Two-loop investigation of new physics effects on M_W from a doublet extension of the SM Higgs sector

Based on

Phys. Lett. B 833 (2022) 137295 (arXiv:2204.05269)

in collaboration with Henning Bahl and Georg Weiglein

Johannes Braathen

MWDays 2023, CERN | 20 April 2023

Outline

- Introduction
- Solving the W-boson mass discrepancy at loop level $\rightarrow$ case of the 2HDM
- Some words about the 2HDM and custodial symmetry
- Our results in the 2HDM
- Conclusions

Introduction: M_W and the CDF result

M_W as an Electroweak Precision Observable (EWPO)

- **Electroweak precision observables**, including
  - **W-boson mass M_W**
  - (Squared sine of) Effective leptonic weak mixing angle $\sin^2\theta_{eff}^{lep}$
  - Z-boson decay width Γ_Z
  - Muon anomalous magnetic moment $(g-2)_\mu$

  etc.

  are **measured** very precisely, and can also be **computed** to high level of accuracy in terms of **G_F, α, M_Z** (most precisely measured EW quantities) and m_h, m_t, α_S, $\Delta\alpha_{had}$, $\Delta\alpha_{lept}$, m_b, etc.

- Allow testing the SM as well as BSM models

- Before April, experimental *world average* was [PDG 2020]
  M_W^{exp} = 80 379 ± 12 MeV

- *SM prediction* (full 1L+2L, partial 3L and 4L, see [Awramik, Czakon, Freitas, Weiglein '03])
  M_W^{SM} = 80 353 ± 6 MeV ([Bagnaschi, Chakraborti, Heinemeyer, Saha, Weiglein '22])
  → **already a small discrepancy!**

CDF M_W result (from the viewpoint of BSM phenomenologists)

- April 7, 2022: CDF result [Science **376**, 170 (2022)]

  M_W^{CDF} = 80 433.5 ± 9.4 MeV

- **~7σ away from SM prediction**
- Not for us (BSM pheno.) to say "this or that measurement is right/wrong"
- Possible issues remain to be discussed about the CDF measurement and its compatibility with previous results → central value could decrease and/or uncertainty could be augmented
- Even so, **inclusion of CDF II into world average will most certainly increase the *already existing* pull from the SM prediction**
- **Strong motivation to investigate BSM contributions to W-boson mass!**

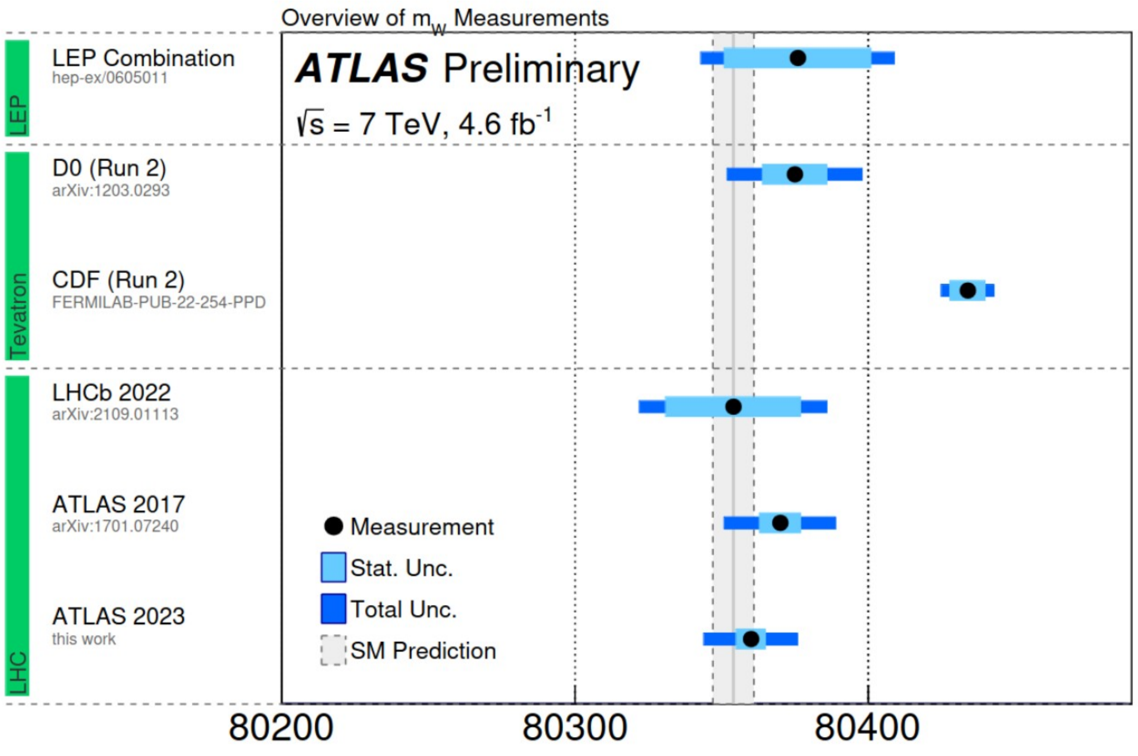


Figure from [ATLAS-CONF-2023-004]

M_W calculation in the SM and beyond

C.f. talk of W. Hollik yesterday

- Base for M_W calculation is the decay of the muon
  - **Extract G_F from muon lifetime τ_μ** by computing τ_μ in the **Fermi theory**
  - Relate M_W, M_Z, α, G_F by **computing muon decay in full theory**, and **matching to Fermi theory result**

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Leftrightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

(in OS scheme)

$\Delta r \equiv \Delta r(M_W, M_Z, m_h, m_t, \ldots)$ denotes corrections to muon decay (w/o finite QED effects)

- Previous relation used to determine M_W as solution, via iterations, of

$$M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

(in OS scheme)

- Inclusion of known higher-order SM corrections *crucial* $\Delta r = \Delta r^{\text{SM}} + \Delta r^{\text{BSM}}$

- Δr^{SM} known to full 1L & 2L + leading 3L & 4L → see [Awramik, Czakon, Freitas, Weiglein '03]

Solving the M_W discrepancy at loop level

M_W^{CDF} = 80 433.5 ± 9.4 MeV

Note 1: solutions at tree level are also possible
(discussed in several talks yesterday & today, e.g. talks by K. Mimasu or C.W. Chiang)

Note 2: many models have been considered at loop level (large number of papers compute the S, T, U parameters at 1L and check if they can reproduce the preferred values obtained by a global fit including the CDF result, see e.g. [Strumia, 2204.04191] or talk by L. Silvestrini yesterday

Some models work, some don't

e.g. singlet extension, c.f. [Sakurai, Takahashi, Yin 2204.04770] which found that $\Delta M_W \leq 5$ MeV

→ in what follows, we will consider whether the **2HDM** can accommodate a value M_W as high as the CDF result (future world-average will certainly be lower, hence needed BSM deviation will be smaller, and easier to reproduce) employing a **2L calculation of M_W**

The 2HDM and custodial symmetry

The Two-Higgs-Doublet Model (2HDM)

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge 1

$$\begin{aligned} V(\Phi_1,\Phi_2) =& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) \\ &+ \frac{1}{2}\Lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\Lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \Lambda_3\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_1\right) + \Lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+ \left[\frac{1}{2}\Lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Lambda_6\Phi_1^\dagger\Phi_1 + \Lambda_7\Phi_2^\dagger\Phi_2\right)\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]. \end{aligned}$$

$$v_1^2 + v_2^2 = v^2 \simeq (246\text{ GeV})^2$$

- **CP-conserving 2HDM**, with softly-broken Z_2 symmetry $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$ to avoid tree-level FCNCs $\rightarrow$ m_{12}^2 and Λ_5 real, $\Lambda_6=\Lambda_7=0$
- **Mass eigenstates**:
  - h, H: CP-even Higgs bosons *(h → 125-GeV SM-like state)*
  - A: CP-odd Higgs boson
  - $H^\pm$: charged Higgs boson
- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H\pm}$, Z_2 soft breaking mass m_{12}^2, angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta=v_2/v_1$)
- We take the **alignment limit α=β-π/2** → all Higgs couplings are SM-like at tree level
  → compatible with current experimental data + no mixing of CP-even scalars!

Custodial symmetry in the scalar sector of the 2HDM I

- In SM (and at 0L) the Higgs potential is *invariant under global transformations of $SU(2)_L \mathrm{x} SU(2)_R$*
- After EWSB, this invariance group is *broken by the Higgs VEV down to* ***$SU(2)_{L+R}$***

  → **custodial symmetry**, which ensures **$\rho^{(0)}=1$**

  → quark sector breaks the custodial symmetry → **$\Delta\rho_{tb}^{SM}\neq 0$**
- What about the 2HDM?
- Using the Higgs basis Φ_{SM}, Φ_{NS} (+ alignment), one can first rewrite the scalar potential as

$$V = V_{\mathrm{I}} + V_{\mathrm{II}} + V_{\mathrm{III}} + V_{\mathrm{IV}};$$

$$V_{\mathrm{I}} = \frac{m_{h^0}^2}{2v^2}\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right)^2 - \frac{1}{2} m_{h^0}^2 \left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right),$$

$$V_{\mathrm{II}} = \left[\frac{1}{2v^2}\left(m_{h^0}^2 + \frac{4}{t_{2\beta}^2}\left(m_{H^0}^2 - \frac{m_{12}^2}{s_\beta c_\beta}\right)\right) + \frac{\Lambda_6\left(2c_{2\beta}-1\right)}{4c_\beta s_\beta^3} - \frac{\Lambda_7\left(2c_{2\beta}+1\right)}{4c_\beta^3 s_\beta}\right]\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)^2$$
$$+ \left(\frac{m_{12}^2}{c_\beta s_\beta} - \frac{m_{h^0}^2}{2}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right),$$

$$V_{\mathrm{III}} = \left(\frac{m_{A^0}^2}{v^2} - \frac{2m_{H^\pm}^2}{v^2} + \frac{m_{H^0}^2}{v^2}\right)\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}}\right)$$
$$+ \left(\frac{m_{H^0}^2}{2v^2} - \frac{m_{A^0}^2}{2v^2}\right)\left(\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}}\right)^2 + \left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right)^2\right)$$
$$+ \left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{SM}}\right)\left(\frac{2m_{H^\pm}^2}{v^2} + \frac{m_{h^0}^2}{v^2} - \frac{2m_{12}^2}{v^2 c_\beta s_\beta}\right),$$

$$V_{\mathrm{IV}} = \left(\frac{2}{v^2 t_{2\beta}}\left(m_{H^0}^2 - \frac{m_{12}^2}{c_\beta s_\beta}\right) - \frac{\Lambda_7}{2c_\beta^2} + \frac{\Lambda_6}{2s_\beta^2}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{NS}}\right)\left(\Phi_{\mathrm{NS}}^\dagger \Phi_{\mathrm{SM}} + \Phi_{\mathrm{SM}}^\dagger \Phi_{\mathrm{NS}}\right).$$

with

$$\Phi_{\mathrm{SM}} = \begin{pmatrix}\phi_{\mathrm{SM}}^+ \\ \phi_{\mathrm{SM}}^0\end{pmatrix} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG)\end{pmatrix}$$

$$\Phi_{\mathrm{NS}} = \begin{pmatrix}\phi_{\mathrm{NS}}^+ \\ \phi_{\mathrm{NS}}^0\end{pmatrix} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(-H + iA)\end{pmatrix}$$

[Hessenberger '18]

Custodial symmetry in the scalar sector of the 2HDM II

- In Higgs basis, one can construct bidoublets

$$\mathcal{M}_\mathrm{X} = \left(i\sigma_2\Phi_\mathrm{X}^*|\Phi_\mathrm{X}\right) = \begin{pmatrix} \phi_\mathrm{X}^{0\,*} & \phi_\mathrm{X}^+ \\ -\phi_\mathrm{X}^- & \phi_\mathrm{X}^0 \end{pmatrix} \quad \text{with X} = \text{SM or NS}$$

  which transform under $SU(2)_L$ x $SU(2)_R$ as

$$\mathcal{M}_\mathrm{SM} \to L\mathcal{M}_\mathrm{SM}R^\dagger \quad \text{and} \quad \mathcal{M}_\mathrm{NS} \to L\mathcal{M}_\mathrm{NS}R'^{\,\dagger} \text{ with } L \in SU(2)_L,\ R, R' \in SU(2)_R$$

- 2 custodial symmetric (even $SU(2)_L$ x $SU(2)_R$) invariant quantities

  → $\mathrm{tr}\big(\mathcal{M}_\mathrm{X}^\dagger\mathcal{M}_\mathrm{X}\big) = 2\Phi_\mathrm{X}^\dagger\Phi_\mathrm{X}, \quad \mathrm{X} = \mathrm{SM}$ or NS

$$\begin{aligned}
V =& V_\mathrm{I} + V_\mathrm{II} + V_\mathrm{III} + V_\mathrm{IV};\\
V_\mathrm{I} =& \frac{m_{h^0}^2}{2v^2}\left(\Phi_\mathrm{SM}^\dagger\Phi_\mathrm{SM}\right)^2 - \frac{1}{2}m_{h^0}^2\left(\Phi_\mathrm{SM}^\dagger\Phi_\mathrm{SM}\right),\\
V_\mathrm{II} =& \left[\frac{1}{2v^2}\left(m_{h^0}^2 + \frac{4}{t_{2\beta}^2}\left(m_{H^0}^2 - \frac{m_{12}^2}{s_\beta c_\beta}\right)\right) + \frac{\Lambda_6\left(2c_{2\beta}-1\right)}{4c_\beta s_\beta^3} - \frac{\Lambda_7\left(2c_{2\beta}+1\right)}{4c_\beta^3 s_\beta}\right]\left(\Phi_\mathrm{NS}^\dagger\Phi_\mathrm{NS}\right)^2\\
&+ \left(\frac{m_{12}^2}{c_\beta s_\beta} - \frac{m_{h^0}^2}{2}\right)\left(\Phi_\mathrm{NS}^\dagger\Phi_\mathrm{NS}\right),
\end{aligned}$$

  → V_I and V_II respect the custodial symmetry

Custodial symmetry in the scalar sector of the 2HDM III

$$V_{\rm III} = \left(\frac{m_{A^0}^2}{v^2} - \frac{2m_{H^\pm}^2}{v^2} + \frac{m_{H^0}^2}{v^2}\right)\left(\Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM}\right)$$
$$+\left(\frac{m_{H^0}^2}{2v^2} - \frac{m_{A^0}^2}{2v^2}\right)\left(\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM}\right)^2 + \left(\Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)^2\right)$$
$$+\left(\Phi_{\rm NS}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm SM}^\dagger\Phi_{\rm SM}\right)\left(\frac{2m_{H^\pm}^2}{v^2} + \frac{m_{h^0}^2}{v^2} - \frac{2m_{12}^2}{v^2 c_\beta s_\beta}\right),$$

→ $V_{\rm III}$ enters Δρ from 1L

$$V_{\rm IV} = \left(\frac{2}{v^2 t_{2\beta}}\left(m_{H^0}^2 - \frac{m_{12}^2}{c_\beta s_\beta}\right) - \frac{\Lambda_7}{2c_\beta^2} + \frac{\Lambda_6}{2s_\beta^2}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM} + \Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)$$

→ $V_{\rm IV}$ enters Δρ from 2L

- $V_{\rm III}$ and $V_{\rm IV}$ involve the non-invariant combinations $\Phi_{\rm NS}^\dagger\Phi_{\rm SM} \pm \Phi_{\rm SM}^\dagger\Phi_{\rm NS}$
  → break custodial symmetry

  → enter scalar corrections to Δρ at 1L and 2L respectively → potential contributions to Δρ and hence Δr and M_W!

- $\Phi_{\rm SM}$ and $\Phi_{\rm NS}$ have same hypercharge Y=1 → R and R' are related, by a matrix X, as $R=X^{-1}R'X$,

  and $\mathrm{tr}\left(\mathcal{M}_{\rm SM}^\dagger\mathcal{M}_{\rm NS}X\right)$ is invariant under $SU(2)_L$ x $SU(2)_R$

Custodial symmetry in the scalar sector of the 2HDM IV

$$V_{\rm III} = \left(\frac{m_{A^0}^2}{v^2} - \frac{2m_{H^\pm}^2}{v^2} + \frac{m_{H^0}^2}{v^2}\right)\left(\Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM}\right) + \left(\frac{m_{H^0}^2}{2v^2} - \frac{m_{A^0}^2}{2v^2}\right)\left(\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM}\right)^2 + \left(\Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)^2\right) + \left(\Phi_{\rm NS}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm SM}^\dagger\Phi_{\rm SM}\right)\left(\frac{2m_{H^\pm}^2}{v^2} + \frac{m_{h^0}^2}{v^2} - \frac{2m_{12}^2}{v^2 c_\beta s_\beta}\right),$$

$$V_{\rm IV} = \left(\frac{2}{v^2 t_{2\beta}}\left(m_{H^0}^2 - \frac{m_{12}^2}{c_\beta s_\beta}\right) - \frac{\Lambda_7}{2c_\beta^2} + \frac{\Lambda_6}{2s_\beta^2}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm NS}\right)\left(\Phi_{\rm NS}^\dagger\Phi_{\rm SM} + \Phi_{\rm SM}^\dagger\Phi_{\rm NS}\right)$$

- From CP invariance in the Higgs sector, we have that X=Id or X=-iσ_3
  - X=Id $\rightarrow \mathrm{tr}\left(\mathcal{M}_{\rm SM}^\dagger\mathcal{M}_{\rm NS}X\right) = \Phi_{\rm NS}^\dagger\Phi_{\rm SM} + \Phi_{\rm SM}^\dagger\Phi_{\rm NS}$ is custodial invariant

    → **V_{IV} invariant and V_{III} invariant if $m_A=m_{H\pm}$**

  - X=-iσ_3 → $\mathrm{tr}\left(\mathcal{M}_{\rm SM}^\dagger\mathcal{M}_{\rm NS}X\right) = -i\Phi_{\rm NS}^\dagger\Phi_{\rm SM} + i\Phi_{\rm SM}^\dagger\Phi_{\rm NS}$ is custodial invariant

    → **V_{III} invariant if $m_H=m_{H\pm}$** while **V_{IV} must vanish → imposes either $m_H^2=m_{12}^2/(s_\beta c_\beta)=M^2$ or $t_\beta=1$**

- E.g. at 1L, where *only terms from V_{III}* enter, we have explicitly

$$\Delta\rho^{(1)}_{\text{non-SM}} = \frac{\alpha}{16\pi^2 s_W^2 M_W^2}\left\{\frac{m_A^2 m_H^2}{m_A^2 - m_H^2}\ln\frac{m_A^2}{m_H^2} - \frac{m_A^2 m_{H\pm}^2}{m_A^2 - m_{H\pm}^2}\ln\frac{m_A^2}{m_{H\pm}^2} - \frac{m_H^2 m_{H\pm}^2}{m_H^2 - m_{H\pm}^2}\ln\frac{m_H^2}{m_{H\pm}^2} + m_{H\pm}^2\right\} \xrightarrow{m_{H\pm}\to m_H \text{ or } m_A} 0$$

M_W in the 2HDM

Calculation of M_W including 2L BSM effects: THDM_EWPOS

- Code (public since 2022) based on [Hessenberger, Hollik '16] and [Hessenberger, Hollik '22]
- Computes Δρ and EWPOs in (aligned) 2HDM as well as IDM to full 1L + leading 2L BSM (+ higher SM)
- Specifically, the computed EWPOs are **M_W** and observables at Z pole, namely
  - **Z-boson width** $\Gamma_Z = \Sigma_f\, \Gamma(Z \to f\bar{f})$ with
  - **Effective leptonic weak mixing angle**

$$\Gamma(Z \to f\bar{f}) = \frac{G_F M_Z^3}{6\sqrt{2}\pi} N_c^f \left[(g_V^f)^2 R_V^f + (g_A^f)^2 R_A^f \right]$$

$$\sin^2\theta_{\rm eff}^{\rm lep} \equiv \frac{1}{4}\left(1 - \frac{g_V^{\rm lep}}{g_A^{\rm lep}}\right)$$

(assuming lepton universality)

N_c^f: colour factor
$g_{V,A}^f$: eff. vector/axial coup. of Z boson to fermion f
$R_{V,A}^f$: radiation factors (final state QCD & QED corr.)

- Corrections to Δρ:
  - **1L**: SM-like top quark piece + BSM scalar piece
  - **2L**: (1L)^2 pieces + **genuine pieces**, i.e. *{top+SM scalars}*, *{top+BSM scalars}*, *{BSM scalars only}*, *{SM+BSM scalars}* – all computed in gaugeless limit
- 2L BSM corrections to Δr, Γ_Z, $\sin^2\theta_{\rm eff}^{\rm lep}$ can always be split between a **reducible part** (i.e. (1L)^2 terms) and an **irreducible part**, which is **proportional to genuine 2L BSM corrections to Δρ**
- Higher order SM corrections to Δr, Γ_Z, $\sin^2\theta_{\rm eff}^{\rm lep}$ included via known parametrisations

  → see details in [Hessenberger '18], [Hessenberger, Hollik '22]

A parameter scan to investigate EWPOs

[Bahl, JB, Weiglein 2204.05269]

- *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types
- Constraints in our parameter scan (all except the last are checked with `ScannerS` [Mühlleitner et al. 2007.02985])

experimental

  - SM-like Higgs measurements with `HiggsSignals`
  - Direct searches for BSM scalars with `HiggsBounds`
  - b-physics constraints, using results from [Gfitter group 1803.01853]

theoretical

  - Vacuum stability
  - Boundedness-from-below of the potential
  - NLO perturbative unitarity, using results from [Grinstein et al. 1512.04567], [Cacchio et al. 1609.01290]

- For points passing these constraints, we compute M_W, $\sin^2\theta_{\text{eff}}^{\text{lep}}$ and Γ_Z using `THDM_EWPOS`
  - **red points** ≡ parameter points that reproduce CDF value for M_W within 1σ, i.e.

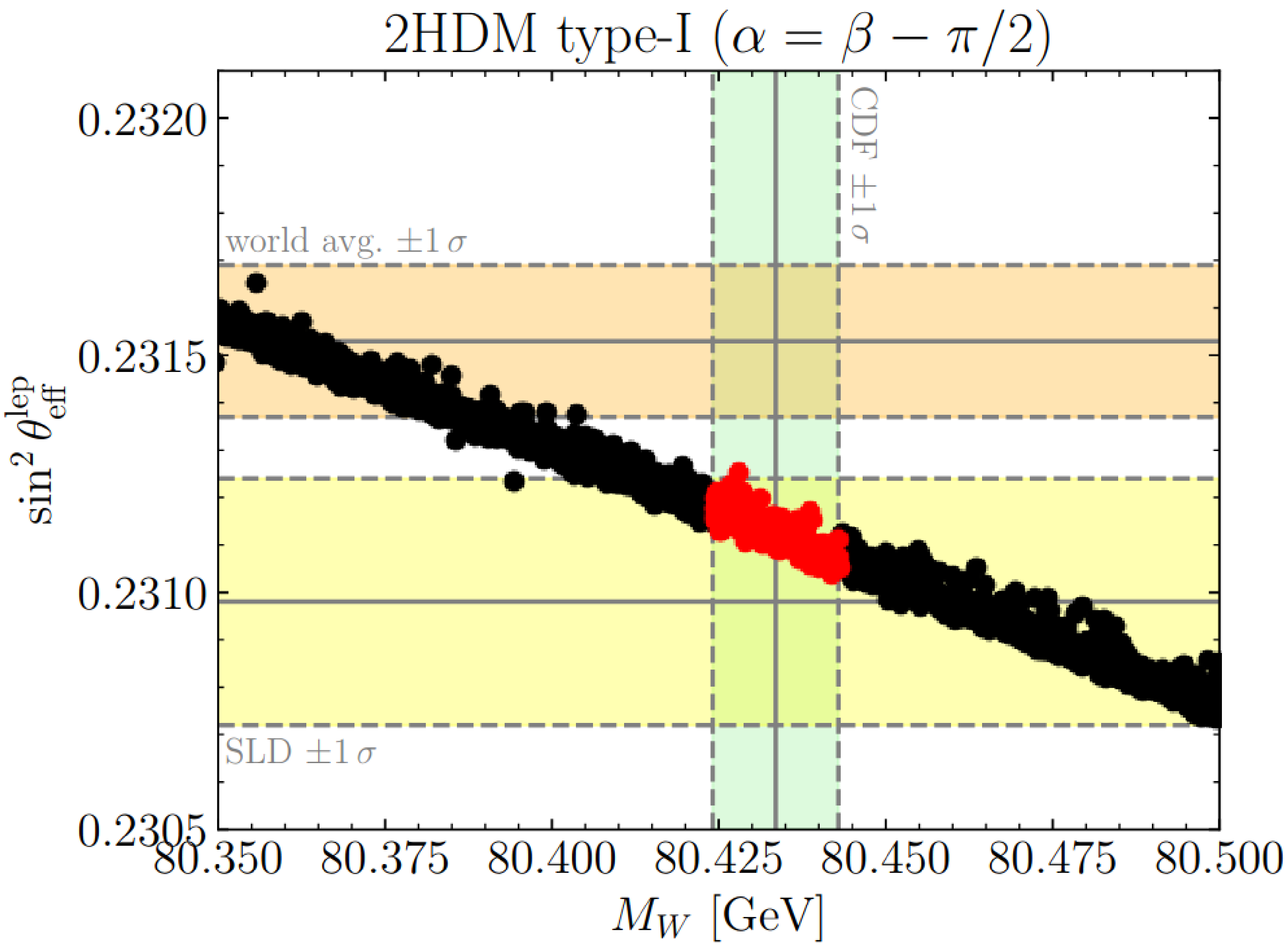
$$\mathbf{80\,424\text{ MeV} \leq (M_W^{(2)})^{\text{2HDM}} \leq 80\,442\text{ MeV}}$$

  - **black points** ≡ all other points

Numerical results

Results: M_W vs $\sin^2\theta_{eff}^{lep}$

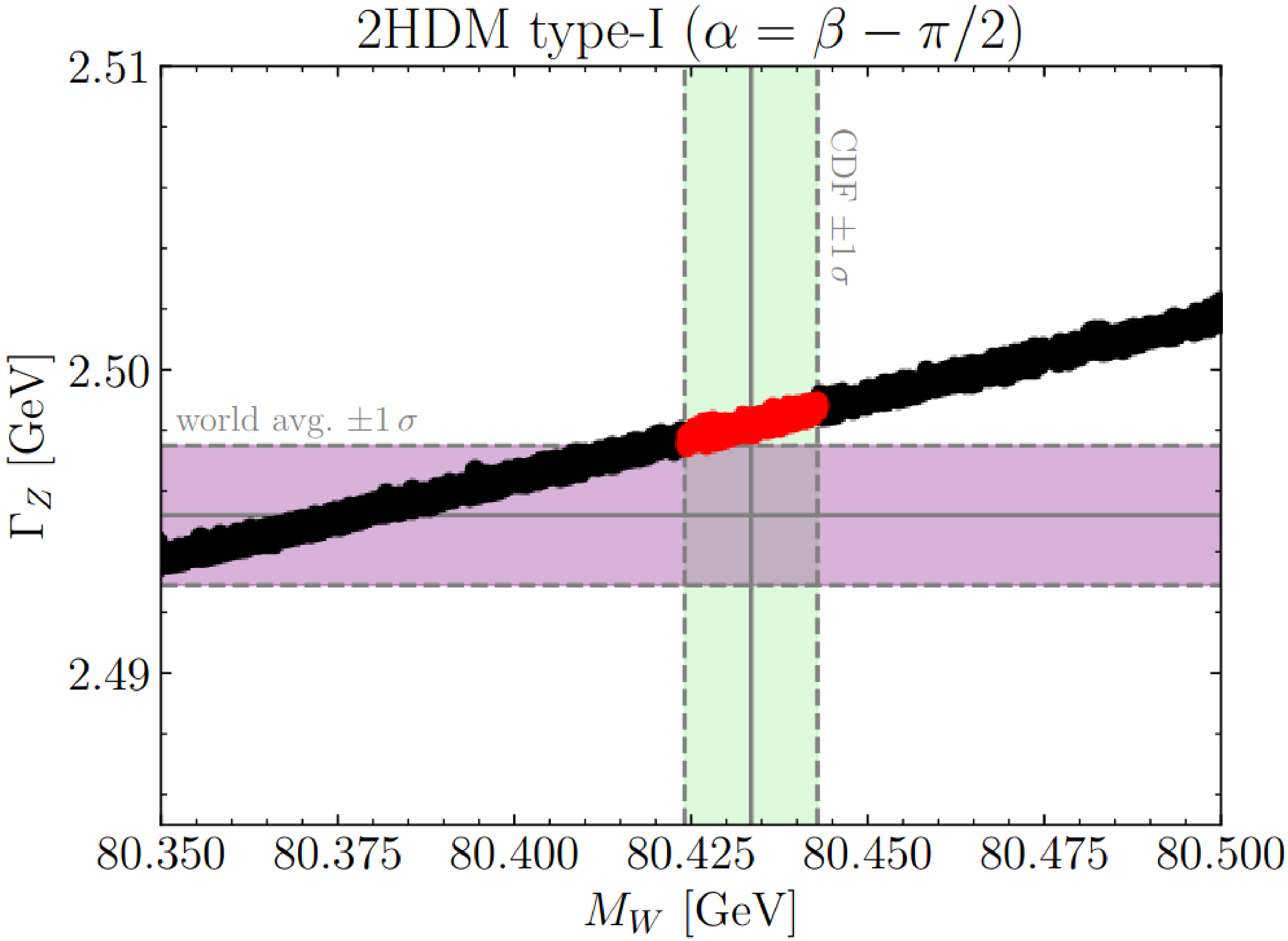
[Bahl, JB, Weiglein 2204.05269]



- **2HDM can explain the discrepancy in M_W!**
- Light tension with world average for $\sin^2\theta_{eff}^{lep}$ but good agreement with SLD result
- ***World average***: using both LEP result (based on forward-backward asymmetry of bottom quarks) + SLD result (based on left-right asymmetry) which show a *3σ discrepancy* between each other
- ***SLD***: most precise *single* measurement of $\sin^2\theta_{eff}^{lep}$ and only depends on leptonic couplings

Results: M_W vs Γ_Z

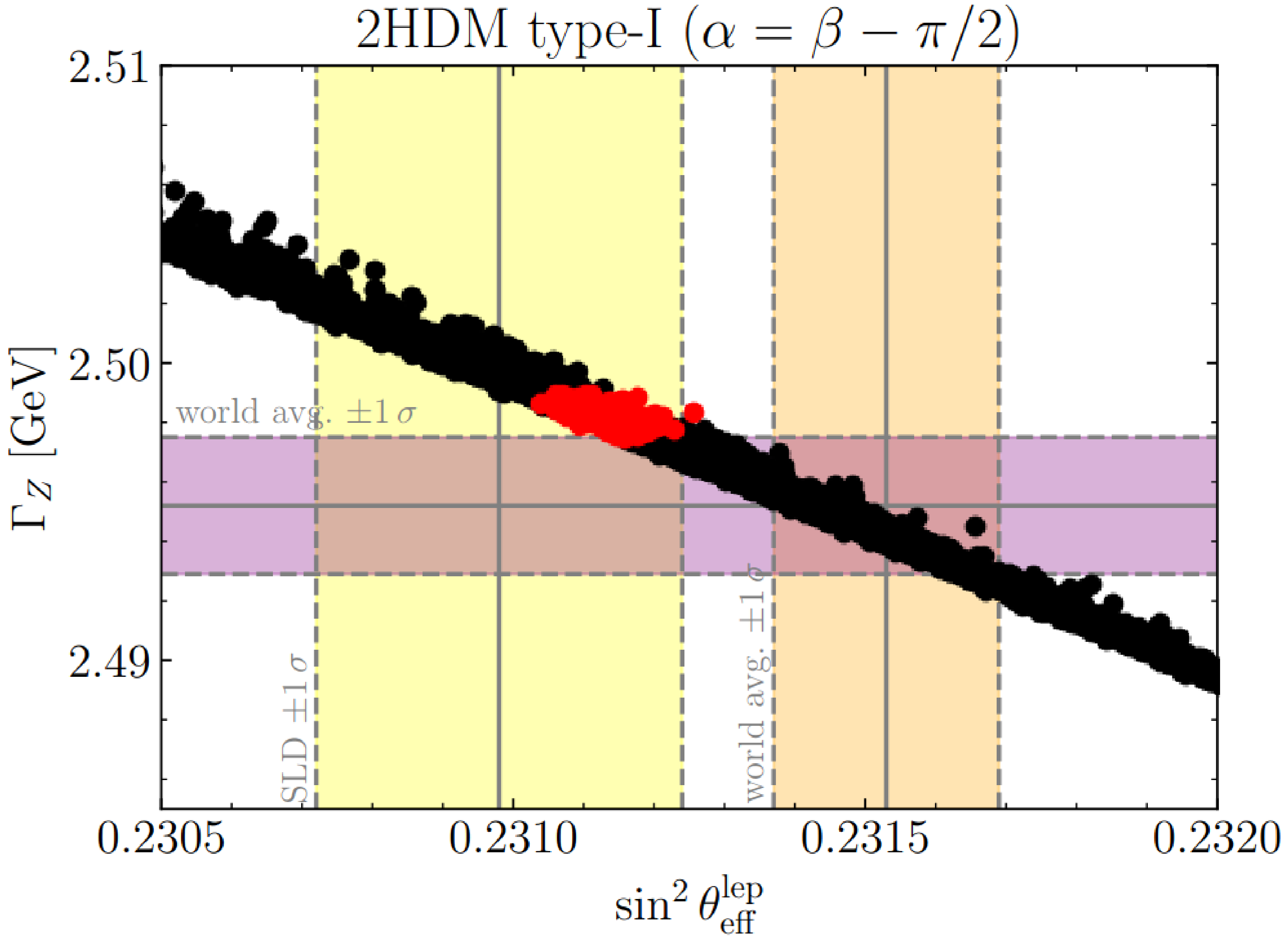
[Bahl, JB, Weiglein 2204.05269]



- Result for Γ_z compatible within 1 – 1.5σ of world average

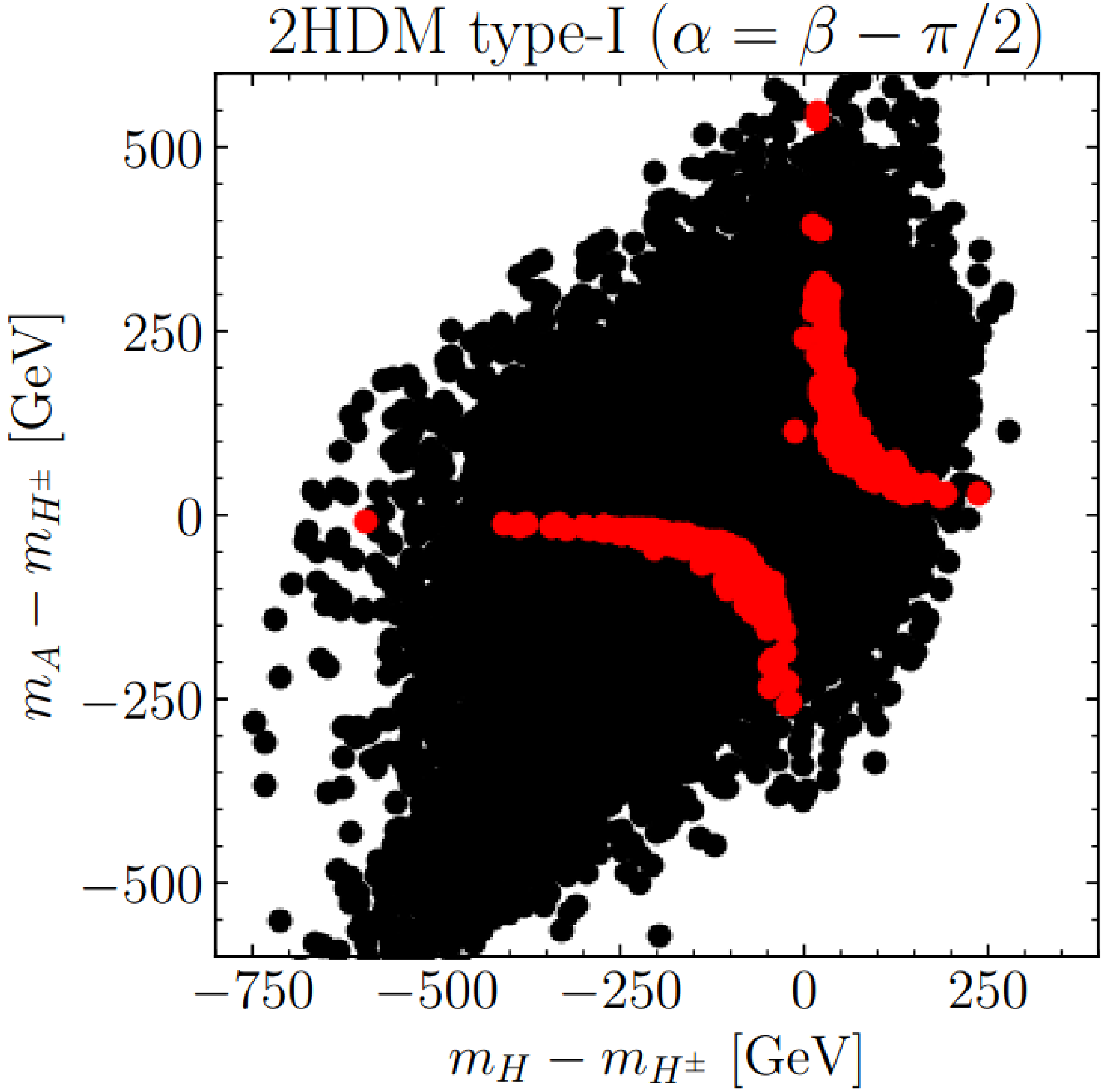
Results: $\sin^2\theta_{eff}^{lep}$ vs Γ_Z

[Bahl, JB, Weiglein 2204.05269]



Results in the (M_H-$M_{H\pm}$, M_A-$M_{H\pm}$) plane

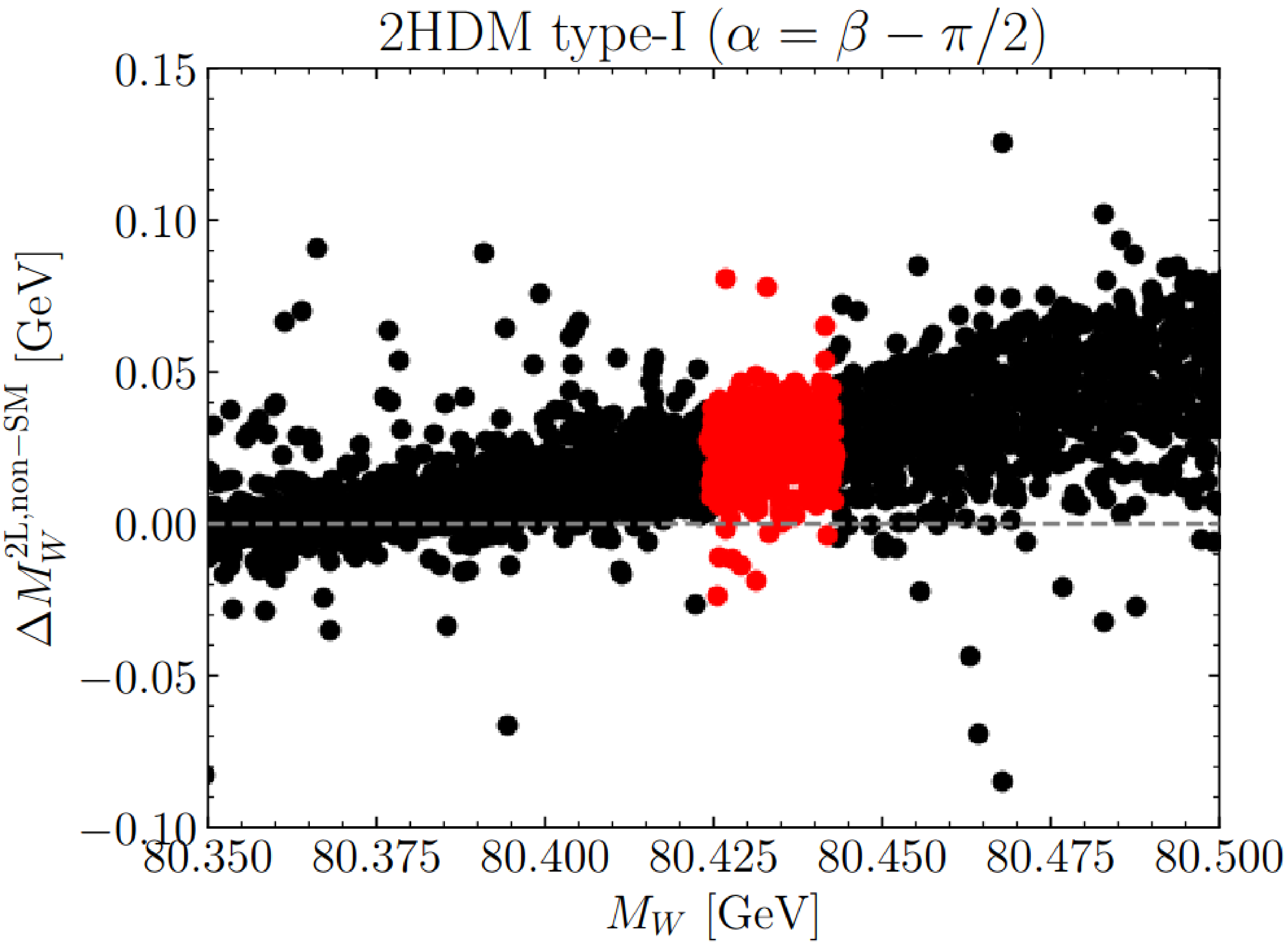
[Bahl, JB, Weiglein 2204.05269]



- **Mass hierarchy where m_H=m_A=$m_{H\pm}$ is no longer allowed because it cannot reproduce M_W!**
- The reason is that in this limit, the *custodial symmetry is restored in the 2HDM scalar sector*
  → scalar contributions to Δρ *vanish*
  → no way of getting a large enough contribution to M_W!
- Need **m_H-$m_{H\pm}$ < 0 and m_A-$m_{H\pm}$ < 0** or **m_H-$m_{H\pm}$ > 0 and m_A-$m_{H\pm}$ > 0** to have a **positive** contribution to Δρ
- Needed mass splitting of ≥ 50 GeV translates into an **upper bound on BSM scalar masses** of O(few TeV) (see also [Heo, Jung, Lee '22])

Impact of two-loop corrections to M_W

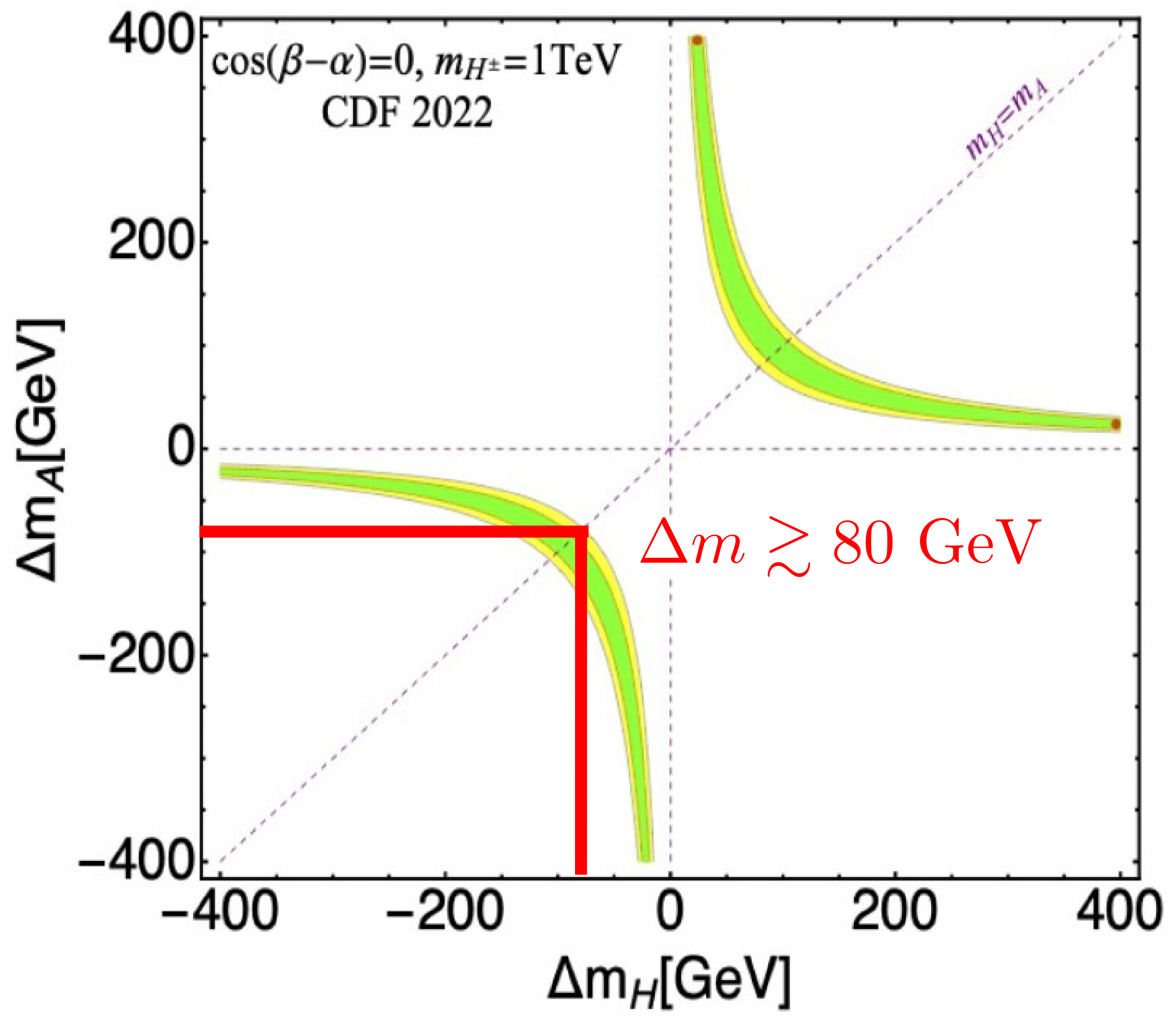
[Bahl, JB, Weiglein 2204.05269]

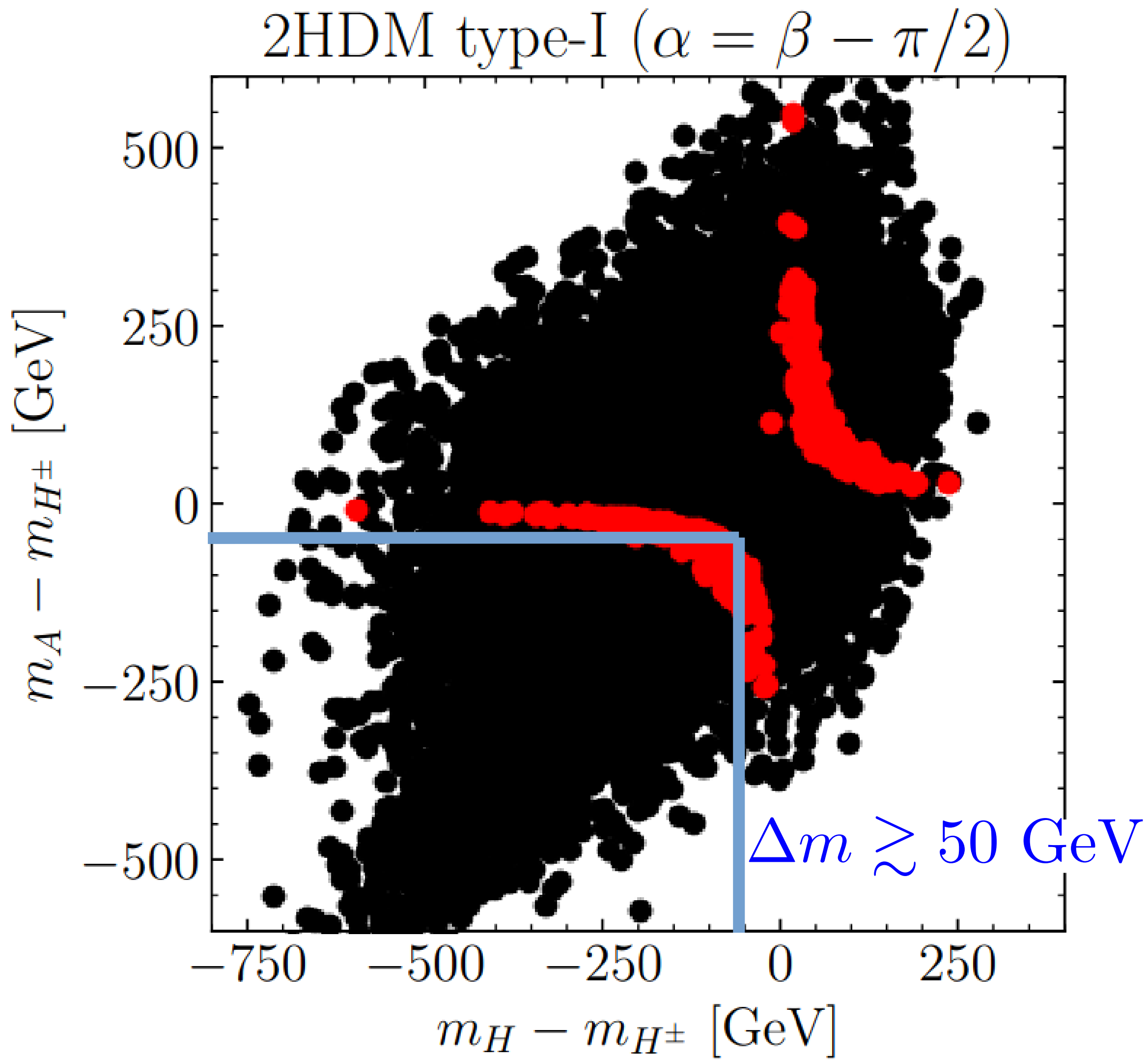


- 2L corrections to M_W often significant, and can play an important role in reaching the values of M_W compatible with the CDF result (especially when custodial sym. is accidentally restored at 1L)
- **Shows the importance of including 2L BSM effects!**

Impact of two-loop corrections to M_W II

[Bahl, JB, Weiglein 2204.05269]

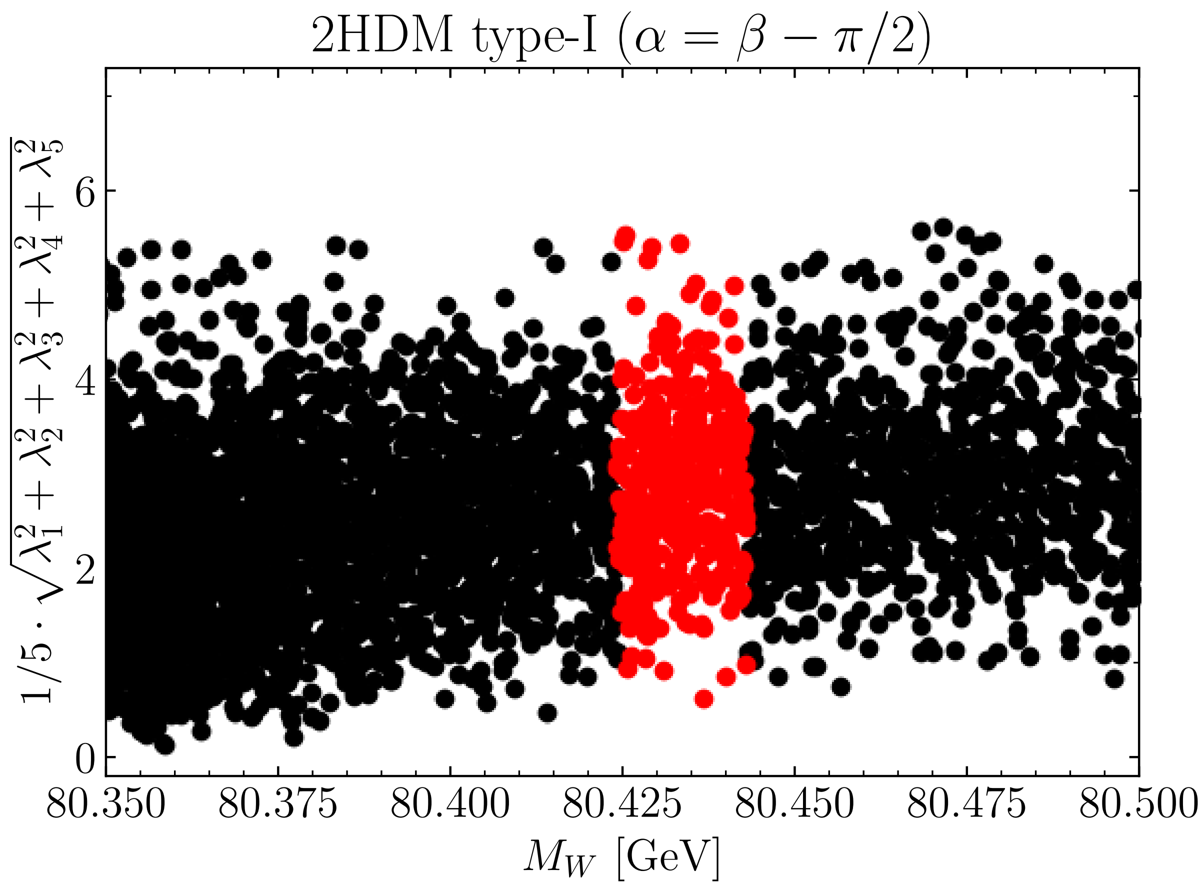


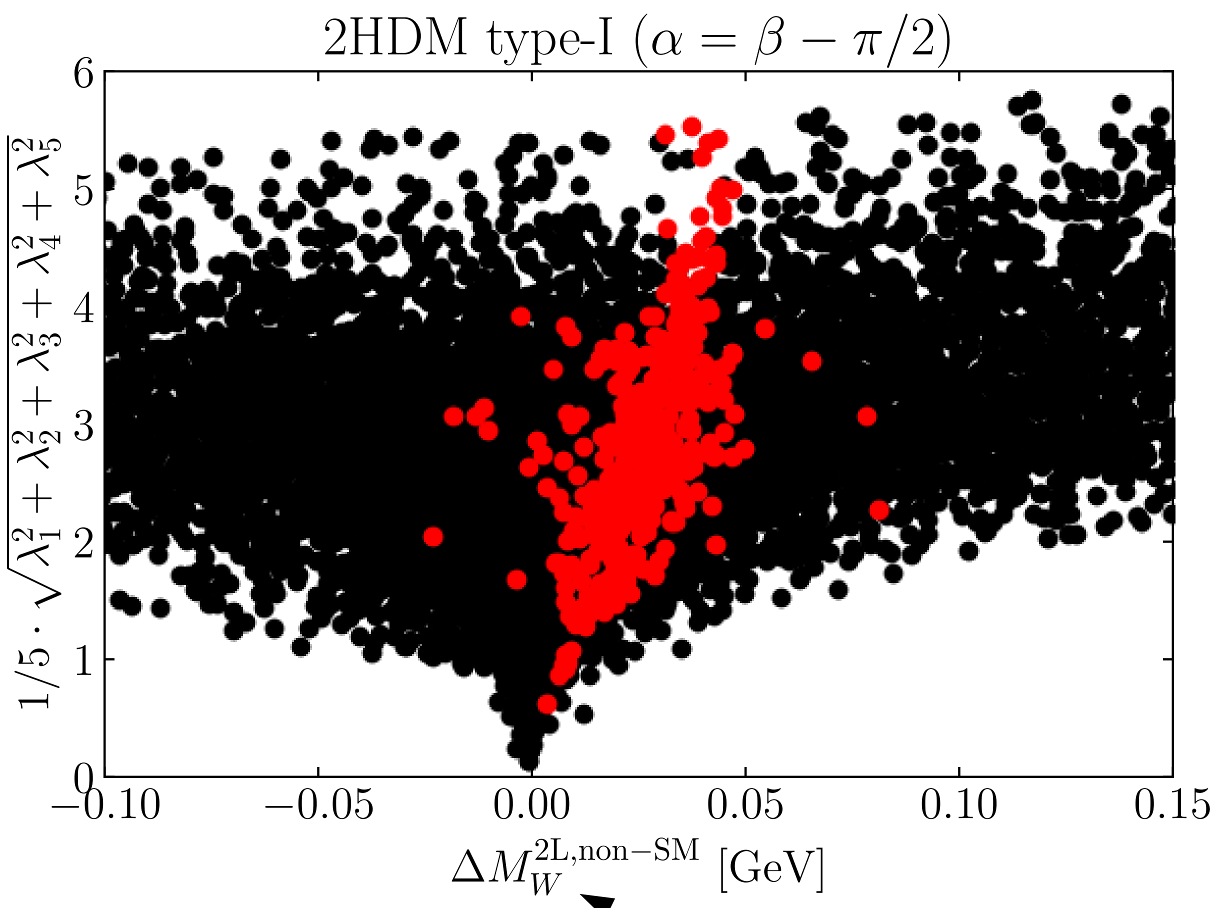


Plot from [Lu, Wu, Wu, Zhu 2204.03796] using 1L S, T, U

Typical couplings sizes

[Bahl, JB, Weiglein 2204.05269]





- Large scalar couplings are **not necessary** to reproduce the CDF value for M_W (with or without large 2L effects)

Summary

- M_W is one of the best measured EWPO, and comparison of theory prediction and experimental results allow stringent tests of SM as well as BSM theories

- Recent excitement related to **CDF result**, seemingly **7σ away from SM → strong motivation to consider BSM contributions to M_W**

- [Bahl, JB, Weiglein 2204.05269] investigated situation in 2HDM, with calculation of M_W including leading 2L BSM (+ h.o. SM) effects using THDM_EWPOS → **2HDM can accommodate M_W discrepancy while keeping satisfactory agreement for $\sin^2\theta_{eff}^{lep}$ and Γ_Z**
(also possible in 2HDM extensions → c.f. talk of G. Arcadi with 2HDMa or [Biekötter, Heinemeyer, Weiglein '22] in N2HDM)

- CDF result is not compatible with degenerate mass hierarchies in 2HDM → **upper bound** on BSM scalar masses

- **Impact of 2L (BSM) corrections to M_W can be significant** (not necessarily related to large couplings)

Thank you for your attention!

Contact

DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

Johannes Braathen
DESY Theory group
johannes.braathen@desy.de

M_W calculation in the SM I

See e.g. [Awramik, Czakon, Freitas, Weiglein '03], [Hessenberger TUM thesis '18]

- Base for M_W calculation is the decay of the muon
  - **Extract G_F from muon lifetime τ_μ** by computing τ_μ in the **Fermi theory**

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(m_e^2/m_\mu^2)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1+\Delta q)$$

with $F(x) \equiv 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$

Tree-level W propagator contributions (not in Fermi th. but numerically tiny)

QED corrections (known to 1L+2L)

  - Relate M_W, M_Z, α, G_F by **computing muon decay in SM**, and **matching to Fermi theory result**

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

OS scheme

$\Delta r \equiv \Delta r(M_W, M_Z, m_h, m_t, \ldots)$ denotes corrections to muon decay (w/o finite QED effects)

  - Previous relation used to determine M_W as solution, via iterations, of

$$M_W^2 = M_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1 + \Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

OS scheme

M_W calculation in the SM II

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r) \qquad M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

- At one loop

$$\Delta r^{(1)} = 2\delta^{(1)}Z_e + \frac{\Sigma_{WW}^{(1)}(p^2=0) - \delta^{(1)}M_W^2}{M_W^2} - \frac{\delta^{(1)}s_W^2}{s_W^2} + \{\text{vertex + box corrections}\}$$

Σ_{WW}: transverse part of the W-boson self-energy, $\delta^{(1)}X$: 1L counterterm to quantity X

- One can show that

$$\delta^{(1)}Z_e \simeq \frac{1}{2}\Delta\alpha + \cdots \quad \text{and} \quad \frac{\delta^{(1)}s_W^2}{s_W^2} \simeq \frac{c_W^2}{s_W^2}\Delta\rho^{(1)}$$

$$\text{with} \quad \Delta\alpha = \frac{\partial}{\partial p^2}\Sigma_{\gamma\gamma}\Big|_{p^2=0} - \frac{\text{Re}\Sigma_{\gamma\gamma}(p^2=M_Z^2)}{M_Z^2}$$

- Leading terms can be rewritten as [Sirlin '80]

$$\Delta r^\alpha = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho^{(1)} + \Delta r_{\text{remainder}}(m_h)$$

with $\Delta\alpha$: contribution from light fermion loops to photon vacuum polarisation
$\Delta\rho$: corrections to the ρ parameter

$$\rho \equiv \frac{G_{\text{NC}}}{G_{\text{CC}}} \quad \Rightarrow \quad \rho^{(0)} = \frac{M_W^2}{c_W^2 M_Z^2} = 1 \text{ and } \Delta\rho^{(1)} = \frac{\Sigma_{ZZ}^{(1)}(p^2=0)}{M_Z^2} - \frac{\Sigma_{WW}^{(1)}(p^2=0)}{M_W^2}$$

M_W calculation in the SM III

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \quad \Rightarrow \quad M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

$$M_W^2 = M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}\left(1+\Delta r(M_W^2, M_Z^2, m_h^2, m_t^2, \cdots)\right)}\right]$$

- At higher orders

$$\Delta r = \Delta r^{\alpha} + \Delta r^{\alpha\alpha_s} + \Delta r^{\alpha\alpha_s^2} + \Delta r^{\alpha\alpha_s^3 m_t} + \Delta r^{\alpha^2}_{\text{ferm}} + \Delta r^{\alpha^2}_{\text{bos}} + \Delta r^{G_F^2\alpha_s m_t^4} + \Delta r^{G_F^3 m_t^6}$$

QCD (2L+3L+approx.4L) *EW (2L)* *leading 3L corr. to Δρ*

- [Awramik, Czakon, Freitas, Weiglein ‘03] gives a parametrisation as

$$\begin{aligned} M_{\rm W} \;=\; & M_{\rm W}^0 - c_1\,{\rm dH} - c_2\,{\rm dH}^2 + c_3\,{\rm dH}^4 + c_4({\rm dh}-1) - c_5\,{\rm d}\alpha + c_6\,{\rm dt} - c_7\,{\rm dt}^2 \\ & - c_8\,{\rm dH}\,{\rm dt} + c_9\,{\rm dh}\,{\rm dt} - c_{10}\,{\rm d}\alpha_{\rm s} + c_{11}\,{\rm dZ}, \end{aligned}$$

with

$${\rm dH} = \ln\left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right), \quad {\rm dh} = \left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right)^2, \quad {\rm dt} = \left(\frac{m_{\rm t}}{174.3\ {\rm GeV}}\right)^2 - 1,$$

$${\rm dZ} = \frac{M_{\rm Z}}{91.1875\ {\rm GeV}} - 1, \quad {\rm d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad {\rm d}\alpha_{\rm s} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.119} - 1,$$

$$\begin{aligned} M_{\rm W}^0 &= 80.3779\ {\rm GeV}, & c_1 &= 0.05263\ {\rm GeV}, & c_2 &= 0.010239\ {\rm GeV}, \\ c_3 &= 0.000954\ {\rm GeV}, & c_4 &= -0.000054\ {\rm GeV}, & c_5 &= 1.077\ {\rm GeV}, \\ c_6 &= 0.5252\ {\rm GeV}, & c_7 &= 0.0700\ {\rm GeV}, & c_8 &= 0.004102\ {\rm GeV}, \\ c_9 &= 0.000111\ {\rm GeV}, & c_{10} &= 0.0774\ {\rm GeV}, & c_{11} &= 115.0\ {\rm GeV}, \end{aligned}$$

- *Note*: Δr also serves to extract the Higgs VEV from G_F

$$v^2 = \frac{1}{\sqrt{2}G_F}(1+\Delta r)$$

M_W calculation beyond the SM

- Idea of the calculation remains the same, but full theory calculation (that is matched with the Fermi theory one) is now done in the **BSM model**

- In BSM models, M_W ($\leftrightarrow$ muon decay) can receive contributions both at **tree level** and at **loop level**. Considering a model with both sources (and turning to $\overline{\text{MS}}$ for simplicity just here), one can write at 1L [Athron et al. 1710.03760, 2204.05285]

$$M_W^2\Big|^{\overline{\text{MS}}} = (M_W^{\text{SM}}|^{\overline{\text{MS}}})^2 \left\{1 + \frac{s_W^2}{c_W^2 - s_W^2}\left[\frac{c_W^2}{s_W^2}(\Delta\rho_{\text{tree}} + \Delta\rho_{\text{loop}}^{\text{BSM}}) - \Delta r_{\text{remainder}}^{\text{BSM}} - \Delta\alpha^{\text{BSM}}\right]\right\}$$

- In the following, we will only discuss models with $\boldsymbol{\rho^{(0)}=1}$, and we stay in **OS scheme**

- Some 2L corrections to $\Delta\rho$ known in BSM models
  - $O(\alpha\alpha_S)$ SUSY corrections in [Djouadi et al. ‘96, ‘98]
  - $O(\alpha_t^2,\alpha_t\alpha_b,\alpha_b^2)$ in MSSM in [Heinemeyer, Weiglein ‘02], [Hastier, Heinemeyer, Stöckinger, Weiglein ‘05]
  - BSM scalar + top quark corrections in (aligned) 2HDM and IDM [Hessenberger, Hollik ‘16]
- Inclusion of known higher-order SM corrections crucial $\Delta r = \Delta r^{\text{SM}} + \Delta r^{\text{BSM}}$
- Calculations of M_W with Δr to full BSM 1L + partial BSM 2L (from resummation and $\Delta\rho$) + SM up to 4L
  - MSSM [Heinemeyer, Hollik, Weiglein, Zeune ‘13]
  - NMSSM [Stål, Weiglein, Zeune ‘15]
  - MRSSM [Diessner, Weiglein ‘19]
  - **2HDM & IDM** [Hessenberger ‘18] (TUM thesis and code `THDM_EWPOS`)

M_W calculation in the 2HDM

$$\Delta r = \Delta r_{\rm SM} + \Delta r_{\rm NS}$$

- Δr at 2L

$$\Delta r^{(2)}_{\rm NS} = \Delta r^{(2)}_{\rm NS,red} + \Delta r^{(2)}_{\rm NS,irr}.$$

where

$$\Delta r^{(2)}_{\rm NS,red} = -2\frac{c_W^2}{s_W^2}\Delta\alpha\,\Delta\rho^{(1)}_{\rm NS} + 4\frac{c_W^4}{s_W^4}\Delta\rho^{(1)}_{\rm NS}\,\Delta\rho^{(1)}_{\rm t} + 2\frac{c_W^4}{s_W^4}\left(\Delta\rho^{(1)}_{\rm NS}\right)^2$$

$$\Delta r^{(2)}_{\rm irr} = -\frac{c_W^2}{s_W^2}\,\delta\rho^{(2)}\,, \qquad \delta\rho^{(2)} = \delta\rho^{(2)}_{\rm t,SM} + \delta\rho^{(2)}_{\rm t,NS} + \delta\rho^{(2)}_{\rm H,NS} + \delta\rho^{(2)}_{\rm H,Mix},$$

- NB: $\Delta\rho$ at 2L

$$\Delta\rho^{(2)} = -\frac{c_W^2}{s_W^2}\left(\Delta\rho^{(1)}\right)^2 + \delta\rho^{(2)}$$

Genuine 2L piece

- Effective leptonic weak mixing angle

$$s_l^2 \; = \; s_W^2\,\kappa = s_W^2\,(1+\Delta\kappa)\,.$$

$$\Delta\kappa^{(1)} = \frac{c_W^2}{s_W^2}\Delta\rho^{(1)} + \cdots \qquad \Delta\kappa^{(2)} = \Delta\alpha\,\frac{c_W^2}{s_W^2}\Delta\rho^{(1)} - \frac{c_W^4}{s_W^4}\left(\Delta\rho^{(1)}\right)^2 + \frac{c_W^2}{s_W^2}\,\delta\rho^{(2)}$$

Fixed vs running width

- OS renormalisation conditions: W- (and Z-) boson mass defined as **real part of the complex pole of the propagator** → gauge invariant definition

- Expanding propagator around complex pole → Breit-Wigner shape with a **fixed width**

- W- (and Z-) boson mass measured experimentally corresponds (usually) to a definition of the mass with a Breit-Wigner shape with **running width**

- Comparison of theory and experiment requires a conversion:

$$M_W^{\text{run. width}} = M_W^{\text{fix. width}} + \frac{\Gamma_W^2}{2M_W^{\text{run. width}}}$$

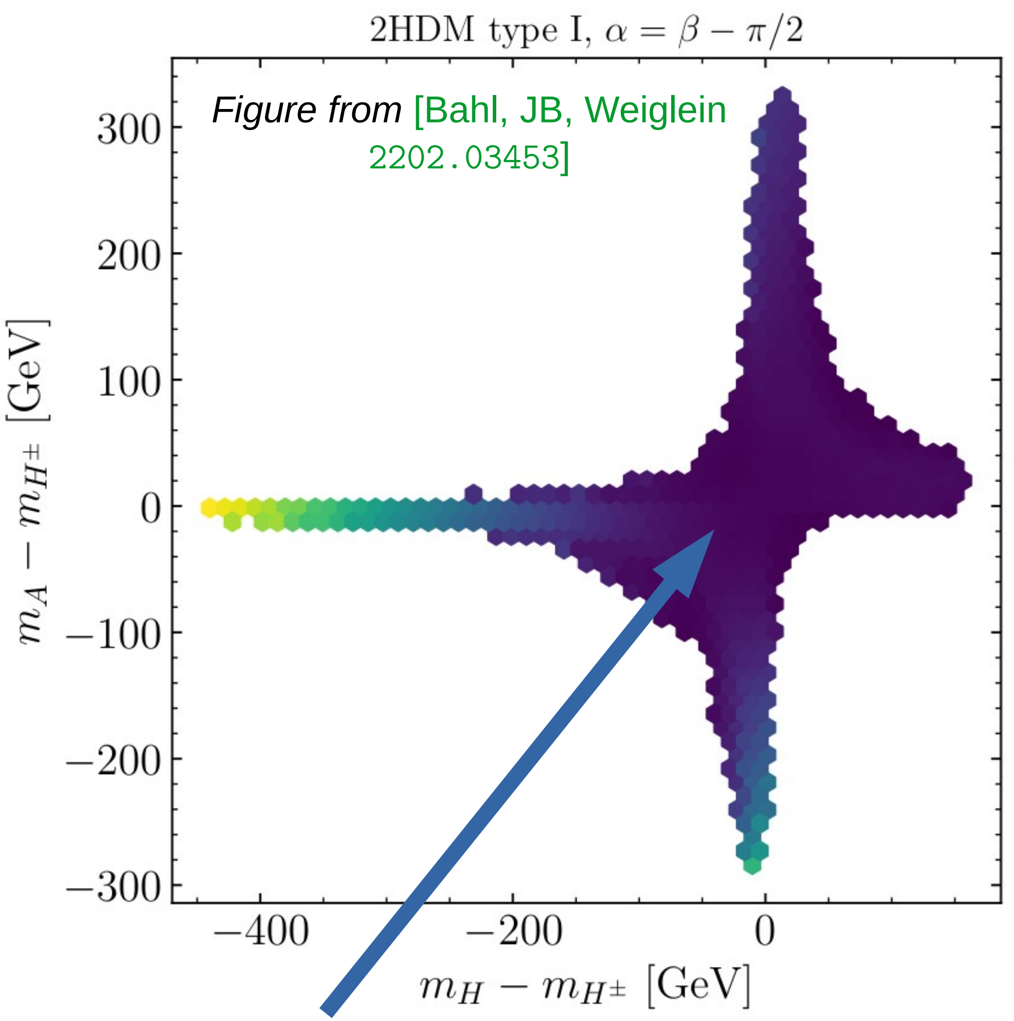
where for the W decay width one uses a result parametrised in terms of G_F and including 1L QCD corrections

$$\Gamma_W = \frac{3G_F(M_W^{\text{run. width}})^3}{2\sqrt{2}\pi}\left(1+\frac{2\alpha_s}{3\pi}\right)$$

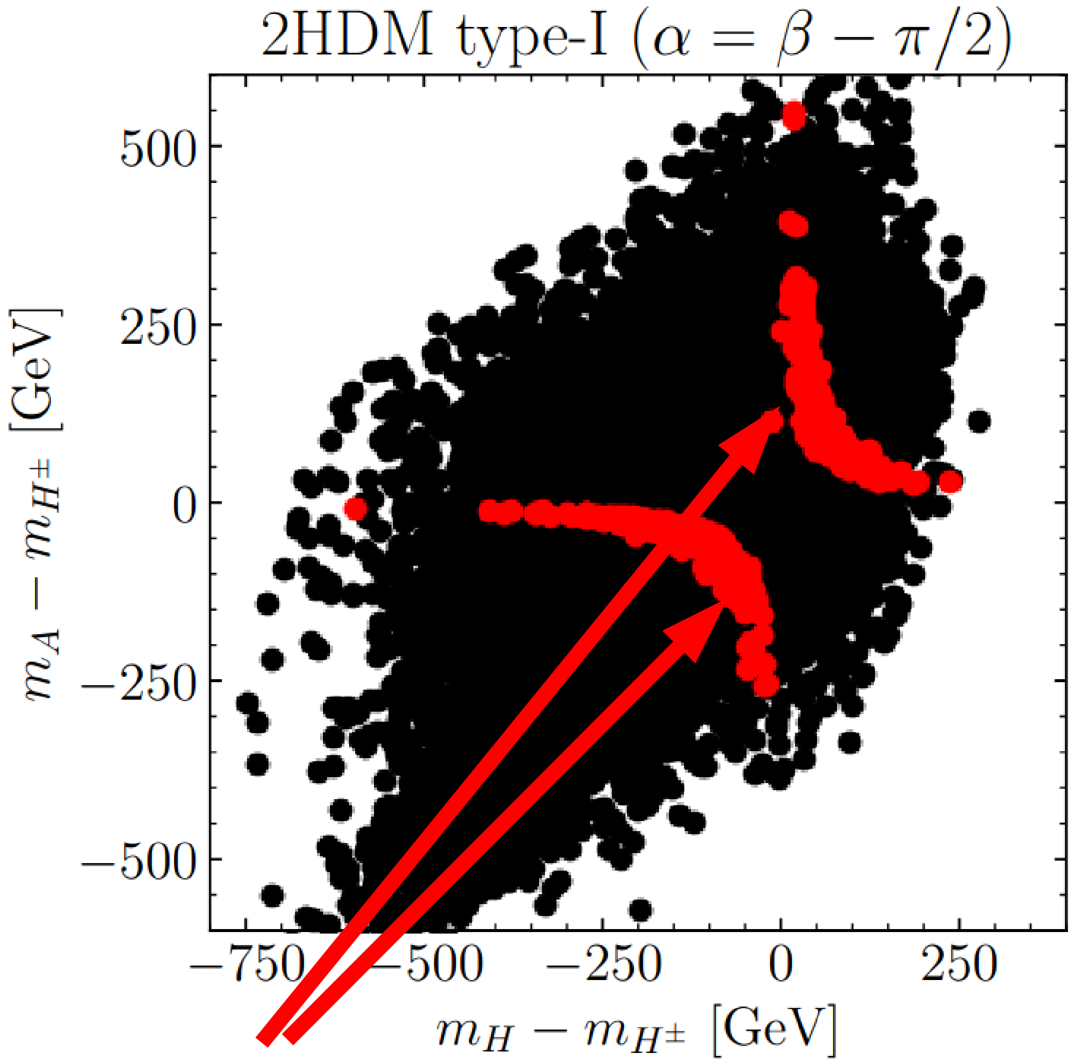
- Resulting shift of **~27 MeV**

Results in the (M_H-$M_{H\pm}$, M_A-$M_{H\pm}$) plane II

[Bahl, JB, Weiglein 2204.05269]

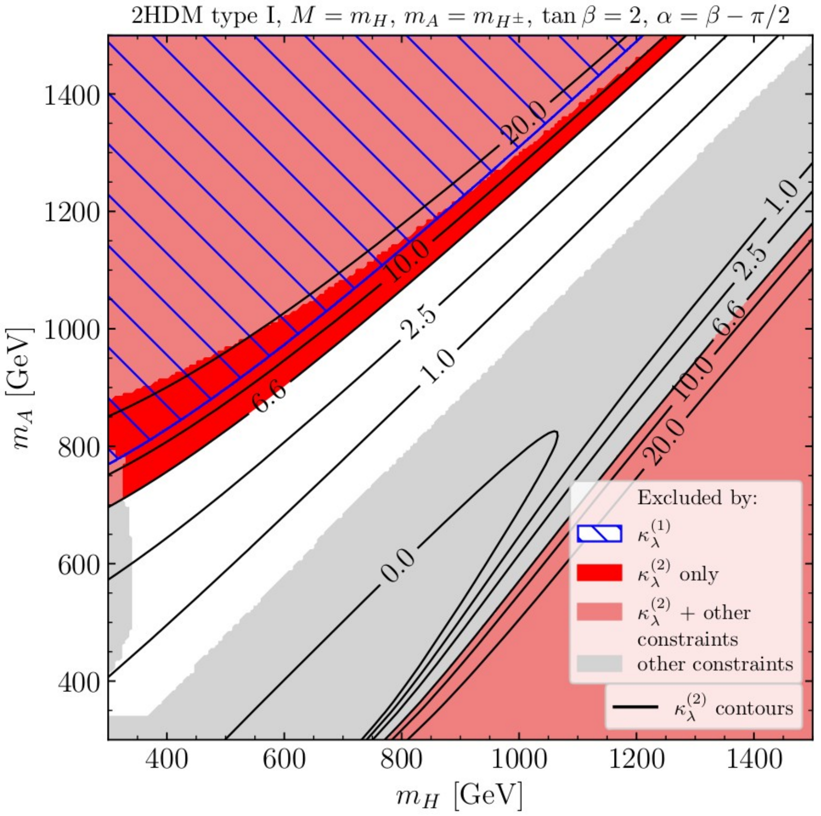


Reproducing the world average value for M_W (w/o CDF)

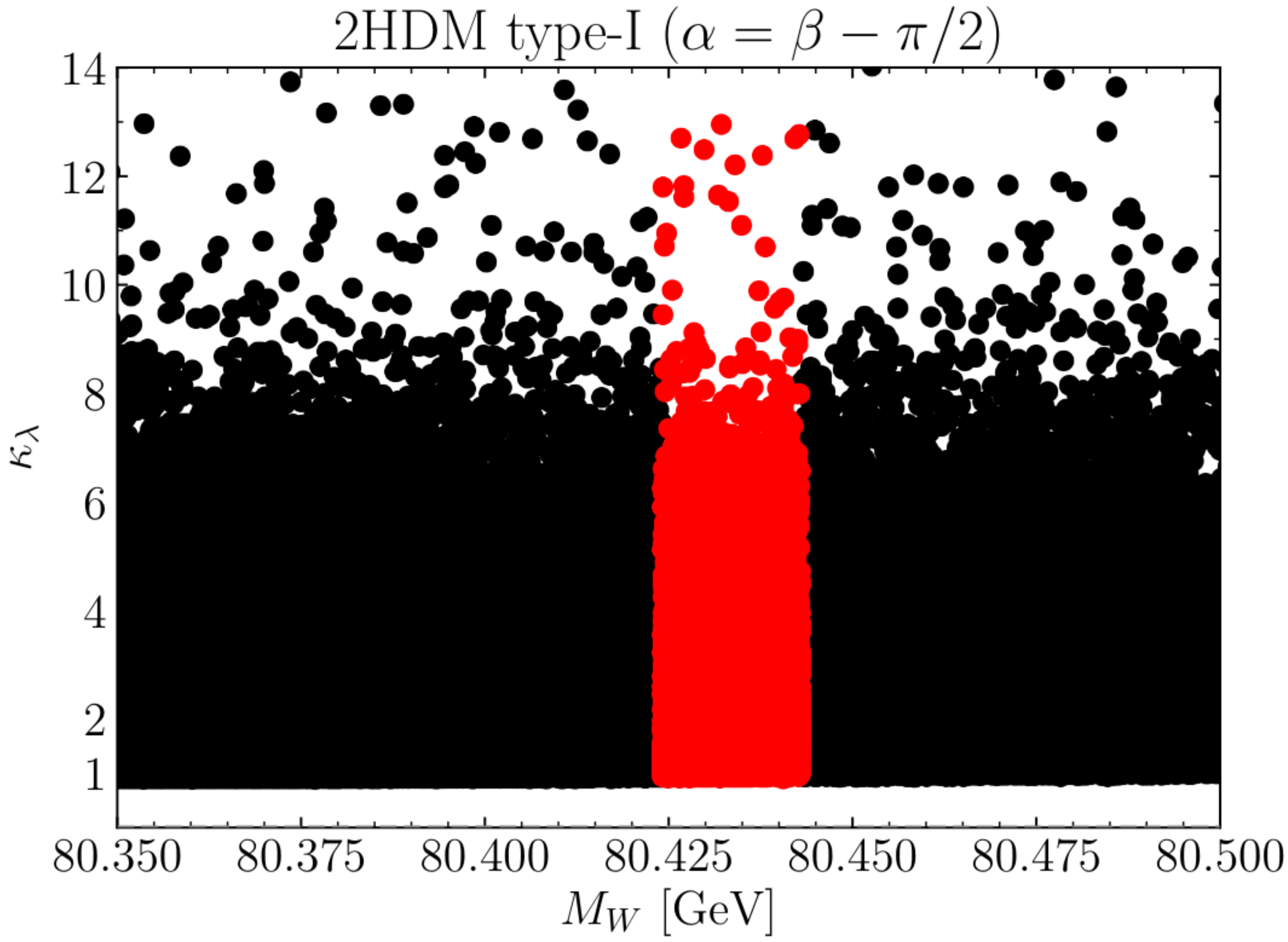


Reproducing the CDF result for M_W

Correlation between M_W and κ_λ



[Bahl, JB, Weiglein 2202.03453]



- No apparent correlation between M_W and κ_λ
- Only few points excluded by $-1.0 < \kappa_\lambda < 6.6$ [ATLAS-CONF-2021-052]