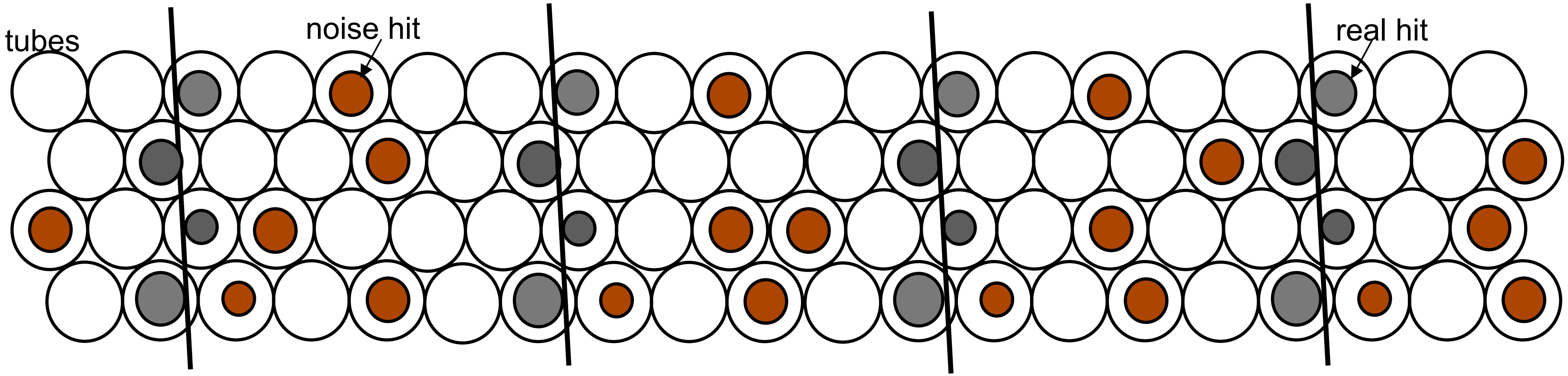
Motivation

Greek Team for WP2.3
17 Feb 2011, AIDA, Kick-off Meeting, CERN

How to perform pattern recognition in Monitored Drift Tube chambers in the case of high noise environment?

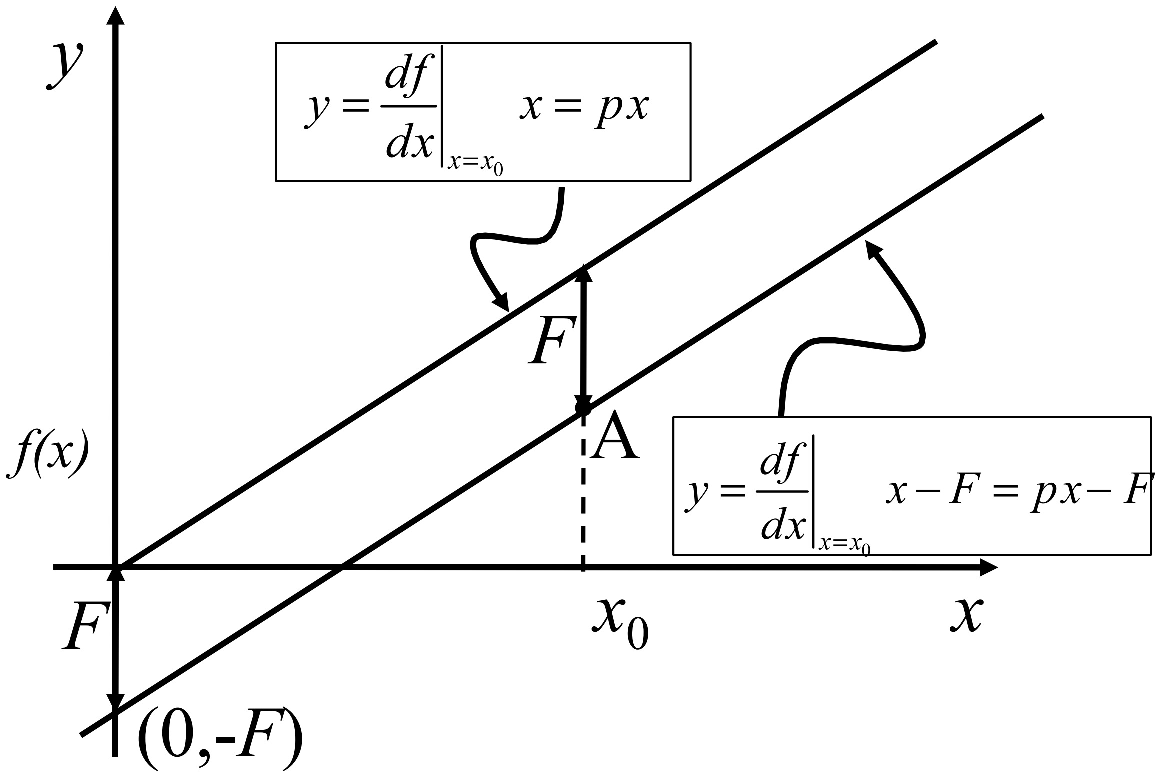


Legendre Transform (LT) of Convex Functions

$$\frac{\mathrm{d}^2 f}{\mathrm{d}^2 x} > 0 \quad \text{convex function}$$

$$f(x) \xleftrightarrow{LT} F(p) \quad \text{Legendre transform pairs}$$

$$F(p) = \sup_x \left[px - f(x) \right] = -\inf_x \left[f(x) - px \right]$$



Calculate LT at a point x_0:

$$p = \left.\frac{df}{dx}\right|_{x=x_0} \Rightarrow x_0 = X(p)$$

$$F(p) = px_0 - f(x_0)$$

$$= pX(p) - f\big(X(p)\big)$$

Legendre Transform

The Legendre transformation is an application of the duality relationship between points and lines.

The functional relationship specified by $f(x)$ can be represented equally :

- as a set of (x, y) points, or
- as a set of tangent lines specified by their slope and intercept values.

Example: Legendre Transform of $x^2/2$

$$f(x) = \frac{x^2}{2}$$

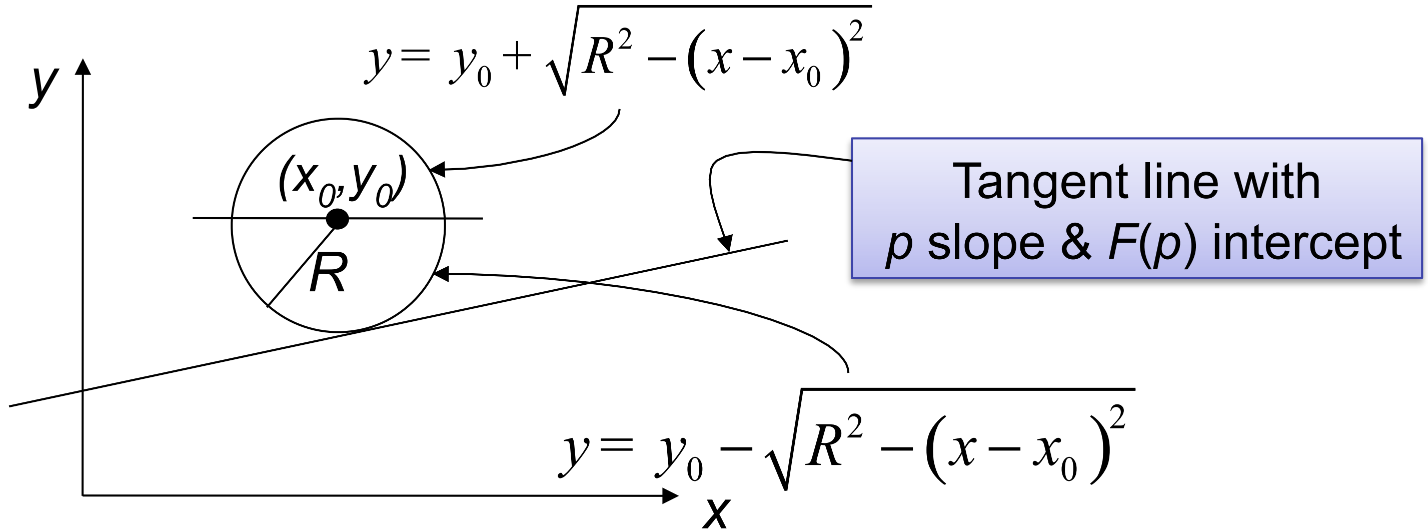
$$p = \frac{df}{dx} = x \Rightarrow x = p$$

$$F(p) = px - f(x) = \frac{p^2}{2}$$

$$\Rightarrow \frac{x^2}{2} \xleftrightarrow{LT} \frac{p^2}{2}$$

A parabola becomes a parabola!

Legendre Transform (LT) of a Circle (1)



$$f(x) = \begin{cases} y_0 + \sqrt{R^2 - (x - x_0)^2} & \text{concave part} \\ y_0 - \sqrt{R^2 - (x - x_0)^2} & \text{convex part} \end{cases}$$

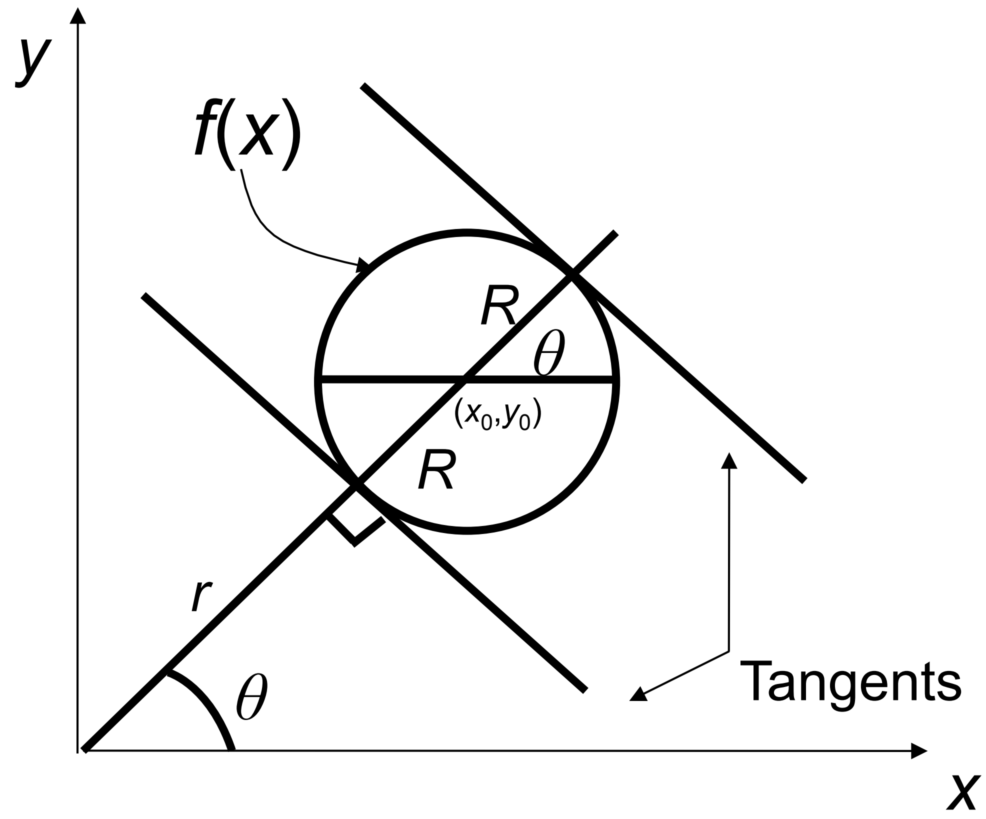
intercept

slope

$$\underset{\text{intercept}}{F}(p) = \begin{cases} +y_0 - x_0 \underset{\text{slope}}{p} + R\sqrt{p^2 + 1} & \text{concave part} \\ -y_0 + x_0 p + R\sqrt{p^2 + 1} & \text{convex part} \end{cases}$$

a circle becomes a hyperbola!

Legendre Transform (LT) of a Circle (2)



Canonical form of a line:

$$r = x\cos\theta + y\sin\theta$$

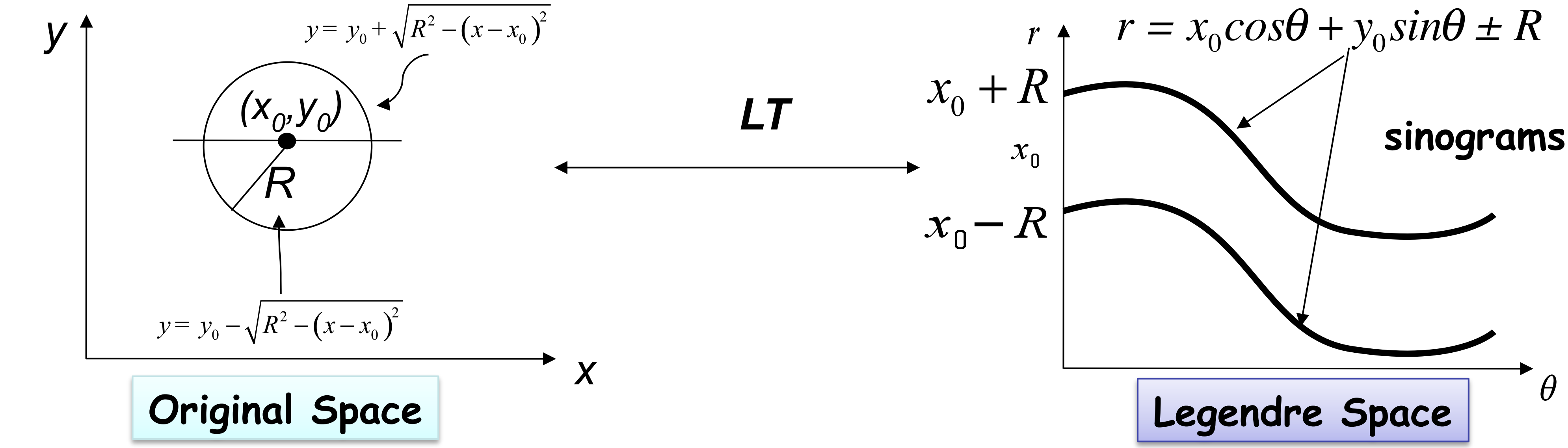
$$\Rightarrow \frac{r}{\sin\theta} + x(-\cot\theta) = y$$

$F(p)$ intercept

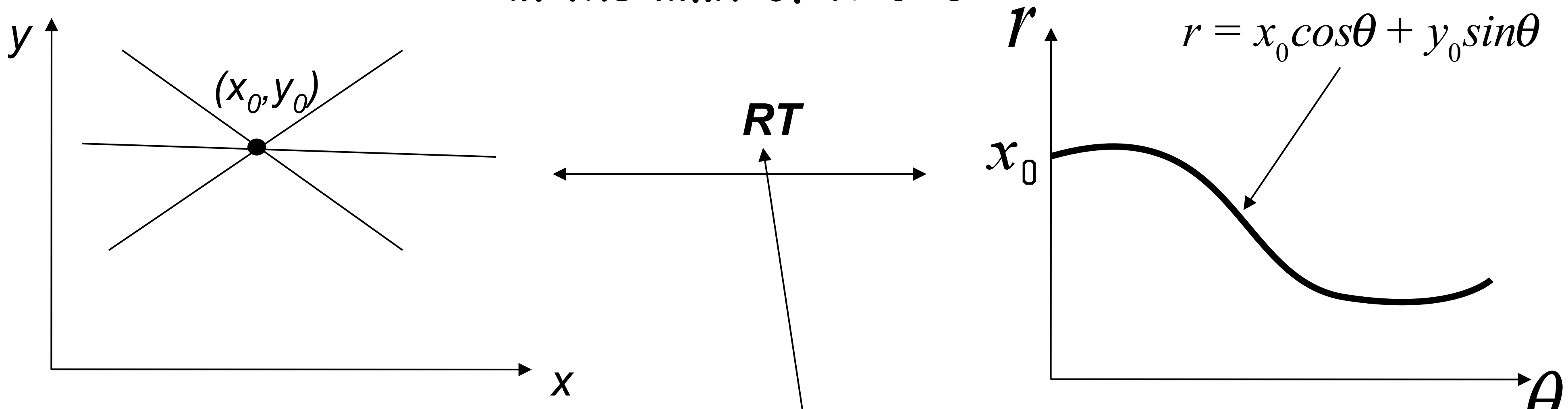
p slope

$$f(x) \xleftrightarrow{LT} F(p) = \begin{cases} r = x_0\cos\theta + y_0\sin\theta + R \\ r = x_0\cos\theta + y_0\sin\theta - R \end{cases}$$

Legendre Transform (LT) of a Circle (3)

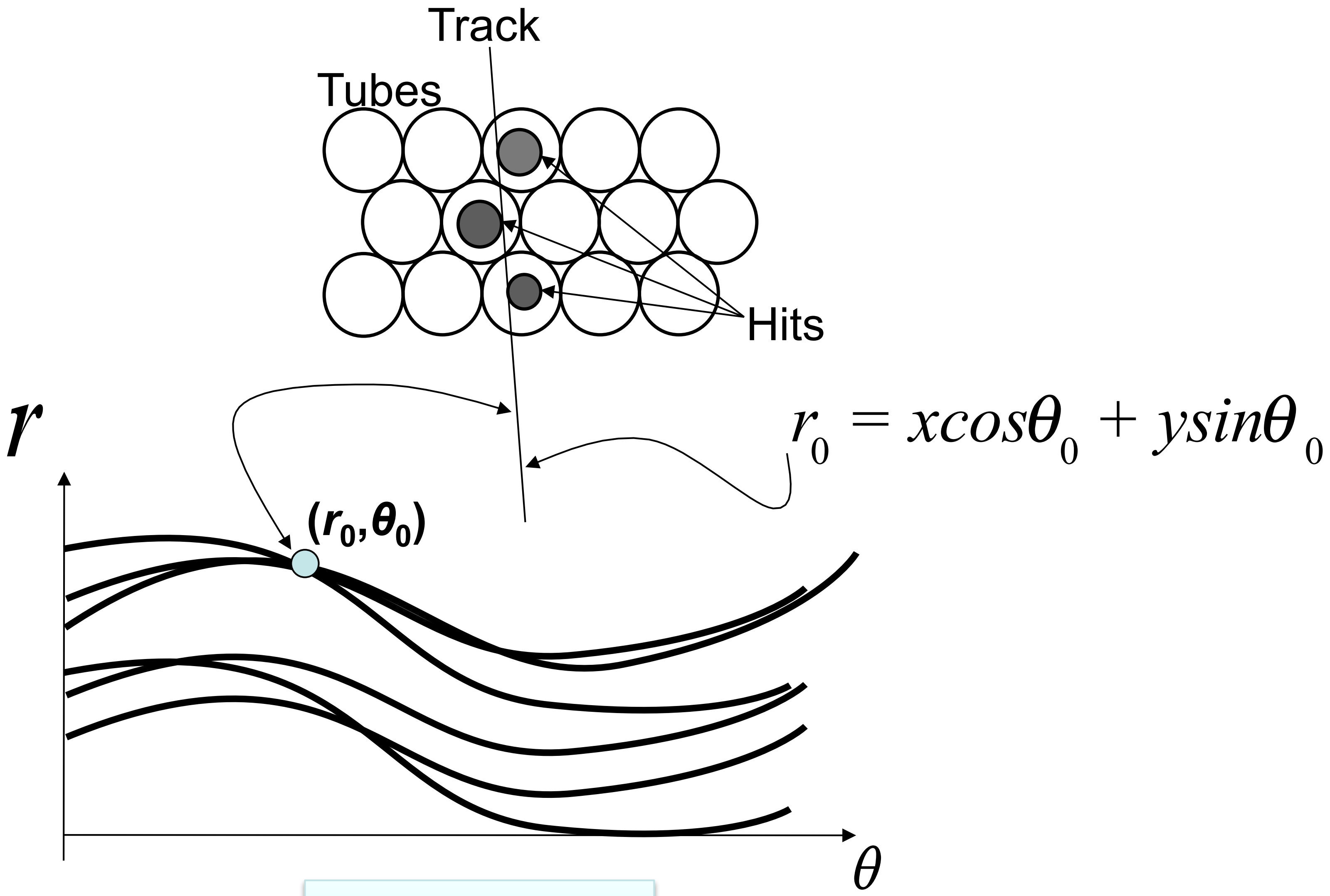


in the limit of $R \rightarrow 0$



we get the Radon/Hough Transform!
extensively used in PET & pattern recognition

Legendre Transform for Tracking (1)



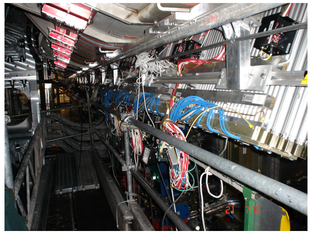
Legendre Space

- To each circle corresponds a couple of sinograms in the Legendre Space.
- The point with the maximum intensity defines the common tangent of all circles.

Performance of Legendre Algorithm - Single Track Events

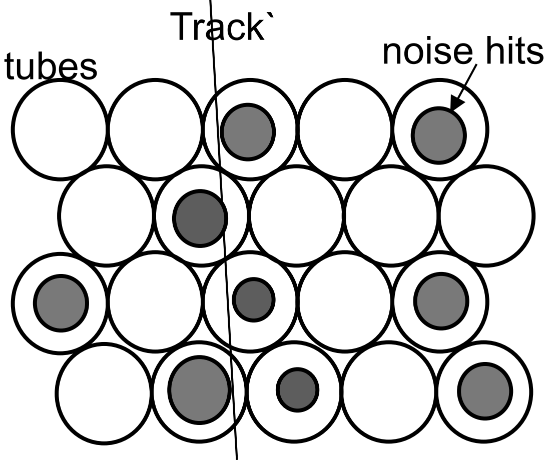
Monte Carlo Events

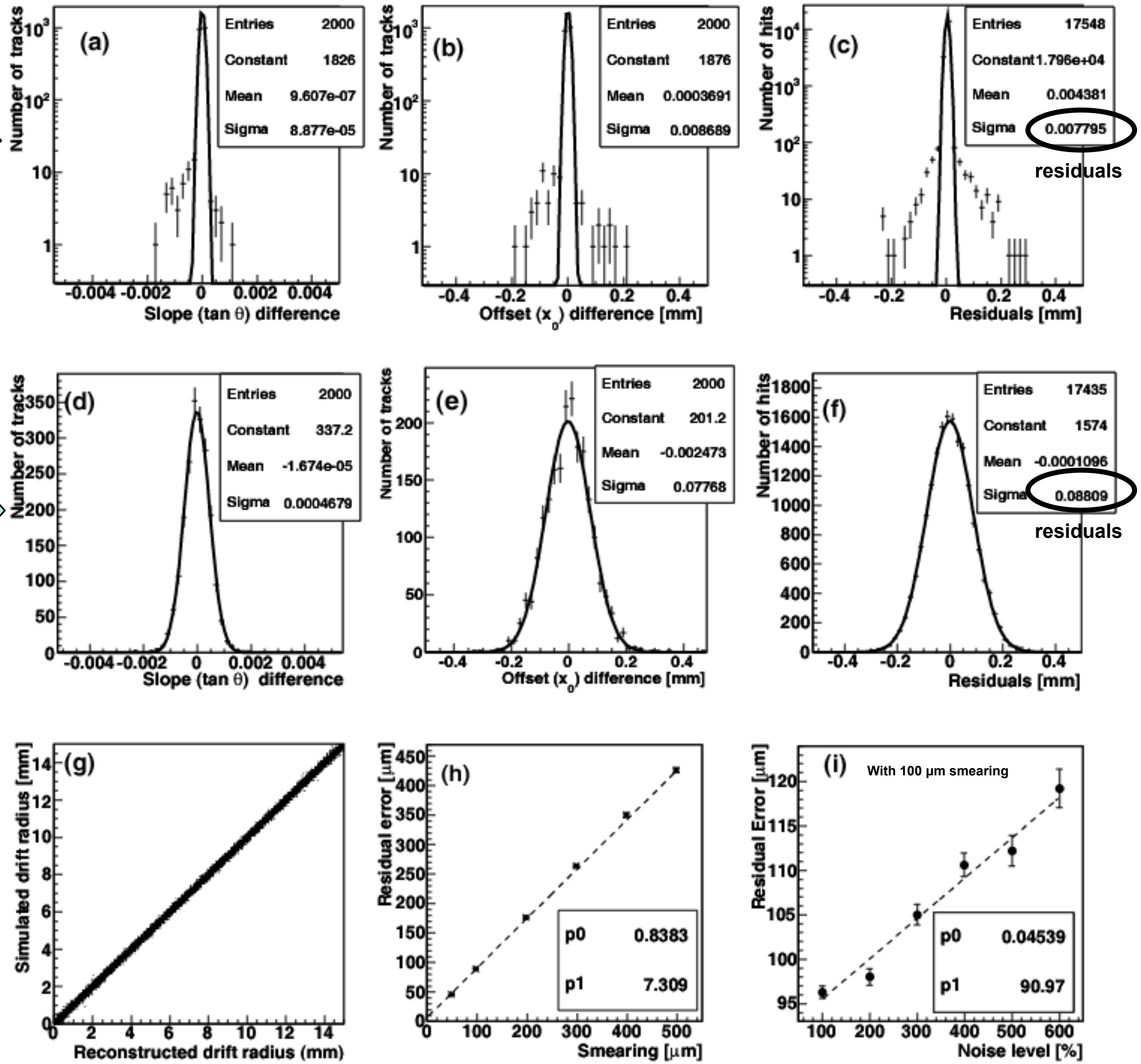
single track events randomly generated in *θ* & *r*.



single track events randomly generated in *θ* & *r* with 100 μm smearing factor.

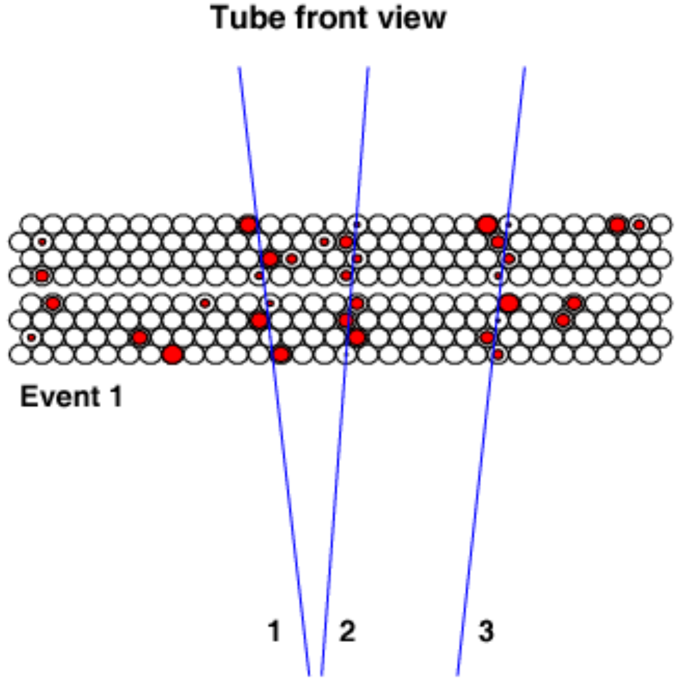
$$y = (x - x_0)\tan\theta$$



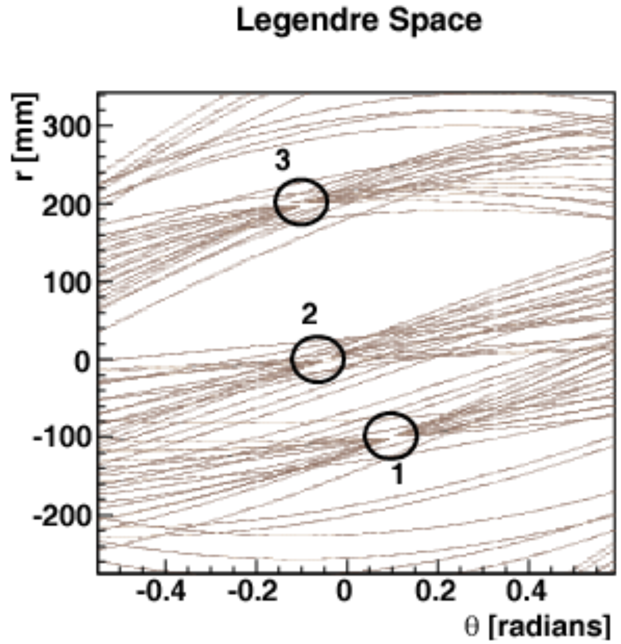


Performance of Legendre Algorithm - Multi Track Events

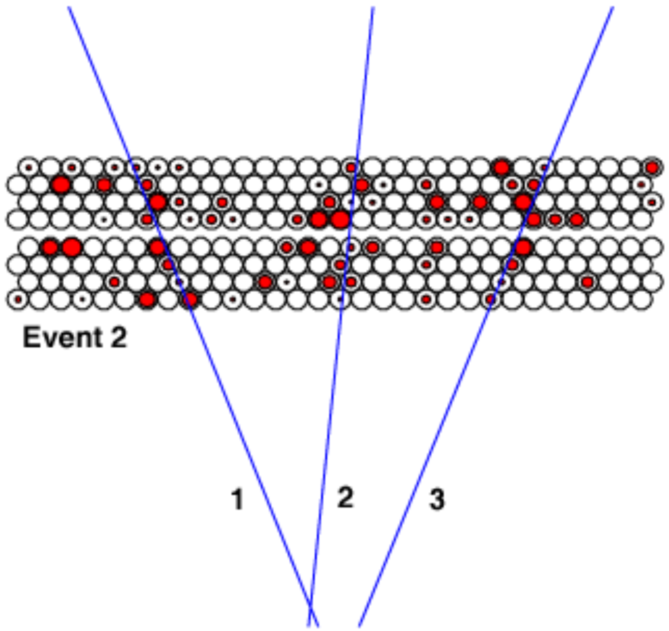
with 50% noise level



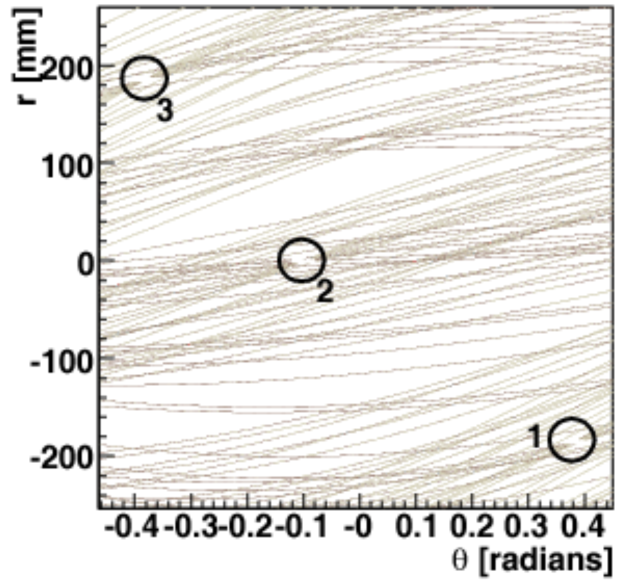
LT



with 200% noise level



LT



Conclusions/Summary

- A new efficient fast tracking method using the Legendre transformation of circles in conjunction with a χ^2 test is developed.

- This method is successfully applied to a set of hits (circles) on a drift chamber using Monte Carlo simulated data.

- In the limit of zero circle radii, the Legendre transform is reduced to the Radon/Hough transform of a set of points providing us with an efficient method of tracking as well.