# Investigating LHC Electron Cloud Instabilities through Linearized Vlasov Method

**Sofia Johannesson**, Giovanni Iadarola, Prof. Mike Seidel, Tatiana Pieloni.

# Outline

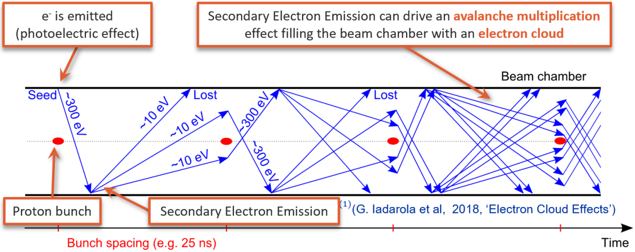
Introduction

E-cloud in a Vlasov formalism

Solutions to the Vlasov equation

First benchmark with measurements

# Electron clouds



e⁻ is emitted (photoelectric effect)

Secondary Electron Emission can drive an **avalanche multiplication** effect filling the beam chamber with an **electron cloud**

Proton bunch

Secondary Electron Emission

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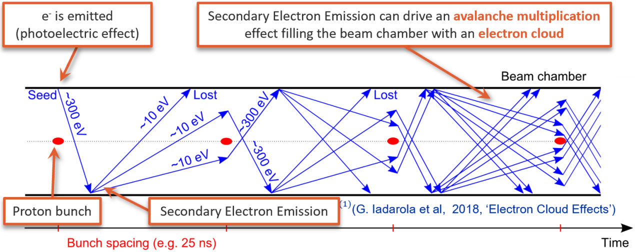
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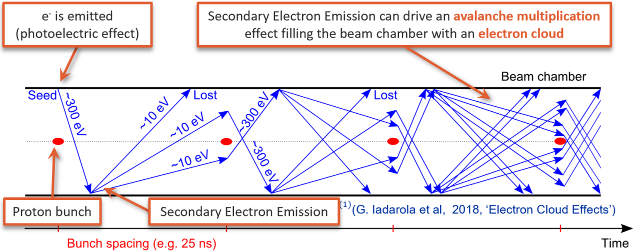

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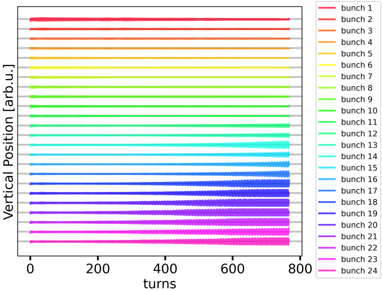
Unwanted effects:

- Transverse instabilities
- Transverse emittance blow-up
- Particle losses
- Heat Loads
- Vacuum Degradation

# Instabilities driven by e-cloud.



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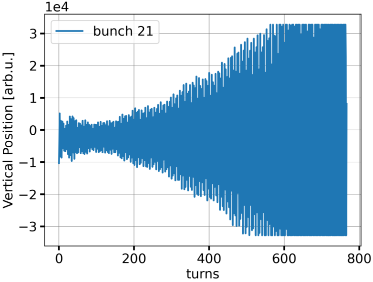

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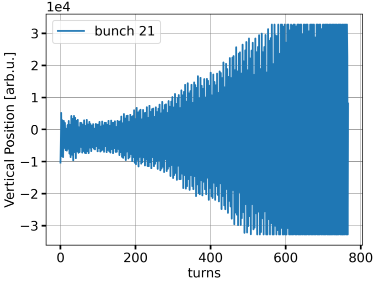

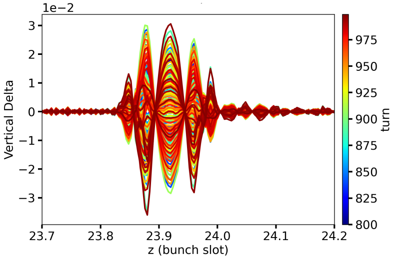

LHC measurements, June 2022

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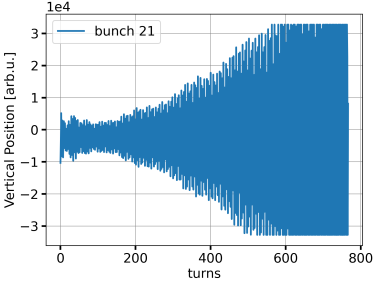

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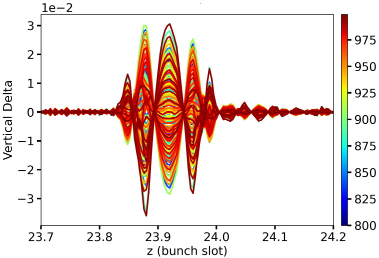

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- Conventional simulations using macroparticle tracking together with the PIC method for e-cloud beam interaction, are very computationally heavy. [7] G. Iadarola. et al., 2017, *Evolution of Python Tools for the Simulation of Electron Cloud Effects*

# Instabilities driven by electron cloud

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Previous attempts of using the Vlasov method to model e-cloud driven instabilities have not included these points together. [10] K. Ohmi et al, 2001, *Wake-Field and Fast Head-Tail Instability Caused by an Electron Cloud.*, [11] E. Perevedentsev, 2002, *Head-Tail Instability Caused by Electron Cloud*

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- This leads to the Linearized Vlasov Equation, which truncated to first order and expressed with Poisson brackets is:

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- The electron cloud forces are contained in $\Delta\mathbf{H}$
- The distortion $\Delta\psi$ is the impact of the perturbation and the **unknown**

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$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0 (Q_{x0} + \Delta Q(r,\phi)) \frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{df_0}{dJ_x} \sqrt{\frac{2 J_x R}{Q_{x0}}} \sin\theta_x F_x^{coh}(z,t)$$

reds are unknowns and greens come from e-cloud forces

# E-cloud in the Vlasov equation - diolar forces

Begin by describing the dipolar e-cloud forces:

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Choose a set of sinusoid beam distortions, $h_n(z)$ where $z$ is the position along the bunch. The sinusoid test functions satisfy the orthogonality condition: $\int h_n(z)h_{n'}(z) = H_n^2\delta_{n,n'}$

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Each distortion, $h_n$, corresponds to a response function $k_n$ calculated from the interaction with e-cloud using single-pass PIC simulations.

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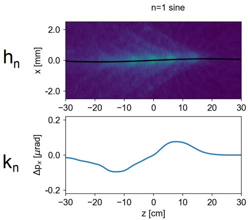

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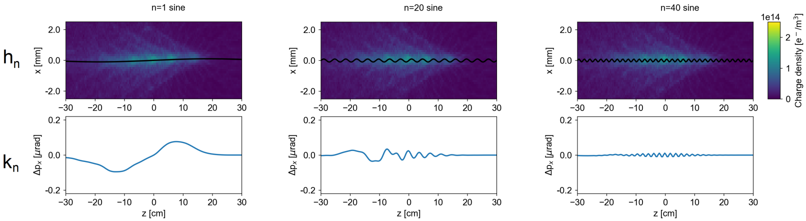

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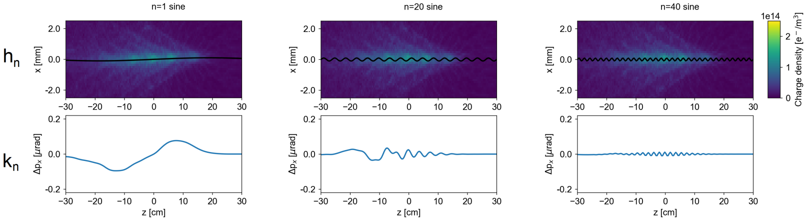

These calculations use the e-cloud in the superconducting quadrupoles of the LHC for a beam energy of 450GeV.

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# E-cloud in the Vlasov Equation - Dipolar Forces

Describe the transverse centroid along the bunch, $\bar{x}(z)$, as a linear combination of test functions $h_n$:

$$\bar{x}(z) = \sum_{n=0}^{\infty} a_n h_n(z); \quad a_n = \frac{1}{H_n^2} \int \bar{x}(z) h_n(z) dz \tag{2}$$

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The kick, $\Delta x'$, of arbitrary distribution $\bar{x}(z)$ is:

$$\Delta x'(z) = \sum_{n=0}^{\infty} a_n k_n(z) \tag{3}$$

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The coherent force can be expressed from the transverse kick $\Delta x'$. Assuming the force is distributed uniformly in the accelerator:

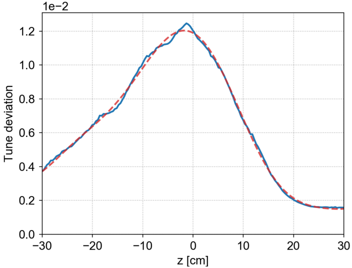
$$F_x^{coh}(z,t) = \frac{m_0 \gamma v^2}{2\pi R} \Delta x' \tag{4}$$

$m_0$ is the proton mass, $\gamma$ is the relativistic gamma, $v$ is the velocity of the protons and $R$ is the total radius of the LHC.

# E-cloud in Vlasov equation - Quadrupolar forces

Model detuning using a polynomial

$$\Delta Q(z) = \sum_{n=0}^{N_p} A_n z^n \qquad (5)$$



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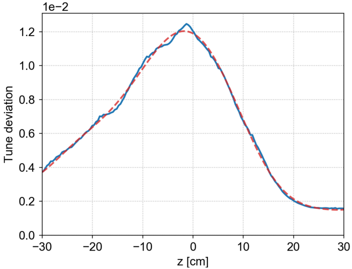

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Generalize by adding chromaticity

$$\Delta Q(z,\delta) = \sum_{n=0}^{N_p} A_n z^n + B_n \delta^n \tag{6}$$

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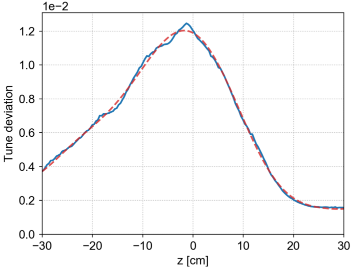

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Generalize by adding chromaticity

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Including only linear chromaticity:

$$\Delta Q(z, \delta) = Q' \delta + \sum_{n=0}^{N_p} A_n z^n \qquad (7)$$

# Solving the Vlasov Equation - Ansatz of $\Delta\psi$

Equation to solve:

$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0 (Q_{x0} + \Delta Q(r,\phi)) \frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{df_0}{dJ_x} \sqrt{\frac{2J_x R}{Q_{x0}}} \sin\theta_x F_x^{coh}(z,t)$$

$\omega_0$ is the angular revolution frequency, $Q_0$ is the unperturbed tune and $Q_s$ is the synchrotron frequency

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dipolar forces from e-cloud

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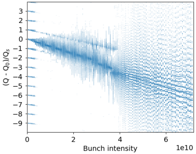
tune shift, $\left(\frac{Re(\Omega)}{\omega_0} - Q_0\right)/Q_s$



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$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0(Q_{x0} + \underset{\uparrow}{\Delta Q(r,\phi)})\frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma}\frac{df_0}{dJ_x}\sqrt{\frac{2J_x R}{Q_{x0}}}\sin\theta_x \underset{\uparrow}{F_x^{coh}(z,t)}$$

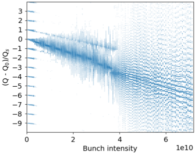
detuning from e-cloud and chromaticity

dipolar forces from e-cloud

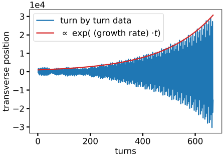
First ansatz:

$\Delta\psi(J_x,\theta_x,r,\phi,t) = e^{j\Omega t}\Delta\psi(J_x,\theta_x,r,\phi)$.

tune shift, $(\frac{Re(\Omega)}{\omega_0} - Q_0)/Q_s$



instability growth rate, $-Im(\Omega)$



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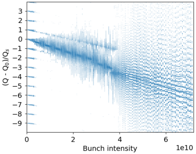

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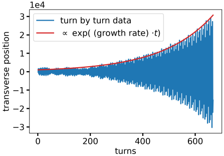

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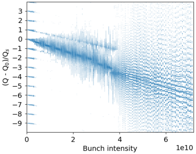

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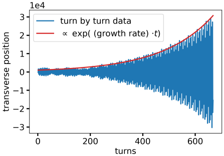

After further calculations linerized Vlasov Equation becomes an **eigenvalue problem**:

$$b_{lm}(\Omega - Q_{x0}\omega_0 - l\omega_s) = \sum_{l'm'}(\mathbf{M}_{lm,l'm'} + \tilde{\mathbf{M}}_{lm,l'm'})b_{l'm'} \qquad (8)$$

Unkowns in red and terms including electron cloud forces in green

$\omega_0$ is the angular revolution frequency, $Q_0$ is the unperturbed tune and $Q_s$ is the synchrotron frequency

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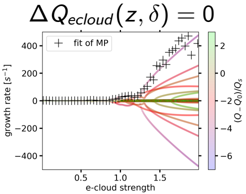

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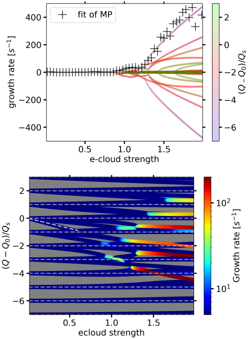

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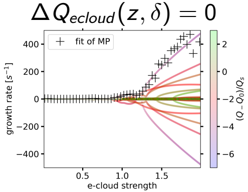

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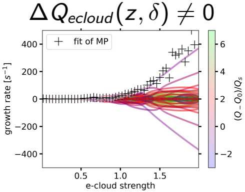

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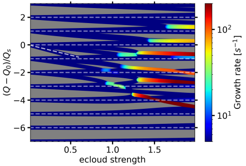

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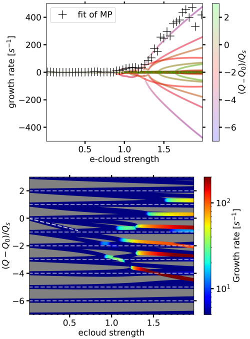

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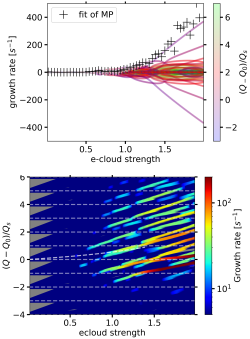

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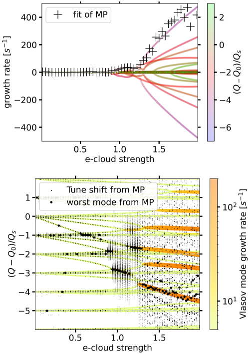

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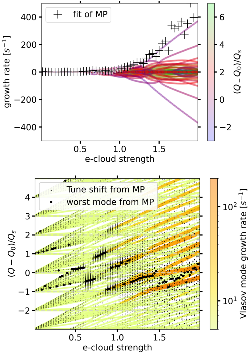

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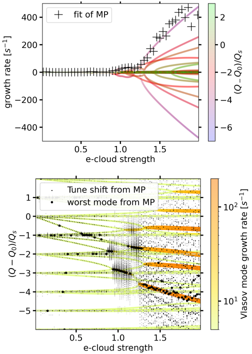

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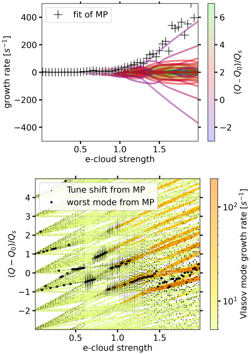

Good agreement with MP simulations!

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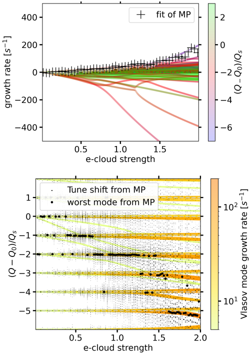

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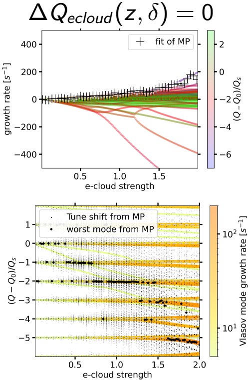
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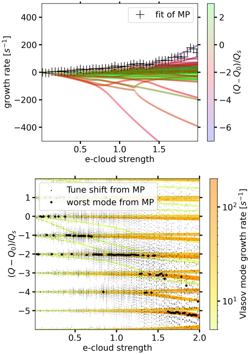

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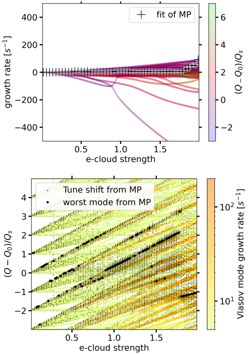

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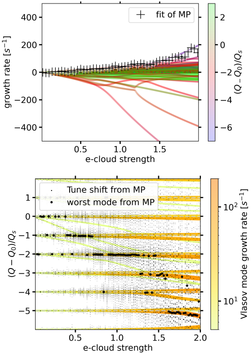

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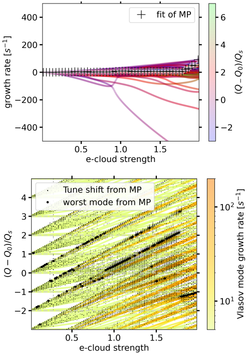

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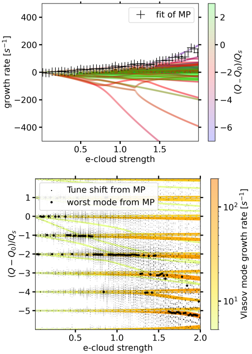

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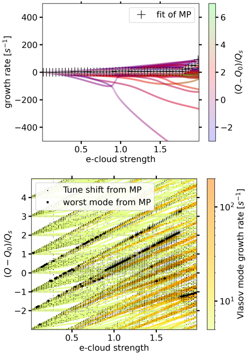

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The tune shift of the Vlasov modes agree well with the macro-particle spectra.

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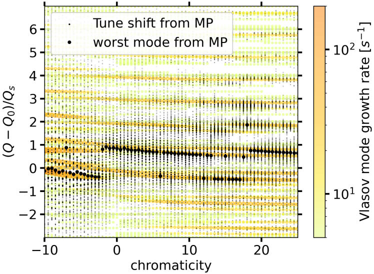

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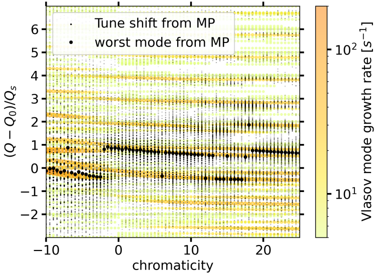

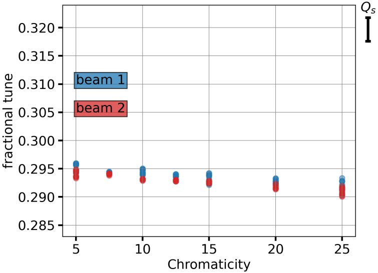

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This is confirmed by measurements.

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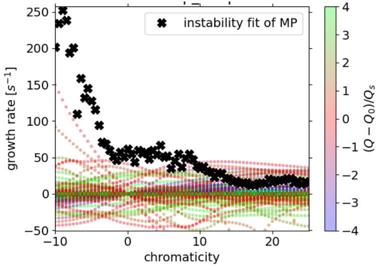

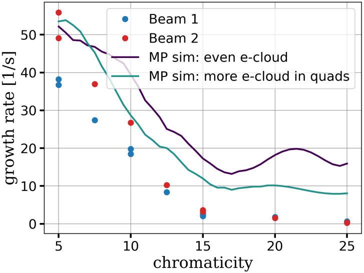

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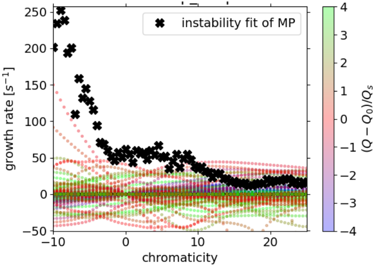

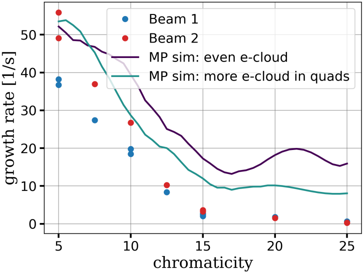

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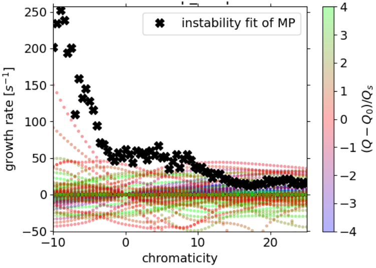

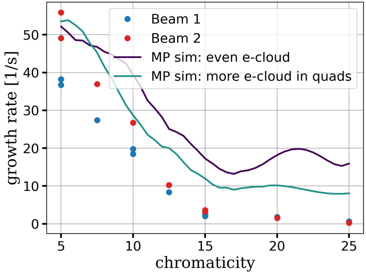

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The macroparticle simulations are damped by incoherent mechanisms not captured by the Vlasov model.

# Conclusions

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Thank you for your attention!

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