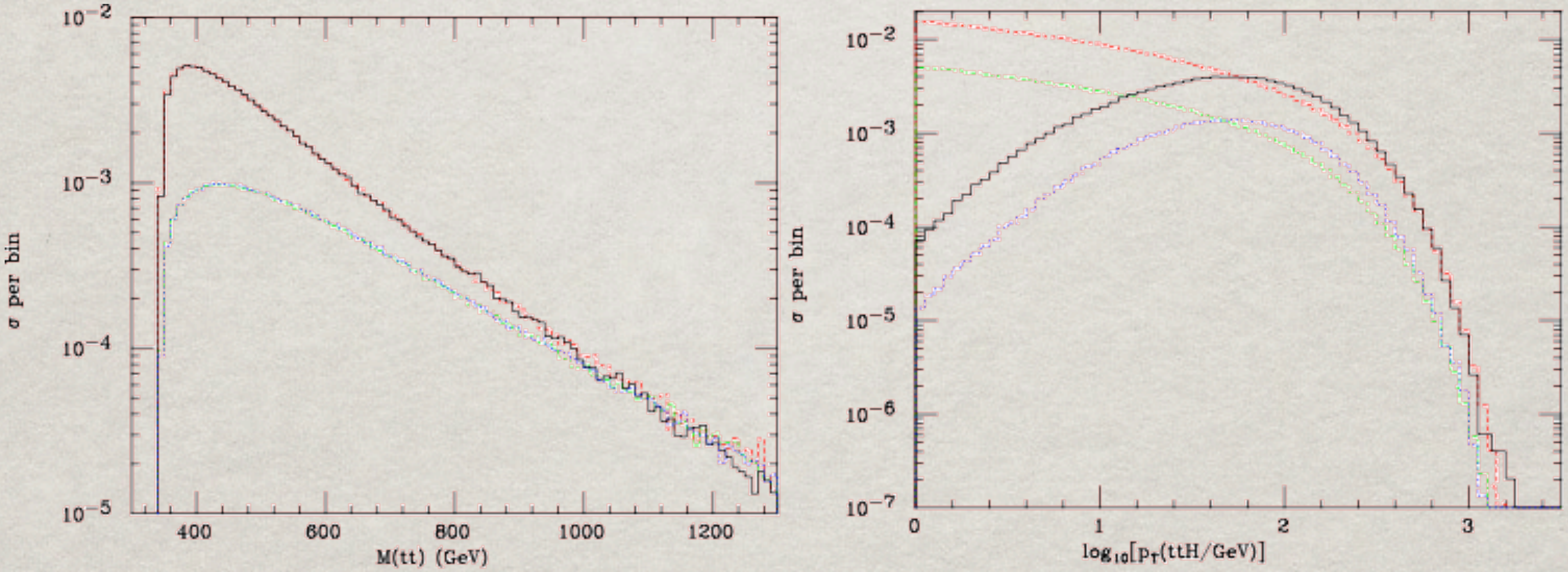
	Process	μ	n_{lf}	Cross section (pb) LO	Cross section (pb) NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^- j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^- jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^- j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+ jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+ j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZj$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

DISTRIBUTIONS

FULL MACHINERY AT WORK

- Case study of $[H/A]t\bar{t}$ with starring actors:

MGv4, CT, MadFKS, MadLoop and aMC@NLO interfaced to Herwig6 !



Solid: aMC@NLO scalar. Dashed: aMC@NLO pseudoscalar

Dotted: NLO scalar. Dotdashed: NLO pseudoscalar

Left: $t\bar{t}$ invariant mass. Right: $t\bar{t}H$ p_T

Future plans

MGv5 to the rescue

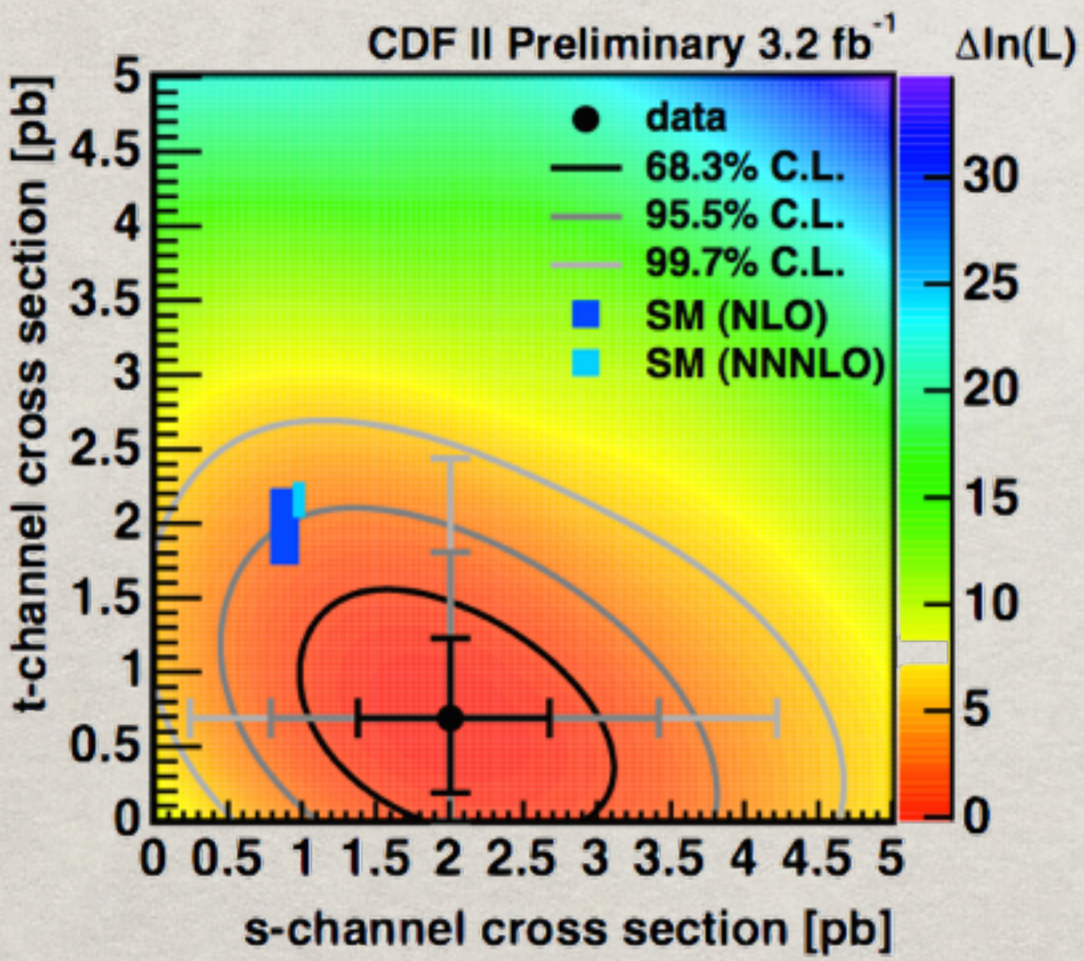
- MG5 much more advantageous:
  - Extreme programming and python leads to modular and flexible code
  - Test routines makes maintenance easy
  - Designed from the beginning for being fast and user-friendly.
  - Output in many formats and automatic writing of HELAS subroutines.
- Short term plans:
  - MadLoop @ MGv5 with the recycling optimizations
  - Get rid of ALL existing constraints.
- Further term plans:
  - Automate generation UV and R2 terms in FeynRules.
  - Recursion relations and computation of color ordered amplitudes.

Thanks!

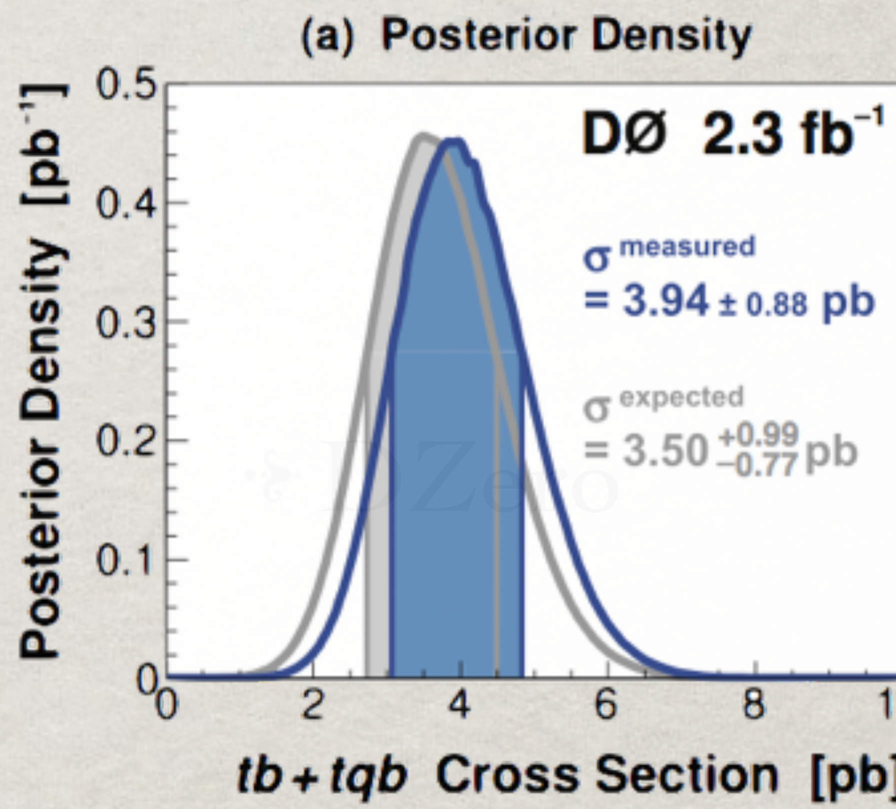
Additional Slides

Single-top analysis

DZero and CDF both observe significant signal for single-top



CDF note 9716



arXiv:0903.0850

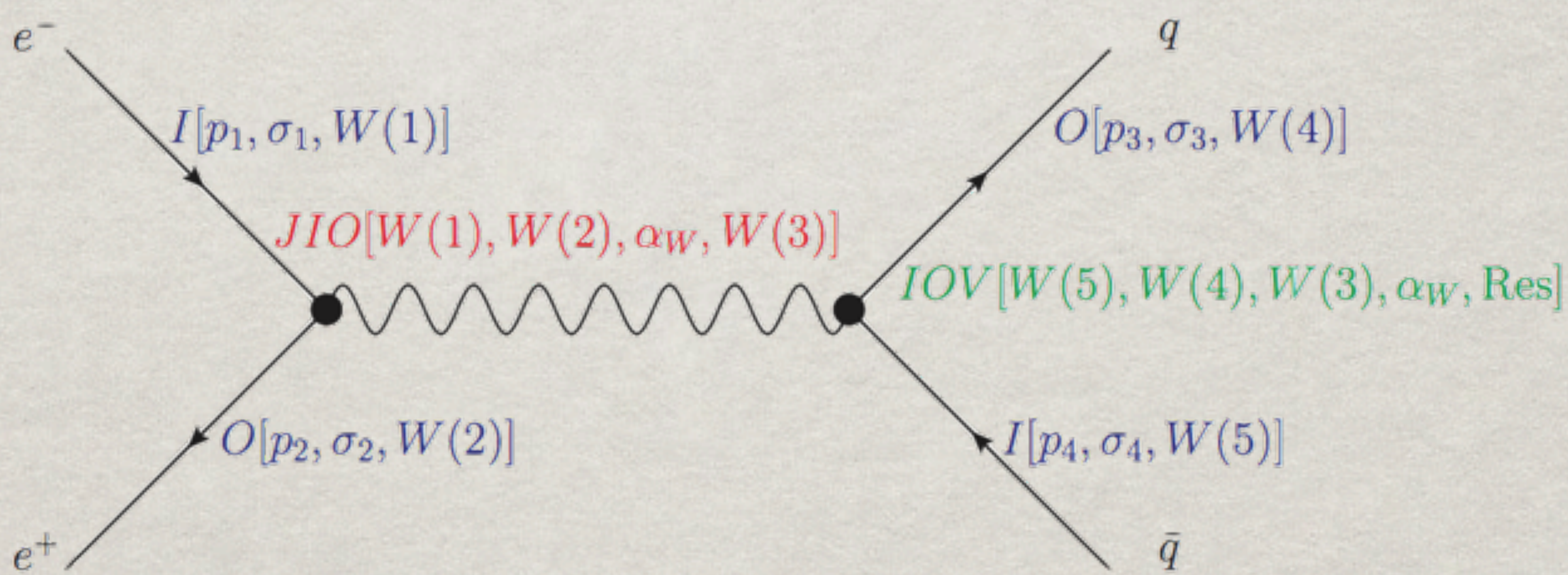
2σ-discrepancy pointed out by CDF on the {s,t}-channel plane

- New Physics, stat. fluctuations?
- Mistake in prediction?
  - ➥ Mistreatment of t-channel in 4F/5F scheme?

MadGraph

The evolutive way of computing tree-diagrams

- First generates all tree-level Feynman Diagrams
- Compute the amplitude of each diagram using a chain of calls to HELAS subroutines



- Finally square all the related amplitude with their right color factors to construct the full LO amplitude