

MadGOLEM and the NLO Subtleties

MadGraph Spring Meeting 2011

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Outline

- Motivations
- Catani & Seymour Subtraction Method for SUSY
- On-Shell Subtraction Method
- Conclusions

Motivations

- Leading order (LO)
 - Finite by definition: external partons well separated in phase space
 - Highly automated:
MadGraph/MadEvent, CompHep, Alpgen, Amegic++/Sherpa...
- Next-to-leading order (NLO)
 - More Precise! Less sensibility from factorization/renormalization scales
 - Not yet fully automated in a process independent approach
 - **MadGOLEM**: Fully automated tool to perform NLO (SUSY-QCD) in a process independent approach (main focus on SUSY models)
 - Under development

Motivations

- MadGOLEM approach against divergencies:
 - IR divergencies
 - On-shell propagators
- Two main technics in the market to overcome these problems:
 - I) Phase space slicing
 - Introduce a **phase space cut δ** to be set very small
 - Very crude and increasingly unpopular method
 - II) Subtraction Methods
 - Introduces a **local counter term** which exactly matches the singular behaviour

Motivations

● MadGOLEM approach against divergencies:

● IR divergencies  Catani-Seymour Subtraction Method

Massless: S. Catani, M.H. Seymour, 1997

Massive: S. Catani, S. Dittmaier, M.H. Seymour, Z. Trocsanyi, 2002

● On-shell propagators  On-Shell Subtraction Method

W. Beenakker, R. Hopker, M. Spira, P.M. Zerwas, Nucl. Phys. B 492, 51 (1997)

● Two main technics in the market to overcome these problems:

I) Phase space slicing

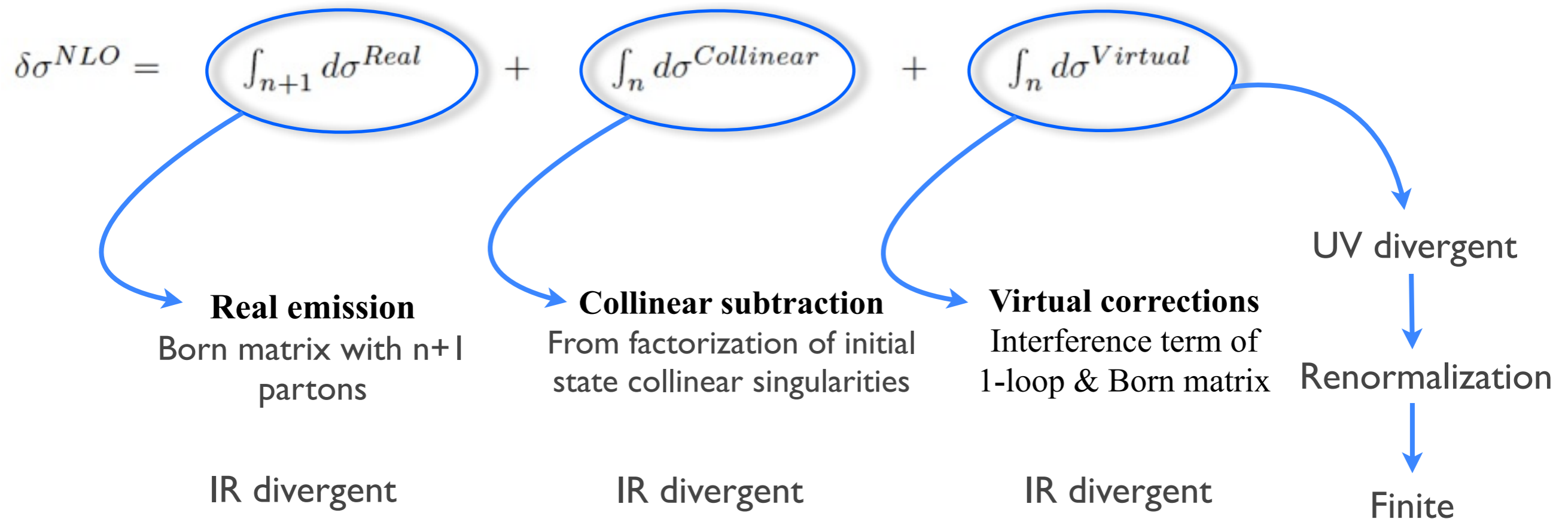
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II) Subtraction Methods

- Introduces a **local counter term** which exactly matches the singular behaviour

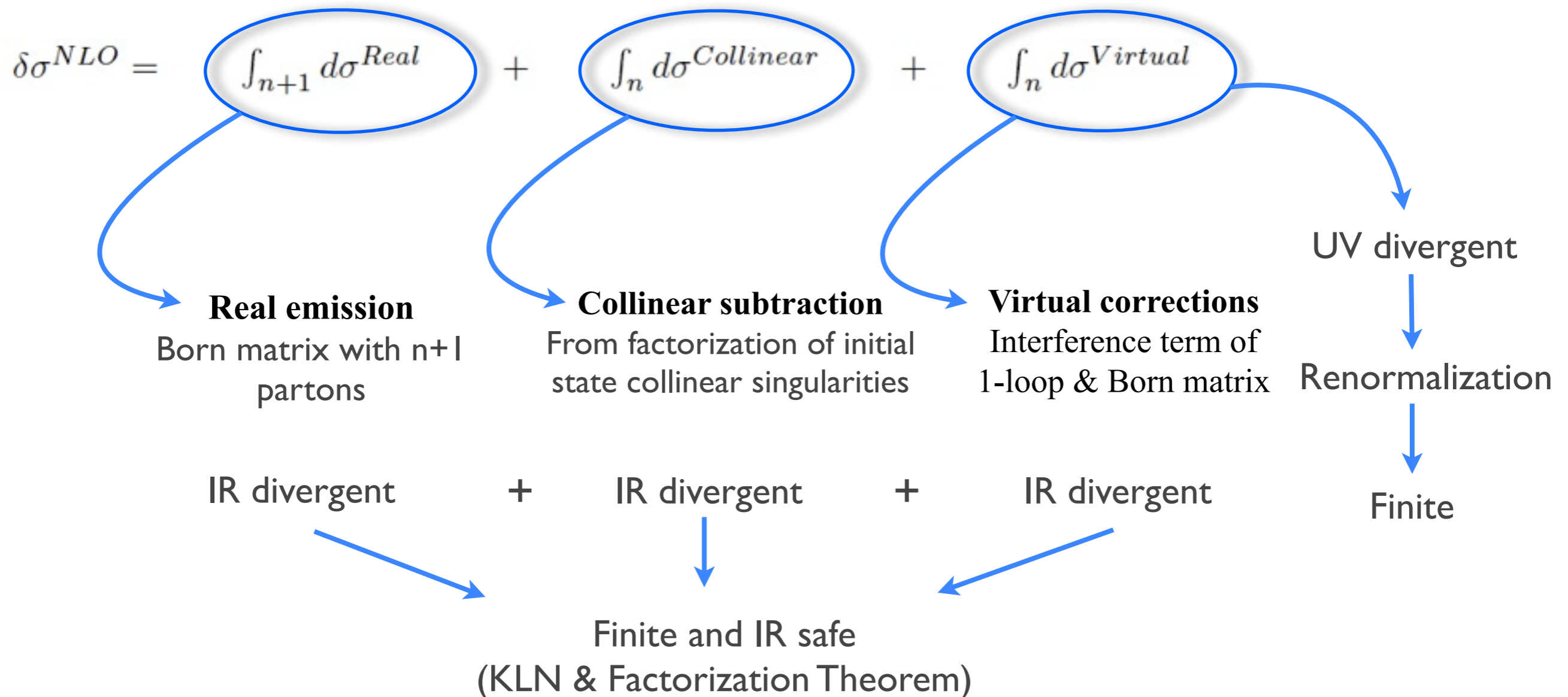
Catani-Seymour Subtraction Method

NLO schematically



Catani-Seymour Subtraction Method

NLO schematically



Problem:

How to get finite individual contributions from MC methods?

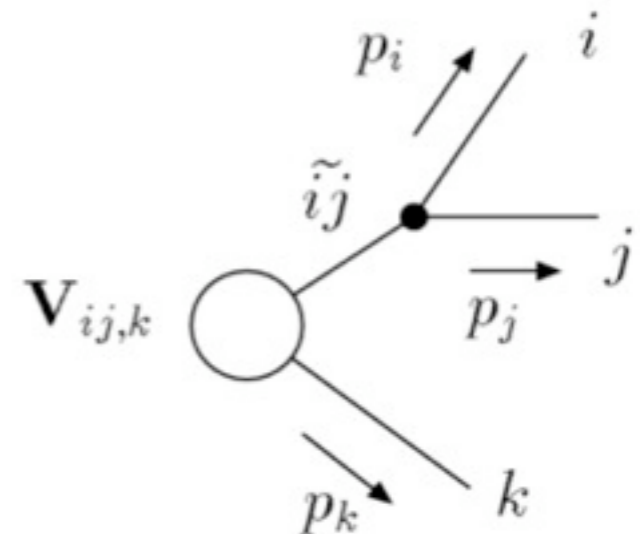
Catani-Seymour Subtraction Method

- CS Subtraction Method: construction of **local counter terms** using the universality of soft and collinear limits

$$|\mathcal{M}_{n+1}|^2 \rightarrow |\mathcal{M}_n|^2 \otimes V_{ij,k} \quad \longrightarrow \quad d\sigma^A \equiv \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

$$\delta\sigma^{NLO} = \int_{n+1} (d\sigma_{\varepsilon=0}^{\text{Real}} - d\sigma_{\varepsilon=0}^A) + \int_n (d\sigma^{\text{Collinear}} + d\sigma^{\text{Virtual}} + \int_1 d\sigma^A)_{\varepsilon=0}$$

- $V_{ij,k}$ is a singular factor, and depends only on the quantum numbers of i, j and k , and on their momenta. It is completely process independent.



Catani-Seymour Subtraction Method

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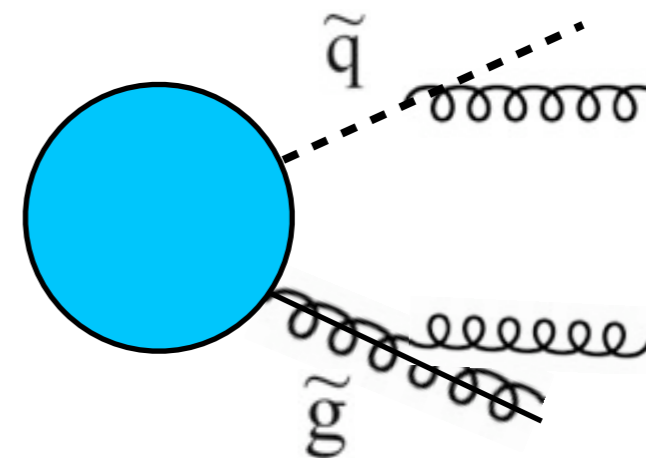
- We use the MadDipole package for the SM dipoles

R. Frederix T. Gehrmann, N. Greiner hep-ph/0808.2128 & 1004.2905

- $|D_\mu \tilde{q}|^2$ & $\tilde{g} \not{D} \tilde{g}$ induces a gluon emission from the squark & gluino legs:

→ IR singularities in the real emission

→ Need of SUSY dipoles



Catani-Seymour Subtraction Method

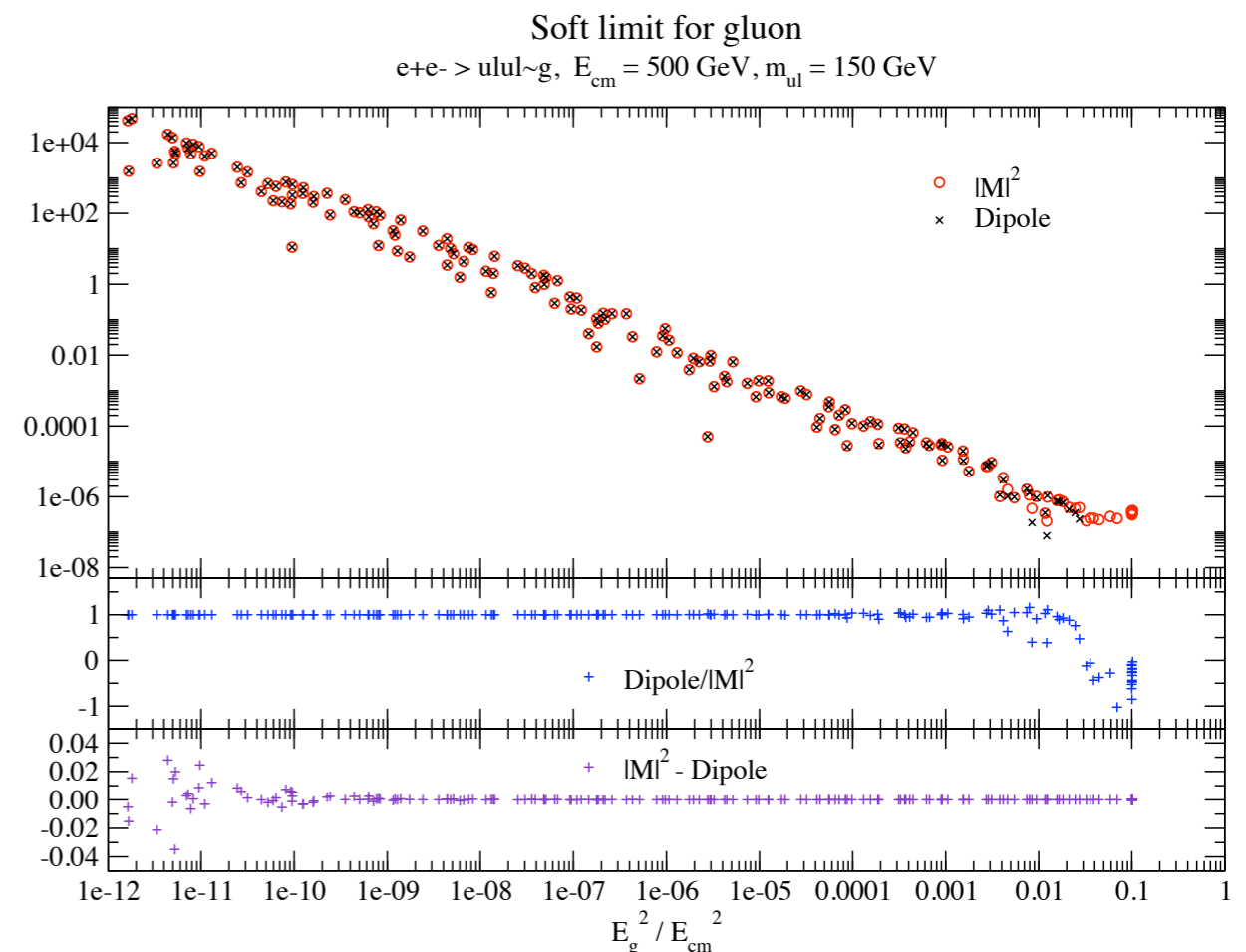
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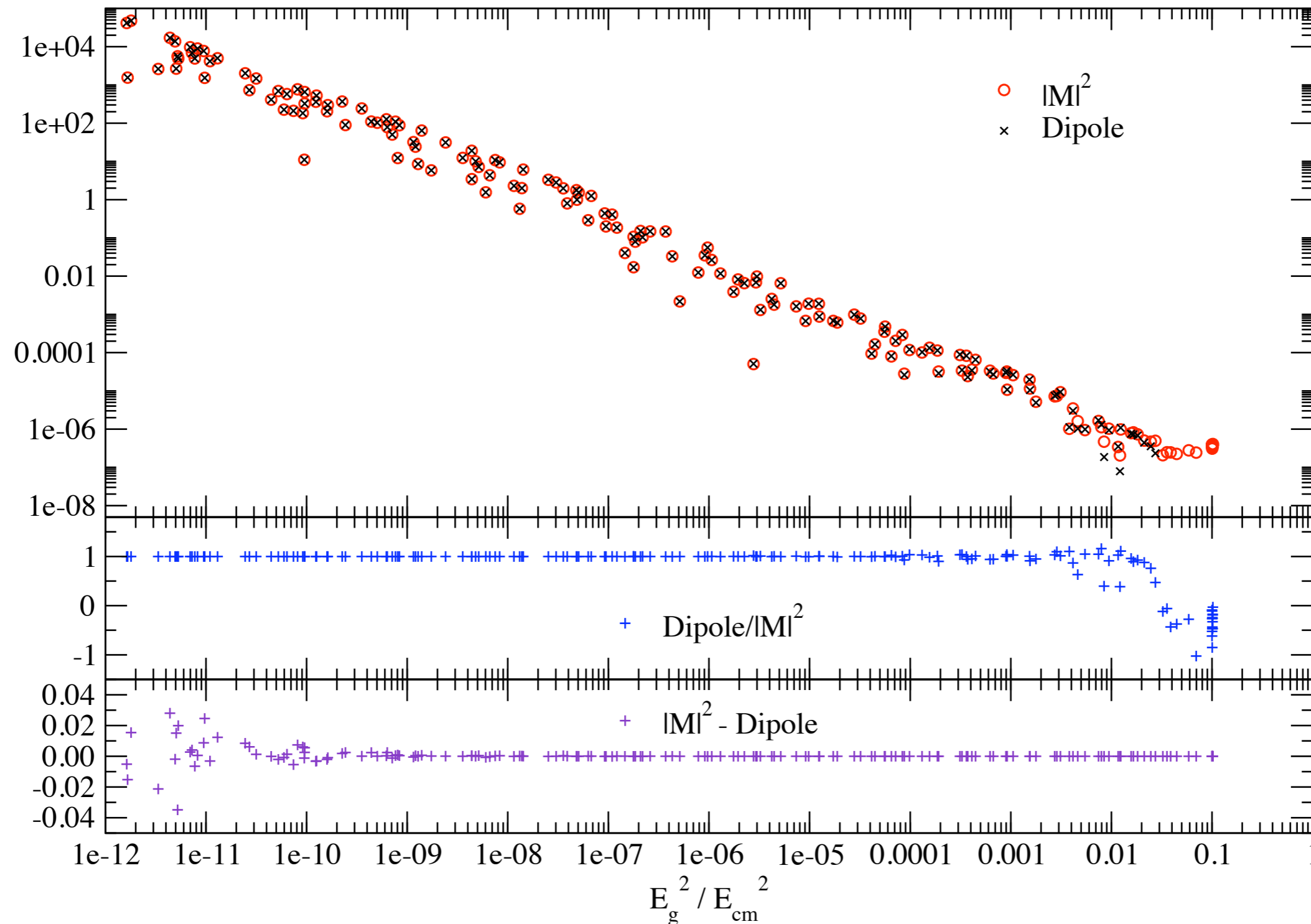
We extend this package with the SUSY dipoles



Catani-Seymour Subtraction Method

Soft limit for gluon

$e^+e^- \rightarrow u\bar{u}l\bar{l}g$, $E_{\text{cm}} = 500 \text{ GeV}$, $m_{ul} = 150 \text{ GeV}$



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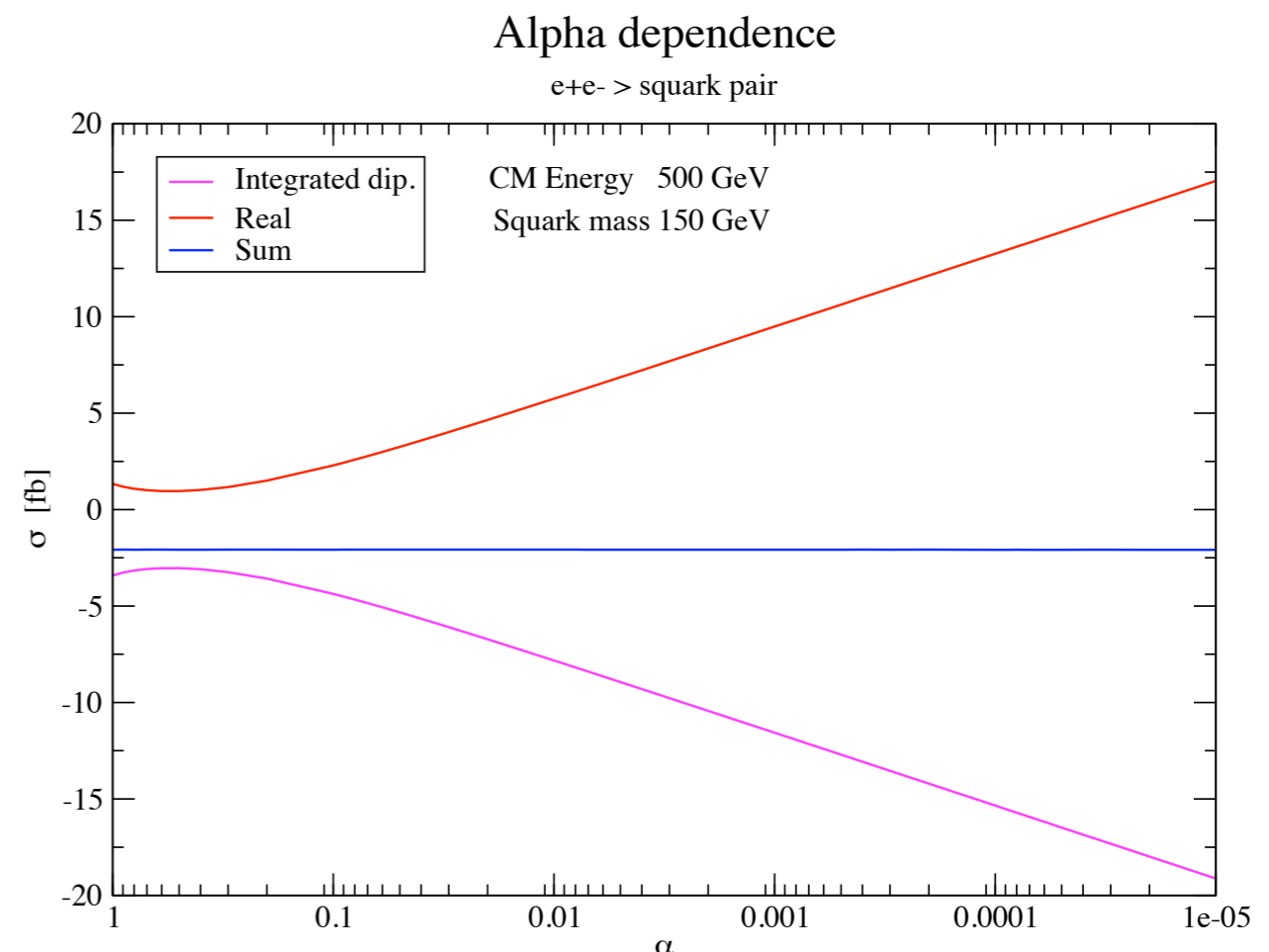
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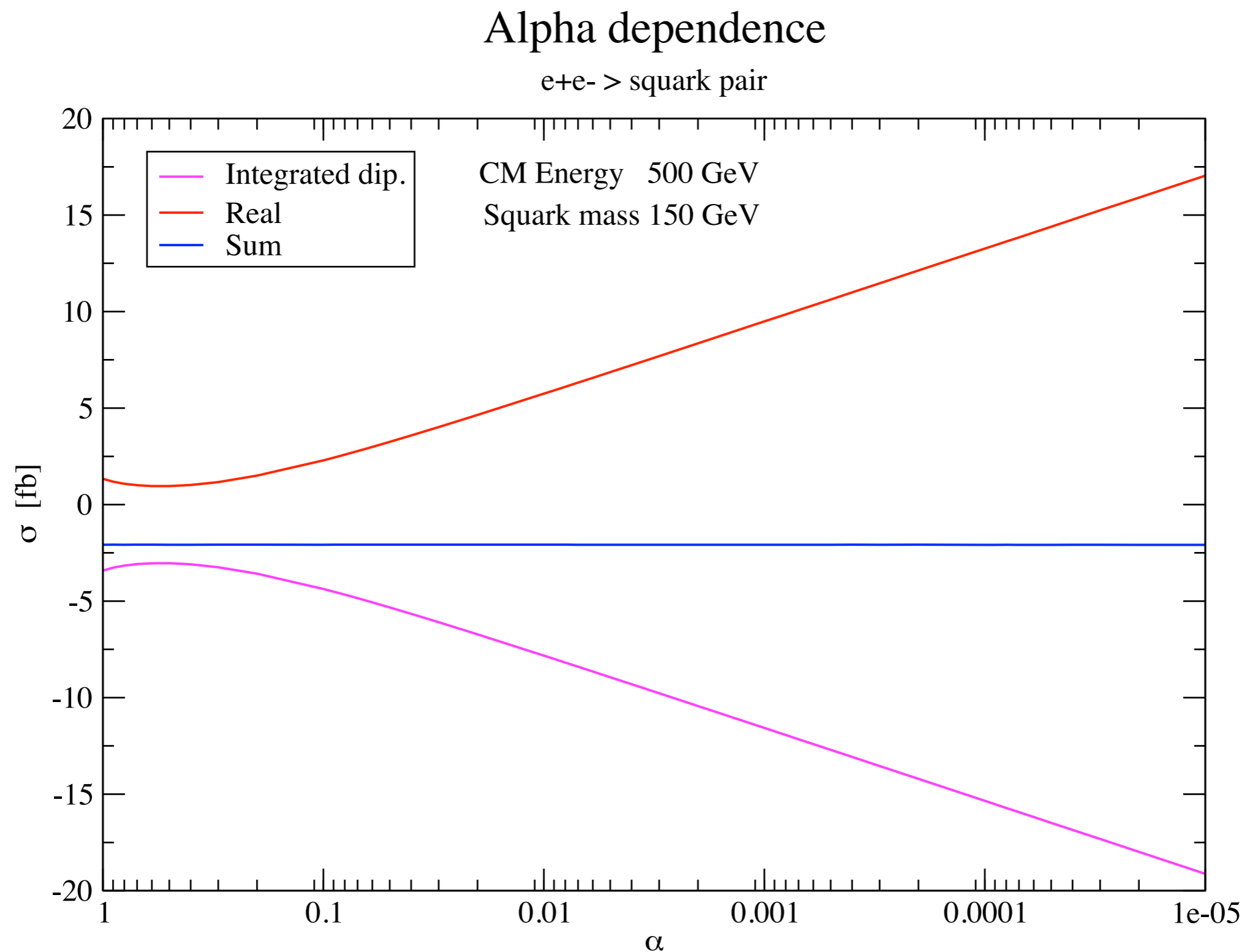
Derivation and implementation of alpha parameter (avoid calculations far from the divergencies)

$$\mathcal{D}'_{ij,k} = \mathcal{D}_{ij,k} \theta(y_{ij,k} < \alpha), \quad \alpha \in [0, 1]$$

Used in the SM case by Z. Nagy, Z. Trocsanyi. [hep-ph/9806317](https://arxiv.org/abs/hep-ph/9806317)



Catani-Seymour Subtraction Method

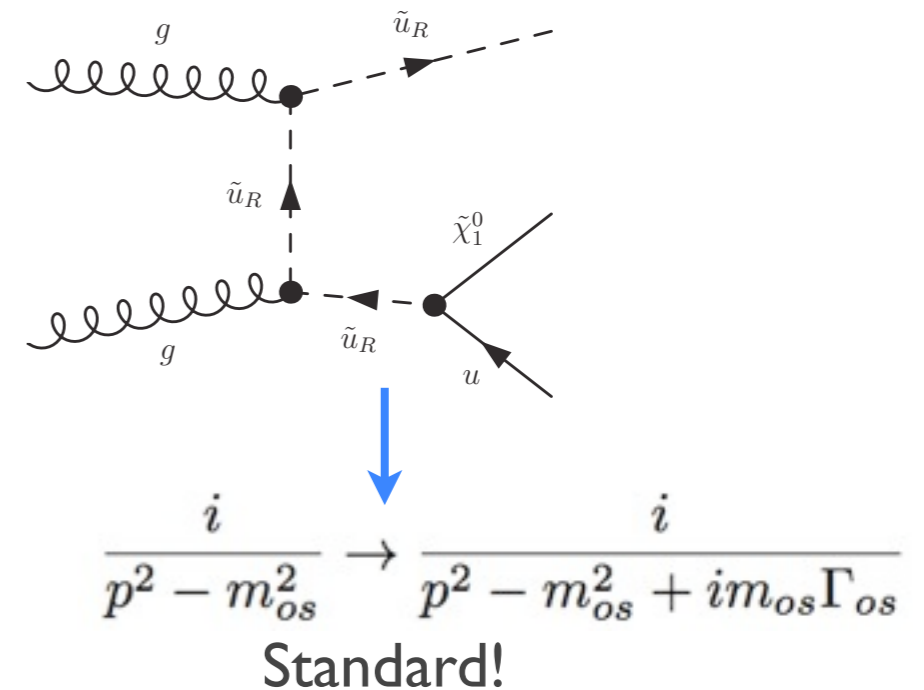
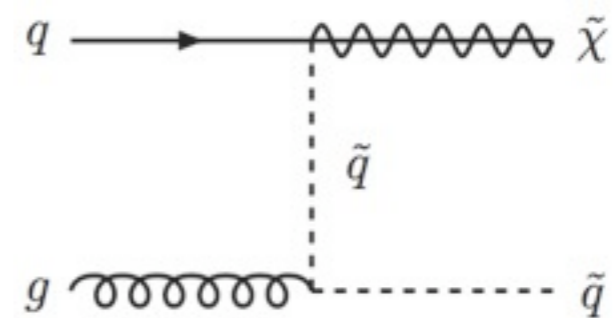


On-Shell Subtraction Method

Beyond **UV** & **IR** div. another type of div. can occur, namely **OS intermediate states**

Ex: $pp \rightarrow sq\chi_1$

@NLO new incoming states for $(sq\chi_1 + \text{jets})$



On-Shell Subtraction Method

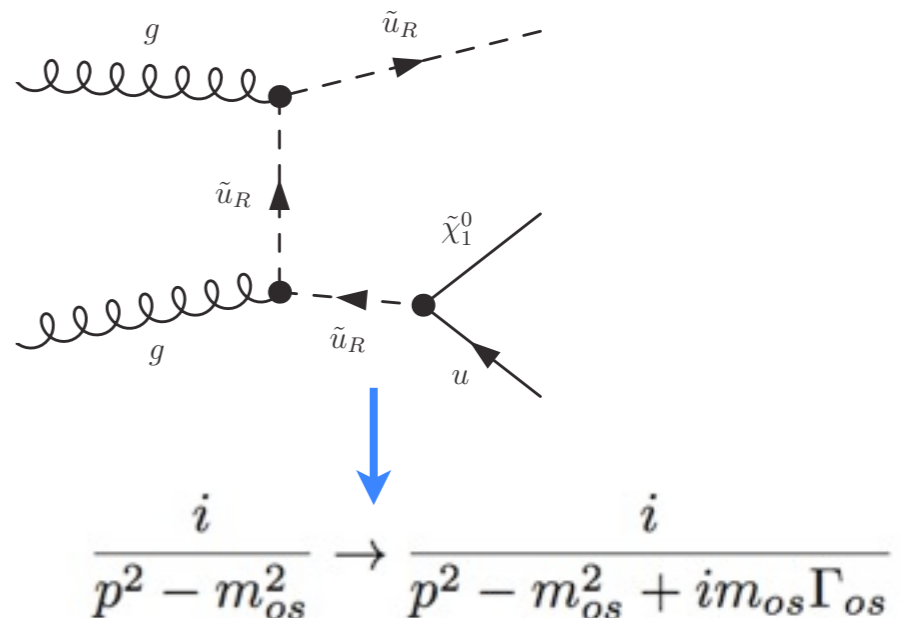
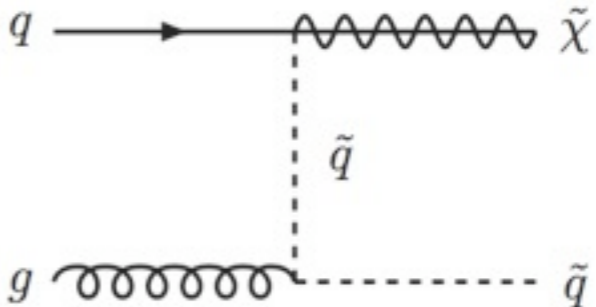
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- @NLO new incoming states
for ($\text{sq}\chi_1 + \text{jets}$)

- Although this pole is showing another feature:
This higher order amplitude is smoothly connected to the Born one

→ Double counting!



On-Shell Subtraction Method

Differentiation between Off & On-shell production to avoid **double counting**

$$gg \rightarrow \tilde{q}\tilde{q}^* \rightarrow \tilde{q}\chi_1\bar{q}$$

Squark neutralino production

$$gg \rightarrow \tilde{q}\tilde{q} * BR(\tilde{q} \rightarrow \chi_1\bar{q})$$

Squark pair production

$$|\mathcal{M}^{OS}_{gg \rightarrow \tilde{q}\chi_1\bar{q}}|^2 \rightarrow |\mathcal{M}^{OS}_{gg \rightarrow \tilde{q}\tilde{q}}|^2 \frac{m_{\tilde{q}}\Gamma_{\tilde{q}}/\pi}{\left(M^2 - m_{\tilde{q}}^2\right)^2 + m_{\tilde{q}}^2\Gamma_{\tilde{q}}^2} BR(\tilde{q} \rightarrow \chi_1\bar{q})$$

$$\xrightarrow{\Gamma_{\tilde{q}} \rightarrow 0} |\mathcal{M}^{OS}_{gg \rightarrow \tilde{q}\tilde{q}}|^2 \delta(M^2 - m_{\tilde{q}}^2) BR(\tilde{q} \rightarrow \chi_1\bar{q})$$

- One way to avoid this double counting is to apply the cut $|M - m_{os}| > n\Gamma$

→ Very crude approximation

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We want a **local counter term** which subtracts the pole in **gauge inv.** approach!

On-Shell Subtraction Method W. Beenakker, R. Hopker, M. Spira, P.M. Zerwas, Nucl. Phys. B 492, 51 (1997)

On-Shell Subtraction Method

On-Shell subtraction method (W. Beenakker, R. Hopker, M. Spira, P.M. Zerwas '97)

Propagator must contain a finite width Γ_{OS} , regarded as a regulator

$$\frac{i}{p^2 - m_{OS}^2} \rightarrow \frac{i}{p^2 - m_{OS}^2 + im_{OS}\Gamma_{OS}}$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{res}|^2 + 2\text{Re}[\mathcal{M}_{res}^* \mathcal{M}_{rem}] + |\mathcal{M}_{rem}|^2$$

With spin correlations!
Gauge Invariant!

The OS Subtraction term can be written as:

$$d\sigma^{os} = \sum_{os\ part.} |\mathcal{M}^{OS}_{res}|^2 (m_{os}\Gamma_{os}\pi) \frac{m_{os}\Gamma_{os}/\pi}{(M^2 - m_{os}^2)^2 + m_{os}^2\Gamma_{os}^2} \theta(m_{os} - m_{os\ decay})$$

$$\sigma^{Real}(\Gamma_{os}) = \int_{n+1} d\Phi_{n+1} [(|\mathcal{M}_{res}|^2 - d\sigma^{os}) + 2\text{Re}[\mathcal{M}_{res}^* \mathcal{M}_{rem}] + |\mathcal{M}_{rem}|^2 - d\sigma^A]$$

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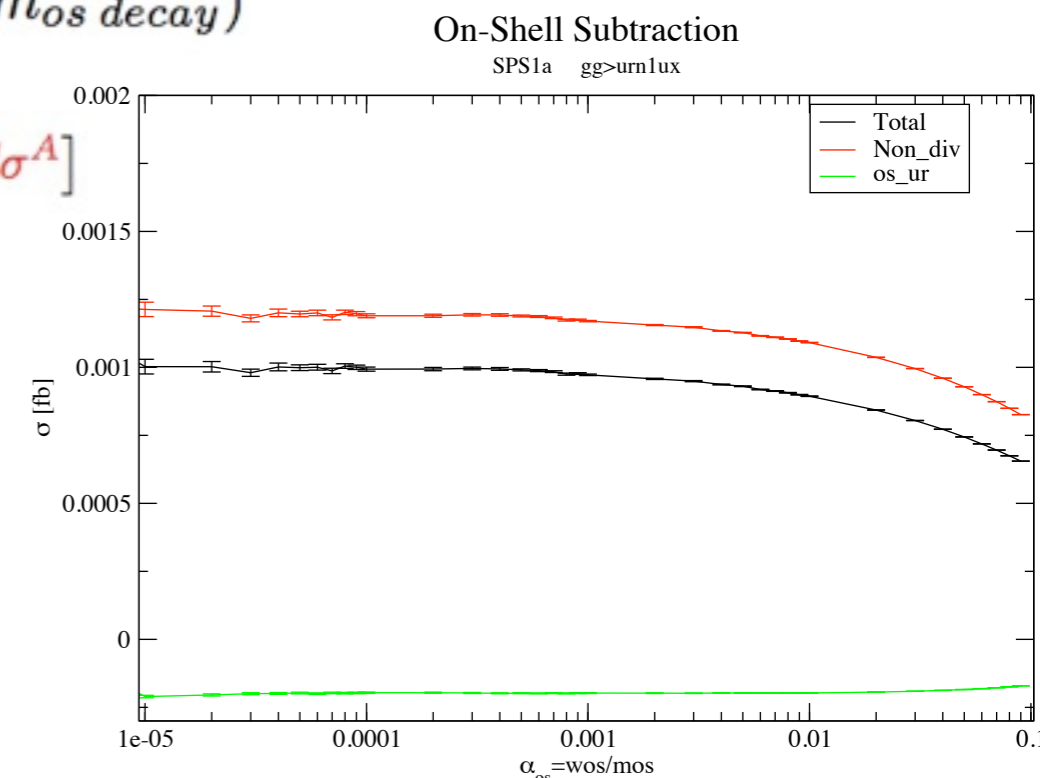
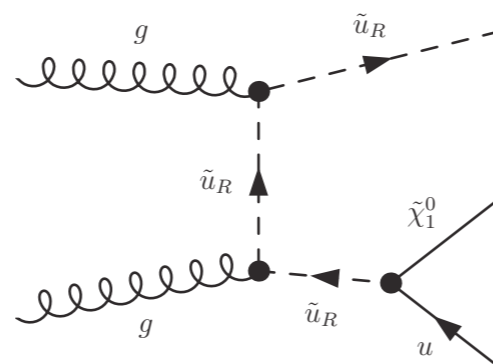
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gg>urnlux

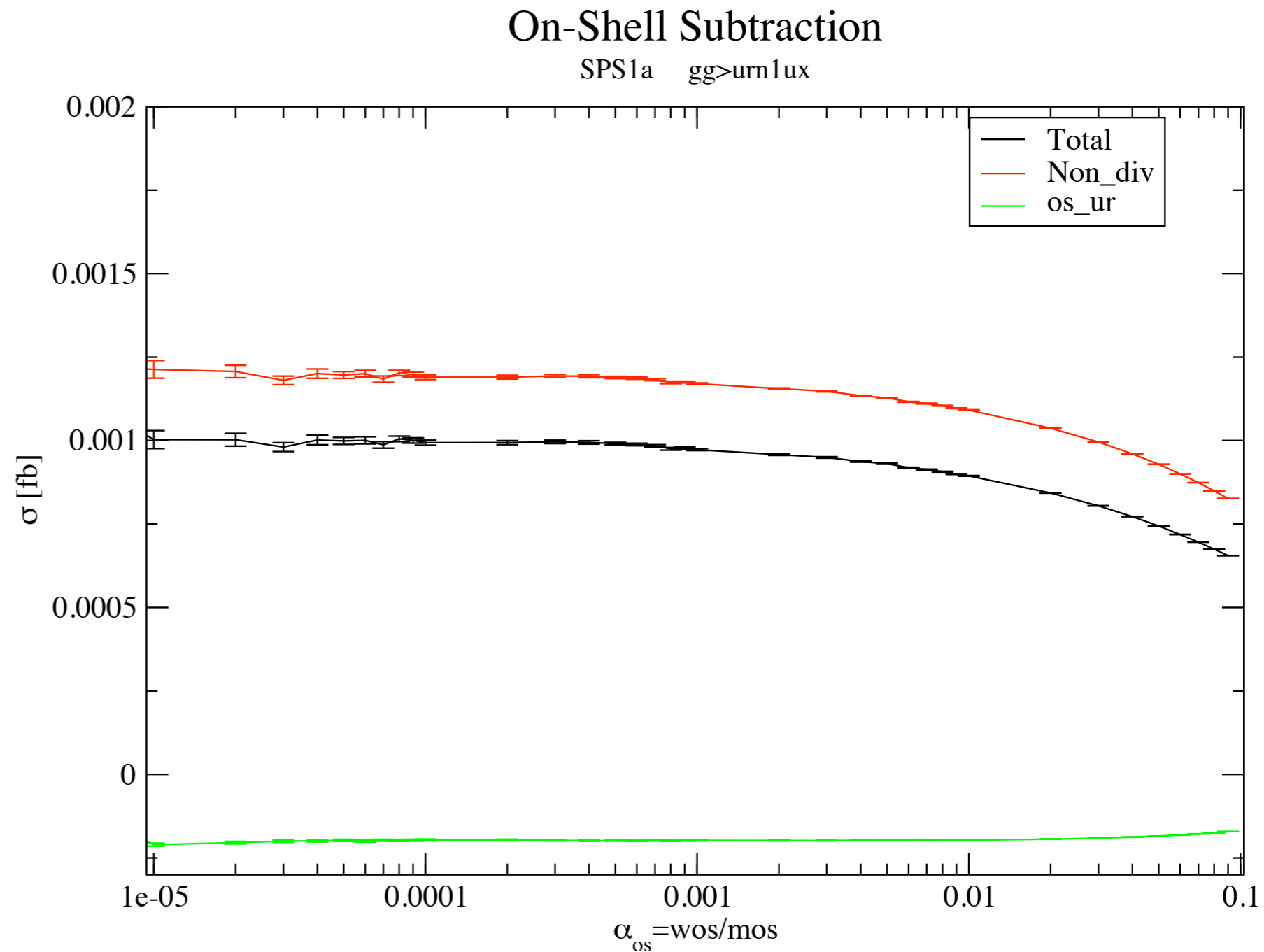
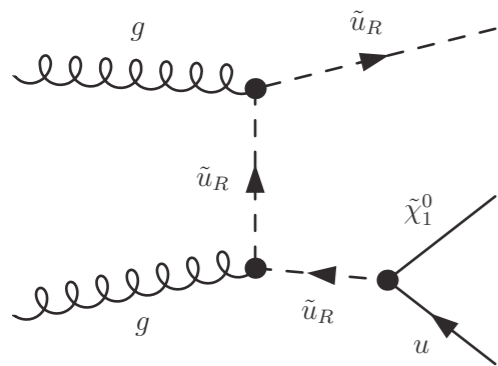
1 possible OS particle: **ur**



On-Shell Subtraction Method

 $gg \rightarrow u\bar{u}l\nu$

1 possible OS particle: $u\bar{u}$



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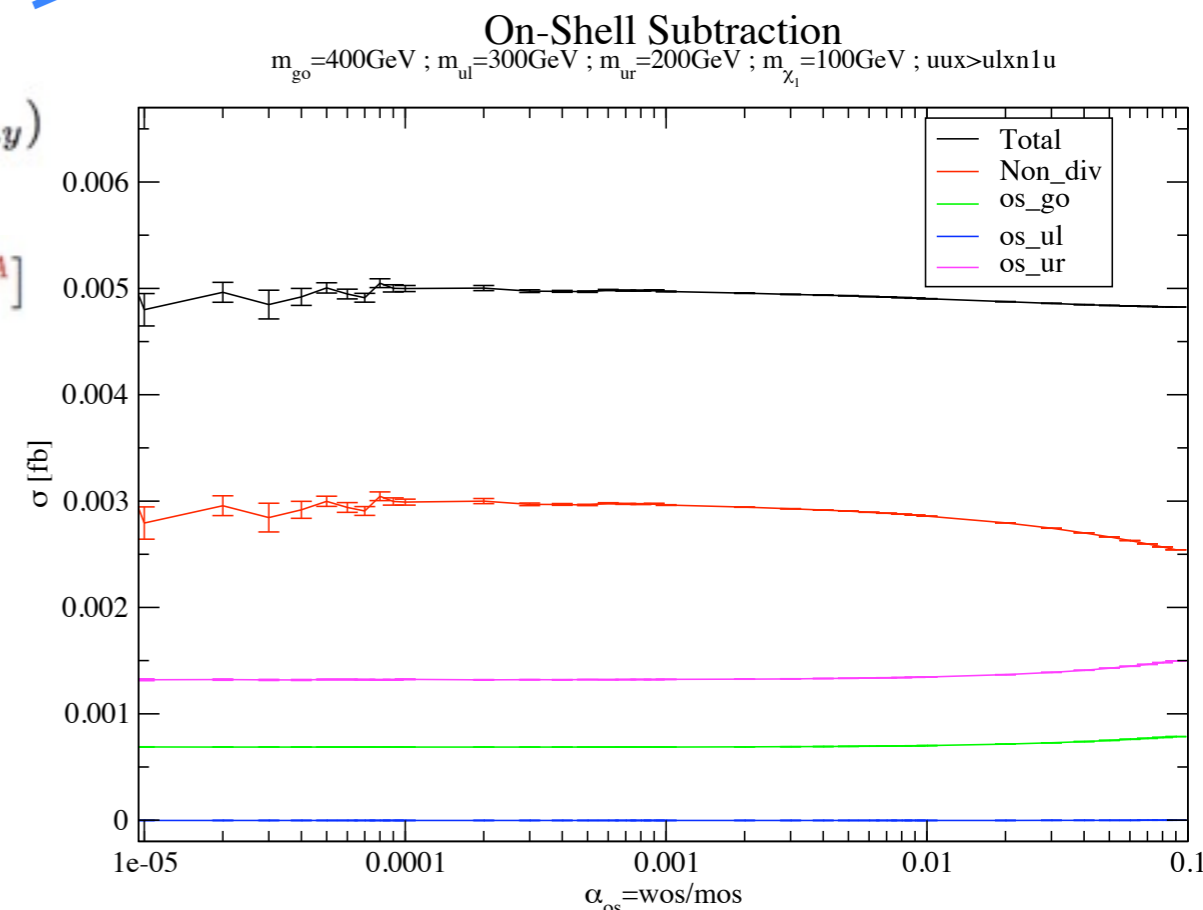
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$uux > ulxnlu$

3 possible OS particles: go, ul, ur

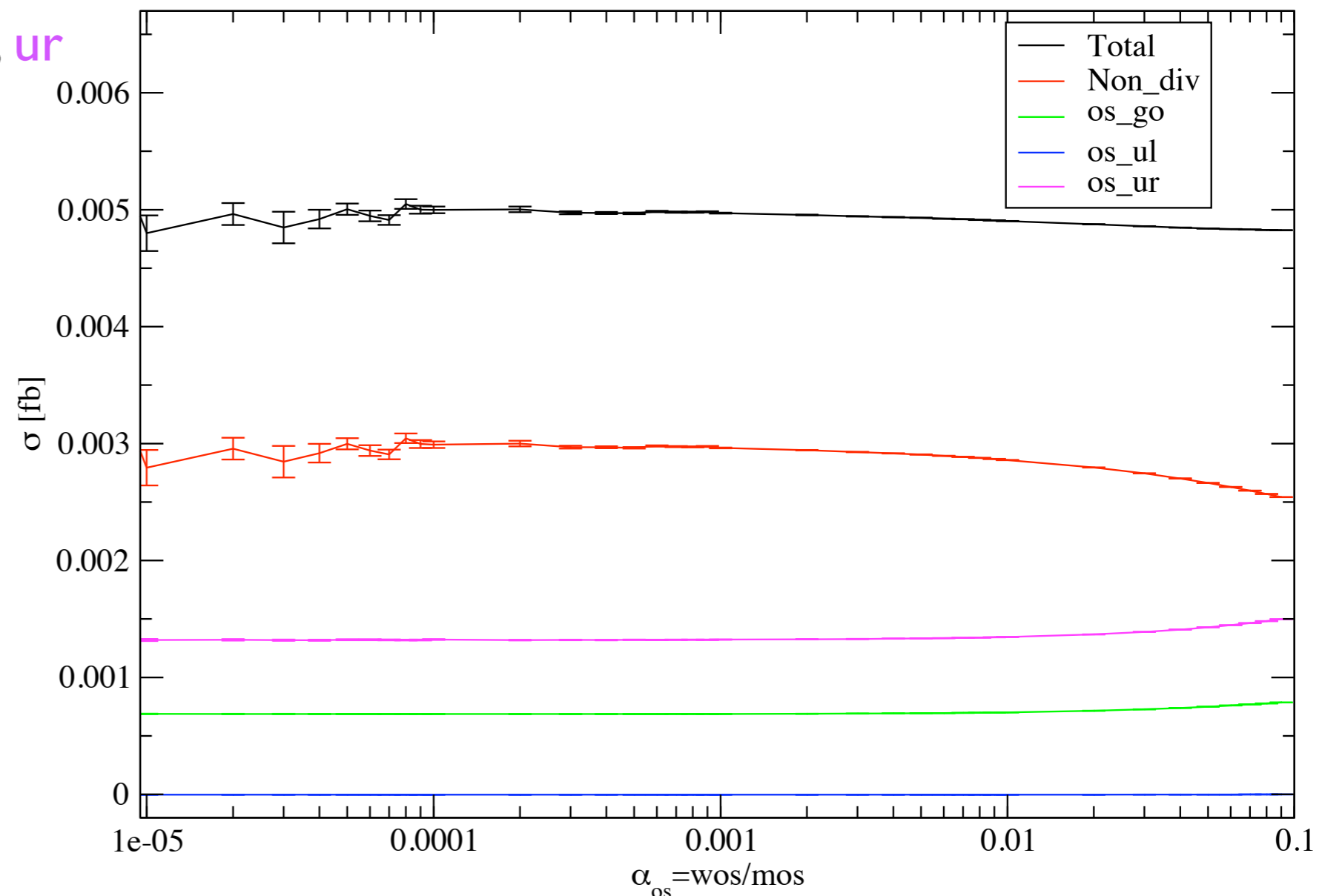


On-Shell Subtraction Method

 $uux > ulxn1u$

On-Shell Subtraction
 $m_{go}=400\text{GeV}$; $m_{ul}=300\text{GeV}$; $m_{ur}=200\text{GeV}$; $m_{\chi_1}=100\text{GeV}$; $uux > ulxn1u$

3 possible OS particles: go , ul , ur



Summary

- $\delta\sigma^{NLO} = \int_{n+1} \left[\left(d\sigma_{\epsilon=0}^{Real} - d\sigma_{\epsilon=0}^A - d\sigma_{\epsilon=0}^{os} \right) \right] + \int_n \left(d\sigma^C + \int_1 d\sigma^A + d\sigma^V \right)_{\epsilon=0}$
- Extension of the MadDipole package with the SUSY dipoles
 - Derivation and implementation of α parameter for SUSY (avoiding calculations far from the divergencies)
 - Checks: IR behaviour and α dependence
- Automation of the On-Shell Subtraction into MadGOLEM
 - Process independent approach
 - Checks: cancelation of div. and α_{os} dependence
- Cross checks against Prospino.
- Work in progress...

Thanks for your attention!

