Prospects for Precision m_{W} Measurements at Future $\mathrm{e}^+\mathrm{e}^-$ Colliders

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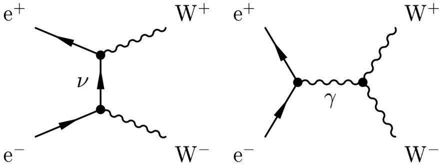
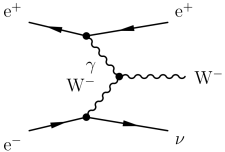


$\mathrm{W}^+\mathrm{W}^-$ production



Single W

Outline of Topics

Introduction

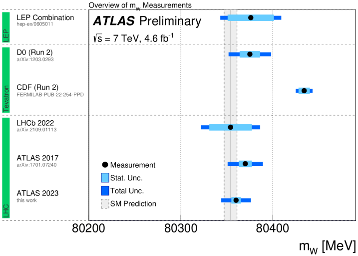
1. Comments on current m_{W} picture
2. Measuring m_{W} in $\mathrm{e^+e^-}$ collisions
3. LEP2 measurements summary

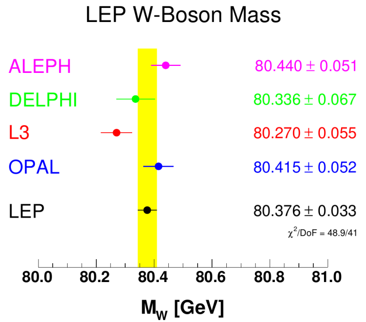
Future $\mathrm{e^+e^-}$ measurements

1. Cross-Sections
2. Single W
3. ILC and Run Plan
4. $\sqrt{s}$ and beamstrahlung
5. Threshold
6. Constrained Reconstruction
7. Leptonic Observables
8. Further Systematics
9. Conclusions

INTRODUCTION

What to think of m_{W} measurements?



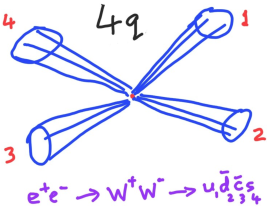


- The LEP results are based on 42 separate measurements with a **healthy** χ^2.
- LEP-combined (33 MeV), LHCb (32 MeV), D0 Run II (23 MeV), ATLAS 2023 (16 MeV) and CDF Run II (9.4 MeV) measurements have a $\chi^2/\mathrm{DoF} = 22.1/4$. p-value of **0.02%** for compatibility (neglecting correlations).
- So rather strong evidence that the ensemble of experimental results are **inconsistent with each other** independent of any SM prediction.
- The PDG procedure adds a scale factor to all measurements to parametrize our ignorance. Scale factor is 2.35. So new error on m_{W} of ≈ 17 MeV.
- May be difficult to measure the same thing in $p\bar{p}$, pp, and $\mathrm{e}^+\mathrm{e}^-$ collisions.

Strong motivation to measure m_{W} precisely with $\mathrm{e}^+\mathrm{e}^-$ collisions!

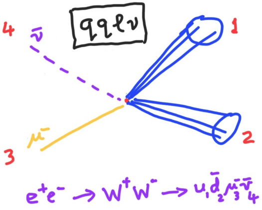
WW Topologies

fully hadronic $q\bar{q}q\bar{q}$



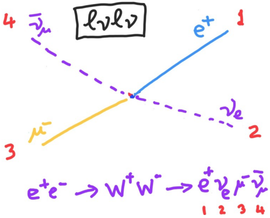
$B_h^2 = 45.4\%$

semi-leptonic $q\bar{q}\ell\nu_\ell$



$6B_\ell B_h = 43.9\%$

fully leptonic $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$



$9B_\ell^2 = 10.6\%$

- Here we take $\ell = e, \mu, \tau$. Events with τ leptons are of some use even for $m_{\rm W}$.
- 100% of the WW final states are potentially useful for $m_{\rm W}$ in $\rm e^+e^-$ collisions not just the 22% of the W final state used in hadron colliders.
- Much of the power of an $\rm e^+e^-$ collider is that one measures the **mass** of the W decay products either directly or by imposing kinematic constraints.

W Mass

m_W is an experimental challenge. Especially so for hadron colliders.

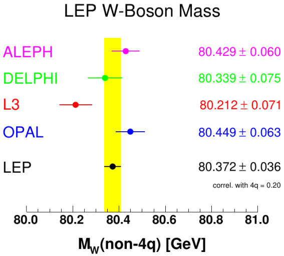
There are several promising approaches to measuring m_W at an $\mathrm{e^+e^-}$ collider:

1. Constrained Reconstruction Kinematically-constrained reconstruction of $\mathrm{W^+W^-}$ using constraints from **4-momentum conservation** and optionally mass-equality: the LEP2 work-horse. Primarily using $q\bar{q}\ell\nu_\ell$ events. Color reconnection disfavors use of $q\bar{q}q\bar{q}$ channel. Use E_b constraint for $q\bar{q}\tau\nu_\tau$.
2. Hadronic Mass Direct measurement of the **hadronic mass**. This can be applied particularly to single-W events decaying hadronically or to the hadronic system in semi-leptonic $\mathrm{W^+W^-}$ events (especially for $q\bar{q}\tau\nu_\tau$).
3. Lepton Endpoints The 2-body decay of each W leads to endpoints in the lepton (or jet) **energy** at $E_\ell = E_\mathrm{b}(1 \pm \beta)/2$ where β is the W velocity. These can be used to infer m_W. Can use for WW events with ≥ 1 prompt lepton.
4. Fully Leptonic Reconstruction **Pseudomass** method (Apply 5 constraints).
5. Threshold Scan Measurement of the $\mathrm{W^+W^-}$ cross-section near **threshold**. Uses all final states. Requires dedicated luminosity well below Higgs threshold and good control of background. ILC benefits from longitudinal polarization for background control. See also recent talk by P. Azzurri.

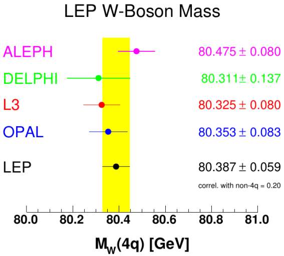
Mini Review of LEP2 m_W Results (arXiv:1302.3415)

Data-taking 1996–2000, with $\sqrt{s}$ =161–209 GeV

$q\bar{q}\ell\nu_\ell$

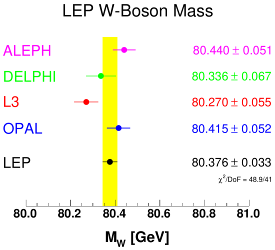


$q\bar{q}q\bar{q}$



Threshold Analysis	
Experiment	m_W[GeV]
ALEPH	80.20 ± 0.34
DELPHI	$80.45^{+0.45}_{-0.41}$
L3	$80.78^{+0.48}_{-0.42}$
OPAL	$80.40^{+0.46}_{-0.43}$

OPAL ($\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$): $80.41 \pm 0.41 \pm 0.13$ GeV

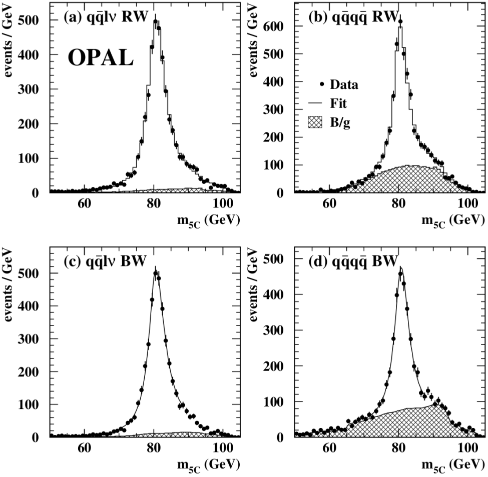


Constrained Reconstruction of m_{W} in WW events

$$P_s(m_{\mathrm{W}}, \Gamma_{\mathrm{W}}, m_{i,\mathrm{rec}}) = S(m_{\mathrm{W}}, \Gamma_{\mathrm{W}}, m_i, s') \otimes ISR(s', s) \otimes R(m_i, m_{i,\mathrm{rec}})$$

- Main LEP2 results were based on applying kinematic constraints to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$ events.
- Here 5C fit. $(\mathrm{E}, \vec{p}) = (\sqrt{s}, \vec{0})$ and $m_{W^+} = m_{W^-}$
- OPAL used a convolution fit (CV), a reweighting MC template technique (RW) and a Breit-Wigner fit (BW). All 3 applied separately to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$.
- CV fit is most powerful - uses per event resolution function.

hep-ex/0508060



LEP Combined m_W Systematics

Source	Systematic Uncertainty in MeV			
	on m_W			on Γ_W
	$\mathrm{q\bar{q}}\ell\nu_\ell$	$\mathrm{q\bar{q}q\bar{q}}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	–	35	8	27
Bose-Einstein Correlations	–	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

- $q\bar{q}q\bar{q}$ events benefit in fitted **mass resolution** from all 4 fermions being visible and detectable, but they also have **combinatorial ambiguities**.
- The color reconnection (CR) phenomenon (well established in other systems) is thought to be a severe limitation for using the $q\bar{q}q\bar{q}$ channel to progress on m_W at future $\mathrm{e^+e^-}$ colliders. LEP2 results use model with **no CR**.

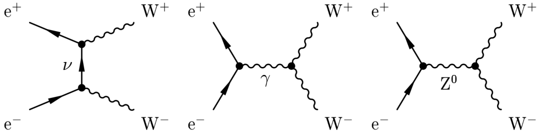
Future e^+e^- measurements of m_W

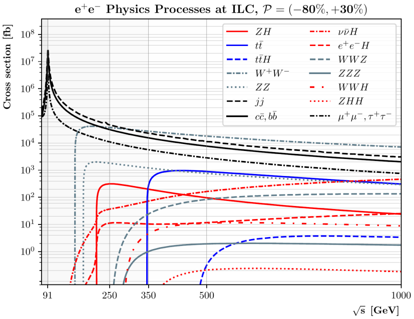
FUTURE e^+e^- MEASUREMENTS OF m_W

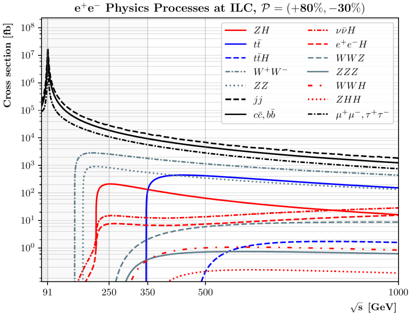
Examples will be mostly drawn from ILC.
Issues are mostly similar for other collider possibilities.
The main differences relevant for m_W are:

- availability of longitudinally polarized beam**s** and potential for higher energies at linear colliders
- potential to use resonant depolarization for beam energy calibration of non-colliding bunches at lower energies for sufficiently large circular colliders.

(Polarized) Cross-Sections




e^+e^- **Physics Processes at ILC,** $\mathcal{P} = (-80\%, +30\%)$

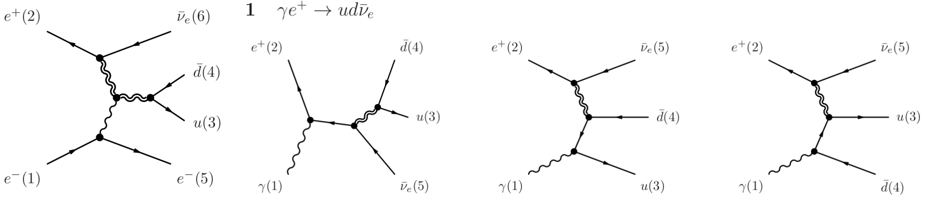

e^+e^- **Physics Processes at ILC,** $\mathcal{P} = (+80\%, -30\%)$

$$\sigma_{WW}\ (\sqrt{s} = 250\ \text{GeV}) = 37\ \text{pb} \qquad \sigma_{WW}\ (\sqrt{s} = 250\ \text{GeV}) = 3\ \text{pb}$$

For (-80%, +30%) expect 75M W bosons per ab^{-1} at $\sqrt{s} = 250$ GeV.

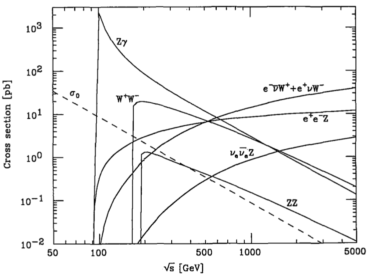
Single W production ($e^+e^- \to We\nu_e$)

4f final state, $ff'e^+\nu_e$ or $ff'e^-\bar{\nu}_e$ with $W \to ff'$. (CC20 diagrams for $W \to q\bar{q}$)



- At higher $\sqrt{s}$, opportunity to produce W and Z in t-channel processes where typically an electron has minimal p_T and is undetected
- Can use hadronic W decays to reconstruct the mass
- Could use hadronic Z decays with similar kinematics for control
- Some benefit from polarization

K. Hagiwara et al. / Weak boson production



ILC and Run Plan

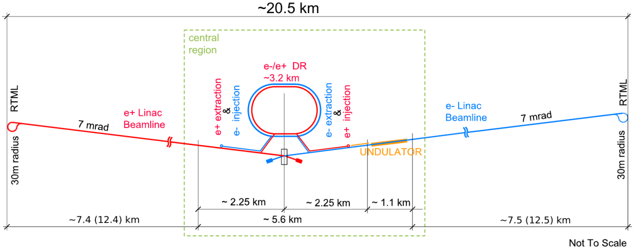
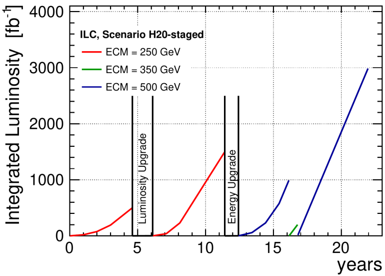


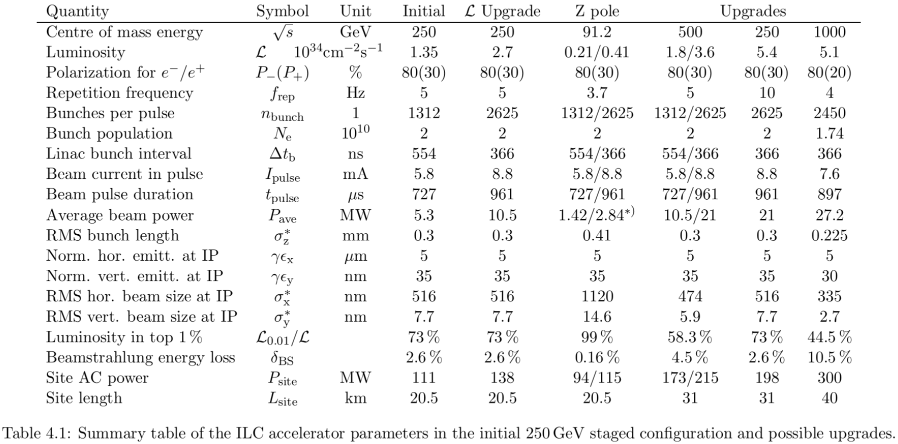
Figure 4.1: Schematic layout of the ILC in the 250 GeV staged configuration.



- (2.0, 0.2, 4.0) ab^{-1} at $\sqrt{s} = (250, 350, 500)$ GeV
- Polarized beams (4 colliders in 1)
- Room for dedicated runs at Z (0.1 ab^{-1}) and at WW threshold (0.5 ab^{-1}) prior to energy upgrade (arXiv:1506.07830)
- Can upgrade to higher energies

ILC Accelerator Parameters

See ILC paper for Snowmass for latest on ILC accelerator, detectors and physics



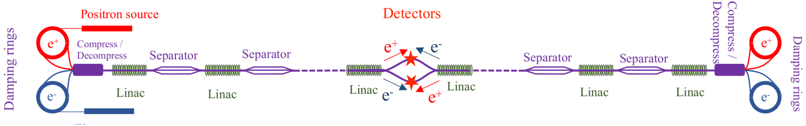
Quantity	Symbol	Unit	Initial	$\mathcal{L}$ Upgrade	Z pole	Upgrades		
Centre of mass energy	$\sqrt{s}$	GeV	250	250	91.2	500	250	1000
Luminosity	$\mathcal{L}$	$10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.35	2.7	0.21/0.41	1.8/3.6	5.4	5.1
Polarization for e^-/e^+	$P_-(P_+)$	%	80(30)	80(30)	80(30)	80(30)	80(30)	80(20)
Repetition frequency	f_{rep}	Hz	5	5	3.7	5	10	4
Bunches per pulse	n_{bunch}	1	1312	2625	1312/2625	1312/2625	2625	2450
Bunch population	N_{e}	10^{10}	2	2	2	2	2	1.74
Linac bunch interval	Δt_{b}	ns	554	366	554/366	554/366	366	366
Beam current in pulse	I_{pulse}	mA	5.8	8.8	5.8/8.8	5.8/8.8	8.8	7.6
Beam pulse duration	t_{pulse}	μs	727	961	727/961	727/961	961	897
Average beam power	P_{ave}	MW	5.3	10.5	$1.42/2.84^{*)}$	10.5/21	21	27.2
RMS bunch length	σ^*_{z}	mm	0.3	0.3	0.41	0.3	0.3	0.225
Norm. hor. emitt. at IP	$\gamma\epsilon_{\mathrm{x}}$	μm	5	5	5	5	5	5
Norm. vert. emitt. at IP	$\gamma\epsilon_{\mathrm{y}}$	nm	35	35	35	35	35	30
RMS hor. beam size at IP	σ^*_{x}	nm	516	516	1120	474	516	335
RMS vert. beam size at IP	σ^*_{y}	nm	7.7	7.7	14.6	5.9	7.7	2.7
Luminosity in top 1 %	$\mathcal{L}_{0.01}/\mathcal{L}$		73 %	73 %	99 %	58.3 %	73 %	44.5 %
Beamstrahlung energy loss	δ_{BS}		2.6 %	2.6 %	0.16 %	4.5 %	2.6 %	10.5 %
Site AC power	P_{site}	MW	111	138	94/115	173/215	198	300
Site length	L_{site}	km	20.5	20.5	20.5	31	31	40

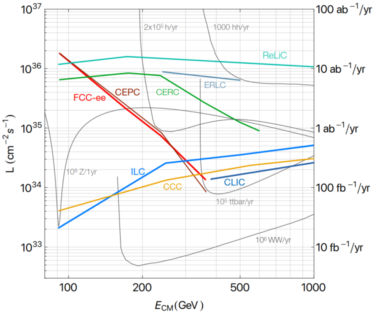
Table 4.1: Summary table of the ILC accelerator parameters in the initial 250 GeV staged configuration and possible upgrades.

Note: $\sqrt{s}$, luminosities, polarizations, BS energy loss, power needs. Potential to run at all center-of-mass energies from 91 to 1000 GeV.

The ultimate e^+e^- collider?

Energy recovery e^+e^- colliders have received attention. Latest Recycling Linear Collider conceptual idea (ReLiC) looks very intriguing!





Plot from EF-ITF (T. Roser et al)

- Scope for much higher lumi and/or power savings
- Really explore HH production
- Potential for high L at high energy
- With polarized beams and low beamstrahlung
- Concurrent data-taking with two detectors
- Linear collider pathway to more sustainable future collider

Any of these machines is revolutionary compared to SLC/LEP.

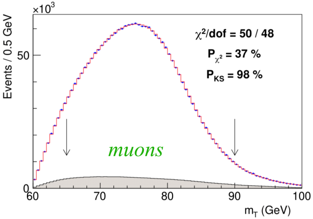
General Remarks

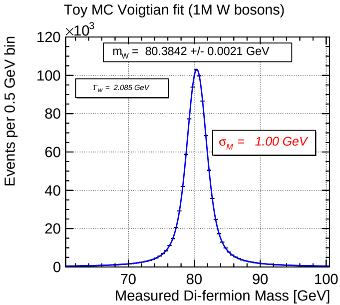
- It is not straightforward to project the performance for measurements that are probably **systematics** limited with ab^{-1} data sets.
- Future $\mathrm{e}^{+}\mathrm{e}^{-}$ collider data sets will benefit from much **better detectors** than at LEP2, the advantages of beam **polarization** (for linear colliders) and an experimental environment conducive to precision measurement (trigger, bunch structure, hermeticity (ILC), detector material).
- Measurements of W mass, were already quite complex at LEP2. Getting to a **realistic** estimate of the eventual performance at a future $\mathrm{e}^{+}\mathrm{e}^{-}$ collider is not trivial.
- We can make educated guesses and identify salient issues.
- In some simpler cases, like the polarized WW threshold scan (ILC) and purely leptonic observables, we can be relatively confident of the experimental projections including systematics.

Sensitivity to m_{W} at hadron and $\mathrm{e}^{+}\mathrm{e}^{-}$ colliders

Hadron colliders rely on the $m_T(\ell,\nu)$ and $p_T(\ell)$ in leptonic decays of singly produced W bosons. In contrast, $\mathrm{e}^{+}\mathrm{e}^{-}$ colliders can reconstruct the mass of the W boson decay products: measure directly $(m_{\mathrm{W}}, \Gamma_{\mathrm{W}})$ from the B-W lineshape.

CDF Run II
2.4M $W \rightarrow \mu\nu_{\mu}$ decays



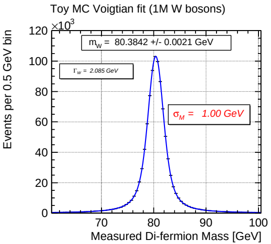


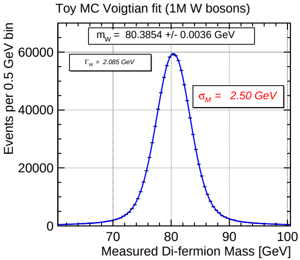
$m_{\mathrm{W}}(m_T) = 80\,446.1 \pm 9.2 \pm 7.3$ MeV

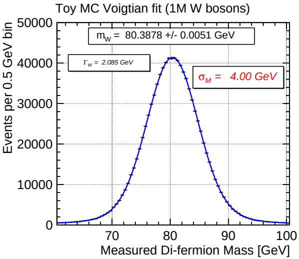
Fit with Breit-Wigner $\otimes$ Gaussian

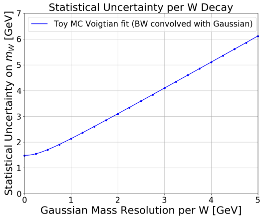
Ultimate sensitivity of a future $\mathrm{e}^{+}\mathrm{e}^{-}$ collider depends on the techniques, channels, mass resolution, and statistics. Could achieve the same m_{W} stat. sensitivity as this CDF plot with **only** 2.2% of the W decays for $\sigma_M = 1.0$ GeV (optimistic).

Intrinsic m_W Sensitivity from Lineshape







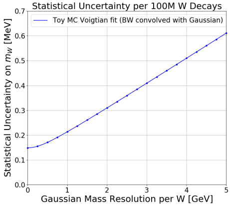


Basic sensitivity

$$\sigma_{m_{\mathrm{W}}} = \frac{f(\sigma_M, \Gamma_{\mathrm{W}})}{\sqrt{N_W}}$$

We will use both:

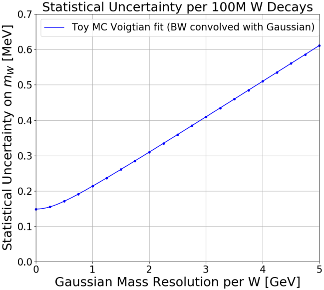
- Per decay m_{W} estimators (m_{ij}).
- Per event estimators: average mass, $\frac{1}{2}(m_{12} + m_{34})$ or $m_{5\mathrm{C}}$ $\{N_{WW}\}$

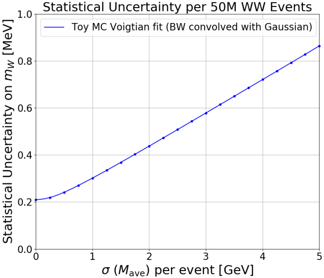


Scaled to ILC-like statistics

Decays or Events

To a very good approximation, the distribution of the averaged mass, follows the same Breit-Wigner distribution. So apply the same curve to WW events.





σ_M (GeV)	Δm_W (MeV)	$\Delta\Gamma_W^a$ (MeV)	$\Delta\Gamma_W^b$ (MeV)
1.0	0.21	0.41	0.63
2.5	0.35	0.63	1.0
4.0	0.50	0.89	1.6

- Fits with 100M W decays and 1, 2 or 3 parameters fitted ($m_{\rm W}$, $\Gamma_{\rm W}$, σ_M).
- Statistical uncertainties only. Note that individual W's and event-averaged masses will have very different resolutions (some excellent).

Center-of-mass Energy

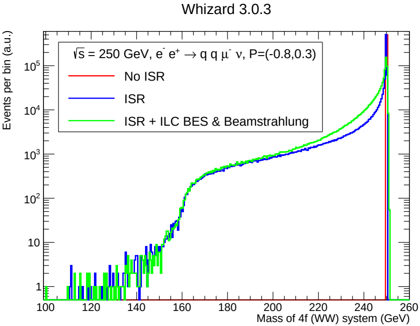
A crucial ingredient for most measurement approaches including threshold lineshape and constrained kinematic fits.
There are two main methods discussed.

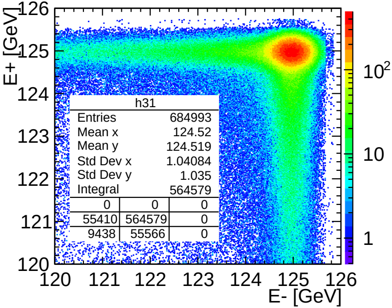
1. Beam energy calibration using resonant spin depolarization as was done at LEP1 (but not feasible at LEP2) for circular colliders. See arXiv:1909.12245 for FCC-ee.
   Beam energy spread means not feasible to use colliding bunches, and more difficult/impossible at higher energy. Need to transport orbital beam energy to collision center-of-mass energy.
2. Data-based center-of-mass energy estimation. Favored channels are $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ and Bhabhas. Premised on exquisite momentum calibration. See arXiv:2209.03281 and backup.

Beamstrahlung (most relevant to linear colliders)

Beam-beam interaction leads to energy loss (radiated photons) prior to collision
Two main issues (more important as $\sqrt{s}$ increases).

1. worsening of the validity of the kinematic constraints (similar to ISR).
2. presence of "overlay" particles from concurrent soft $\gamma\gamma$ and eγ collisions





h31	
Entries	684993
Mean x	124.52
Mean y	124.519
Std Dev x	1.04084
Std Dev y	1.035
Integral	564579

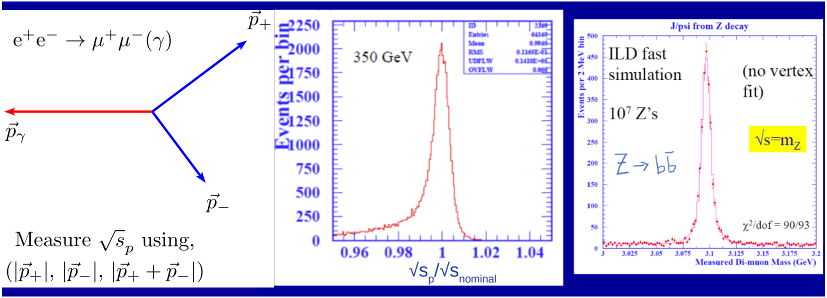
0	0	0
55410	564579	0
9438	55566	0

- Idealized: $< M >= 250.0$ GeV
- ISR only: $< M >= 242.9$ GeV
- ISR+BES+BS: $< M >= 240.3$ GeV

Need to use medium-angle Bhabhas and $\mathrm{e}^+\mathrm{e}^- \rightarrow \mu^+\mu^-$ to measure the luminosity spectrum (essentially the beam structure functions).

$\sqrt{s_p}$ Method for Absolute Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.

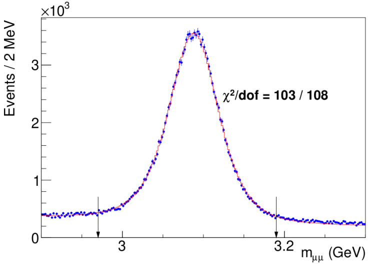


Tie detector *p*-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

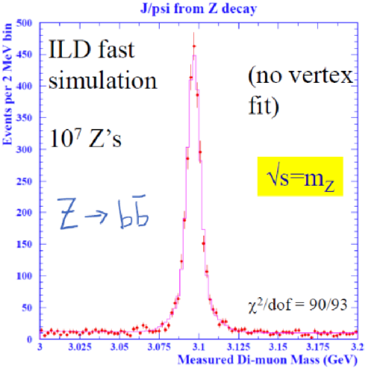
Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on *p*-scale per 1.2M $J/\psi \rightarrow \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc). (Links to more details in backup)
- relies on excellent modeling of QED effects (**ISR** and FSR)

Compare J/ψ Mass Resolution (CDF vs ILD for ILC)



Source	J/ψ (ppm)	Υ (ppm)	Correlation (%)
QED	1	1	100
Magnetic field non-uniformity	13	13	100
Ionizing material correction	11	8	100
Resolution model	10	1	100
Background model	7	6	0
COT alignment correction	4	8	0
Trigger efficiency	18	9	100
Fit range	2	1	100
$\Delta p/p$ step size	2	2	0
World-average mass value	4	27	0
Total systematic	29	34	16 ppm
Statistical NBC (BC)	2	13(10)	0
Total	29	36	16 ppm



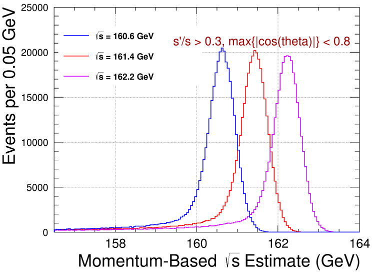
Much better mass resolution at ILC. Can measure momentum scale to 1 ppm stat. with 4.2B hadronic Z's. Systematics should be better than CDF (eg. no trigger). Previous "conservative" estimate of 10 ppm for ILC seems too conservative.

Center-of-Mass Energy near WW Threshold

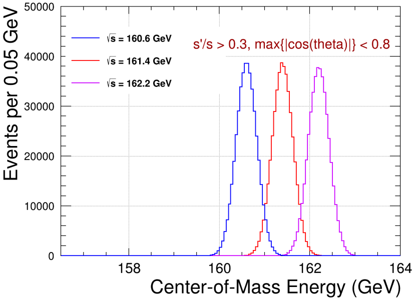
Study with $e^+e^- \to \mu^+\mu^-(\gamma)$

- Use KKMCee with energy spread of 0.203% (ILC-like)
- No beamstrahlung for now
- Tail from radiative effects
- 44.7% of events pass muon cuts
- Plots: $\Delta(\sqrt{s})/\sqrt{s} = 5.0$ ppm stat.

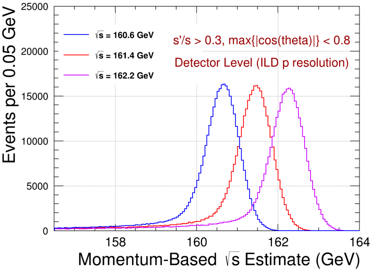
1M produced dimuons



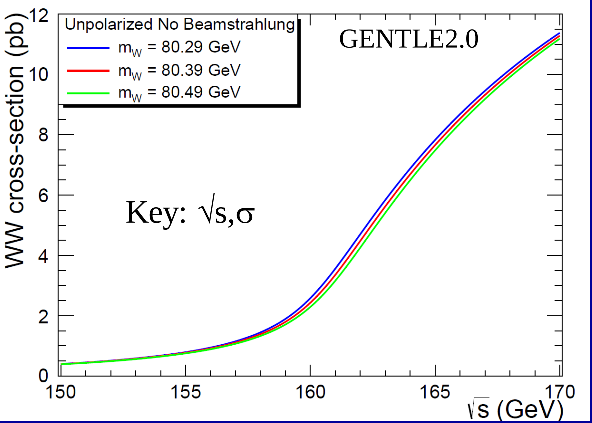
1M produced dimuons

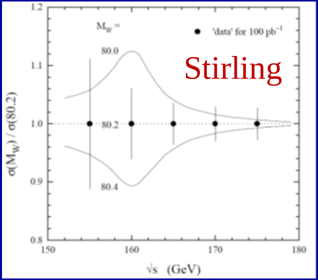


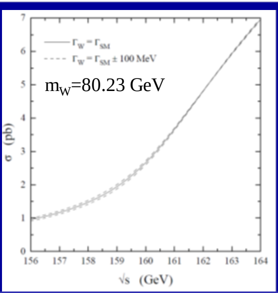
1M produced dimuons



m_W from cross-section close to threshold







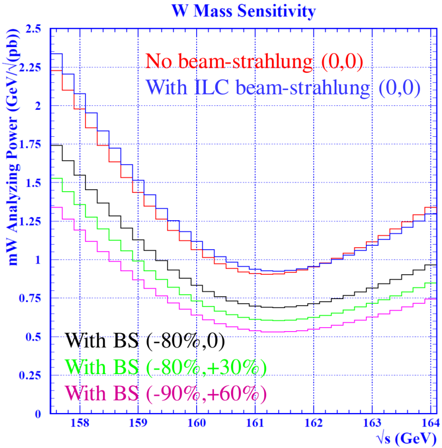
$\sigma_t \sim \beta$

$\sigma_s \sim \beta^3$

$$\Delta M_{\mathrm{sys}}^{\mathrm{bkgd}} = 470 \text{ MeV} \left[\frac{\Delta\sigma}{1 \text{ pb}}\right]$$

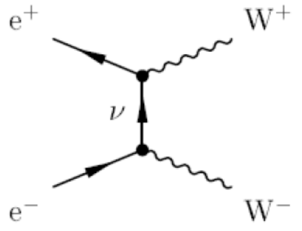
Threshold sensitivity to m_{W}

$$\Delta M_{\mathrm{stat}} = \left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1} \Delta\sigma = \left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1} \sqrt{\frac{\sigma}{\varepsilon p \mathcal{L}}} = \frac{K}{\sqrt{Q\mathcal{L}}} \text{ with } Q \equiv \varepsilon p$$



- Following Stirling, Nucl. Phys. B456 (1995) 3
- Plot shows $K = \sqrt{\sigma}\left|\frac{\mathrm{d}\sigma}{\mathrm{d}M}\right|^{-1}$
- For $\varepsilon, p = 100\%$, $\mathcal{L} = 0.5\ \mathrm{ab}^{-1}$ and (-80%, +30%) polarizations, optimum is $\Delta M_{\mathrm{stat}} = 0.85$ MeV
- Polarization of e^- and e^+ beams at ILC (with beamstrahlung) offers **much** better sensitivity per unit of integrated luminosity than the LEP-like unpolarized case appropriate for FCC-ee/CEPC.

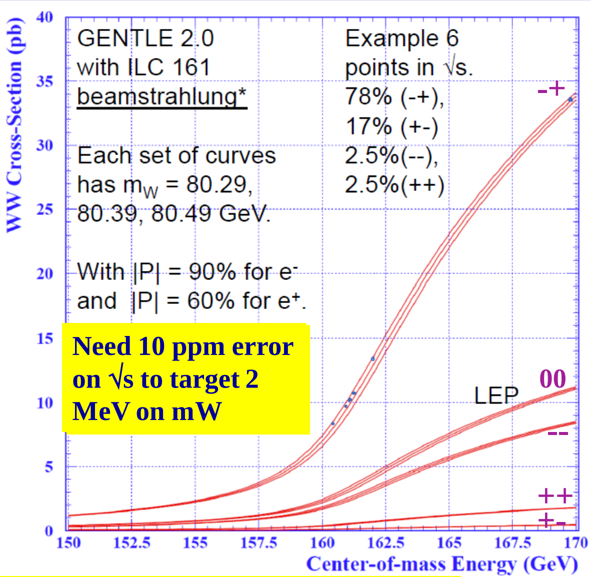
ILC Polarized Threshold Scan



Use (-+) helicity combination of e^- and e^+ to enhance WW.

Use (+-) helicity to suppress WW and measure background.

Use (--) and (++) to control polarization (also use 150 pb Z-like events)



GENTLE 2.0
with ILC 161
beamstrahlung*

Each set of curves has m_W = 80.29, 80.39, 80.49 GeV.

With |P| = 90% for e^- and |P| = 60% for e^+.

Example 6 points in √s.
78% (-+),
17% (+-)
2.5%(--),
2.5%(++)

Need 10 ppm error on √s to target 2 MeV on mW

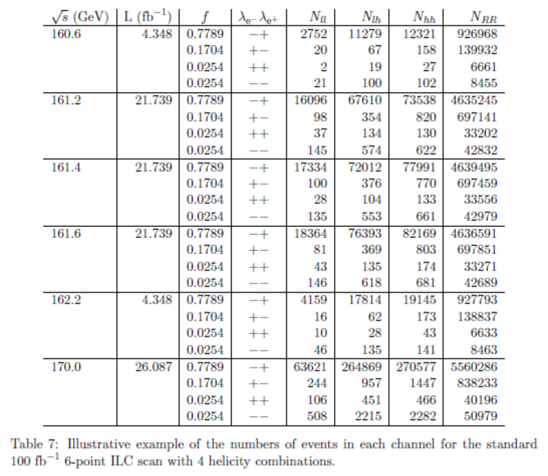
Experimentally very robust. Measure pol., bkg. in situ

ILC Polarized Scan Counting Experiment

Example: 6 point scan (index i), (90% e-, 60% e+ polarization) with -+, +-, ++ and - - helicity combinations (index k)

Count events in 3 WW candidate categories (lvlv, qqlv, qqqq – index j) with expectation μ_{ijk} and one Z-like category (radiative return and f fbar) with expectation ν_{ik}.

96 event counts



$\sqrt{s}$ (GeV)	L (fb^{-1})	f	$\lambda_{e^-}\lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	−+	2752	11279	12321	926968
		0.1704	+−	20	67	158	139932
		0.0254	++	2	19	27	6661
		0.0254	−−	21	100	102	8455
161.2	21.739	0.7789	−+	16096	67610	73538	4635245
		0.1704	+−	98	354	820	697141
		0.0254	++	37	134	130	33202
		0.0254	−−	145	574	622	42832
161.4	21.739	0.7789	−+	17334	72012	77991	4639495
		0.1704	+−	100	376	770	697459
		0.0254	++	28	104	133	33556
		0.0254	−−	135	553	661	42979
161.6	21.739	0.7789	−+	18364	76393	82169	4636591
		0.1704	+−	81	369	803	697851
		0.0254	++	43	135	174	33271
		0.0254	−−	146	618	681	42689
162.2	4.348	0.7789	−+	4159	17814	19145	927793
		0.1704	+−	16	62	173	138837
		0.0254	++	10	28	43	6633
		0.0254	−−	46	135	141	8463
170.0	26.087	0.7789	−+	63621	264869	270577	5560286
		0.1704	+−	244	957	1447	838233
		0.0254	++	106	451	466	40196
		0.0254	−−	508	2215	2282	50979

Table 7: Illustrative example of the numbers of events in each channel for the standard 100 fb^{-1} 6-point ILC scan with 4 helicity combinations.

Data could also be taken with other helicity combinations (00, -0,+0,0-,0+) if warranted. (eg. further checks of polarization model)

Results from updated ILC study (arXiv:1603.06016)

Fit essentially includes experimental systematics. Main one: **background** determination.

Fit parameter	Value	Error
m_W (GeV)	80.388	3.77×10^{-3}
f_l	1.0002	0.924×10^{-3}
ε (lvlv)	1.0004	0.969×10^{-3}
ε (qqlv)	0.99980	0.929×10^{-3}
ε (qqqq)	1.0000	0.942×10^{-3}
σ_B (lvlv) (fb)	10.28	0.92
σ_B (qqlv) (fb)	40.48	2.26
σ_B (qqqq) (fb)	196.37	3.62
A^B_{LR} (lvlv)	0.15637	0.0247
A^B_{LR} (qqlv)	0.29841	0.0119
A^B_{LR} (qqqq)	0.48012	4.72×10^{-3}
$\lvert P(e^-)\rvert$	0.89925	1.27×10^{-3}
$\lvert P(e^+)\rvert$	0.60077	9.41×10^{-4}
σ_{Z} (pb)	149.93	0.052
A^{Z}_{LR}	0.19062	2.89×10^{-4}

Example 6-point ILC scan with 100 fb^{-1}

Note 125 inv fb/yr now feasible!
(1908.08212, Yokoya, Kubo, Okogi).
2-point scan estimates

$\lvert P(e^-)\rvert$	$\lvert P(e^+)\rvert$	100 fb^{-1}	500 fb^{-1}
80 %	30 %	6.02	**2.88**
90 %	30 %	5.24	2.60
80 %	60 %	4.05	2.21
90 %	60 %	3.77	2.12

Total m_{W} experimental uncertainty (MeV)

High $\lvert P(e^+)\rvert$ very helpful!

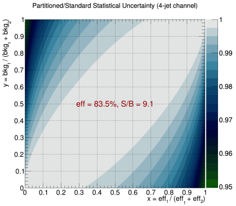
$$\Delta m_{\mathrm{W}}(\mathrm{MeV}) = 2.4\ (\mathrm{stat}) \oplus 3.1\ (\mathrm{syst}) \oplus 0.4\ (\sqrt{\mathrm{s}}) \oplus \mathrm{theory}$$

($\sqrt{s}$ uncertainty revised to 5 ppm given recent developments)

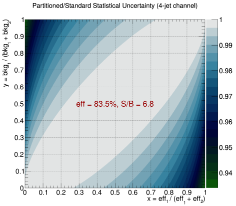
Improving Event Selection Performance

- The m_{W} statistical uncertainty is driven by, $Q \equiv \varepsilon p$, given that $\Delta m_{\mathrm{W}}(\mathrm{stat}) = K(\sqrt{s})/\sqrt{Q\,\mathcal{L}}$. There is scope to use multi-variate classifiers for event selection especially in the 4-jet channel.
- How much to gain? Assess by partitioning the event selection into a higher S/B and a lower S/B region.
- Current assumptions give purities =90.1, 87.2, 31.3% (LR-polarized, unpolarized, RL-polarized)

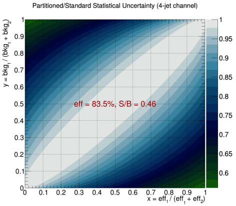
Plots show the 4-jet $\sqrt{Q_0/(Q_1+Q_2)}$ m_{W} stat. uncertainty improvement factor for the 3 cases for all potential (not necessarily feasible) partitionings. Guess 1–2% improvement may be possible this way. Further progress needs a looser selection.



(-90,+60)% Polarized



Unpolarized

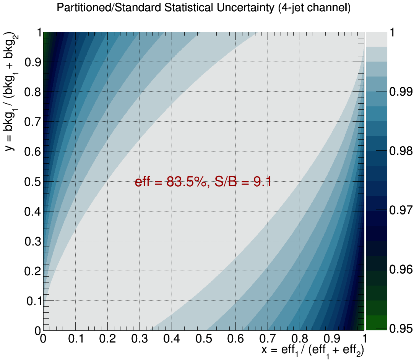


(+90,-60)% Polarized

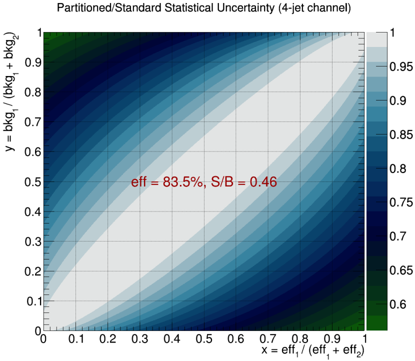
Zoomed plots

Plots show the 4-jet $\sqrt{Q_0/(Q_1+Q_2)}$ m_{W} stat. uncertainty improvement factor for the 2 extreme cases for all potential (not necessarily feasible) partitionings. Guess 1–2% improvement may be possible this way. Likely makes sense to move to a 10-channel ansatz, although need to then care about cross-talk. Further progress needs a complementary looser selection.

Note that the right plot applies to the small sensitivity to m_{W} of the RL helicities data-set.



(-90,+60)% Polarized



(+90,-60)% Polarized

New Luminosity Treatment for ILC

- Prior study assumed that the absolute luminosity can be measured to 0.1% using low angle Bhabhas (LABH). Main issues: theory and acceptance definition (including beam-beam).

Now,

- Model LABH with $\sigma_{161} = 12$ nb. Use for relative luminosity (per scan point).
- Use QED process $\mathrm{e^+e^-} \to \gamma\gamma$ for absolute luminosity. Currently assume $\sigma_{161} = 37.5$ pb (35 mrad), and systematic precision of 0.01%. (See CCMNP arXiv:1906.08056 for theoretical justifications). At WW threshold, the $\mathrm{e^+e^-} \to \gamma\gamma$ cross-section is about 10 times the unpolarized WW one.
- Model the event counting statistics for Bhabhas and $\mathrm{e^+e^-} \to \gamma\gamma$.

Key Complementary Features of $\mathrm{e^+e^-} \to \gamma\gamma$

- Lowest angle acceptance not so critical. $d\sigma/d\cos\theta \sim \frac{1+\cos^2\theta}{\sin^2\theta}$
- $A_{LR} = 0$. But only $-+$ and $+-$ beam helicities. ($\mathrm{e^+e^-}$ also has $--, ++$). So LR, RL cross-sections are 75 pb each!
- Aids in measuring polarization! Constraints on Bhabha backgrounds to $\gamma\gamma$.
- No beam-beam. Will need e/γ discrimination in LCAL at low angle.

Threshold m_W Systematics Tables (units are MeV)

100 fb^{-1} 6-point scan. (90%, 60%) beam polarizations. See 1603.06016.

Fit type	Uncertainty source	ΔM_W	ΔM_W (syst.)
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	A^B_{LR}	3.86	0.80
fixall	Statistical	2.43	
	Systematic		3.10
standard	Total Error	**3.94**	

New 500 fb^{-1} 6-point scan with (90%, 60%) beam polarizations. (for now, same sharings in $\sqrt{s}$ and helicity configurations).

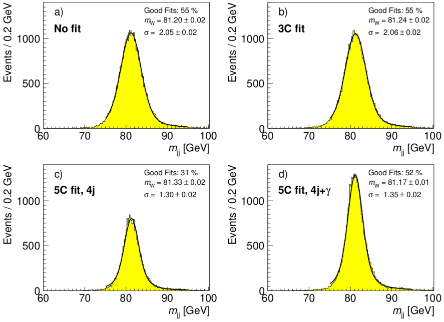
Fit type	Uncertainty source	ΔM_W	ΔM_W (syst.)
fixbkg	Background	1.47	1.33
fixpol	Polarization	1.92	0.51
fixeff	Efficiency	1.83	0.75
fixalum	New Luminosity	1.98	0.09
fixALRB	A^B_{LR}	1.81	0.81 (?)
fixall	Statistical	1.09	
fixallp	(Rel. lumi stats)	0.008	
Systematic		1.66	
standard	Total Error	**1.98**	

Constrained Fits

Some ideas and progress

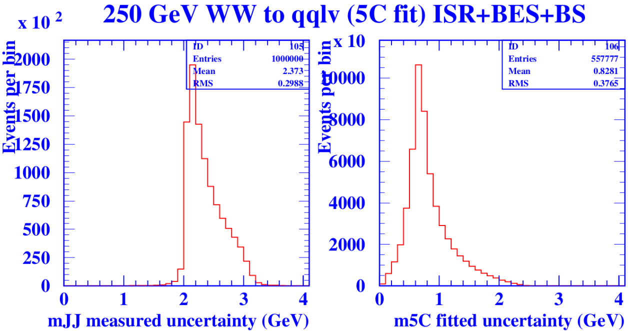
1. Photon radiation treatment in kinematic fits (M. Beckmann, B. List and J. List) arXiv:1006.0436 Applied to $q\bar{q}q\bar{q}$ at $\sqrt{s} = 500$ GeV.
2. Jet specific energy resolution studies (Wilson, IWLC 2010).
3. "ErrorFlow" studies: parametrizing jet uncertainties (A. Ebrahimi thesis)
4. Kinematic Fitting for Particle Flow Detectors at Future Higgs Factories (Y.Radkhorrami, J.List), arXiv:2111.14775
5. Kinematic reconstruction at FCC-ee* (M. Béguin thesis) - also near threshold.

- BLL - do simplified study of $q\bar{q}q\bar{q}$ reconstruction at $\sqrt{s} = 500$ GeV without "overlay".
- Shown is the average di-jet mass and its resolution (Voigtian fit).
- 4j+γ method adds an ISR photon as an additional "measured" object with large error
- Estimate 1.35 GeV mass resolution for 52% of events.



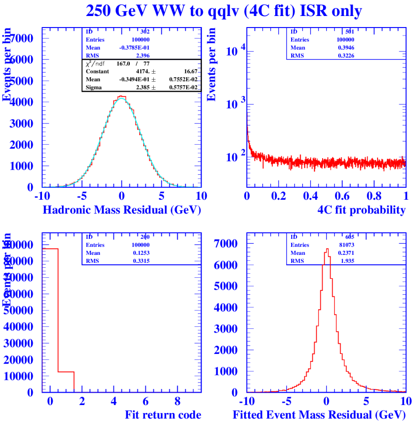
Toy study of constrained fitting for $q\bar{q}\ell\nu_\ell$ (ILC250)

- Looked at $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ events generated with Whizard 3.0.3.
- 3 configurations examined: no ISR, ISR only, ISR + ILC-BES&BS
- Used jet energy and angular resolution parametrization from D. Ward and W. Yan (from 2009). Neglected jet masses. $m_{\rm had}$ resolution ≈ 2.4 GeV.
- Used APLCON (V. Blobel) implementation
- Treat neutrino as unmeasured. Both 4C and 5C fits (1 dof & 2 dof).
- Method works perfectly with no ISR.
- Lots of room for improvement by using event-by-event fitted uncertainties.
- Issues with BLL photon method – may not work for $q\bar{q}\ell\nu_\ell$? (less constraints)

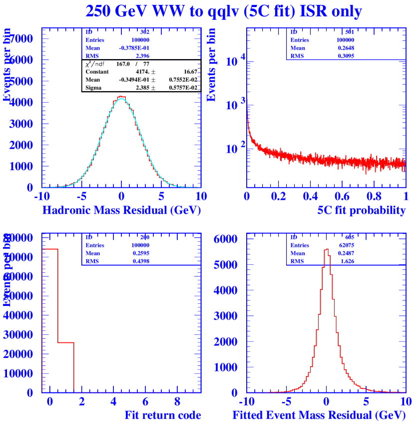


Fit $q\bar{q}\ell\nu_\ell$ ($\ell = e, \mu$) with ISR only (not even BES)

Successful fits defined as converging and having $p_{\mathrm{fit}} > 0.02$
(Residual $= m_{\mathrm{estimate}} - m_{\mathrm{generator}}$)



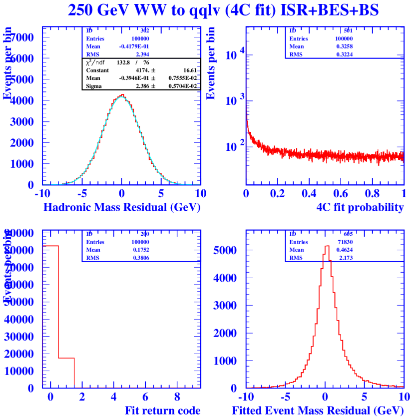
$\varepsilon_{\mathrm{fit}} = 81\%$, "$\sigma$"$=$1.94 GeV



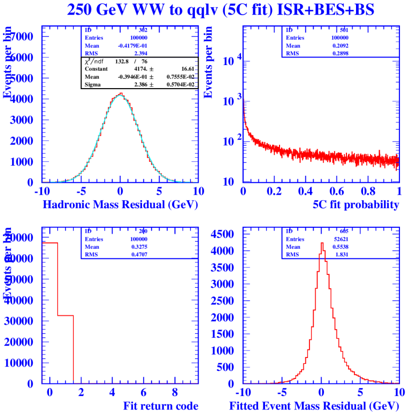
$\varepsilon_{\mathrm{fit}} = 62\%$, "$\sigma$"$=$1.63 GeV

Fit $q\bar{q}\ell\nu_\ell$ ($\ell = e, \mu$) with ILC beam effects

Successful fits defined as converging and having $p_{\mathrm{fit}} > 0.02$
(Residual $= m_{\mathrm{estimate}} - m_{\mathrm{generator}}$)



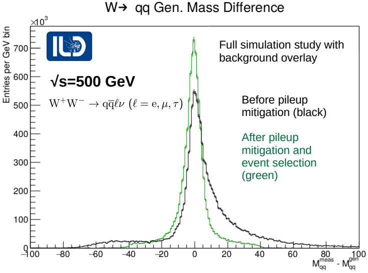
$\varepsilon_{\mathrm{fit}} = 72\%$, "$\sigma$"=2.17 GeV



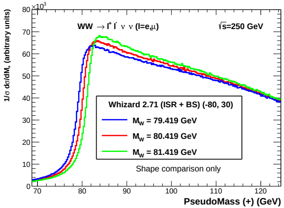
$\varepsilon_{\mathrm{fit}} = 55\%$, "$\sigma$"=1.83 GeV

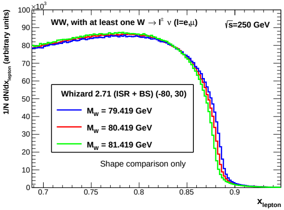
On average, the fit does not appear to improve much over the hadronic mass

m_{W}, Γ_{W} measurements concurrent with Higgs program



- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta m_{\mathrm{W}} = 2.4$ MeV for $1.6\ \mathrm{ab}^{-1}$ (-80%, +30%).
- Can be improved, but m_{had}-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set





Sensitivity to m_{W} with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta m_{\mathrm{W}} = 4.4$ MeV for 2 ab^{-1} (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell / E_{\mathrm{b}}$. Exptl. systematics small.

m_{W} Measurement Using Leptons

One complementary method for measuring $\mathrm{M_W}$ at LEP was the measurement by OPAL (hep-ex/020326) using $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$ events. Results were modest. Limited by the integrated luminosity of 0.67 fb^{-1} (unpolarized), and the poor momentum resolution ($\Delta p/p$). ILC will be much better for L, P and $\Delta p/p$. Disadvantages: higher $\sqrt{s}$ and beamstrahlung.

Method uses lepton $\vec{\mathrm{p}}$ measurement:

- The prompt (e, μ)-lepton energy spectrum in ee, $\mu\mu$, eμ, eτ, $\mu\tau$ events with endpoints at $E_\pm = \frac{1}{2} E_{\mathrm{b}}(1 \pm \beta)$. Can also apply to $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$.
- The positive pseudo-mass (M_+) solution in ee, $\mu\mu$, eμ events.

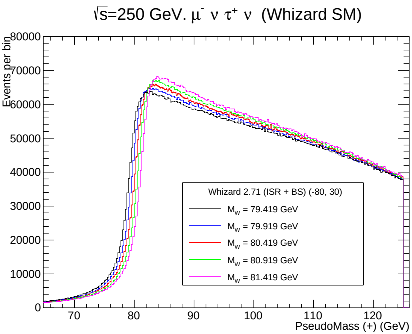
Latter assumes 4-momentum conservation, equal (l-ν) masses, and guesses that the neutrinos are in the same plane as the di-lepton.

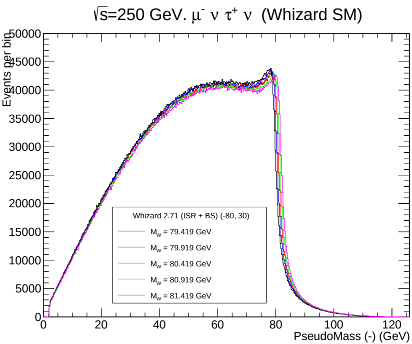
$$M_\pm^2 = \frac{2}{|\vec{p}_\ell + \vec{p}_{\ell'}|^2}\Big((P\ \vec{p}_{\ell'} - Q\ \vec{p}_\ell)\cdot(\vec{p}_\ell + \vec{p}_{\ell'}) \pm\sqrt{|\vec{p}_\ell \times \vec{p}_{\ell'}|^2[|\vec{p}_\ell + \vec{p}_{\ell'}|^2(E_{\mathrm{b}} - E_\ell)^2 - (P+Q)^2]}\Big), \tag{1}$$

where

$$P = E_{\mathrm{b}}E_\ell - E_\ell^2 + \frac{1}{2}m_\ell^2, \qquad Q = -E_{\mathrm{b}}E_{\ell'} - \vec{p}_\ell\cdot\vec{p}_{\ell'} + \frac{1}{2}m_{\ell'}^2.$$

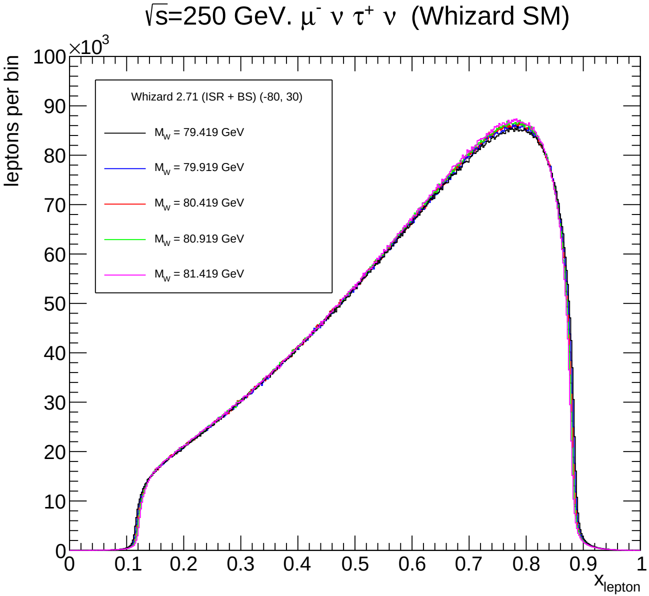
PseudoMasses (10M events per sample) (-80,+30)





- Study just uses changes in the shape. The total cross sections should be relatively insensitive to m_{W} well above threshold (depends on SM parameter scheme implementation though ....).
- Plots are at generator level (no detector smearing).
- Find that **both** pseudomasses are sensitive to m_{W}.

Lepton Endpoint (20M leptons per sample) (-80,+30)



Estimated m_{W} statistical uncertainties from leptons

Use 2.0 ab^{-1} with all beam polarizations (45%/45%/5%/5%) at generator level at $\sqrt{s} = 250$ GeV incl. beamstrahlung. Detector resolution neglected ($\sigma \ll \Gamma_{\mathrm{W}}$). Estimates based on ensemble test fits.

1. M_+: 1.50M prompt dilepton events = 8.8 MeV
2. M_-: 1.50M prompt dilepton events = 11.2 MeV
3. Pseudomasses combined: 1.50M prompt dilepton events = 6.9 MeV (assuming uncorrelated)
4. Endpoints: 4.50M leptons (from dileptons)= 11.0 MeV
5. Combined: Fully leptonic (M and endpoints) = 5.9 MeV (neglects possible correlation (+11% in OPAL case))
6. Semi-leptonic endpoints (12.6M leptons) = 6.6 MeV
7. Grand total = 4.4 MeV

Other Methods

Fully hadronic channel has huge statistical power, but thought plagued by color reconnection (CR) systematics.

Christiansen and Sjöstrand (arXiv:1506.09085) show that CR effects could be diagnosed using W mass measurements at various $\sqrt{s}$.

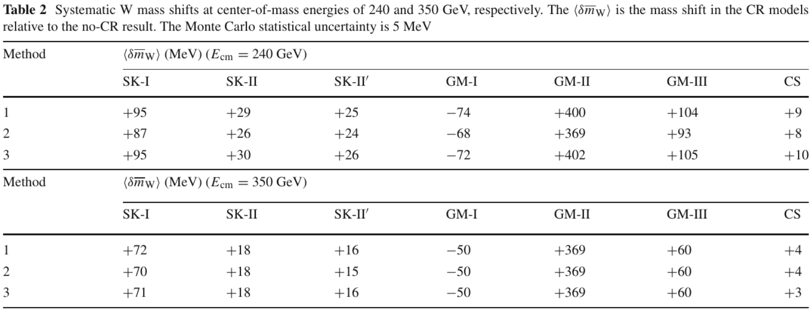


Table 2 Systematic W mass shifts at center-of-mass energies of 240 and 350 GeV, respectively. The $\langle \delta \overline{m}_W \rangle$ is the mass shift in the CR models relative to the no-CR result. The Monte Carlo statistical uncertainty is 5 MeV

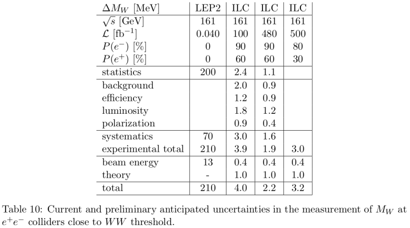
Method	$\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm} = 240$ GeV)						
	SK-I	SK-II	SK-II′	GM-I	GM-II	GM-III	CS
1	+95	+29	+25	−74	+400	+104	+9
2	+87	+26	+24	−68	+369	+93	+8
3	+95	+30	+26	−72	+402	+105	+10
Method	**$\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm} = 350$ GeV)**						
	SK-I	SK-II	SK-II′	GM-I	GM-II	GM-III	CS
1	+72	+18	+16	−50	+369	+60	+4
2	+70	+18	+15	−50	+369	+60	+4
3	+71	+18	+16	−50	+369	+60	+3

But this is not really at all well established and very model dependent.

Note that jet reconstruction in the 4q channel normally tries to reduce the potential size of such effects

Updated m_W Prospects Tables (for ILC)

1: Polarized threshold scan

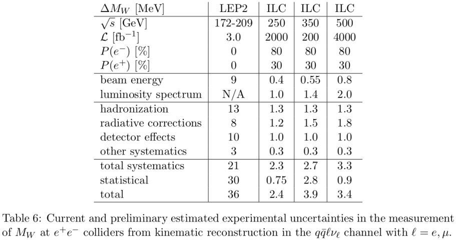


ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	161	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480	500
$P(e^-)$ [%]	0	90	90	80
$P(e^+)$ [%]	0	60	60	30
statistics	200	2.4	1.1	
background		2.0	0.9	
efficiency		1.2	0.9	
luminosity		1.8	1.2	
polarization		0.9	0.4	
systematics	70	3.0	1.6	
experimental total	210	3.9	1.9	3.0
beam energy	13	0.4	0.4	0.4
theory	-	1.0	1.0	1.0
total	210	4.0	2.2	3.2

Table 10: Current and preliminary anticipated uncertainties in the measurement of M_W at e^+e^- colliders close to WW threshold.

- Changes wrt Snowmass 2013
- Update with current ILC run plan integrated luminosities
- Halve beam energy uncertainty (10 ppm → 5 ppm)
- Include guessed theory uncertainty in threshold total

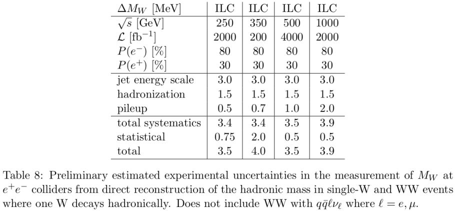
2: $q\bar{q}\ell\nu_\ell$



ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	172-209	250	350	500
$\mathcal{L}$ [fb^{-1}]	3.0	2000	200	4000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
beam energy	9	0.4	0.55	0.8
luminosity spectrum	N/A	1.0	1.4	2.0
hadronization	13	1.3	1.3	1.3
radiative corrections	8	1.2	1.5	1.8
detector effects	10	1.0	1.0	1.0
other systematics	3	0.3	0.3	0.3
total systematics	21	2.3	2.7	3.3
statistical	30	0.75	2.8	0.9
total	36	2.4	3.9	3.4

Table 6: Current and preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from kinematic reconstruction in the $q\bar{q}\ell\nu_\ell$ channel with $\ell = e, \mu$.

3: Hadronic mass



ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	2000	200	4000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	0.75	2.0	0.5	0.5
total	3.5	4.0	3.5	3.9

Table 8: Preliminary estimated experimental uncertainties in the measurement of M_W at e^+e^- colliders from direct reconstruction of the hadronic mass in single-W and WW events where one W decays hadronically. Does not include WW with $q\bar{q}\ell\nu_\ell$ where $\ell = e, \mu$.

Summary

- Several methods to measure the W mass with precision of a few MeV.
- Systematics are and will be complementary to some extent.
- Estimate overall experimental uncertainty of 2.0 MeV.
- This could be reduced further to about 1.5 MeV combined with dedicated 500 fb^{-1} run at threshold.
- Scope for complementary m_{W} measurements with similar precision from standard ILC running.
- Fully leptonic events statistical estimate is 5.9 MeV.
- Constrained reconstruction - very promising - but needs more detailed study.
- Experimental strategies for controlling systematics associated with $\sqrt{s}$, polarization, luminosity spectrum are worked out.
- Momentum scale is a key. Enabled by precision low material tracker. Can also open up a measurement of m_{Z}.
- An accelerator is needed. Let's make this happen!

Backup Slides

Recent studies related to $\sqrt{s}_p$ method

- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

Recently,

- Several talks on $\sqrt{s}_p$ and $\sqrt{s}$ issues. Latest ones, ILCX, ILC-WG3 and ILC-MDI
- Includes a more careful look at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$. Include crossing angle, full simulation and reconstruction with ILD, track error matrices, vertex fitting, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- Also a look at colliding beam-energy/interaction-vertex correlations and more of a focus on dL/d$\sqrt{s}$ issues.
- Prospects for Z lineshape with a polarized scan including energy systematics.

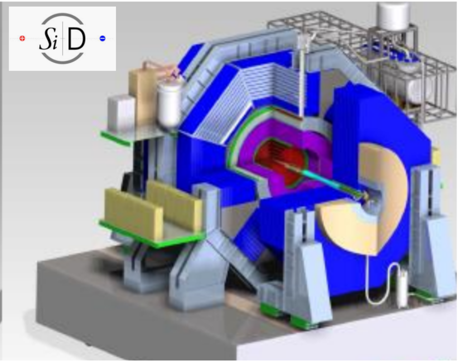
ILC Detectors

Modern detectors designed for ILC [5]

ILD = International Large Detector

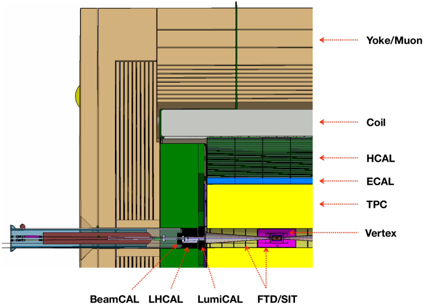
(also ILD Interim Design Report (IDR) [6])

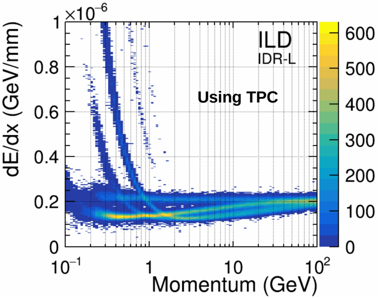
SiD = Silicon Detector



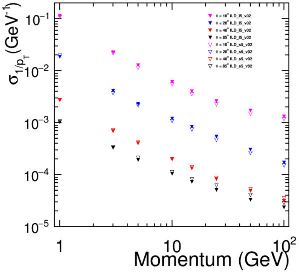
- B=3.5–5T. Particle-flow for hadronic jets. **Very hermetic**.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

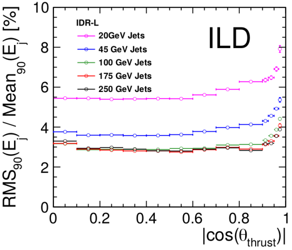
ILD Detector (See IDR)

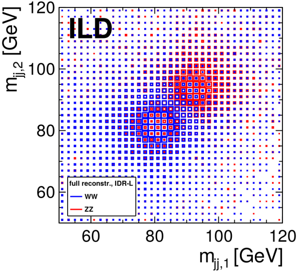




Momentum Resolution







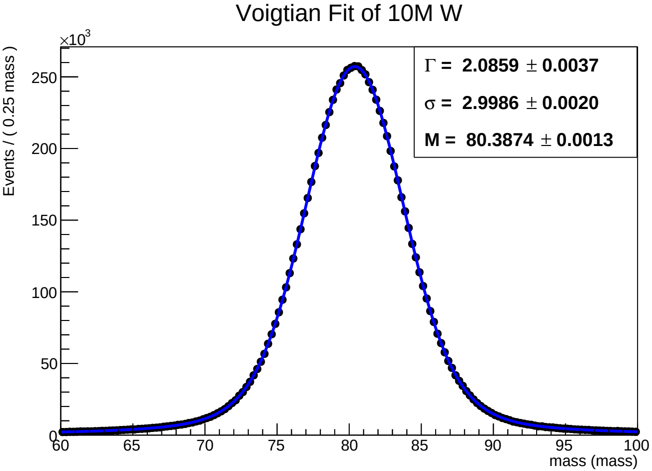
Fits to W Lineshape (M, Γ, σ_M)

Higgs factory machines like ILC likely systematics dominated for m_{W} and Γ_{W}. Statistical uncertainties for m_{W} and Γ_W for 10^7 W bosons.

σ_M (GeV)	Δm_{W} (MeV)	$\Delta\Gamma_W^a$ (MeV)	$\Delta\Gamma_W^b$ (MeV)
1.0	0.67	1.3	2.0
2.0	0.98	1.7	2.7
2.5	1.1	2.0	3.2
3.0	1.3	2.3	3.7
4.0	1.6	2.8	5.0

Estimated from a simple parametric fit of the Breit-Wigner lineshape convolved with a range of constant Gaussian experimental mass resolutions, σ_M. The m_{W} uncertainty is evaluated with a one parameter fit with the width and mass resolution fixed. The corresponding uncertainties on the Γ_W width are evaluated either with the mass resolution fixed and known perfectly from a 2-parameter fit (Γ_W^a), or more realistically, from a 3-parameter fit (Γ_W^b) that also fits for the mass resolution.

Toy MC Example. (Has $\chi^2/\text{ndf} = 152/157.$)



I had wrongly assumed that one needed to know σ very well to extract Γ, but this is not the case. Of course with no constraint on σ, the uncertainty on Γ is larger. In reality, σ varies from W to W. So for a similar approach to work, one needs well understood event by event errors. Use by categorizing events with varying quality levels.

Kinematic Reconstruction in Fully Leptonic Events

See Appendix B of Hagiwara et al., Nucl. Phys. B. 282 (1987) 253 for full production and decay 5-angle reconstruction in fully leptonic events ($\ell \nu_\ell \ell' \bar{\nu}_{\ell'}$) without taus as motivated by TGC analyses.

The technique applies energy and momentum conservation. One solves for the anti-neutrino 3-momentum, decomposed into its components in the dilepton plane, and out of it. Additional assumptions are:

- the energies of the two W's are equal to $E_{\rm b}$, so $\mathrm{m}(\mathrm{W}^+) = \mathrm{m}(\mathrm{W}^-)$.
- a specified value for $m_{\rm W}$

$$\vec{p}_{\bar{\nu}} = a\ \vec{p}_\ell + b\ \vec{p}_{\ell'} + c\ \vec{p}_\ell \times \vec{p}_{\ell'}$$

By specifying, $m_{\rm W}$, one can find a, b and c^2, so there are two solutions.

The alternative pseudomass technique, does not assume $m_{\rm W}$, but sets $c = 0$, and similarly has two solutions (a_+, b_+) and (a_-, b_-).

Hadronization Systematics

How does a W, Z, H, t decay hadronically?

Models like PYTHIA, HERWIG etc have been tuned extensively to data. Not expected to be a complete picture.
Inclusive measurements of **identified particle rates** and **momenta spectra** are an essential ingredient to describing hadronic decays of massive particles.
ILC could provide comprehensive measurements with up to 1000 times the published LEP statistics and with a much better detector with Z running.
High statistics with W events.

Why?

Measurements based on hadronic decays, such as hadronic mass, jet directions underlie much of what we do in energy frontier experiments.
Key component of understanding jet energy scales and resolution.
Important to also understand flavor dependence: u-jets, d-jets, s-jets, c-jets, b-jets, g-jets.

Momentum Scale Calibration (essential for $\sqrt{s}$)

Most obvious: use $J/\psi \to \mu^+\mu^-$. Event rate limited unless sizeable Z running.

Particle	n_{Zhad}	Decay	BR (%)	$n_{\mathrm{Zhad}} \cdot$ BR	Γ/M	PDG ($\Delta M/M$)
J/ψ	0.0052	$\mu^+\mu^-$	5.93	0.00031	3.0×10^{-5}	1.9×10^{-6}
$\mathrm{K^0_S}$	1.02	$\pi^+\pi^-$	69.2	0.71	1.5×10^{-14}	2.6×10^{-5}
Λ	0.39	$\pi^-\mathrm{p}$	63.9	0.25	2.2×10^{-15}	5.4×10^{-6}
$\mathrm{D^0}$	0.45	$\mathrm{K}^-\pi^+$	3.88	0.0175	8.6×10^{-13}	2.7×10^{-5}
K^+	2.05	various	-	-	1.1×10^{-16}	3.2×10^{-5}
π^+	17.0	$\mu^+\nu_\mu$	100	-	1.8×10^{-16}	2.5×10^{-6}

Candidate particles for momentum scale calibration and abundances in Z decay

Sensitivity of mass-measurement to p-scale (α) depends on daughter masses and decay

$$m_{12}^2 = m_1^2 + m_2^2 + 2p_1p_2\left[(\beta_1\beta_2)^{-1} - \cos\psi_{12}\right]$$

Particle	Decay	$<\alpha>$	max α	σ_M/M	$\Delta p/p$ (10 MZ)	$\Delta p/p$ (GZ)	PDG limit
J/ψ	$\mu^+\mu^-$	0.99	0.995	7.4×10^{-4}	13 ppm	1.3 ppm	1.9 ppm
$\mathrm{K^0_S}$	$\pi^+\pi^-$	0.55	0.685	1.7×10^{-3}	1.2 ppm	0.12 ppm	38 ppm
Λ	$\pi^-\mathrm{p}$	0.044	0.067	2.6×10^{-4}	3.7 ppm	0.37 ppm	80 ppm
$\mathrm{D^0}$	$\mathrm{K}^-\pi^+$	0.77	0.885	7.6×10^{-4}	2.4 ppm	0.24 ppm	30 ppm

Estimated momentum scale statistical errors ($p = 20$ GeV)

Use of J/ψ would decouple $\sqrt{s}$ determination from m_{Z} knowledge.

Opens up possibility of improved m_{Z} measurements.

Full Simulation + Kalman Filter

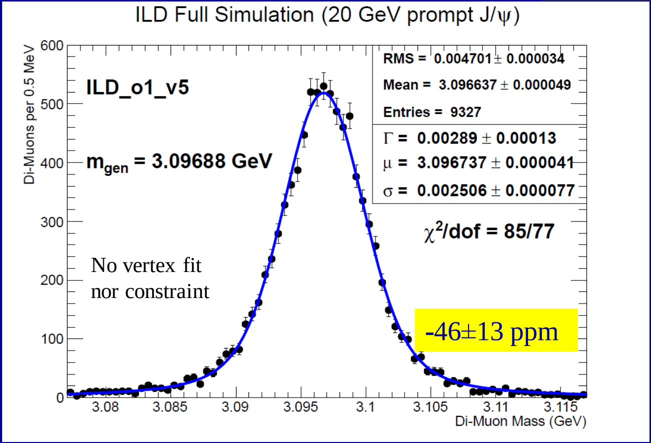
$\sqrt{s}=m_Z$

10k "single particle events"

Work in progress – likely need to pay attention to issues like energy loss model and FSR.

Preliminary statistical precision similar.

More realistic material, energy loss and multiple scattering.



m_{gen} = 3.09688 GeV

χ^2/dof = 85/77

No vertex fit nor constraint

-46±13 ppm

Empirical Voigtian fit.

Need consistent material model in simulation AND reconstruction

m_W Prospects

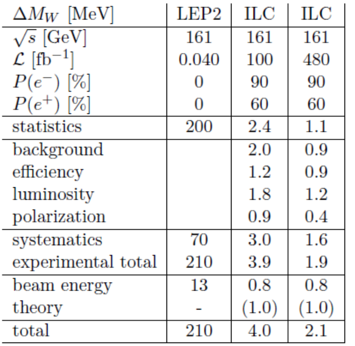
1. Polarized Threshold Scan
2. Kinematic Reconstruction
3. Hadronic Mass

Method 1: Statistics limited.

Method 2: With up to 1000 the LEP statistics and much better detectors. Can target factor of 10 reduction in systematics.

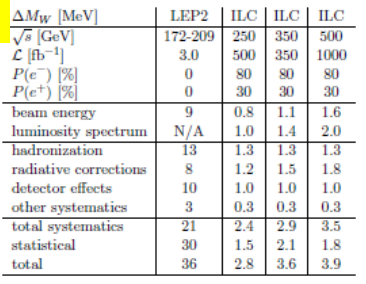
Method 3: Depends on di-jet mass scale. Plenty Z's for 3 MeV.

1



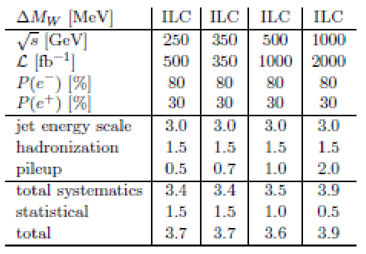
ΔM_W [MeV]	LEP2	ILC	ILC
$\sqrt{s}$ [GeV]	161	161	161
$\mathcal{L}$ [fb^{-1}]	0.040	100	480
$P(e^-)$ [%]	0	90	90
$P(e^+)$ [%]	0	60	60
statistics	200	2.4	1.1
background		2.0	0.9
efficiency		1.2	0.9
luminosity		1.8	1.2
polarization		0.9	0.4
systematics	70	3.0	1.6
experimental total	210	3.9	1.9
beam energy	13	0.8	0.8
theory	-	(1.0)	(1.0)
total	210	4.0	2.1

2



ΔM_W [MeV]	LEP2	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	172-209	250	350	500
$\mathcal{L}$ [fb^{-1}]	3.0	500	350	1000
$P(e^-)$ [%]	0	80	80	80
$P(e^+)$ [%]	0	30	30	30
beam energy	9	0.8	1.1	1.6
luminosity spectrum	N/A	1.0	1.4	2.0
hadronization	13	1.3	1.3	1.3
radiative corrections	8	1.2	1.5	1.8
detector effects	10	1.0	1.0	1.0
other systematics	3	0.3	0.3	0.3
total systematics	21	2.4	2.9	3.5
statistical	30	1.5	2.1	1.8
total	36	2.8	3.6	3.9

3

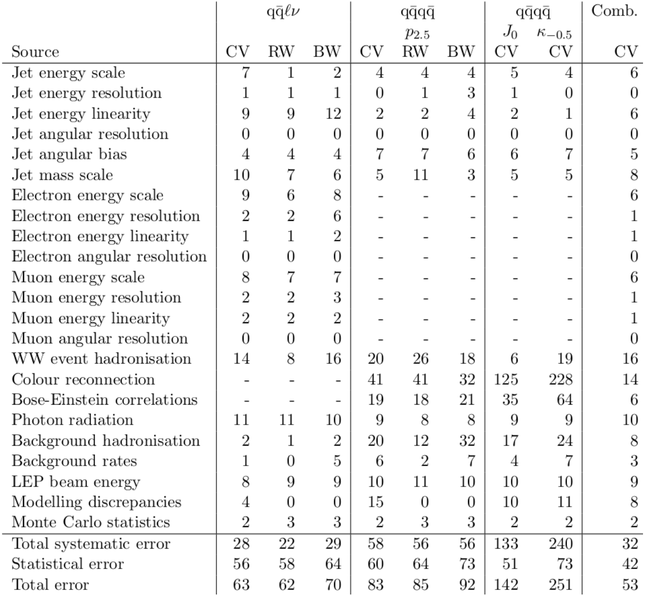


ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

See Snowmass document for more details

Bottom-line: 3 different methods with prospects to measure mW with error < 5 MeV

OPAL m_W Systematics



	$q\bar{q}\ell\nu$			$q\bar{q}q\bar{q}$			$q\bar{q}q\bar{q}$		Comb.
					$p_{2.5}$		J_0	$\kappa_{-0.5}$	
Source	CV	RW	BW	CV	RW	BW	CV	CV	CV
Jet energy scale	7	1	2	4	4	4	5	4	6
Jet energy resolution	1	1	1	0	1	3	1	0	0
Jet energy linearity	9	9	12	2	2	4	2	1	6
Jet angular resolution	0	0	0	0	0	0	0	0	0
Jet angular bias	4	4	4	7	7	6	6	7	5
Jet mass scale	10	7	6	5	11	3	5	5	8
Electron energy scale	9	6	8	-	-	-	-	-	6
Electron energy resolution	2	2	6	-	-	-	-	-	1
Electron energy linearity	1	1	2	-	-	-	-	-	1
Electron angular resolution	0	0	0	-	-	-	-	-	0
Muon energy scale	8	7	7	-	-	-	-	-	6
Muon energy resolution	2	2	3	-	-	-	-	-	1
Muon energy linearity	2	2	2	-	-	-	-	-	1
Muon angular resolution	0	0	0	-	-	-	-	-	0
WW event hadronisation	14	8	16	20	26	18	6	19	16
Colour reconnection	-	-	-	41	41	32	125	228	14
Bose-Einstein correlations	-	-	-	19	18	21	35	64	6
Photon radiation	11	11	10	9	8	8	9	9	10
Background hadronisation	2	1	2	20	12	32	17	24	8
Background rates	1	0	5	6	2	7	4	7	3
LEP beam energy	8	9	9	10	11	10	10	10	9
Modelling discrepancies	4	0	0	15	0	0	10	11	8
Monte Carlo statistics	2	3	3	2	3	3	2	2	2
Total systematic error	28	22	29	58	56	56	133	240	32
Statistical error	56	58	64	60	64	73	51	73	42
Total error	63	62	70	83	85	92	142	251	53

Fit the Event Counts to Model Expectations

$$x \equiv |P(\mathrm{e}^-)|, \;\; y \equiv |P(\mathrm{e}^+)|$$

Event count expectations:

$$\mu_{ijk} = \left(f_S^k(x, y, A_{LR}^{WW})\, \sigma_i(m_W, \alpha_S)\, \varepsilon_j B_j + g_B^k(x, y, A_{LR}^B)\, \sigma_B^j \right) f_l L_{ik}$$

$$\nu_{ik} = g_Z^k(x, y, A_{LR}^Z)\, \sigma_Z^i\; f_l L_{ik}$$

Signal, background, and Z-control sample spin factors:

$$f_S^{-+}(x, y, A) = 1 + xy + A(x + y)$$

$$f_S^{+-}(x, y, A) = 1 + xy - A(x + y)$$

$$f_S^{++}(x, y, A) = 1 - xy - A(x - y)$$

$$f_S^{--}(x, y, A) = 1 - xy + A(x - y)$$

$$g_{B,Z}^{-+}(x, y, A) = 1 + xy + A(x + y)$$

$$g_{B,Z}^{+-}(x, y, A) = 1 + xy - A(x + y)$$

$$g_{B,Z}^{++}(x, y, A) = 1 - xy - A(x - y)$$

$$g_{B,Z}^{--}(x, y, A) = 1 - xy + A(x - y)$$

Set A=0.99 for WW (estimate of 0.992 (Wopper), 0.988 (Racoon))

References I

[1] K. Fujii et al., [LCC Physics Working Group],
ILC Study Questions for Snowmass 2021, arXiv:2007.03650

[2] K. Fujii et al. [LCC Physics Working Group],
Tests of the Standard Model at the International Linear Collider
arXiv:1908.11299

[3] T. Behnke et al.
The International Linear Collider Technical Design Report - Volume 1:
Executive Summary, arXiv:1306.6327

[4] P. Bambade et al.
The International Linear Collider: A Global Project, arXiv:1903.01629

[5] T. Behnke et al.
The International Linear Collider Technical Design Report - Volume 4:
Detectors, arXiv:1306.6329

[6] H. Abramowicz et al. [ILD Concept Group],
International Large Detector: Interim Design Report, arXiv:2003.01116

References II

[7] G. W. Wilson, Precision Electroweak Measurements at a Future e^+e^- Linear Collider, PoS **ICHEP2016**, 688 (2016)

[8] K. Yokoya, K. Kubo and T. Okugi,
Operation of ILC250 at the Z-pole, arXiv:1908.08212

[9] K. Fujii et al. [LCC Physics Working Group],
The role of positron polarization for the initial 250 GeV stage of the International Linear Collider, arXiv:1801.02840

[10] G. W. Wilson,
Updated Study of a Precision Measurement of the W Mass from a Threshold Scan Using Polarized e^- and e^+ at ILC, arXiv:1603.06016

[11] R. Hawkings and K. Monig,
Electroweak and CP violation physics at a linear collider Z factory, Eur. Phys. J. direct **1**, 1 (2000)

References III

[12] A. B. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. W. Grunewald, K. Monig, S. Riemann and T. Riemann,
ZFITTER: A semi-analytical program for fermion pair production in e^+e^- annihilation from version 6.21 to version 6.42, Comput. Phys. Commun. **174**, 728-758 (2006)

[13] K. Fujii et al. [LCC Physics Working Group],
"Physics Case for the 250 GeV Stage of the International Linear Collider," arXiv:1710.07621

[14] A. Babich, P. Osland, A. Pankov and N. Paver,
New physics signatures at a linear collider: Model independent analysis from 'conventional' polarized observables, Phys. Lett. B **518**, 128-136 (2001)