Bell inequalities and quantum entanglement in weak gauge boson production at the LHC and future colliders

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based on: **arXiv:** 2302.00683 [hep-ph]

Introduction

- "Quantum Entanglement" between two systems is a pure quantum phenomena
- it violates Bell's inequalities (set of correlation measurements)

  J.S. Bell, "On the EPR Paradox" , Phys 1 (1964) 195
- incompatible with any prediction based on classical physics or local realism (EPR, hidden variables theories)
- to test these inequalities, pairs of two outcome measurements is required
- Experimental tests of Bell's inequalities violation

  R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki Rev. Mod. Phys 81 (2009) quant-ph/0702225

  - **pair of photons** Freedman-Clauser, PRL 28 (1972); Aspect-Dalibard-Roger, PRL 49 (1982)
  - **ions** M.A. Rowe *et. Al* , Nature 409 (2001)
  - **superconductive systems** M. Ansmann *et al.* , Nature 461 (2009)
  - **nitrogen vacancy centers** W. Pfaff et al. , Nature Physics 9 (2013)
  - **pairs of three-outcome measurements with photons** A.Vaziri *et al* , PRL 89 (2002)
- High energy collisions can give rise to quantum entanglement !

  (not yet tested)

bipartite systems → two entangled particles

discreet degrees of freedoms in each system

fundamental fermions: spin ½
massless spin 1 (photon): — 2 → **qubit**

massive spin 1 (W, Z) — 3 → **qutrit**

Probing entanglement at colliders

Polarization mainly studied for heavy fermions, the decays of which act as their own polarimeters

(previous works on Bell's inequalities at high energy)

$e^+e^- \to \Lambda\bar{\Lambda} \to \pi^- p \pi^+ \bar{p}$ — Tornqvist, Found Phys. 11 (1981); Nabel, Dittmar, Dreiner, PLB 280 (1992)

- **Neutral K meson systems** — Benatti, Floreanini, EPJC 13 (2000); Bertlmann, Grimus, Hiesmayr, PLA 289 (2001)
- **Positronium** — Acin, Latorre, Pascual, PRA 63 (2001); Li-Qiao, PLA 373 (2009)
- **Charmonium decays** $\to \Lambda\bar{\Lambda}$ — Baranov, J. Phys. G 35 (2008); Chen *et al* PTEP 2013, 1302.6438 [hep-ph]; Qian *et al.* PRD 101 (2020) 2002.04283 [quant-ph]
- **Neutrino oscillations** — Banerjee et al, EPJC 75 (2015) 1508.03480 [hep-ph]

Probing entanglement at LHC and future colliders

(recent activity → starting from 2021)

- **top-quark pair production**

SM →

- **Afik, de Nova,** Euro Phys. J Plus 136 (2021) 2003.02280 [quant-ph]
  Fabbrichesi, Floreanini, Panizzo, PRL 127 (2021), 2102.11883 [hep-ph]
  Severi, Boschi, Maltoni, Sioli, EPJC 82 (2022), 2110.10112 [hep-ph]
  Afik, de Nova, Quantum 6 (2022), 2203.05582 [quant-ph]]
  Aguilar-Saavedra, Casas, EPJC 82 (2022), 2205.00542 [hep-ph]

New Physics →

- **Fabbrichesi, EG, Floreanini**, EPJC 83 (2023), 2302.00683 [hep-ph]
  Aoude, Madge, Maltoni, Mantani, PRD 106 (2022), 2203.05619 [hep-ph]
  Severi, Vryonidou, JHEP 01 (2023), 2210.09339 [hep-ph]

- **tau-pair production (Drell-Yan)**
  - **Fabbrichesi, EG, Floreanini**, EPJC 83 (2023), 2302.00683 [hep-ph]

- **Λ-hyperons**
  - **Gong, Parida, Tu, Venugopalan**, 2107.13007 [hep-ph]

- **Higgs boson** → tau pair, two photons
  - **Fabbrichesi, Gabrielli, Floreanini**, EPJC 83 (2023), 2208.11723 [hep-ph]
    Altakach, Lambda, Maltoni, Mawatari, Sakurai, 2211.10513 [hep-ph]

→ weak gauge-boson pairs

- **Alan Barr**, PLB 285 (2022), 2106.01377 [hep-ph]
  Barr, Caban, Rembielinski, 2204.11063 [hep-ph]
  Aguilar-Saavedra, Bernal, Casas, Moreno, 2209.13441 [hep-ph]
  Aguilar-Saavedra, 2209.14033 [hep-ph]
  Fabbrichesi, Floreanini. EG, Marzola, 2302.00683 [hep-ph]

- **WW, ZZ, WZ** (Drell-Yan)
  - **Ashby-Pickering, Barr, Wierzchucka**, 2209.13990 [quant-ph]
    Fabbrichesi, Floreanini. EG, Marzola, 2302.00683 [hep-ph]

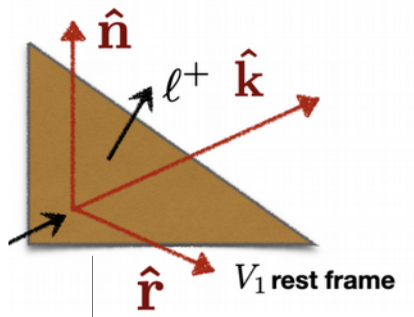
Quantum tomography of two Vector Boson production

- Requires the knowledge of the **polarization density matrix** for two vector bosons (WW, ZZ, WZ)
- it can be fully reconstructed from the angular distributions of the **VB decay products**
- so far experimental analysis have been focused on the density matrix of two spin ½ particles
- for instance for top-quark pairs (not exactly the same as analyzing spin-correlations)
- no experimental studies so far at LHC for the density matrix of two Vector Boson production
- knowledge of the full polarization density matrix allows to study many interesting phenomena

> ▶ Quantum Entanglement
>
> ▶ Violation of Bell's inequalities
>
> ▶ Sensitivity to New Physics

Density matrix of one spin-1 particle V_1

(covariant formalism)



decay plane of V_1 at rest

right-handed basis → $\left\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\right\} \qquad \hat{\mathbf{n}} = \hat{\mathbf{r}} \times \hat{\mathbf{k}}$

Spin-1 eigenstates

- **on rest frame**

$$\psi_\pm = -\frac{1}{\sqrt{2}}(\pm\hat{\mathbf{n}} + i\hat{\mathbf{r}}) \qquad \psi_0 = \hat{\mathbf{k}}$$

corresponding to eigenvalues $\lambda = \pm 1, 0$

direction of spin-axis quantization

- In a **more general frame** (performing a Lorentz boost along $\hat{\mathbf{k}}$)

$$p^\mu = E(1, \hat{\mathbf{k}}\beta)$$

velocity

particle energy

boosted (n,r,k) basis → $(n_1^\mu, n_2^\mu, n_3^\mu)$

$$\left\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\right\} \Rightarrow n_1^\mu = (0,\, \hat{\mathbf{n}})\,,\; n_2^\mu = (0,\, \hat{\mathbf{r}})\,,\; n_3^\mu = \frac{E}{M}(\beta,\, \hat{\mathbf{k}})$$

$$n_0^\mu = E/M(1,\, \hat{\mathbf{k}}\beta) \Rightarrow g_{\mu\nu}\, n_m^\mu n_n^\nu = -\delta_{mn}$$

{*n*,*m*}=0,1,2,3

orthogonal to the particle 4-momentum

$$n_m^\mu p_\mu = 0$$

rest frame limit

$$\varepsilon^{\mu}(p,\lambda)\xrightarrow[(\beta\to 0)]{}\psi_{\pm}\,,\;\psi_0$$

covariant polarization vector of spin-1

$$\varepsilon^{\mu}(p,\lambda)=-\frac{1}{\sqrt{2}}|\lambda|\,(\lambda\,n_1^{\mu}+i\,n_2^{\mu})+\Big(1-|\lambda|\Big)n_3^{\mu}$$

helicity

$\lambda=\pm 1,0$

Covariant Projector

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p)\;=\;\varepsilon^{\mu}(p,\lambda)^{\star}\varepsilon^{\nu}(p,\lambda')$$

master formula

$$=\frac{1}{3}\left(-g^{\mu\nu}+\frac{p^{\mu}p^{\nu}}{M^2}\right)\delta_{\lambda\lambda'}-\frac{i}{2M}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}n^{i}_{\beta}\,(S_i)_{\lambda\lambda'}-\frac{1}{2}n_i^{\mu}n_j^{\nu}\,(S_{ij})_{\lambda\lambda'}$$

H.S. Song, Lett. Nuovo Cim. 25 (1979)
S.Y. Choi, T. Lee, H.S. Song, PRD 40 (1989)
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

$$S_{ij}=S_iS_j+S_jS_i-\frac{4}{3}\mathbb{1}\,\delta_{ij}\qquad\qquad \varepsilon^{0123}=1$$

$S_i,\; i\in\{1,2,3\}$ rotation matrices for spin-1 particle (see backup slides)

basis correspondence for $(S_i)_{\lambda\lambda'}$

$$|+\rangle=\begin{pmatrix}1\\0\\0\end{pmatrix},\quad |0\rangle=\begin{pmatrix}0\\1\\0\end{pmatrix},\quad |-\rangle=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

corresponding to eigenevalues $\lambda=\pm 1,0$

Matrix element for a spin-1 emission

$$\mathcal{M}(\lambda) \quad = \quad \mathcal{M}_\mu \varepsilon^{\mu\star}(p,\lambda)$$

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \quad = \quad \varepsilon^\mu(p,\lambda)^\star \varepsilon^\nu(p,\lambda')$$

Density matrix

covariant expression

$$\rho(\lambda,\lambda') \quad = \quad \frac{\mathcal{M}(\lambda)\mathcal{M}^\dagger(\lambda')}{|\overline{\mathcal{M}}|^2} \quad = \quad \frac{\mathcal{M}_\mu \mathcal{M}^\dagger_\nu \mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p)}{|\overline{\mathcal{M}}|^2}$$

unpolarized square amplitude

Useful to project density matrix
on the Gell-Mann basis

$$\rho(\lambda,\lambda') = \left(\frac{1}{3}\mathbb{1} + \sum_{a=1}^{8} v^a T^a\right)_{\lambda\lambda'}$$

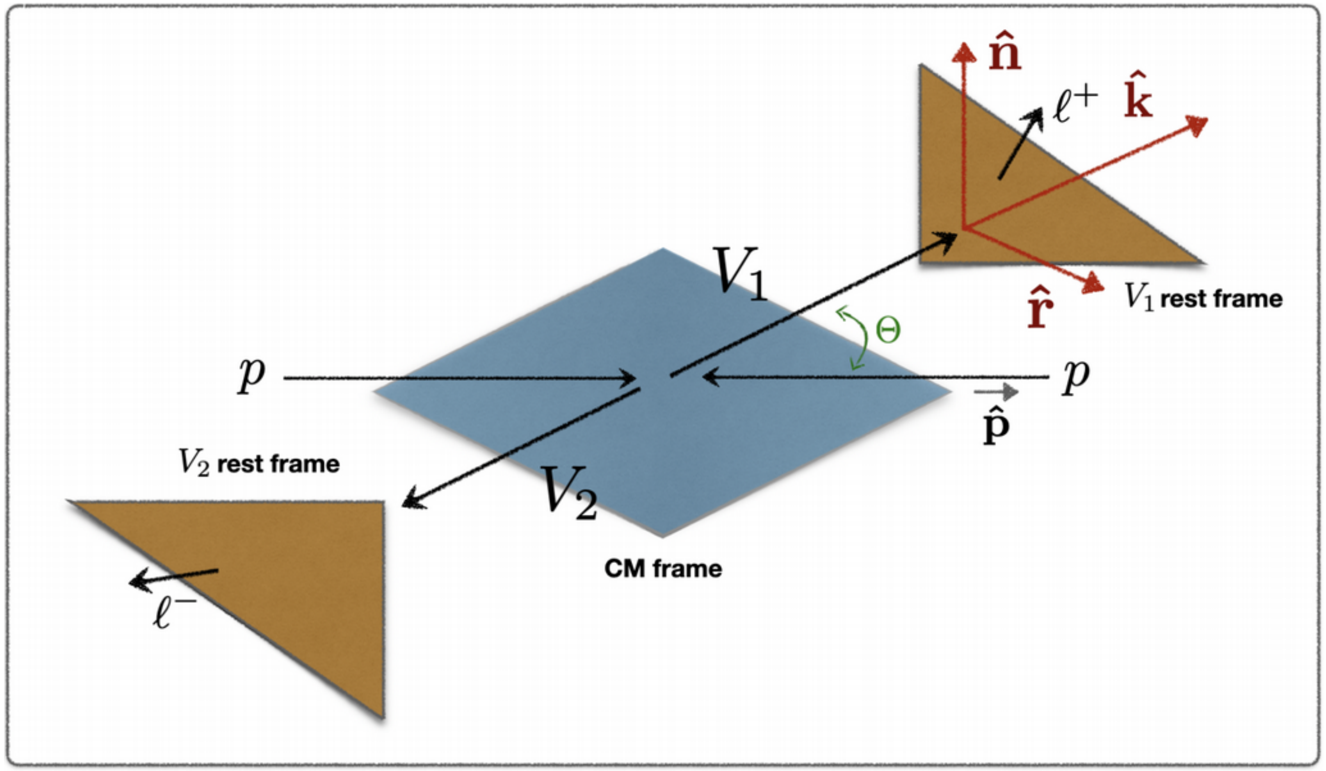
T^a 3x3 Gell-Mann matrices

$$v^a \quad = \quad \frac{1}{2}\mathrm{Tr}\left[\rho T^a\right]$$

Density matrix of two spin-1 particles $V_1 V_2$

$$\bar{q}(p_1)\, q(p_2) \to V_1(k_1, \lambda_1)\, V_2(k_2, \lambda_2)$$

in the center of mass frame



$$\hat{\mathbf{r}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} - \cos\Theta\hat{\mathbf{k}}\right), \qquad \hat{\mathbf{n}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} \times \hat{\mathbf{k}}\right)$$

scattering angle

Kinematic relations (same mass)

c.m. energy

$$p_1^\mu = E\,(1,\, \hat{\mathbf{p}}), \quad p_2^\mu = E\,(1,\, -\hat{\mathbf{p}}), \quad k_1^\mu = E\,(1,\, \beta\hat{\mathbf{k}}), \quad k_2^\mu = E\,(1,\, -\beta\hat{\mathbf{k}})$$

velocity in c.m. frame

$$n_1^\mu(1) = n_1^\mu(2) = (0,\, \hat{\mathbf{n}}), \quad n_2^\mu(1) = n_2^\mu(2) = (0,\, \hat{\mathbf{r}})$$
$$n_3^\mu(1) = \gamma\,(\beta,\, \hat{\mathbf{k}}), \quad n_3^\mu(2) = \gamma\,(-\beta,\, \hat{\mathbf{k}}),$$

Matrix element of **two-Vector Boson** production

$$\bar{q}(p_1)\, q(p_2) \to V_1(k_1, \lambda_1)\, V_2(k_2, \lambda_2)$$

$$\mathcal{M}(\lambda_1, \lambda_2) \quad = \quad \mathcal{M}_{\mu\nu} \varepsilon^{\mu\star}(k_1, \lambda_1) \varepsilon^{\nu\star}(k_2, \lambda_2)$$

Density matrix

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \frac{\mathcal{M}_{\mu\nu} \mathcal{M}^{\dagger}_{\mu'\nu'} \mathscr{P}^{\mu\nu}_{\lambda_1 \lambda_1'}(k_1) \mathscr{P}^{\mu\nu}_{\lambda_2 \lambda_2'}(k_2)}{|\overline{\mathcal{M}}|^2}$$

9x9 matrix

unpolarized matrix element square

ρ depends on scalar products of $n^{\mu}_{m}(1)$, $n^{\mu}_{m}(2)$

with the momenta of the reaction with $m = 1, 2, 3$

$$\mathscr{P}^{\mu\nu}_{\lambda\lambda'}(p) \quad = \quad \varepsilon^{\mu}(p, \lambda)^{\star} \varepsilon^{\nu}(p, \lambda')$$

Density matrix for two-QUTRITS

(WW , ZZ , WZ)

useful to decompose the **density matrix** on the basis of tensor products of Gell-Mann matrices

$$\{\mathbb{1}\otimes\mathbb{1},\, \mathbb{1}\otimes T^a,\, T^a\otimes\mathbb{1},\, T^a\otimes T^b\}$$

T^a 3x3 Gell-Mann matrices

given by the Kronecker product of the matrix representations $[A\otimes B]_{ii'jj'} = A_{ii'}B_{jj'}$

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}[\mathbb{1}\otimes\mathbb{1}] + \sum_a f_a[\mathbb{1}\otimes T^a] + \sum_a g_a[T^a\otimes\mathbb{1}] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

9x9 matrix

$$f_a = \frac{1}{6}\,\mathrm{Tr}\,[\rho\,(\mathbb{1}\otimes T^a)]\,,\quad f_b = \frac{1}{6}\,\mathrm{Tr}\,[\rho\,(T^a\otimes\mathbb{1})]\,,\quad h_{ab} = \frac{1}{4}\,\mathrm{Tr}\left[\rho\left(T^a\otimes T^b\right)\right]$$

- these are scalar quantities that depend on VV' invariant mass and scattering angle Θ in c.m. frame
- we can also extract them from data, using the decay products of final VB (see next slides)

Reconstructing the correlation coefficients from the data

Ashby-Pickering, Barr, Wierzchucka, 2209.13990 [quant-ph]

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$$

missing energy due to the presence of neutrinos

- These process include also the production of VB via the resonant Higgs boson channel as well as via quark-fusion (Drell-Yan)

- The momenta of the final leptons provide a measurement of the VB polarizations

- These momenta are the only information we need to extract from the numerical simulation or from the data to reconstruct the polarization density matrix

W W

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+\,d\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \text{Tr}\left[\rho_{V_1V_2}\left(\Gamma_+ \otimes \Gamma_-\right)\right]$$

depend on the invariant mass m_{VV} (or velocity β) and scattering angle Θ in the V_1V_2 cm frame

Rahaman, Singh, NPB 984 (2022), 2109.09345 [hep-ph]

$$d\Omega^\pm = \sin\theta^\pm d\theta^\pm\, d\phi^\pm$$

- $d\Omega^\pm$: solid angle of $\ell^\pm$
- $\theta^\pm$: polar angle
- $\phi^\pm$: azimuthal angle

→ phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1V_2}$ = density matrix of V_1V_2

$\Gamma_\pm$ → Density matrices that describe the polarization of the two decaying W into final leptons (the charged ones assumed to be massless)

these are projectors in the case of the W-bosons because of their chiral couplings to leptons can be computed by rotating to an arbitrary polar axis the spin ± 1 states of gauge bosons taken in the z-direction

$$\Gamma_\pm = \frac{1}{3}\mathbb{1} + \sum_{i=1}^{8} \mathfrak{q}_\pm^a\, T^a$$

→ Density matrices for W-bosons

the functions $\mathfrak{q}_\pm^a$ can be written in terms of the respective spherical coordinates

↓ some polynomial of spherical coordindates

$$\begin{aligned} h_{ab} &= \frac{1}{\sigma}\int\int \frac{d\sigma}{d\Omega^+\, d\Omega^-}\, \mathfrak{p}_+^a\, \mathfrak{p}_-^b\, d\Omega^+ d\Omega^- \\ f_a &= \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^+}\, \mathfrak{p}_+^a\, d\Omega^+ \\ g_a &= \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^-}\, \mathfrak{p}_-^a\, d\Omega^- \end{aligned}$$

$\mathfrak{p}_\pm^n$ a particular set of orthogonal functions ⇒

$$\left(\frac{3}{4\pi}\right)\int \mathfrak{p}_\pm^n\, \mathfrak{q}_\pm^m\, d\Omega^\pm = \delta^{nm}$$

For the ZZ production the density matrices $\Gamma_{\pm}$ are not projector due to the Z boson coupling

$$\mathcal{L} \supset -i \frac{g}{\cos\theta_W} \left[g_L (1-\gamma^5)\gamma_\mu + g_R (1+\gamma^5)\gamma_\mu \right] Z^\mu$$

$$\tilde{\mathfrak{q}}^n = \frac{1}{g_R^2 + g_L^2} \left[g_R^2\, \mathfrak{q}_+^n + g_L^2\, \mathfrak{q}_-^n \right]$$

$$\tilde{\mathfrak{p}}^n = \sum_m \mathfrak{a}_m^n \mathfrak{p}_+^m$$

$$\mathfrak{a}_m^n = \frac{1}{g_L^2 - g_R^2} \begin{pmatrix} g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 & 0 \\ 0 & g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 \\ 0 & 0 & g_R^2 - \frac{1}{2}\, g_L^2 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2}\, g_L^2 \\ 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 \\ g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 & 0 \\ 0 & g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2}\, g_L^2 & 0 & 0 & 0 & 0 & \frac{1}{2}\, g_L^2 - g_R^2 \end{pmatrix} .$$

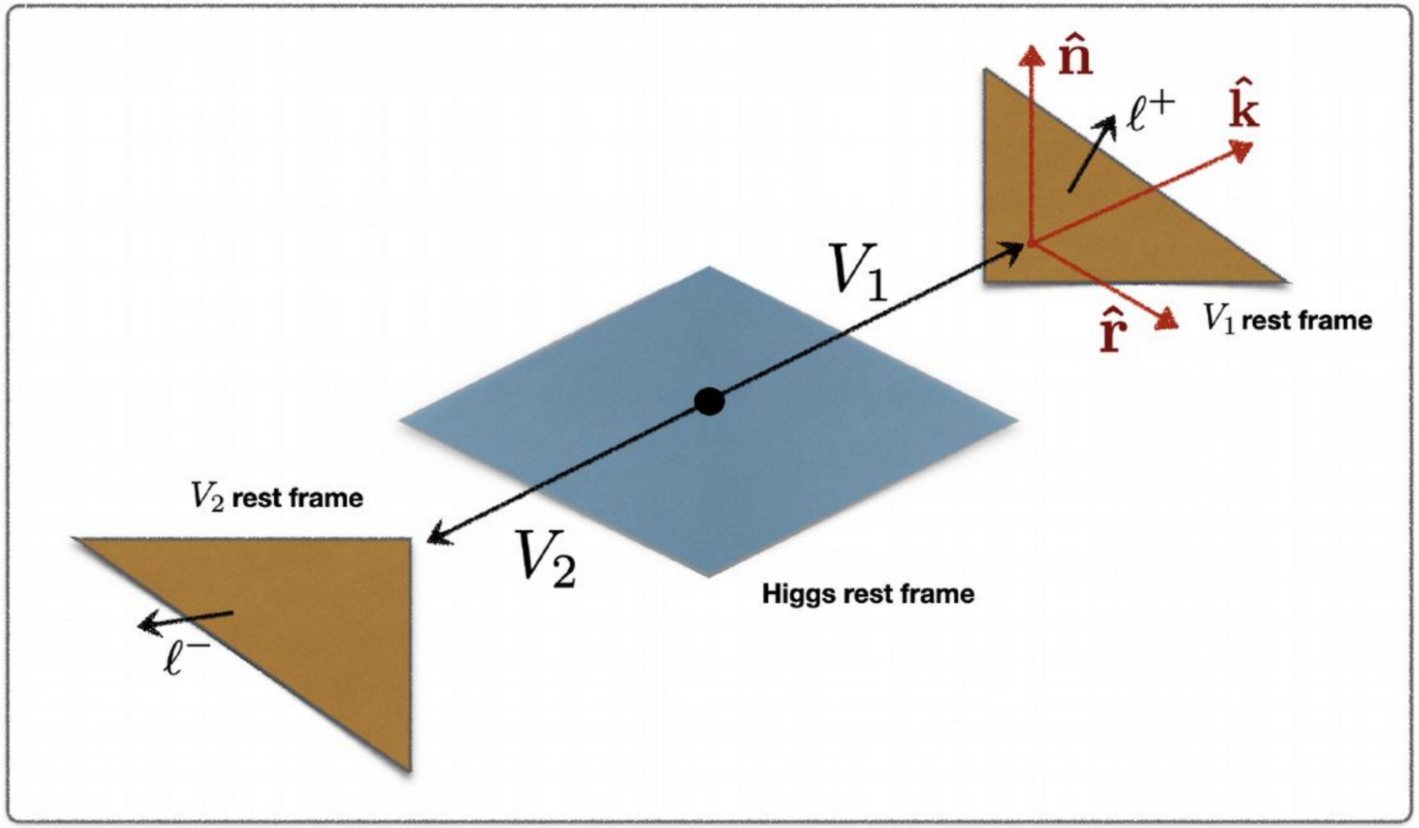
Wigner's *Q symbols*

$$
\begin{aligned}
\mathfrak{q}^1_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(\cos\theta^\pm \pm 1\Big) \cos\phi^\pm \,, \\
\mathfrak{q}^2_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(\cos\theta^\pm \pm 1\Big) \sin\phi^\pm \,, \\
\mathfrak{q}^3_\pm &= \frac{1}{8} \Big(1 \pm 4\,\cos\theta^\pm + 3\cos 2\theta^\pm\Big) \,, \\
\mathfrak{q}^4_\pm &= \frac{1}{2} \sin^2\theta^\pm \cos 2\,\phi^\pm \,, \\
\mathfrak{q}^5_\pm &= \frac{1}{2} \sin^2\theta^\pm \sin 2\,\phi^\pm \,, \\
\mathfrak{q}^6_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(-\cos\theta^\pm \pm 1\Big) \cos\phi^\pm \,, \\
\mathfrak{q}^7_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(-\cos\theta^\pm \pm 1\Big) \sin\phi^\pm \,, \\
\mathfrak{q}^8_\pm &= \frac{1}{8\sqrt{3}} \Big(-1 \pm 12\cos\theta^\pm - 3\cos 2\theta^\pm\Big) \,,
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{p}^1_\pm &= \sqrt{2}\,\sin\theta^\pm \Big(5\,\cos\theta^\pm \pm 1\Big) \cos\phi^\pm \,, \\
\mathfrak{p}^2_\pm &= \sqrt{2}\,\sin\theta^\pm \Big(5\,\cos\theta^\pm \pm 1\Big) \sin\phi^\pm \,, \\
\mathfrak{p}^3_\pm &= \frac{1}{4} \Big(5 \pm 4\,\cos\theta^\pm + 15\,\cos 2\theta^\pm\Big) \,, \\
\mathfrak{p}^4_\pm &= 5\,\sin^2\theta^\pm \cos 2\,\phi^\pm \,, \\
\mathfrak{p}^5_\pm &= 5\,\sin^2\theta^\pm \sin 2\,\phi^\pm \,, \\
\mathfrak{p}^6_\pm &= \sqrt{2}\,\sin\theta^\pm \Big(-5\,\cos\theta^\pm \pm 1\Big) \cos\phi^\pm \,, \\
\mathfrak{p}^7_\pm &= \sqrt{2}\,\sin\theta^\pm \Big(-5\cos\theta^\pm \pm 1\Big) \sin\phi^\pm \,, \\
\mathfrak{p}^8_\pm &= \frac{1}{4\sqrt{3}} \Big(-5 \pm 12\cos\theta^\pm - 15\cos 2\theta^\pm\Big) \,.
\end{aligned}
$$

Di-boson production in Higgs boson decays

$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$$



In the Higgs boson rest frame the density matrix of the bipartite VV* system does not depend on the scattering angle, but only by the Higgs mass, the V mass and the off-shell V* mass

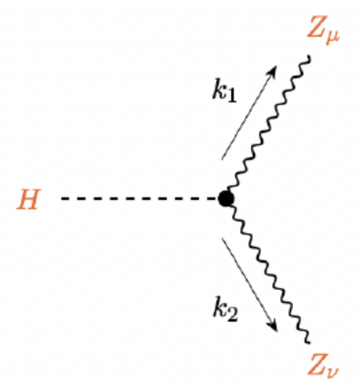
Di-boson production in Higgs boson decays

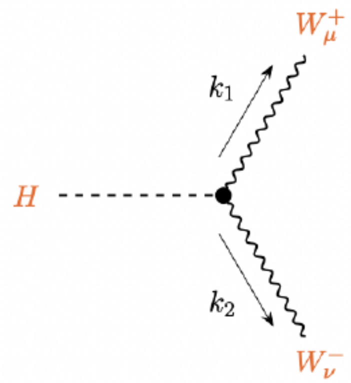
A. Barr, PLB 825 (2022) , 2106.01377 [hep-ph]

Aguilar-Saavedra *et al*, 2209.1344` [hep-ph]

$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$$

Results based on
arXiv: 2302.00683 [hep-ph]





$\xi_W = 1$, and $\xi_Z = 1/(2c_W)$, with $c_W = \cos\theta_W$ → Weinberg angle

g is the weak coupling

V^* regarded as an off-shell vector boson with mass $M_V^* = f M_V$ $\quad 0 < f < 1$

mass of V boson M_V

Quantum Amplitude

$$\mathcal{M}_H(\lambda_1, \lambda_2) = g\, M_V\, \xi_V\, g_{\mu\nu} \varepsilon^{\mu\star}(k_1, \lambda_1) \varepsilon^{\nu\star}(k_2, \lambda_2)$$

$$\mathcal{M}_H(\lambda_1, \lambda_2)\mathcal{M}_H(\lambda_1', \lambda_2')^\dagger = g^2\, M_V^2\, \xi_V^2\, g_{\mu\nu} g_{\mu'\nu'} \mathscr{P}^{\mu\mu'}_{\lambda_1\lambda_1'}(k_1)\, \mathscr{P}^{\nu\nu'}_{\lambda_2\lambda_2'}(k_2)$$

By projecting ↓ into the Gell-Mann matrix basis one can easily get the **f,g** and **h** correlation coefficients

Unpolarized square amplitude

$$|\overline{\mathcal{M}}_H|^2 = \frac{g^2\xi_V^2}{4f^2M_V^2}\left[m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4\right]$$

The **non-vanishing** **f,g** and **h** elements

$$g_a = f_a \text{ for } a \in \{1,\dots,8\}$$

$$f_3 = \frac{1}{6}\,\frac{-m_H^4 + 2(1+f^2)m_H^2M_V^2 + (1-f^2)^2M_V^4}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4}\,,$$

$$f_8 = -\frac{1}{\sqrt{3}}f_3\,,$$

$$h_{16} = h_{61} = h_{27} = h_{72} = \frac{fM_V^2(-m_H^2 + (1+f^2)M_V^2)}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4}\,,$$

$$h_{33} = \frac{1}{4}\,\frac{(m_H^2 - (1+f^2)M_V^2)^2}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4}\,,$$

$$h_{38} = h_{83} = -\frac{1}{4\sqrt{3}}$$

$$h_{44} = h_{55} = \frac{2f^2M_V^4}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4}\,,$$

$$h_{88} = \frac{1}{12}\,\frac{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1-14f^2+f^4)M_V^4}{m_H^4 - 2(1+f^2)m_H^2M_V^2 + (1+10f^2+f^4)M_V^4}\,,$$

Inserting the f,g and h into the Gell-Mann basis decomposition

$$\rho_H = 2\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathrm{Tr}\left[\rho_H\right] = 1$$

density matrix is idempotent

$$\rho_H^2 = \rho_H$$

Signaling that H → VV* is a **pure state**

$$\rho_H = |\Psi_H\rangle\langle\Psi_H|$$

Aguilar-Saavedra *et al*,
2209.1344` [hep-ph]

using the basis

$$|\lambda\,\lambda'\rangle = |\lambda\rangle \otimes |\lambda'\rangle \text{ with } \lambda, \lambda' \in \{+, 0, -\}$$

where the pure state is

arXiv: 2302.00683 [hep-ph]

$$|\Psi_H\rangle = \frac{1}{\sqrt{2+\varkappa^2}}\left[|+-\rangle - \varkappa\,|0\,0\rangle + |-+\rangle\right]$$

$$\varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

Bell's inequalities (for qutrits)

- consider the following correlator $\mathcal{I}_3$ for probability measurements

Collins, Gisin, Linden, Massar, Popescu, PRL 88 (2002)

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right]$$

with $\mathcal{B}$ a suitable Bell operator
Depending on the measurements

Generalized Bell's inequalities for two-qutrits

For deterministic local models

CGLMP
$$\mathcal{I}_3 \leq 2$$

→ **QM qutrits can violate this inequality** with upper bound = 4

- For the case of maximally entangled state $\rho = |\Psi_+\rangle\langle\Psi_+|$ optimal choice of measurements has been found $\rightarrow$ giving a specific form of $\mathcal{B}$

Acin, Durt, Gisin, Latorre, PRA 65 (2002), quant-ph/0111143

- still freedom to modify measured observables through **unitary transformations U,V** on $\mathcal{B}$

$$\mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

→ U,V unitary 3x3 matrices depending on the kinematic of the process

in order to **maximize** the **violation** of CGLMP **Bell's inequality** for two-qutrits

- Notice that, maximal violation of Bell's inequality obtained with $\mathcal{B}$ is for a density matrix which is NOT maximally entangled

Quantifying entanglement

- Concurrence $\mathcal{C}[\rho]$ vanishes for separable states (see backup slides for definition)

Rungta, Buzek, Caves, Hillery, Milburn, PRA 64 (2001)

For qutrits

- analytical solution exists only for the lower bound

$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho]$$

Mintert, Buchleitner, PRL 98 (2007)

$$\mathscr{C}_2[\rho] = 2\max\left(0,\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_A)^2\right],\ \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[(\rho_B)^2\right]\right)$$

- If non-vanishing unequivocally signal the presence of entanglement (witness of entanglement) → used in our analysis of entanglement for WW, ZZ and WZ productions

- Solution for upper bound

$$\left(\mathcal{C}[\rho]\right)^2 \leq 2\min\left(1 - \mathrm{Tr}\left[(\rho_A)^2\right],\ 1 - \mathrm{Tr}\left[(\rho_B)^2\right]\right)$$

- maximum value of Concurrence can be obtained for the maximum symmetric state

for two-qutrits → $\mathcal{C}[|\Psi_+\rangle] = 2/\sqrt{3}$ corresponding to the state $|\Psi_+\rangle = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}|i\rangle\otimes|i\rangle$

- On the Gell-Mann basis, lower bound of Concurrence can be easily computed

$$\mathscr{C}_2 = 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,, \\ -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]\,,$$

- If the bipartite (A,B) system is a pure state (as in the $H \to VV$ case)

  it is possible to quantify its entanglement by computing

Entropy of entanglement

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

- in terms of the von Neumann entropy of either the two component subsystems A and B with corresponding reduced polarization density submatrices ρ_A and ρ_B

(see also backup slides)

$$0 \leq \mathscr{E}[\rho] \leq \ln 3$$

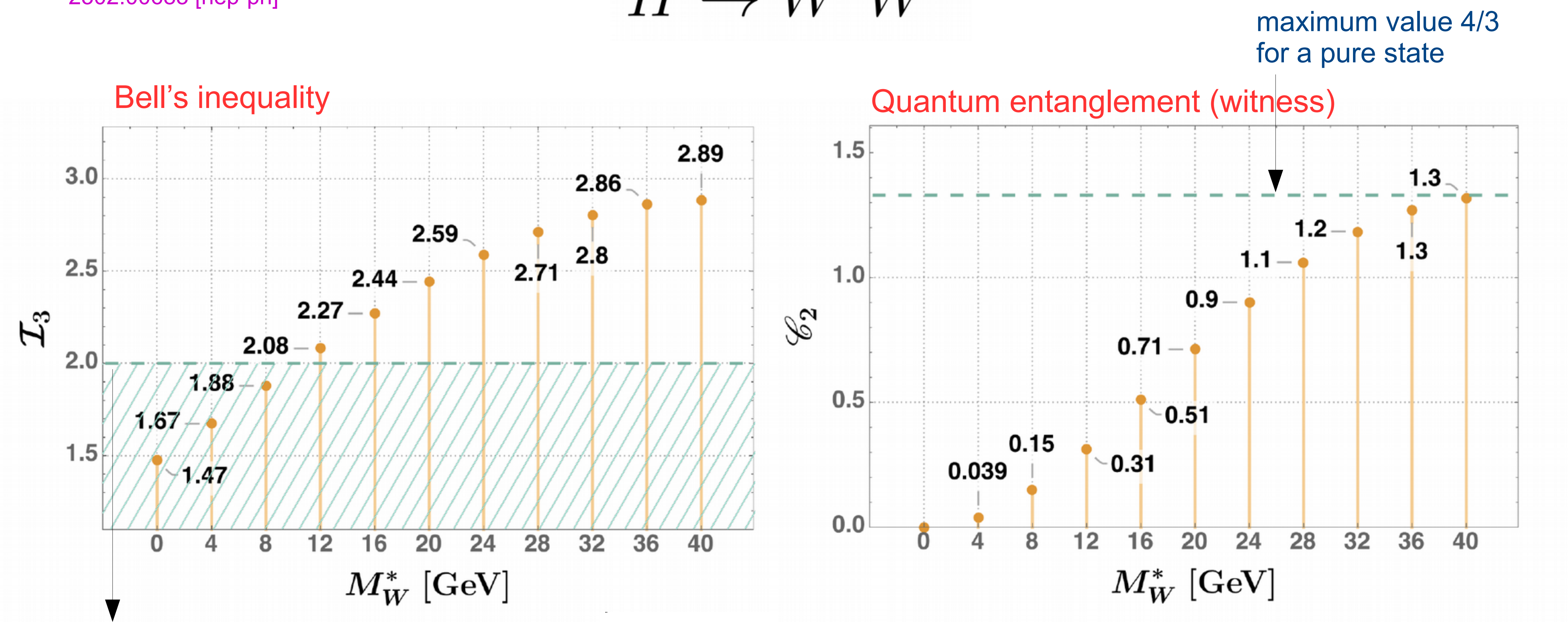
for a two-qutrit system

equality holds if and only if the bipartite is separable

corresponding to the maximally entangled state

2302.00683 [hep-ph]

$H \to W\ W^*$



Bell's inequality violation condition $\mathcal{I}_3 > 2$

Maximization of $\mathcal{I}_3$ performed point by point, since it depends on M_{W^*}

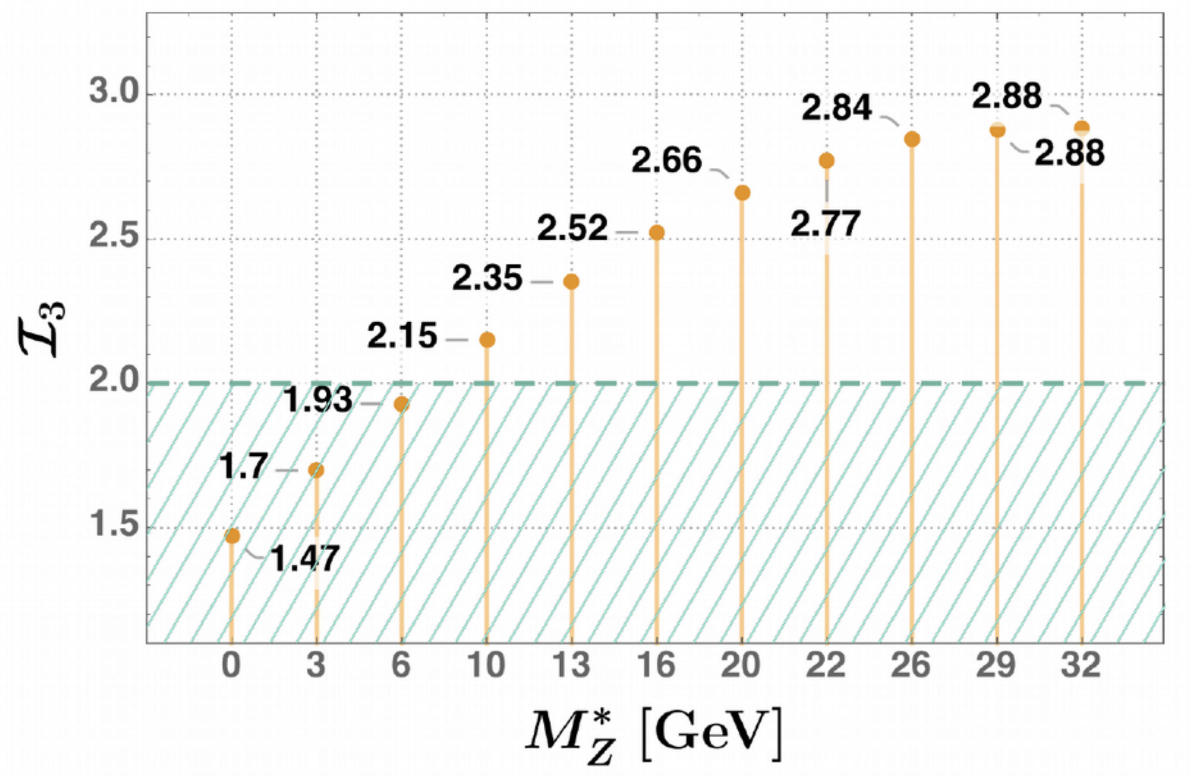
(see backup slides for optimized U,V matrices in the region of max entanglement)

$$\mathscr{C}_2 = \frac{32 f^2 M_V^4 \left[m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1+4f^2+f^4) M_V^4 \right]}{\left[m_H^4 - 2(1+f^2) m_H^2 M_V^2 + (1+10f^2+f^4) M_V^4 \right]^2}$$

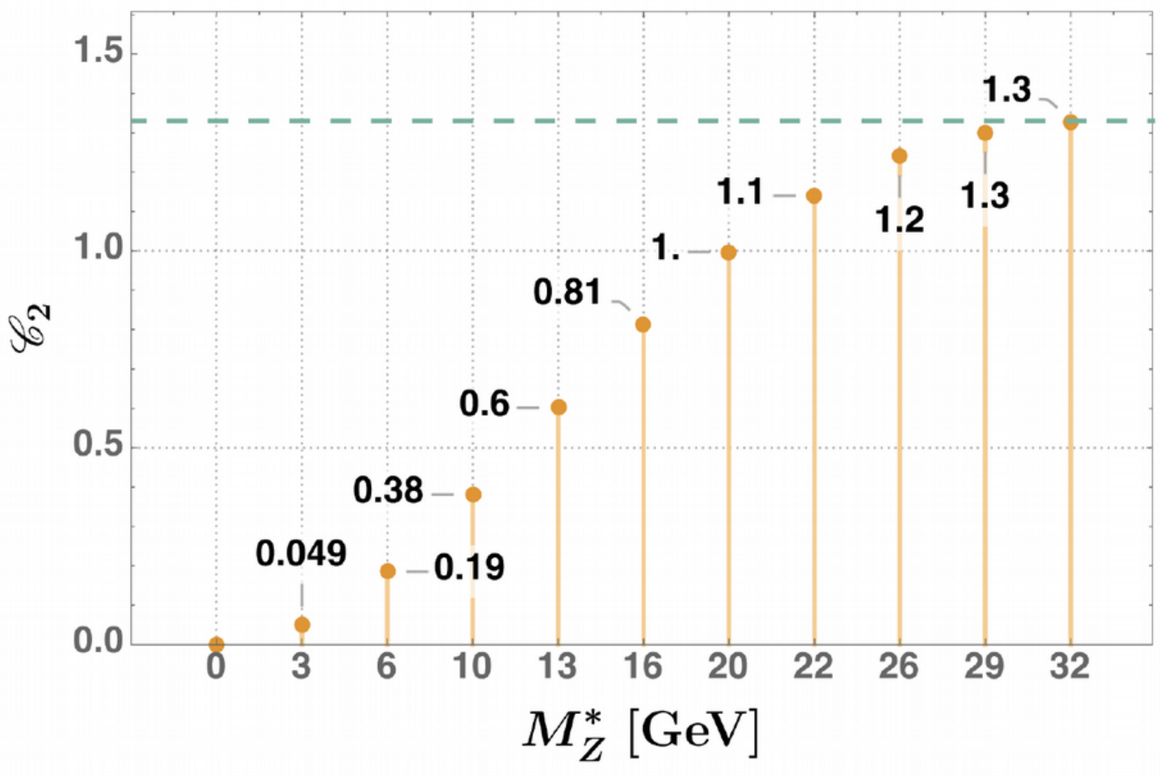
arXiv: 2302.00683 [hep-ph]

$H \to Z\ Z^*$

Bell's inequality



Quantum entanglement (witness)

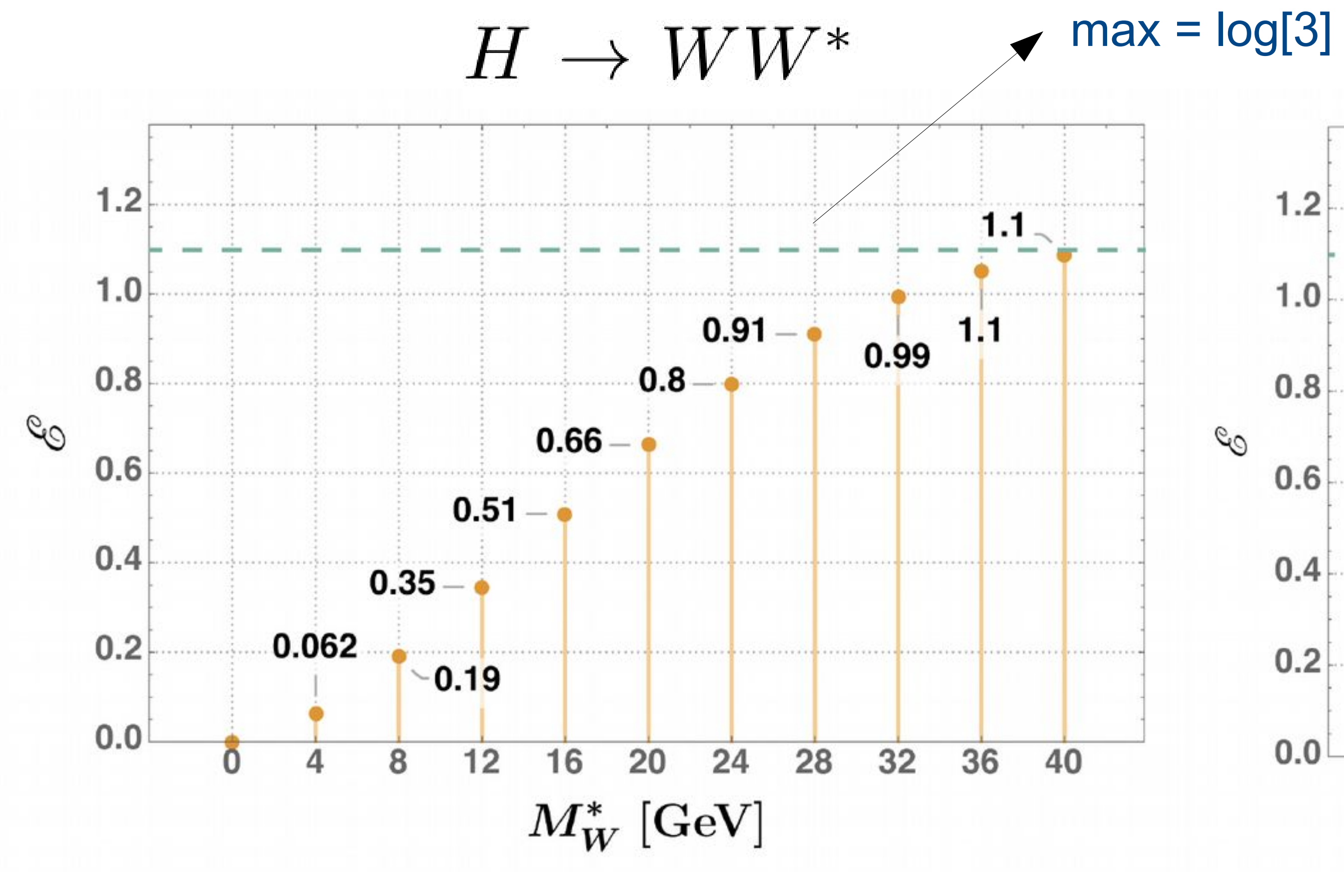


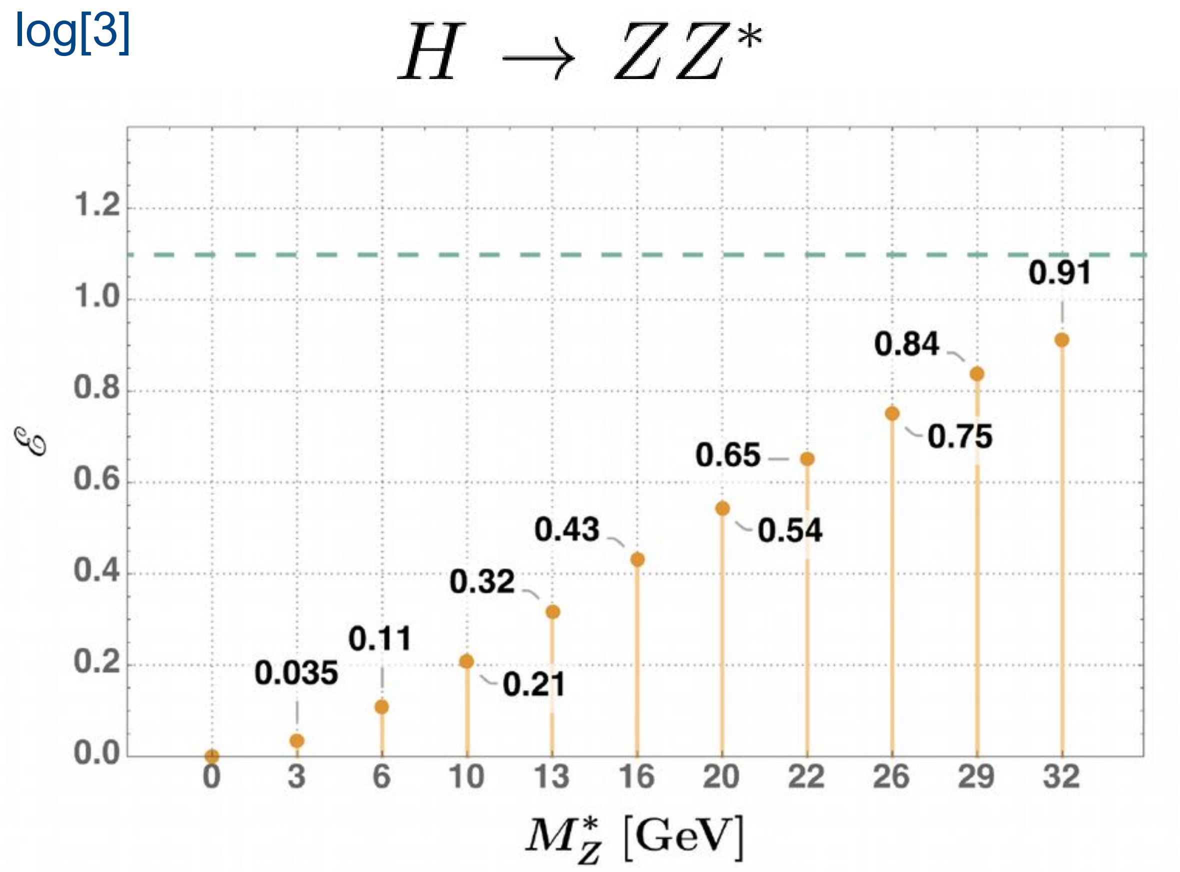
Maximization of $\mathcal{I}_3$ performed point by point, since it depends on M_{Z^*}

(see backup slides for optimized U,V matrices in the region of max entanglement)

Entropy of entanglement for $H \to VV^*$

arXiv: 2302.00683 [hep-ph]

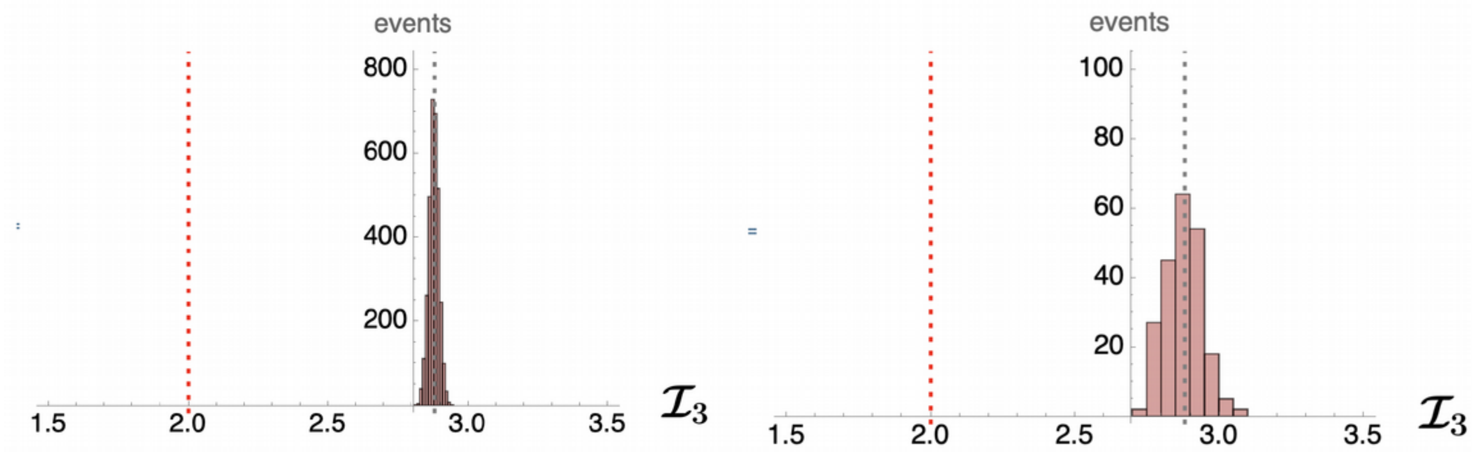




Events and sensitivity

Using MADGRAPH5 @ LO corrected by k factors at N3LO + N3LL

$H \to WW^*$ $\qquad$ $H \to ZZ^*$



	$W^+\ell^-\bar{\nu}_\ell$	$Z\ell^-\ell^-$
events	3237	217

Figure 5: Gaussian distribution of the 3237 events for the $H \to W^+\ell^-\bar{\nu}_\ell$ process and of the 217 of the $H \to Z\ell^+\ell^-$ process. Both sets of events have mean value $\mathcal{I}_3 = 2.88$. The threshold value of 2 for Bell inequality violation is shown as a dashed red line.

- Only fully leptonic decays used. Number of events reduced by 25% to account in efficiency of identification of final leptons
- **Significance** for rejecting the null hypothesis $\mathcal{I}_3 \leq 2$ is **50** for **WW*** and **13** for **ZZ***
- Results confirm numerical simulations for WW* and ZZ* of **A. Barr,** PLB 825 (2022) , 2106.01377 [hep-ph]
- Fully realistic estimate of the uncertainty is missing, as systematic uncertainties due to unfolding, background, and detector have been only modeled partially
- Results for ZZ are also consistent with corresponding ones in **Aguilar-Saavedra *et al***, 2209.1344` [hep-ph]

Di-boson production in pp collisions
Drell-Yan processes

For two VB produced in proton collisions, density matrix is given by the convex combination of the density matrices of the involved parton contributions $\rho^{q_1\bar{q}_2}$

$$\rho = \sum_{\{q_1\bar{q}_2\}} w^{q_1\bar{q}_2}\, \rho^{q_1\bar{q}_2}$$

with $\sum_{\{q_1,\bar{q}_2\}} w^{q_1\bar{q}_2} = 1$

Sum includes both configuration where the anti-quark originate from either protons

This relation holds $\rho^{\bar{q}_2 q_1}(\Theta) = \rho^{q_1\bar{q}_2}(\Theta+\pi)$

where

$$w^{q_1\bar{q}_2} = \frac{L^{q_1\bar{q}_1}\, |\overline{\mathcal{M}}^{q_1\bar{q}_2}_{V_1V_2}|^2}{\sum_{\{q_1\bar{q}_2\}} L^{q_1\bar{q}_1}\, |\overline{\mathcal{M}}^{q_1\bar{q}_2}_{V_1V_2}|^2}$$

parton luminosity of the initial $q_1\bar{q}_2$ state

$$L^{q_1\bar{q}_1}(\tau) = \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{\mathrm{d}z}{z}\, q_{q_1}(\tau\, z)\, q_{\bar{q}_2}\left(\frac{\tau}{z}\right)$$

$|\overline{\mathcal{M}}^{q_1\bar{q}_2}_{V_1V_2}|^2$ = unpolarized square amplitude of the partonic process $q_1\,\bar{q}_2 \to V_1V_2$

Decomposing the matrix density into the Gell-Mann matrix basis

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_a f_a\left[\mathbb{1}\otimes T^a\right] + \sum_a g_a\left[T^a\otimes\mathbb{1}\right] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

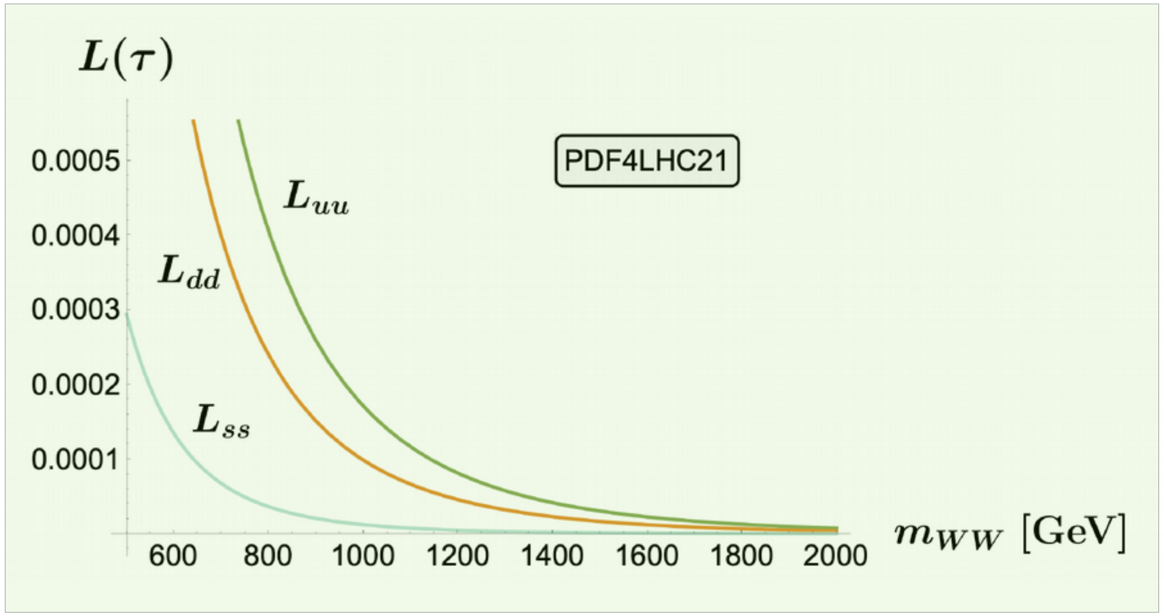
we obtain for the **h** correlations coefficients in VV production → depend on scattering angle

$$h_{ab}[m_{VV},\Theta] = \frac{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(\tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta] + \tilde{h}^{q\bar{q}}_{ab}[m_{VV},\Theta+\pi]\right)}{\sum_{q=u,d,s} L^{q\bar{q}}(\tau)\left(A^{q\bar{q}}[m_{VV},\Theta] + A^{q\bar{q}}[m_{VV},\Theta+\pi]\right)}$$

and analogously for the f_a and g_a correlation coefficients, where

$$A^{q\bar{q}} = |\overline{\mathcal{M}}^{q\bar{q}}_{WW}|^2$$ and $$\tilde{h}_{ab} = A^{q\bar{q}} h_{ab}$$

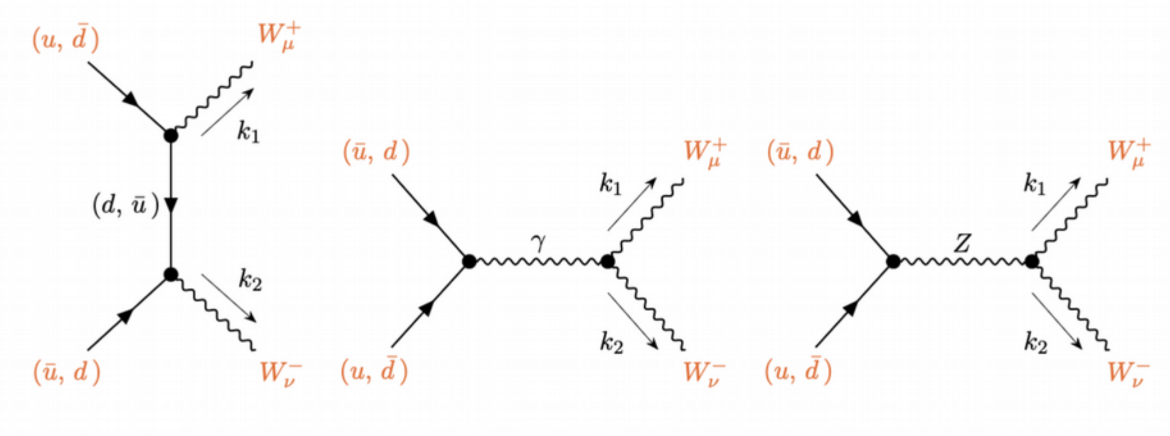
Parton luminosity



- main uncertainty on the correlation coefficients comes from the missing higher order QCD corrections
- giving approx a 10% uncertainty on the main entanglement observables
- other theoretical uncertainties, mainly from PDF and top-quark mass, is negligible → of the order of permille effect

arXiv: 2302.00683 [hep-ph]

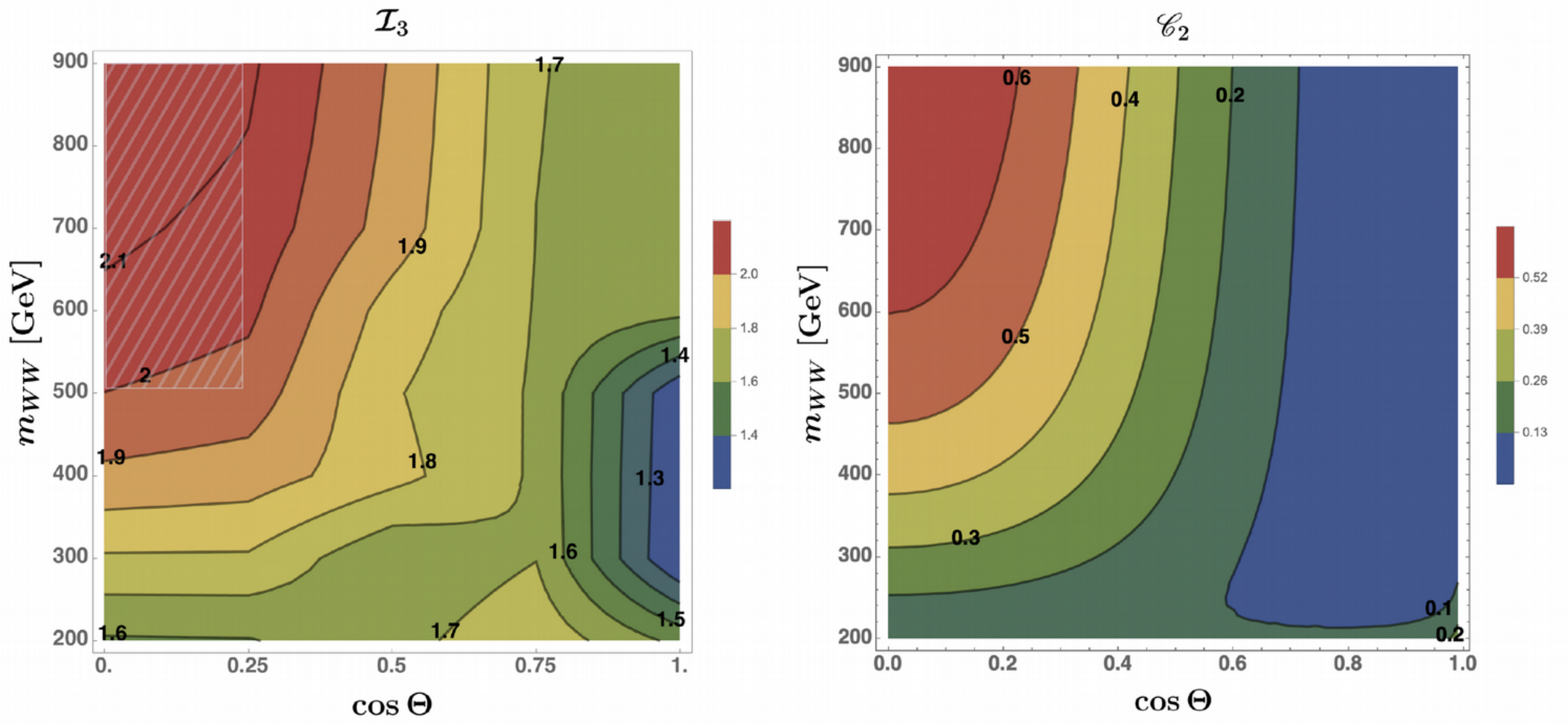
$pp \to W^+W^-$



optimization for maximum value of $\mathcal{I}_3$ of Bell's inequality violation is employed point by point in the Θ m_{WW} space

$$\mathcal{B} \to (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(see backup slides for their expressions in hatched area)



hatched area in the left-plot for $\mathcal{I}_3$ >2 indicates bin used as reference for our estimation of the significance (see next slides)

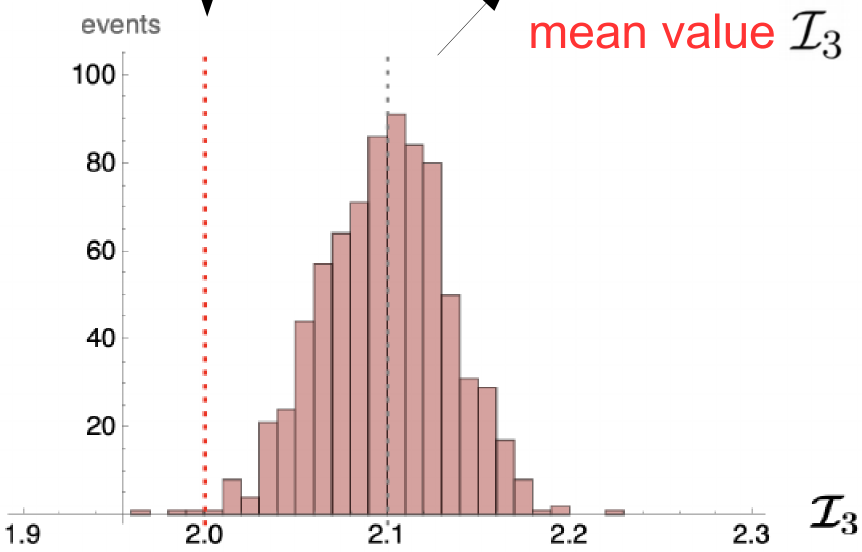
Events and sensitivity $pp \to W^+W^-$

	(run2) $\mathcal{L} = 140$ fb^{-1}	(Hi-Lumi) $\mathcal{L} = 3$ ab^{-1}
events	36	777

Number of expected events in the kinematic region $m_{WW} > 500$ GeV and $\cos\Theta < 0.25$ at the LHC with $\sqrt{s} = 13$ TeV and luminosity $\mathcal{L}$ =140 fb^{-1} (run2) and luminosity $\mathcal{L} = 3$ ab^{-1} (Hi-lumi). A benchmark efficiency of 0.25 is assumed.

Limit Bell's inequality

Gaussian distribution of the 777 events with mean value $\mathcal{I}_3$ =2.1



- estimated by using MADGRAPH5 @ LO for cross sections, corrected by the k-factors at the NNLO
- N. events reduced of 25% due to efficiency in identification of final leptons

Hi-Lumi runs $\to$ significance ~ 5 to reject the null hypothesis $\mathcal{I}_3 \leq 2$

arXiv: 2302.00683 [hep-ph]

$$pp \to W^+W^-$$

In the maximum entangled region $m_{WW} = 900 \text{ GeV and } \cos\Theta = 0,$

$$\rho = \alpha\, |\Psi_{+-}\rangle\langle\Psi_{+-}| + \beta\, |\Psi_{+-0}\rangle\langle\Psi_{+-0}| + \gamma\, |00\rangle\langle 00| + \delta\, |\Psi_{0-}\rangle\langle\Psi_{0-}|$$

$$\alpha \simeq 0.72, \quad \beta \simeq 0.18, \quad \gamma \simeq 0.07 \text{ and } \delta \simeq 0.02$$

$$\alpha + \beta + \gamma + \delta = 1$$

$$|\Psi_{+-}\rangle = \frac{1}{\sqrt{2}}\big(|++\rangle - |--\rangle\big)\,,$$
$$|\Psi_{0-}\rangle = \frac{1}{\sqrt{2}}\big(|0-\rangle + |-0\rangle\big)\,,$$
$$|\Psi_{+-0}\rangle = \frac{1}{\sqrt{3}}\big(|++\rangle - |--\rangle + |00\rangle\big)$$

$$|a\,b\rangle = |a\rangle \otimes |b\rangle \text{ with } a, b \in \{+,\, 0,\, -\}$$

matrix density is a mixture

dominant contribution comes from the state $|\Psi_{+-}\rangle$

explaining why $\mathscr{C}_2$ is large but far from maximum value $2/\sqrt{3} \simeq 1.15$

	(run2) $\mathcal{L} = 140\ \text{fb}^{-1}$	(Hi-Lumi) $\mathcal{L} = 3\ \text{ab}^{-1}$
events	1	20

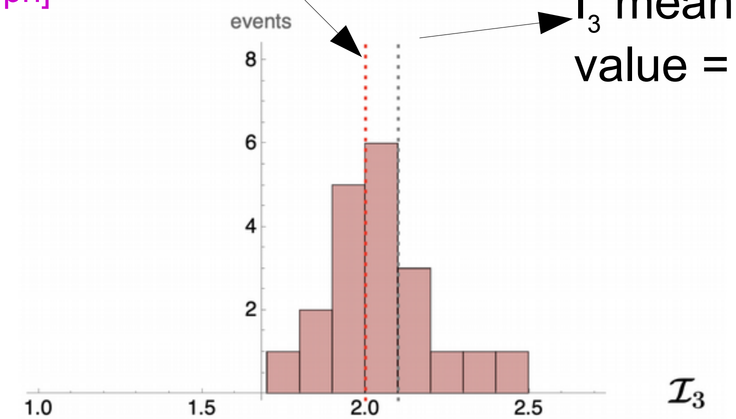
$p\,p \to Z Z$

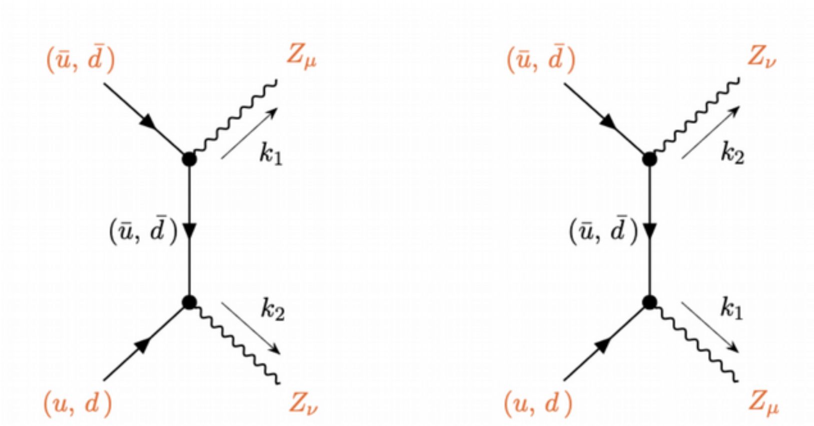
arXiv: 2302.00683 [hep-ph]

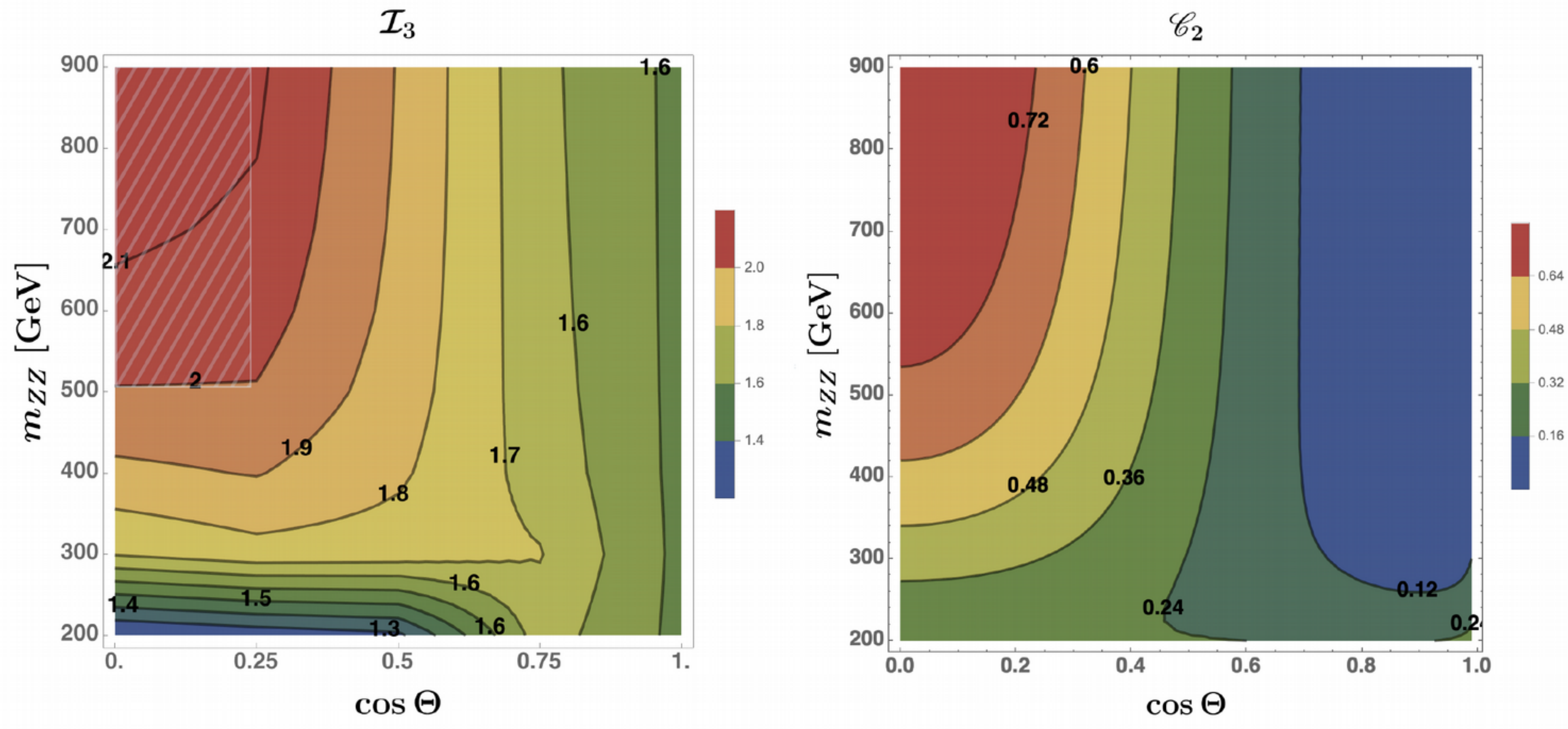
Gaussian distribution for 20 events

Bell's inequality

I_3 mean value = 2.1



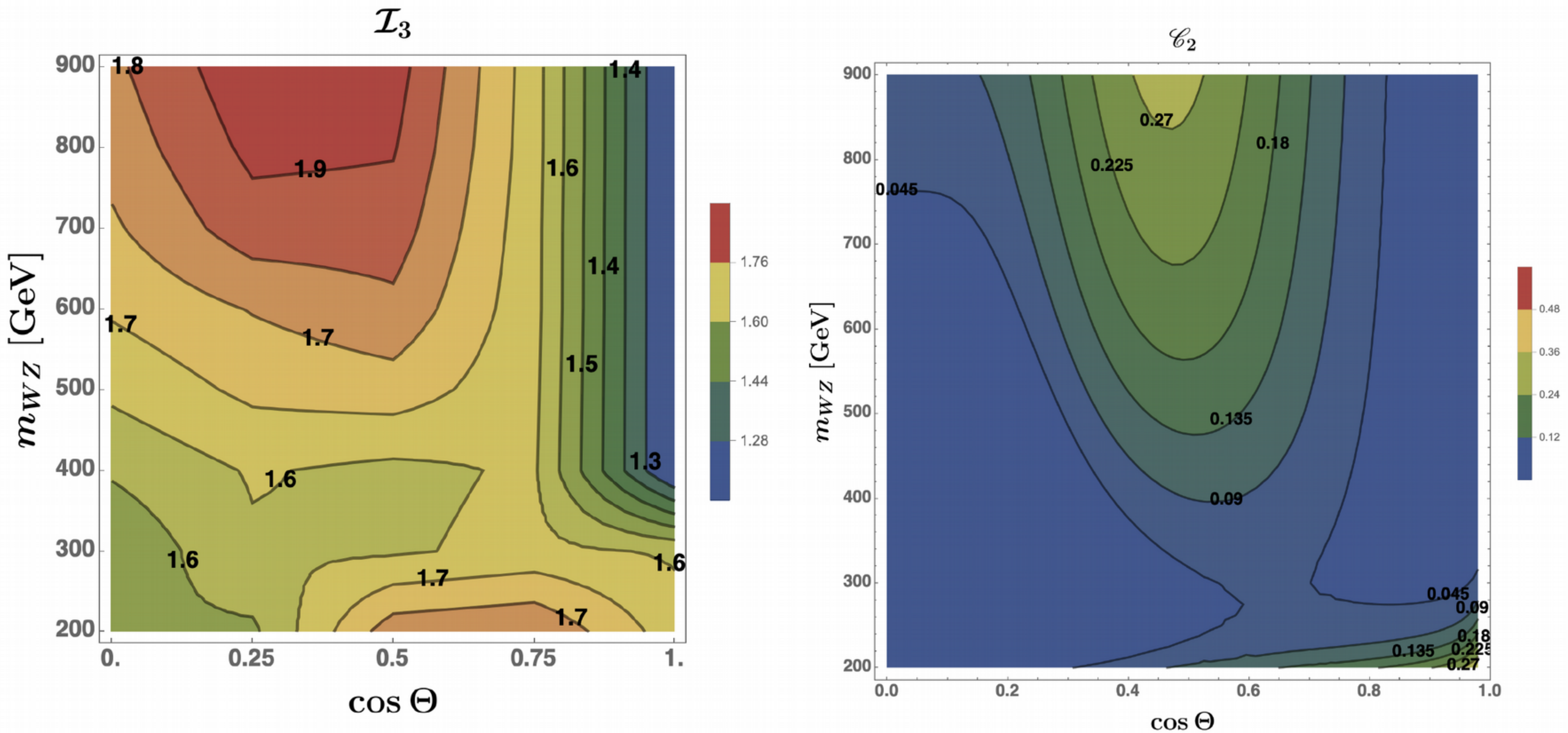




significance 0.6 for rejecting the null hypothesis

$$p\,p \to W^+ Z$$

arXiv: 2302.00683 [hep-ph]



No violation of Bell's inequalities in the relevant kinematic regions (m_{WZ} ~ 1 TeV) .

Same conclusions for entanglement.

WW, ZZ, WZ production analyzed also in 2209.13990 [quant-ph] using full simulation at partonic level.
conclusions differ from our results (possible underestimated errors..)

Constraining HWW and HZZ anomalous couplings with Quantum Tomography at the LHC

Fabbrichesi, Floreanini, EG, Marzola (preliminary)

We use polarization density matrix of the processes

$$H \to WW^*$$ $$H \to ZZ^*$$

CP-even

CP-odd

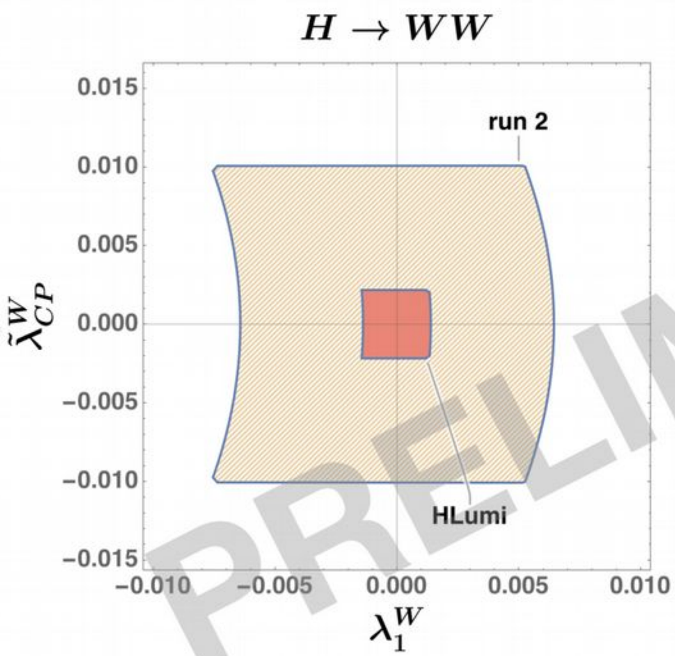
to constrain anomalous Higgs couplings to WW and ZZ

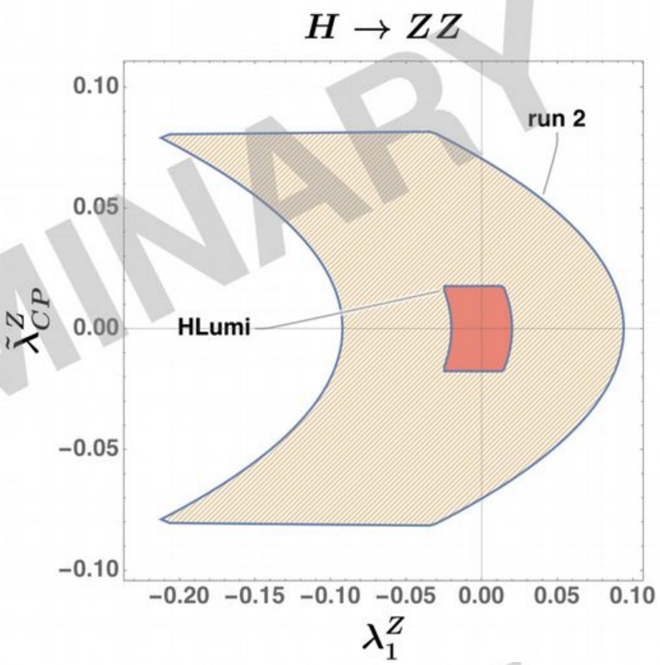
$V^{\mu\nu}$ → Field strength , V=W,Z

$\tilde{V}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}$ (dual)

Effective Higgs-VV Lagrangian (including SM)

$$\begin{aligned}\mathcal{L}_{HVV} \;=\; & g\, m_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} m_Z Z_\mu Z^\mu H \\ & - \frac{g}{m_W}\Bigg[\frac{\lambda_1^W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \lambda_2^W \Big(W^{+\nu}\partial^\mu W^-_{\mu\nu} + \text{H.c.}\Big) + \frac{\widetilde{\lambda}^W_{CP}}{4} W^+_{\mu\nu}\widetilde{W}^{-\mu\nu} \\ & + \frac{\lambda_1^Z}{2} Z_{\mu\nu} Z^{\mu\nu} + \lambda_2^Z Z^\nu \partial^\mu Z_{\mu\nu} + \frac{\widetilde{\lambda}^Z_{CP}}{4} Z_{\mu\nu}\widetilde{Z}^{\mu\nu}\Bigg] H\,,\end{aligned}$$

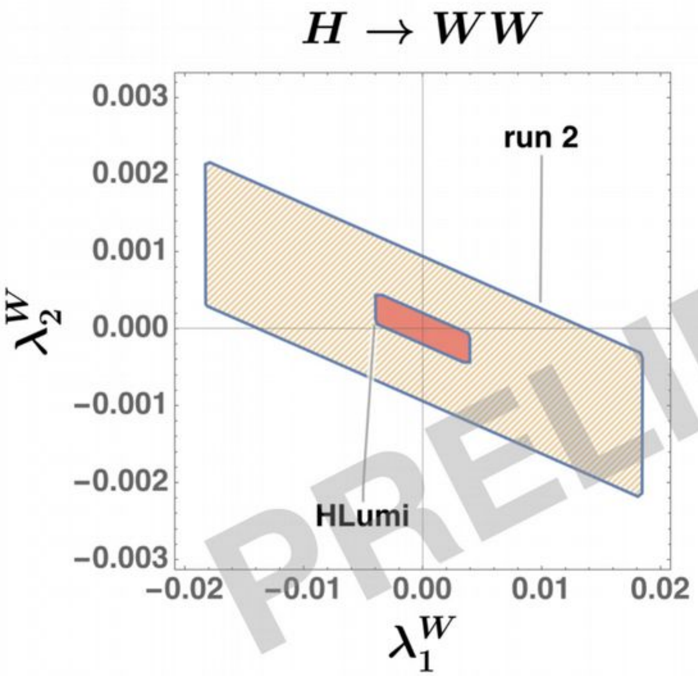


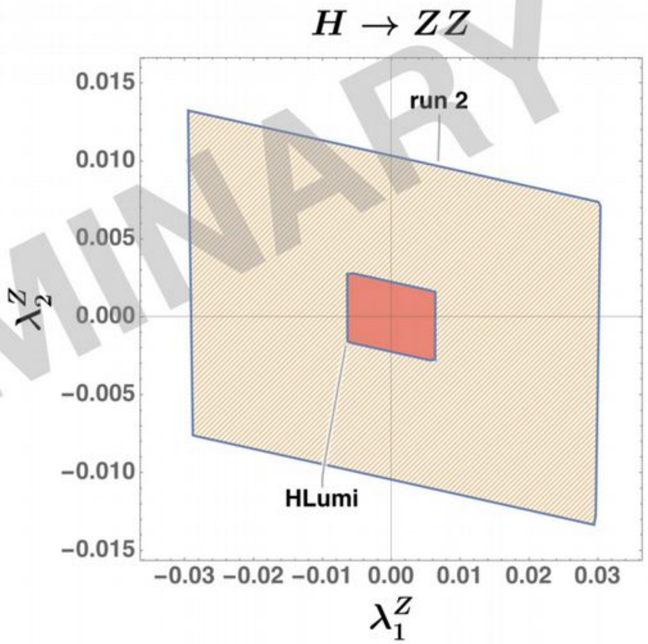


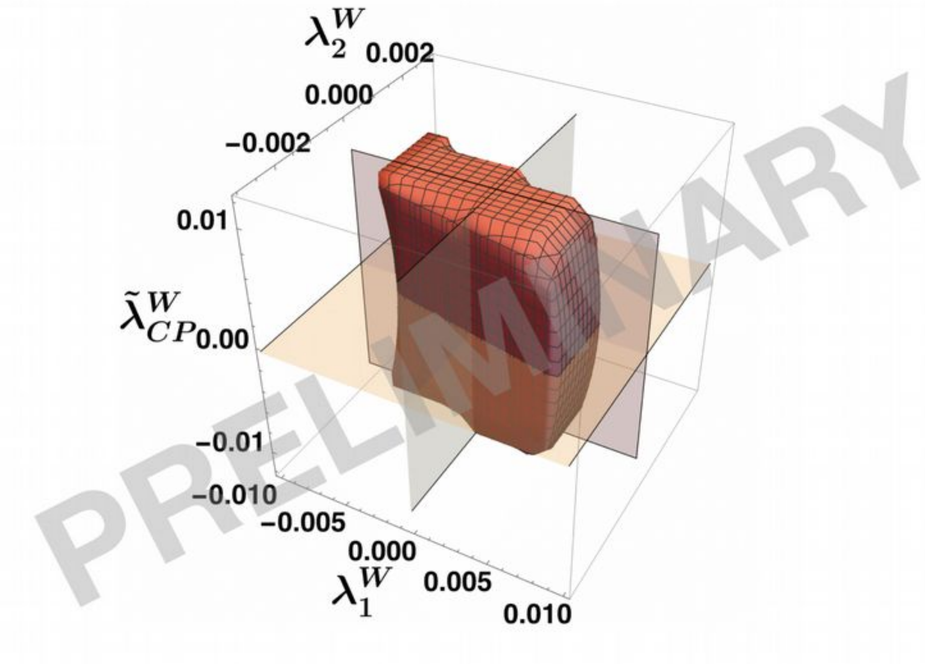
All limits are at the 95% C.L .

LHC run 2 ($\mathcal{L} = 140$ fb^{-1})

HLumi ($\mathcal{L} = 3$ ab^{-1})



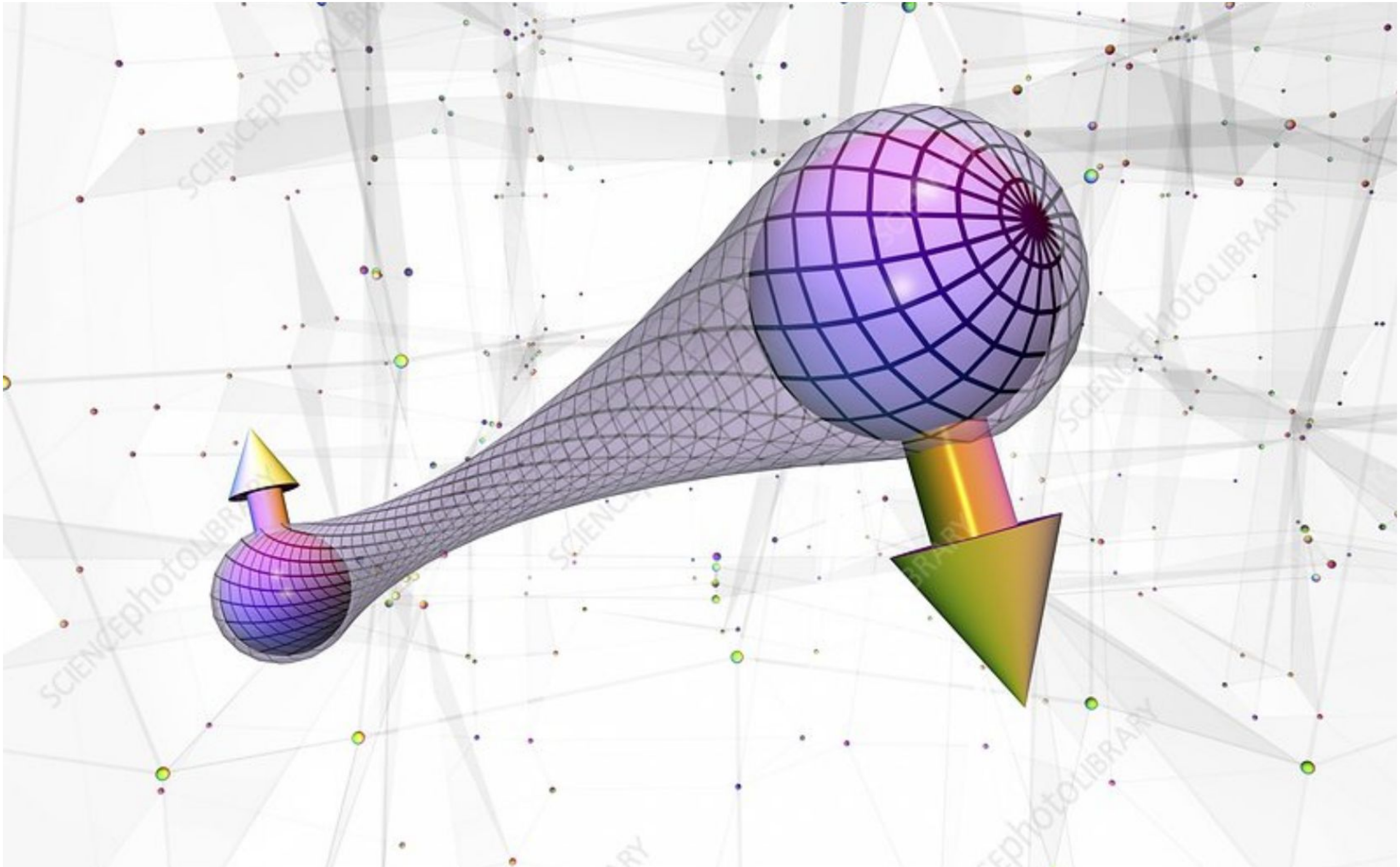




Bounds stronger than 1-2 order of magnitude with respect to present CMS bounds **[CMS Coll]**, 1901.00174 [hep-ex]

Competitive even with projected bounds from future linear colliders **Sharma, Shivaji**, 2207.03862 [hep-ph]

Thank you !



backup slides

- A suitable observable (concurrence) to quantify entanglement in a bipartite (A,B) system for a pure state $|\Psi\rangle$ (with matrix density $\rho = |\Psi\rangle\langle\Psi|$) is defined as

Rungta, Buzek, Caves, Hillery, Milburn, PRA 64 (2001)

concurrence

$$\mathcal{C}[|\Psi\rangle] = \sqrt{1 - \mathrm{Tr}\left[(\rho_r)^2\right]}$$

$r = A$ or B

vanishes for separable states $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$ (pure states)

$$\rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right]$$ and similar for ρ_B

Trace performed in the subsystem B

- For mixed states with matrix density $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, $p_i \geq 0$, $\sum_i p_i = 1$

$$\mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i \, \mathcal{C}[|\Psi_i\rangle]$$

infimum taken over all possible decompositions in pure states

$\mathcal{C}[\rho]$ vanishes for separable states

Acin, Durt, Gisin, Latorre, PRA 65 (2002), quant-ph/0111143

Bell operator is

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

written on the basis of spin-operators, where

S_3 spin operator → diag$\{1, 0, -1\}$

U,V matrices maximizing the Bell observable $\mathcal{I}_3$ in H → WW*, ZZ*

for region $M_W^* = 40\ \mathrm{GeV}$ and $M_Z^* = 32\ \mathrm{GeV}$

$$U_W = \begin{pmatrix} \frac{4}{11} + \frac{i}{14} & \frac{1}{6} + \frac{9i}{13} & \frac{3}{5} + \frac{i}{14} \\ -\frac{1}{9} - \frac{6i}{7} & 0 & \frac{1}{10} + \frac{i}{2} \\ \frac{4}{11} + \frac{i}{12} & -\frac{1}{7} - \frac{7i}{10} & \frac{3}{5} + \frac{i}{10} \end{pmatrix}, \quad V_W = \begin{pmatrix} -\frac{1}{7} - \frac{7i}{12} & -\frac{7}{10} - \frac{i}{10} & -\frac{1}{9} - \frac{6i}{17} \\ \frac{11}{21} + \frac{i}{17} & 0 & -\frac{6}{7} - \frac{i}{26} \\ -\frac{1}{8} - \frac{3i}{5} & \frac{7}{10} + \frac{i}{8} & -\frac{1}{10} - \frac{5i}{14} \end{pmatrix}$$

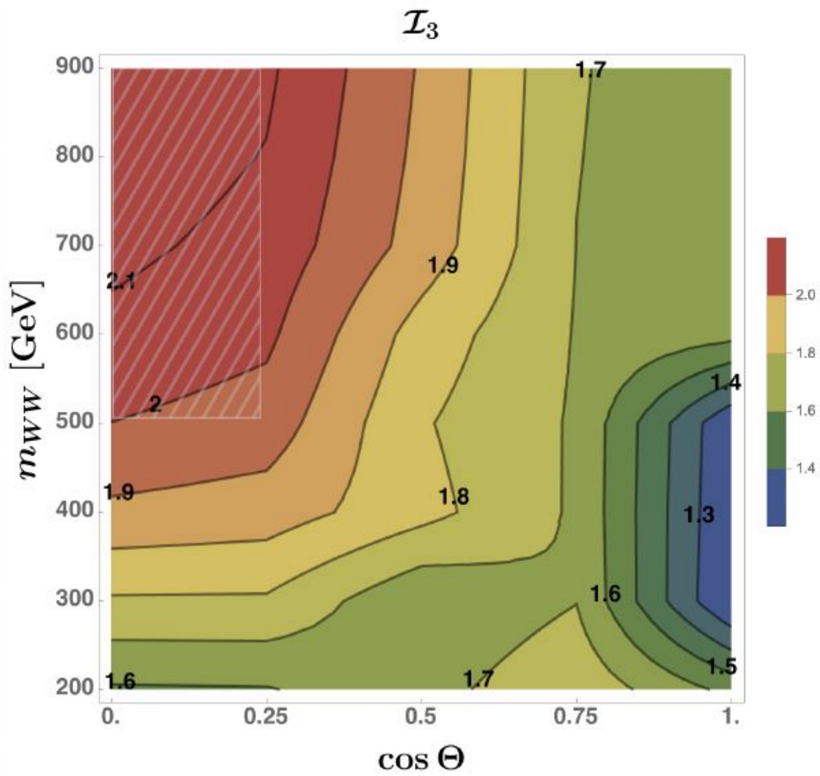
$$U_Z = \begin{pmatrix} -\frac{1}{2} + \frac{3i}{11} & \frac{7}{13} + \frac{5i}{11} & \frac{4}{13} - \frac{3i}{10} \\ -\frac{1}{2} + \frac{3i}{8} & 0 & -\frac{15}{31} + \frac{5i}{8} \\ -\frac{1}{5} + \frac{10i}{19} & -\frac{5}{7} & +\frac{1}{22} - \frac{3i}{7} \end{pmatrix}, \quad V_Z = \begin{pmatrix} -\frac{1}{7} - \frac{5i}{12} & \frac{7}{11} + \frac{2i}{7} & \frac{1}{25} - \frac{5i}{9} \\ \frac{2}{11} + \frac{10i}{13} & 0 & \frac{2}{7} + \frac{6i}{11} \\ \frac{1}{6} + \frac{2i}{5} & -\frac{11}{16} + \frac{i}{5} & -\frac{1}{3} - \frac{4i}{9} \end{pmatrix}$$

approximated matrices within 1% , unitary barring $O(10^{-2})$

U,V matrices maximizing the Bell observable $\mathcal{I}_3$ in pp → WW

corresponding to the hatched area (see plot below)

$$U_W = \begin{pmatrix} \frac{1}{50} - \frac{5i}{9} & -\frac{1}{6} + \frac{3i}{7} & -\frac{1}{13} + \frac{9i}{13} \\ \frac{1}{4} - \frac{4i}{7} & \frac{2}{9} - \frac{5i}{7} & \frac{1}{5} + \frac{i}{12} \\ \frac{2}{5} - \frac{2i}{5} & -\frac{1}{9} + \frac{4i}{9} & \frac{1}{3} - \frac{3i}{5} \end{pmatrix}, \quad V_W = \begin{pmatrix} -\frac{1}{16} - \frac{4i}{7} & -\frac{2}{11} + \frac{3i}{7} & -\frac{1}{8} + \frac{2i}{3} \\ -\frac{2}{13} + \frac{3i}{5} & -\frac{3}{11} + \frac{5i}{7} & -\frac{1}{5} - \frac{i}{13} \\ \frac{1}{3} - \frac{4i}{9} & -\frac{1}{8} + \frac{3i}{7} & \frac{3}{8} - \frac{3i}{5} \end{pmatrix}$$



approximated matrices within 1% , unitary barring $O(10^{-2})$

Spin-1 matrices

$$S_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Expressed as a function of Gell-Mann matrices

$$S_1 = \frac{1}{\sqrt{2}}\left(T^1 + T^6\right), \quad S_2 = \frac{1}{\sqrt{2}}\left(T^2 + T^7\right), \quad S_3 = \frac{1}{2}T^3 + \frac{\sqrt{3}}{2}T^8$$

$$\begin{aligned} S_{31} = S_{13} &= \frac{1}{\sqrt{2}}\left(T^1 - T^6\right), \\ S_{12} = S_{21} &= T^5, \\ S_{23} = S_{32} &= \frac{1}{\sqrt{2}}\left(T^2 - T^7\right) \\ S_{11} &= \frac{1}{2\sqrt{3}}T^8 + T^4 - \frac{1}{2}T^3, \\ S_{22} &= \frac{1}{2\sqrt{3}}T^8 - T^4 - \frac{1}{2}T^3, \\ S_{33} &= T^3 - \frac{1}{\sqrt{3}}T^8, \end{aligned}$$

Gell-Mann basis

$$T^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$T^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$\mathbb{1}$ being the 3×3 unit matrix