Precise QCD+QED resummed predictions

For the Drell-Yan process

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CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Leandro Cieri

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MWDays23 Workshop

Outline

Precise resummed QCD+QED predictions

- Motivation
- Brief TH intro
- QCD resummation at N4LLa [DYTurbo]
  [2303.12781, 2202.10343, 2111.14509, 2103.04974, 1910.07049]
- QED+QCD resummation at NLL [DYqT]
  [2302.05403, 1805.11948]
- Outlook

In collaboration with: Andrea Autieri, Stefano Camarda, Giancarlo Ferrera, German Sborlini

Motivation

Drell-Yan process

- Standard candle for precision measurements and theory at the LHC

- Detector calibration

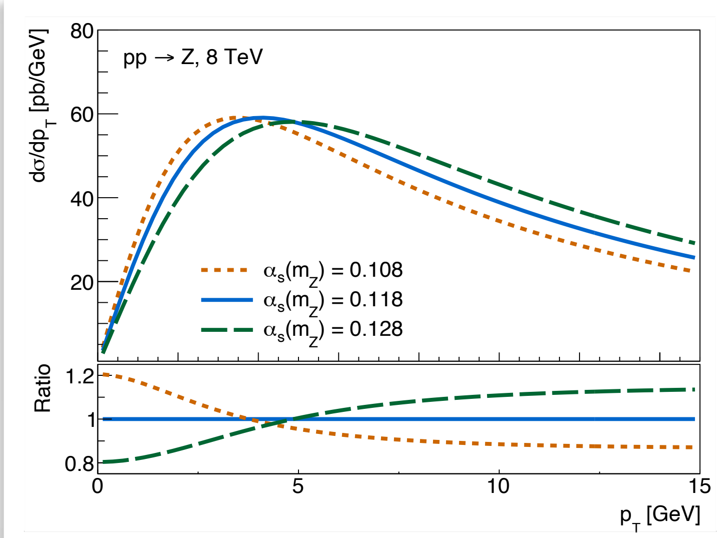
- Extraction of PDFs

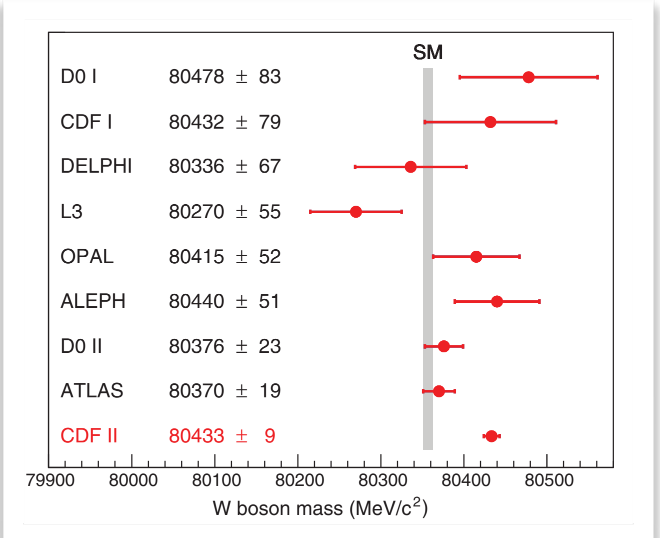
- Precise measurement of the strong coupling

- Precise measurement and determination of MW

⋮

- …





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Data at small transverse momentum is very relevant

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Drell-Yan process

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Data at small transverse momentum is very relevant

Transverse momentum resummation

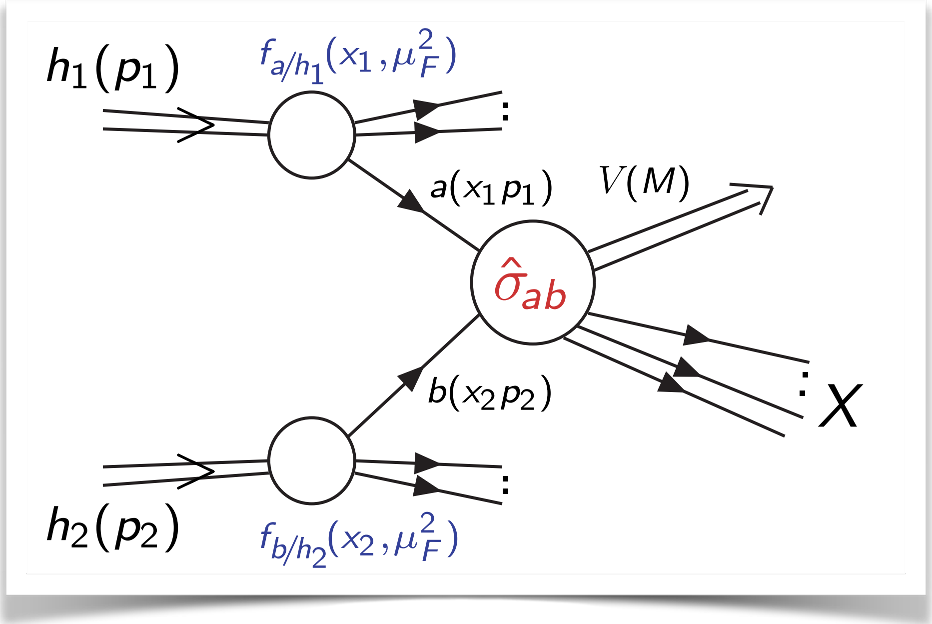
QCD q_T resummation at N4LLa

[DYTurbo]

Transverse momentum resummation

up to N4LL+N4LO accuracy

Camarda, LC, Ferrera [2023]



$$h_1 + h_2 \to V + X \to l_3 + l_4 + X$$

$$\frac{d\sigma_{h_1h_2\to l_3l_4}}{d^2\mathbf{q_T} dM^2 dy d\mathbf{\Omega}}(\mathbf{q_T}, M^2, y, s, \mathbf{\Omega}) = \sum_{a_1,a_2} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a_1/h_1}(x_1,\mu_F^2)\, f_{a_2/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{a_1a_2\to l_3l_4}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}}$$

- $f_{a/h_1}(x_1,\mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim M$.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion in the strong coupling $\alpha_S(M)$ ($M \gg \Lambda_{QCD} \sim 1$ GeV).
- This framework relies in the QCD factorization property of the cross sections

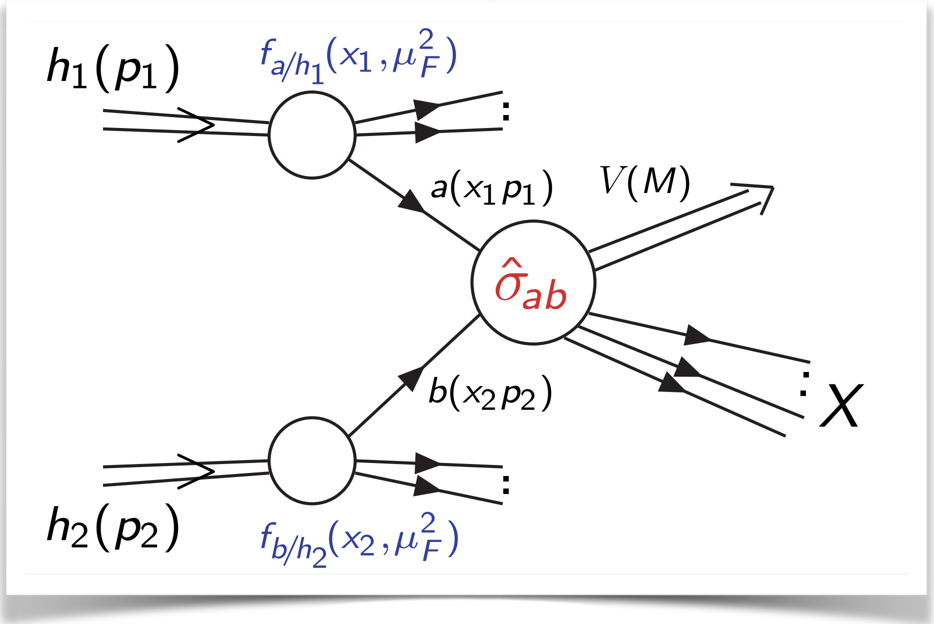
Collins, Soper, Sterman [1988] Aybat, Sterman [2008]

[DYTurbo]

Transverse momentum resummation

Camarda, LC, Ferrera [2023]

up to N4LL+N4LO accuracy



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  Collins, Soper, Sterman [1988]   Aybat, Sterman [2008]

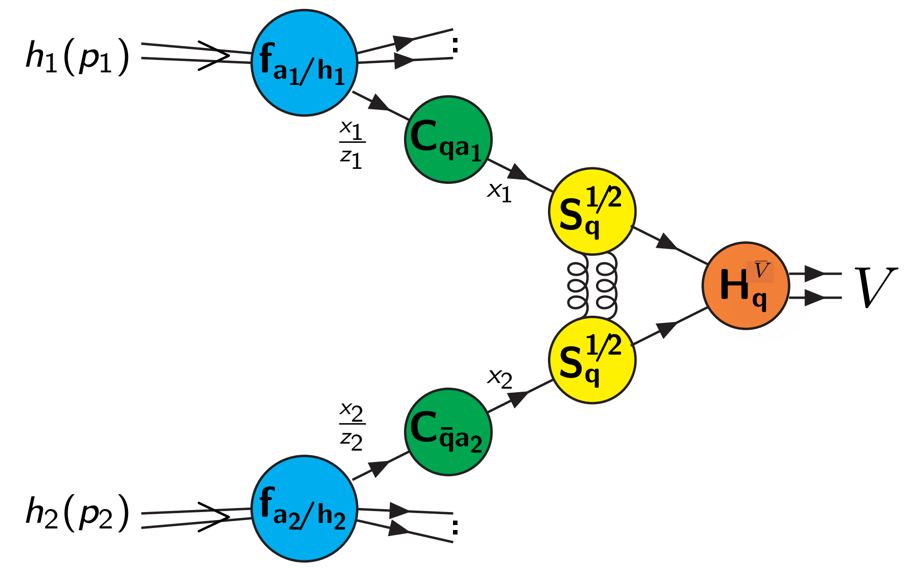
  Beware! violation of strict collinear factorization beyond N3LO (two loop amplitudes with 5 external legs and n ≥ 4 QCD partons)

  Catani, de Florian, Rodrigo [2011]   Forshaw, Seymour, Siodmok [2012]

Transverse momentum resummation

up to N4LL+N4LO accuracy

Camarda, LC, Ferrera [2023]



The cross section can be decomposed as

$$[d\hat{\sigma}_{a_1 a_2 \to l_3 l_4}] = [d\hat{\sigma}^{(\mathrm{res.})}_{a_1 a_2 \to l_3 l_4}] + [d\hat{\sigma}^{(\mathrm{fin.})}_{a_1 a_2 \to l_3 l_4}]$$

$$\left[d\hat{\sigma}^{(\mathrm{res.})}_{a_1 a_2 \to l_3 l_4}\right] = \sum_{b_1, b_2 = q, \bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1 b_2 \to l_3 l_4}}{d\mathbf{\Omega}} \frac{1}{\hat{s}} \int_0^\infty \frac{db}{2\pi}\, b\, J_0(b q_T)\, \mathcal{W}_{a_1 a_2, b_1 b_2 \to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2)$$

Bozzi, Catani, de Florian, Grazzini [2005] Bozzi, Catani, de Florian, Ferrera, Grazzini [2011] Catani, de Florian, Ferrera, Grazzini [2015]

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\,(\alpha_S; M/\mu_R, M/\mu_F, M/Q) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$\mathcal{H}_V(\alpha_S) = H_V(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)$$

$$\mathcal{H}_V(\alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_V^{(n)}$$

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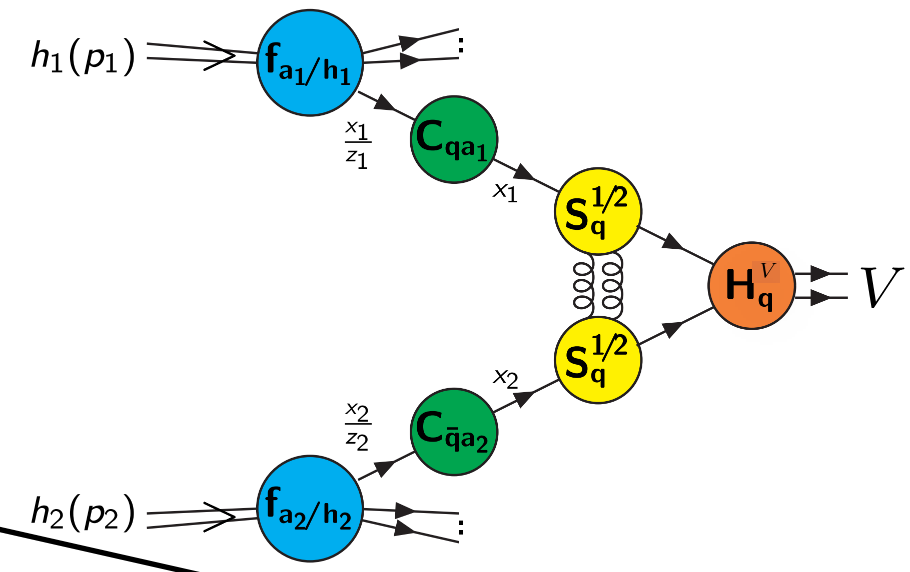
Transverse momentum resummation

up to N4LL+N4LO accuracy

Camarda, LC, Ferrera [2023]



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$$\lim_{Q_T\to 0}\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(\mathrm{fin.})}_{F\,ab}}{dq_T^2}\right]_{\mathrm{f.o.}} = 0$$

$$\left[d\hat{\sigma}^{(\mathrm{res.})}_{a_1a_2\to l_3l_4}\right] = \sum_{b_1,b_2=q,\bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1b_2\to l_3l_4}}{d\boldsymbol{\Omega}}\,\frac{1}{\hat{s}}\int_0^\infty \frac{db}{2\pi}\, b\, J_0(bq_T)\, \mathcal{W}_{a_1a_2,b_1b_2\to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2)$$

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Hard-virtual factor

Sudakov

$$\mathcal{H}_V(\alpha_S) = H_V(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)$$

Collinear coefficient functions

$$\mathcal{H}_V(\alpha_S) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_V^{(n)}$$

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Transverse momentum resummation

up to N4LL+N4LO accuracy

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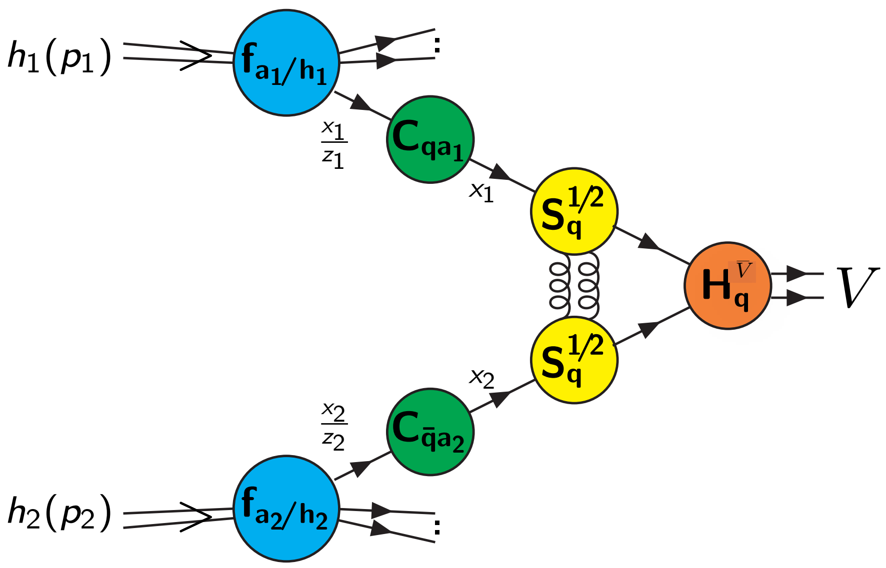

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We are interested in the impact of the resummation

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$L \equiv \ln(Q^2b^2/b_0^2)$$

$$\mathcal{H}_V(\alpha_S) = H_V(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)$$

Missing N4LO f.o contribution

$$\mathcal{H}_V(\alpha_S) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_V^{(n)}$$

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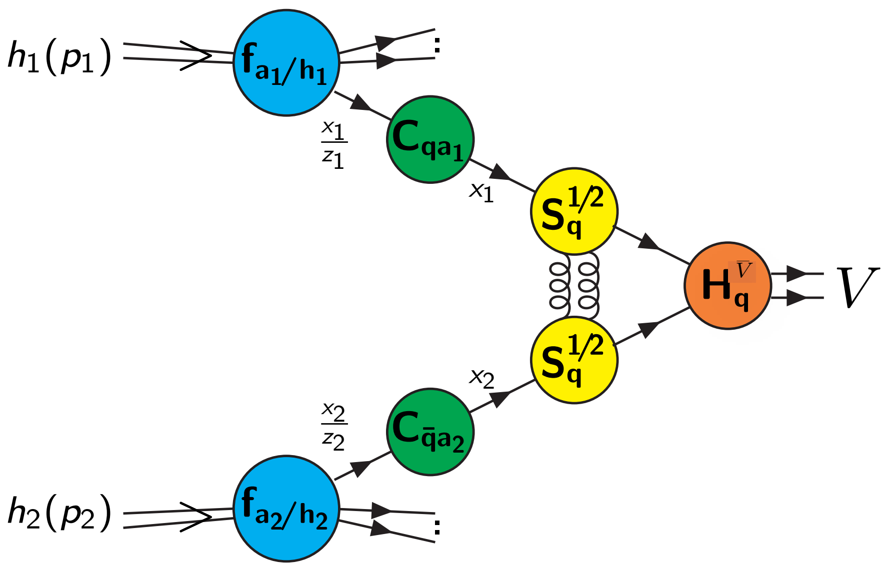
Without matching: artificial estimation of Q variation with +1 prescription

For a discussion about the f.o, resummation and the matching please see Alex and Tobias talks

Transverse momentum resummation

up to N4LL+N4LO accuracy

Camarda, LC, Ferrera [2023]



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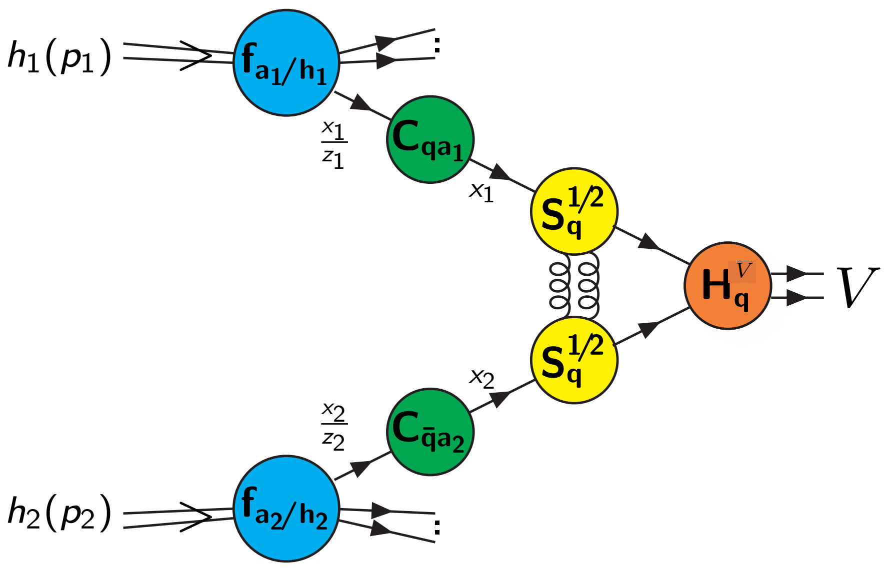

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Collinear coefficient functions

Resummation scheme dependent statement!!!!

Partially known

Unknown

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[DYTurbo]

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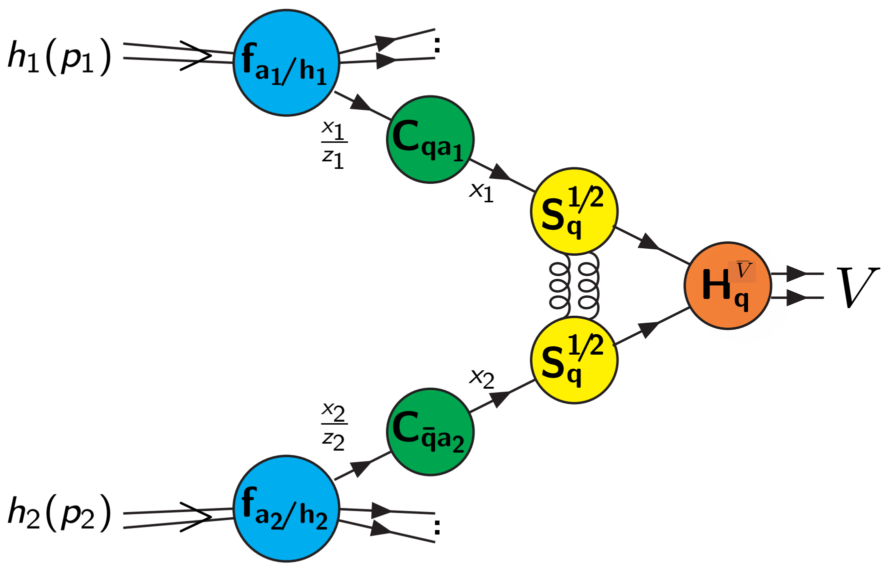

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$$\mathcal{H}_V(\alpha_S) = H_V(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)$$

δ(1-z) contribution requires the definition of subtraction operators I at N4LO → we postpone this topic to the discussion session

Resummation scheme independent statement!!!!

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\end{aligned}$$

δ(1-z) contribution: partially known

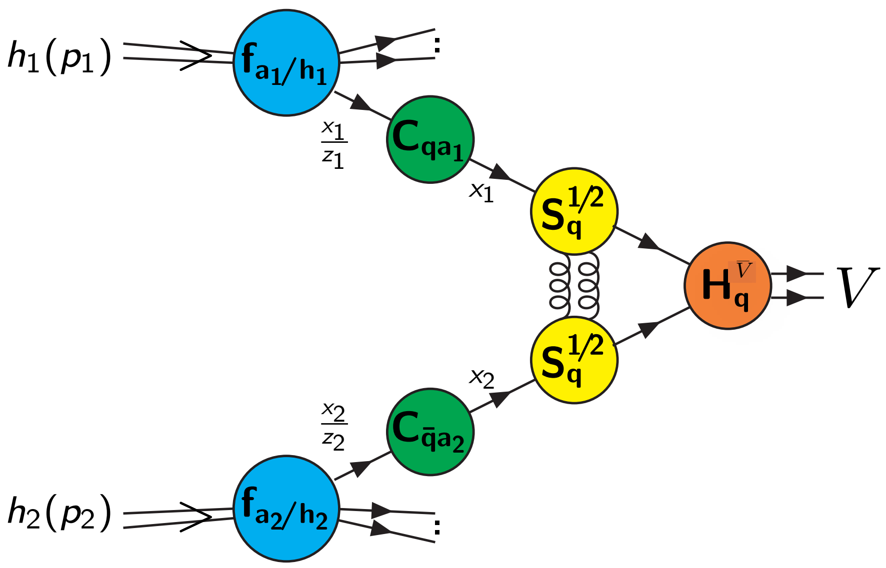
Non δ(1-z) contribution: unknown

[DYTurbo]

Transverse momentum resummation

Camarda, LC, Ferrera [2023]

up to N4LL+N4LO accuracy



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$$\mathcal{H}_V(\alpha_S) = H_V(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)$$

The δ(1-z) contribution can be computed from the four-loop quark form factor

Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser [2022]

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Transverse momentum resummation

up to N4LL+N4LO accuracy

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$$\begin{aligned} \mathcal{G}(\alpha_S, L) &= -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + \widetilde{B}(\alpha_S(q^2))\right] \\ &= L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n g^{(n+2)}(\alpha_S L) \end{aligned}$$

g(n) controls and resums the $\alpha_S L^k$ (k≥1) logarithmic terms

$$\widetilde{B}(\alpha_S) = B(\alpha_S) + 2\beta(\alpha_S)\frac{d\ln C(\alpha_S)}{d\ln\alpha_S} + 2\gamma(\alpha_S)$$

$$\lambda = \frac{1}{\pi}\beta_0\,\alpha_S(\mu_R^2)\,L \quad , \quad \overline{B}^{(n)} = \widetilde{B}^{(n)} + A^{(n)}\ln\frac{M^2}{Q^2}$$

- At N4LL we need the resummation coefficients
- A5 : 1–3·10⁻³ relative uncertainty
- B4 : negligible uncertainty
- C4 : 1–2·10⁻³ relative uncertainty
- γ4 singlet : 1–3·10⁻³ relative uncertainty (non-singlet negligible)

$$\begin{aligned} g^{(5)}(\alpha_S L) = & -\frac{A^{(5)}}{12\beta_0^2}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4} - \frac{\overline{B}^{(4)}}{3\beta_0}\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3} \\ & + \frac{A^{(4)}}{3\beta_0}\Bigg(\frac{\beta_1}{\beta_0^2}\Bigg[\frac{\lambda(-12+42\lambda-28\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Bigg] \\ & + \frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(3)}\Bigg(\frac{\beta_1}{\beta_0^2}\Bigg[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\Bigg] \\ & + \frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2}\Bigg) + A^{(3)}\Bigg(-\frac{\beta_2}{4\beta_0^3}\frac{\lambda^3(4-\lambda)}{(1-\lambda)^4} \\ & + \frac{\beta_1^2}{\beta_0^4}\Bigg[\frac{\lambda(12-24\lambda+52\lambda^2-13\lambda^3)}{36(1-\lambda)^4} + \frac{\ln(1-\lambda)}{3(1-\lambda)^3} + \frac{1-4\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda)\Bigg] \\ & + \frac{\beta_1}{\beta_0^2}\Bigg[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Bigg]\ln\frac{Q^2}{\mu_R^2} \\ & - \frac{\lambda^2(6-4\lambda+\lambda^2)}{2(1-\lambda)^4}\ln^2\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(2)}\Bigg(-\frac{\beta_2}{3\beta_0^2}\frac{(3-\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^3}\Bigg(\frac{(3-\lambda)\lambda^2}{3(1-\lambda)^3} \\ & - \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Bigg) - \frac{2\beta_1}{\beta_0}\frac{\ln(1-\lambda)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2} - \beta_0\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln^2\frac{Q^2}{\mu_R^2}\Bigg) \\ & + A^{(2)}\Bigg(-\frac{\beta_3}{12\beta_0^3}\frac{\lambda^3(8-5\lambda)}{(1-\lambda)^4} + \frac{\beta_1\beta_2}{3\beta_0^4}\Bigg(\frac{\lambda(6-21\lambda+44\lambda^2-20\lambda^3)}{6(1-\lambda)^4} \\ & + \frac{1-4\lambda+9\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Bigg) + \frac{\beta_1^3}{\beta_0^5}\Bigg(\frac{\lambda(-12+42\lambda-64\lambda^2+25\lambda^3)}{36(1-\lambda)^4} \\ & - \frac{(1-4\lambda+9\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Bigg) \\ & + \Bigg[\frac{\beta_2}{3\beta_0^2}\frac{(3+4\lambda-\lambda^2)\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{\beta_0^3}\Bigg(-\frac{(3+4\lambda-\lambda^2)\lambda^2}{3(1-\lambda)^4} - \frac{2\lambda}{(1-\lambda)^4}\ln(1-\lambda) \\ & - \frac{1-4\lambda}{(1-\lambda)^4}\ln^2(1-\lambda)\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{\beta_0}\Bigg[-\frac{\lambda}{(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Bigg]\ln^2\frac{Q^2}{\mu_R^2} \\ & + \frac{\beta_0}{3}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln^3\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(1)}\Bigg(-\frac{\beta_3}{6\beta_0^2}\frac{(3-2\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1\beta_2}{\beta_0^3}\Bigg(\frac{(3-2\lambda)\lambda^2}{3(1-\lambda)^3} \\ & + \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda)\Bigg) + \frac{\beta_1^3}{\beta_0^4}\Bigg(-\frac{(3-2\lambda)\lambda^2}{6(1-\lambda)^3} - \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda) - \frac{\ln^2(1-\lambda)}{2(1-\lambda)^3} \\ & + \frac{\ln^3(1-\lambda)}{3(1-\lambda)^3}\Bigg) + \Bigg[\frac{\beta_2}{\beta_0}\frac{\lambda}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^2}\Bigg(-\frac{\lambda}{(1-\lambda)^3} - \frac{\ln(1-\lambda)}{(1-\lambda)^3} + \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2} \\ & + \beta_1\Bigg[-\frac{\lambda(3-3\lambda+\lambda^2)}{2(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\Bigg]\ln^2\frac{Q^2}{\mu_R^2} + \beta_0^2\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3}\ln^3\frac{Q^2}{\mu_R^2}\Bigg) \\ & + A^{(1)}\Bigg(\frac{\beta_2^2}{3\beta_0^4}\Bigg(\frac{\lambda(-12+42\lambda-52\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \ln(1-\lambda)\Bigg) \\ & + \frac{\beta_4}{3\beta_0^3}\Bigg(\frac{\lambda(12-42\lambda+40\lambda^2-13\lambda^3)}{12(1-\lambda)^4} + \ln(1-\lambda)\Bigg) + \frac{\beta_1\beta_3}{6\beta_0^4}\Bigg(-\frac{\lambda(2-5\lambda)}{3}\frac{(3-3\lambda+\lambda^2)}{(1-\lambda)^4} \\ & - \frac{2-8\lambda+9\lambda^2-10\lambda^3+4\lambda^4}{(1-\lambda)^4}\ln(1-\lambda)\Bigg) + \frac{\beta_1^2\beta_2}{\beta_0^5}\Bigg(\frac{\lambda(12-42\lambda+52\lambda^2+5\lambda^3)}{36(1-\lambda)^4} \\ & - \frac{(-1+3\lambda-3\lambda^2+3\lambda^3)}{3(1-\lambda)^3}\ln(1-\lambda) - \frac{3\lambda^2}{2(1-\lambda)^4}\ln^2(1-\lambda)\Bigg) + \frac{\beta_1^4}{2\beta_0^6}\Bigg(-\frac{\lambda^3(2+3\lambda)}{6(1-\lambda)^4} \\ & + \frac{\lambda^2(-3+2\lambda-2\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-6\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda) \\ & + \frac{1-4\lambda}{6(1-\lambda)^4}\ln^4(1-\lambda)\Bigg) + \Bigg[-\frac{\beta_3}{6\beta_0^2}\frac{\lambda^2(-3-2\lambda+2\lambda^2)}{(1-\lambda)^4} - \frac{\beta_1\beta_2}{\beta_0^3}\Bigg(\frac{2\lambda^3}{3(1-\lambda)^3} + \frac{3\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Bigg) \\ & + \frac{\beta_1^3}{\beta_0^4}\Bigg(-\frac{\lambda^2(3-2\lambda+2\lambda^2)}{6(1-\lambda)^4} - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln(1-\lambda) - \frac{1-6\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda) \\ & + \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2} + \Bigg[-\frac{3\beta_2}{2\beta_0}\frac{\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{2\beta_0^2}\Bigg(-\frac{(1-3\lambda)\lambda}{(1-\lambda)^4} - \frac{(1-6\lambda)}{(1-\lambda)^4}\ln(1-\lambda) \\ & + \frac{(1-4\lambda)}{(1-\lambda)^4}\ln^2(1-\lambda)\Bigg)\Bigg]\ln^2\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{3}\Bigg[\frac{\lambda(2+6\lambda-4\lambda^2+\lambda^3)}{2(1-\lambda)^4} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Bigg]\ln^3\frac{Q^2}{\mu_R^2} \\ & - \frac{\beta_0^2}{12}\frac{(6-4\lambda+\lambda^2)\lambda^2}{(1-\lambda)^4}\ln^4\frac{Q^2}{\mu_R^2}\Bigg)\,, \end{aligned}$$

g(5) still fits in a slide!

Transverse momentum resummation

up to N4LL+N4LO accuracy

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$\mathcal{G}(\alpha_S, L) = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\ln\frac{M^2}{q^2} + \widetilde{B}(\alpha_S(q^2))\right]$$

$$= L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n g^{(n+2)}(\alpha_S L)$$

g(n) controls and resums the $\alpha_S L^k$ (k≥1) logarithmic terms

$$\widetilde{B}(\alpha_S) = B(\alpha_S) + 2\beta(\alpha_S)\frac{d\ln C(\alpha_S)}{d\ln\alpha_S} + 2\gamma(\alpha_S)$$

$$\lambda = \frac{1}{\pi}\beta_0\,\alpha_S(\mu_R^2)\,L \quad , \quad \overline{B}^{(n)} = \widetilde{B}^{(n)} + A^{(n)}\ln\frac{M^2}{Q^2}$$

- At N4LL we need the resummation coefficients
- A5 : 1–3·10⁻³ relative uncertainty
- B4 : negligible uncertainty
- C4 : 1–2·10⁻³ relative uncertainty
- γ4 singlet : 1–3·10⁻³ relative uncertainty (non-singlet negligible)

$$g^{(5)}(\alpha_S L) = -\frac{A^{(5)}}{12\beta_0^2}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4} - \frac{\overline{B}^{(4)}}{3\beta_0}\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}$$
$$+ \frac{A^{(4)}}{3\beta_0}\Bigg(\frac{\beta_1}{\beta_0^2}\left[\frac{\lambda(-12+42\lambda-28\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\right]$$
$$+ \frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(3)}\Bigg(\frac{\beta_1}{\beta_0^2}\left[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\right]$$
$$+ \frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2}\Bigg) + A^{(3)}\Bigg(-\frac{\beta_2}{4\beta_0^3}\frac{\lambda^3(4-\lambda)}{(1-\lambda)^4}$$
$$+ \frac{\beta_1^2}{\beta_0^4}\left[\frac{\lambda(12-24\lambda+52\lambda^2-13\lambda^3)}{36(1-\lambda)^4} + \frac{\ln(1-\lambda)}{3(1-\lambda)^3} + \frac{1-4\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda)\right]$$
$$+ \frac{\beta_1}{\beta_0^2}\left[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\right]\ln\frac{Q^2}{\mu_R^2}$$
$$- \frac{\lambda^2(6-4\lambda+\lambda^2)}{2(1-\lambda)^4}\ln^2\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(2)}\Bigg(-\frac{\beta_2}{3\beta_0^2}\frac{(3-\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^3}\Bigg(\frac{(3-\lambda)\lambda^2}{3(1-\lambda)^3}$$
$$- \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Bigg) - \frac{2\beta_1}{\beta_0}\frac{\ln(1-\lambda)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2} - \beta_0\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln^2\frac{Q^2}{\mu_R^2}\Bigg)$$
$$+ A^{(2)}\Bigg(-\frac{\beta_3}{12\beta_0^3}\frac{\lambda^3(8-5\lambda)}{(1-\lambda)^4} + \frac{\beta_1\beta_2}{3\beta_0^4}\Bigg(\frac{\lambda(6-21\lambda+44\lambda^2-20\lambda^3)}{6(1-\lambda)^4}$$
$$+ \frac{1-4\lambda+9\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Bigg) + \frac{\beta_1^3}{\beta_0^5}\Bigg(\frac{\lambda(-12+42\lambda-64\lambda^2+25\lambda^3)}{36(1-\lambda)^4}$$
$$- \frac{(1-4\lambda+9\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Bigg)$$
$$+ \Bigg[\frac{\beta_2}{3\beta_0^2}\frac{(3+4\lambda-\lambda^2)\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{\beta_0^3}\Bigg(-\frac{(3+4\lambda-\lambda^2)\lambda^2}{3(1-\lambda)^4} - \frac{2\lambda}{(1-\lambda)^4}\ln(1-\lambda)$$
$$- \frac{1-4\lambda}{(1-\lambda)^4}\ln^2(1-\lambda)\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{\beta_0}\left[-\frac{\lambda}{(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\right]\ln^2\frac{Q^2}{\mu_R^2}$$
$$+ \frac{\beta_0}{3}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln^3\frac{Q^2}{\mu_R^2}\Bigg) + \overline{B}^{(1)}\Bigg(-\frac{\beta_3}{6\beta_0^2}\frac{(3-2\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1\beta_2}{\beta_0^3}\Bigg(\frac{(3-2\lambda)\lambda^2}{3(1-\lambda)^3}$$
$$+ \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda)\Bigg) + \frac{\beta_1^3}{\beta_0^4}\Bigg(-\frac{(3-2\lambda)\lambda^2}{6(1-\lambda)^3} - \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda) - \frac{\ln^2(1-\lambda)}{2(1-\lambda)^3}$$
$$+ \frac{\ln^3(1-\lambda)}{3(1-\lambda)^3}\Bigg) + \Bigg[\frac{\beta_2}{\beta_0}\frac{\lambda}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^2}\Bigg(-\frac{\lambda}{(1-\lambda)^3} - \frac{\ln(1-\lambda)}{(1-\lambda)^3} + \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2}$$
$$+ \beta_1\left[-\frac{\lambda(3-3\lambda+\lambda^2)}{2(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\right]\ln^2\frac{Q^2}{\mu_R^2} + \beta_0^2\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3}\ln^3\frac{Q^2}{\mu_R^2}\Bigg)$$
$$+ A^{(1)}\Bigg(\frac{\beta_2^2}{3\beta_0^4}\Bigg(\frac{\lambda(-12+42\lambda-52\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \ln(1-\lambda)\Bigg)$$
$$+ \frac{\beta_4}{3\beta_0^3}\Bigg(\frac{\lambda(12-42\lambda+40\lambda^2-13\lambda^3)}{12(1-\lambda)^4} + \ln(1-\lambda)\Bigg) + \frac{\beta_1\beta_3}{6\beta_0^4}\Bigg(-\frac{\lambda(2-5\lambda)}{3}\frac{(3-3\lambda+\lambda^2)}{(1-\lambda)^4}$$
$$- \frac{2-8\lambda+9\lambda^2-10\lambda^3+4\lambda^4}{(1-\lambda)^4}\ln(1-\lambda)\Bigg) + \frac{\beta_1^2\beta_2}{\beta_0^5}\Bigg(\frac{\lambda(12-42\lambda+52\lambda^2+5\lambda^3)}{36(1-\lambda)^4}$$
$$- \frac{(-1+3\lambda-3\lambda^2+3\lambda^3)}{3(1-\lambda)^3}\ln(1-\lambda) - \frac{3\lambda^2}{2(1-\lambda)^4}\ln^2(1-\lambda)\Bigg) + \frac{\beta_1^4}{2\beta_0^6}\Bigg(-\frac{\lambda^3(2+3\lambda)}{6(1-\lambda)^4}$$
$$+ \frac{\lambda^2(-3+2\lambda-2\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-6\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)$$
$$+ \frac{1-4\lambda}{6(1-\lambda)^4}\ln^4(1-\lambda)\Bigg) + \Bigg[-\frac{\beta_3}{6\beta_0^2}\frac{\lambda^2(-3-2\lambda+2\lambda^2)}{(1-\lambda)^4} - \frac{\beta_1\beta_2}{\beta_0^3}\Bigg(\frac{2\lambda^3}{3(1-\lambda)^3} + \frac{3\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Bigg)$$
$$+ \frac{\beta_1^3}{\beta_0^4}\Bigg(-\frac{\lambda^2(3-2\lambda+2\lambda^2)}{6(1-\lambda)^4} - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln(1-\lambda) - \frac{1-6\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda)$$
$$+ \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Bigg)\Bigg]\ln\frac{Q^2}{\mu_R^2} + \Bigg[-\frac{3\beta_2}{2\beta_0}\frac{\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{2\beta_0^2}\Bigg(-\frac{(1-3\lambda)\lambda}{(1-\lambda)^4} - \frac{(1-6\lambda)}{(1-\lambda)^4}\ln(1-\lambda)$$
$$+ \frac{(1-4\lambda)}{(1-\lambda)^4}\ln^2(1-\lambda)\Bigg)\Bigg]\ln^2\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{3}\left[\frac{\lambda(2+6\lambda-4\lambda^2+\lambda^3)}{2(1-\lambda)^4} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\right]\ln^3\frac{Q^2}{\mu_R^2}$$
$$- \frac{\beta_0^2}{12}\frac{(6-4\lambda+\lambda^2)\lambda^2}{(1-\lambda)^4}\ln^4\frac{Q^2}{\mu_R^2}\Bigg) ,$$

g(5) still fits in a slide!

Transverse momentum resummation

up to N4LL+N4LO accuracy

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$\begin{aligned} \mathcal{G}(\alpha_S, L) &= -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + \widetilde{B}(\alpha_S(q^2))\right] \\ &= L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n g^{(n+2)}(\alpha_S L) \end{aligned}$$

g(n) controls and resums the $\alpha_S L^k$ ($k \geq 1$) logarithmic terms

$$\widetilde{B}(\alpha_S) = B(\alpha_S) + 2\beta(\alpha_S)\frac{d\ln C(\alpha_S)}{d\ln\alpha_S} + 2\gamma(\alpha_S)$$

$$\lambda = \frac{1}{\pi}\beta_0\,\alpha_S(\mu_R^2)\,L \;\;,\;\; \overline{B}^{(n)} = \widetilde{B}^{(n)} + A^{(n)}\ln\frac{M^2}{Q^2}$$

- At N4LL we need the resummation coefficients
- A5 : 1–3·10^{-3} relative uncertainty
- B4 : negligible uncertainty
- C4 : 1–2·10^{-3} relative uncertainty
- γ4 singlet : 1–3·10^{-3} relative uncertainty (non-singlet negligible)

$$\begin{aligned}
g^{(5)}(\alpha_S L) = & -\frac{A^{(5)}}{12\beta_0^2}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4} - \frac{\overline{B}^{(4)}}{3\beta_0}\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3} \\
& + \frac{A^{(4)}}{3\beta_0}\Big(\frac{\beta_1}{\beta_0^2}\Big[\frac{\lambda(-12+42\lambda-28\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Big] \\
& + \frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln\frac{Q^2}{\mu_R^2}\Big) + \overline{B}^{(3)}\Big(\frac{\beta_1}{\beta_0^2}\Big[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\Big] \\
& + \frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2}\Big) + A^{(3)}\Big(-\frac{\beta_2}{4\beta_0^3}\frac{\lambda^3(4-\lambda)}{(1-\lambda)^4} \\
& + \frac{\beta_1^2}{\beta_0^4}\Big[\frac{\lambda(12-24\lambda+52\lambda^2-13\lambda^3)}{36(1-\lambda)^4} + \frac{\ln(1-\lambda)}{3(1-\lambda)^3} + \frac{1-4\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda)\Big] \\
& + \frac{\beta_1}{\beta_0^2}\Big[\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Big]\ln\frac{Q^2}{\mu_R^2} \\
& - \frac{\lambda^2(6-4\lambda+\lambda^2)}{2(1-\lambda)^4}\ln^2\frac{Q^2}{\mu_R^2}\Big) + \overline{B}^{(2)}\Big(-\frac{\beta_2}{3\beta_0^2}\frac{(3-\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^3}\Big(\frac{(3-\lambda)\lambda^2}{3(1-\lambda)^3} \\
& - \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Big) - \frac{2\beta_1}{\beta_0}\frac{\ln(1-\lambda)}{(1-\lambda)^3}\ln\frac{Q^2}{\mu_R^2} - \beta_0\frac{\lambda(3-3\lambda+\lambda^2)}{(1-\lambda)^3}\ln^2\frac{Q^2}{\mu_R^2}\Big) \\
& + A^{(2)}\Big(-\frac{\beta_3}{12\beta_0^3}\frac{\lambda^3(8-5\lambda)}{(1-\lambda)^4} + \frac{\beta_1\beta_2}{3\beta_0^4}\Big(\frac{\lambda(6-21\lambda+44\lambda^2-20\lambda^3)}{6(1-\lambda)^4} \\
& + \frac{1-4\lambda+9\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Big) + \frac{\beta_1^3}{\beta_0^5}\Big(\frac{\lambda(-12+42\lambda-64\lambda^2+25\lambda^3)}{36(1-\lambda)^4} \\
& - \frac{(1-4\lambda+9\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Big) \\
& + \Big[\frac{\beta_2}{3\beta_0^2}\frac{(3+4\lambda-\lambda^2)\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{\beta_0^3}\Big(-\frac{(3+4\lambda-\lambda^2)\lambda^2}{3(1-\lambda)^4} - \frac{2\lambda}{(1-\lambda)^4}\ln(1-\lambda) \\
& - \frac{1-4\lambda}{(1-\lambda)^4}\ln^2(1-\lambda)\Big)\Big]\ln\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{\beta_0}\Big[-\frac{\lambda}{(1-\lambda)^4} - \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Big]\ln^2\frac{Q^2}{\mu_R^2} \\
& + \frac{\beta_0}{3}\frac{\lambda^2(6-4\lambda+\lambda^2)}{(1-\lambda)^4}\ln^3\frac{Q^2}{\mu_R^2}\Big) + \overline{B}^{(1)}\Big(-\frac{\beta_3}{6\beta_0^2}\frac{(3-2\lambda)\lambda^2}{(1-\lambda)^3} + \frac{\beta_1\beta_2}{\beta_0^3}\Big(\frac{(3-2\lambda)\lambda^2}{3(1-\lambda)^3} \\
& + \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda)\Big) + \frac{\beta_1^3}{\beta_0^4}\Big(-\frac{(3-2\lambda)\lambda^2}{6(1-\lambda)^3} - \frac{\lambda}{(1-\lambda)^3}\ln(1-\lambda) - \frac{\ln^2(1-\lambda)}{2(1-\lambda)^3} \\
& + \frac{\ln^3(1-\lambda)}{3(1-\lambda)^3}\Big) + \Big[\frac{\beta_2}{\beta_0}\frac{\lambda}{(1-\lambda)^3} + \frac{\beta_1^2}{\beta_0^2}\Big(-\frac{\lambda}{(1-\lambda)^3} - \frac{\ln(1-\lambda)}{(1-\lambda)^3} + \frac{\ln^2(1-\lambda)}{(1-\lambda)^3}\Big)\Big]\ln\frac{Q^2}{\mu_R^2} \\
& + \beta_1\Big[-\frac{\lambda(3-3\lambda+\lambda^2)}{2(1-\lambda)^3} + \frac{\ln(1-\lambda)}{(1-\lambda)^3}\Big]\ln^2\frac{Q^2}{\mu_R^2} + \beta_0^2\frac{\lambda(3-3\lambda+\lambda^2)}{3(1-\lambda)^3}\ln^3\frac{Q^2}{\mu_R^2}\Big) \\
& + A^{(1)}\Big(\frac{\beta_2^2}{3\beta_0^4}\Big(\frac{\lambda(-12+42\lambda-52\lambda^2+7\lambda^3)}{12(1-\lambda)^4} - \ln(1-\lambda)\Big) \\
& + \frac{\beta_4}{3\beta_0^3}\Big(\frac{\lambda(12-42\lambda+40\lambda^2-13\lambda^3)}{12(1-\lambda)^4} + \ln(1-\lambda)\Big) + \frac{\beta_1\beta_3}{6\beta_0^4}\Big(-\frac{\lambda(2-5\lambda)}{3}\frac{(3-3\lambda+\lambda^2)}{(1-\lambda)^4} \\
& - \frac{2-8\lambda+9\lambda^2-10\lambda^3+4\lambda^4}{(1-\lambda)^4}\ln(1-\lambda)\Big) + \frac{\beta_1^2\beta_2}{\beta_0^5}\Big(\frac{\lambda(12-42\lambda+52\lambda^2+5\lambda^3)}{36(1-\lambda)^4} \\
& - \frac{(-1+3\lambda-3\lambda^2+3\lambda^3)}{3(1-\lambda)^3}\ln(1-\lambda) - \frac{3\lambda^2}{2(1-\lambda)^4}\ln^2(1-\lambda)\Big) + \frac{\beta_1^4}{2\beta_0^6}\Big(-\frac{\lambda^3(2+3\lambda)}{6(1-\lambda)^4} \\
& + \frac{\lambda^2(-3+2\lambda-2\lambda^2)}{3(1-\lambda)^4}\ln(1-\lambda) - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln^2(1-\lambda) - \frac{1-6\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda) \\
& + \frac{1-4\lambda}{6(1-\lambda)^4}\ln^4(1-\lambda)\Big) + \Big[-\frac{\beta_3}{6\beta_0^2}\frac{\lambda^2(-3-2\lambda+2\lambda^2)}{(1-\lambda)^4} - \frac{\beta_1\beta_2}{\beta_0^3}\Big(\frac{2\lambda^3}{3(1-\lambda)^3} + \frac{3\lambda^2}{(1-\lambda)^4}\ln(1-\lambda)\Big) \\
& + \frac{\beta_1^3}{\beta_0^4}\Big(-\frac{\lambda^2(3-2\lambda+2\lambda^2)}{6(1-\lambda)^4} - \frac{(1-3\lambda)\lambda}{(1-\lambda)^4}\ln(1-\lambda) - \frac{1-6\lambda}{2(1-\lambda)^4}\ln^2(1-\lambda) \\
& + \frac{1-4\lambda}{3(1-\lambda)^4}\ln^3(1-\lambda)\Big)\Big]\ln\frac{Q^2}{\mu_R^2} + \Big[-\frac{3\beta_2}{2\beta_0}\frac{\lambda^2}{(1-\lambda)^4} + \frac{\beta_1^2}{2\beta_0^2}\Big(-\frac{(1-3\lambda)\lambda}{(1-\lambda)^4} - \frac{(1-6\lambda)}{(1-\lambda)^4}\ln(1-\lambda) \\
& + \frac{(1-4\lambda)}{(1-\lambda)^4}\ln^2(1-\lambda)\Big)\Big]\ln^2\frac{Q^2}{\mu_R^2} + \frac{\beta_1}{3}\Big[\frac{\lambda(2+6\lambda-4\lambda^2+\lambda^3)}{2(1-\lambda)^4} + \frac{1-4\lambda}{(1-\lambda)^4}\ln(1-\lambda)\Big]\ln^3\frac{Q^2}{\mu_R^2} \\
& - \frac{\beta_0^2}{12}\frac{(6-4\lambda+\lambda^2)\lambda^2}{(1-\lambda)^4}\ln^4\frac{Q^2}{\mu_R^2}\Big),
\end{aligned}$$

g(5) still fits in a slide!

Transverse momentum resummation

Camarda, LC, Ferrera [2023]

up to N4LL+N4LO accuracy

$$\mathcal{W}_V(b,M;\alpha_S,\mu_R^2,\mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$\begin{aligned}\mathcal{G}(\alpha_S,L) &= -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\ln\frac{M^2}{q^2} + \widetilde{B}(\alpha_S(q^2))\right] \\ &= L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n g^{(n+2)}(\alpha_S L)\end{aligned}$$

g(n) controls and resums the $\alpha_S L^k$ (k≥1) logarithmic terms

$$\widetilde{B}(\alpha_S) = B(\alpha_S) + 2\beta(\alpha_S)\frac{d\ln C(\alpha_S)}{d\ln\alpha_S} + 2\gamma(\alpha_S)$$

$$\lambda = \frac{1}{\pi}\beta_0\,\alpha_S(\mu_R^2)\,L \quad , \quad \overline{B}^{(n)} = \widetilde{B}^{(n)} + A^{(n)}\ln\frac{M^2}{Q^2}$$

- At N4LL we need the resummation coefficients
- A5 : 1–3·10⁻³ relative uncertainty
- B4 : negligible uncertainty
- C4 : 1–2·10⁻³ relative uncertainty
- γ4 singlet : 1–3·10⁻³ relative uncertainty (non-singlet negligible)

Herzog, Moch, Ruijl, Ueda, Vermaseren, Vogt [2019]
Henn, Korchemsky, Mistlberger [2020]
von Manteuffel, Panzer, Schabinger [2020]
Moult, Xing Zhu, Jiao Zhu [2022]

Moch, Ruijl, Ueda, Vermaseren, Vogt [2017]
Falcioni, Herzog, Moch, Vogt [2023] Moch, Ruijl, Ueda, Vermaseren, Vogt [2022]

Transverse momentum resummation

up to N4LL+N4LO accuracy

Camarda, LC, Ferrera [2023]

$$\mathcal{W}_V(b,M;\alpha_S,\mu_R^2,\mu_F^2) = \mathcal{H}_V\,(\alpha_S;M/\mu_R,M/\mu_F,M/Q) \times \exp\{\mathcal{G}(\alpha_S,L;M/\mu_R,M/Q)\}$$

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We rely on the Levin transform assigning 100% uncertainty → We assume that the Levin transform estimates the correct sign and order of magnitude.

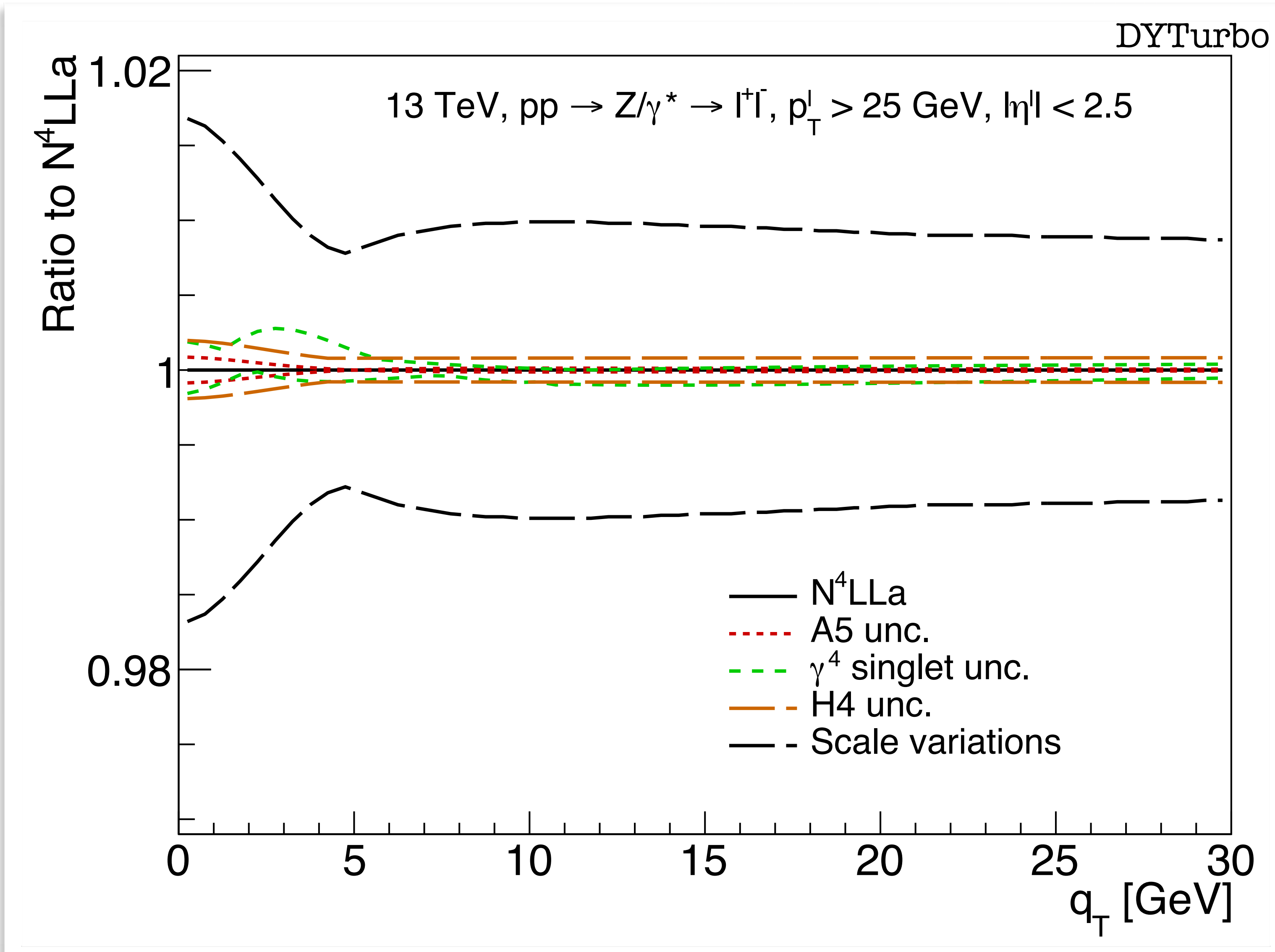
David Levin [1972]

Transverse momentum resummation

Camarda, LC, Ferrera [2023]

up to N4LL+N4LO accuracy

Anticipating our results



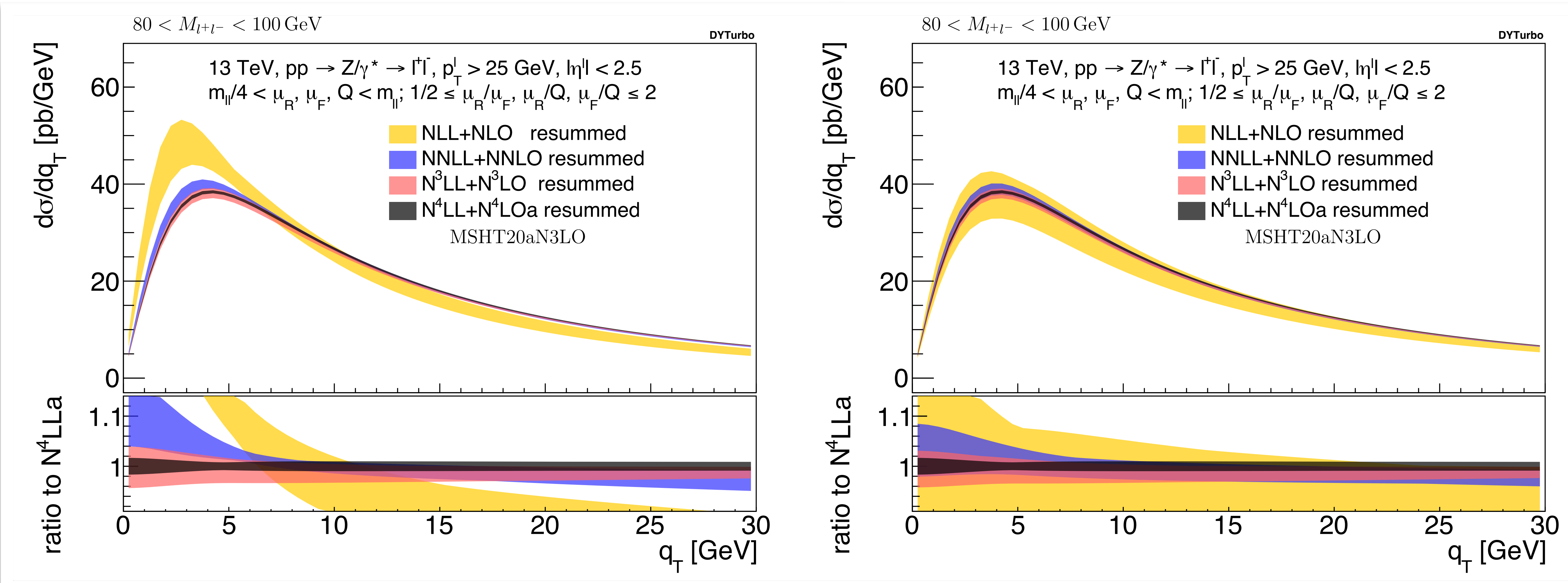
- The uncertainties in the N4LL+N4LO approximation are found to be 5 to 10 times smaller compared to the missing higher order uncertainties estimated through scale variations.
- All “main” channels already present at NNLO : qqbar, qg, gg.
- N4LL is the first order at which all the combination of the channels are opened: {q,qb,qp,qbp,g} x {q,qb,qp,qbp,g} (all combinations)

qT resummation up to N4LLa accuracy

Results

Camarda, LC, Ferrera [2023]

The qT spectrum of Z/γ* bosons with lepton selection cuts at the LHC (√s = 13 TeV) at various perturbative orders



Formal mismatch between the N³LO Altarelli-Parisi evolution as encoded in the N³LO parton densities functions and the corresponding NᵏLO evolution included in the N$^{(k+1)}$LL partonic resummed formula.

The order of Altarelli-Parisi evolution in the resummed prediction is equal to the order of the parton densities

- Negligible impact at N3LL and N4LL on the choice → we apply this strategy in the next slides
- Scale dependence reduced a factor 2 from N3LL to N4LL → N4LL accuracy is at the 1%-1.5%

qT resummation up to N4LLa accuracy

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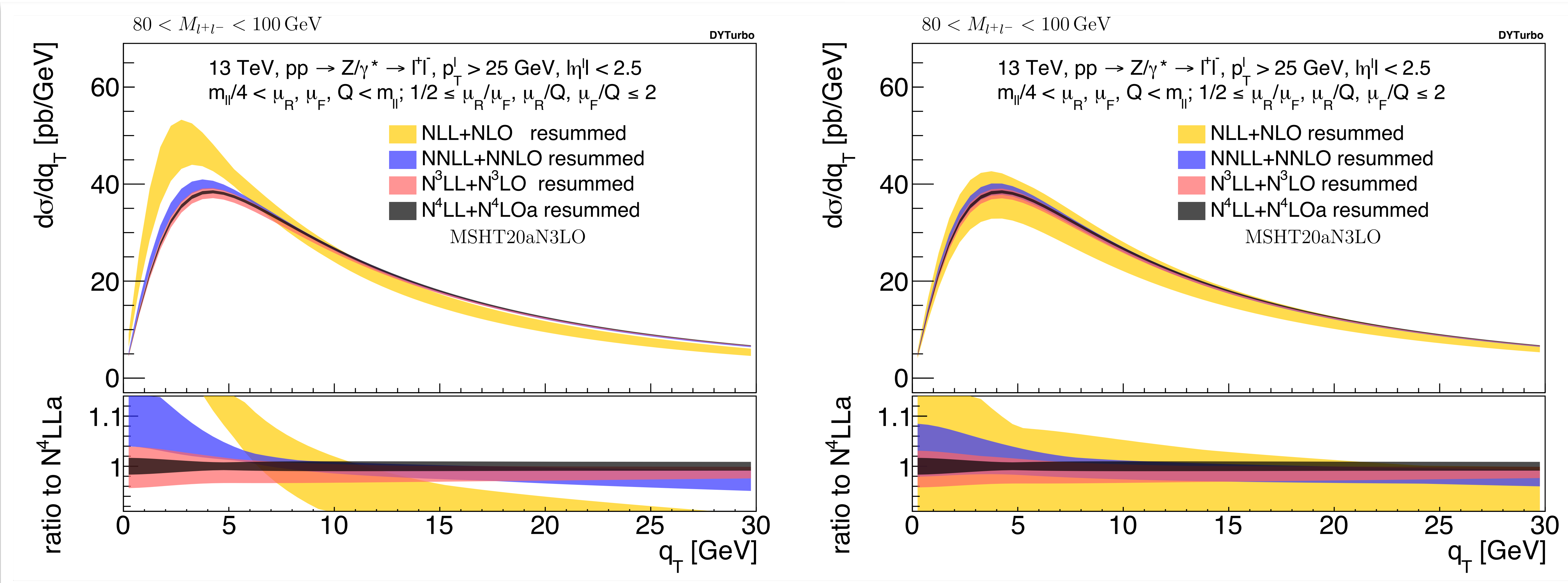
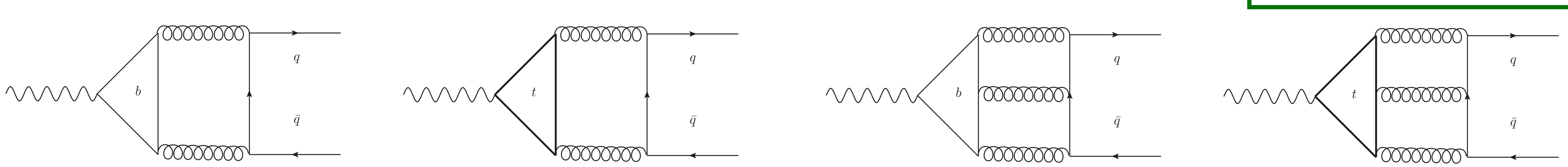


- Negligible impact at N3LL and N4LL on the choice → we apply this strategy in the next slides
- Scale dependence reduced a factor 2 from N3LL to N4LL → N4LL accuracy is at the 1%-1.5%
- Effect of a finite top-quark mass including the singlet contributions mediated by heavy-quark loops at NNLO and N³LO included

–0.04% at NNLO and less than +0.001% at N3LO



Rijken, van Neerven [1995]

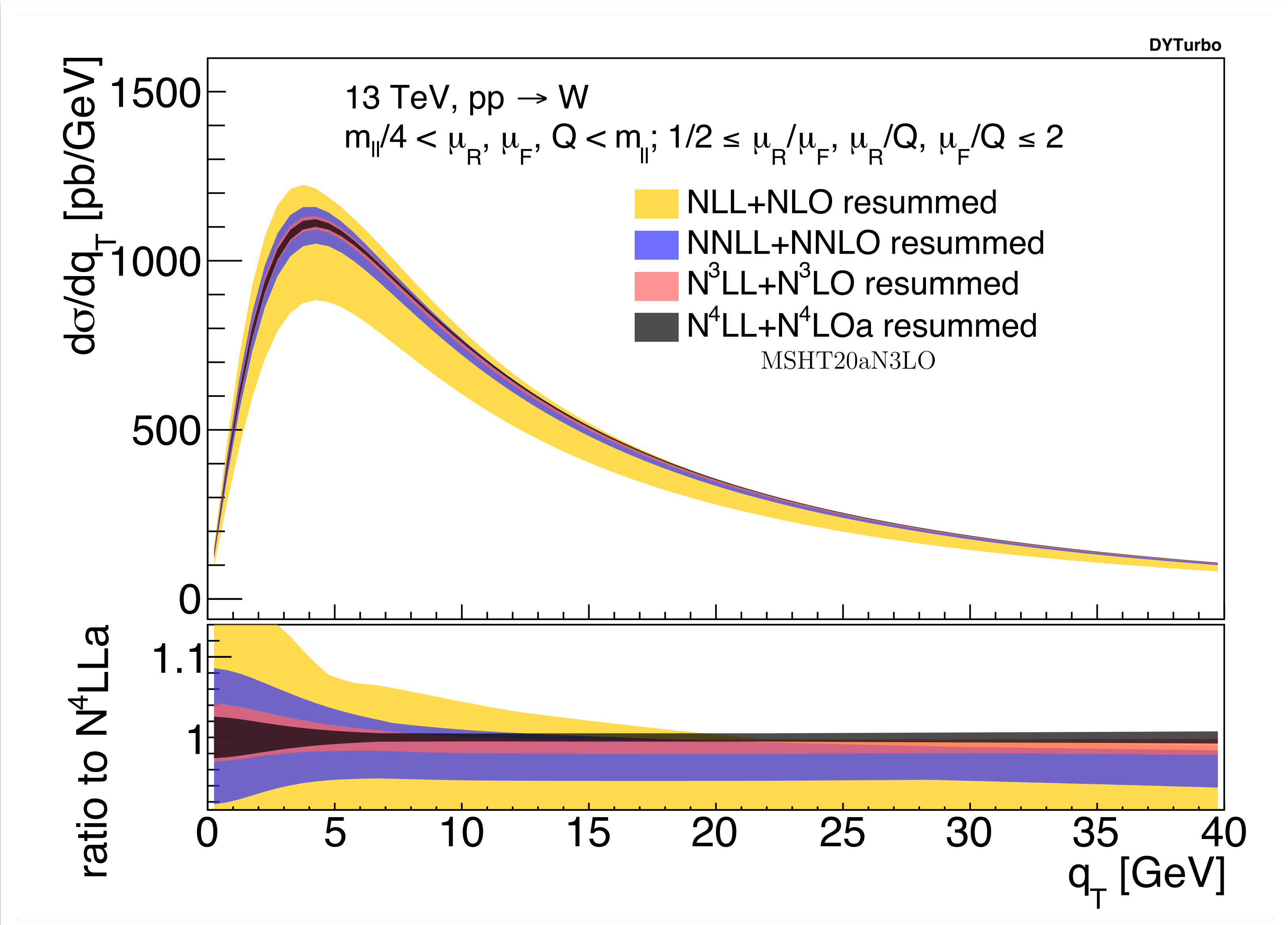
Chen, Czakon, Niggetiedt [2021]

qT resummation up to N4LLa accuracy

Results

Camarda, LC, Ferrera [2023]

The qT spectrum of W+ and W– bosons with inclusive leptonic decay at the LHC (√s = 13 TeV) at various perturbative orders

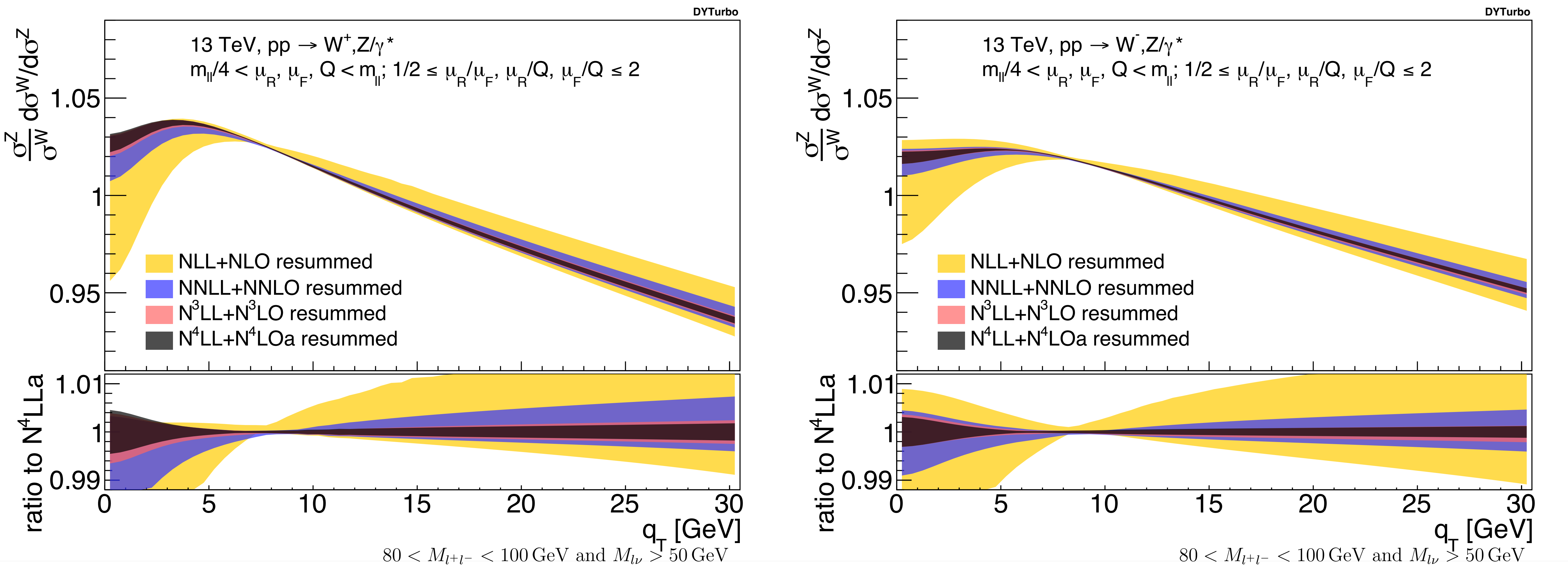


The scale variation at N4LLa accuracy is around ±2% at qT ~ 1GeV, then it reduces at ±1% level at the peak (qT ~ 4GeV), it decreases further to ±0.5% for qT ~ 7 GeV and remains below ±1% level up to qT ~ 30 GeV.

qT resummation up to N4LLa accuracy

Results

Camarda, LC, Ferrera [2023]



OK if the mechanisms in the numerator and denominator are roughly the same

$$R(q_T) = \frac{\sigma_Z}{\sigma_W}\frac{d\sigma_W}{dq_T}\Big/\frac{d\sigma_Z}{dq_T}$$

- Correlated scale variation
- N4LL very relevant removing uncertainties in the W/Z pT distribution ratio
- However, analysis is not complete : flavour-dependent intrinsic kT, **process-dependent EW effects**

Next part of the talk

QED+QCD q_T resummation at NLL+NLO

QED+QCD qT resummation at NLL+NLO

Main differences respect to pure QCD case

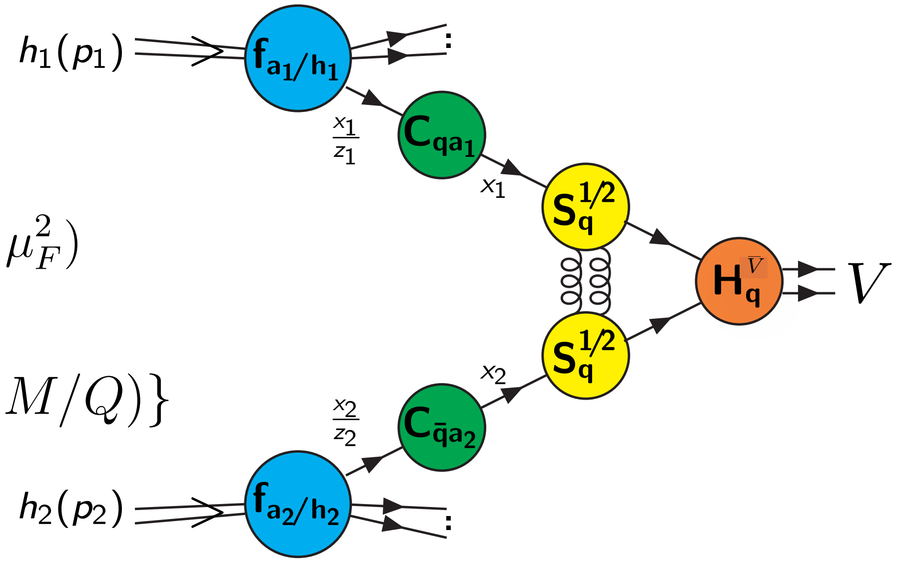
Autieri, LC, Ferrera, Sborlini [2023]

On-shell Z and W production

The cross section can be decomposed as

Catani, Grazzini, Torre [2014]

$$[d\hat{\sigma}_{a_1a_2\to l_3l_4}] = [d\hat{\sigma}^{(\text{res.})}_{a_1a_2\to l_3l_4}] + [d\hat{\sigma}^{(\text{fin.})}_{a_1a_2\to l_3l_4}]$$



$$\left[d\hat{\sigma}^{(\text{res.})}_{a_1a_2\to l_3l_4}\right] = \sum_{b_1,b_2=q,\bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1b_2\to l_3l_4}}{d\Omega} \frac{1}{\hat{s}} \int_0^\infty \frac{db}{2\pi}\, b\, J_0(bq_T)\, \mathcal{W}_{a_1a_2,b_1b_2\to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2)$$

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$

$$L = \log\left(\frac{b^2\,Q^2}{b_0^2} + 1\right)$$

Here we include the f.o predictions at NLO (QED and QCD)
Unitary constraint

- Z on-shell at NLL+NLO: colourless and chargeless final state LC, Ferrera, Sborlini [2018]
- W on-shell at NLL+NLO: colourless and charged final state → New Autieri, LC, Ferrera, Sborlini [2023]

The resummation formalism can be obtained with plain abelianization of the QCD results and it will be not presented in detail here

Naive abelianization of the QCD results does not work. Apart from this fact, the (more involved) abelizanization procedure has to be applied to the QCD resummation for ttbar final state

QED+QCD qT resummation at NLL+NLO

Main differences respect to pure QCD case

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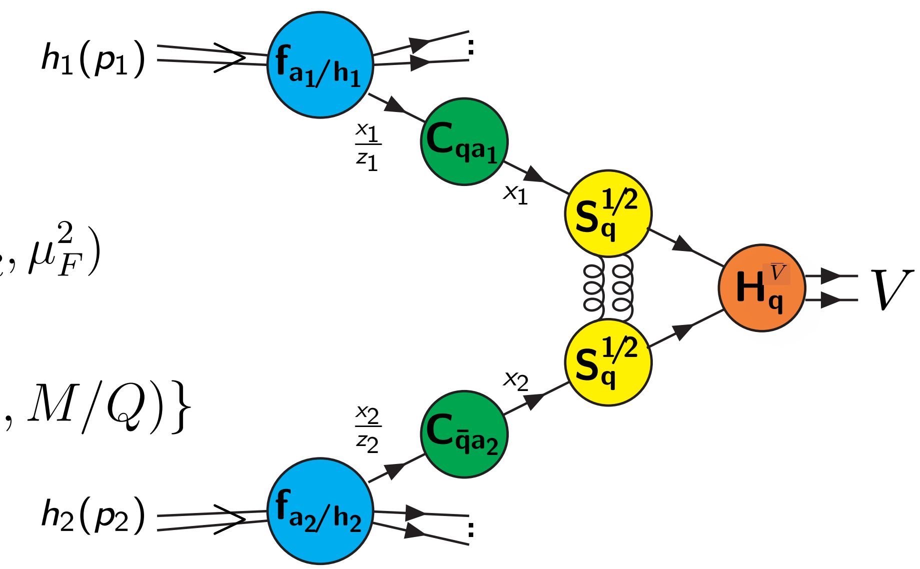

- W on-shell at NLL+NLO: colourless and charged final state → New   Autieri, LC, Ferrera, Sborlini [2023]

$$\mathcal{G}'_N(\alpha, L) = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left(A'(\alpha(q^2))\log\left(\frac{M^2}{q^2}\right) + \widetilde{B}'_N(\alpha(q^2)) + D'(\alpha(q^2))\right)$$

$$D'(\alpha) = \frac{\alpha}{\pi} D'^{(1)} + \sum_{n=2}^{+\infty}\left(\frac{\alpha}{\pi}\right)^n D'^{(n)}$$

$$\mathcal{G}'_N(\alpha_S, \alpha, L) = \mathcal{G}_N(\alpha_S, L) + L\, g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{+\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g_N'^{(n)}(\alpha L)$$

$$+\, g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{+\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L)$$

New linear logarithmic term

It is specific of charged high-mass system production and it is due to QED soft non-collinear (wide angle) radiation from the underlying subprocess

$$D'^{(1)} = -\frac{e_V^2}{2}$$

QED+QCD qT resummation at NLL+NLO

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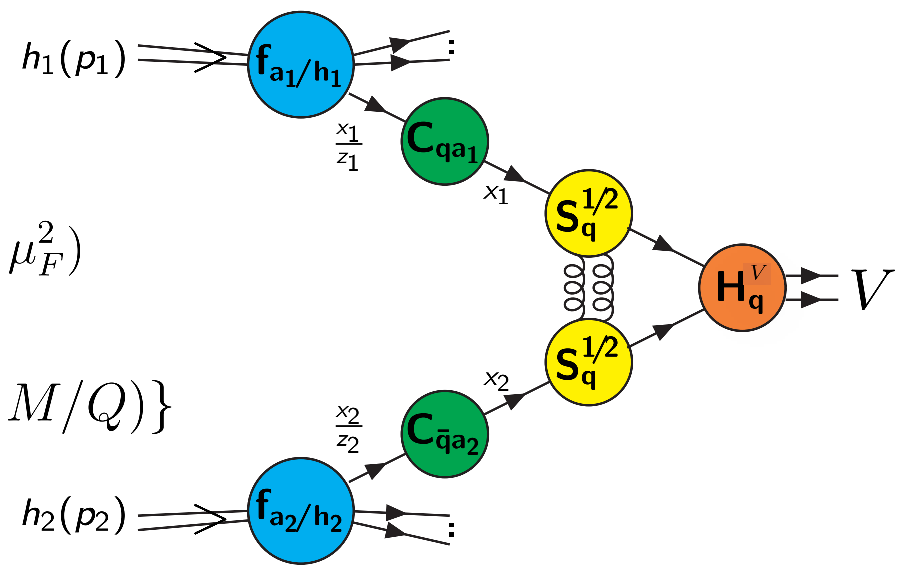

$$\left[d\hat{\sigma}^{(\mathrm{res.})}_{a_1a_2\to l_3l_4}\right] = \sum_{b_1,b_2=q,\bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1b_2\to l_3l_4}}{d\Omega} \frac{1}{\hat{s}} \int_0^\infty \frac{db}{2\pi}\, b\, J_0(bq_T)\, \mathcal{W}_{a_1a_2,b_1b_2\to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2)$$

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$$D'(\alpha) = \frac{\alpha}{\pi} D'^{(1)} + \sum_{n=2}^{+\infty}\left(\frac{\alpha}{\pi}\right)^n D'^{(n)}$$

$$g'^{(1)}(\alpha L) = \frac{A_q'^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1-\lambda')}{\lambda'}\,,$$

$$g_N'^{(2)}(\alpha L) = \frac{\widetilde{B}'^{(1)}_{q,N}}{\beta_0'}\ln(1-\lambda') - \frac{A_q'^{(2)}}{\beta_0'^2}\left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda')\right) + \frac{A_q'^{(1)}\beta_1'}{\beta_0'^3}\left(\frac{1}{2}\ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}\right)$$

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)}\beta_{0,1}}{\beta_0^2\beta_0'} h(\lambda, \lambda') + \frac{A_q'^{(1)}\beta_{0,1}'}{\beta_0'^2\beta_0} h(\lambda', \lambda)$$

$$h(\lambda,\lambda') = -\frac{\lambda'}{\lambda-\lambda'}\ln(1-\lambda) + \ln(1-\lambda')\left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'}\right)\right] - \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'}\right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'}\right),$$

"New" mixed contribution not present by trivial abelianization of QCD results

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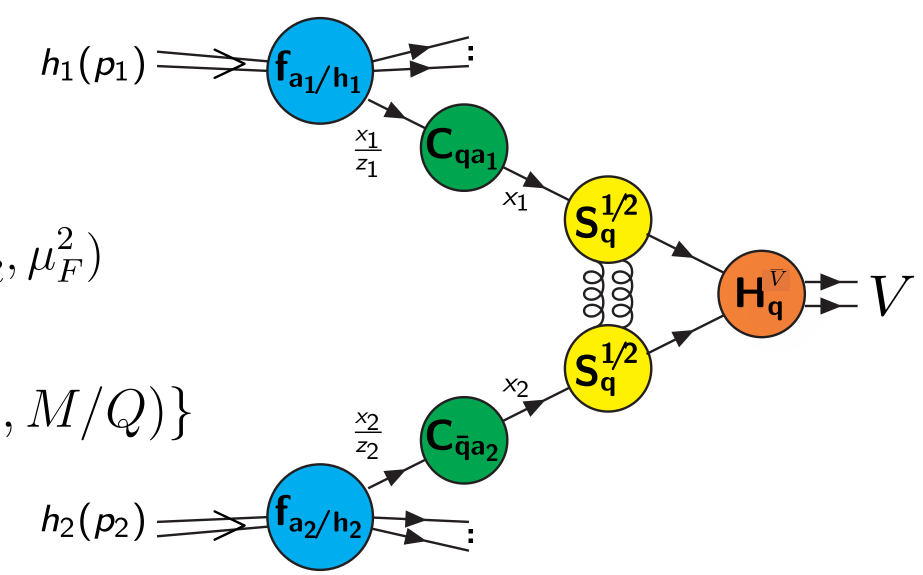

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New linear logarithmic term

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$$A'^{(1)} = \frac{e_{q_f}^2 + e_{\bar{q}_{f'}}^2}{2},$$

$$A'^{(2)} = -\frac{5}{9} \frac{e_{q_f}^2 + e_{\bar{q}_{f'}}^2}{2} N^{(2)},$$

$$\widetilde{B}'^{(1)}_N = B'^{(1)} + \gamma'^{(1)}_{q_f q_f, N} + \gamma'^{(1)}_{\bar{q}_{f'} \bar{q}_{f'}, N},$$

$$N^{(2)} = 3 \sum_{q=1}^{n_f} e_q^2 + \sum_{l=1}^{n_l} e_l^2,$$

$$B'^{(1)} = -\frac{3}{2} \frac{e_{q_f}^2 + e_{\bar{q}_{f'}}^2}{2},$$

$$\gamma'^{(1)}_{qq,N} = e_q^2 \left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1) \right)$$

$$\gamma'^{(1)}_{q\gamma,N} = \frac{3}{2} e_q^2 \frac{N^2 + N + 2}{N(N+1)(N+2)},$$

$$D'^{(1)} = -\frac{e_V^2}{2}$$

[DYqT]

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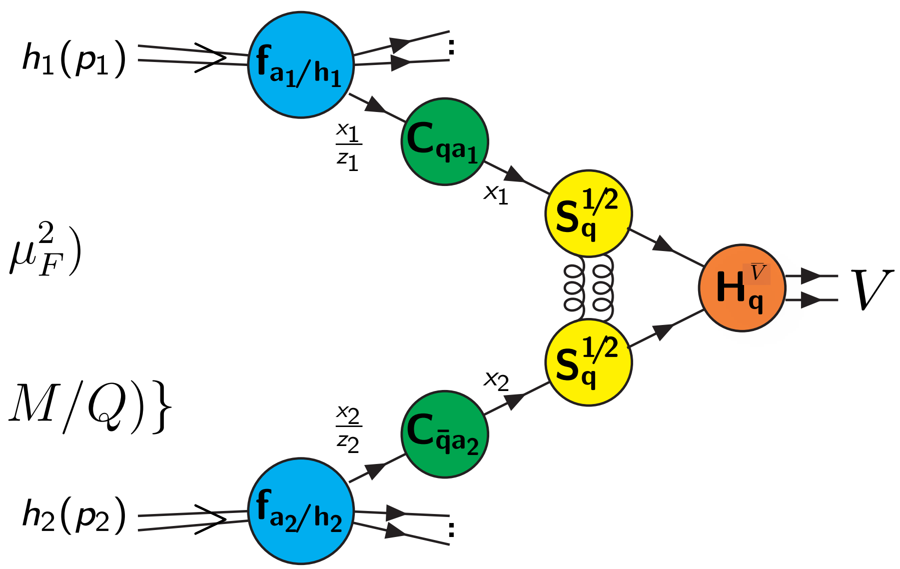
On-shell Z and W production

The cross section can be decomposed as

$$[d\hat{\sigma}_{a_1 a_2 \to l_3 l_4}] = [d\hat{\sigma}^{(\text{res.})}_{a_1 a_2 \to l_3 l_4}] + [d\hat{\sigma}^{(\text{fin.})}_{a_1 a_2 \to l_3 l_4}]$$

$$\left[d\hat{\sigma}^{(\text{res.})}_{a_1 a_2 \to l_3 l_4}\right] = \sum_{b_1, b_2 = q, \bar{q}} \frac{d\hat{\sigma}^{(0)}_{b_1 b_2 \to l_3 l_4}}{d\Omega} \frac{1}{\hat{s}} \int_0^\infty \frac{db}{2\pi}\, b\, J_0(b q_T)\, \mathcal{W}_{a_1 a_2, b_1 b_2 \to V}(b, M, \hat{y}, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2)$$

$$\mathcal{W}_V(b, M; \alpha_S, \mu_R^2, \mu_F^2) = \mathcal{H}_V\left(\alpha_S; M/\mu_R, M/\mu_F, M/Q\right) \times \exp\{\mathcal{G}(\alpha_S, L; M/\mu_R, M/Q)\}$$



$$\mathcal{H}'^{V\,(1)}_{q_f \bar{q}_{f'} \leftarrow q_f \bar{q}_{f'}, N} = \frac{e^2_{q_f} + e^2_{\bar{q}_{f'}}}{2} \left(\frac{1}{N(N+1)} + H'^{V\,(1)} \right)$$

$$\mathcal{H}'^{V\,(1)}_{q_f \bar{q}_{f'} \leftarrow \gamma \bar{q}_{f'}, N} = \frac{3\, e^2_{q_f}}{(N+1)(N+2)},$$

$$\mathcal{H}'^{V\,(1)}_{q_f \bar{q}_{f'} \leftarrow q_f \gamma, N} = \frac{3\, e^2_{\bar{q}_{f'}}}{(N+1)(N+2)},$$

We include the full set of one-loop EW virtual scattering amplitudes. Not only for the W, but for the Z for the sake of completeness.

Originally not included in → LC, Ferrera, Sborlini [2018]

Wackeroth, Hollik [1997]
Behring, Buccioni, Caola, Delto, Jaquier, Melnikov et al. [2020]
Bonciani, Buccioni, Rana, Vicini [2021]

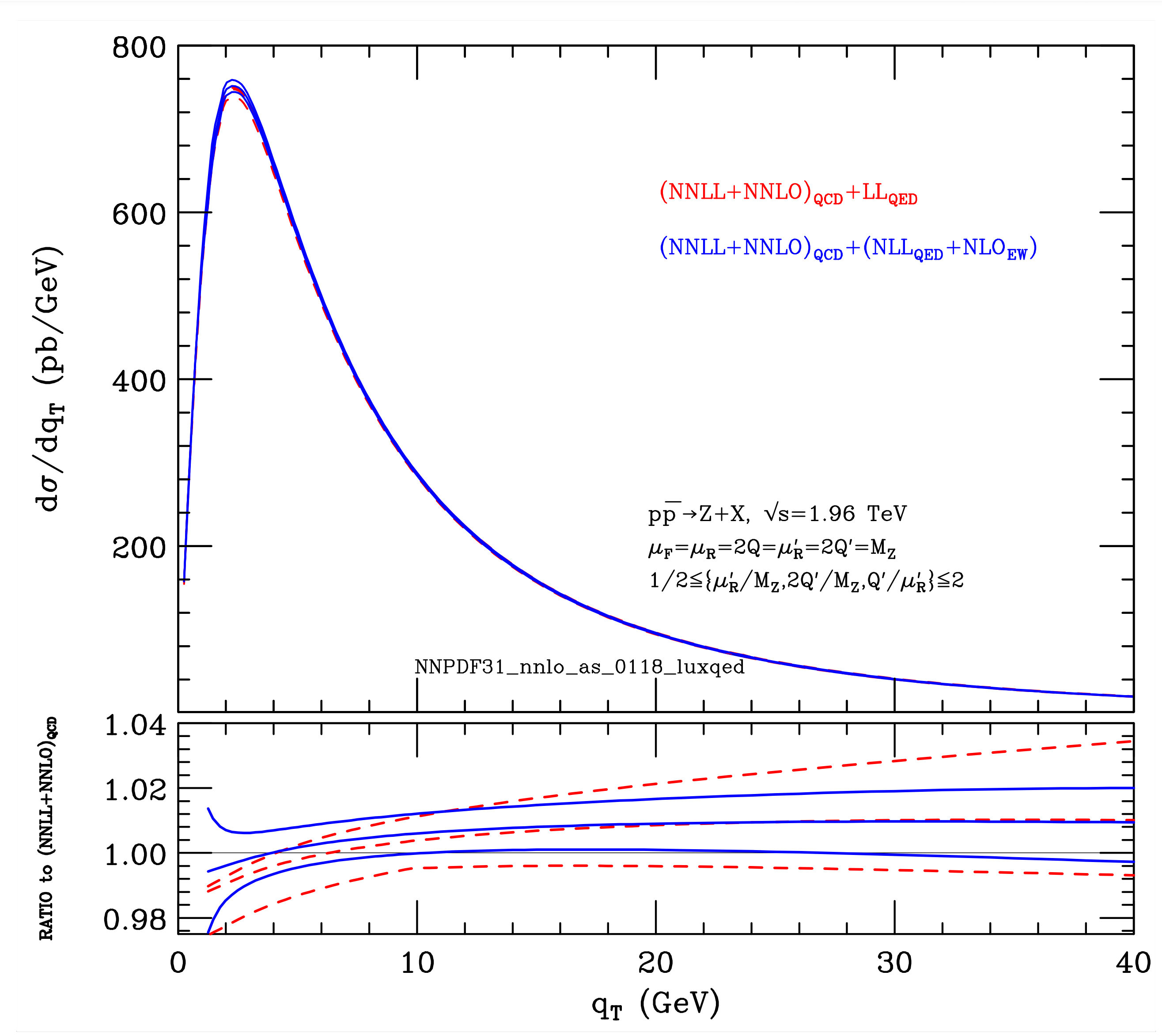
- The hard virtual factor H'^{V} requires the definition of subtraction operators I, suitable to treat massive and charged final states → we left this topic to the discussion session
- The expansion of the f.o contribution served as a check for the involved abelianization procedure → we left this topic to the discussion session (also the linear power corrections)

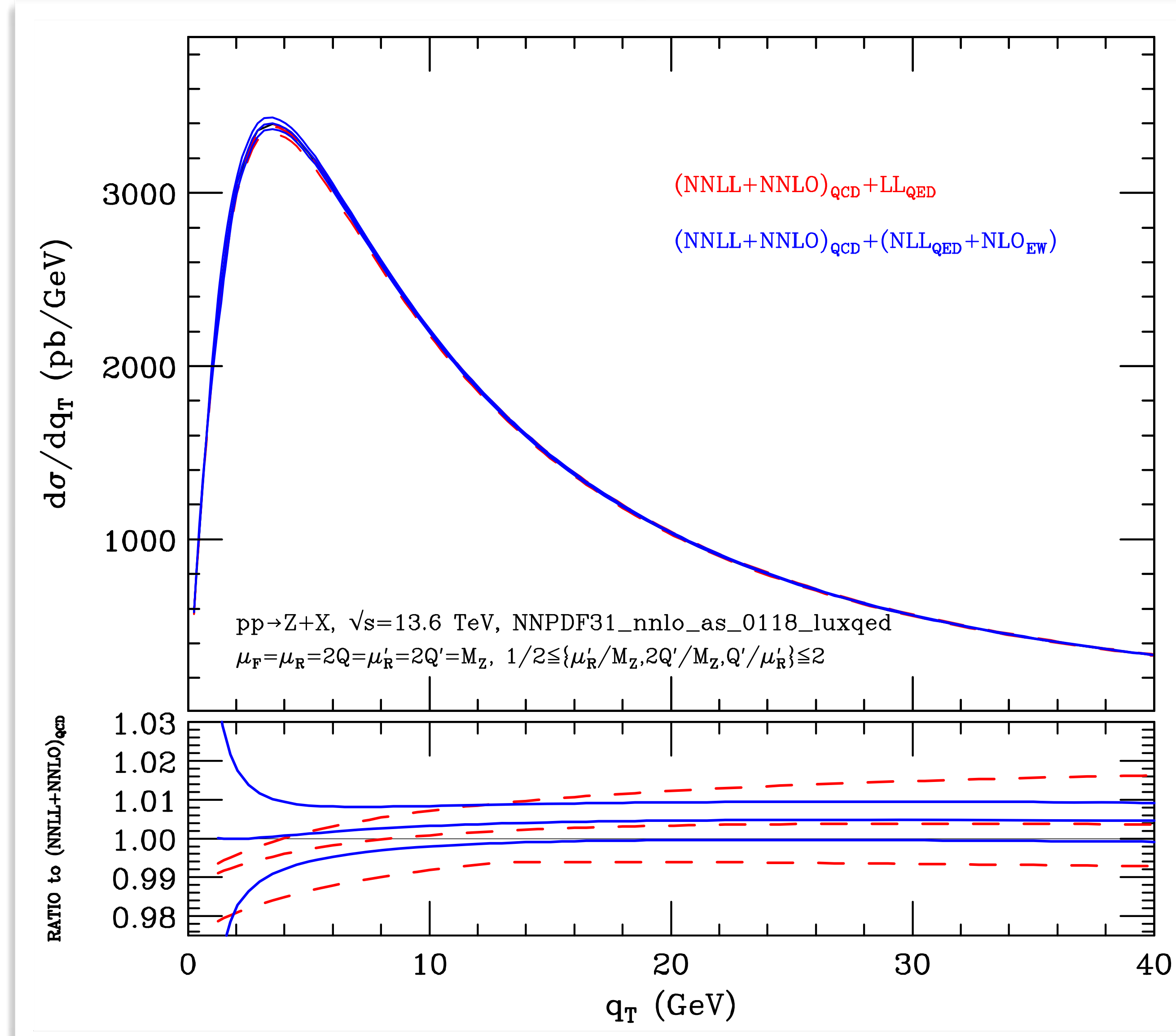
[DYqT]

QED+QCD qT resummation at NLL+NLO

Results

Autieri, LC, Ferrera, Sborlini [2023]



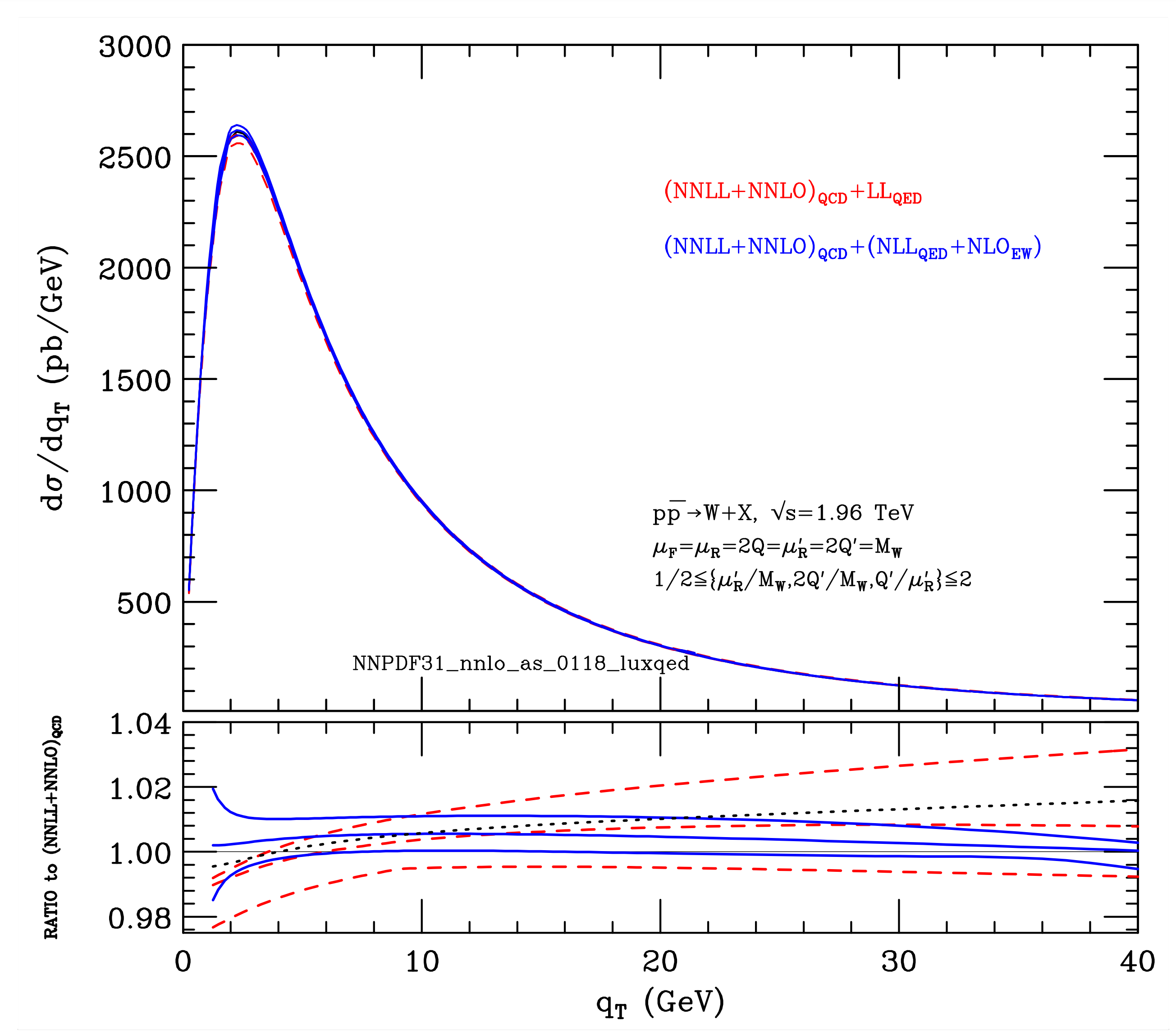


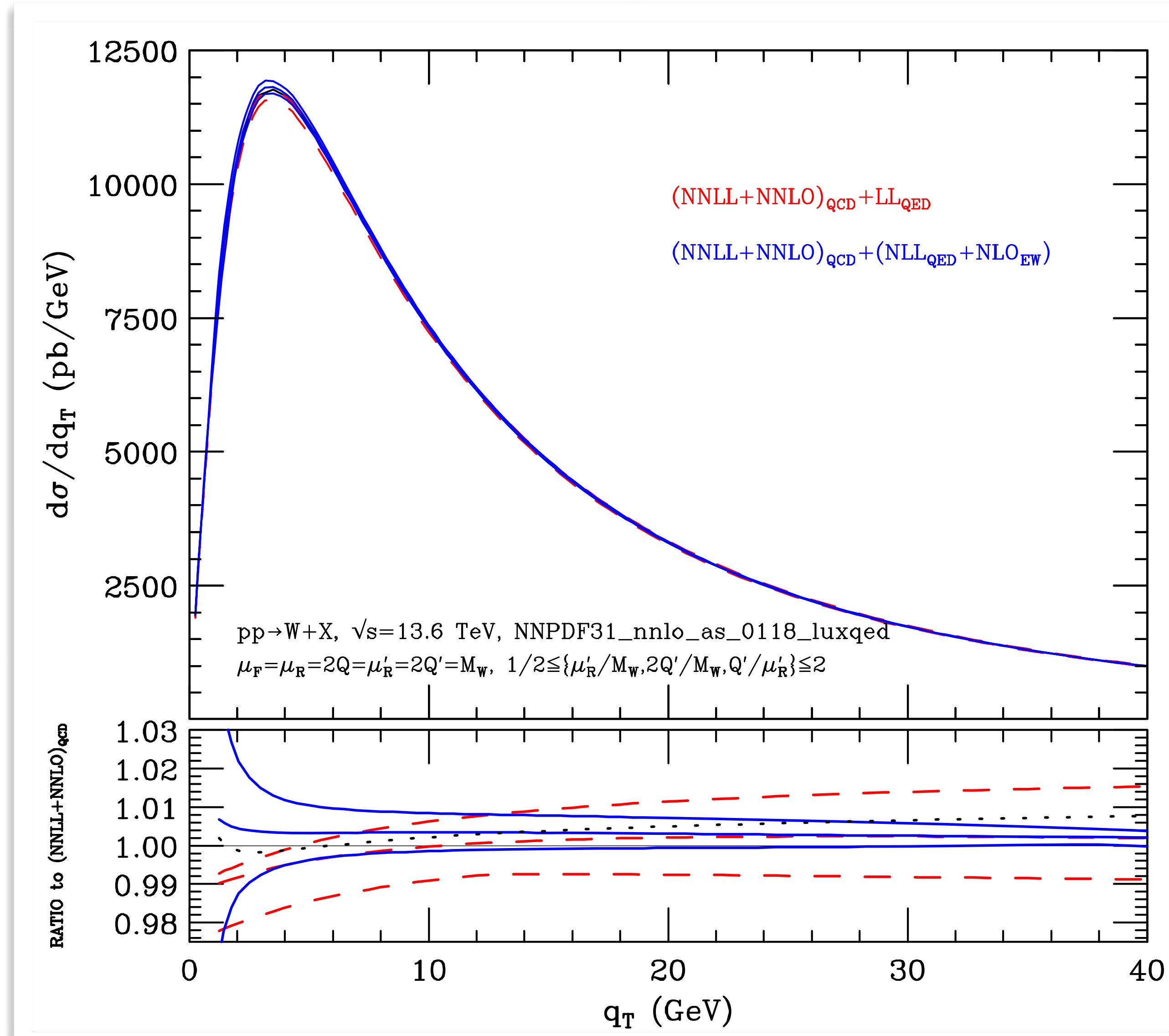
- The scale variation band is reduced by roughly a factor 2 with the inclusion of the NLL+NLO corrections
- At the Tevatron and at the LHC, QED uncertainty is dominated by the renormalization scale at LL accuracy and resummation scale at NLL+NLO LC, Ferrera, Sborlini [2018]
- The effect of EW loop corrections is extremely small (per-mille level effect)
- Overall order 0.5% at the LHC at NLL

QED+QCD qT resummation at NLL+NLO

Results

Autieri, LC, Ferrera, Sborlini [2023]



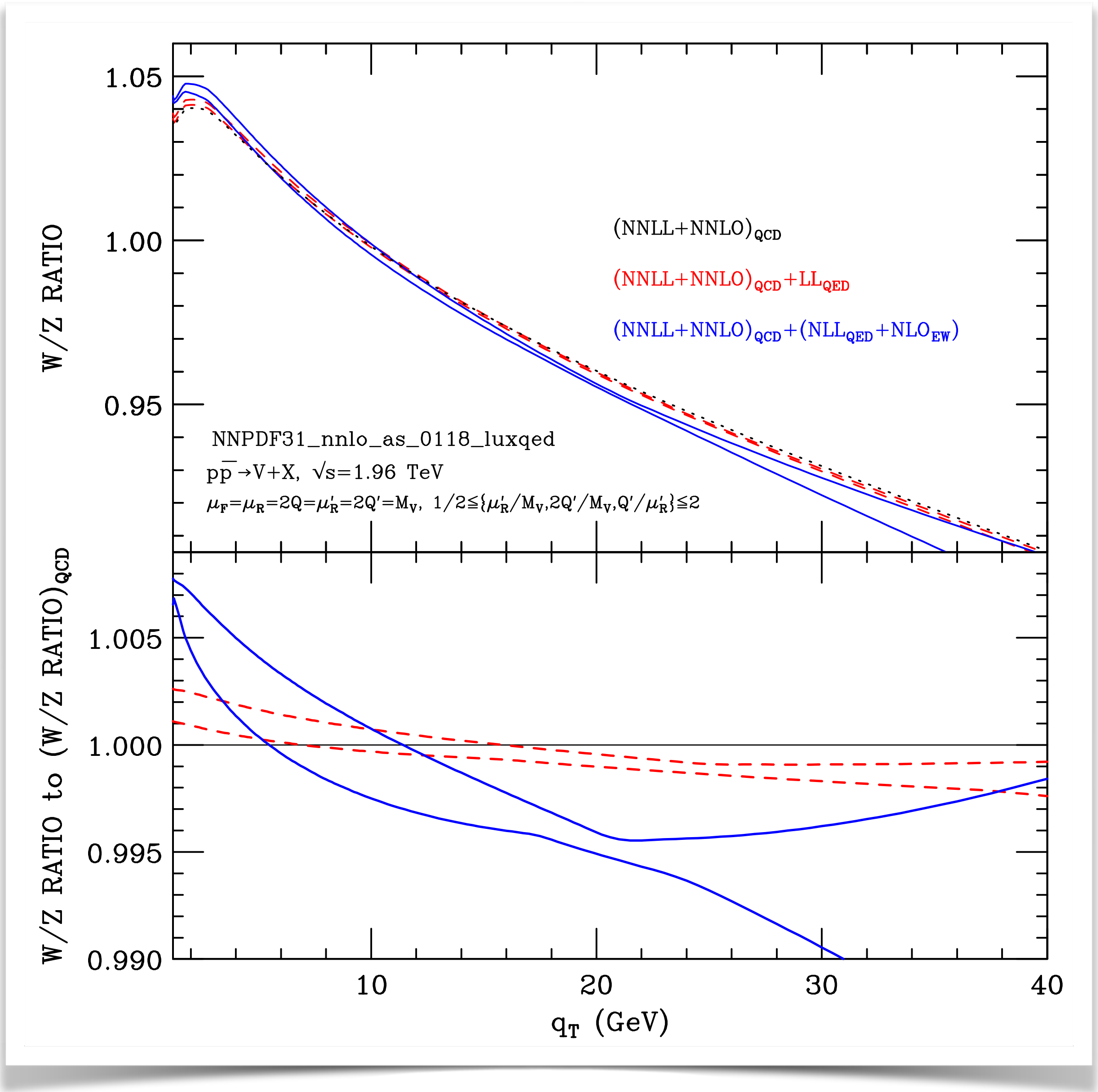


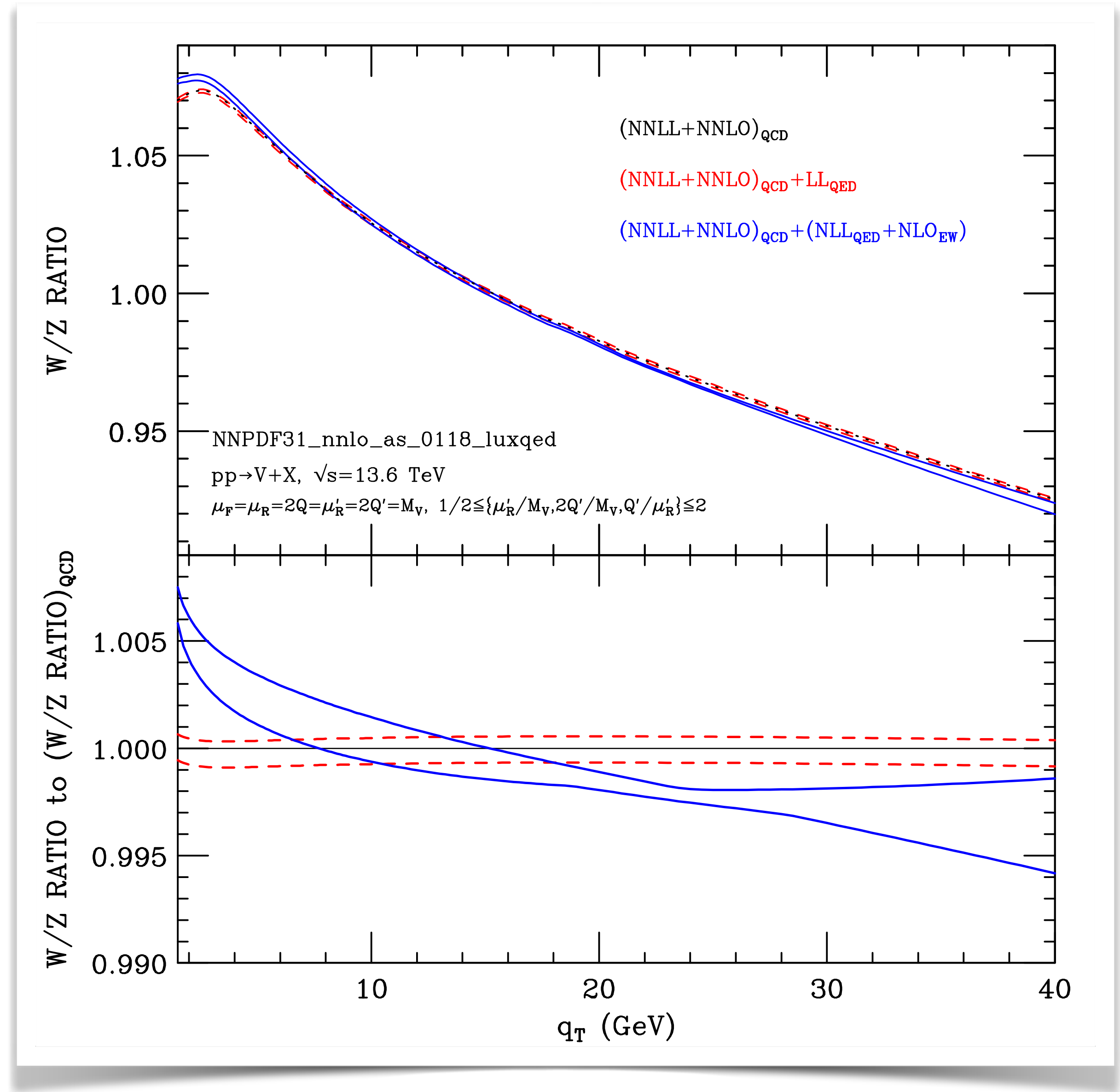
- The NLL+NLO prediction without the effect of soft wide-angle QED radiation (black dotted curve)
- NLL+NLO scale variation band reduction factor 1.5-2 for qT ≲ 20GeV and up to a factor 3 for qT ≳ 30GeV
- Overall order 0.5% at the LHC at NLL

QED+QCD qT resummation at NLL+NLO

Results

Autieri, LC, Ferrera, Sborlini [2023]





- Correlated scale variation → use the difference between the prediction at NLL+NLO and the LL?
- The impact of NLL+NLO QED corrections is to make the distribution softer at O(0.5 – 1%) level
- This is the combined effect of the W distribution slightly softer and the Z distribution harder

Example of what happen with different mechanisms in numerator and denominator

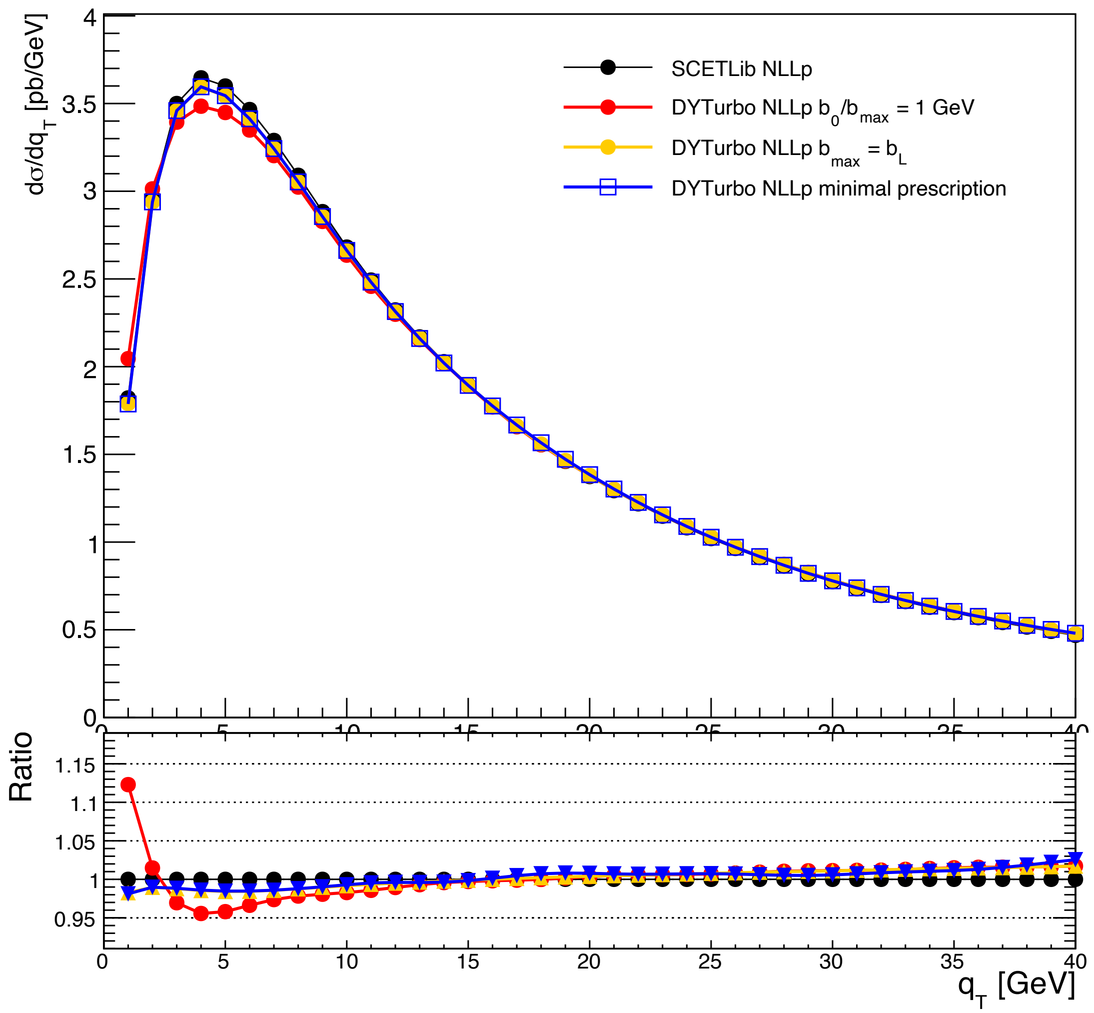
Outlook

- N4LL QCD plays a relevant role removing uncertainties in the W/Z pT distribution ratio
- NLL+NLO QCD+QED corrections to on shell Z and W boson production introduce non negligible effects for the W/Z pT distribution ratio
- QCD resummation at N4LL is implemented in the public code DYTurbo
- NLL+NLO QCD+QED corrections to on shell Z and W boson production are encoded in DYqT. (very soon in DYTurbo)
- Full NLL+NLO QCD+QED corrections to Z and W boson production with decays $\rightarrow$ very soon in DYTurbo

Thank you!!!

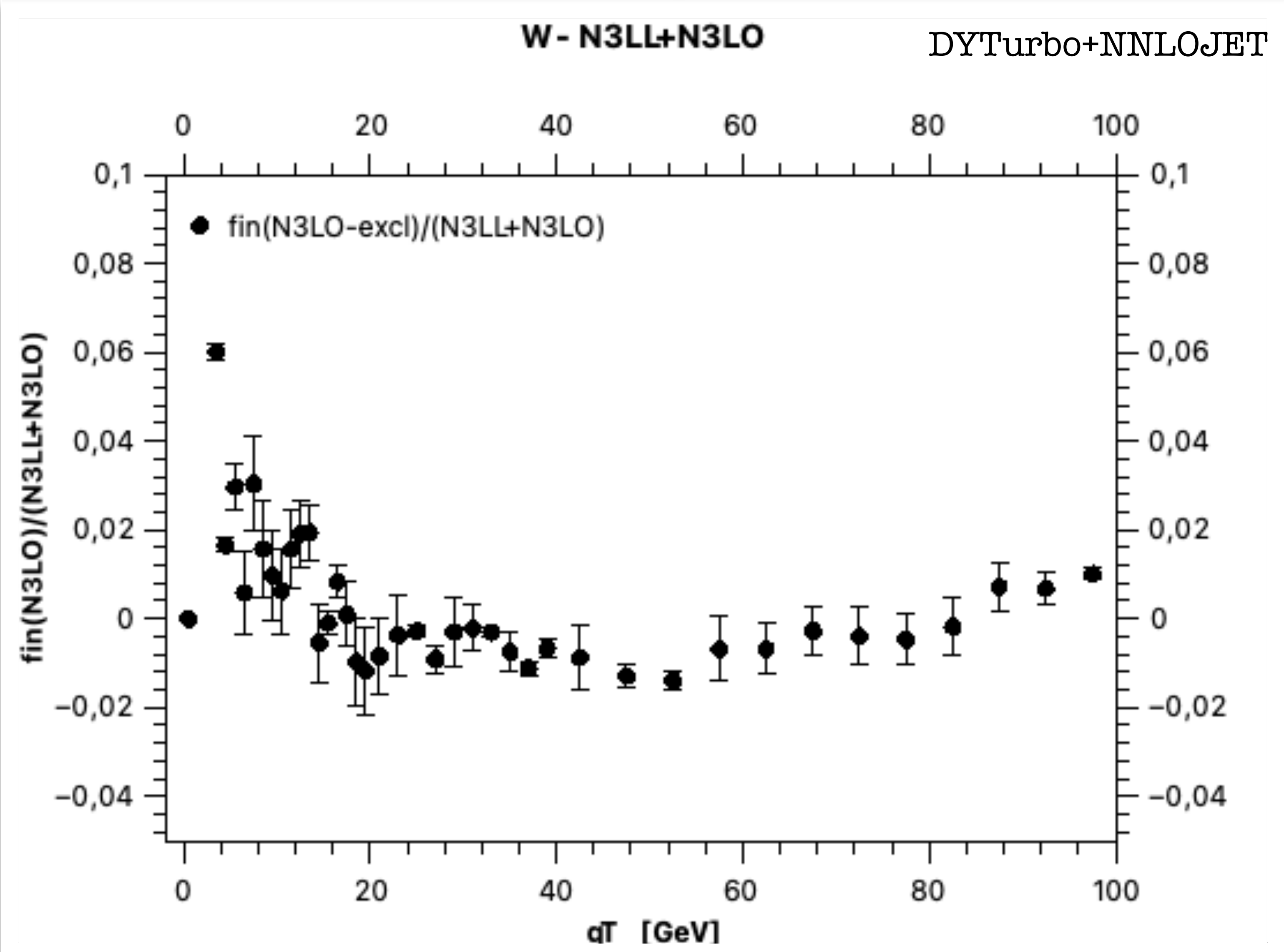
Backup slides

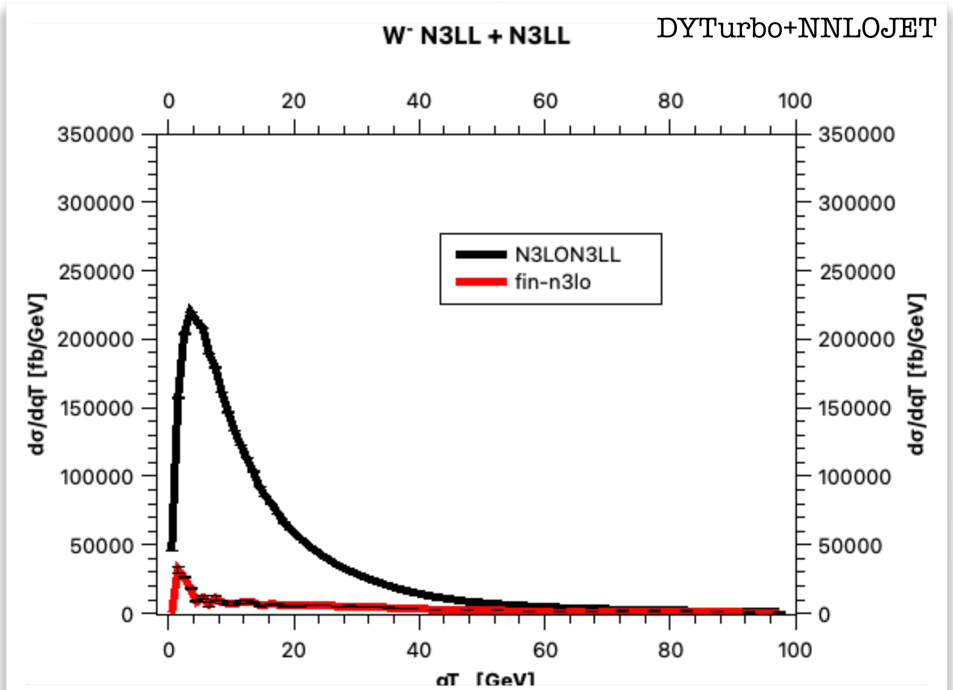
Comparison of b* and minimal prescription



- keep bstar with bmax = b0/1GeV to evaluate PDFs, but integrate up to or beyond the Landau pole in the Sudakov
- In one prescription bmax = bL with bL = b0 · exp(1/(2αs β0))
- In the other prescription the path of integration is deformed in the complex plane (minimal prescription)

Size of the finite part W-





Non perturbative model used in the N4LLa

For the non-perturbative (NP) effects at very small transverse momenta we introduced, in the conjugated b-space, a NP form factor of the form

Collins, Rogers [2015]

$$S_{NP}(b) = \exp\{-g_1 b^2 - g_K(b)\ \ln(M^2/Q_0^2)\}$$

$$g_K(b) = g_0 \left(1 - \exp\left[-\frac{C_F \alpha_S((b_0/b_\star)^2) b^2}{\pi g_0 b_{\rm lim}^2}\right]\right)$$

$$g_1 = 0.5\ \text{GeV}^2,\ Q_0 = 1\ \text{GeV},\ g_0 = 0.3,\ b_{\rm lim} = 1.5\ \text{GeV}^{-1}$$

$$b_\star^2 = b^2 b_{\rm lim}^2/(b^2 + b_{\rm lim}^2)$$

Other choices available in the code

$$S_{NP}(b) = \exp\left\{ - \left(g_1 + g_2 \log\left(\frac{m}{Q_0}\right) + g_3 \log\left(\frac{100m}{\sqrt{s}}\right)\right) b^2 \right\}$$