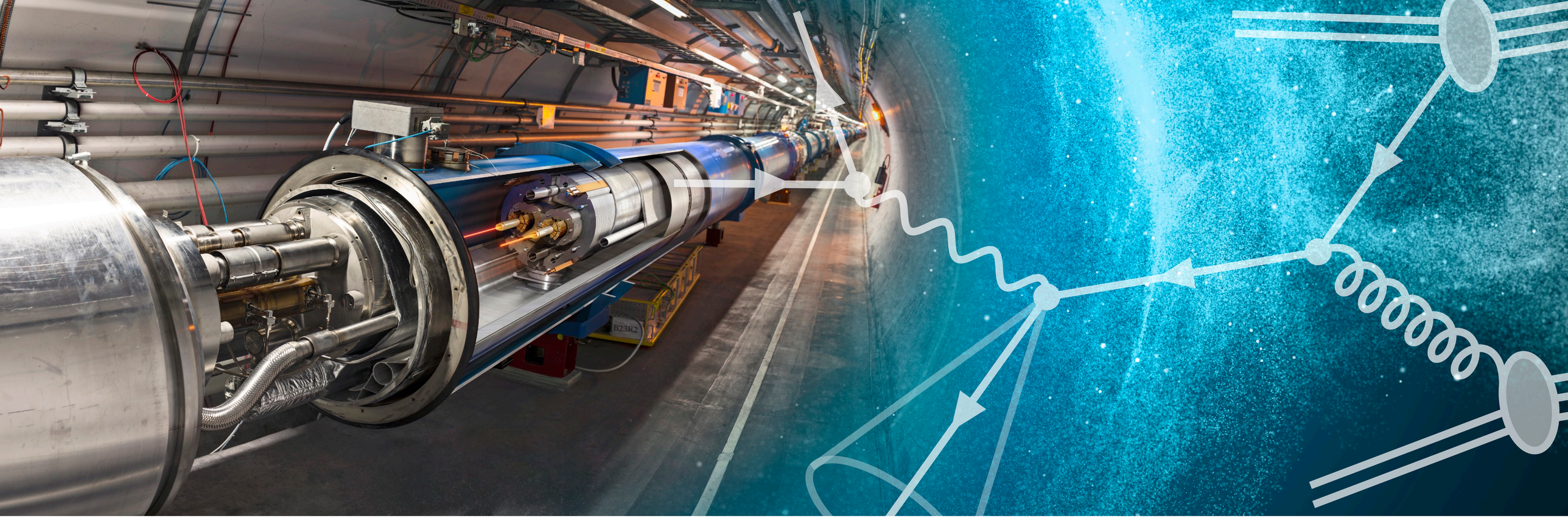


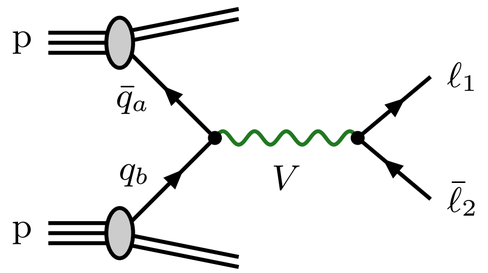
PRECISION QCD PREDICTIONS FOR THE DRELL-YAN PROCESS

NNLOJET ⊕ T-Z. Yang, H.X. Zhu ⊕ RadISH

Alexander Huss

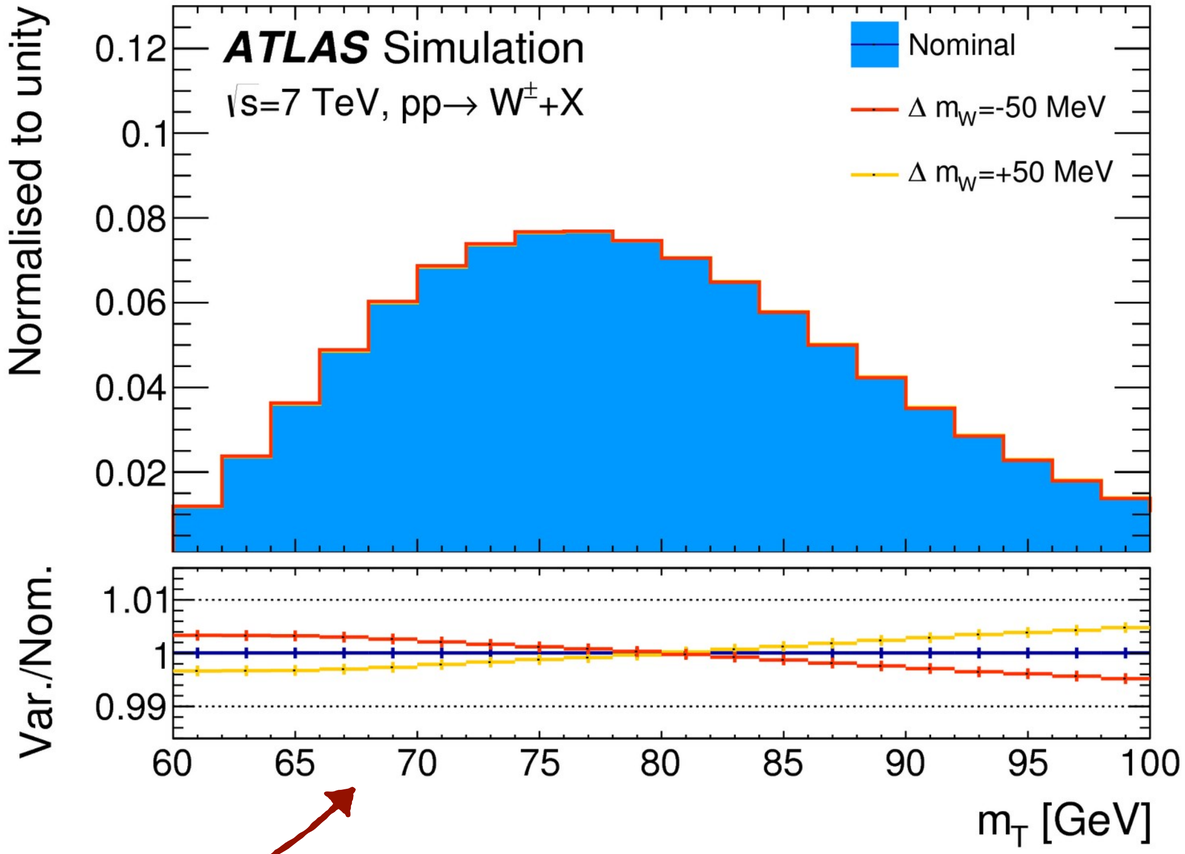
MWDays23 Workshop — CERN, Geneva — April 18th 2023

Drell Yan — a Standard Candle



- clean signature $(\ell^{\pm}, E_{\mathrm{T}}^{\mathrm{miss}})$
  & large cross section:
  (~ 1000 **Z** & ~ 4000 $\mathbf{W^{\pm}}$) / sec *
- detector calibration, BSM searches, luminosity monitor, quark PDFs, …
- precision measurements: $\sin^2(\theta_{\mathrm{w}})$, M_{W}
  ↪ $\Delta M_{\mathrm{W}} \simeq 10\,\mathrm{MeV}$
  ⇝ control *shape* at few ‰

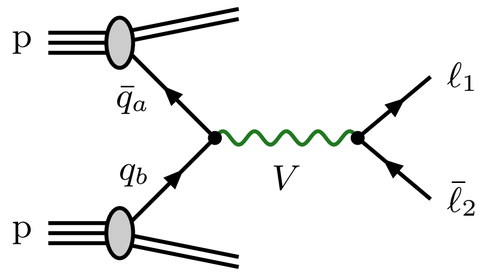
Talk by Camarda



big TH challenge!

* $\mathscr{L} = 2 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

Drell Yan — a Standard Candle & the Transverse Momentum



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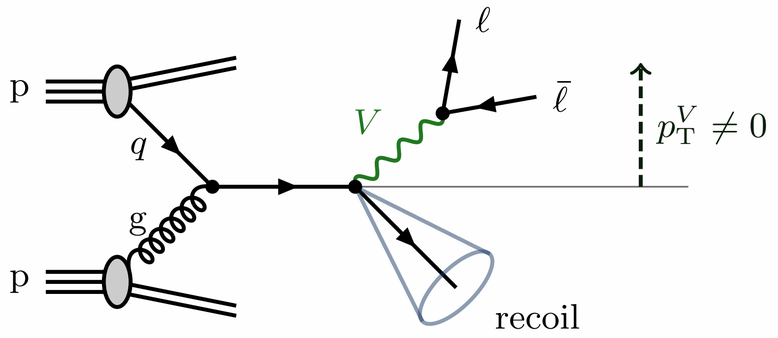


- recoil ⇒ direct sensitivity to α_{s}
  & PDFs (high-x gluon)
- precision QCD tests
  ↪ non-perturbative QCD, resummation, fixed-order, EW Sudakovs, …

$\mathcal{O}(\alpha_{\mathrm{s}}^{n})$ for Drell-Yan ↭ $\mathrm{N}^{n}\mathrm{LO}_{0}$ for e.g. σ^{tot}, $M_{\ell\ell}$, Y_V, p_{T}^{ℓ}, … ↭ $\mathrm{N}^{n-1}\mathrm{LO}_{1}$ for p_{T}^{V}, …

* $\mathscr{L} = 2 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

Outline.

- **fixed-order N³LO predictions**

  [NNLOJET + T-Z. Yang, H.X. Zhu]

  ↪ inclusive cross sections

  ↪ rapidity distributions

  ↪ fiducial $W^{\pm}$ predictions for CDF

- **transverse momentum resummation**

  [NNLOJET + RadISH]

  ↪ p_T^Z/p_T^W ratio

  ↪ fiducial cross sections

  & linear power corrections

  ↪ fiducial distributions

[CuTe + MCFM] ⇝ Tobias

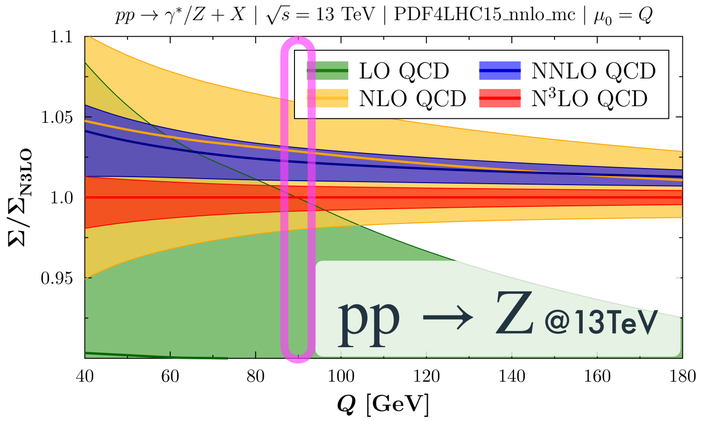
[DYTurbo] ⇝ Leandro

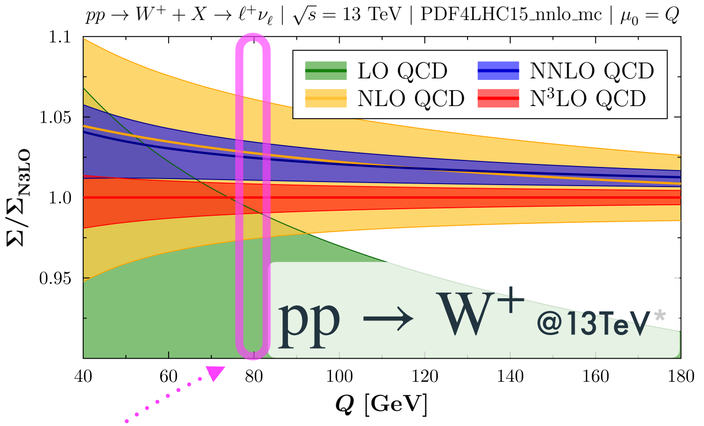
Inclusive Drell – Yan @ N³LO

[Baglio, Duhr, Mistlberger, Szafron '22]

- all $\sigma^{\text{tot}}_{\text{DY}}$ known to N³LO
  - $pp \to \gamma^*$, $pp \to W^{\pm}$
    [C. Duhr, F. Dulat, B. Mistlberger '20]
  - $pp \to \gamma^*/Z$
    [C. Duhr, B. Mistlberger '21]
  - public code: `n3loxs`
- similar features
  $\forall\ \text{DY}_{\text{procs}}$ @ LHC
  ↪ $K_{\text{N}^3\text{LO}} \sim -2\,\%$
  ↪ non-overlapping bands
  ↪ $\Delta^{\text{NNLO}}_{\text{scl}} \simeq \Delta^{\text{N}^3\text{LO}}_{\text{scl}}$
- *origin:* NNLO likely underestimated ($q\bar{q}$ vs. qg)

DY Processes



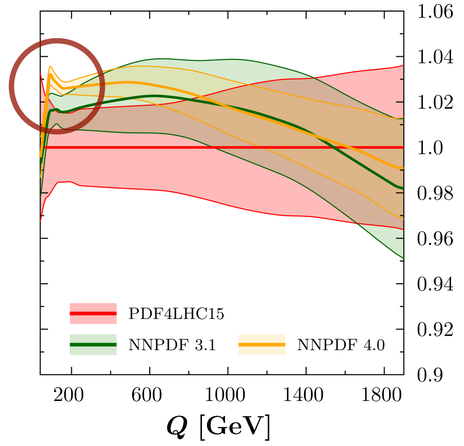


* similar for W^-

resonance region

Uncertainties

- scales ⇝ $< 1\,\%$
- PDFs ⇝ $\sim \pm 2\,\%$



improvable with more data

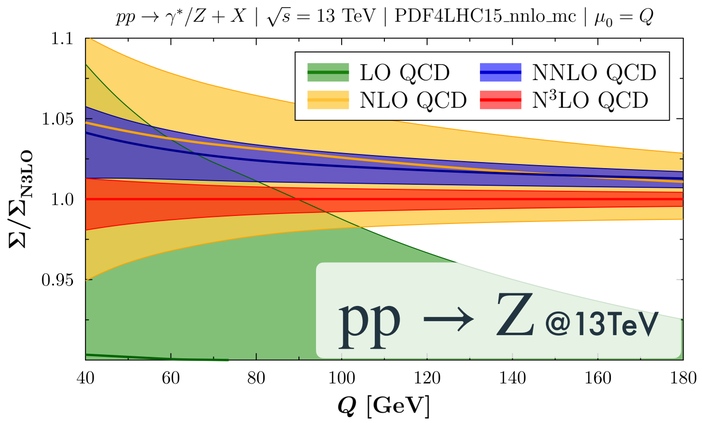
recently: aN³LO PDFs from MSHT

- missing N³LO PDFs

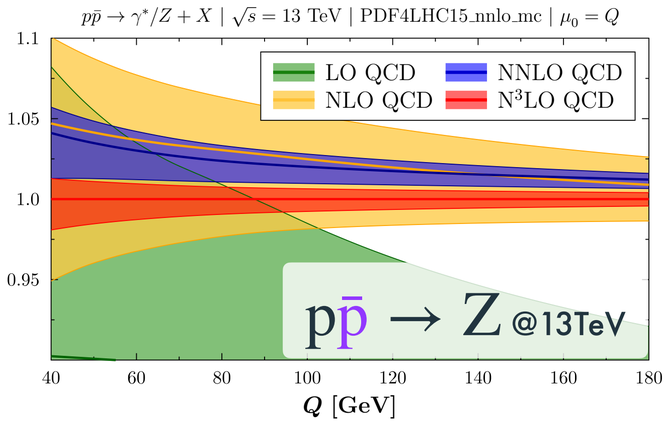
$$\frac{1}{2}\frac{\left|\sigma^{\text{NNLO}}(\text{NNLO PDF}) - \sigma^{\text{NNLO}}(\text{NLO PDF})\right|}{\sigma^{\text{NNLO}}(\text{NNLO PDF})}$$

⇝ $\sim \pm 2.5\,\%$

INCLUSIVE DRELL YAN @ N³LO – COLLIDER VS. ENERGY



COLL

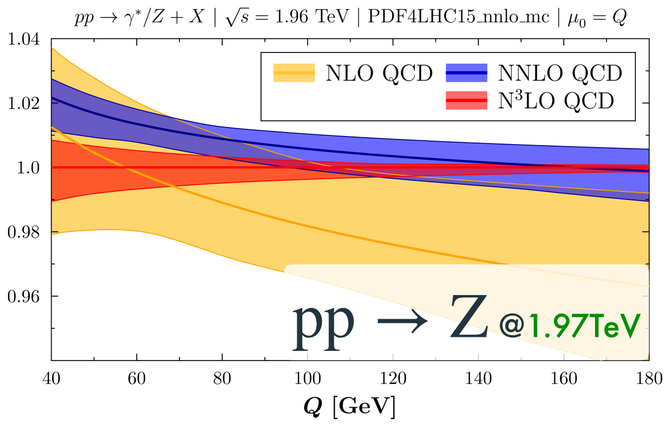


only minor impact from pp ⟶ pp̄

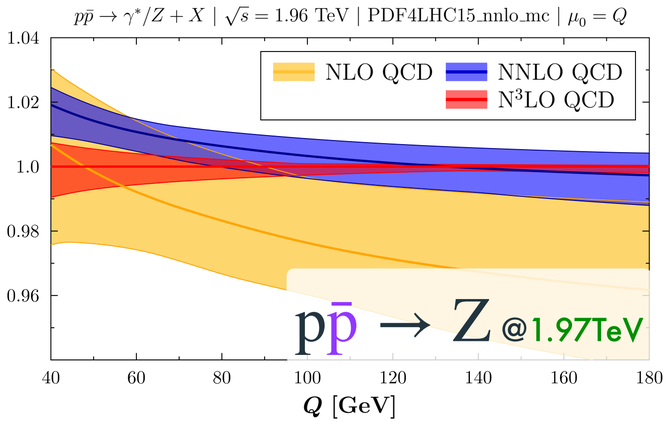
ENGY

ENGY

better pert. behaviour for lower collider energies



COLL



* similar for $W^{\pm}$

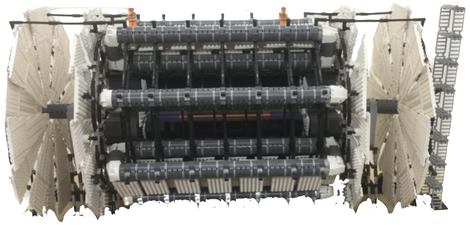
Going Differential @ N³LO — q_T Subtraction

Fully Inclusive

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}(\text{sec})$

Fully Differential

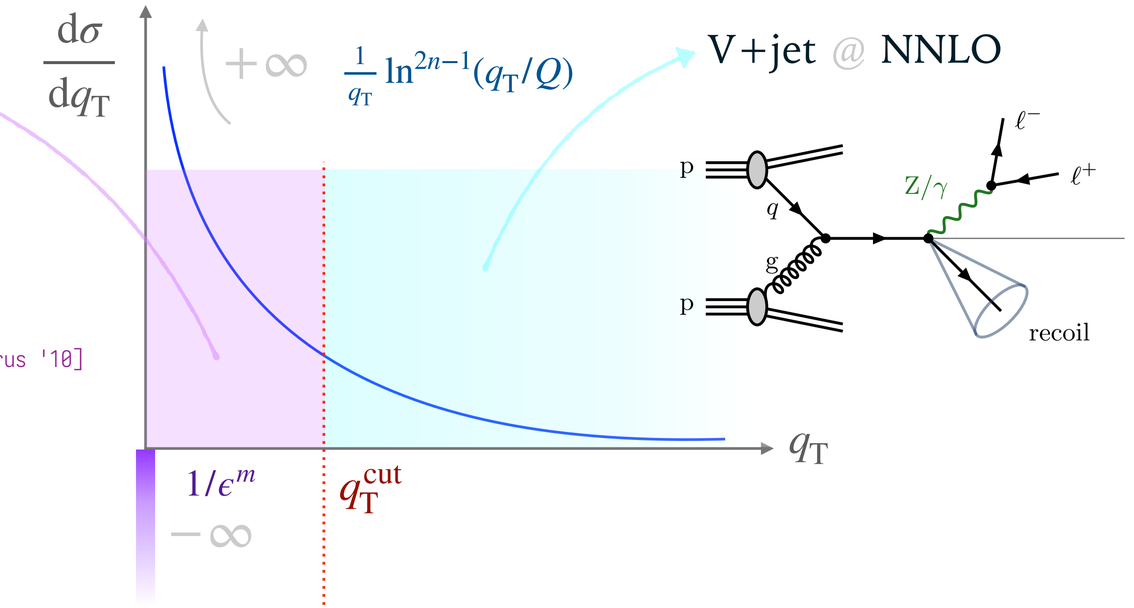


✓ $\mathrm{d}\sigma$ ⇝ fiducial cuts, arbitrary distributions, …

✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)\,\text{h}$

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$ [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$ [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$ [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO

[Catani, Grazzini '07]

$$\mathrm{d}\sigma^V_{\text{N}^3\text{LO}} = \mathrm{d}\sigma^V_{\text{N}^3\text{LO}}\Big|_{q_T<q_T^{\text{cut}}} + \mathrm{d}\sigma^V_{\text{N}^3\text{LO}}\Big|_{q_T>q_T^{\text{cut}}}$$

$$= \mathscr{H}^V_{\text{N}^3\text{LO}} \otimes \mathrm{d}\sigma^V_{\text{LO}} + \left[\mathrm{d}\sigma^{V+\text{jet}}_{\text{NNLO}} - \mathrm{d}\sigma^{V,\text{CT}}_{\text{N}^3\text{LO}}\right]_{q_T>q_T^{\text{cut}}} + \mathcal{O}\left((q_T^{\text{cut}}/Q)^n\right)$$

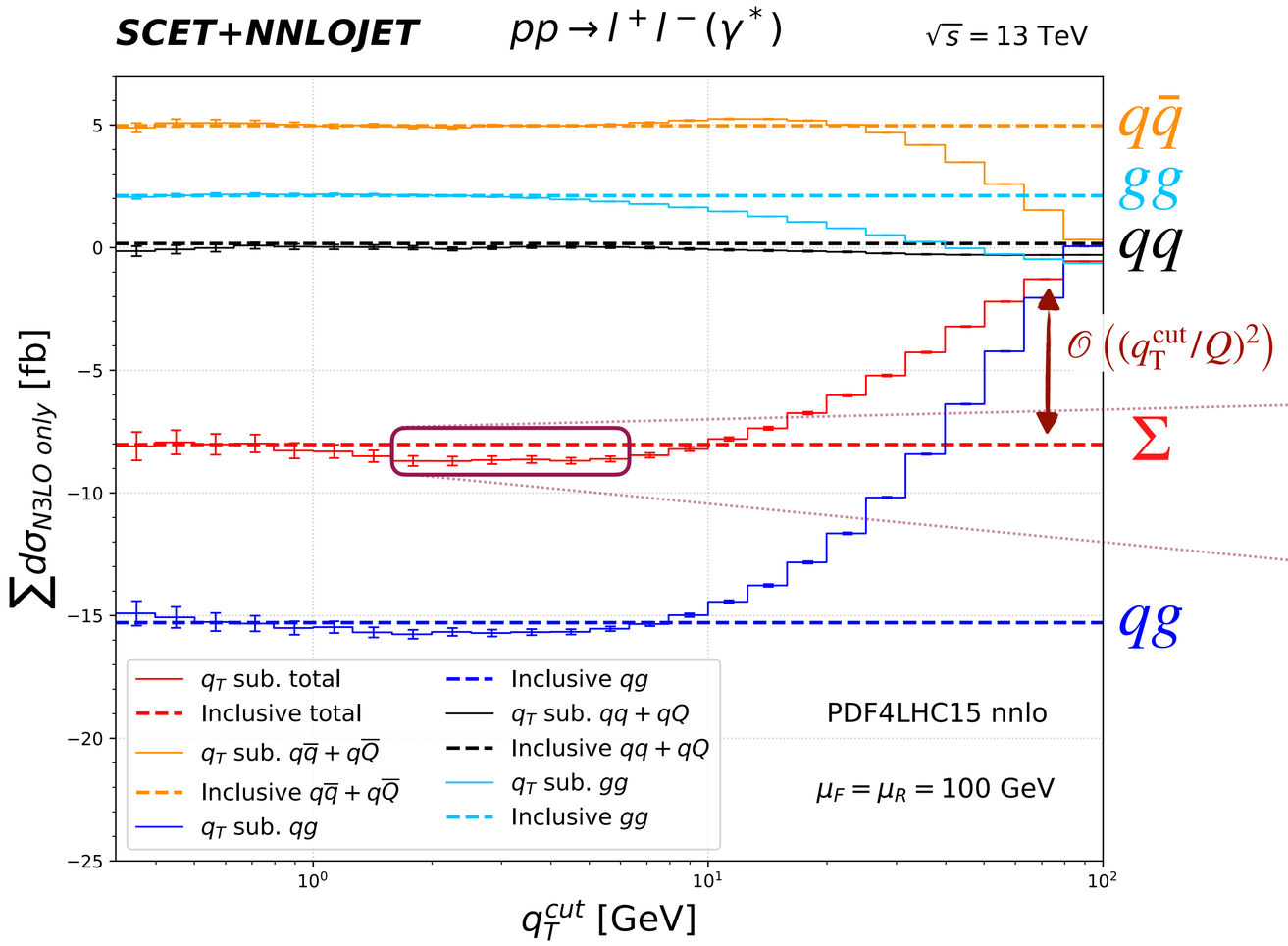
slicing error

q_T^{cut} as small as possible ⇜⇝ q_T^{cut} as large as possible

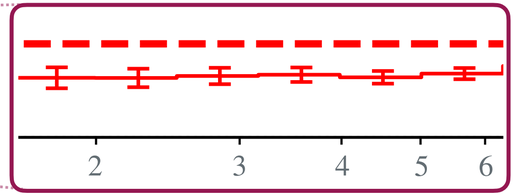
↪ suppress power corrections ↪ numerical stability & efficiency

STEP 0 — VALIDATION

[Chen, Gehrmann, Glover, AH, Yang Zhu '21, '22]



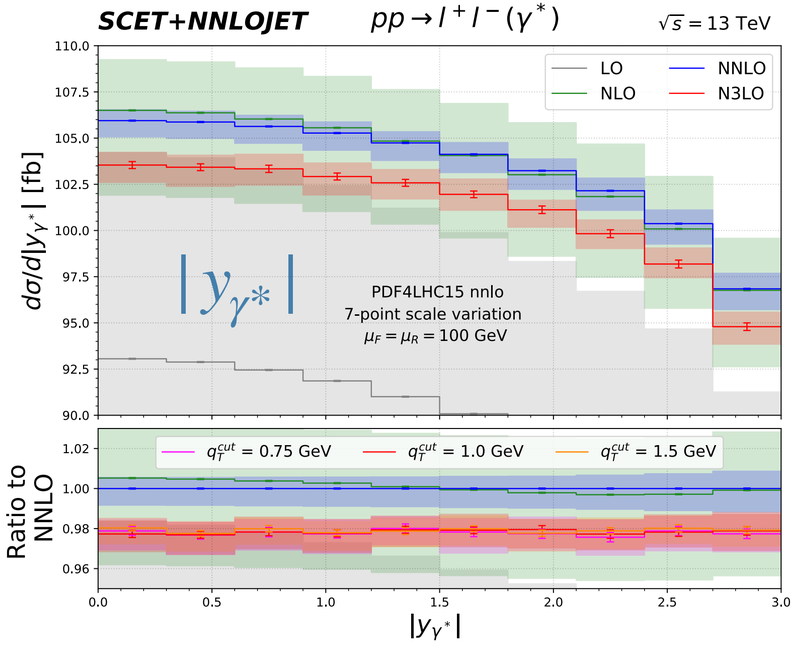
- *fully independent* calculation of the inclusive cross section
- ---- ⇝ analytic result [Duhr, Dulat, Mistlberger '20]
- "fake" plateau: $q_T^{\text{cut}} \in [2, 5]\,\text{GeV}$
  ↪ 12% error on $\delta\text{N}^3\text{LO}$!

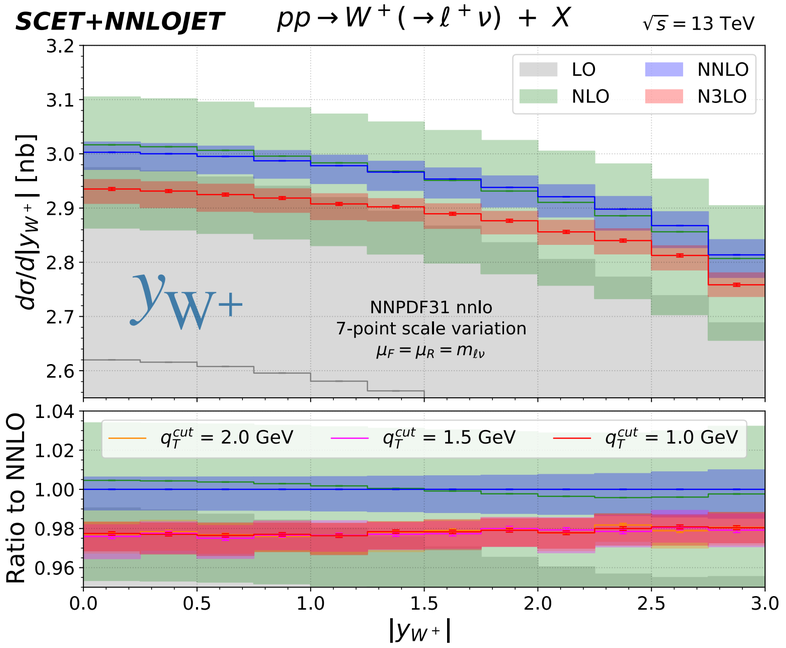


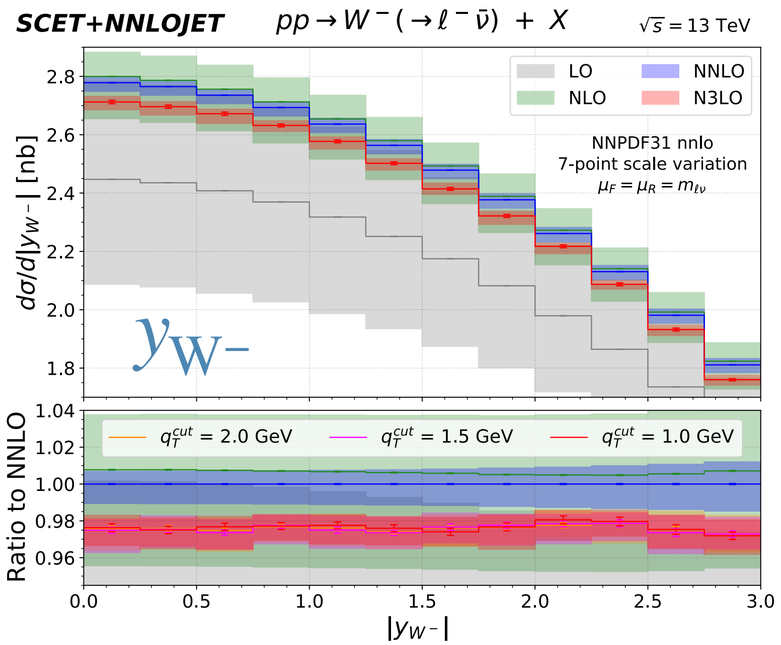
- converges to correct result for $q_T^{\text{cut}} \lesssim 1\,\text{GeV}$
- fit & extrapolate?
  ⇝ marginal gains for potentially uncontrolled systematics

Drell Yan @ N³LO — Y_V Distributions

[Chen, Gehrmann, Glover, AH, Yang, Zhu '21,'22]

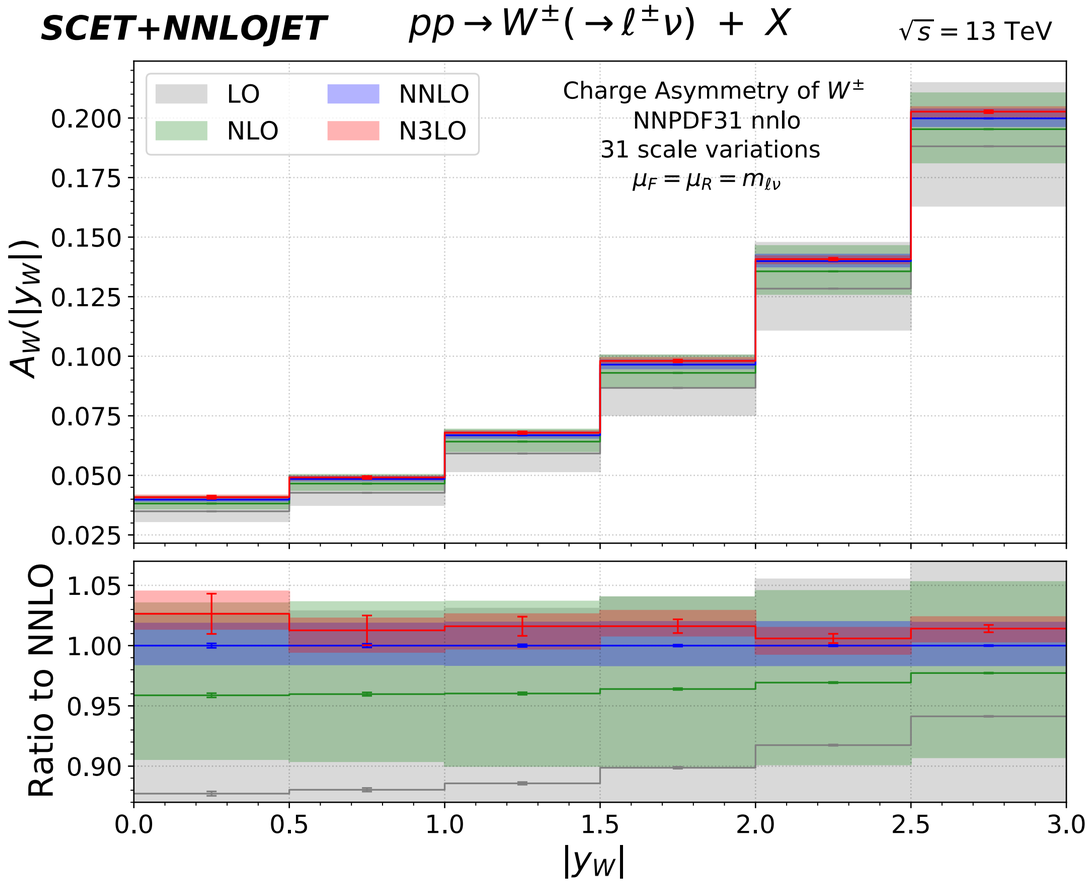






- same collider @ 13 TeV
  - ⇝ almost universal / flat NNLO → N³LO corrections! (disparate x_i)
- NC & CC$^{\pm}$ processes probe different parton content across Y_V (valence u vs. d, …)

W Charge Asymmetry @ N³LO



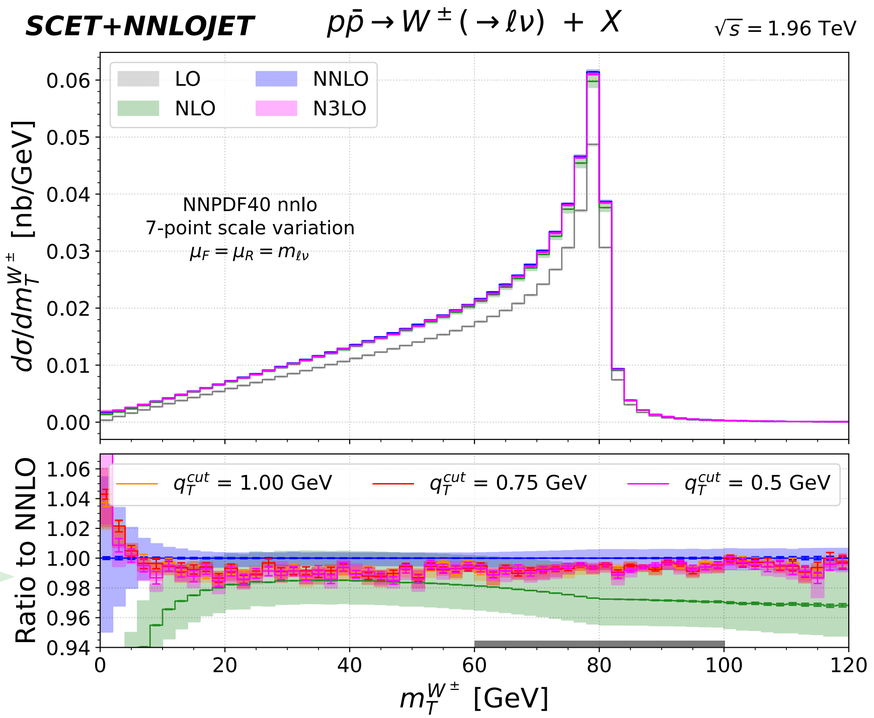
$$A_{\mathrm{W}}(|y_{\mathrm{W}}|) \equiv \frac{\mathrm{d}\sigma^{\mathrm{W}^+}/\mathrm{d}|y_{\mathrm{W}^+}| - \mathrm{d}\sigma^{\mathrm{W}^-}/\mathrm{d}|y_{\mathrm{W}^-}|}{\mathrm{d}\sigma^{\mathrm{W}^+}/\mathrm{d}|y_{\mathrm{W}^+}| + \mathrm{d}\sigma^{\mathrm{W}^-}/\mathrm{d}|y_{\mathrm{W}^-}|}$$

- ratio ⇝ cancellation of systematics (lumi, …)
  - ↪ *scales*: uncorrelated 31-point var.
- N³LO corrections of about +2%
- good perturbative convergence ($\Delta^{\mathrm{N^3LO}}_{\mathrm{scl}} \sim \pm 1.5\,\%$)
- direct sensitivity to $u(x)/d(x')$ ratio
  - ↪ extend to $\ell^{\pm}$ (measured)

W Production – Absolute Spectrum

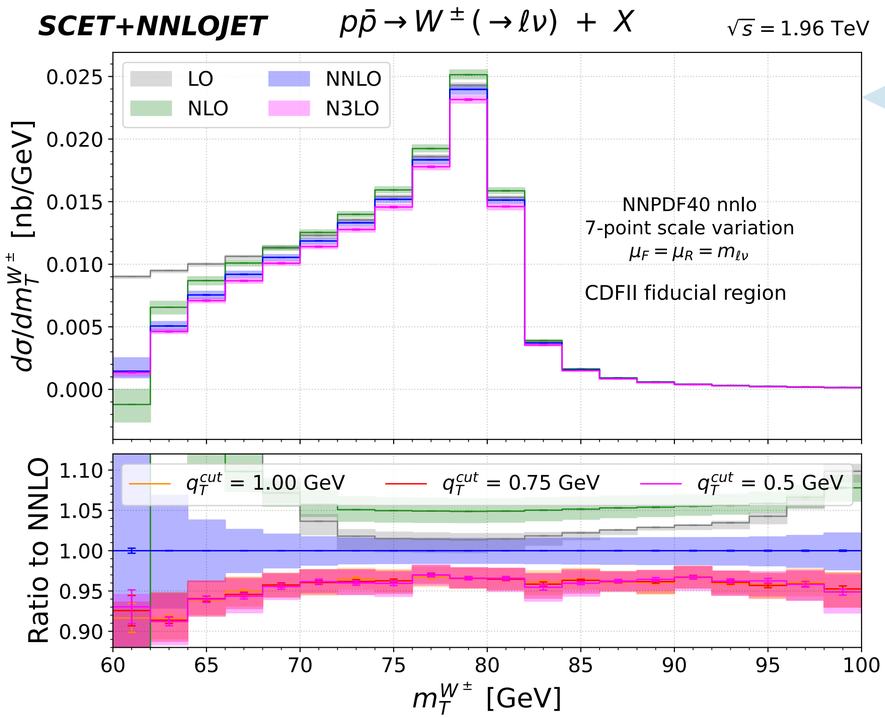
$$M_{\mathrm{T}}^{\mathrm{W}} \equiv \sqrt{E_{\mathrm{T}}^{\ell}\, E_{\mathrm{T}}^{\nu}\left(1-\cos\Delta\varphi_{\ell\nu}\right)}$$

Inclusive



1.96TeV – better convergence as seen in σ^{tot}

Fiducial (CDF II)



$p_{\mathrm{T}}^{\ell},\, E_{\mathrm{T}}^{\mathrm{miss}} \in [30, 55]\,\mathrm{GeV}$

$|\eta^{\ell}| < 1$

$p_{\mathrm{T}}^{\mathrm{W}} < 15\,\mathrm{GeV}$

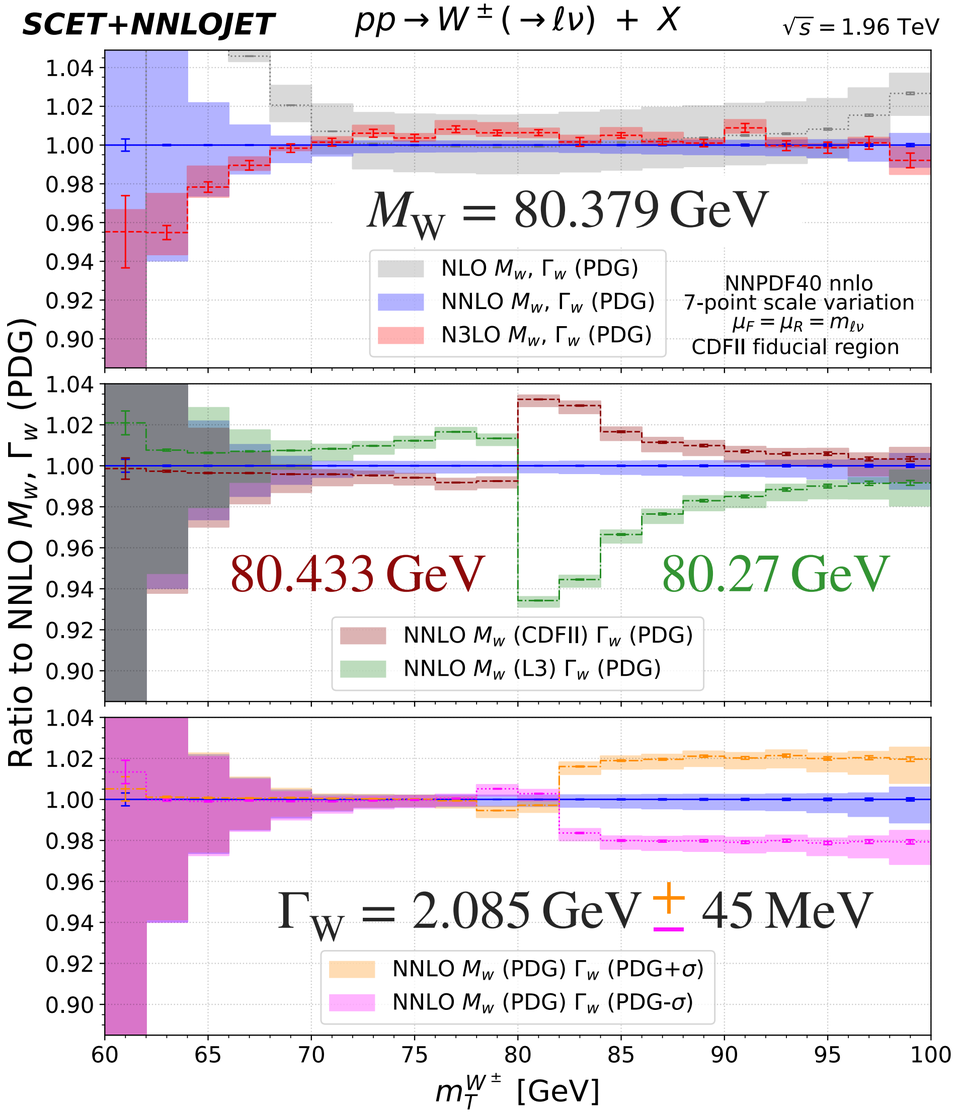
LO
NLO
NNLO
N3LO

- remain largely flat around peak; larger corrections at low $M_{\mathrm{T}}^{\mathrm{W}}$
- fiducial cuts impact pattern of radiative corrections
- larger N³LO corrections ($-1\,\%$ [inc.] vs. $-4\,\%$ [fid.])

fiducial power corrections?

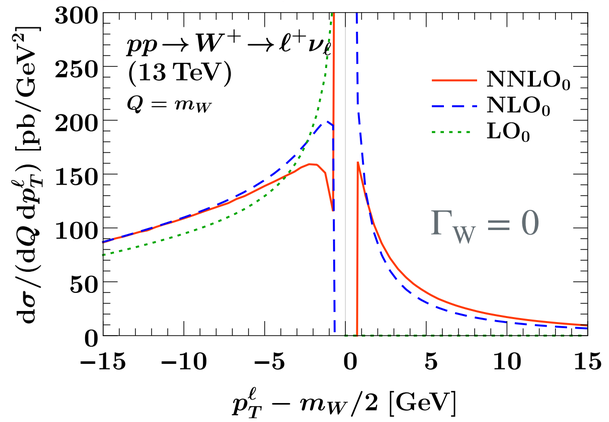
W Production – Normalized Spectrum

$$(1/\sigma)\,(\mathrm{d}\sigma/\mathrm{d}M_\mathrm{T}^\mathrm{W})$$



- normalisation compensates h.o. corrections
  - ↪ NNLO → N³LO: $\lesssim 1\,\%$ (peak region)
- impact of N³LO *smaller* than
  - ↪ M_W variation (CDF, L3) 3–6 %
  - ↪ Γ_W variation (PDG error) 2 %
- *here:* pure fixed-order
  - ↪ resummation? (M_T rather insensitive to ISR)
- what about p_T^ℓ?
  - ↪ Jacobian peak $\sim M_\mathrm{W}/2$
  - ↪ resummation mandatory

[Ebert, Michel, Stewart, Tackmann '20]



Outline.

- **fixed-order N³LO predictions**

  [NNLOJET + T-Z. Yang, H.X. Zhu]

  ↪ inclusive cross sections

  ↪ rapidity distributions

  ↪ fiducial $W^{\pm}$ predictions for CDF

- **transverse momentum resummation**

  [NNLOJET + RadISH]

  ↪ p_T^Z/p_T^W ratio

  ↪ fiducial cross sections

  & linear power corrections

  ↪ fiducial distributions

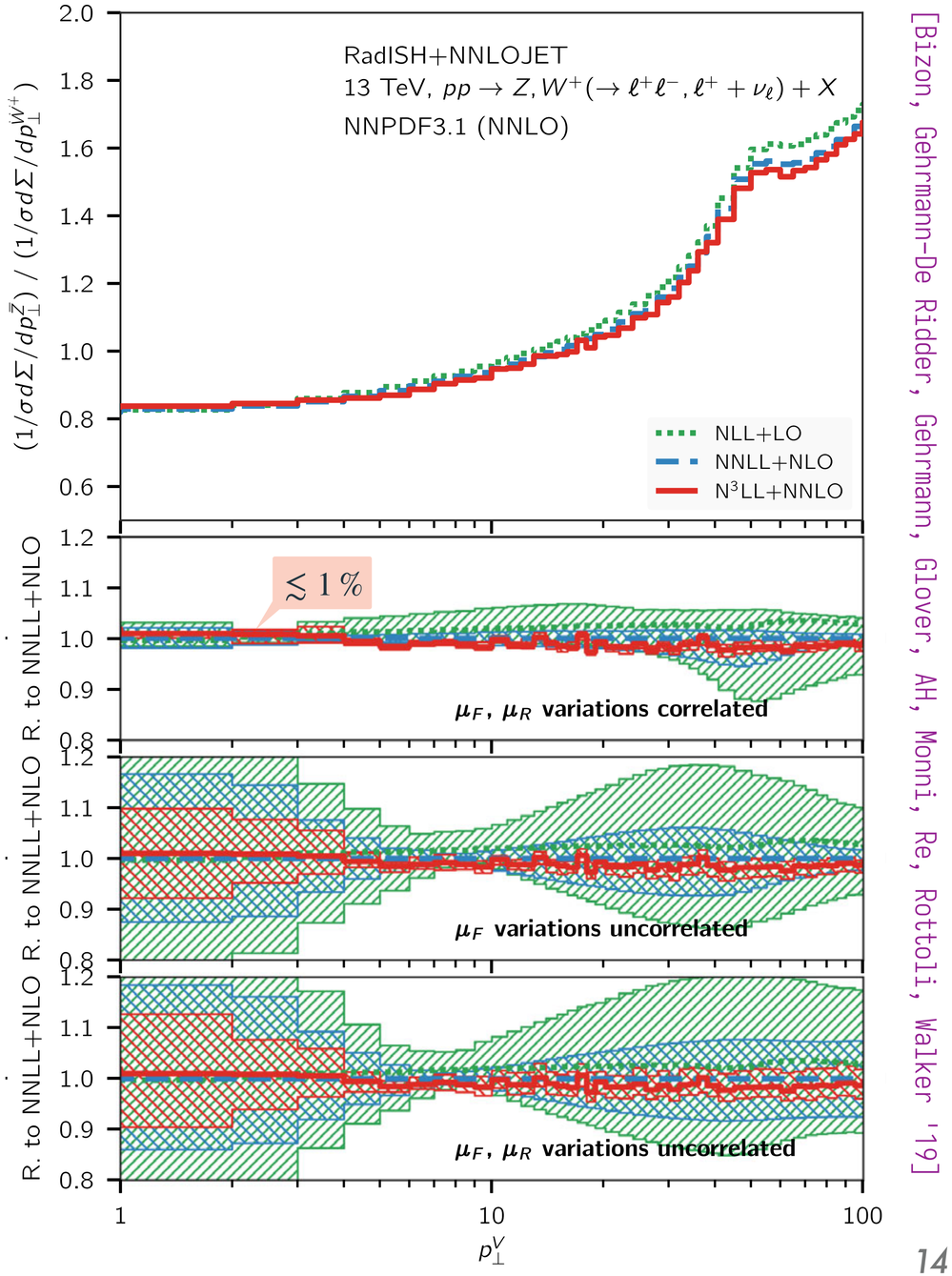
Z / W Ratio @ N³LL+NNLO₁

$$\frac{\frac{1}{\sigma^{Z}}\left(\mathrm{d}\sigma^{Z}/\mathrm{d}p_{\mathrm{T}}^{Z}\right)}{\frac{1}{\sigma^{W^+}}\left(\mathrm{d}\sigma^{W^+}/\mathrm{d}p_{\mathrm{T}}^{W^+}\right)}$$

NLL+LO
NNLL+NLO
N³LL+NNLO

- central in $p_{\mathrm{T}}^{Z} \rightsquigarrow p_{\mathrm{T}}^{W}$ modelling
- ratio of quantities [num/den.] $(\mu_{\mathrm{F}}, \mu_{\mathrm{R}}, Q)$
  ↔ how to assess perturbative uncertainties?
  - ↪ matching scale Q — correlated (same γ_i)
  - ↪ $\mu_{\mathrm{F}}, \mu_{\mathrm{R}}$ — vary correlation (different PDFs)

  ⇒ stable ratio ⇜⇝ strong correlation between sources of perturbative uncertainties



[Bizon, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Walker '19]

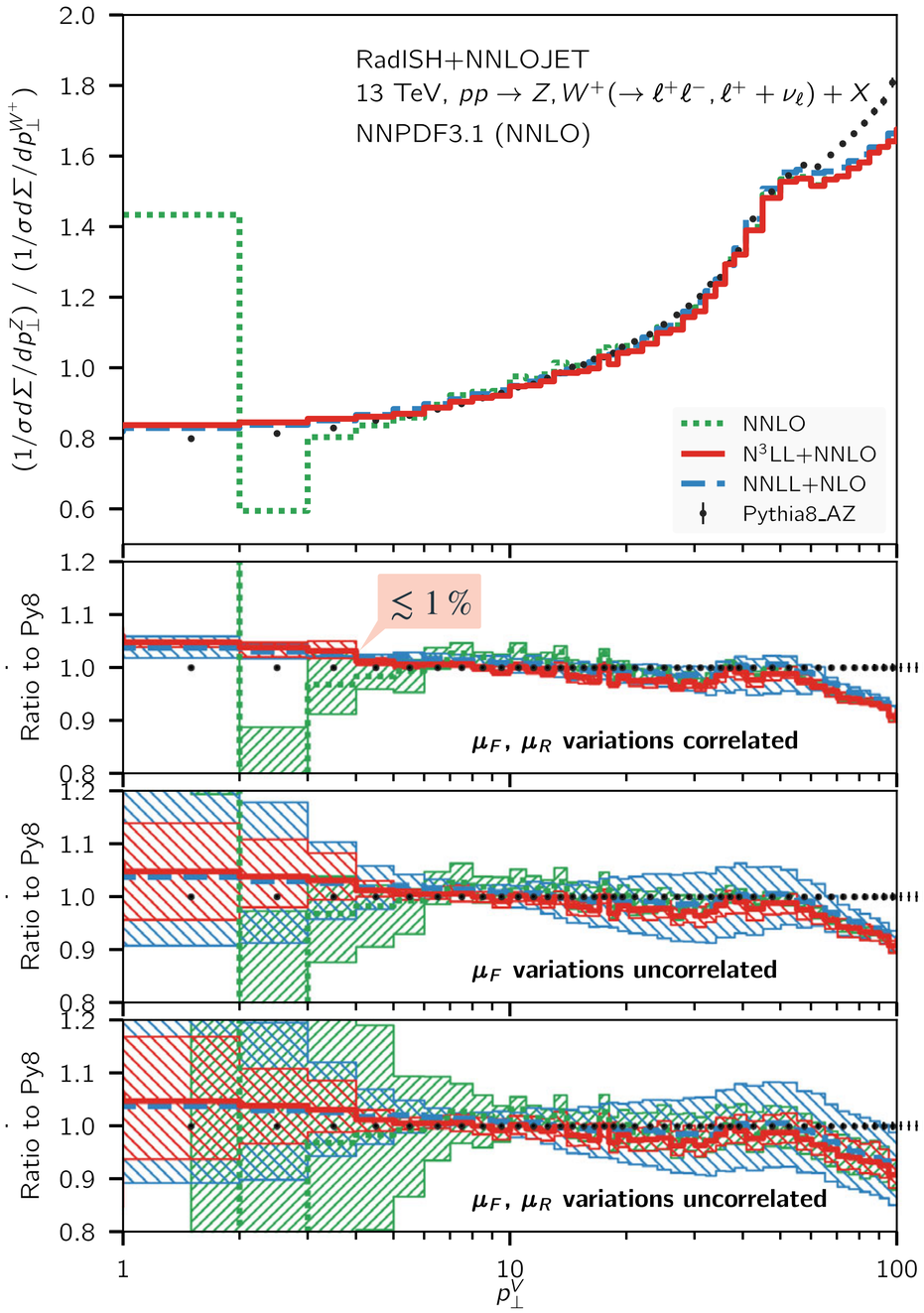
Z / W Ratio @ N³LL+NNLO₁

$$\frac{\frac{1}{\sigma^{Z}}\left(\mathrm{d}\sigma^{Z}/\mathrm{d}p_{\mathrm{T}}^{Z}\right)}{\frac{1}{\sigma^{W^+}}\left(\mathrm{d}\sigma^{W^+}/\mathrm{d}p_{\mathrm{T}}^{W^+}\right)}$$

NNLO
N³LL+NNLO
NNLL+NLO
Pythia8_AZ

- central in $p_{\mathrm{T}}^{Z} \rightsquigarrow p_{\mathrm{T}}^{W}$ modelling
- ratio of quantities [num/den.] $(\mu_{\mathrm{F}}, \mu_{\mathrm{R}}, Q)$
  ↔ how to assess perturbative uncertainties?
  - ↪ matching scale Q — correlated (same γ_i)
  - ↪ $\mu_{\mathrm{F}}, \mu_{\mathrm{R}}$ — vary correlation (different PDFs)

  ⇒ stable ratio ⇜⇝ strong correlation between sources of perturbative uncertainties
- differences to `Pythia8_AZ`
  - ↪ *missing:* PDF unc., QED/EW, mass effects(?), …
  - ↪ *but also:* "Pythia is not QCD" [K.Melnikov — QCD@LHC '16]



[Bizon, Gehrmann-De Ridder, Gehrmann, Glover, AH, Monni, Re, Rottoli, Walker '19]

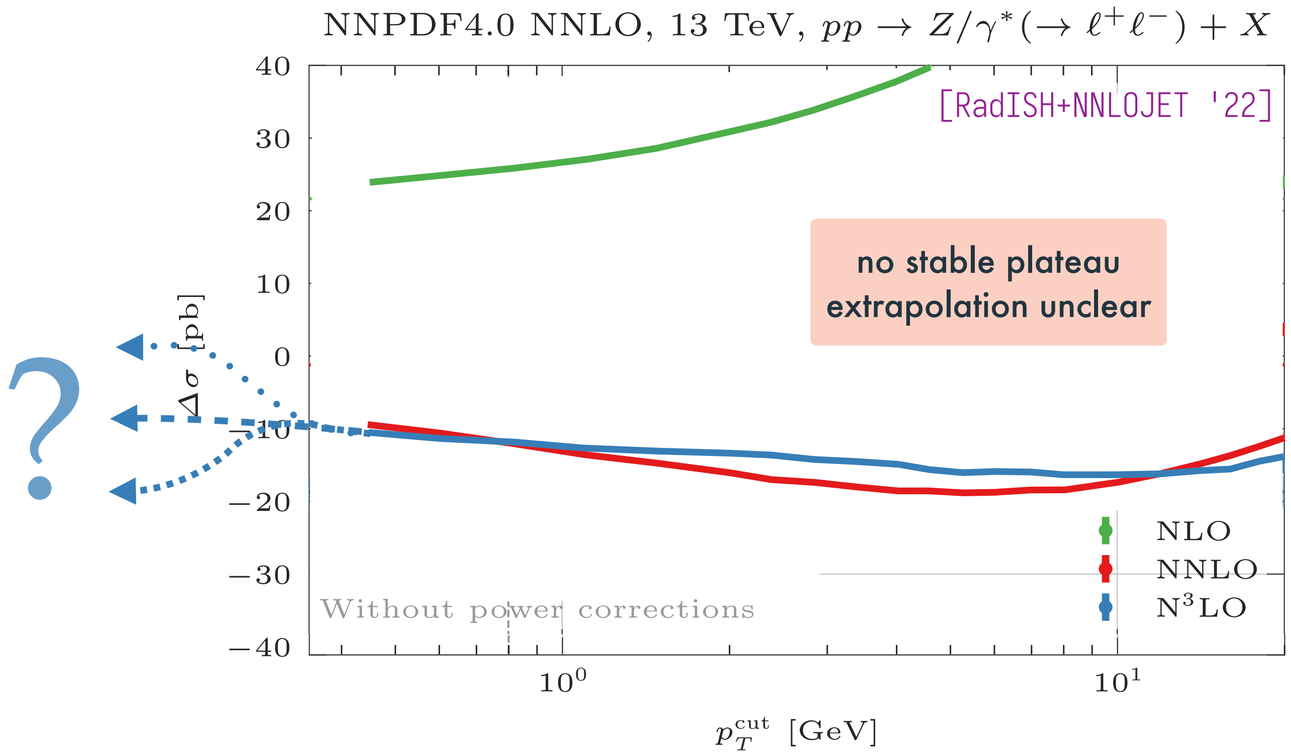
Fiducial Cuts and Linear Power Corrections — N³LO Slicing

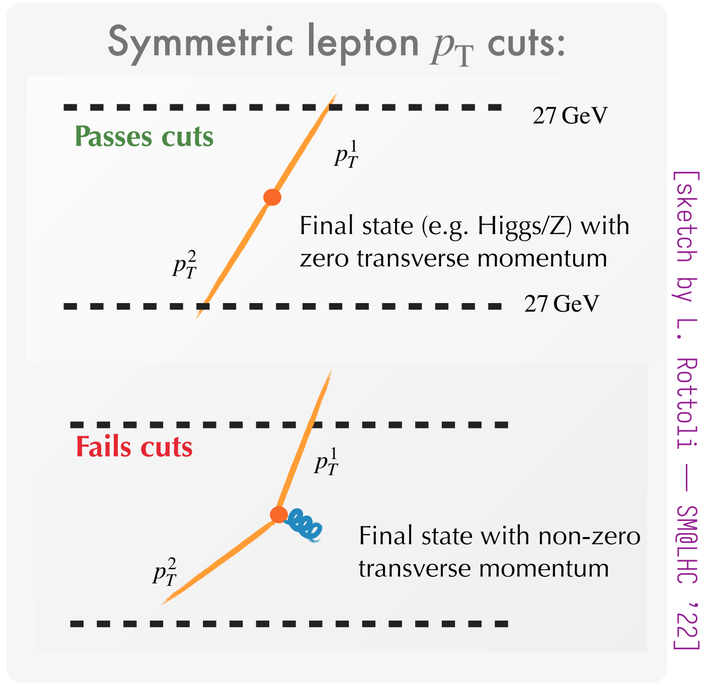
- fiducial cuts ⇝ can induce linear power corrections

  [Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]

  - can jeopardise q_T slicing $\mathcal{O}\left((q_T^{\rm cut}/Q)^2\right)$ ⇝ $\mathcal{O}\left(q_T^{\rm cut}/Q\right)$

    $[q_T^{\rm cut} \lesssim 1\,\text{GeV}]$ $[q_T^{\rm cut} \lesssim 10^{-2}\,\text{GeV}\ ?!]$





[sketch by L. Rottoli — SM@LHC '22]

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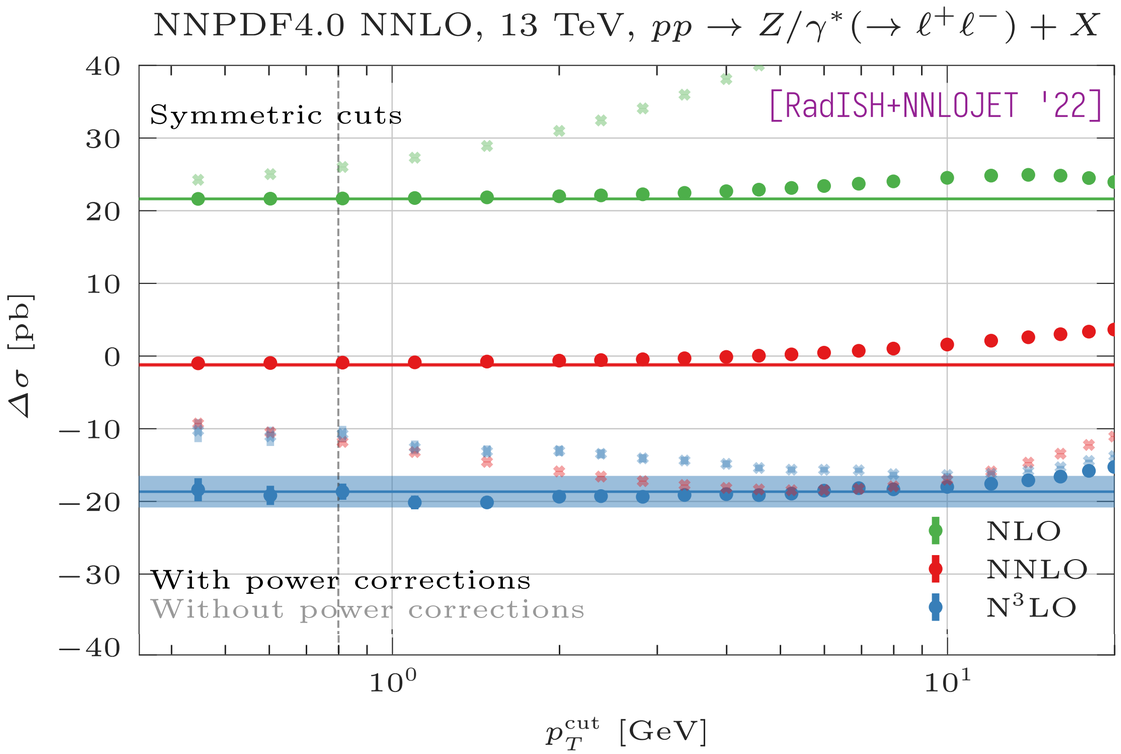
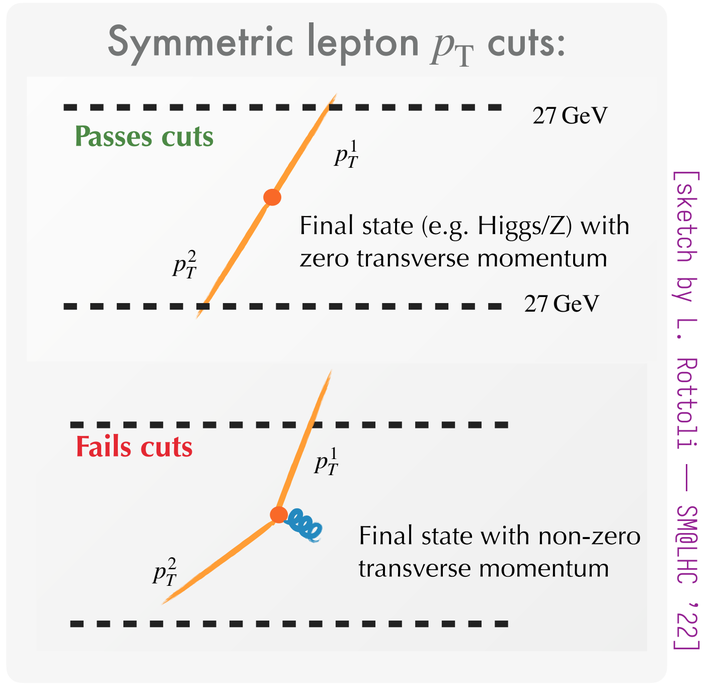




[sketch by L. Rottoli — SM@LHC '22]

can *compute* & *subtract* the linear term:

↪ simple boost of $V \to \ell\bar{\ell}$ system

(pure kinematics & acceptance effect)

[Catani, de Florian, Ferrera, Grazzini '15]
[Ebert, Michel, Stewart, Tackmann '21]

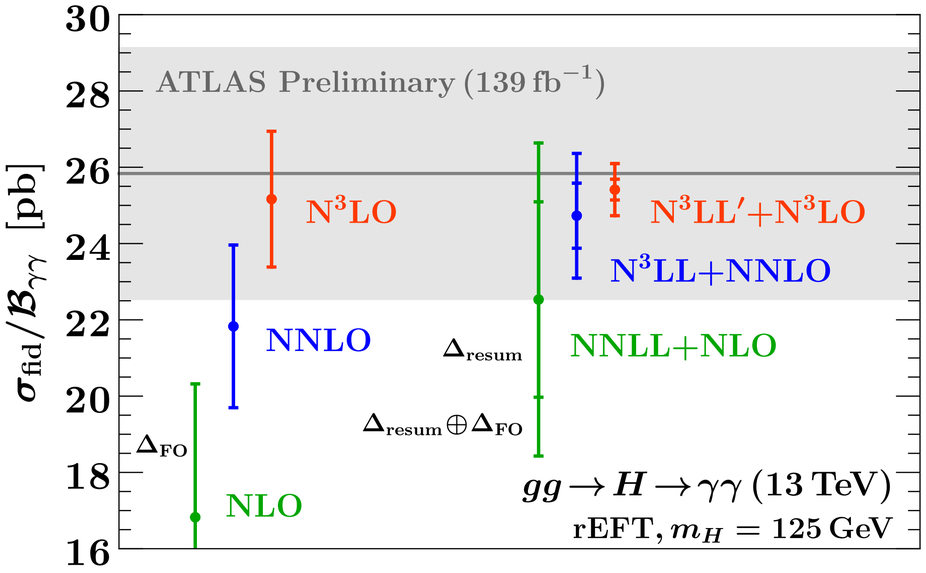
Fiducial Cuts and Linear Power Corrections — Inclusive Quantities

- can compute the N3LO cross section reliably ✓
  but: potential sensitivity on soft physics in inclusive quantities ?
  c.f. $gg \to H \to \gamma\gamma$

$$\sigma^{\text{FO}}_{\text{fid}}/\mathcal{B}_{\gamma\gamma} = 6.928\,[1 + (1.300 + 0.129_{\text{fpc}}) + (0.784 - 0.061_{\text{fpc}}) + (0.331 + 0.150_{\text{fpc}})]\,\text{pb}$$

linear fiducial power corrections

[Billis, Dehnadi, Ebert, Michel, Tackmann '21]



- can be resummed to all orders
  with the same recoil prescription [Ebert, Michel, Stewart, Tackmann '21]

- How strongly is the Drell-Yan fiducial cross section impacted by this?

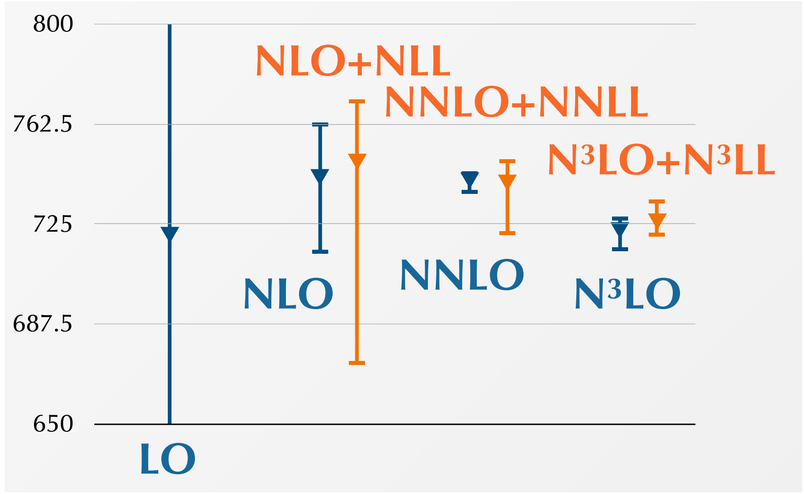
Fiducial Z Cross Section @ N³LO+N³LL

Symmetric cuts:

$p_{\mathrm{T}}^{\ell^\pm} > 27\,\mathrm{GeV}$

[Chen, Gehrmann, Glover, AH, Monni, Rottoli, Re, Torrielli '22]

Order	σ [pb] Symmetric cuts	
k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$



- $K_{\mathrm{N^3LO}} \sim -2.5\,\%$; outside scale bands (fixed order)
- fixed order vs. +resummation — similar central values

  Γ_V ⇜⇝ regulator?

  ⇝ smaller fiducial power corrections than $gg \to H \to \gamma\gamma$; nonetheless, not negligible
- N³LO+N³LL more robust error estimate (matching scale Q)

get rid of these completely by solving the problem at its core

Fiducial Z Cross Section @ N³LO+N³LL

Product cuts:

[Salam, Slade '21]

$$\sqrt{p_{\mathrm{T}}^{\ell^+} p_{\mathrm{T}}^{\ell^-}} > 27\,\mathrm{GeV}$$

$$\min\left\{p_{\mathrm{T}}^{\ell^\pm}\right\} > 20\,\mathrm{GeV}$$

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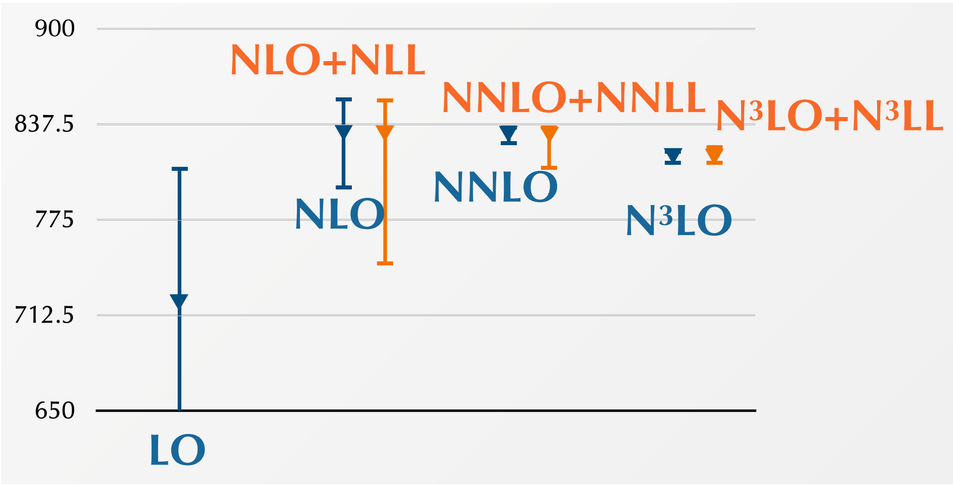
Order	σ [pb] Product cuts	
k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

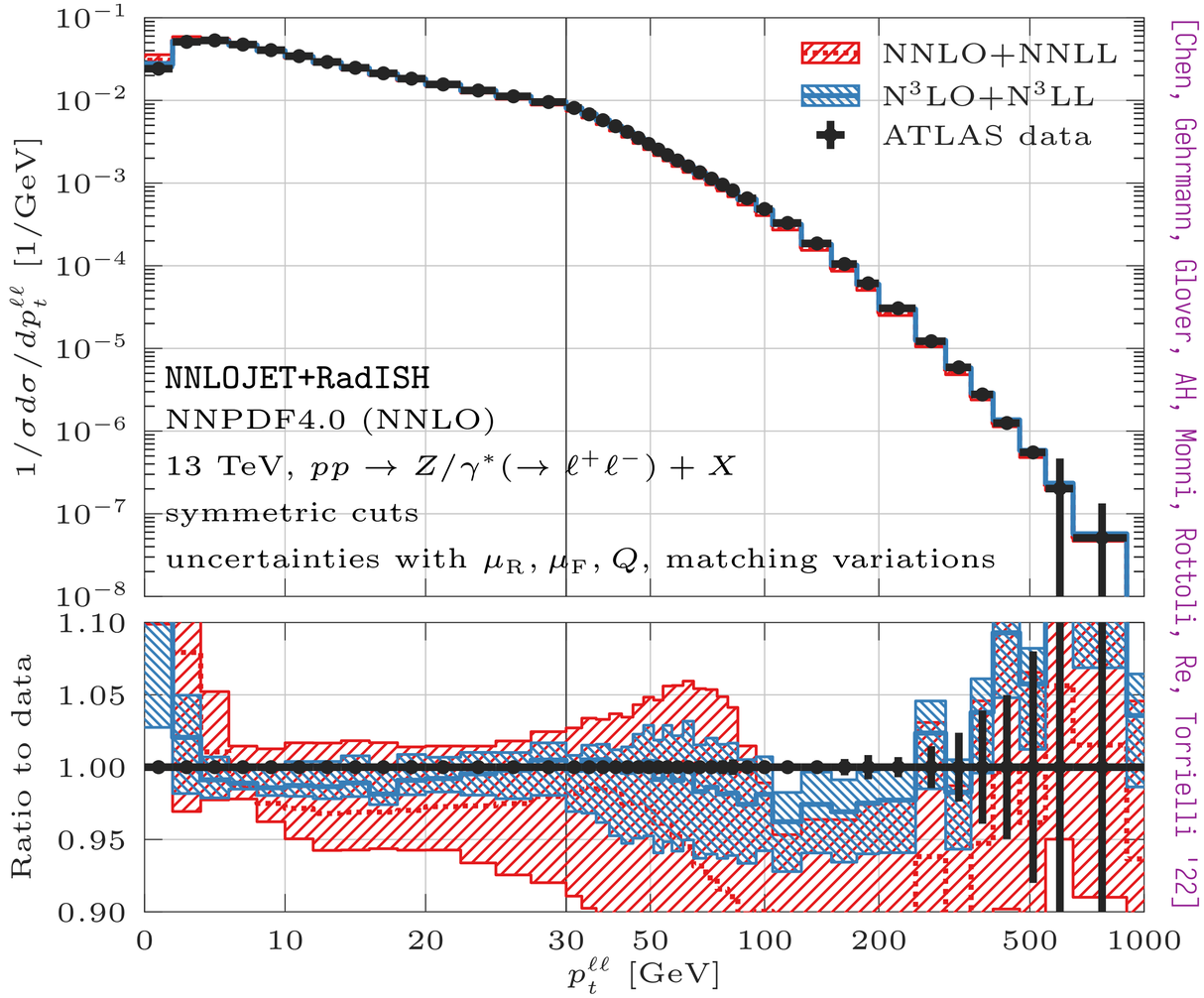


- $K_{\mathrm{N^3LO}} \sim -2\,\%$; outside scale bands (fixed order)
- fixed order vs. +resummation — virtually identical central values
  - ⇝ basically no linear fiducial power corrections; very robust
- N³LO+N³LL more robust error estimate (matching scale Q)

Z-Boson Transverse Momentum Distribution @ N³LO+N³LL

$$\mathrm{d}\sigma_V^{\mathrm{N^3LO+N^3LL}} \equiv \mathrm{d}\sigma_V^{\mathrm{N^3LL}} + \mathrm{d}\sigma_{V+\mathrm{jet}}^{\mathrm{NNLO}} - \left[\mathrm{d}\sigma_V^{\mathrm{N^3LL}}\right]_{\mathcal{O}(\alpha_s)}$$

- excellent agreement with data across p_{T}^{Z} spectrum
  - ↪ lowest bin susceptible to non-perturbative effects
  - ↪ *high-p_{T}*: missing EW corrections
- uncertainties — ***few percent*** ($p_{\mathrm{T}} \gtrsim 50\,\mathrm{GeV} \nearrow 5\%$)
  - ↪ μ_{R}, μ_{F}: 7-point variation
  - ↪ matching scale Q: $+2$ variation
  - ↪ matching scheme: $+3 \times 9 = 27$
  - ⇒ envelope of 36 variations



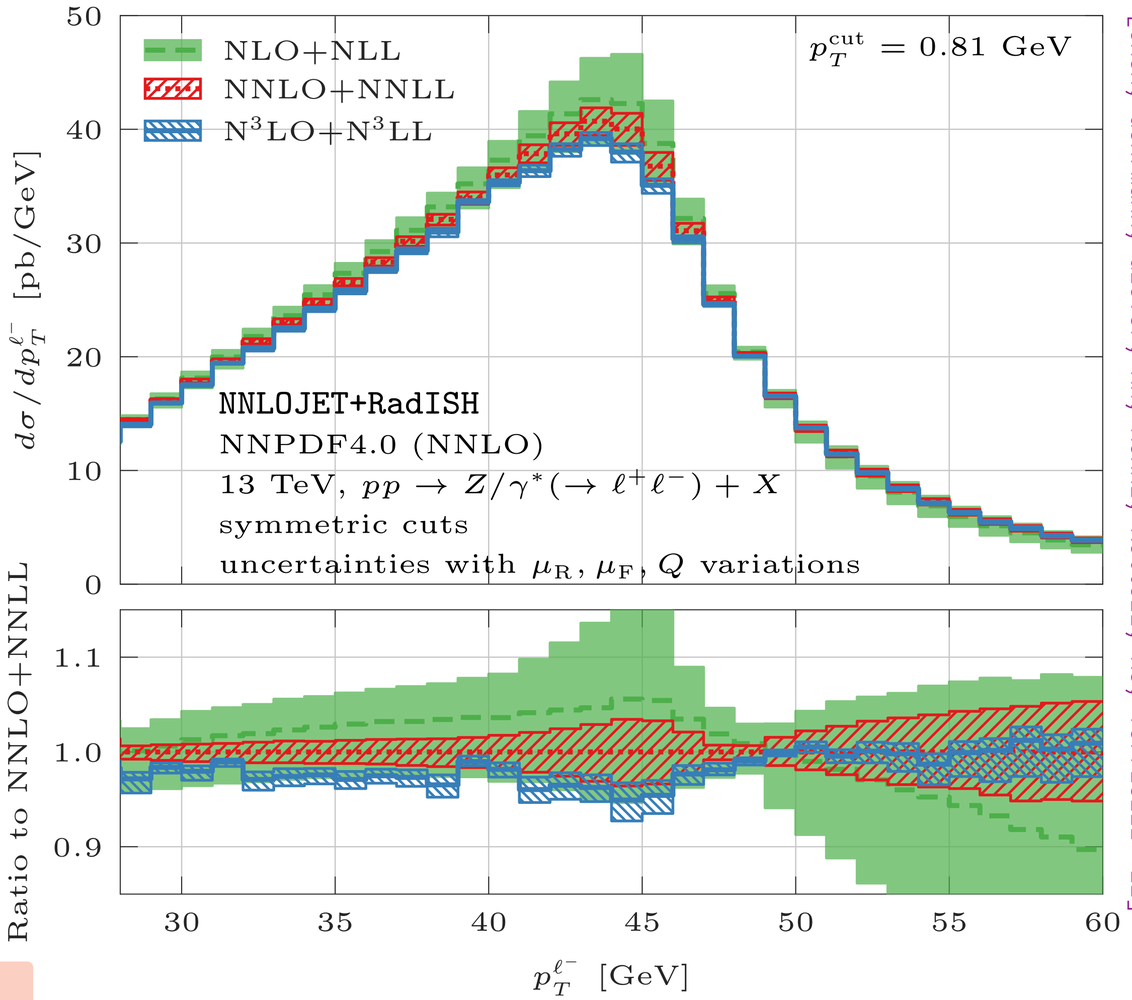
[Chen, Gehrmann, Glover, AH, Monni, Rottoli, Re, Torrielli '22]

Lepton Transverse Momentum Distribution @ N³LO+N³LL

$$\mathrm{d}\sigma_V^{\mathrm{N^3LO+N^3LL}} \equiv \mathrm{d}\sigma_V^{\mathrm{N^3LL}} + \mathrm{d}\sigma_{V+\mathrm{jet}}^{\mathrm{NNLO}} - \left[\mathrm{d}\sigma_V^{\mathrm{N^3LL}}\right]_{\mathcal{O}(\alpha_s)}$$

- fully differential calculation
  - ↪ access to fiducial observables
    & decay kinematics
- lepton transverse momentum
  - ↪ important in *M_{W} extraction*
  - ↪ challenging due to Jacobian peak
    @ $p_{\mathrm{T}}^{\ell} \sim M_V/2$ (integrable singularity)
  - ↪ resummation mandatory
- reduced uncertainties
  & some impact on shape

how would it translate to M_{W} shift in $\mathrm{W}^{\pm}$ production?



[Chen, Gehrmann, Glover, AH, Monni, Rottoli, Re, Torrielli '22]

Conclusions & Outlook

- N³LO predictions for Drell-Yan processes:
  ↪ $\Delta_{\text{scl}} < 1\,\%$ (non-overlapping), $\Delta_{\text{PDF}} \sim \pm 2\,\%$ (improvable), $\Delta_{\text{PDF–TH}} \sim \pm 2.5\,\%$
  but: corrections rather *flat*; compensate to a large extent in ratios (shape)

- Fiducial cuts ⇝ linear power corrections
  [1] can jeopardise N³LO slicing calculation
  [2] can introduce soft sensitivity to inclusive quantities
  ↪ *solutions*: [1] compute & subtract
  [2] resummation of fiducial power corrections
  [1+2] adjust fiducial cuts ⇝ more robust predictions

- N³LO+N³LL ⇝ distributions at *few-percent level*
  ↪ *next:* fiducial $W^{\pm}$ @ N³LO+N³LL

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Thank you!