Electroweak Corrections to W/Z Resonances

Stefan Dittmaier

universität freiburg

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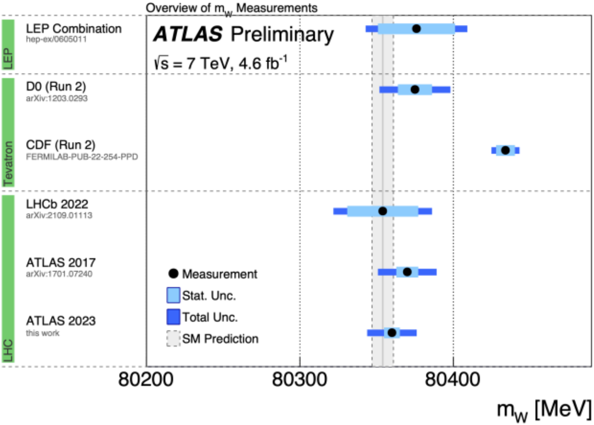
Introduction

M_W precision measurements:

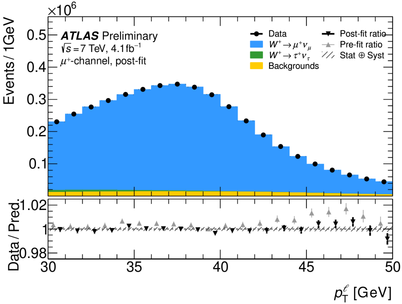
ATLAS '23



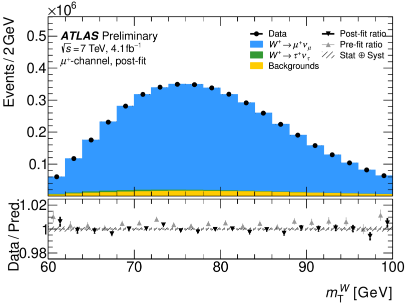
Most precise measurements via resonance distributions

$\hookrightarrow$ Precise description of resonance shapes required

Most important for M_W determination st hadron colliders: $p_{\mathrm{T},\ell}$ and $M_{\mathrm{T},\nu_\ell \ell}$ distributions



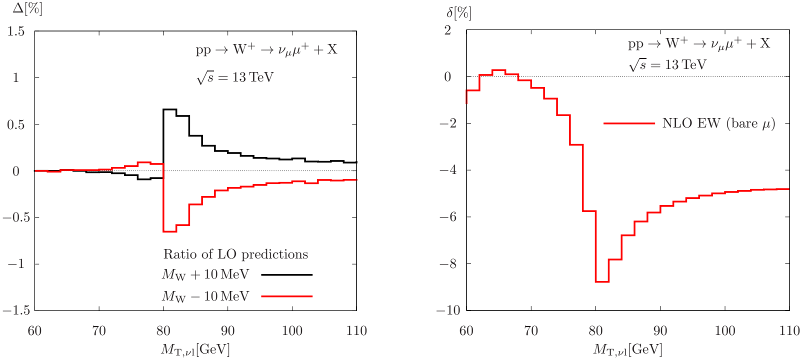
ATLAS '23



Control of radiative corrections and off-shell/finite-width effects is crucial.

But: Non-trivial issues with gauge invariance in predictions!

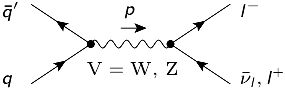
Sensitivity of distributions to M_W versus NLO EW corrections:



Shape prediction at the level of few 0.1% required!

$\hookrightarrow$ Proper inclusion of EW corrections at NLO + beyond crucial!

Parametrization of resonances – running versus fixed W/Z widths



$$\hat{\sigma}(p^2) \sim \left| \frac{R}{p^2 - M_{\rm V}^2 + {\rm i} M_{\rm V} \Gamma_{\rm V}(p^2)} \right|^2$$

Possible parametrizations in fits to data:

- running width: $\Gamma_{\rm V}(p^2) = \Gamma_{\rm V}^{\rm run} \times \dfrac{p^2}{M_{\rm V}^2}$ for $p^2 > 0$

  $\hookrightarrow$ used at LEP, Tevatron and the LHC,
  i.e. $M_{\rm V}^{\rm LEP} \equiv M_{\rm V}^{\rm run}$, $\Gamma_{\rm V}^{\rm LEP} \equiv \Gamma_{\rm V}^{\rm run}$

- fixed width: $\Gamma_{\rm V}(p^2) = \Gamma_{\rm V}^{\rm fix} =$ const.

  $\hookrightarrow$ $(M_{\rm V}^{\rm fix})^2 - {\rm i} M_{\rm V}^{\rm fix} \Gamma_{\rm V}^{\rm fix} =$ location of propagator pole,
  i.e. $M_{\rm V}^{\rm pole} \equiv M_{\rm V}^{\rm fix}$, $\Gamma_{\rm V}^{\rm pole} \equiv \Gamma_{\rm V}^{\rm fix}$

Note: equivalence of the two parametrizations: Bardin et al '88; Beenakker et al '96

$$R^{\rm fix} = \frac{R^{\rm run}}{1 + {\rm i}\gamma}, \quad M_{\rm V}^{\rm fix} = \frac{M_{\rm V}^{\rm run}}{\sqrt{1+\gamma^2}}, \quad M_{\rm V}^{\rm fix}\Gamma_{\rm V}^{\rm fix} = \frac{M_{\rm V}^{\rm run}\Gamma_{\rm V}^{\rm run}}{1+\gamma^2}, \quad \gamma = \frac{\Gamma_{\rm V}^{\rm run}}{M_{\rm V}^{\rm run}}$$

Explicit numbers: $M_{\rm W}^{\rm run} - M_{\rm W}^{\rm fix} \approx 28\,{\rm MeV}$, $M_{\rm Z}^{\rm run} - M_{\rm Z}^{\rm fix} \approx 34\,{\rm MeV}$

Mass and width definitions in QFT:

- (real) on-shell definition:
  - M_V^2 as zero in Re{inverse propagator}
  - widely used for predictions at LEP
  - naturally leads to a running width (after Dyson of summation!)
  - – gauge-dependent parametrization of observables beyond NLO Sirlin '91

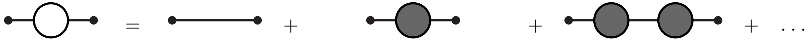
- pole definition:
  - $\mu_\mathrm{V}^2 = M_\mathrm{V}^2 - \mathrm{i} M_\mathrm{V} \Gamma_\mathrm{V} =$ location of propagator pole
    $\hookrightarrow$ gauge-independent definition Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01
  - μ_V^2 possible as input parameter
    $\hookrightarrow$ Γ_V in propagator without extra Dyson summation,
    e.g. via *complex-mass scheme* Denner et al '99,'05
  - + gauge-independent parametrization of observables possible

Gauge-invariance issues with W/Z resonances

Gauge invariance and treatment of resonances

for more details, see Denner, SD, 1912.06823 and refs. therein

Dyson summation of propagators mixes perturbative orders.



$$
\begin{aligned}
G(p^2) &= \frac{\mathrm{i}}{p^2-M^2} + \frac{\mathrm{i}}{p^2-M^2}\,\mathrm{i}\Sigma_{\mathrm{R}}(p^2)\,\frac{\mathrm{i}}{p^2-M^2} + \quad \dots \\
&= \frac{\mathrm{i}}{p^2 - M^2 + \Sigma_{\mathrm{R}}(p^2)}, \qquad \Sigma_{\mathrm{R}}(M^2) = \mathrm{i}M\Gamma
\end{aligned}
$$

But:

Consistency of pert. calculations often requires complete fixed orders.

$\hookrightarrow$ Consistency jeopardized if no special care is taken!

Gauge-invariance requirements:

- proper cancellation of gauge-parameter dependences
  (relations between self-energies, vertex corrections, boxes, etc.)
- validity of (internal) Ward identities
  (e.g. ruling cancellations for forward scattering of $\mathrm{e}^{\pm}$ or at high energies)

Required: schemes to introduce width Γ

- without breaking gauge invariance
- maintaining (at least) NLO accuracy everywhere in phase space

Some incidental remarks:

The issue of gauge invariance goes

- beyond the definition of M and Γ and also
- beyond the question of parametrizing the resonance!

It is about the consistency of amplitudes everywhere in phase space, i.e.

- on resonance,
- in off-shell regions, and
- in the transition region between on-/off-shell domains.

Width schemes for corrections in a nutshell:

for more details, see Denner, SD, 1912.06823 and refs. therein

- Naive schemes

$$\frac{1}{p^2 - M^2} \to \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma(p^2)}$$ in all or at least in resonant props.

Fixed-width scheme: $\Gamma(p^2) = \text{const.}$

$\hookrightarrow$ breaks gauge invariance only "mildly",
but partial inclusion of Γ in loops screws up consistency

Running-width scheme: $M\Gamma(p^2) = \mathrm{Im}\{\Sigma_\mathrm{R}(p^2)\} \neq \text{const.}$

$\hookrightarrow$ crude breaking of gauge invariance in off-shell regions,
often completely wrong results

- "Factorization Scheme" (FS)

Global correction factor (limit $\Gamma \to 0$) times gauge-invariant LO XS, e.g.:

$$\mathrm{d}\hat{\sigma}^{\mathrm{res}}_{\mathrm{virt}} = \delta_{\mathrm{virt}}\big|_{\Gamma\to 0} \times \mathrm{d}\hat{\sigma}^{\mathrm{res}}_{\mathrm{LO}}\big|_{\Gamma\neq 0}$$

$\hookrightarrow$ gauge invariant, simple for DY,
but problematic for radiation, not simple (impossible?) beyond NLO

Note: NLO on and off resonance, but transition region interpolated.

Width schemes for corrections in a nutshell: (continued)

- **Pole Scheme (PS)** Stuart '91; Aeppli et al. '93, '94; etc.

  Isolate resonance in a gauge-invariant way and introduce Γ only there:

$$\mathcal{M} = \frac{R(p^2)}{p^2-M^2} + N(p^2) = \frac{R(M^2)}{p^2-M^2} + \frac{R(p^2)-R(M^2)}{p^2-M^2} + N(p^2)$$

$$\to \underbrace{\frac{\tilde{R}(M^2-\mathrm{i}M\Gamma)}{p^2-M^2+\mathrm{i}M\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2)-R(M^2)}{p^2-M^2}}_{\text{non-res./non-fact. corrs.}} + \underbrace{\tilde{N}(p^2)}_{\text{non-resonant}} .$$

  ↪ consistent, gauge invariant, NLO everywhere possible, but subtle and cumbersome in practice (complex kinematics, pole location is branch point rather than pole, IR structure of radiation)

- **Leading pole approximation (PA)**

  Take term with highest resonance enhancement of pole expansion = leading term of Pole Scheme

  ↪ consistent, gauge invariant, straightforward, but valid only in resonance neighbourhood, rel. uncertainty for EW corrections $= \frac{\alpha}{\pi} \times \mathcal{O}(\Gamma/M)$

  ↪ in general not sufficient at NLO for high-precision DY, but e.g. good basis for higher-order corrections such as $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$

Width schemes for corrections in a nutshell: (continued)

- Complex-Mass Scheme (CMS) Denner et al. '99,'05; Denner, SD '19

  Complex masses for $V = Z, W$ from

  $$\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V = \text{location of complex poles in } V \text{ propagators.}$$

  Complex (on-shell) weak mixing angle via

  $$c_\mathrm{W} = \mu_\mathrm{W}/\mu_\mathrm{Z}.$$

  Perturbative calculation as usual (with complex on-shell renormalization).

  All algebraic relations expressing gauge invariance hold exactly (gauge-parameter cancellation, Ward identities).

  $\hookrightarrow$ General and systematic, gauge invariant, NLO everywhere, but all one-loop integrals with complex masses needed (known!)

- Further schemes (not in use for DY processes):

  Effective Field Theories Beneke et al. '03,'04; Hoang, Reisser '04

  $\hookrightarrow$ related to Pole Scheme and Leading Pole Approximation

  Schemes based on resummations of propagator corrections

  $\hookrightarrow$ still not fully gauge invariant or no full inclusion of NLO corrections

Electroweak corrections to W/Z production at the LHC

NLO electroweak corrections + h.o. improvements to Drell–Yan processes

$\hookrightarrow$ several independent calculations/codes available:
FEWZ (Gavin et al '12); HORACE (Carloni Calame et al '06,'07,...); POWHEG BOX (Barze et al '12,'13); RADY (SD et al '01,'09,...); SANC (Arbuzov et al '05,'07,...); WINHAC (Placzek et al '03,'09,...); WZGRAD (Baur et al '98,'01,...)

Features of RADY: (private code, used in the following)

hep-ph/0109062; 0710.3309; 0911.2329; 1403.3216; 1511.08016; 2009.02229

- Processes: $\mathrm{pp} \to \ell^+\ell^- + X$ and $\mathrm{pp} \to \ell^+\nu_\ell/\ell^-\bar{\nu}_\ell + X$
- Corrections:
  - NLO EW+QCD
  - universal EW corrections beyond NLO
  - higher-order FSR via structure functions
  - dominant $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ corrections in resonance regions
  - off-shell $\mathcal{O}(N_f\alpha_\mathrm{s}\alpha)$ corrections
- Models: SM, MSSM, THDM, SESM
- Special features:
  - NLO corrections to $\gamma\gamma \to \ell^+\ell^-$ channel
  - various EW input schemes: $\{G_\mu, M_\mathrm{W}, M_\mathrm{Z}\}$, $\{\alpha(M_\mathrm{Z}), M_\mathrm{W}, M_\mathrm{Z}\}$, etc.
  - different gauge-invariant resonance schemes:
    complex-mass scheme, pole scheme, factorization scheme
  - optional: leading pole expansion

Gauge invariance and DY amplitudes

W production:

- LO:

  Gauge invariance ok if $\mathcal{M}_{\mathrm{LO}}$ is parametrized in terms of α, s_{W}, and μ_{W}^2
  (dependent: $c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2$, $M_{\mathrm{Z}} = \mu_{\mathrm{W}}/cw$, but M_{Z} does not appear).

  $\hookrightarrow$ FS definition of $\hat{\sigma}_{\mathrm{LO}}^{q\bar{q}'\to\mathrm{W}/\gamma\to\ell\nu}$ in RADY.

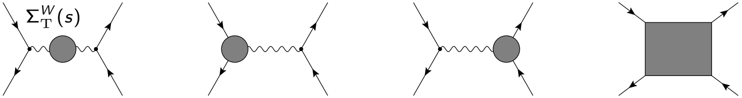
- NLO EW:

  No gauge-invariant decomposition of EW corrections into photonic and weak parts!

  $\hookrightarrow$ dedicated CMS and FS implementations in RADY

FS treatment of W production: SD, Krämer '01

Virtual corrections:



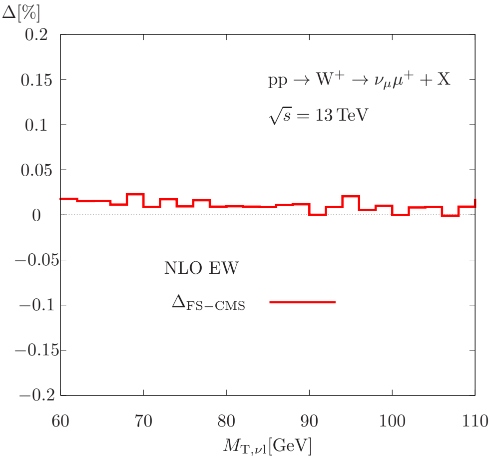
W self-energy | $Wq\bar{q}'$ and $W\nu_l l$ vertex corrections | box diagrams

$$\mathrm{d}\sigma^{\mathrm{FS}}_{\mathrm{virt}}(\hat{s},\hat{t}) = \underbrace{\mathrm{d}\sigma_{\mathrm{LO}}}_{\propto \frac{1}{|\hat{s}-M_{\mathrm{W}}^2+\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}|^2}} \times \underbrace{\Big[\delta_{WW}(\hat{s}) + \delta_{Wdu}(\hat{s}) + \delta_{W\nu_l l}(\hat{s}) + \delta_{\mathrm{box}}(\hat{s},\hat{t})\Big]}_{\Gamma_{\mathrm{W}} \neq 0 \text{ only in } \log(\hat{s}-M_{\mathrm{W}}^2+\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}})}$$

Real photonic corrections:

- amplitude gauge invariant for complex W-boson mass μ_{W} and real s_{W}
- IR divergences exactly match between $\mathrm{d}\sigma^{\mathrm{FS}}_{\mathrm{virt}}$ and $\mathrm{d}\sigma^{\mathrm{FS}}_{\mathrm{real}}$
- running $\Gamma_{\mathrm{W}}(\hat{s})$ could be introduced in gauge-invariant way by adjusting $\mathcal{M}$ in $\mathrm{d}\sigma^{\mathrm{FS}}_{\mathrm{real}}$

Comparison of width schemes for W production at NLO EW



Consistency between the FS and CMS at the level of $\Delta_{\mathrm{FS-CMS}} = \frac{\mathrm{d}\sigma_{\mathrm{FS}}}{\mathrm{d}\sigma_{\mathrm{CMS}}} - 1 \sim 0.02\%$!

Gauge invariance and DY amplitudes

Z production:

- LO and photonic corrections:

  Gauge invariance ok if $\mathcal{M}_{\mathrm{LO}}$ is parametrized in terms of α, s_{W}, and μ_{Z}^2
  (dependent: $c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2$, $M_{\mathrm{W}} = c_{\mathrm{W}}\mu_{\mathrm{Z}}$, but M_{W} does not appear).

  $\hookrightarrow$ PS/FS definition of $\hat{\sigma}_{\mathrm{LO}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}$ and $\delta_{\mathrm{phot}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}$ in RADY.

- Weak corrections: more complicated!

  $\hookrightarrow$ dedicated CMS, PS, and FS implementations in RADY

FS treatment of weak corrections to Z production: SD, Huber '09

$$\mathrm{d}\hat{\sigma}_{\mathrm{weak}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell} = \delta_{\mathrm{weak}}\Big|_{\Gamma_{\mathrm{Z}}=0} \times \mathrm{d}\hat{\sigma}_{\mathrm{LO}}^{q\bar{q}\to\mathrm{Z}/\gamma\to\ell\ell}\Big|_{\Gamma_{\mathrm{Z}}\neq 0}$$

Note:

- $\gamma\mathrm{Z}$ interference in δ_{weak} near $\hat{s} \sim M_{\mathrm{Z}}^2$ not correct (NNLO effect), due to non-resonant LO diagram (γ exchange) in contrast to W case
- IR matching of photonic corrections would be problematic

PS treatment of weak corrections to Z production: SD, Huber '09

Matrix element for weak corrections:

$$\mathcal{M}_{\rm weak}^{\sigma\tau} = f_{\rm weak}^{{\rm virt},\,\sigma\tau} \underbrace{\mathcal{A}^{\sigma\tau}}_{\text{Dirac structures for chiralities } \sigma,\tau}, \qquad f_{\rm weak}^{\sigma\tau} = f_{\rm weak}^{{\rm self},\,\sigma\tau}(\hat{s}) + f_{\rm weak}^{{\rm vert},\,\sigma\tau}(\hat{s}) + f_{\rm weak}^{{\rm box},\,\sigma\tau}(\hat{s},\hat{t})$$

Consistent isolation of resonant parts in formfactors $f_{\rm weak}^{\sigma\tau}$:

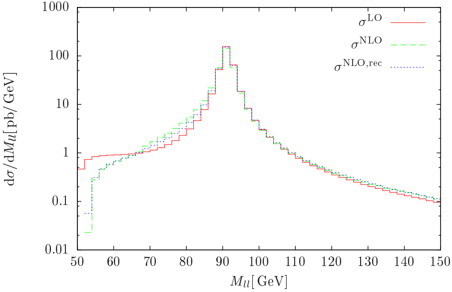
$$\begin{aligned}
f_{\rm LO}^{\sigma\tau}(\hat{s}) + f_{\rm weak}^{{\rm self},\,\sigma\tau}(\hat{s}) = {}& -e^2 \left\{ \frac{Q_{\rm q}\,Q_\ell}{\hat{s}} \left[1 - \frac{\Sigma_{\rm T}^{AA}(\hat{s})}{\hat{s}}\right] + \frac{g_{qqZ}^{\sigma}\,g_{llZ}^{\tau}}{\hat{s}-M_{\rm Z}^2}\left[1 - \frac{\Sigma_{\rm T}^{ZZ}(\hat{s})}{\hat{s}-M_{\rm Z}^2}\right] + \frac{Q_\ell\,g_{qqZ}^{\sigma} + Q_{\rm q}\,g_{llZ}^{\tau}}{\hat{s}} \frac{\Sigma_{\rm T}^{AZ}(\hat{s})}{\hat{s}-M_{\rm Z}^2} \right\} \\
\to {}& -e^2 \left\{ \frac{Q_{\rm q}\,Q_\ell}{\hat{s}} \left[1 - \frac{\Sigma_{\rm T}^{AA}(\hat{s})}{\hat{s}}\right] + g_{qqZ}^{\sigma}\,g_{llZ}^{\tau} \left[\frac{1 - \Sigma_{\rm T}'^{ZZ}(M_{\rm Z}^2)}{\hat{s}-\mu_{\rm Z}^2} \right.\right. \\
& \left. - \frac{\Sigma_{\rm T}^{ZZ}(\hat{s}) - \Sigma_{\rm T}^{ZZ}(M_{\rm Z}^2) - (\hat{s}-M_{\rm Z}^2)\Sigma_{\rm T}'^{ZZ}(M_{\rm Z}^2)}{(\hat{s}-M_{\rm Z}^2)^2} \right] \\
& \left. + \left(Q_\ell\,g_{qqZ}^{\sigma} + Q_{\rm q}\,g_{llZ}^{\tau}\right) \left[\frac{1}{\hat{s}-\mu_{\rm Z}^2} \frac{\Sigma_{\rm T}^{AZ}(M_{\rm Z}^2)}{M_{\rm Z}^2} + \frac{1}{\hat{s}-M_{\rm Z}^2} \left(\frac{\Sigma_{\rm T}^{AZ}(\hat{s})}{\hat{s}} - \frac{\Sigma_{\rm T}^{AZ}(M_{\rm Z}^2)}{M_{\rm Z}^2} \right) \right] \right\}
\end{aligned}$$

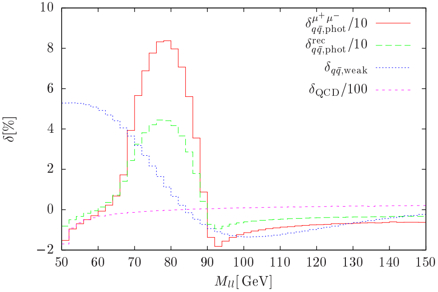
... similarly for vertex corrections

- γZ interference in $f_{\rm weak}^{\sigma\tau}$ near $\hat{s} \sim M_{\rm Z}^2$ described properly (as in CMS)
- IR matching of photonic corrections would be problematic (as in FS)

Short reminder to electroweak corrections to Z production

SD, Huber '09





- NLO QED corrections (mostly FSR) several 10%
  [maximally $\sim$ 40%(80%) for dressed leptons (bare muons)]
- Mulit-γ effects still at the few-% level
- Weak NLO corrections at the few-% level
  $\hookrightarrow$ most sensitive to width scheme

Short reminder to electroweak corrections to Z production

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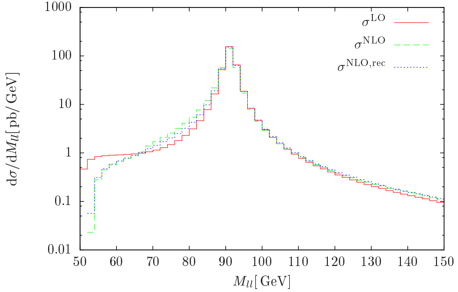

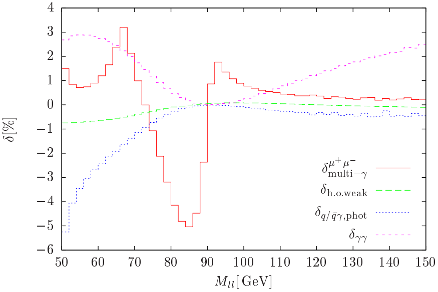

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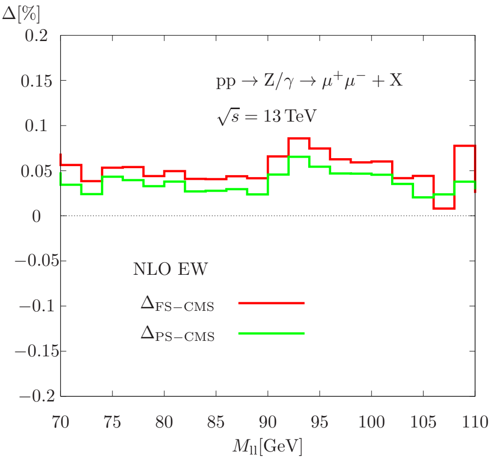
Comparison of width schemes for Z production at NLO EW



Consistency between the PS, FS, and CMS at the level of
$\Delta_{\mathrm{FS/PS-CMS}} = \frac{\mathrm{d}\sigma_{\mathrm{FS/PS}}}{\mathrm{d}\sigma_{\mathrm{CMS}}} - 1 \lesssim 0.1\%$!

Integrated cross sections for Z production

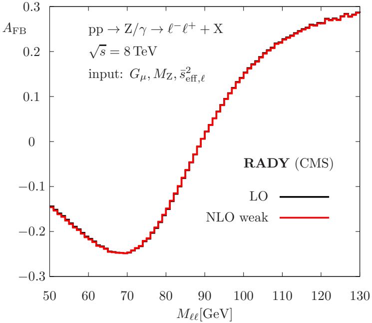
$M_{\ell\ell}$ = invariant mass of $\ell^+\ell^-$ pair (bare μ); $\sqrt{s} = 8\,\mathrm{TeV}$; input: $G_\mu, M_\mathrm{Z}, M_\mathrm{W}$

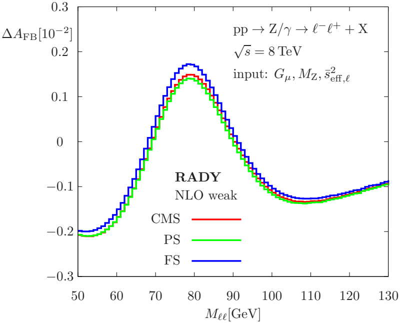
Scheme:	$89 < M_{\ell\ell}[\mathrm{GeV}] < 93$	$60 < M_{\ell\ell}[\mathrm{GeV}] < 81$	$81 < M_{\ell\ell}[\mathrm{GeV}] < 101$	$101 < M_{\ell\ell}[\mathrm{GeV}] < 150$
		$\sigma(\mathrm{LO})[\mathrm{pb}]$		
RADY CMS	612.456(1)	46.8732(1)	880.420(2)	30.86266(6)
RADY PS	612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
RADY FS	612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
rel. diff.	0.01%	0.005%	0.01%	0.02%
		$\sigma(\mathrm{NLO})/\sigma(\mathrm{LO})$		
RADY CMS	0.99102(1)	1.02786(1)	0.99143(1)	0.98908(1)
RADY PS	0.99131(1)	1.02843(1)	0.99172(1)	0.98917(1)
RADY FS	0.99148(1)	1.02864(1)	0.99189(1)	0.98924(1)
diff.	0.05%	0.08%	0.05%	0.02%
		$\sigma(\mathrm{NLO+HO})/\sigma(\mathrm{LO})$		
RADY CMS	0.99131(1)	1.02508(1)	0.99168(1)	0.98898(1)
RADY PS	0.99161(1)	1.02568(1)	0.99198(1)	0.98907(1)
RADY FS	0.99179(1)	1.02589(1)	0.99216(1)	0.98915(1)
diff.	0.05%	0.08%	0.05%	0.02%

$\hookrightarrow$ Width scheme dependence $\lesssim$ 0.1%

FB asymmetry $A_{\rm FB}$ in Z production – comparison of width schemes

$A_{\rm FB}$ defined via Collins–Soper angles $\quad\to\quad$ sensitivity to $\sin^2\theta^{\rm lept}_{\rm eff}$



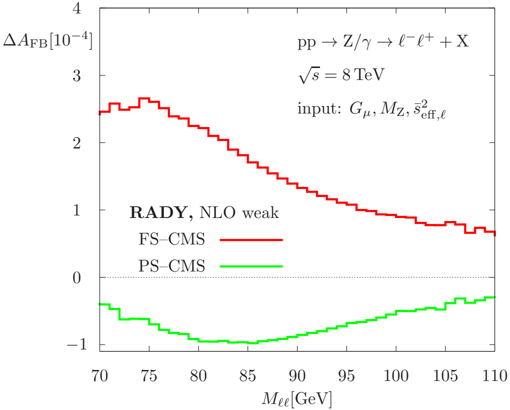


Some experimental uncertainties:

- Z resonance at LEP: $\quad \Delta A^{\rm b}_{\rm FB} = 0.0016, \quad \Delta A^{\ell}_{\rm FB} = 0.0010$

  $\hookrightarrow\ \Delta\sin^2\theta^{\rm lept}_{\rm eff} = 0.00029$ from $\Delta A^{\rm b}_{\rm FB}$

FB asymmetry A_{FB} – differences of width schemes differentially



$\hookrightarrow$ $|\text{PS-CMS}| \lesssim 10^{-4}$

FS less accurate (theoretically not as solid as PS/CMS)

Precision target for experiment: $\Delta A_{\mathrm{FB}} \lesssim 10^{-4}$

Note:
Proper extensions of width schemes beyond NLO should reduce differences!

Comparison of width schemes – FB asymmetry $A_{\rm FB}$

$A_{\rm FB}$ defined via Collins–Soper angles; $\sqrt{s} = 8\,\mathrm{TeV}$; input: $G_\mu, M_Z, \sin^2\theta^{\rm lept}_{\rm eff}$

(for Powheg and input scheme see Chiesa, Piccinini, Vicini, arXiv:1906.11569)

Code/scheme:	$89<M_{\ell\ell}$[GeV]<93	$60<M_{\ell\ell}$[GeV]<81	$81<M_{\ell\ell}$[GeV]<101	$101<M_{\ell\ell}$[GeV]<150
		$A_{\rm FB}$(LO)		
RADY/CMS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/CMS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/PS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/PS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/FS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/FS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
|X−CMS|	0	0	0	0
		$A_{\rm FB}$(NLO weak)		
RADY/CMS	0.030459(3)	−0.214082(4)	0.028738(4)	0.219509(5)
Powheg/CMS	0.03046(2)	−0.21408(2)	0.02873(2)	0.219506(25)
RADY/PS	0.030376(3)	−0.214136(4)	0.028658(4)	0.219475(5)
Powheg/PS	0.03038(2)	−0.21413(2)	0.02865(2)	0.219472(25)
RADY/FS	0.030589(3)	−0.213854(4)	0.028871(4)	0.219573(5)
Powheg/FS	0.03059(2)	−0.21385(2)	0.02886(2)	0.219571(25)
|PS−CMS|	0.00008	0.00005	0.00008	0.00003
|FS−CMS|	0.0001	0.0002	0.0001	0.00006

$\hookrightarrow$ |PS-CMS| $\lesssim$ 0.00008

FS less accurate (theoretically not as solid as PS/CMS)

Precision target for experiment: $\Delta A_{\rm FB} \lesssim 10^{-4}$

Going beyond NLO?!

- Universal corrections

  $\Delta\alpha$, $\Delta\rho$, multi-photon radiation

  $\hookrightarrow$ mostly unproblematic, implemented in most DY NLO codes

- $\mathcal{O}(\alpha_s\alpha)$ NNLO corrections

  - known pole approximation sufficient for W/Z resonance physics ($\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, M_{W}) SD, Schwinn, Huss '14,'16
    on-shell W/Z: Bonciani et al '19,'20; Delto et al '19; Buccioni et al '20; Cieri et al '20

  - known for off-shell Z production (approximately for W),
    Behring et al '20, '21; Buonocore et al '21; Bonciani et al '21; Armadillo et al '22; Buccioni et al '22

    complex-mass scheme unproblematic at $\mathcal{O}(\alpha_s\alpha)$ SD, Schmidt, Schwarz '20

- $\mathcal{O}(\alpha^2)$ NNLO corrections

  - full off-shell calculation not urgently needed,
    complex-mass scheme not yet available at NNLO

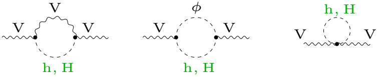
  - pole approximation should be adequate approach for LHC physics
    $\hookrightarrow$ first steps taken for virtual corrections to Z production
    Freitas, Chen '22

    But: care required when matching full NLO to pole-approximated NNLO parts!

BSM effects in W/Z production

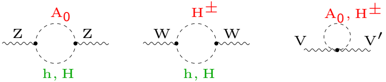
Considered SM extensions:

- **SESM** (Higgs singlet extension)
  additional CP-even Higgs boson H mixing with SM-like Higgs boson h
  $\hookrightarrow$ WW, ZZ self-energy corrections (also in vertex counterterms)



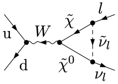
$\mathrm{V} = \mathrm{W}, \mathrm{Z}$

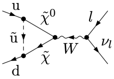
- **THDM**
  additional CP-even Higgs boson H mixing with SM-like Higgs boson h
  $+$ additional Higgs bosons A_0 (CP odd) and $\mathrm{H}^{\pm}$
  $\hookrightarrow$ WW, ZZ, $\gamma\mathrm{Z}$, $\gamma\gamma$ self-energy corrections ($+$ vertex counterterms)

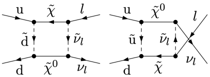
$+$  $+ \ldots$

$\mathrm{V}, \mathrm{V}' = \mathrm{W}, \mathrm{Z}, \gamma$

- **MSSM**
  THDM-like Higgs bosons $+$ gauginos $\tilde{\chi}$ $+$ sfermions $\tilde{f}$
  $\hookrightarrow$ additional self-energy, vertex, and box contributions

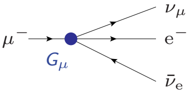
$\ldots +$ 

$+ \ldots +$ 

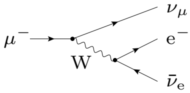
$+ \ldots +$ 

$+ \ldots$

Preliminary consideration:



+ QED corrections $\Longleftrightarrow$



+ EW corrections

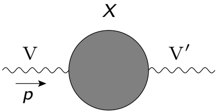
$\hookrightarrow$ Relation between G_μ, $\alpha(0)$, M_W, and M_Z including corrections:

$$\alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \alpha(0)(1+\Delta r)$$

Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

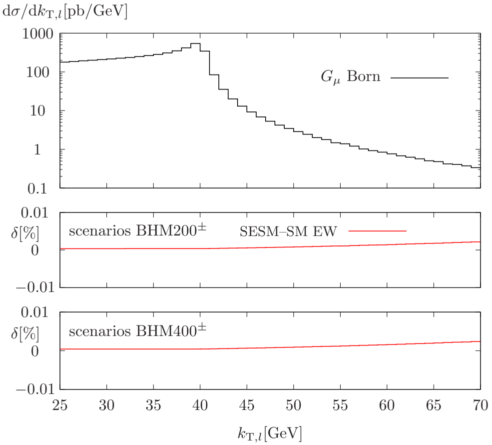
Sirlin '80, Marciano, Sirlin '80

G_μ input parameter scheme should absorb major parts of heavy-particle effects into LO coupling α_{G_μ}:



$\rightsquigarrow \quad \Delta r \quad \rightsquigarrow \quad \alpha_{G_\mu} \quad$ for $M_X^2 \gg M_\mathrm{W}^2, |p^2|$

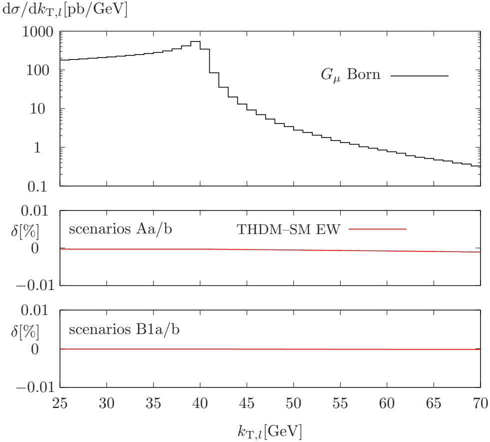
SESM NLO effects to W production at the LHC



SESM corrections negligible near W resonance in G_μ scheme!

(Difference SESM–SM stays $\lesssim$ 0.01% for $k_{T,l}$ in TeV range.)

THDM NLO effects to W production at the LHC

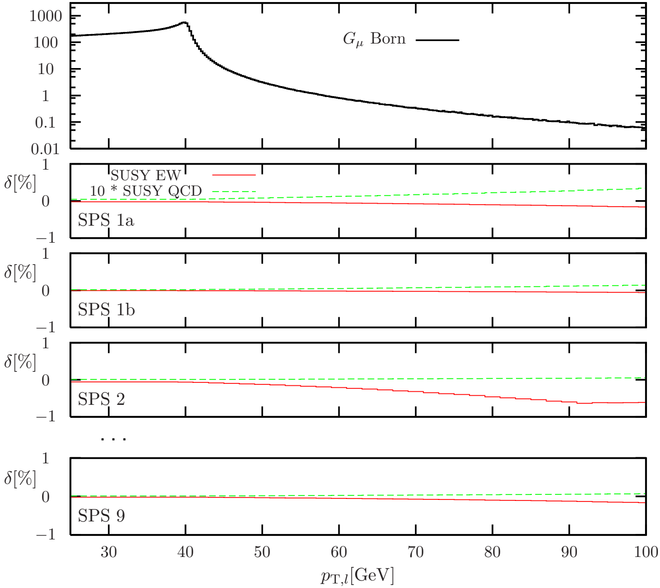


THDM corrections negligible near W resonance in G_μ scheme!

(Difference THDM–SM stays $\lesssim 0.1\%$ for $k_{T,l}$ in TeV range.)

SUSY NLO effects to W production in the MSSM at the LHC

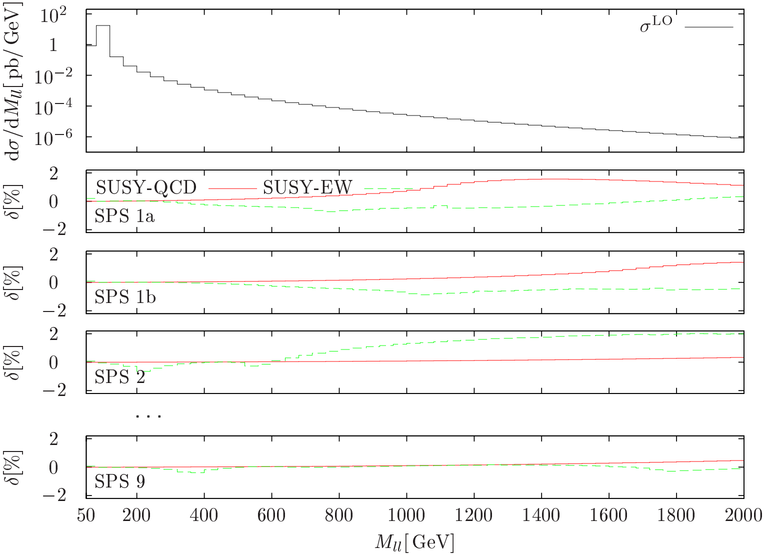
Brensing et al
arXiv:0710.3309



SUSY corrections < 0.1% near W resonance for viable MSSM scenarios!

SUSY NLO effects to Z production in the MSSM at the LHC

SD, Huber
arXiv:0911.2329



SUSY corrections $< 0.1\%$ near Z resonance for viable MSSM scenarios!

Conclusions and outlook

Predictions for W/Z production at the LHC

- Required precision:
  - for M_W: shape predictions at the 0.1% level of accuracy
  - for $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$: predictions for A_FB with $\Delta A_\mathrm{FB} \lesssim 10^{-4}$

  $\hookrightarrow$ issue of width schemes for describing W/Z resonances
- Width schemes respecting gauge invariance:
  - complex-mass scheme (CMS)
  - pole scheme (PS)
  - factorization schemes (FS)

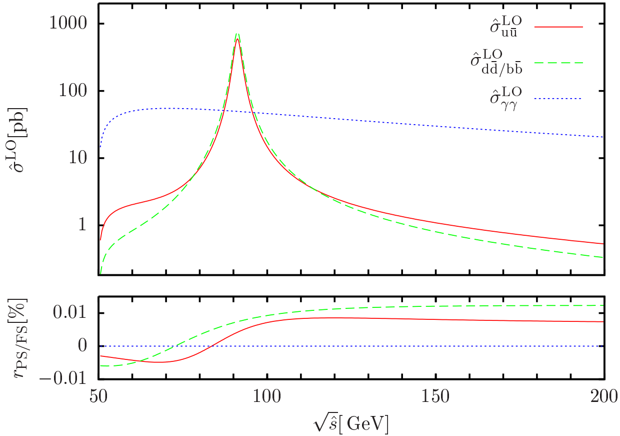
  $\hookrightarrow$ agreement within $\lesssim 0.1\%$ in $M_{\mathrm{T},\nu l}$ and M_{ll} shapes at NLO,
  agreement between CMS and PS for A_FB with $\Delta A_\mathrm{FB} < 10^{-4}$
- Going beyond NLO?
  - full NNLO EW corrections still out of reach
  - first step: NN...LO effects via pole expansions (corrected residues)

    but: take care of consistent matching with NLO parts!
- Impact of BSM effects on W/Z resonances?

  $\hookrightarrow$ effects calculated for MSSM, THDM, SESM (Higgs singlet)
  and found negligible (in the G_μ scheme)

Backup slides

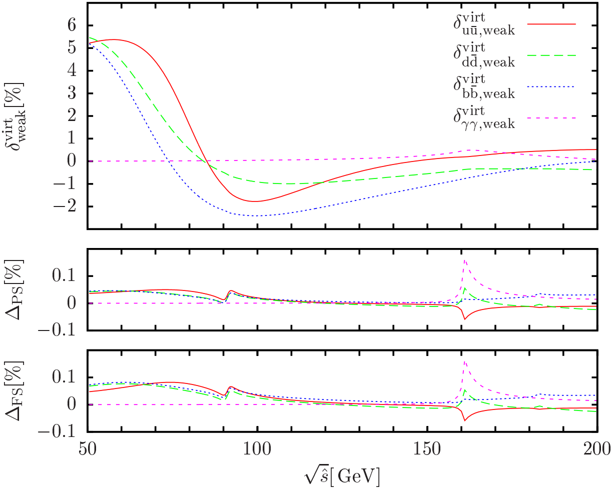
Partonic LO neutral-current Drell–Yan cross sections



SD, Huber
arXiv:0911.2329

Rel. difference: $r_X = \frac{\hat{\sigma}^{\rm LO}|_X}{\hat{\sigma}^{\rm LO}|_{\rm CMS}} - 1 \lesssim 0.01\%$ $\quad X = \mathrm{PS/FS}$

Weak corrections to partonic LO nuetral-current DY XS

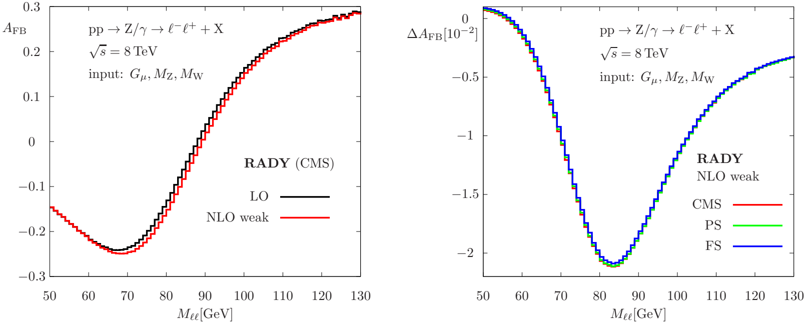


SD, Huber
arXiv:0911.2329

Difference: $\Delta_X = \delta^{\rm virt}_{\rm weak}\big|_X - \delta^{\rm virt}_{\rm weak}\big|_{\rm CMS} \lesssim$ 0.1%, X = PS/FS

FB asymmetry A_{FB} in Z production – comparison of width schemes

A_{FB} defined via Collins–Soper angles $\quad \rightarrow$ sensitivity to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$

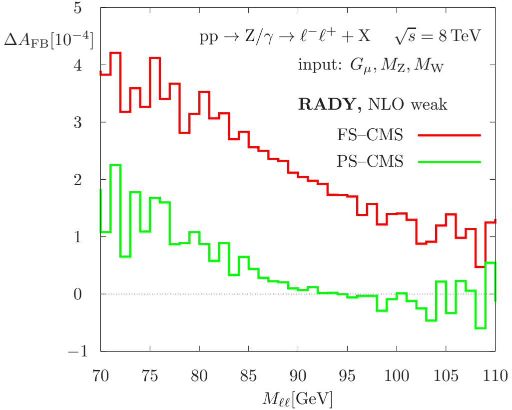


Some experimental uncertainties:

- Z resonance at LEP: $\quad \Delta A_{\mathrm{FB}}^{\mathrm{b}} = 0.0016, \quad \Delta A_{\mathrm{FB}}^{\ell} = 0.0010$

  $\hookrightarrow \; \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = 0.00029$ from $\Delta A_{\mathrm{FB}}^{\mathrm{b}}$

FB asymmetry A_{FB} – differences of width schemes differentially



Comparison of width schemes – FB asymmetry $A_{\rm FB}$

$A_{\rm FB}$ defined via Collins–Soper angles; $\sqrt{s} = 8\,\mathrm{TeV}$; input: G_μ, M_Z, M_W

Scheme:	$89 < M_{\ell\ell[GeV]} < 93$	$60 < M_{\ell\ell[GeV]} < 81$	$81 < M_{\ell\ell[GeV]} < 101$	$101 < M_{\ell\ell[GeV]} < 150$
		$A_{\rm FB}(\mathrm{LO})$		
RADY CMS	0.046551(4)	−0.202894(5)	0.044817(4)	0.226101(5)
RADY PS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
RADY FS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
|X−CMS|	$<$ 0.00001	0.00006	$<$ 0.00001	0.00001
		$A_{\rm FB}(\mathrm{NLO}) - A_{\rm FB}(\mathrm{LO})$		
RADY CMS	−0.01736(1)	−0.01233(1)	−0.01735(1)	−0.00689(1)
RADY PS	−0.01735(1)	−0.01220(1)	−0.01734(1)	−0.00691(1)
RADY FS	−0.01717(1)	−0.01199(1)	−0.01716(1)	−0.00681(1)
|PS−CMS|	$\lesssim$ 0.00001	0.0001	$\lesssim$ 0.00001	$\lesssim$ 0.00002
|FS−CMS|	0.0002	0.0003	0.0002	0.0001
		$A_{\rm FB}(\mathrm{NLO+HO}) - A_{\rm FB}(\mathrm{LO})$		
RADY CMS	−0.01615(1)	−0.01131(1)	−0.01614(1)	−0.00656(1)
RADY PS	−0.01614(1)	−0.01118(1)	−0.01613(1)	−0.00657(1)
RADY FS	−0.01595(1)	−0.01096(1)	−0.01594(1)	−0.00647(1)
|PS−CMS|	$\lesssim$ 0.00001	0.0001	$\lesssim$ 0.00001	$\lesssim$ 0.00001
|FS−CMS|	0.0002	0.0003	0.0002	0.0001

$\hookrightarrow$ |PS-CMS| $\lesssim$ 0.00001 in resonance window

FS less accurate (theoretically not as solid as PS/CMS)

Precision target for experiment: $\Delta A_{\rm FB} \lesssim 10^{-4}$

Singlet Extension of the SM (SESM)

Lagrangian: restriction to real, $\mathbb{Z}_2$-symmetric case!

$$\mathcal{L}_{\rm Higgs} = (D_\mu\Phi)^\dagger(D^\mu\Phi) + \tfrac{1}{2}(\partial\sigma)^2 - V(\Phi,\sigma),$$

$$V = -\mu_2^2\Phi^\dagger\Phi + \tfrac{1}{4}\lambda_2(\Phi^\dagger\Phi)^2 + \lambda_{12}\sigma^2\Phi^\dagger\Phi - \mu_1^2\sigma^2 + \lambda_1\sigma^4$$

Complex scalar SU(2) doublet & real scalar singlet: $v_{1,2}$ = vevs

$$\Phi = \begin{pmatrix} \phi^+ \\ (\eta_2 + i\chi + v_2)/\sqrt{2} \end{pmatrix}, \qquad \sigma = v_1 + \eta_1, \qquad Y_{\rm W}(\Phi) = 1$$

$\hookrightarrow$ “mass basis” h, H: $\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\alpha)\begin{pmatrix} H \\ h \end{pmatrix}, \quad R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Transformation of input parameters:

original set: $\{\lambda_1, \lambda_2, \lambda_{12}, \mu_1^2, \mu_2^2, v_1, v_2, g_1, g_2\}$

$\downarrow$

mass basis: $\{\underbrace{M_{\rm H}, M_{\rm h}, M_{\rm W}, M_{\rm Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_{12}}_{\overline{\rm MS}}, \alpha, \underbrace{t_{\rm H}, t_{\rm h}}_{\text{tadpoles} \to 0}\}$

Renormalization:
Bojarski et al. '15
Kanemura et al. '15,'17
Denner et al. '17,'18
Altenkamp et al. '18
SD, Rzehak '22

SESM scenarios:

Scenario	$M_\mathrm{H}\,[\,\mathrm{GeV}]$	$\sin\alpha$	λ_{12}
BHM200$^\pm$	200	± 0.29	± 0.07
BHM400$^\pm$	400	± 0.26	± 0.17
BHM600$^\pm$	600	± 0.22	± 0.23
BHM800$^\pm$	800	± 0.20	± 0.26

Two-Higgs-Doublet Model (THDM)

Lagrangian: restriction to CP-conserving case!

$$\mathcal{L}_{\mathrm{Higgs}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2),$$
$$D_\mu = \partial_\mu - \mathrm{i} g_2 I_{\mathrm{W}}^a W_\mu^a + \tfrac{\mathrm{i}}{2} g_1 Y_{\mathrm{W}} B_\mu$$

Higgs potential:

$$\begin{aligned} V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ & + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \tfrac{1}{2}\lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right] \end{aligned}$$

Two complex scalar SU(2) doublets: $v_{1,2} = \text{vevs}$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(\eta_1 + i\chi_1 + v_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\eta_2 + i\chi_2 + v_2) \end{pmatrix}, \quad Y_{\mathrm{W}}(\Phi_{1,2}) = 1$$

Transition to the "mass basis":

CP-even neutral fields: $\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}$

CP-odd neutral fields: $\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = R(\beta) \begin{pmatrix} G_0 \\ A_0 \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}$

charged fields: $\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$

Higgs potential after diagonalization:

$$V = -t_{\rm h} h - t_{\rm H} H + \frac{1}{2} M_{\rm h}^2 h^2 + \frac{1}{2} M_{\rm H}^2 H^2 + \frac{1}{2} M_{\rm A_0}^2 A_0^2 + M_{\rm H^+}^2 H^+ H^- + \dots$$

Transformation of input parameters:

original set: $\{\lambda_1, \dots, \lambda_5, m_{11}^2, m_{22}^2, m_{12}^2, v_1, v_2, g_1, g_2\}$

$\downarrow$

mass basis: $\{\underbrace{M_{\rm H}, M_{\rm h}, M_{\rm A_0}, M_{\rm H^+}, M_{\rm W}, M_{\rm Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_5}_{\overline{\rm MS}}, \alpha, \beta, \underbrace{t_{\rm H}, t_{\rm h}}_{\text{tadpoles} \to 0}\}$

Renormalization:
Santos/Barroso '97; Kanemura et al. '04; Lopez-Val/Sola '09; Degrande '14; Krause et al. '16; Denner et al. '16,'18; Altenkamp '17; SD, Rzehak '22

THDM scenarios:

Scenario	$M_{\mathrm{h}}\,[\mathrm{GeV}]$	$M_{\mathrm{H}}\,[\mathrm{GeV}]$	$M_{\mathrm{A/H^{\pm}}}$	$\tan\beta$	$\cos\beta-\alpha$	λ_5
Aa/b	125	300	460	2.0	± 0.10	-1.9
B1a/b	125	600	690	4.5	± 0.10	-1.9
B2a/b	125	600	690	1.5	± 0.10	-2.4

Low-energy input for the MSSM SPS scenarios:

	SPS 1a	SPS 1b	SPS 2	SPS 3	SPS 4	SPS 5	SPS 6	SPS 7	SPS 8	SPS 9
$\tan\beta$	10	30	10	10	50	5	10	15	15	10
λ[GeV]	393.6	525.5	1443.0	572.4	404.4	693.9	463.0	377.9	514.5	911.7
μ[GeV]	352.4	495.6	124.8	508.6	377.0	639.8	393.9	300.0	398.3	869.9
M_1[GeV]	99.1	162.8	120.4	162.8	120.8	121.4	195.9	168.6	140.0	-550.6
M_2[GeV]	192.7	310.9	234.1	311.4	233.2	234.6	232.1	326.8	271.8	-175.5
$m_{\tilde{g}}$[GeV]	595.2	916.1	784.4	914.3	721.0	710.3	708.5	926.0	820.5	1275.2
A_τ[GeV]	-254.2	-195.8	-187.8	-246.1	-102.3	-1179.3	-213.4	-39.0	-36.7	1162.4
A_t[GeV]	-510.0	-729.3	-563.7	-733.5	-552.2	-905.6	-570.0	-319.4	-296.7	-350.3
A_b[GeV]	-772.7	-987.4	-797.2	-1042.2	-729.5	-1671.4	-811.3	-350.5	-330.3	216.4
$\mu_R^{\overline{\mathrm{DR}}}$[GeV]	454.7	706.9	1077.1	703.8	571.3	449.8	548.3	839.6	987.8	1076.1
$M_{\mathrm{q}L}$[GeV]	539.9	836.2	1533.6	818.3	732.2	643.9	641.3	861.3	1081.6	1219.2
$M_{\mathrm{d}R}$[GeV]	519.5	803.9	1530.3	788.9	713.9	622.9	621.8	828.6	1029.0	1237.6
$M_{\mathrm{u}R}$[GeV]	521.7	807.5	1530.5	792.6	716.0	625.4	629.3	831.3	1033.8	1227.9
$M_{\ell L}$[GeV]	196.6	334.0	1455.6	283.3	445.9	252.2	260.7	257.2	353.5	316.2
M_{eR}[GeV]	136.2	248.3	1451.0	173.0	414.2	186.8	232.8	119.7	170.4	300.0
$M_{\mathrm{q}L}^{3G}$[GeV]	495.9	762.5	1295.3	760.7	640.1	535.2	591.2	836.3	1042.7	1111.6
$M_{\mathrm{d}R}^{3G}$[GeV]	516.9	780.3	1519.9	785.6	673.4	620.5	619.0	826.9	1025.5	1231.7
$M_{\mathrm{u}R}^{3G}$[GeV]	424.8	670.7	998.5	661.2	556.8	360.5	517.0	780.1	952.7	1003.2
$M_{\ell L}^{3G}$[GeV]	195.8	323.8	1449.6	282.4	394.7	250.1	259.7	256.8	352.8	307.4
$M_{\mathrm{e}R}^{3G}$[GeV]	133.6	218.6	1438.9	170.0	289.5	180.9	230.5	117.6	167.2	281.2