W mass calculation in the Standard Model

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CDF result 2022 [Science 276 (2022)]

references therein]. Many of these hypotheses include a source of dark matter, which is currently believed to comprise ~84% of the matter in the universe (*10*) but cannot be accounted for in the SM. Evidence for dark matter is provided by the abnormally high speeds of revolution of stars at large radii in galaxies, the velocities of galaxies in galaxy clusters, x-ray emissions sensing the temperature of hot gas in galaxy clusters, and the weak gravitational lensing of background galaxies by clusters [(*13*, *14*) and references therein]. The additional symmetries and fields in these extensions to the SM would modify (*15*–*24*) the estimated mass of the *W* boson (Fig. 1) relative to the SM expectation (*10*) of $M_W = 80{,}357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}}$ MeV (*25*). The SM expectation is derived from a combination of analytical relations from perturbative expansions on the basis of the internal symmetries of the theory and a set of high-precision measurements of observables, including the *Z* and Higgs boson masses, the top-quark mass, the electromagnetic (EM) coupling, and the muon lifetime, which are used as inputs to the analytical relations. The uncertainties in the SM expectation arise from uncertainties in the data-constrained input parameters (*10*) and from missing higher-order terms in the perturbative SM calculation (*26*, *27*). An example of a nonsupersymmetric SM extension is a modified Higgs sector that includes an additional scalar field with no SM gauge interactions, which predicts an M_W shift of up to ~100 MeV (*17*), depending on the mass of the additional scalar particle and its inter-

photons" (*19*), restoration of parity conservation in the weak interaction (*20*), the possible composite nature of the Higgs boson (*21*), and model-independent modifications of the Higgs boson's interactions (*22*–*24*) have also been evaluated.

Previous analyses (*28*–*44*) yield a value of $M_W = 80{,}385 \pm 15$ MeV (*45*) from the combination of Large Electron-Positron (LEP) collider and Fermilab Tevatron collider measurements. The ATLAS Collaboration has recently reported a measurement, $M_W = 80{,}370 \pm 19$ MeV (*46*, *47*), that is comparable in precision to the Tevatron results. The LEP, Tevatron, and ATLAS measurements have not yet been combined, pending evaluation of uncertainty correlations.

CDF experiment at Tevatron

The Fermilab Tevatron produced high yields of *W* bosons from 2002 to 2011 through quark-antiquark annihilation in collisions of protons (p) and antiprotons ($\bar{p}$) at a center-of-mass

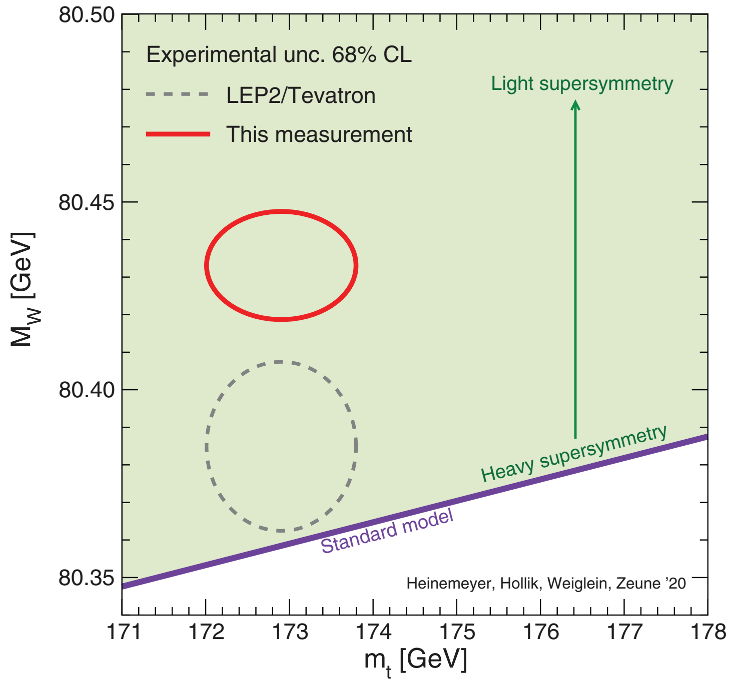


Fig. 1. Experimental measurements and theoretical predictions for the *W* boson mass. The red continuous ellipse shows the M_W measurement reported in this paper and the global combination of top-quark mass measurements, $m_t = 172.89 \pm 0.59$ GeV (*10*). The correlation between the M_W and m_t measurements is negligible. The gray dashed ellipse, updated (*16*) from (*15*), shows the 68% confidence level (CL) region allowed by the previous LEP-Tevatron combination $M_W = 80{,}385 \pm 15$ MeV (*45*) and m_t (*10*). That combination includes the M_W measurement published by CDF in 2012 (*41*, *43*), which this

SM expectation

PDG 2020, Erler and Freitas p. 28

. . .

Figure 10.4: Fit result and one-standard-deviation (39.35% for the closed contours and 68% for the others) uncertainties in M_H as a function of m_t for various inputs, and the 90% CL region ($\Delta\chi^2 = 4.605$) allowed by all data. $\alpha_s(M_Z) = 0.1185$ is assumed except for the fits including the Z lineshape. The width of the horizontal dashed band is not visible on the scale of the plot.

parameters. $\widehat{m}_c$, $\widehat{m}_b$, and $\Delta\alpha^{(3)}_{\rm had}$ are also allowed to float in the fits, subject to the theoretical constraints [28, 51] described in Sec. 10.2, and are correlated with α_s, which in turn is determined mainly through R_ℓ, Γ_Z, $\sigma_{\rm had}$, and τ_τ. The global fit to all data, including the hadron collider m_t average in Eq. (10.13), yields the results in Table 10.7, while those for the weak mixing angle in various schemes are summarized in Table 10.2.

Removing the kinematic constraint on M_H from LHC gives the loop-level determination from the precision data,

$$M_H = 90^{+18}_{-16}\ \text{GeV}\ , \tag{10.64}$$

which is 1.8 σ below the value in Eq. (10.52). The latter is also slightly outside the 90% central confidence range,

$$64\ \text{GeV} < M_H < 122\ \text{GeV}\ . \tag{10.65}$$

This is mostly a reflection of the Tevatron determination of M_W, which is 1.6 σ higher than the SM best fit value in Table 10.4. This is shown in Fig. 10.4 where one sees that the precision data together with M_H from the LHC prefer m_t to be closer to the upper end of its 1 σ allowed range.

Conversely, one can remove the explicit M_W and Γ_W constraints from the global fit and use $M_H = 125.30 \pm 0.13$ GeV to obtain $M_W = 80.357 \pm 0.006$ GeV, which is 1.7 σ below the world average in Eq. (10.51). Finally, one can carry out a fit without including the direct constraint, $m_t = 172.89 \pm 0.59$ GeV, from the hadron colliders. One obtains $m_t = 176.3 \pm 1.9$ GeV, which

Outline

- Introduction
- M_W – M_Z interdependence
- calculation of M_W at one-loop order
- calculation at higher-order
- evaluation and current status
- uncertainties of the predictions

- the SM does not predict any mass

$$M_W, M_Z, M_H, m_f \quad \longleftrightarrow \quad g_2, g_1, v, \mu, g_f$$

- masses are correlated with other measureable quantities

$$M_W, M_Z, \alpha, G_F, \sin^2\theta_W$$

- M_W, M_Z can be obtained from $G_F, \alpha, \sin^2\theta_W$
  since 1973: $\sin^2\theta_W$ known from neutrino scattering
  first calculations at one-loop order done in 1980
  Veltman; Antonelli, Consoli, Corbo
  limited precision, $\Delta\sin^2\theta_W \sim 0.0016$

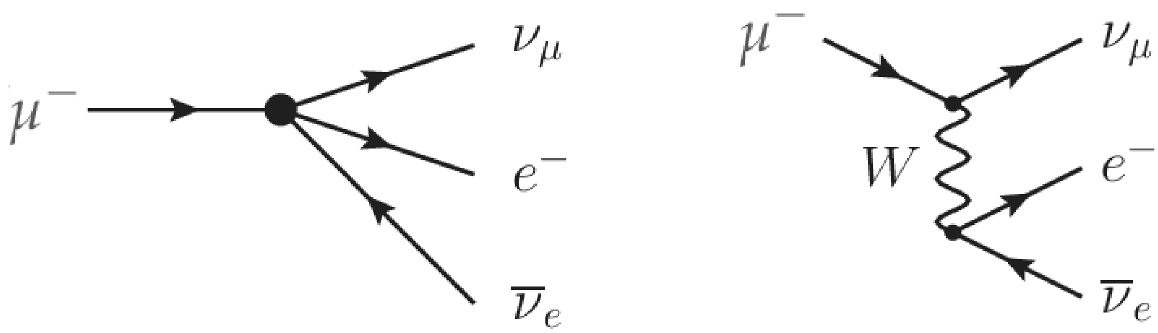
- M_W and M_Z are correlated via G_F and α
  allows to calculate M_W when M_Z is known
  since 1983 *UA1, UA2*
  since 1989 *LEP and SLC experiments*
  $\Rightarrow$ calculate M_W from M_Z, G_F, α (and more)

vector-boson mass correlation

Sirlin 1980

- M_W, M_Z *pole masses*
- $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}, \qquad \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W}\left(1 + \Delta r\right)$
- *“on-shell scheme”*

Fermi constant and W–Z mass correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)}\,(1+\Delta r)$$

loop corrections $\Delta r = \Delta r(M_W, M_Z, \ldots)$

central relation

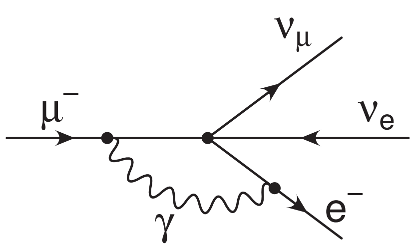
- determines M_W
- needs precise value of G_F
- needs precise calculation of Δr

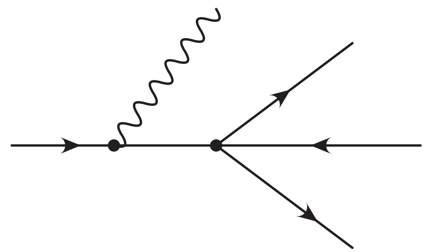
- Fermi constant from muon lifetime

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\, F\Big(\frac{m_e^2}{m_\mu^2}\Big)\,\big(1 + \delta_{\rm QED}\big)$$

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$ *phase space*

$\delta_{\rm QED}$: *QED corrections in the Fermi model*





$\mathcal{O}(\alpha)$

Sirlin, Kinoshita 1959

$\mathcal{O}(\alpha^2)$

Stuart, van Ritbergen 1998,1999

Seidensticker, Steinhauser 1999

Pak, Czarnecki 2008

$\rightarrow\ 1.4 \cdot 10^{-7}$

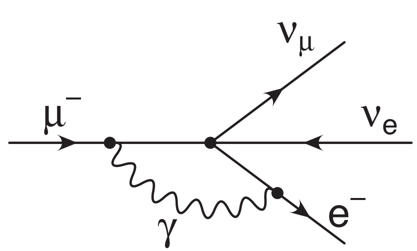
theoretical uncertainty

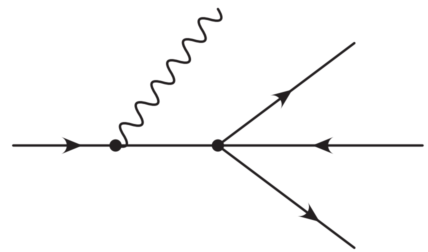
- Fermi constant from muon lifetime

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \delta_{\text{QED}}\right)$$

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x \qquad \textit{phase space}$$

δ_{QED} : *QED corrections in the Fermi model*





$\mathcal{O}(\alpha)$

Sirlin, Kinoshita 1959

$\mathcal{O}(\alpha^2)$

Stuart, van Ritbergen 1998,1999

Seidensticker, Steinhauser 1999

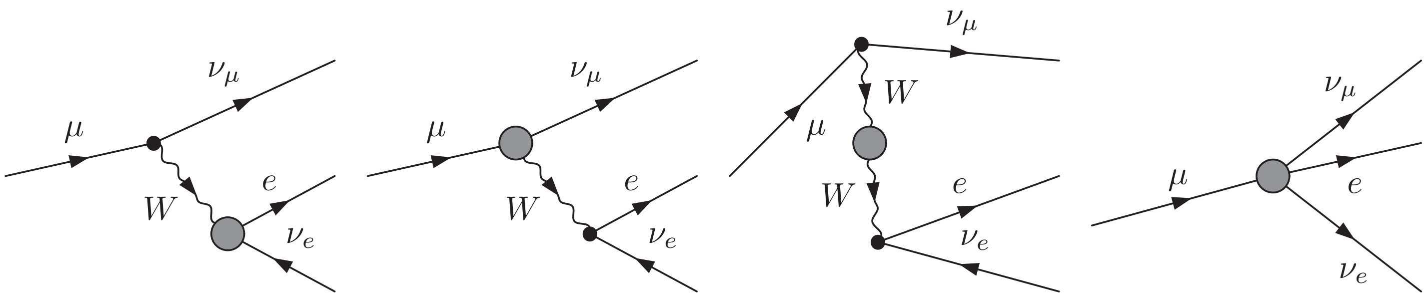
Pak, Czarnecki 2008

- exp: *MuLan Collab. 2011*

$$\boxed{G_F = 1.166\,378\,7(6) \cdot 10^{-5}\,\text{GeV}^{-2}}$$

Δr at one-loop

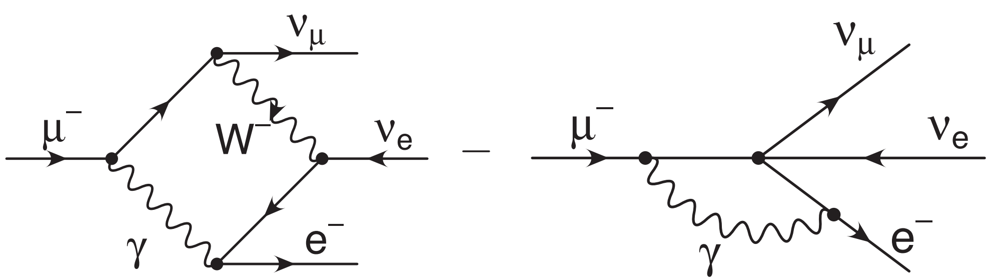
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W}\left[1 + \Delta r(M_W, M_Z, m_t, M_H)\right]$$



$$\Delta r = \text{loop diagrams} + \text{counterterms}$$

$$\text{counterterms} = \frac{\delta\alpha}{\alpha} - \frac{\delta M_W^2}{M_W^2} - \frac{\delta \sin^2\theta_W}{\sin^2\theta_W}$$

- *separation of Fermi model QED corrections*



$$\gamma W \,\text{box graph} \; - \; R_V \qquad \to \; \Delta r$$

$R_V \;\to$ *virtual QED corrections in Fermi model*

renormalization scheme defines counterterms

$$\left[\text{counterterms} = \frac{\delta\alpha}{\alpha} - \frac{\delta M_W^2}{M_W^2} - \frac{\delta \sin^2\theta_W}{\sin^2\theta_W}\right]$$

on-shell scheme: $s_W^2 = 1 - \dfrac{M_W^2}{M_Z^2}$

- $\delta M_{W,Z}^2 = \Sigma_{W,Z}(M_{W,Z}^2)$ *defines pole masses*
- $\dfrac{\delta s_W^2}{s_W^2} = \dfrac{c_W^2}{s_W^2}\left(\dfrac{\delta M_Z^2}{M_Z^2} - \dfrac{\delta M_W^2}{M_W^2}\right)$
- $\dfrac{\delta\alpha}{\alpha} = \Pi^\gamma(0) - 2\dfrac{s_W}{c_W}\dfrac{\Sigma^{\gamma Z}(0)}{M_Z^2}$ *defines $\alpha_{Thomson}$*

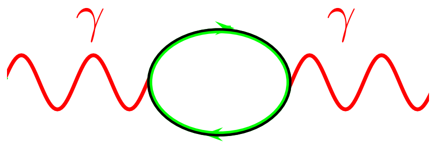
$\overline{MS}$ scheme: UV divergent parts only

hybrid scheme: keep $M_{W,Z}$ as pole masses on-shell scheme: dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\Delta\rho + \Delta r_{\mathrm{rem}}$$

$$\Delta\alpha = \Pi^{\gamma}_{\mathrm{ferm}}(M_Z^2) - \Pi^{\gamma}_{\mathrm{ferm}}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

photon vacuum polarization



$$\Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0)\,|_{5\,\rm quarks} \;=\; \Delta\alpha \qquad \to \; \log\frac{m_f}{M_Z}$$

$$\Delta\alpha \;=\; \Delta\alpha_{\rm lept} + \Delta\alpha^{(5)}_{\rm had},$$

$$\Delta\alpha_{\rm lept} \;=\; 0.031498\;(4-\text{loop})$$ Steinhauser 1998; Sturm 2013

$$\Delta\alpha^{(5)}_{\rm had} \;=\; 0.02760 \pm 0.00010$$ Davier et al. 2019

$$= \; 0.02761 \pm 0.00011$$ Keshavarzi et al. 2019

$$= \; 0.02766 \pm 0.00007$$ PDG 2020 [Erler, Freitas]

significant parametric uncertainty

on-shell scheme: dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\Delta\rho + \Delta r_\mathrm{rem}$$

$$\Delta\alpha = \Pi^\gamma_\mathrm{ferm}(M_Z^2) - \Pi^\gamma_\mathrm{ferm}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \ \to \ \frac{1}{\left(1-\Delta\alpha\right)\left(1+\frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\Delta\rho\right) + \cdots}$$

$$\rho = 1 + \Delta\rho \ \to \ \frac{1}{1-\Delta\rho}$$

Consoli, WH, Jegerlehner 1989

$\Rightarrow$ other representation of W–Z mass interdependence

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\,(1+\Delta r)$$

$$\longrightarrow \frac{\pi\,\alpha(M_Z)}{2M_W^2\left(1-\frac{M_W^2}{\rho M_Z^2}\right)}\,(1+\cdots)$$

$\Rightarrow$ other representation of W–Z mass interdependence

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}(1+\Delta r)$$

$$\longrightarrow \frac{\pi\,\alpha(M_Z)}{2M_W^2\underbrace{\left(1-\frac{M_W^2}{\rho M_Z^2}\right)}}(1+\cdots)$$

$$\sin^2\overline{\theta}_W = \sin^2\theta_W + \cos^2\theta_W\,\Delta\rho$$

$\approx \sin^2\theta_{\rm eff}$ *in Z couplings*

- similar structure in $\overline{MS}$ calculations

Degrassi, Sirlin 1990

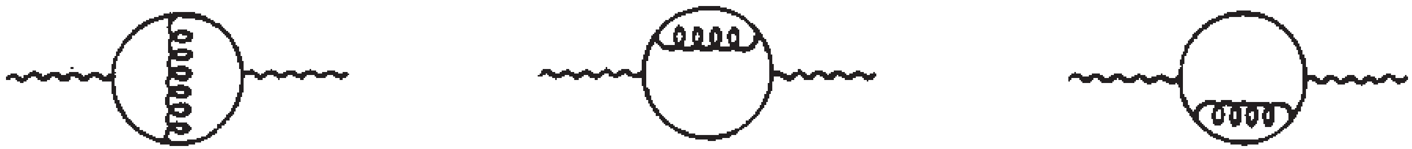
Degrassi, Fanchiotti, Sirlin 1990

Degrassi, Gambino, Giardino 2014

Δr at two and three loops: QCD

$\mathcal{O}(\alpha_s)$ corrections to vector boson self-energies

= $\mathcal{O}(\alpha\alpha_s)$ contribution to Δr $\longrightarrow$ $\Delta r(M_W, M_Z, m_t, M_H, \alpha_s)$



- light-quark loops $\rightarrow$ $\Delta\alpha$ + remnant
- top–bottom loops $\rightarrow$ $\Delta\rho$ + remnant (*dominant part*)
- at $\mathcal{O}(\alpha\alpha_s^2)$: top–bottom loop contributions only

$\mathcal{O}(\alpha\alpha_s)$

Djouadi, Verzegnassi 1987 ($\Delta\rho$)

Kniehl 1990

Halzen, Kniehl 1990

Kniehl, Sirlin 1992

Djouadi, Gambino 1994

$\mathcal{O}(\alpha\alpha_s^2)$

Avdeev, Fleischer, Mikhailov, Tarasov 1994 ($\Delta\rho$)

Chetyrkin, Kühn, Steinhauser 1995

Δr at three and four loops through $\Delta\rho$

$$\Delta r^{(3+4)} = -\frac{c_W^2}{s_W^2}\left(\Delta\rho^{(\alpha^2\alpha_s)} + \Delta\rho^{(\alpha^3)} + \Delta\rho^{(\alpha\alpha_s^3)}\right)$$

- three-loop $\mathcal{O}(\alpha^2\alpha_s)$

  van der Bij, Chetyrkin, Faisst, Kühn, Seidensticker, Veretin 2001
  Faisst, Kühn, Seidensticker, Veretin 2003
  Schröder, Steinhauser 2005

- three-loop $\mathcal{O}(\alpha^3)$

  Faisst, Kühn, Seidensticker, Veretin 2003

- 4-loop $\mathcal{O}(\alpha\alpha_s^3)$

  Chetyrkin, Faisst, Kühn, Maierhofer, Sturm 2006
  Boughezal, Czakon 2006

available for on-shell and $\overline{MS}$ scheme

Δr at electroweak two loops

- fermionic two-loop contributions, on-shell scheme
  - at least one closed fermion loop
    contains also reducible two-loop terms from $\Delta\alpha$ and $\Delta\rho$
  - numerically dominating part

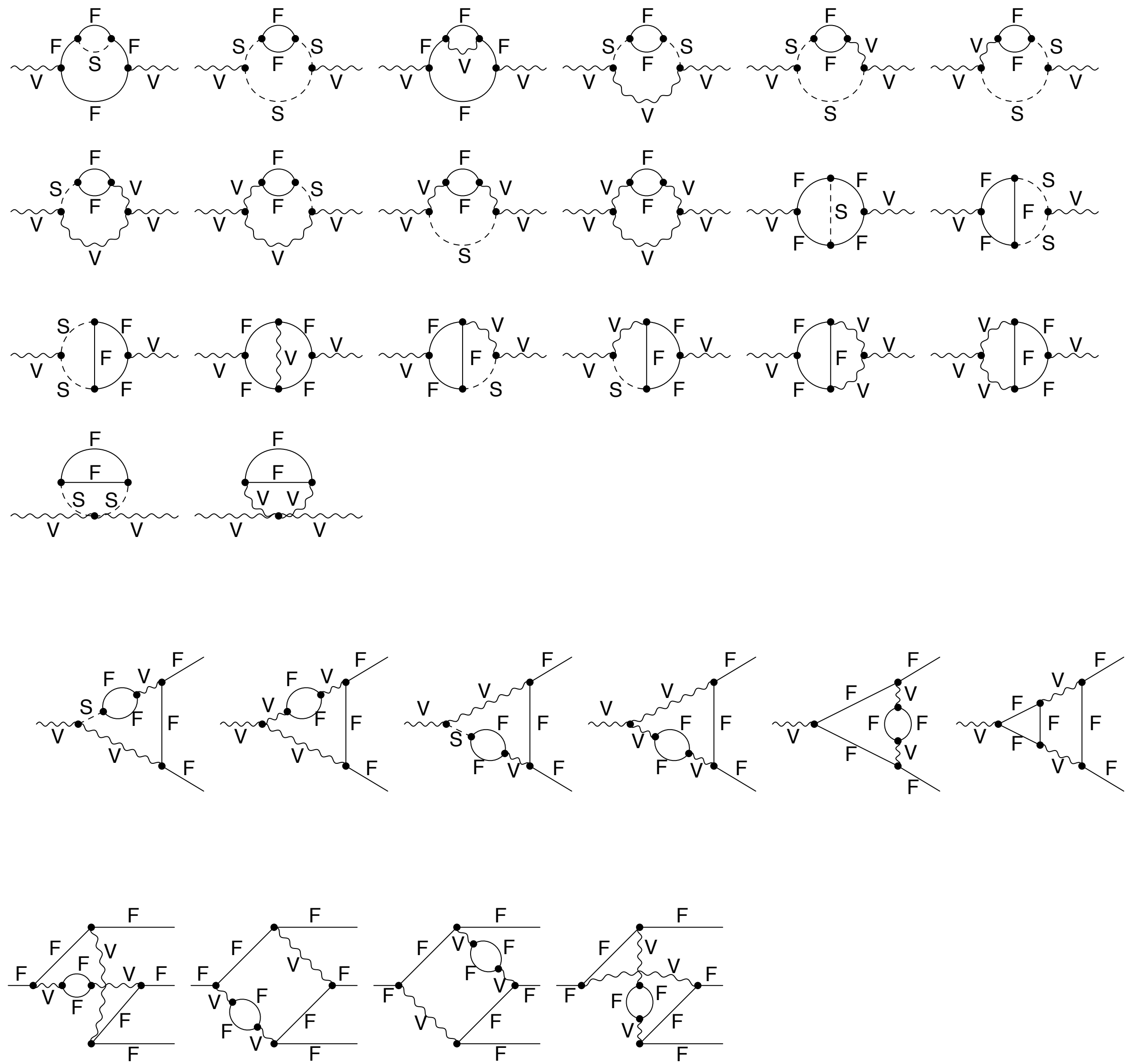
Freitas, WH, Walter, Weiglein 2000, 2002
Awramik, Czakon 2003

- bosonic two-loop contributions, on-shell scheme
  - without a closed fermion loop

Awramik, Czakon 2002
Onishchenko, Veretin 2002

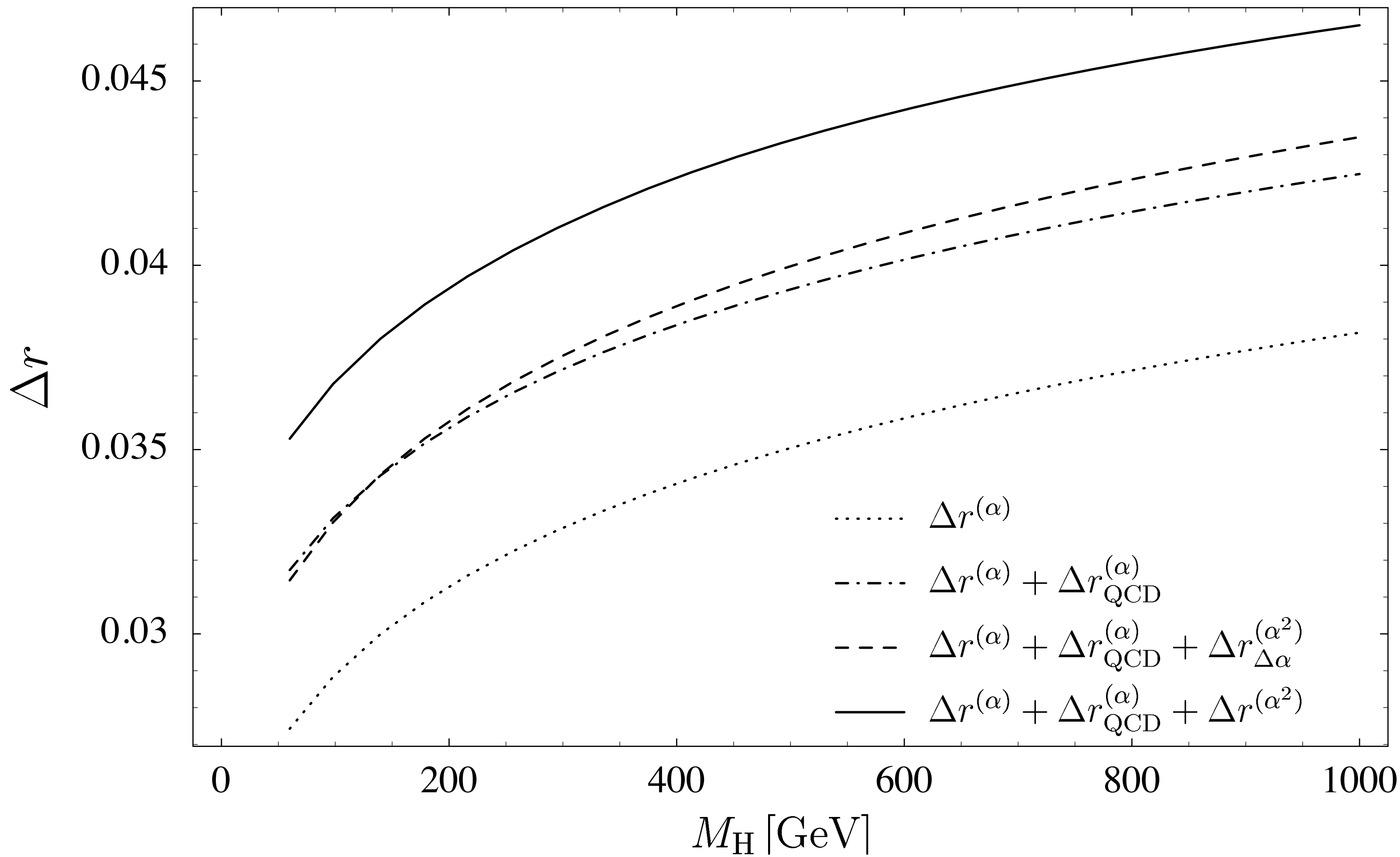
- fermionic and bosonic contributions, $\overline{MS}$-scheme

Degrassi, Gambino, Giardino 2014



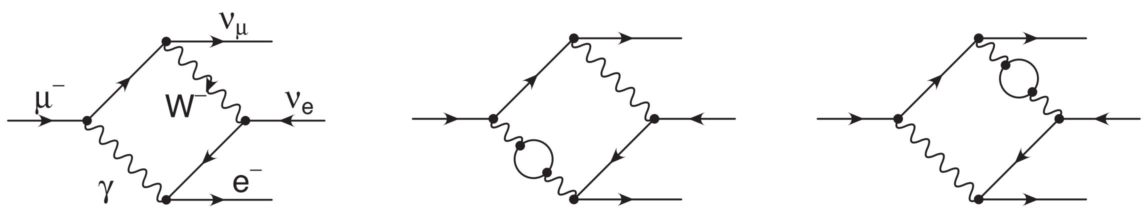
2-loop fermionic diagrams

effects of higher-order terms on Δr



specific aspects of two-loop calculations

- separation of QED corrections in the Fermi model



- definition of W, Z masses
  - complex pole of propagator $\mathcal{M}^2 = \overline{M}^2 - i\,\overline{M}\,\overline{\Gamma}$
    $\left(s - \overline{M}^2 + i\,\overline{M}\,\overline{\Gamma}\right)^{-1}$ *propagator with constant width*
  - conventional masses $M_{W,Z}$
    $\left(s - M^2 + i\,\Gamma s/M\right)^{-1}$ *propagator with s-dependent width*
  - different at two-loop order: $\overline{M} = M - \Gamma^2/2M$
    $\overline{M}_Z = M_Z - 34\,MeV, \qquad \overline{M}_W = M_W - 27\,MeV$

- mass renormalization

dressed propagator $D(s) = \left[s - M_0^2 + \Sigma(s)\right]^{-1}$

bare mass $M_0^2 = \begin{cases} \overline{M}^2 + \delta\overline{M}^2 & (a) \\ M^2 + \delta M^2 & (b) \end{cases}$

renormalization conditions

$(a) \quad D^{-1}(\mathcal{M}^2) = 0 : \qquad \delta\overline{M}^2 = \mathrm{Re}\,\Sigma(\mathcal{M}^2)$

$(b) \quad \mathrm{Re}\,D^{-1}(M^2) = 0 : \qquad \delta M^2 = \mathrm{Re}\,\Sigma(M^2)$

$\delta\overline{M}^2 - \delta M^2 = \mathrm{Im}\,\Sigma'(M^2)\,\mathrm{Im}\,\Sigma(M^2)$ *gauge dependent*

complex pole ($\rightarrow \overline{M}$) is gauge invariant

- renormalization of $\sin^2\theta_W$

$s_W^{0\,2} = s_W^2 + \delta s_W^2$: counterterm is different for M and $\overline{M}$

for $\overline{M}_{W,Z}$: δs_W^2 is gauge independent

- $\overline{M}_{W,Z}$ adapted for two-loop calculations *(translated from/to $M_{W,Z}$)*

$\overline{MS}$ scheme: pole masses $M_{W,Z}$, but running $\hat{\alpha}, \hat{s}^2$ (*scale* μ)

- pole-mass conditions for counterterms

$$\delta M^2_{W,Z} = \mathrm{Re}\,\Sigma_{W,Z}(M^2_{W,Z})$$

  define $M_{W,Z}$ as the conventional masses

- charge renormalization

$$\frac{\delta\hat{\alpha}}{\hat{\alpha}} = \Pi^\gamma(0)\,|_{\mathrm{div}}\ , \quad \Delta\hat{\alpha} = \Pi^\gamma(0)\,|_{\mathrm{fin},\mu=M_Z}\ , \quad \hat{\alpha}(M_Z) = \frac{\alpha}{1-\Delta\hat{\alpha}}$$

- $\overline{MS}$ mixing angle

$$\hat{s}^2 = 1 - \hat{c}^2, \qquad \frac{M_W^{0\,2}}{M_Z^{0\,2}} = \hat{c}^2 + \delta\hat{c}^2, \qquad \hat{\rho} = \frac{M_W^2}{\hat{c}^2\, M_Z^2} = \frac{1}{1-\Delta\hat{\rho}}$$

- M_W–M_Z correlation

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\hat{\alpha}(M_Z)}{2m_W^2\left(1-\frac{M_W^2}{\hat{\rho}M_Z^2}\right)}(1+\Delta\hat{r})$$

$\hat{\alpha}(M_Z)$ *and* $\hat{\rho}$ *accommodate large contributions* $\leftrightarrow$ OS *scheme* $\alpha(M_Z)$ *and* ρ

⇒ other representation of W–Z mass interdependence

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\left(1+\Delta r\right)$$

$$\longrightarrow \frac{\pi\,\alpha(M_Z)}{2M_W^2\underbrace{\left(1-\frac{M_W^2}{\rho M_Z^2}\right)}}\left(1+\cdots\right)$$

$$\sin^2\overline{\theta}_W = \sin^2\theta_W + \cos^2\theta_W\,\Delta\rho$$

$$\approx \sin^2\theta_{\text{eff}} \qquad \textit{in } Z \textit{ couplings}$$

- similar structure in $\overline{MS}$ calculations

Degrassi, Sirlin 1990

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Degrassi, Gambino, Giardino 2014

Numerical evaluation

- different contributions to Δr in the on-shell scheme

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha^2)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha\alpha_s^3)} + \Delta r^{(\alpha^2\alpha_s)} + \Delta r^{(\alpha^3)}$$

(in units of 10^{-4})

$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha^2)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha^3)$
302.36	28.93	34.67	6.97	1.22	-1.44	-0.13

$M_Z = 91.1876,\ m_t = 172.76,\ M_H = 125.09$ (*in GeV*)

$\alpha_s(M_Z) = 0.1179,\ \Delta\alpha_{\rm had}^{(5)} = 0.02766$

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- parametrizations of $M_W(m_t, M_H, \alpha_s, \Delta\alpha)$
  - on-shell scheme — *Awramik, Czakon, Freitas, Weiglein, 0311148v3*

    $M_W = 80.355\, GeV$
  - $\overline{MS}$ scheme — *Degrassi, Gambino, Giardino, 1411.4070*

    $M_W = 80.351\, GeV$

Comments (1)

- update of on-shell evaluation

  improvement in the $\mathcal{O}(\alpha\alpha_s)$ contribution

  not in the parametrization formula

  available as Fortran code(s) linked to `FeynHiggs`

  new result

$$M_W = 80.353\, GeV$$

2203.15710 Bagnaschi et al.

Comments (2)

- OS: does not contain reducible terms beyond 2-loop order (no resummation of $\Delta\alpha$ and $\Delta\rho$)

  $\overline{MS}$: contains resummation via $\hat{\alpha}$ and $\hat{\rho}$

- OS: reducible terms from $\Delta\alpha$ and $\Delta\rho$ at 3-loop order

  $$\Delta r^{(3)} = 0.7 \cdot 10^{-4} \quad \Rightarrow \quad \Delta M_W = -1.2\,\text{MeV}$$

- all fermionic 3-loop contributions (3 closed fermion loops)

  $$\Delta r^{(3)} = 0.7 \cdot 10^{-4}$$

  Stremplat, Dipl. Thesis Karlsruhe 1998; Weiglein 1998

  with masses $\overline{M}_Z$ and $\overline{M}_W$ *Chen, Freitas 2020*

  $$\Delta r^{(3)} = 0.25 \cdot 10^{-4} \quad \Rightarrow \quad \Delta M_W = -0.4\,\text{MeV}$$

- $\overline{MS}$ scheme resums fermion loops (and more)

  with masses M_W and M_Z

not contained in present calculations

- $\mathcal{O}(\alpha^2\alpha_s)$ 3-loop fermionic contributions *Chen, Freitas 2020*

  diagrams with two closed fermion loops and a gluon exchange

  $$\Delta r^{(3)} = -1.09 \cdot 10^{-4} \quad \Rightarrow \quad \Delta M_W = 1.7\,\text{MeV}$$

  in terms of the $\overline{MS}$ top mass:

  $$\Delta r^{(3)} = -0.50 \cdot 10^{-4} \quad \Rightarrow \quad \Delta M_W = 0.8\,\text{MeV}$$

  difference contributes to current theoretical uncertainty

- $m_b \neq 0$ in $\mathcal{O}(\alpha\alpha_s)$ contributions *Djouadi, Gambino 1993*

  $$\Delta r^{(2)}_{m_b} = -0.90 \cdot 10^{-4} \quad \Rightarrow \quad \Delta M_W = 1.5\,\text{MeV}$$

Uncertainties of W mass predictions

- parametric uncertainties

uncertainty of M_W from each parameter (*all masses in MeV*)

	δm_t	δM_H	$\delta\alpha$	$\delta\alpha_s$	δM_Z
1σ	300	140	0.0001	0.0010	2.1
δM_W	1.8	< 0.1	1.8	0.7	2.6

total uncertainty: $\delta M_W = 3.7$ *MeV*

Uncertainties of W mass predictions

- parametric uncertainties

uncertainty of M_W from each parameter (*all masses in MeV*)

	δm_t	δM_H	$\delta\alpha$	$\delta\alpha_s$	δM_Z
1σ	300	140	0.00007	0.0010	2.1
δM_W	1.8	< 0.1	1.3	0.7	2.6

total uncertainty: $\delta M_W = 3.5$ *MeV*

Uncertainties of W mass predictions

- parametric uncertainties

uncertainty of M_W from each parameter (*all masses in MeV*)

	δm_t	δM_H	$\delta\alpha$	$\delta\alpha_s$	δM_Z
1σ	300	140	0.00007	0.0010	2.1
δM_W	1.8	< 0.1	1.3	0.7	2.6

total uncertainty: $\delta M_W = 3.5$ *MeV*

special case: top quark mass *"usually interpreted as the pole mass, but the theoretical uncertainty is hard to quantify" [PDG '20]*

- $m_t = 172.76 \pm 0.30$ *GeV* MC mass
- $m_t = 172.4 \pm 0.7$ *GeV* pole mass

$\Rightarrow \delta M_W = 4.2$ *MeV*

total uncertainty: $\delta M_W = 5.3$ *MeV*

Uncertainties of W mass predictions

- theoretical uncertainties

  intrinsic uncertainties from unknown higher-order corrections
  no systematic procedure, estimates from various options

  - $\overline{MS}$ scheme: 3 MeV

    from variation of renormalization scale

  - on-shell scheme: 4 MeV

    from missing $\mathcal{O}(\alpha^2\alpha_s), \mathcal{O}(\alpha^2\alpha_s^2)$ and $\mathcal{O}(\alpha^3)$

    current $\mathcal{O}(\alpha^2\alpha_s)$ $\Rightarrow$ 3 MeV shift via $\Delta\rho$

    beyond: may be of similar size (experience from ew 2-loop)

    two fermion loops: Chen, Freitas 2019 $\Rightarrow$ 1.7 MeV

    current $\mathcal{O}(\alpha^3)$ $\Rightarrow$ 1 MeV shift via $\Delta\rho$

    3 fermion loops small (0.4 MeV), accidental cancellations

    from W width (enters at 2-loop) 1 MeV unertainty

Summary

- most accurate calculation of the W mass from

$$\alpha, G_F, M_Z, m_t, M_H, \alpha_s$$

- current best predictions from PDG 2020 input
  - on-shell scheme $M_W = 80.353 \pm 0.004\, GeV$
  - $\overline{MS}$ scheme $M_W = 80.351 \pm 0.003\, GeV$

  with theoretical uncertainties

- parametric uncertainty from input parameters

  $\delta M_W = 0.005\, GeV$

- improvement of theoretical uncertainty needs more 3- and 4-loop calculations