MadDM: Dark Matter Calculations using Madgraph 5

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University of Kansas

In collaboration with
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MadGraph Spring 2011

Collider Physics using MadGraph

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\bar{\Psi}\left(\not{D} - m\right)\Psi + \left(D^{\mu}\phi\right)^{*} D_{\mu}\phi + \ldots$$

Lagrangian

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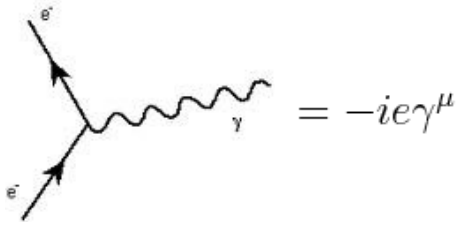
Lagrangian

FeynRules

Model files



$$= -ie\gamma^{\mu}$$

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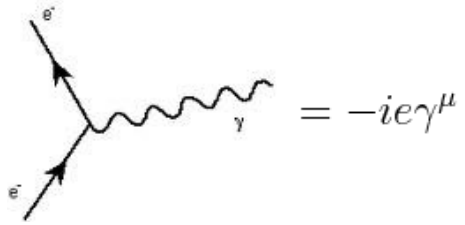

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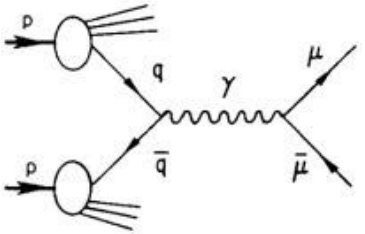

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MadGraph/MadEvent

Parton Level Events

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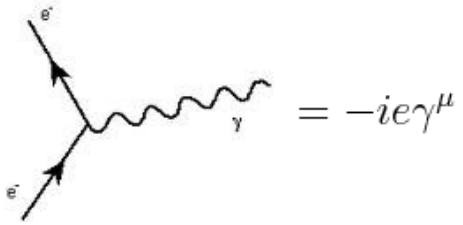

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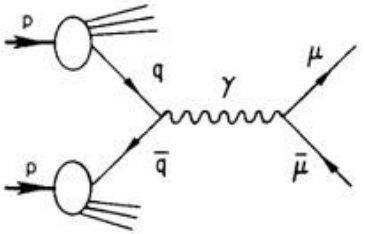

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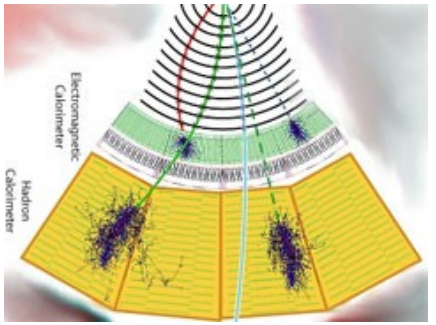

ISR / Parton Showering

Collider Physics using MadGraph

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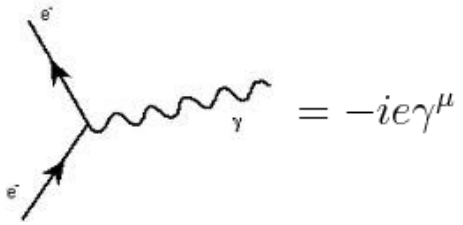

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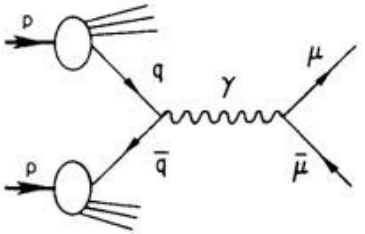

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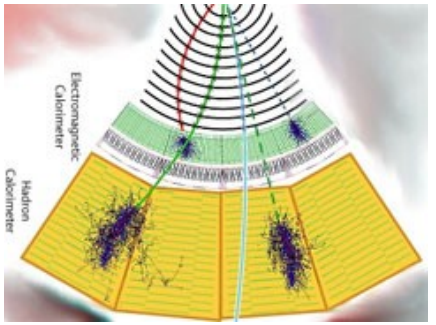

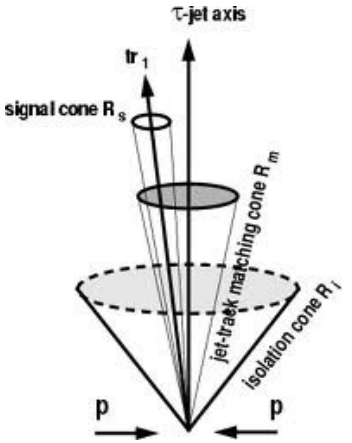

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Jet Reconstruction

Collider Physics using MadGraph

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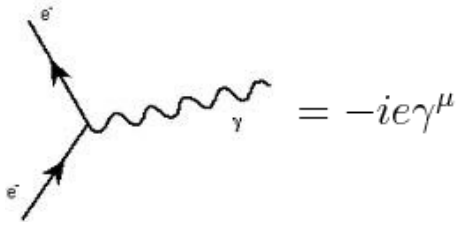

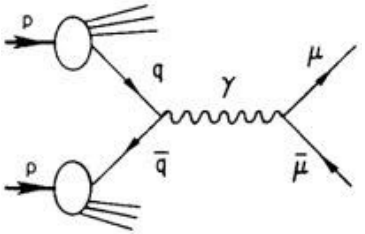

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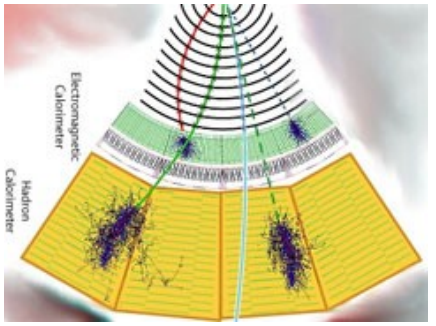
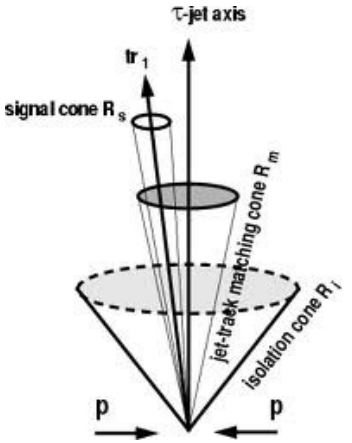


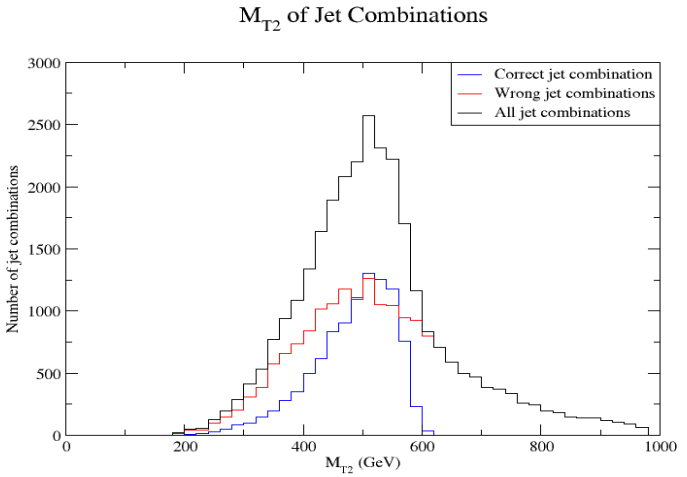


ISR / Parton Showering

PGS

Jet Reconstruction

MadAnalysis
Root



Pretty Plots

Collider Physics using MadGraph

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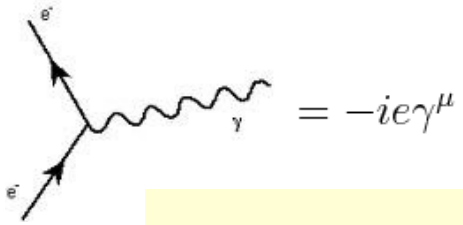

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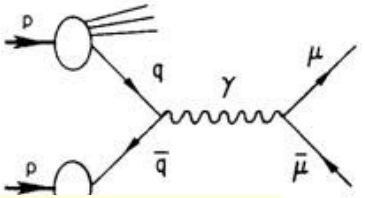

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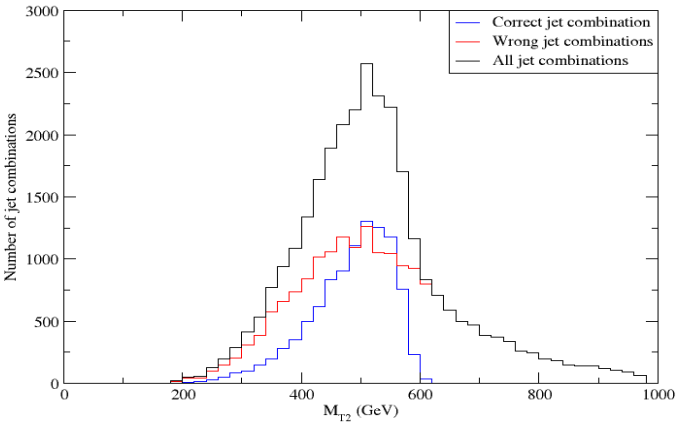


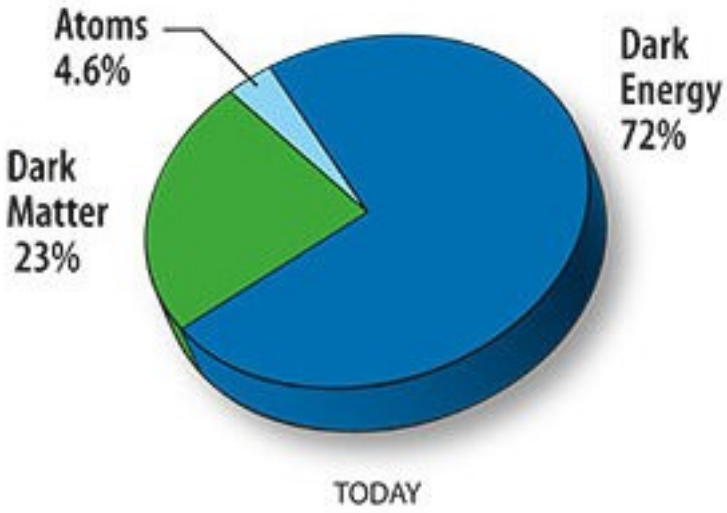
Pretty Plots

????

Profit

What about Dark Matter?

- Many BSM models have a potential DM candidate
  - MSSM (neutralino)
  - UED (KK photon)
- Calculations use the matrix elements Madgraph provides.
  - Relic abundance
  - Direct detection cross sections
  - Indirect detection via DM annihilation



Motivation for Dark Matter Calculations in MadGraph

- MadGraph is very popular among theorists and experimentalists alike.
- Allows users to easily get Dark Matter information (relic density, scattering cross sections, annihilation rates) along with collider events.
- New models with complex DM processes can be easily calculated in the MadGraph framework.
- Cross check with other programs.
- Makes the MadGraph package more complete for doing collider physics.

MadGraph 5

- New python based interface.
- Loads all model information into memory
- Python structure allows for very fast generation of Feynman Diagrams and matrix elements.
- Even on the programming level the only necessary commands to calculate relic density
  - Loading a model with a DM candidate
    - Automatically creates a python object that determines which particle in the model is the DM candidate
  - Set a parameter card with approriate values
  - Run the relic density calculator

Determining the DM candidate

- Scans through the loaded model file and looks for the following criteria:
  - Non-SM particle (abs(particle ID) > 25)
  - No electric charge
  - No color charge
  - Zero width

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Sample output: MSSM

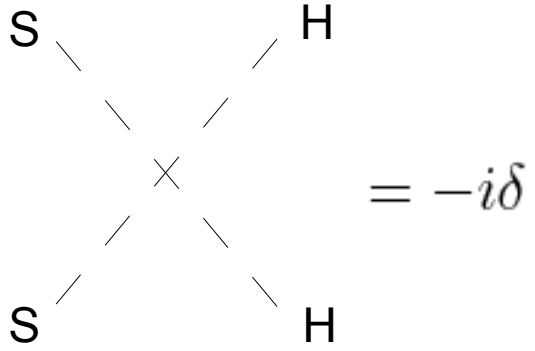
```
mccaskey@Mathew-McCaskeys-MacBook-/madgraph5/maddm:
 python dm_class_test.py
DARK MATTER CANDIDATES:
----------------------
[{
    'name': 'n1',
    'antiname': 'n1',
    'spin': 2,
    'color': 1,
    'charge': 0.00,
    'mass': 'Mneu1',
    'width': 'Wneu1',
    'pdg_code': 1000022,
    'texname': 'n1',
    'antitexname': 'n1',
    'line': 'straight',
    'propagating': True,
    'is_part': True,
    'self_antipart': True
}]
------------------------------
```

Toy DM Model

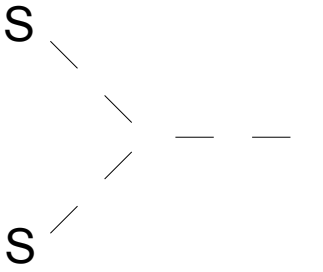
$$V_{\rm DM} = \frac{m^2}{2} H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + \frac{\delta}{2} H^\dagger H S^2 + \frac{m_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4$$

- Add a real scalar singlet to the SM the contains a S → -S symmetry.
- S only talks to the rest of the SM through the Higgs.
- Only two parameters needed to set the DM mass and the interaction stength.
  - δ and m_S are free parameters
- We will take the study point:
  - Higgs mass = 120 GeV

Interactions and Diagrams

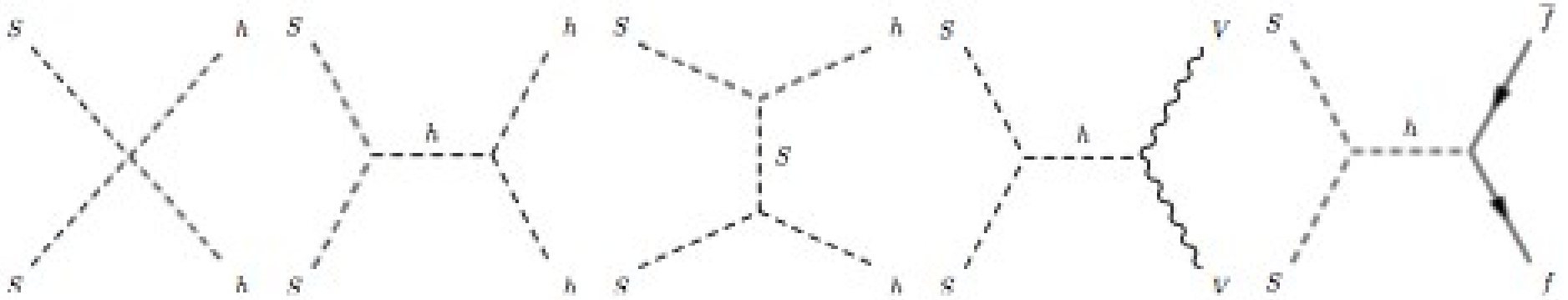


$= -i\delta$



$= -i\delta v$

- DM annihilation Feynman Diagrams



Chemical Rate Equation

$$\frac{dn_\psi}{dt} + 3Hn_\psi = -\langle\sigma|v|\rangle(n_\psi^2 - (n_\psi^{EQ})^2)$$

- DM relic density is governed by the expansion of the universe and the pair annihilation or creation that keeps the DM in equilibrium.
- Transformation to simplify the equation is to use the number density per comoving volume.

$$Y \equiv n_\psi / s$$

- Yet another transformation is to use temperature instead of time.

$$x \equiv m_\psi / T$$

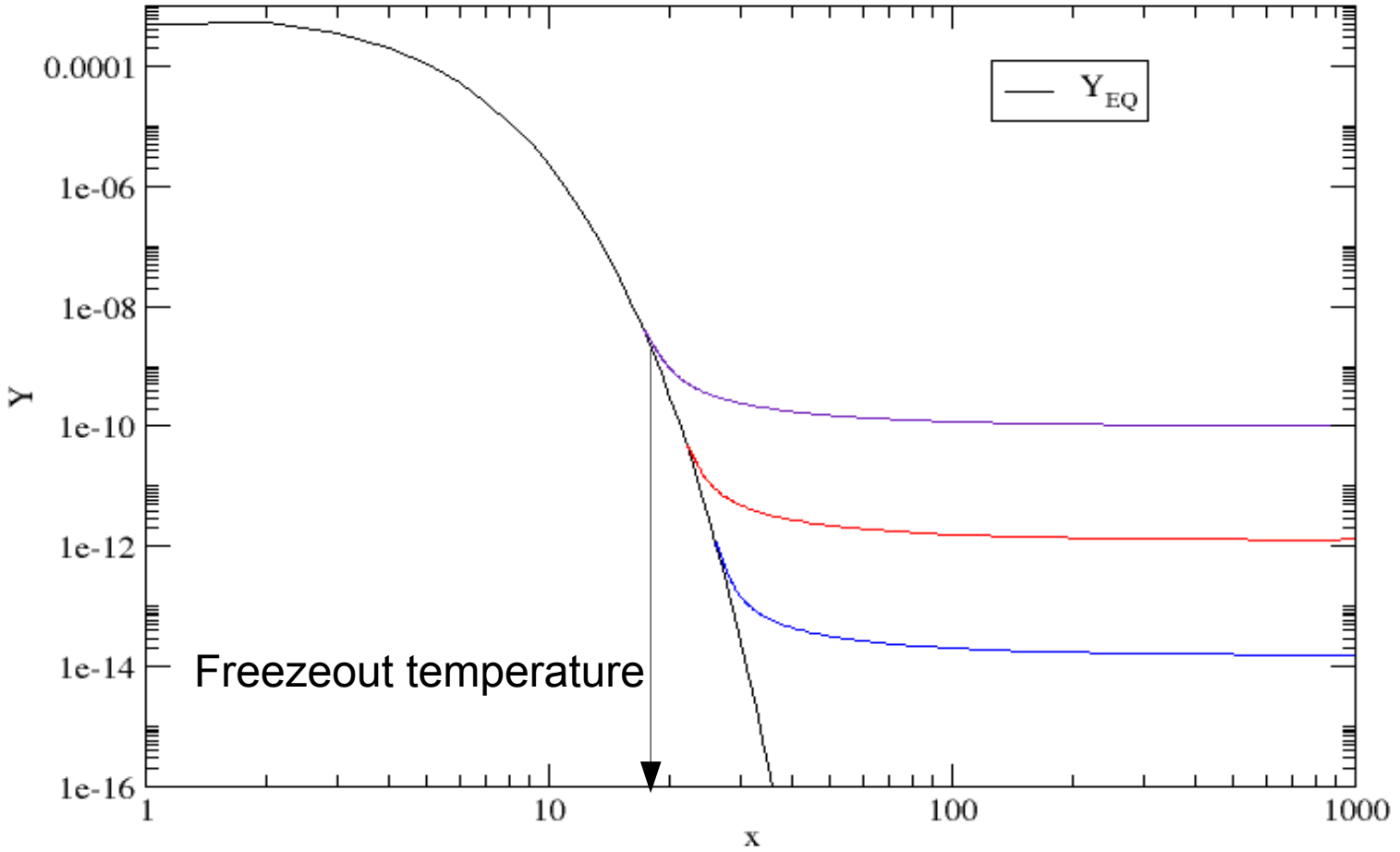
Chemical Rate Equation

- After a few pages of algebra (and several references to cosmology textbooks).

$$\frac{dY}{dx} = -\sqrt{\frac{45}{4\pi^3}}\frac{m_{pl}\ x}{\sqrt{g_*}\ m^2}\langle\sigma_A|v|\rangle s[Y^2 - Y_{EQ}^2]$$

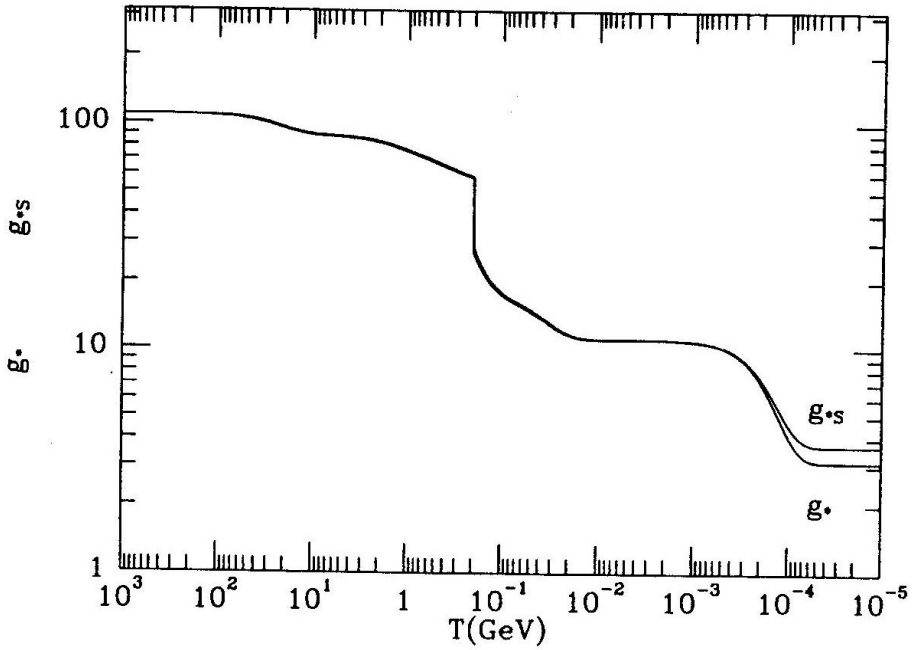
- This is integrated from the early universe (x<<1) to today (x>>1).
- Using the entropy density today and the ciritcal density in cosmology, one can calculate the relic abundance in Ωh^2

Integrated Chemical Rate Equation



Relativistic Degrees of Freedom

- Needed to calculate the entropy density (s) at a given temperate and appears in the conversion between time and temperature.
- In practical purposes only the relativistic d.o.f. at temperatures near freezeout (~10 GeV) are needed



$$\sum_{T>m} a(2s+1)N_c$$

$$a = 1 \ (bosons)$$

$$a = 7/8 \ (fermions)$$

Thermally Averaged Annihilation Cross Section

- Kolb and Turner

$$\langle \sigma_{\psi\bar{\psi}\to X\bar{X}}|v|\rangle \equiv \left(n_{\psi}^{EQ}\right)^{-2} \int d\Pi_{\psi} d\Pi_{\bar{\psi}} d\Pi_{X} d\Pi_{\bar{X}} (2\pi)^{4}$$
$$\delta^{4}(p_{\psi} + p_{\bar{\psi}} - p_{X} - p_{\bar{X}}) |\mathcal{M}|^{2} \exp(-E_{\psi}/T) \exp(-E_{\bar{\psi}}/T)$$

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$$\delta^{4}(p_{\psi} + p_{\bar{\psi}} - p_{X} - p_{\bar{X}}) \overline{|\mathcal{M}|^{2}} \exp(-E_{\psi}/T) \exp(-E_{\bar{\psi}}/T)$$

— The thermal equlibrium density of DM

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The thermal equlibrium density of DM

The matrix element integrated over the final state phase space. Matrix elements can be obtained by Madgraph and in a $2 \to 2$ process can be easily integrated manually.

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The thermal equlibrium density of DM

The matrix element integrated over the final state phase space. Matrix elements can be obtained by Madgraph and in a $2 \to 2$ process can be easily integrated manually.

After a few pages of algebra and variable transformations, this part of the expression can be written in terms of the center of mass energy of the process, or more conveniently in terms of a relative velocity, beta.

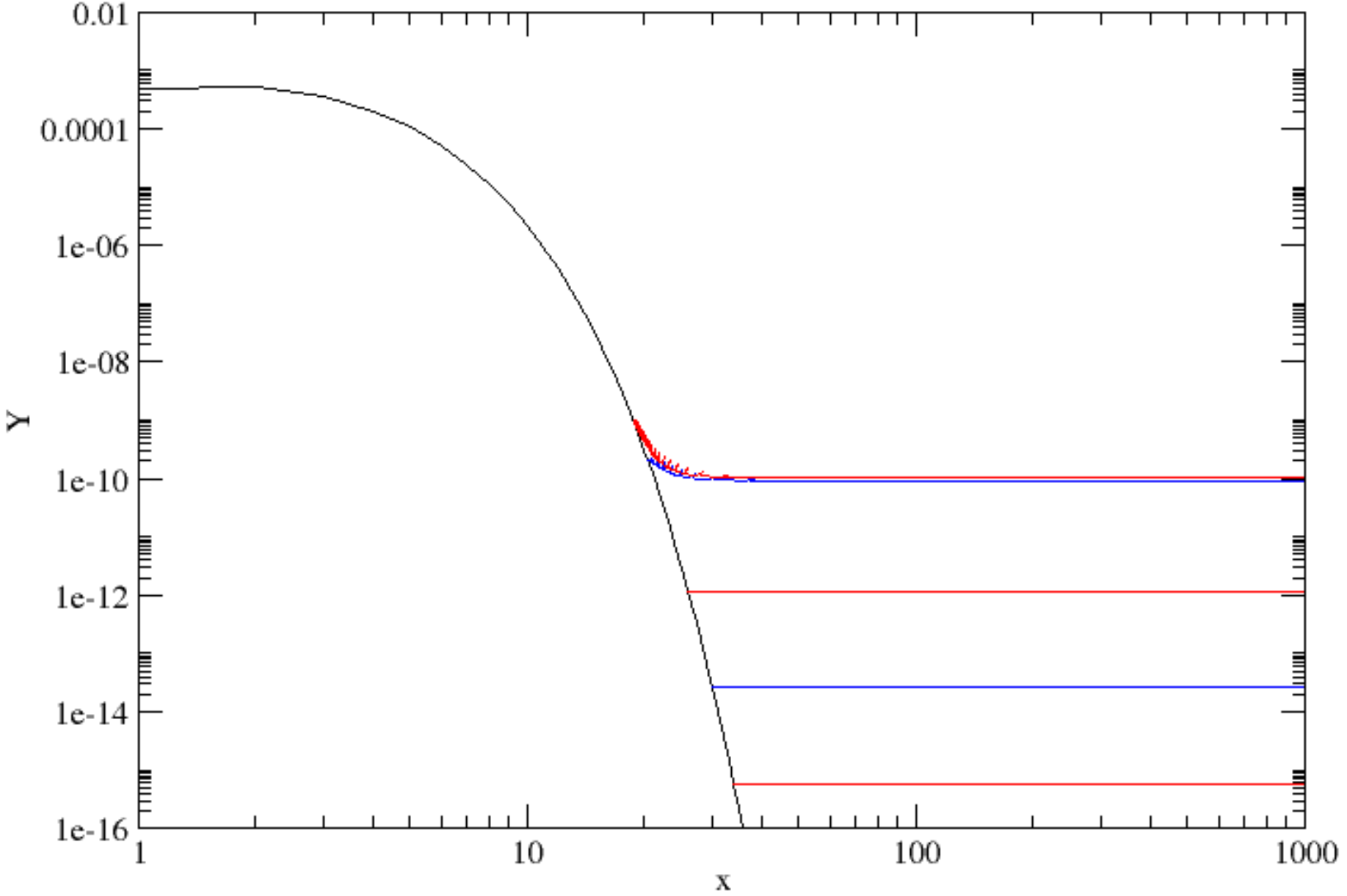
TAACS Simplified

$$\langle \sigma_A |v| \rangle = \frac{x^{3/2}}{m^2\sqrt{\pi}} \int_0^1 d\beta \, \frac{\beta^2}{(1-\beta^2)^{9/4}} \, e^{-2x\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)} \int |\mathcal{M}|^2 dPS_n$$

- Use Madgraph for the matrix element evaluation
- For a 2→2 process the integral over final state phase space is only a 1-D integral.
- Because the evaluation of the matrix elements takes the most amount of time this is evaluated at a limited number of betas and interpolated.
  - Leave the option open for exact evaluation in the case of a narrow resonance.
  - Takes more time

Practical Integration of Chemical Rate Equation

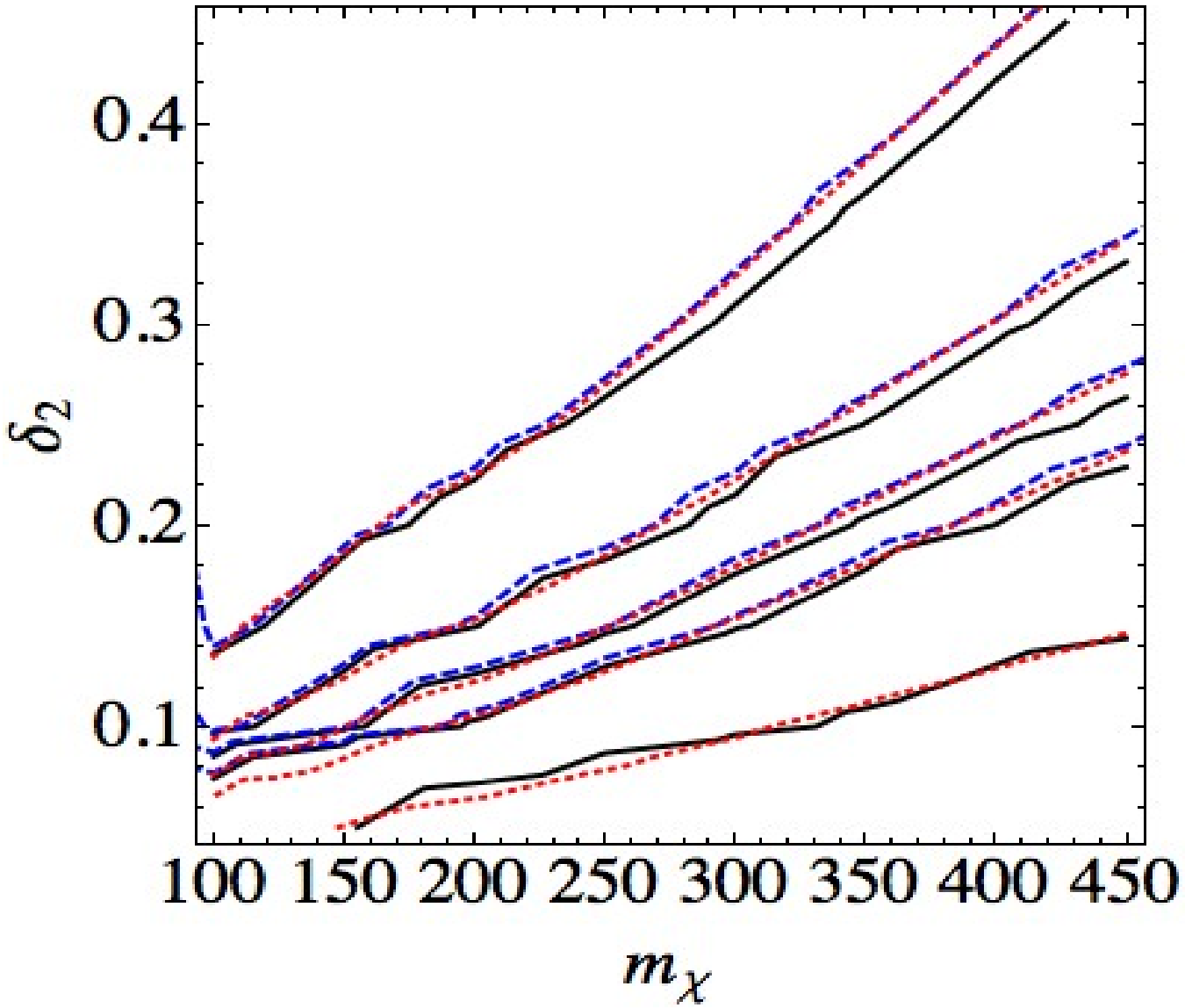
- Numerically it is impractical to integrate the Chemical Rate Equation from a small x
- Ideally we would want to start integrating just before the freezeout temperature.
- Since we do not know the freezeout temperature a priori, we start from a large x and work backwards until two solutions match within a certain tolerance
  - Since all the ODE integrations are near or after freezeout, this process is very fast.


0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14
1e-16
Y
1
10
100
1000
x

Preliminary Comparison with micrOmegas

- We perform a parameter scan over the DM mass and interaction strength.
- Calculate relic density using our code and compare it with the results given by micrOmegas.
- For the most part there is very good agreement between the two programs.
  - The largest error being ~8%
  - Needs to be investigated.

Preliminary Comparison with micrOmegas



Improvements

- All calculations are currently done entirely in python.
  - Possible to improve speed by evaluating the matrix element and do numerical calculations in Fortran.
- More efficient way to get the correct temperature to start the Chemical Rate Equation integration.
  - Starting temperature is currently decremented by one each iteration.
- Need to come up with a better name?
  - Suggestions are welcome.

Additional features to be implemented in the future

- Coannihilations
- 3 body final state
  - Requires monte carlo integration
- DM annihilation rates
  - Indirect detection
- Paramter scans
- Direct detection cross sections
  - Spin-independent / Spin-dependent
- Web based interface (similar to that of MadGraph)

Conclusions

- We have motivated the potential use of a DM calculations tool in the MadGraph framework.
- Have a pretty good relic density calculator that agrees well with other similar programs.
- Plans for more calculations to be implemented making MadDM a much more powerful tool.
- More updates to come!

The End

Thank you!