LHC Schottky Spectrum from Macro-particle Simulations

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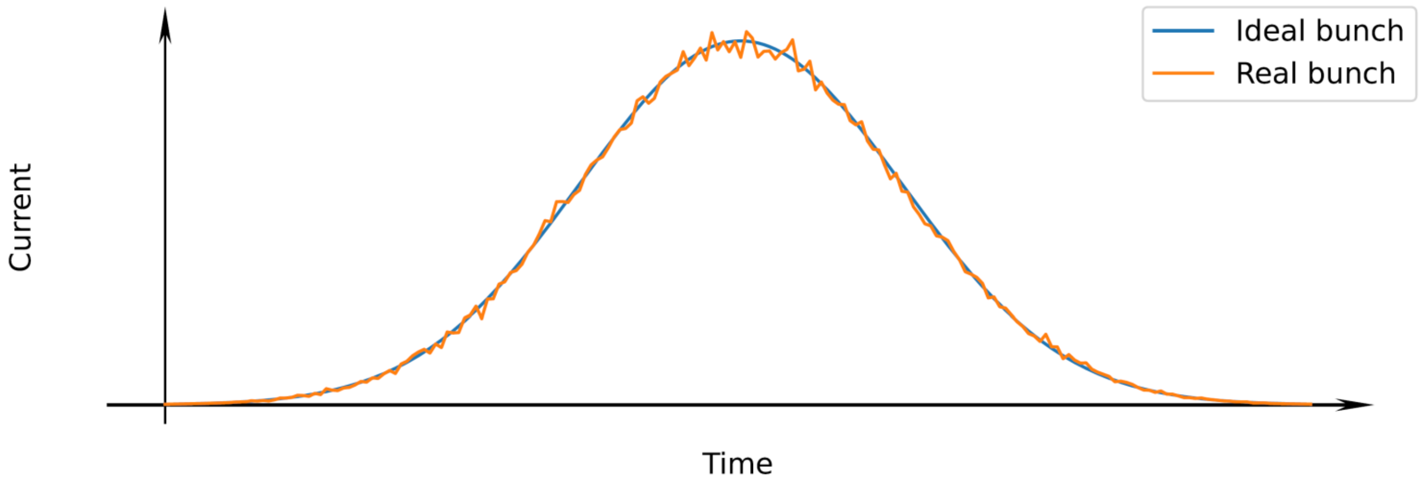
Acknowledgements: Diogo Alves, Kacper Lasocha, Nicolas Mounet, Tatiana Pieloni

Introduction

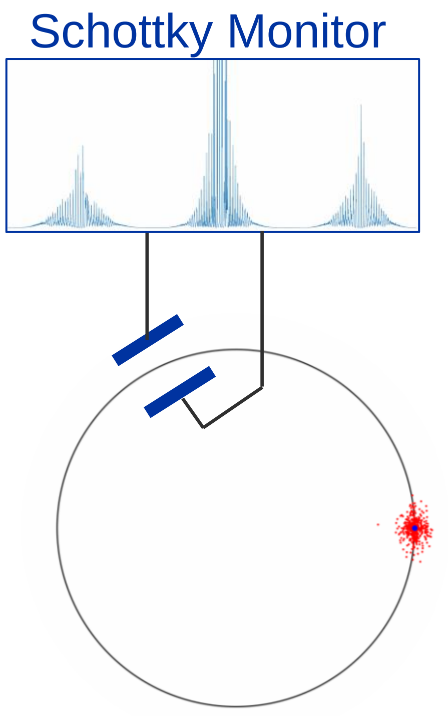
- The beam current in an accelerator is subject to intrinsic fluctuations, which results from the discrete number of particles in the beam.

- These fluctuations in the beam current, called the Schottky noise, are used to obtain information on machine parameters.



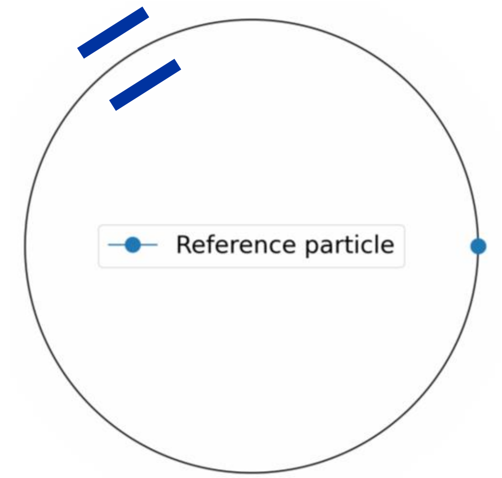
Introduction

- Schottky spectrum is the power spectral density of the beam current in the longitudinal plane and the dipole moment in the transverse planes.
- There are 4 Schottky monitors in the LHC, one for each plane (H & V) of the two beams.
- Contains information on various beam and machine parameters:
  - Betatron and synchrotron tunes
  - Chromaticities
  - Longitudinal bunch profile
- Important non-invasive method for beam diagnostics. For example, the Schottky monitors allow to monitor the drift of the chromaticity over time (in a non-destructive manner), which is an important parameter for the impedance-related instabilities.

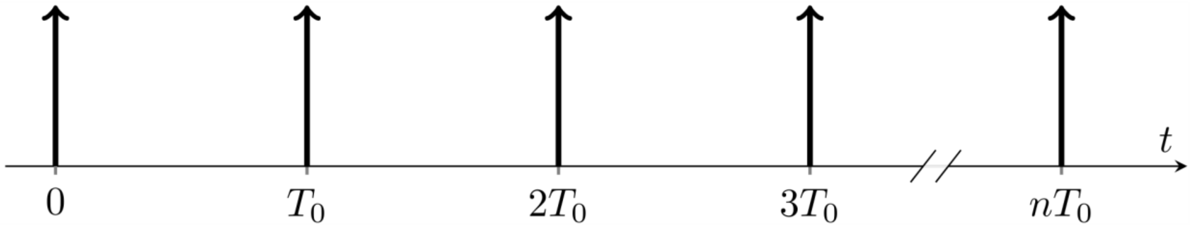


Longitudinal Schottky spectrum

Synchronous particle

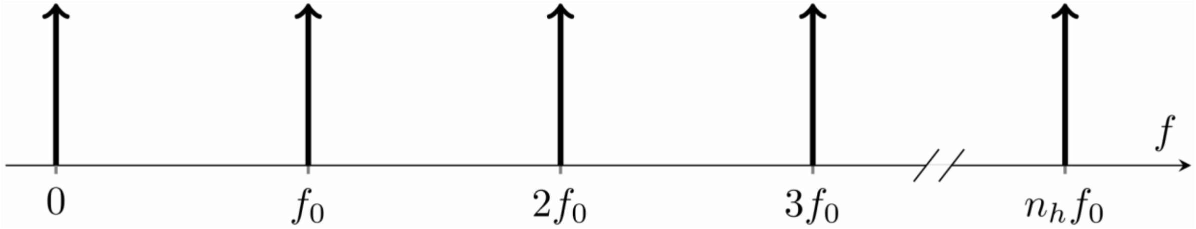


- Time domain:



$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

- Frequency domain:



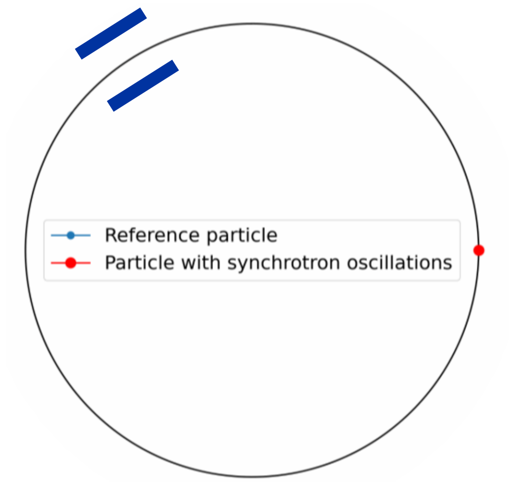
$$\widetilde{i_i(f)} = \frac{q f_0}{2\pi} \sum_{n_h=-\infty}^{\infty} \delta(f - n_h f_0)$$

With:

- q: charge of the particle
- T_0: revolution period
- f_0: revolution frequency
- n_h: harmonic number

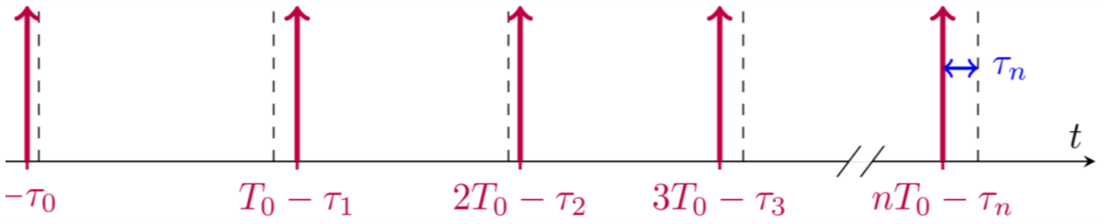
Longitudinal Schottky spectrum

Particle with synchrotron oscillation



For LHC: $Q_s \sim 5 \times 10^{-3}$
(animation for illustration purpose)

- Time domain:



$\tau_{n,i}$: time difference between particle i and the synchronous particle at turn n.

$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta\Big(t - (nT_0 - \tau_{n,i})\Big)$$

- Frequency domain:

$\tau_{n,i} = ?$

Theory: Analytical expression for $\tau_i(t)$

Simulation: Numerical values for $\tau_{n,i}$

FFT algorithm

Analytical Fourier transform

Longitudinal Schottky spectrum

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Theory: Analytical expression for $\tau_i(t)$

Ref. 1: D. Boussard, "Schottky noise and beam transfer function diagnostics".

Simulation: Numerical values for $\tau_{n,i}$

FFT algorithm

Analytical Fourier transform

Limitation of theoretical approach for Schottky spectra

- The theory of Schottky spectra for bunched beams is limited to simplified beam dynamics and does not include, e.g., collective effects, non-linearities and beam interaction with the vacuum chamber.

- For proton beams (and ion beams under certain conditions), these effects distort the Schottky spectra, which prevents the extraction of useful information from the Schottky monitors.

  ➔ Study their impact on Schottky spectra using macro-particle simulations.

- Develop a post-processing method able to compute the Schottky spectrum from macro-particle simulation to investigate the impact of collective effects and non-linearities on the Schottky spectra.

- Once the macro-particle simulations will by validated on measurements in simple conditions (e.g. ion beam at injection), the more complex effects could easily be added to the simulations since the macro-particle code already implements these effects.

Longitudinal Schottky spectrum

Particle with synchrotron oscillation

For LHC: $Q_s \sim 5 \times 10^{-3}$
(animation for illustration purpose)

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$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta\Big(t - (nT_0 - \tau_{n,i})\Big)$$

- Frequency domain:

$\tau_{n,i} = ?$

Theory: Analytical expression for $\tau_i(t)$ → Analytical Fourier transform

Simulation: Numerical values for $\tau_{n,i}$ → FFT algorithm

Simulated longitudinal Schottky spectrum (FFT)

Numerical value for $\tau_{n,i}$

$$i(t) = \sum_{i=1}^{N_p} i_i(t) = q \sum_{i=1}^{N_p} \sum_{n=0}^{N_t} \delta\Big(t - (nT_0 + \tau_{n,i})\Big)$$

FFT algorithm

- Sample each passage of the bunch with 100 points over 10’000 turns.
- Array of 10^{11} samples or 1 TB of data.

➔ **While manageable for smaller accelerator, the FFT is too complex memory wise for the LHC.**

Longitudinal Schottky spectrum

Particle with synchrotron oscillation

For LHC: $Q_s \sim 5 \times 10^{-3}$
(animation for illustration purpose)

- Time domain:

$\tau_{n,i}$: time difference between particle i and the synchronous particle at turn n.

$$i_i(t) = q \sum_{n=-\infty}^{\infty} \delta\big(t - (nT_0 - \tau_{n,i})\big)$$

- Frequency domain:

$\tau_{n,i} = ?$

Theory: Analytical expression for $\tau_i(t)$

Simulation: Numerical values for $\tau_{n,i}$

~~FFT algorithm~~

Analytical Fourier transform

Simulated longitudinal Schottky spectrum (Analytical FT)

- Time domain:

$$i(t) = \sum_{i=0}^{N_p} i_i(t) = q \sum_{i=1}^{N_p} \sum_{n=0}^{N_t} \delta\Big(t - (nT_0 + \tau_{n,i})\Big)$$

- Frequency domain:

$$\widetilde{i(\omega)} = \int_{-\infty}^{\infty} i(t) e^{j\omega t} dt = q \sum_{n=0}^{N_t} \sum_{i=1}^{N_p} e^{j\omega(nT_0 + \tau_{n,i})}$$

➔ still requires a lot of computational resources.

- Expanding the exponential function with its a **Taylor series** and **inverting the order of summation** allows to greatly reduce the computational requirements.

$$\widetilde{i(\omega)} = q \sum_{n=0}^{N_t} e^{j\omega n T_0} \sum_{l=0}^{N_l} \frac{j^l (\omega - \omega_c)^l}{l!} \sum_{i=1}^{N_p} e^{j\omega_c \tau_{n,i}} (\tau_{n,i})^l$$

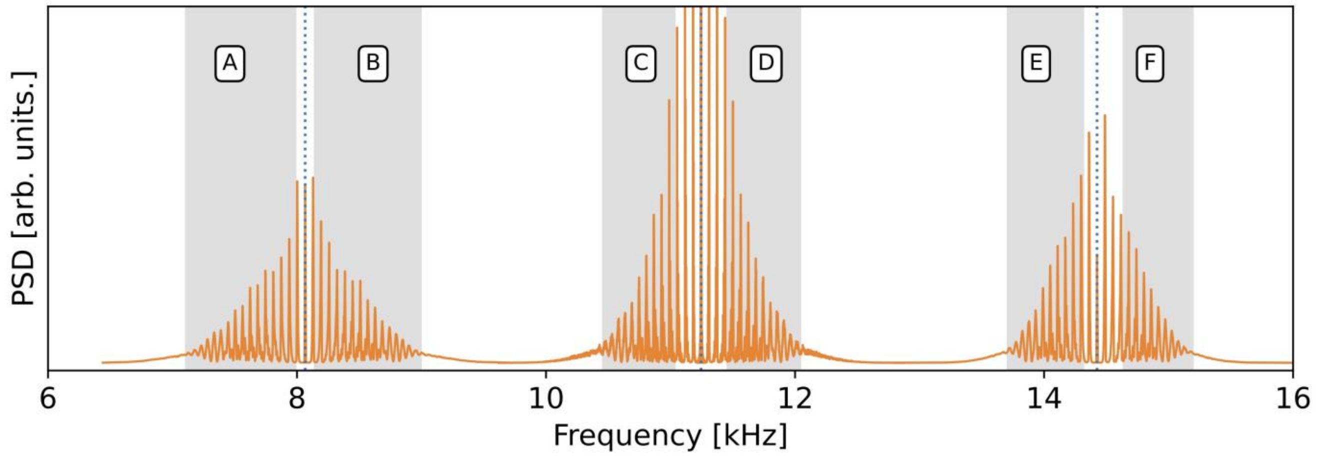
Computational requirements:

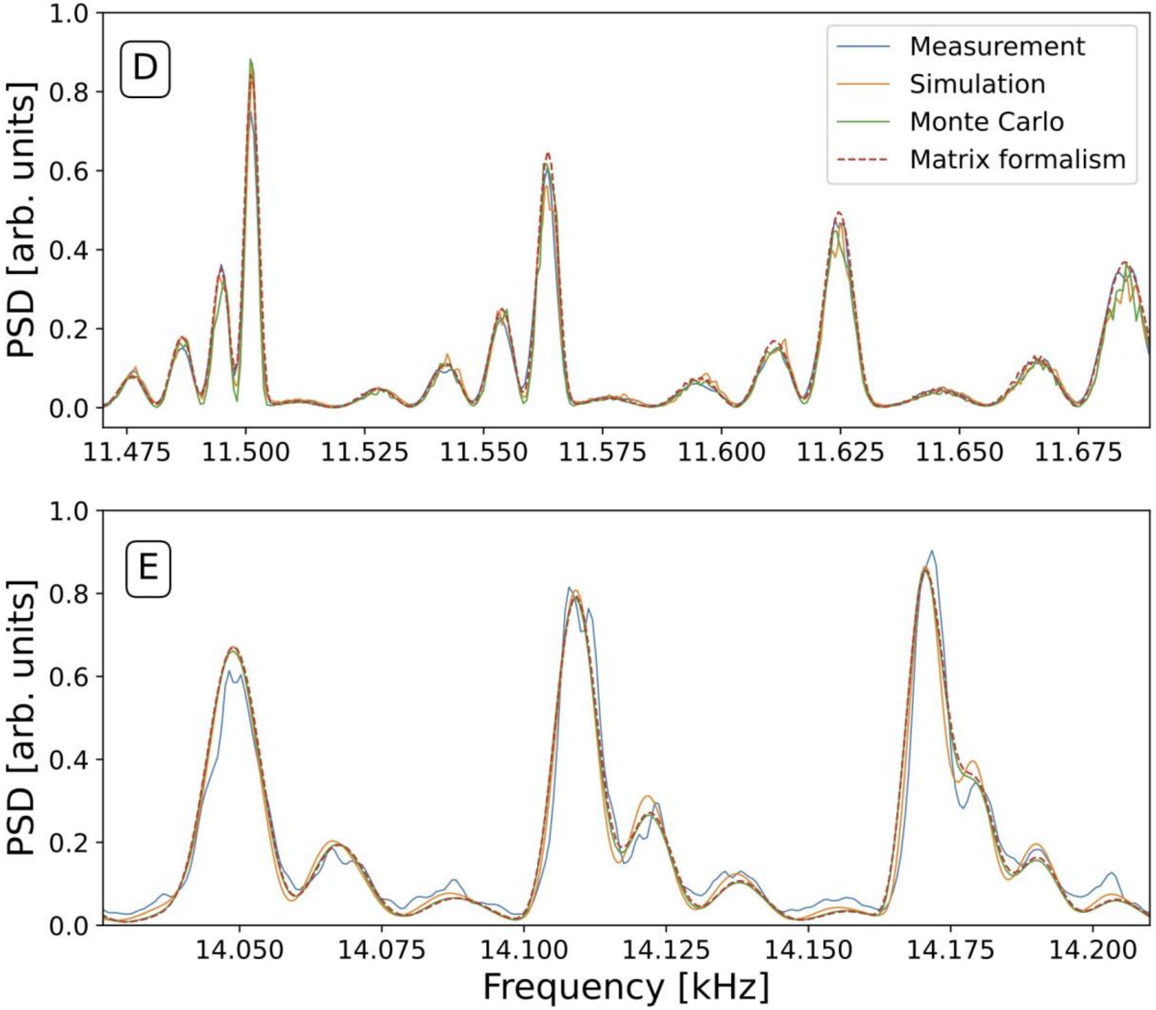
- Evaluate $\mathcal{O}(N_t N_p N_f)$ 10^{14} exponentials.
- Store in memory $N_t \times N_p$ number $\tau_{n,i}$ 100 Gb.

Computational requirements:

- Evaluate $\mathcal{O}(N_t N_p)$ exponentials 10^{10}.
- No need to store in memory the $\tau_{n,i}$ as the Schottky spectrum is calculated on the fly along with the macro-particle simulation.

Comparison with LHC experimental measurements





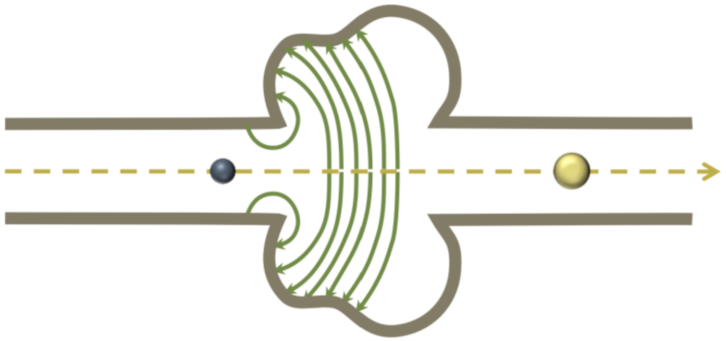
- The simulation method has been **validated on experimental measurements** from a LHC ion beam at injection energy **and with two theoretical formalisms[1]** .
- Collective effects are supposed to be negligeable for this particular case.
- The **different methods are in very good agreement with each other** and reproduce the overall shape of the spectrum as well as the detailed internal structure of the synchrotron satellites.

1: K. Lasocha and D. Alves, "Estimation of transverse bunch characteristics in the LHC using Schottky-based diagnostics".

Schottky spectrum simulation with impedance

- **Simulation method validated for simple situations** where collective effects are negligeable,
  ➔ Explore how collective effects, such as impedance, affect the Schottky spectrum.

- When the beam is surrounded by a vacuum pipe, the particles can interact with the walls and produce electromagnetic field that can impact the trailing particles.



- This collective effect is called **beam coupling impedance** and has a significant impact on proton beams, distorting the Schottky spectra and **preventing the extraction of information from the Schottky monitors.**

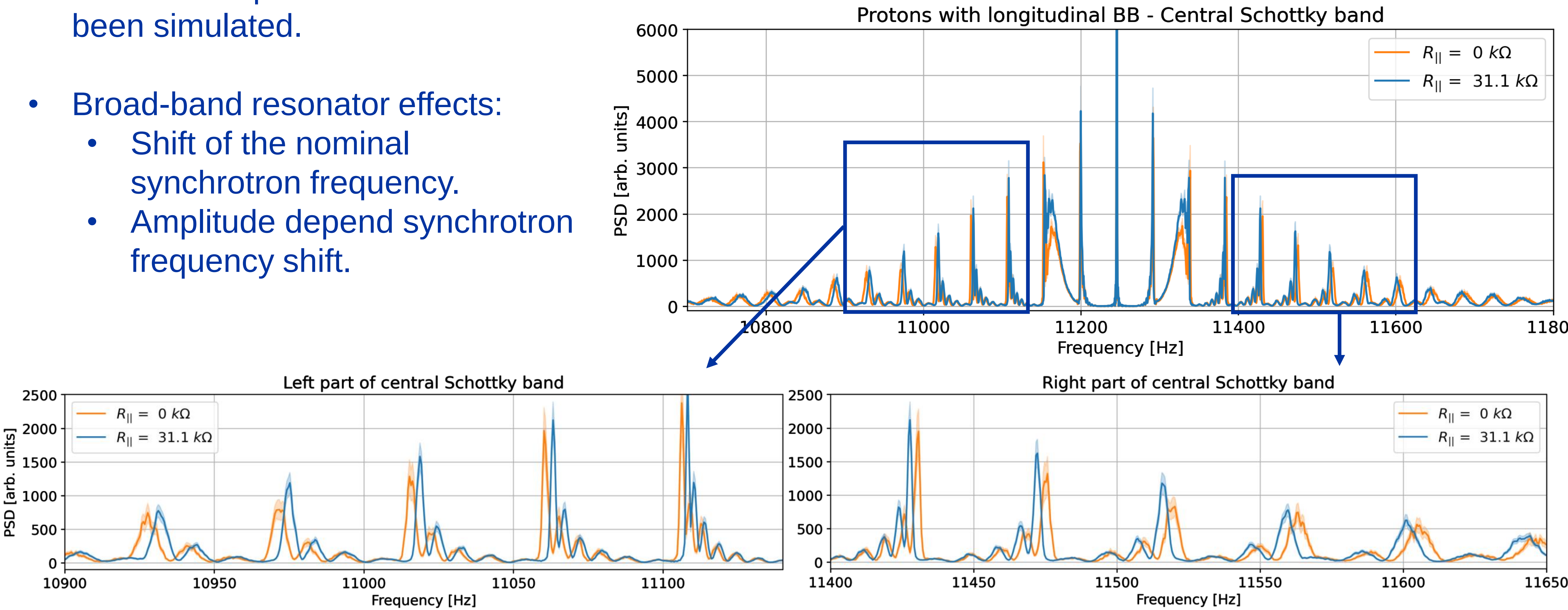
- Due to the complex theoretical description of impedance, it is most suitable to use macro-particle simulation code (where impedance effects are already implemented) combined with the newly developed post-processing method.

Figure from: "Collective effect II", Kevin Li, CAS: Introduction to Accelerator Physics, 2022

Schottky spectrum simulation with impedance

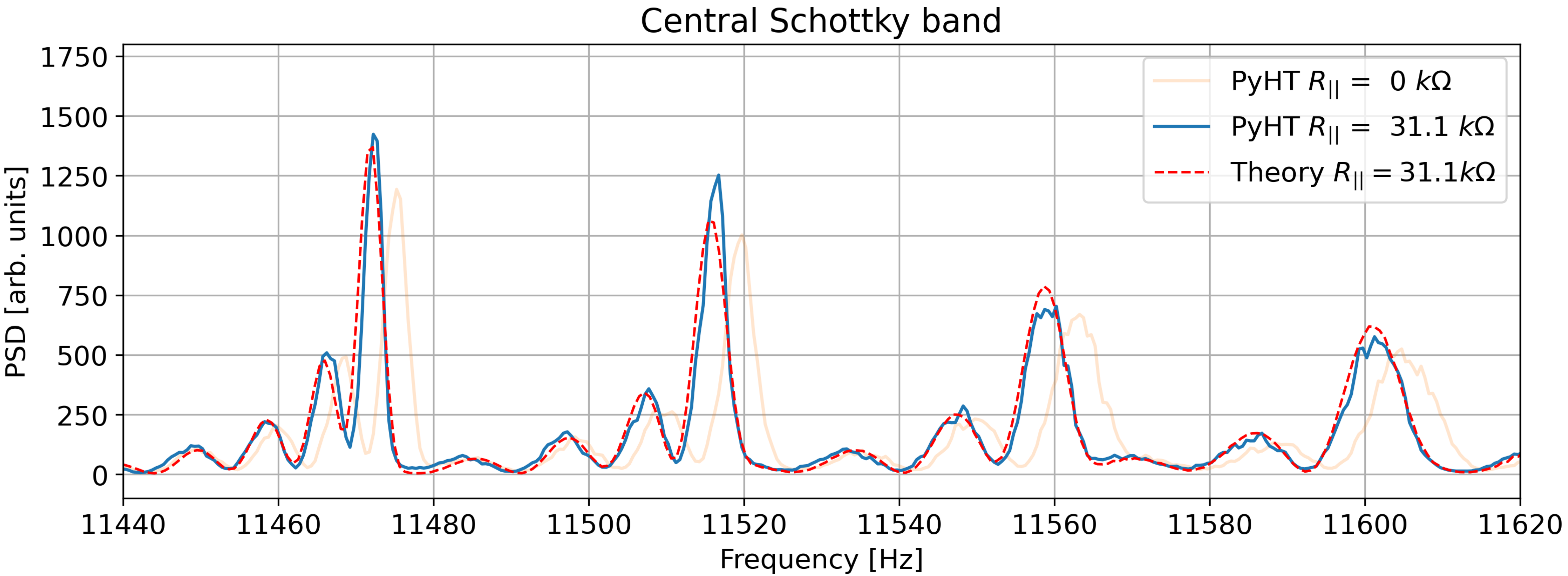
- The simple case of a broad-band resonator impedance model has been simulated.

- Broad-band resonator effects:
  - Shift of the nominal synchrotron frequency.
  - Amplitude depend synchrotron frequency shift.



Schottky spectrum with impedance

- For the relatively simple case of a broad-band resonator impedance model, theoretical expression of the Schottky spectrum can still be derived.
- Agreement between macro-particle simulation and theory is still very good.



Conclusion

- A post-processing method was developed to compute Schottky spectra from macro-particle simulation.

- Good agreement between macro-particle simulation, theory, and experimental measurements were obtained for situation where collective effects are negligeable.

- Effect of impedance have been investigated through simulations for the simple case of a broad-band resonator ➔ Good agreement with theory was obtained.

- The newly developed post-processing method will allow investigation (through simulations) of more complex situations where no theory is currently available.
  - More complex impedance model
  - Transverse non-linearities
  - Beam-beam effects
  - Electron cloud effects
  - …

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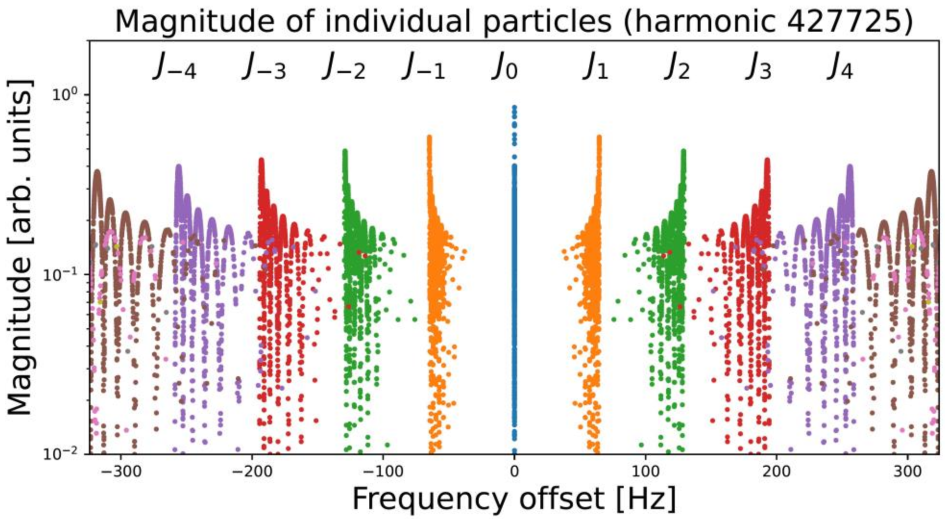
Theoretical longitudinal Schottky spectrum

$\tau_{n,i} = ?$ → **Theory**: Analytical expression for $\tau_i(t)$ → $\tau_i(t) = \hat{\tau}_i \sin(\Omega_{s_i} t + \varphi_{s_i})$

$$i(t) = \sum_{i=1}^{N_p} i_i(t) = q \sum_{i=1}^{N_p} \sum_{n=0}^{N_t} \delta\Big(t - (nT_0 + \tau_{n,i})\Big)$$

$$i(t) = q f_0 \sum_{i=1}^{N_p} \sum_{n,p=-\infty}^{\infty} \underbrace{J_p(n\omega_0 \hat{\tau}_i)}_{\text{Amplitude}} e^{j\left[\underbrace{(n\omega_0 + p\Omega_{s_i})t}_{\text{Frequency}} + \underbrace{p\varphi_{s_i}}_{\text{Phase}}\right]}$$



Ref. 1: D. Boussard, "Schottky noise and beam transfer function diagnostics", doi:10.5170/CERN-1987-003-V2.416