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Systematic effects in the search for the muon EDM using the frozen-spin method

On behalf of the muonEDM collaboration
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- Relativistic spin precession of a charged particle (Thomas-BMT equation) →

- By applying an appropriate radial E-field to the muon we negate the *g-2* term.

- Ideally any observed spin precession would be due to a non-zero EDM.

- Asymmetry between positrons emitted along or opposite the main B-field.

- **Some asymmetry could still be observed due to systematic effects**

$$\vec{\Omega} = -\frac{e}{m_0}\left[a\vec{B} + \left(\frac{1}{\gamma^2-1} - a\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

g-2 term   EDM term
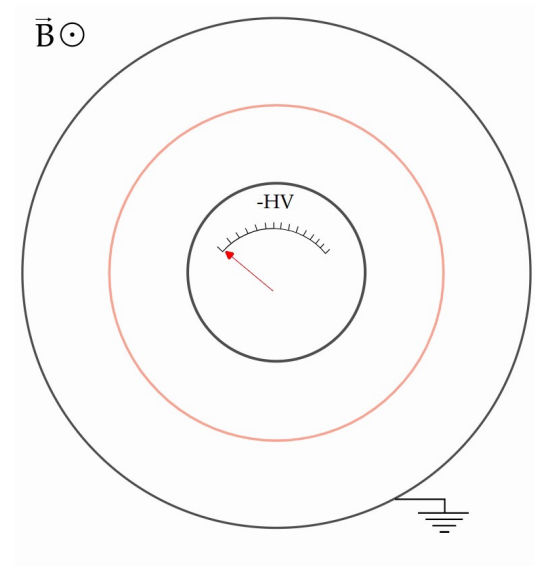
Systematic effects

- Effects that lead to a _real_ or _apparent_ precession of the spin around the radial (E-field) axis that are not related to the EDM.

- Types of systematic effects:

  - Coupling of the anomalous magnetic moment with the EM fields of the experimental setup **(real)**

    - Dynamical phase – the usual spin precession

    - Geometric phase – parallel transport of a vector

  - Early to late variation of detection efficiency of the EDM detectors **(apparent)**

Dynamic phase systematic effects

- To limit systematic effects related to dynamic phases we require that precession due to the anomalous magnetic moment is smaller than the target EDM signal:

$$\vec{\Omega} = -\frac{e}{m_0}\left[a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right)\right]$$

AMM   EDM

- Where $\eta \sim 10^{-9}$ for $d_e = 10^{-23}\,e.\mathrm{cm}$

- It has a similar meaning as the anomalous magnetic moment $a \sim 10^{-3}$

- One can expect that effects due to the anomalous magnetic moment should be suppressed to a level better than $\sim 10^{-6}$

What are geometric phases?

- The geometric phase is a phase difference acquired over the course of a cycle in parameter space.

- Parallel transport of a vector around a closed loop.

- The angle by which it twists is proportional to the area inside the loop:

    - In classical parallel transport it's equal.

    - In quantum mechanics it's -½ (fermions).

- If oscillations around two axes are combined we can observe a phase shift (false EDM)
even if the the time average of each oscillation is zero.
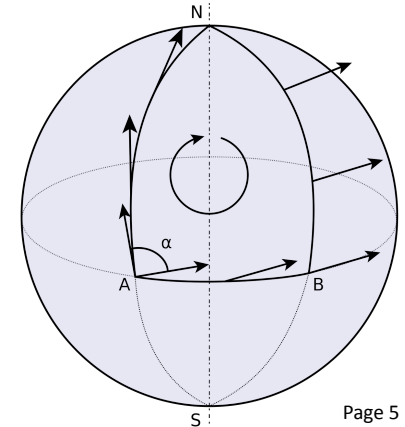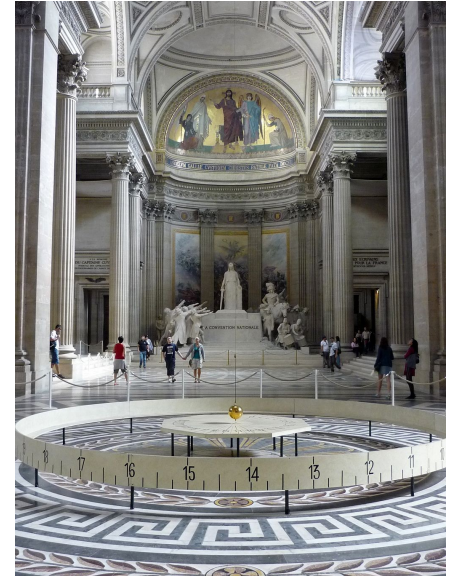
Calculation of geometric phases

- Muons experience cyclotron and betatron motion in the storage ring that leads to oscillations of the spin around different axes.

- Spin precesses around axis **x** with amplitude C_1 and frequency Ω_x, and around **y** with amplitude C_2 and frequency Ω_y. Phase difference between the two β_0.

$$x = C_1 \sin(\Omega_x t), \quad y = C_2 \sin(\Omega_y t + \beta_0)$$

- The movement of the spin encloses an area A on some abstract surface. The area can be calculated from Green's theorem:

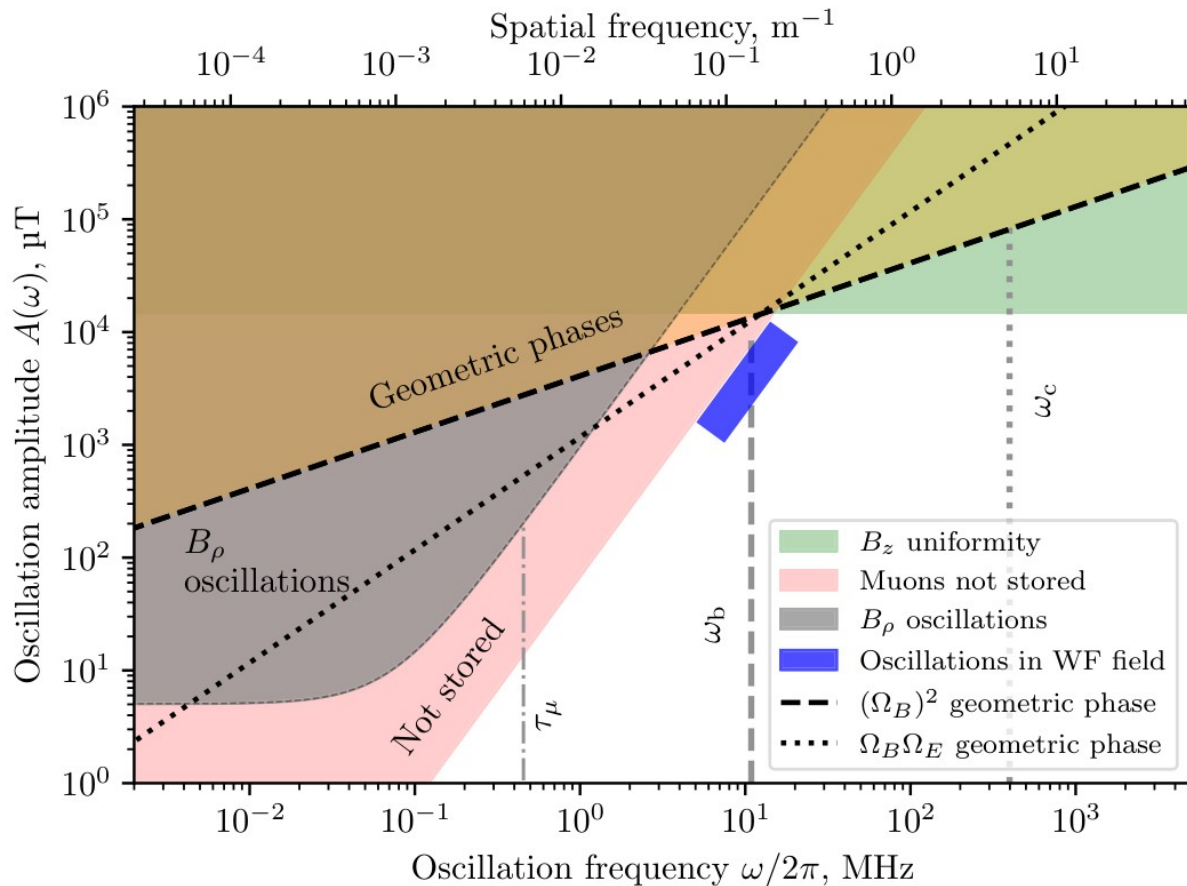$$A = \frac{1}{2} \int_{t_0}^{t_1} (xy' - yx')dt$$

- In the case where the two in-phase oscillations have the same frequency the geometric phase is:

$$A = -\frac{1}{2\omega} \Omega_x \Omega_y t$$

Magnetic field stability and uniformity

$$\vec{\Omega} = -\frac{e}{m_0}\left[a\vec{B} + \left(\frac{1}{\gamma^2-1} - a\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$
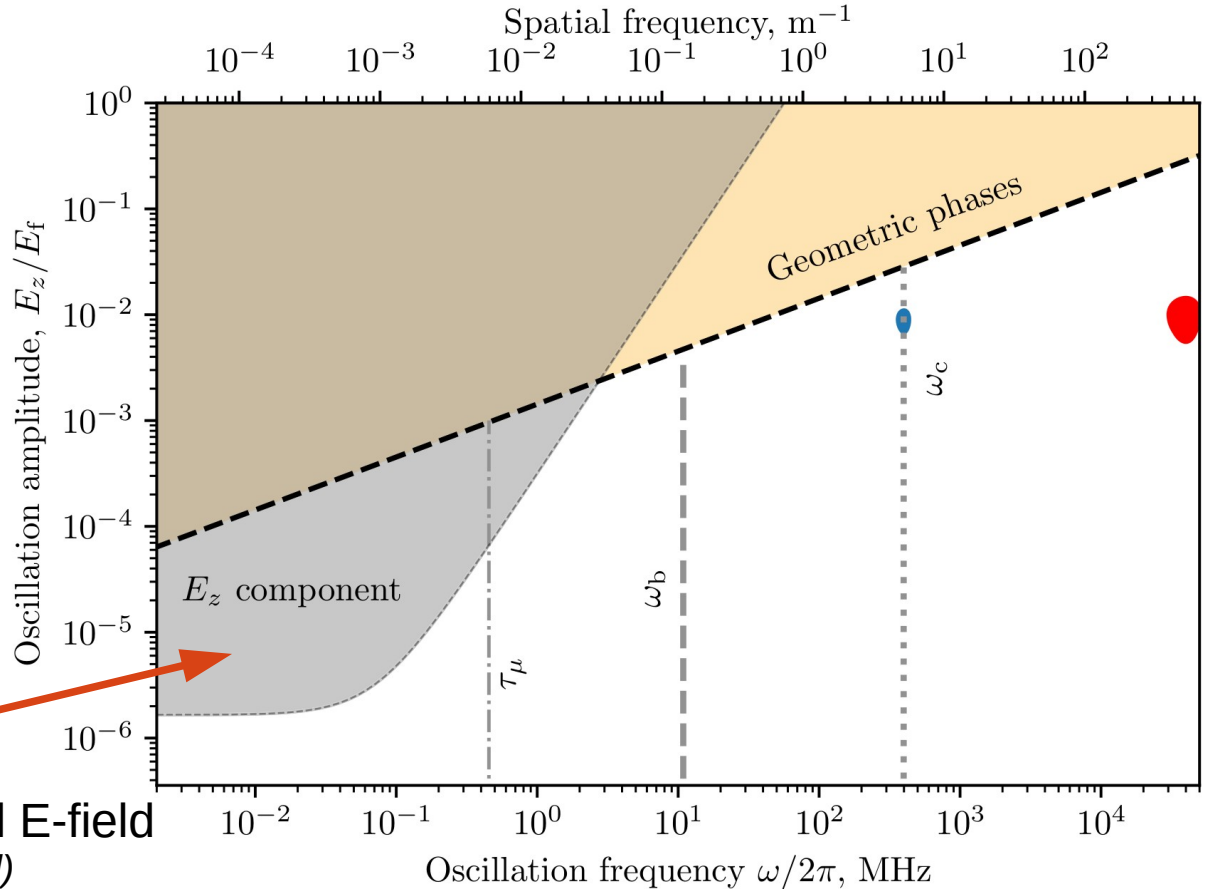
$$A = -\frac{1}{2\omega}\Omega_x\Omega_y t$$

Electric field stability and uniformity

$$\vec{\Omega} = -\frac{e}{m_0}\left[a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$
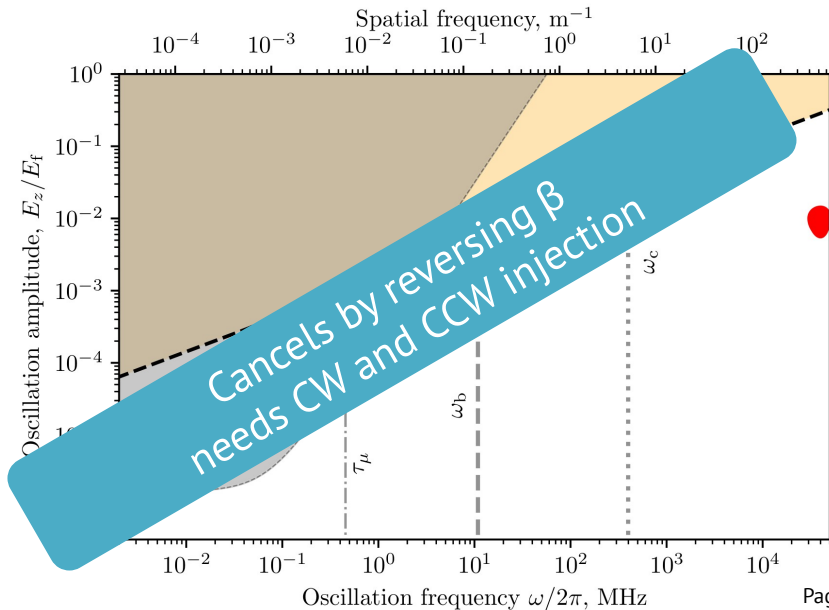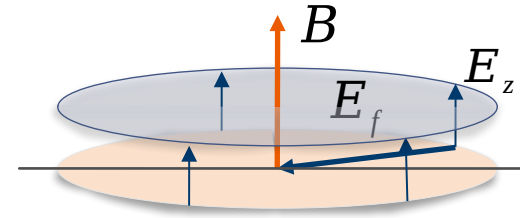
$$A = -\frac{1}{2\omega}\Omega_x \Omega_y t$$

Tight limit on the longitudinal E-field
(E-field component parallel to B-field)

Systematics related to the electric field

- E-field component parallel to the B-field will move orbit out of central plane until: $\langle B_r{}'\rangle = -\langle E_z/\beta\gamma\rangle$

- **Effect cancels if particles are injected alternatively CW and CCW and subtracting counts in the detectors.**

- CW and CCW orbit directions are done by switching the B-field direction.

$$\vec{\Omega} = -\frac{e}{m_0}\left[a\vec{B} + \left(\frac{1}{\gamma^2 - 1} - a\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

Cancels by reversing β needs CW and CCW injection

Kinematics of Michel decay positrons (rest frame)

- Angular distribution of the positrons from the decay

- For high positron energies – preferentially emitted in the direction of the muon spin.

muEDM kinematics
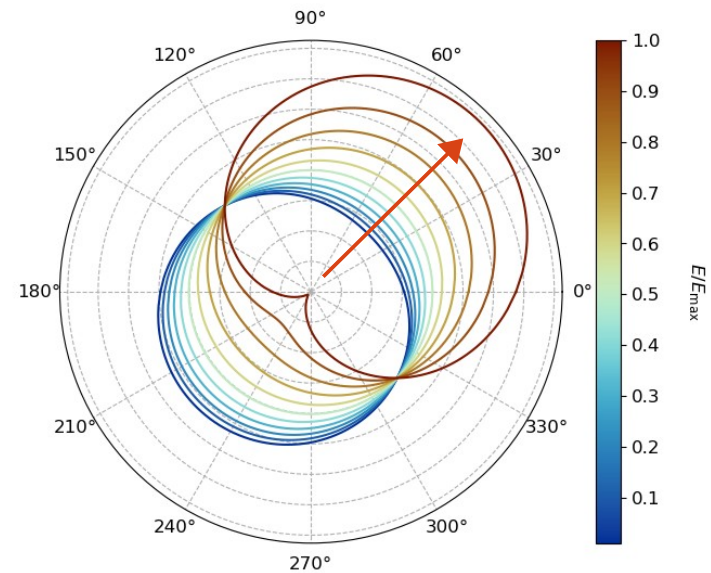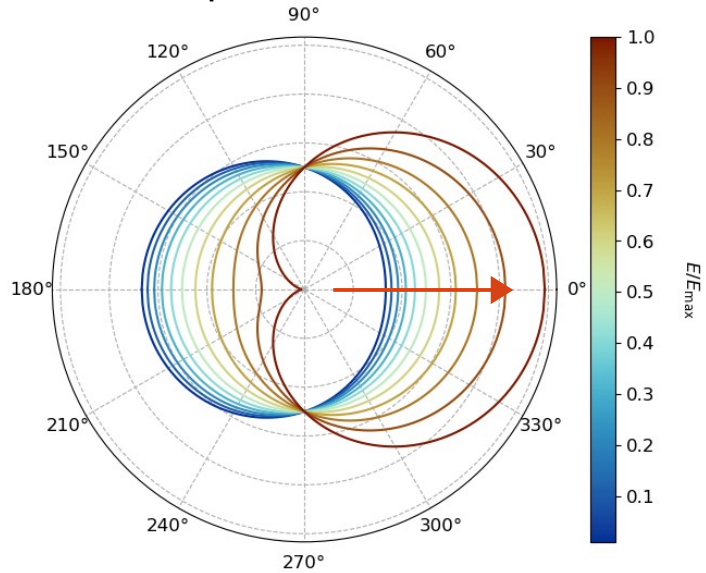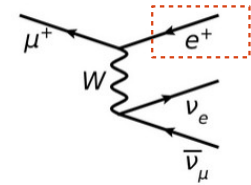
- The first stage – 28 MeV/*c* surface muons. Frozen spin – anti-parallel to the momentum.

- The distribution of decay positrons is Lorentz boosted along the muon momentum direction.

- Precession due to EDM can be measured from the direction of emitted positrons:

  - Asymmetry in emissions along or opposite the B-field.

- High-energy positrons (above 26 MeV) contribute the most to the asymmetry.

t = 0

t > 0

Boost

B-field

Boost
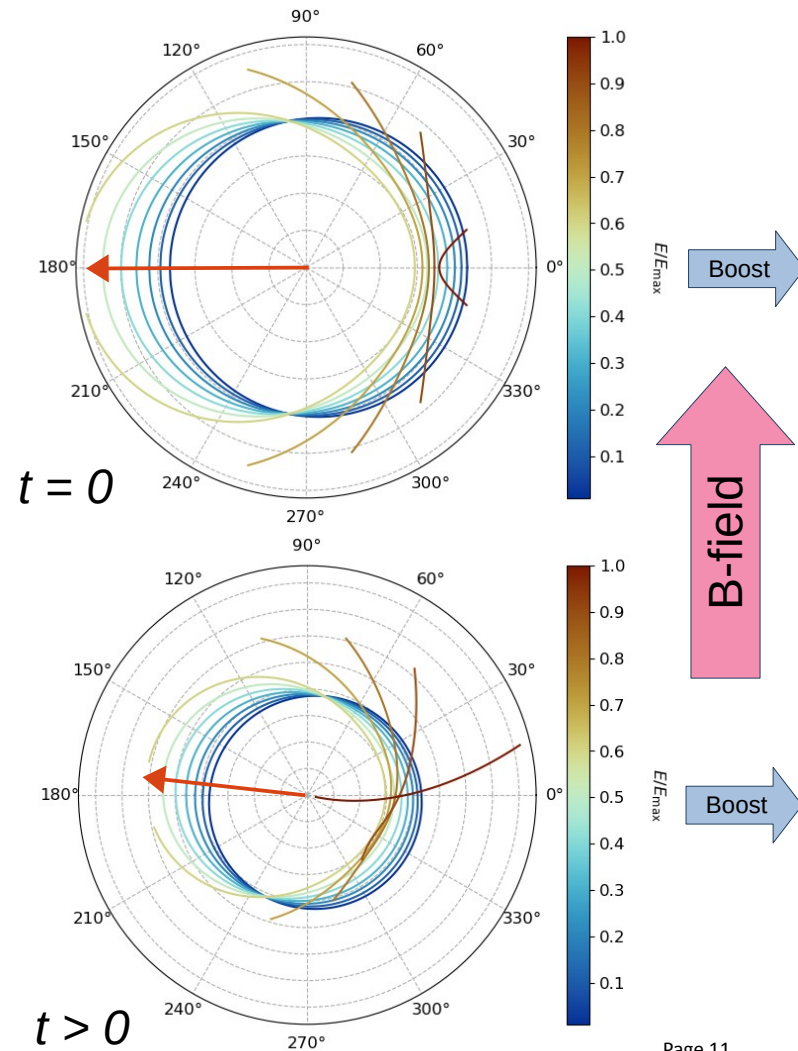
- Static differences in up and down detectors would not lead to a systematic effect.

- **Time-dependent changes in the overall detection efficiency of a set of detectors will be seen as a false EDM signal.**

  - Strong pulsed magnetic field → eddy currents, noise, heat in detectors and associated electronics.

- The studies of this and other apparent systematic effects is currently underway

EDM detectors

B-field

$N_u = \varepsilon_u N$

$N_d = \varepsilon_d N$

$\varepsilon = \varepsilon(t)$
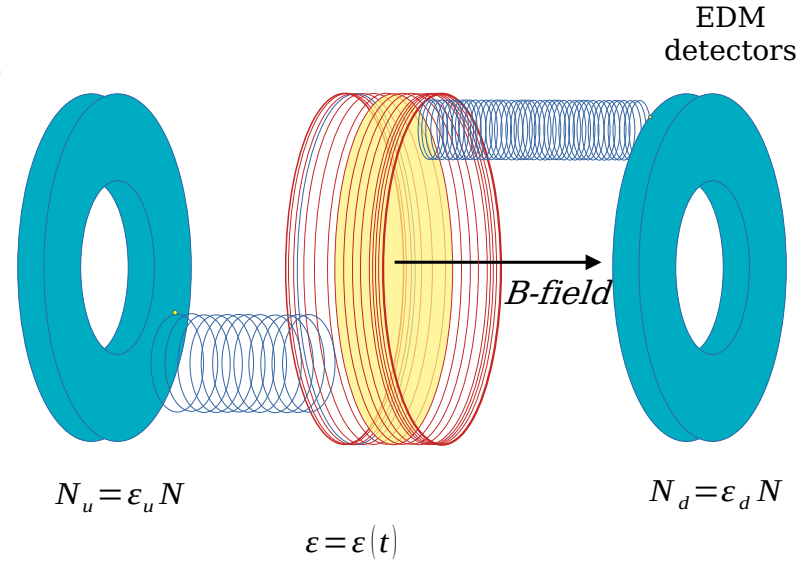
$$A = \frac{N_u - N_d}{N_u + N_d}$$

Conclusions

- Major sources of possible systematic effects in the experiment are:

- Non-zero longitudinal E-field component (i.e. parallel to B-field):

  - Effect cancels if we alternate between CW and CCW injections, determined by the polarity of the main solenoid.

- An analytical description of the geometric phases was developed

  - Places constraints on the EM-field temporal and spatial uniformity.

- Early-to-late variations in the detection efficiency of the EDM detectors:

  - Studies on the effect are underway

  - First steps completed: understanding the boosted decay kinematics
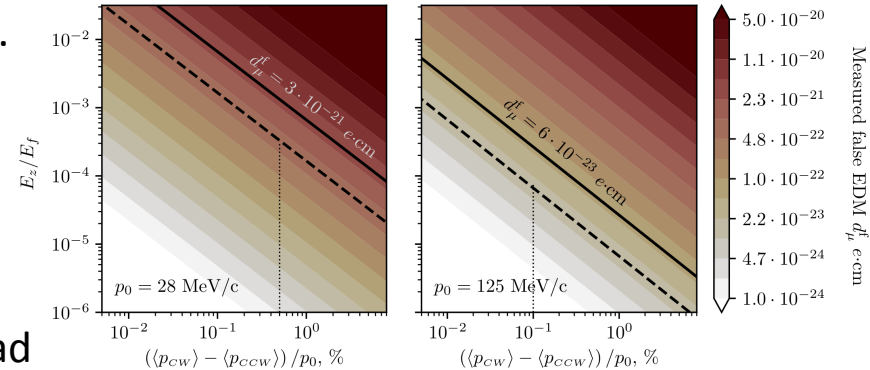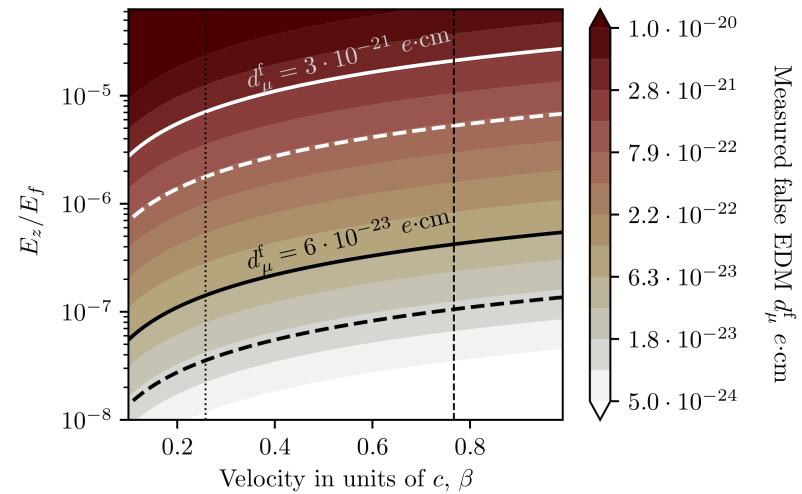
# Thank you for the attention!

Net longitudinal E-field

- Very stringent limit on the net longitudinal E-field.

- The effect changes sign with β.
  (change in the longitudinal B-field direction)

- This places limits on the average muon momentum for CW and CCW injections.

- The cancellation works only if:

  - the muon orbits are in the same place for CW/CCW

  - E-field non-uniformity does not lead to too different average E_z fields for CW/CCW injections.

$$\left\langle \left(\vec{\Omega}_{\mathrm{MDM}} \right)_\rho \right\rangle = -\frac{ea}{mc} \left\langle \left(1 - \frac{1}{a(\gamma^2 - 1)} - \frac{1}{\beta_\theta^2} \right) \beta_\theta \right\rangle \left\langle E_z \right\rangle$$

Artificial longitudinal E-field

- One can generate a longitudinal E-field with controllable intensity.

- Could be used to verify that we can observe EDM-like signal.

- Could be tuned to cancel the intrinsic average E-field of an imperfect electrode, e.g.:

  - tune field until no signal is seen for CW, then measure CCW.

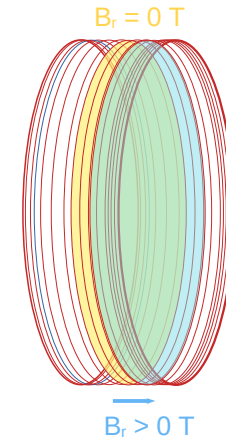 - would increase real signal to systematic effect ratio.
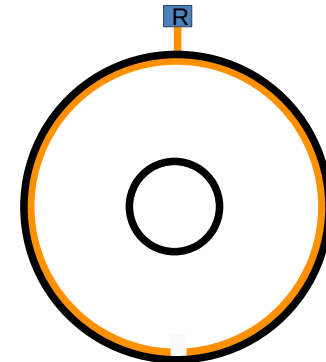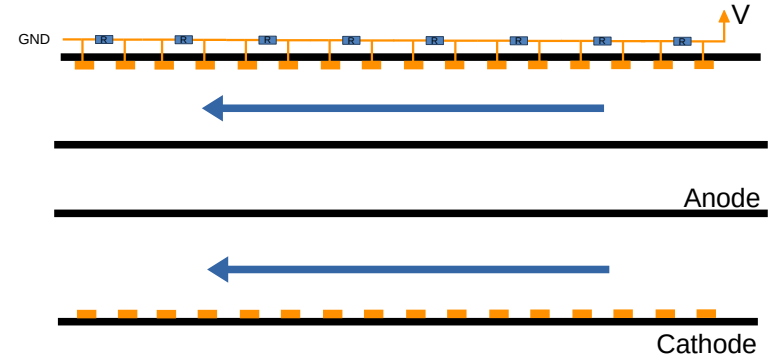
Limits on real spin precession effects

Systematic effect	Constraints	Phase I		Phase II	
		Expected value	Syst. ($\times 10^{-21} e{\cdot}$cm)	Expected value	Syst. ($\times 10^{-23} e{\cdot}$cm)
Cone shaped electrodes (longitudinal E-field)	Up-down asymmetry in the electrode shape	$\Delta_R < 30$ µm	0.75	$\Delta_R < 7$ µm	1.5
Electrode local smoothness (longitudinal E-field)	Local longitudinal electrode smoothness	$\delta_R < 3$ µm	0.75	$\delta_R < 0.7$ µm	1.5
Residual B-field from kick	Decay time of kicker field	< 50 ns	$< 10^{-2}$	< 50 ns	0.5
Net current flowing muon orbit area	Wiring of electronics inside the orbit	< 10 mA	$< 10^{-2}$	< 10 mA	0.3
Early-to-late detection efficiency change	Shielding and cooling of detectors	$-$		$-$	
Resonant geometrical phase accumulation	Misalignment of central axes	Pitch < 1 mrad Offset < 2 mm	2×10^{-2}	Pitch < 1 mrad Offset < 2 mm	0.15
TOTAL			1.1		2.2

*assuming electrode shape does not depend on magnetic field orientation

Time-variable radial B-field

$B_r = 0$ T

$B_r > 0$ T

- A static radial B-field would create an offset in the position of the equilibrium orbit.

  – Not a problem since we are interested in the change in the phase with time.*

- A time-variable B_r can come from the magnetic kick or eddy currents caused by it.

- Assuming an exponentially decaying residual B-field after the kicker pulse one can determine limits on the decay constant as a fucntion of the analysis start time.

- Radial B-fields of less than 5 μT would not induce a significant spin precession.

$$B_\rho(t) = k B_\rho^{(\text{peak})} e^{-t/\tau_B}$$

$d_\mu^f = 3 \cdot 10^{-21}\ e\cdot cm$

Analysis start time, μs

Time-constant τ_B, μs

Measured false EDM $d_\mu^f\ e\cdot cm$

$5.0 \cdot 10^{-20}$	
$1.3 \cdot 10^{-20}$	
$3.6 \cdot 10^{-21}$	
$9.7 \cdot 10^{-22}$	
$2.6 \cdot 10^{-22}$	
$6.9 \cdot 10^{-23}$	
$1.9 \cdot 10^{-23}$	
$5.0 \cdot 10^{-24}$	
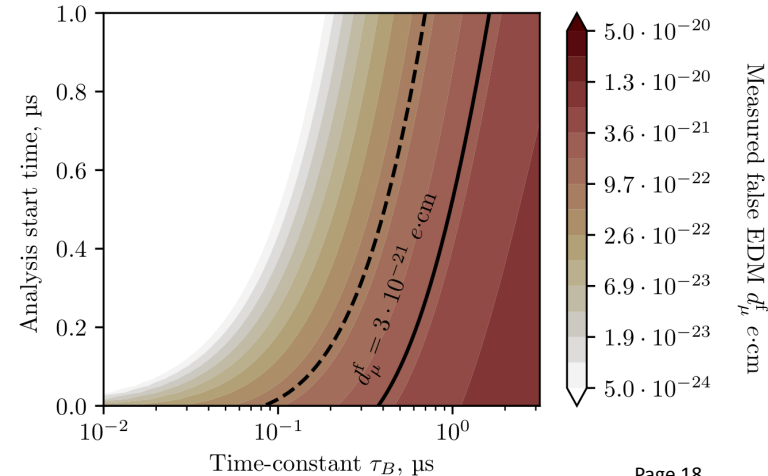
* may or may not induce Berry's phases

Sources of E_z field: electrode alignment

- The E field of an infinitely long coaxial cylinders is:

$$\vec{E}(\vec{r}) = \frac{V}{\log \frac{b}{a}} \begin{pmatrix} x/r^2 \\ y/r^2 \\ 0 \end{pmatrix}$$

-

Shifting the field by r_0 and rotating by α gives:

$$\vec{E}' = R_y(\alpha)\vec{E}(R_y^{-1}(\alpha)\vec{r} + \vec{r}_0) = V_0 \begin{pmatrix} \frac{\xi}{\rho^2}\cos\alpha \\ \frac{\upsilon}{\rho^2} \\ -\frac{\xi}{\rho^2}\sin\alpha \end{pmatrix}$$
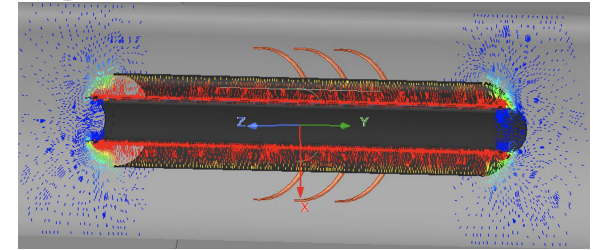
$$V_0 = \frac{V}{\log\frac{b}{a}}, \ \upsilon = y + y_0, \ \xi = x_0 + x\cos\alpha - z\sin\alpha \text{ and } \rho^2 = \xi^2 + \upsilon^2$$

Misalignment of the electric field

- Then average the new field out over a circular orbit:

$$\langle E'(\rho, \zeta)\rangle = \frac{1}{2\pi}\int_0^{2\pi} E' d\phi$$

- It can be shown (numerically for now) that:

$$\langle E'(\rho, \zeta)\rangle = \langle E(\rho, \zeta)\rangle$$
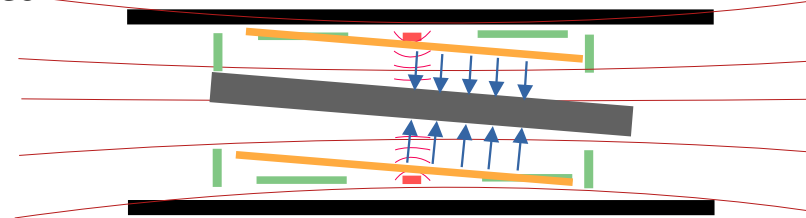
- For a circular orbit the misalignment of the anode or cathode cannot introduce a net horizontal E-field (that was not there before)

- It also does not affect the 'frozen spin' condition

Sources of E_z field: fringe fields

- The assumption for infinite coaxial cylinders holds if there are negligible fringe field in the region of interest

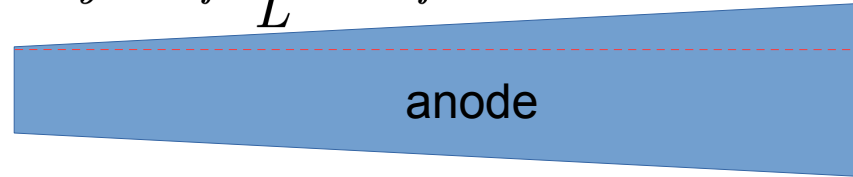- ANSYS Maxwell simulations show less than 0.1 ppm horizontal component in ±20 cm region around ideal orbit
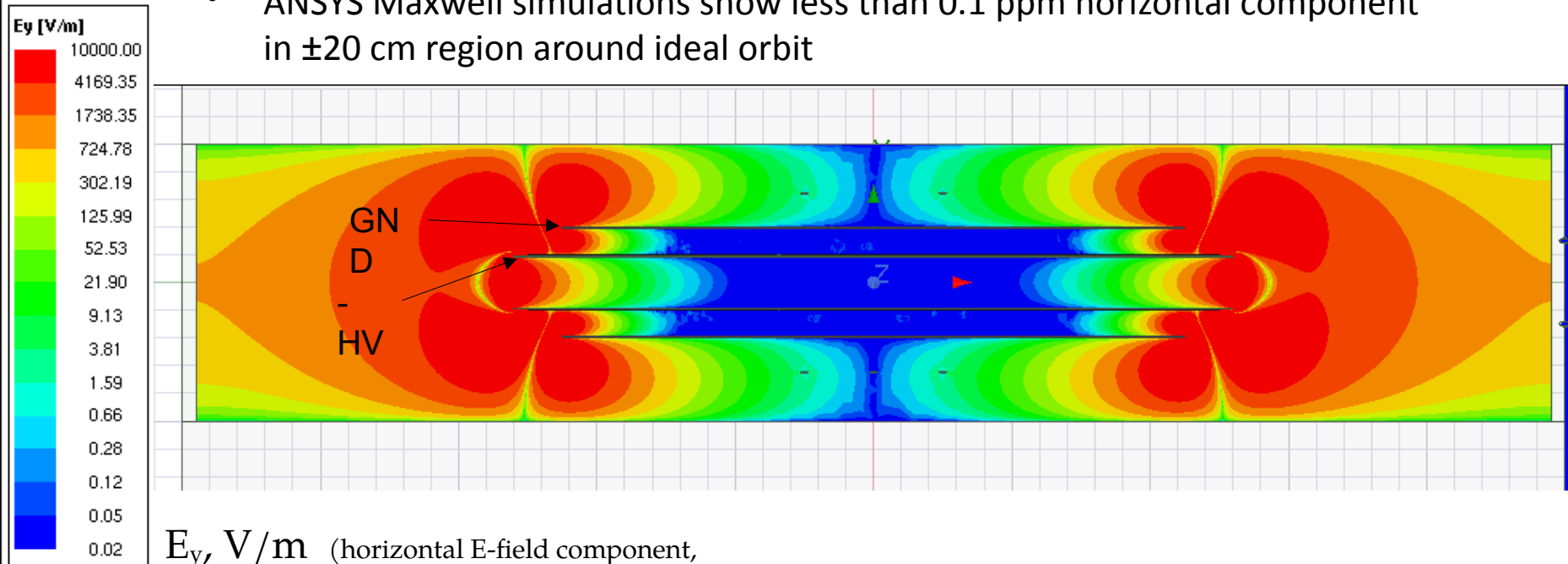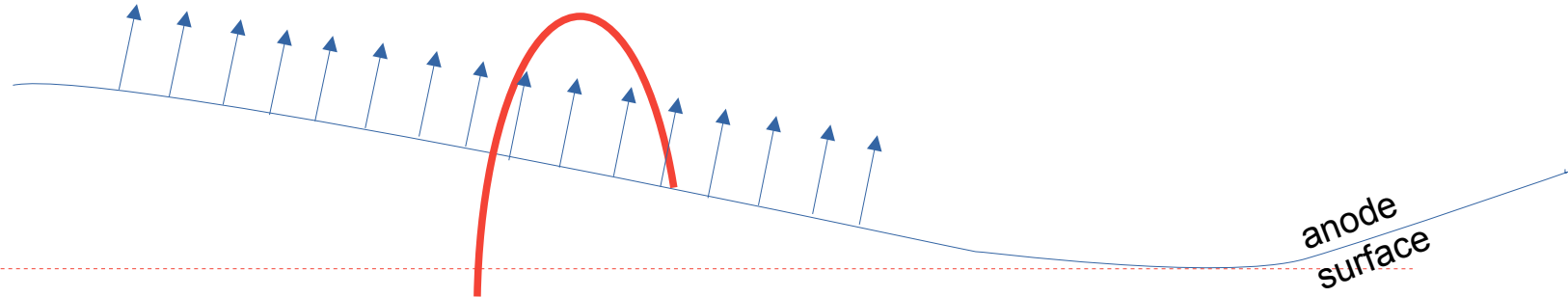

E_y, V/m (horizontal E-field component, logscale)

Sources of E_y field: electrode non-uniformity

- Tapered cone shaped electrodes

$$E_y \approx E_f \frac{\Delta_R}{L} \approx E_f \alpha$$

anode

- Smoothness of the electrodes close to the muon orbit (few centimeters)

anode surface

- Generally sub-micrometer surface smoothness is possible with common machining and polishing techniques

- Cylindricity in the order of 50 nm is measurable even on large samples

Requirements for electrode uniformity

- The condition to assume ideal uniformity is such that E_z is kept below the 0.5 ppm from E_f target

- If the radius of the anode or cathode is larger on one side than on the other then there will be a field component in the vertical direction

  - Using a small angle approximation:

    $$E_y \approx E_f \frac{\Delta_R}{L} \approx E_f \alpha$$

  - The radius of the anode or cathode should not change with more than Δ_R=150 nm (precursor) and Δ_R=300 nm (final) along its length (L ~ 50 cm)

Requirements for electrode uniformity

- Another source of a net vertical field would be a small protrusion on the surface of the electrode

- The E-field of a cylinder with radius R and surface charge density σ at a distance r is:
$$E_f(r) = \frac{\sigma R}{\varepsilon_0 r}$$

- The E-field from a small protrusion with area A at a distance Δ_z from the storage ring can be calculated from Coulomb's law:
$$E_y(\Delta_z) = \frac{1}{4\pi\varepsilon_0}\frac{\sigma A}{\Delta_z^2}$$

- Thus the maximum allowed area for a protrusion is:
$$A = 4\pi\frac{\Delta_z^2}{r}\frac{E_y}{E_f}$$

- Assuming a spherical cap like protrusion at 2 cm from the storage ring – maximum allowed height $h = 40\,\mu m$

Limit on the B-field parallel to the momentum

- Non-zero average B_z field if there is electric current flowing through the area enclosed by the muon orbit

- Write net current!

- From Biot-Savart's law we can give a limit on the systematics due to such current

- Assuming non-insulated wire at the center of the orbit:

  - Precursor: I < 250 mA

  - Final experiment: I < 40 mA

$$\langle \Theta_{\hat{z}} \rangle = -\frac{ea}{m_0} \langle B_z \rangle t$$
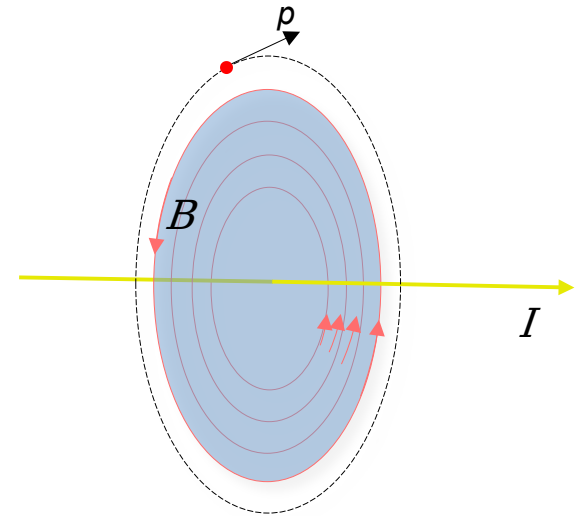
Limit on the radial B-field

- Limit on the kicker field decay time with relation to the injection angle

- Assumptions:

  - half-sine kicker field intensity

  - end of the kick is considered to be at the 10% from maximum livel

  - exponential decay of the ringing signal with time constant τ_B

  - the limit is such that the influence of the residual field is less than a given d_e at ~400 ns time

- *Note: the constraint is lower for later times and stronger for earlier times*
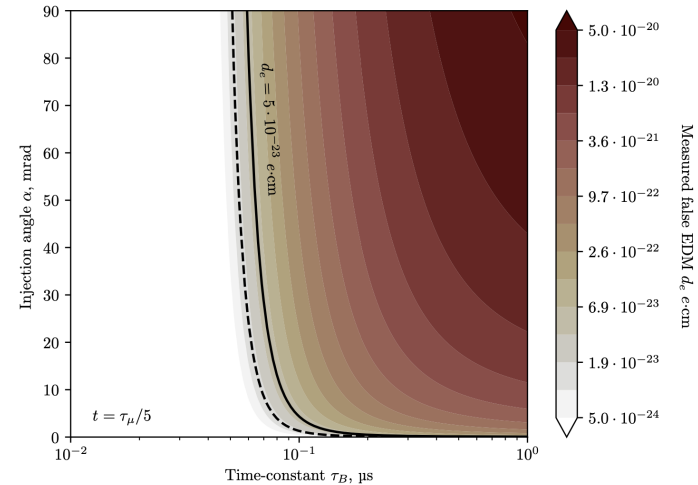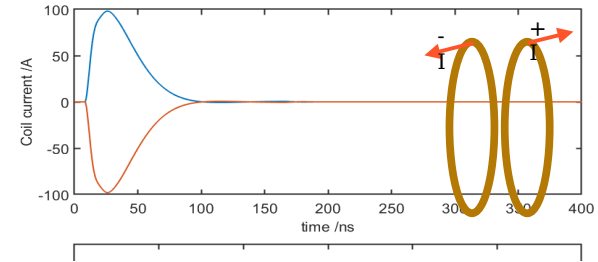
Limit on the radial B-field

- Simulated short current pulse for the two anti-Helmholtz coils

- The solid black line shows the limiting decay time for an exponentially decaying pulse that goes below the limit at 400 ns (overshoot or undershoot)

- The influence of the simulated kicker field to the observed spin precession is negligible after 200 ns
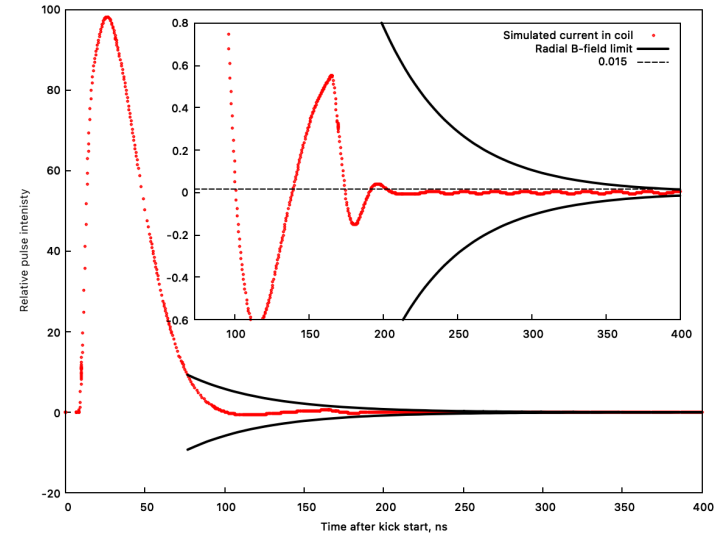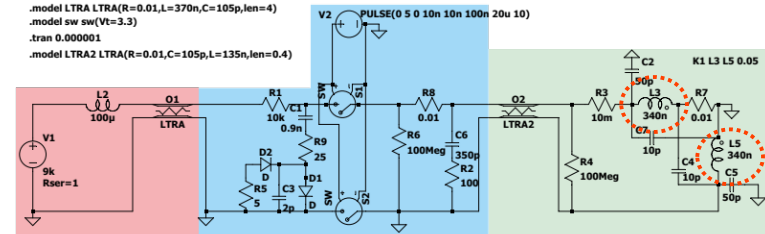
Non-circular muon orbit

- A non-zero average E_y field can be generated if the orbit of the muons is eliptical and at the same time it is not perpendicular to the axis of the anode.

  - the average field will be zero if the center of the orbit lies on the x axis

  - it is positive if it lies on the z axis above zero and negative if below zero

- In the general case the orbit will be eccentric due to the inward radial Lorentz force from the freeze field
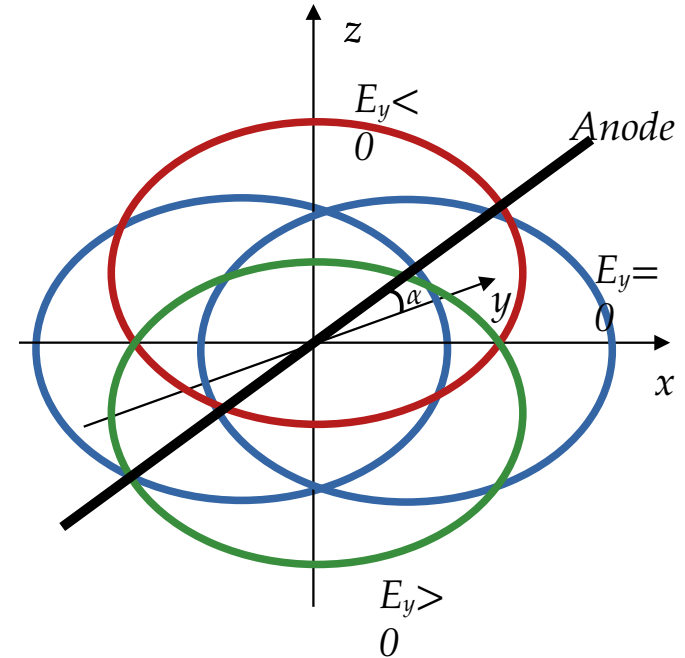
Non-circular muon orbit

- The effect was observed also in the Geant4 simulations and is consisten with the analytical estimate

- Calculations with the analytical equations show that for $\alpha = 0.1°$ and orbit displacements up to 5 mm the eccentricity of the orbit should be kept below 0.1

- The eccentricity caused by the freeze field is significantly lower and does not pose a problem

- This effect could constrain the magnetic field uniformity (analysis pending)
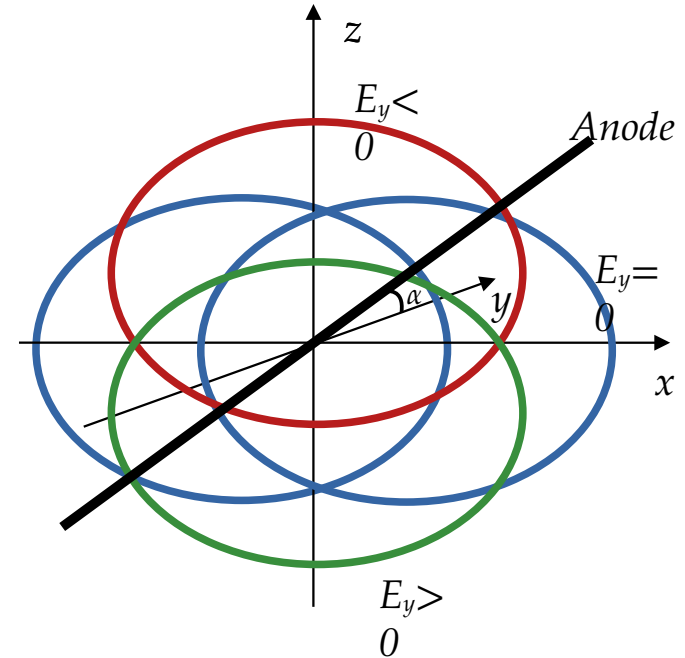
Comparison with G4

- Compared the analytical equations with a Geant4 simulation with the same parameters (weakly focusing coil current, radius; inital spin vector; etc...)

- In both attempts the frozen spin condition is not perfectly met (for illustration)

- Top: $E_y = 0$; Bottom: $E_y = E_{freeze}/10^6$

    - Note: bottom trend is similar to EDM of 10^{-21} e.cm
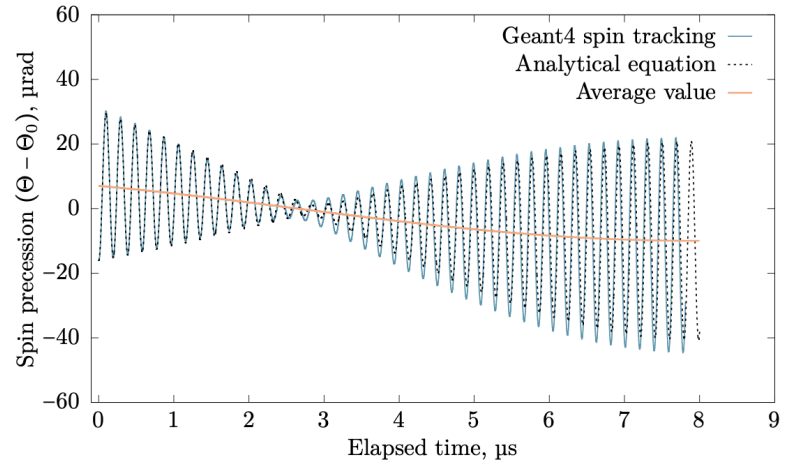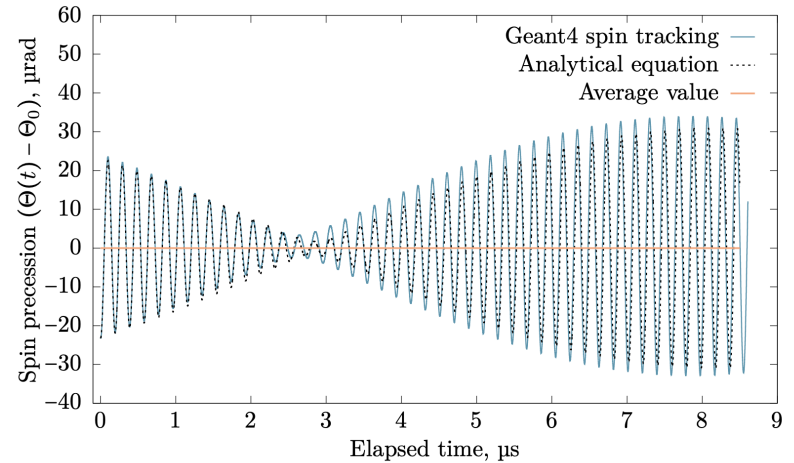
Comparison with G4

- Note that the oscillation frequency is not perfect as the fields are described by first order approximation

- Nevertheless, the equations describe the spin precession well in a very general scenario
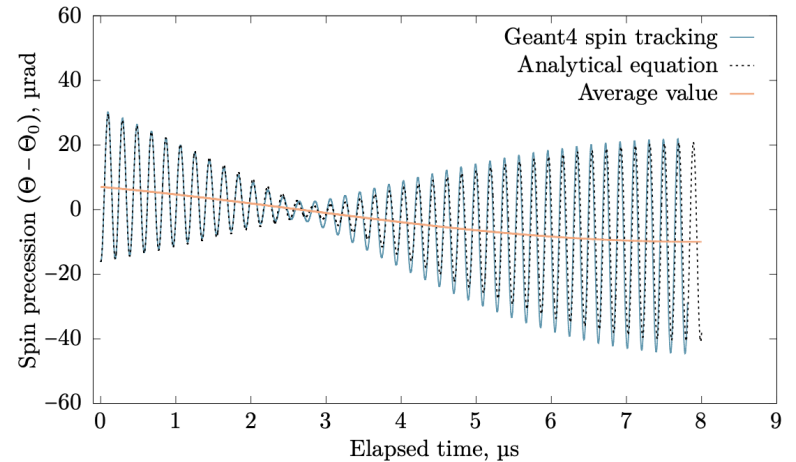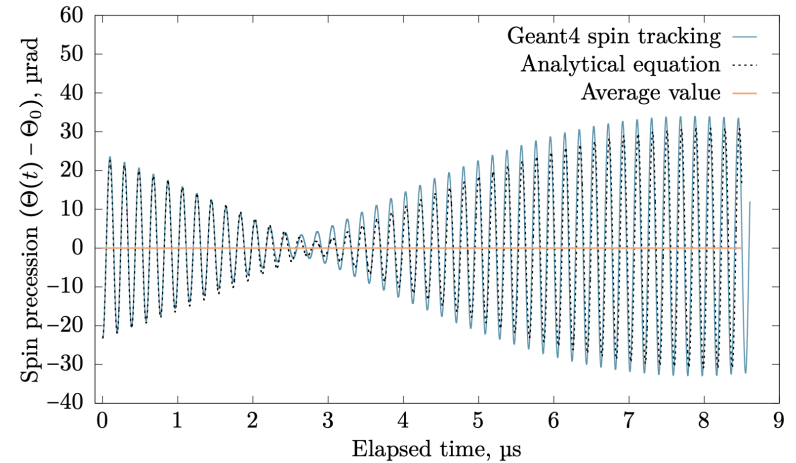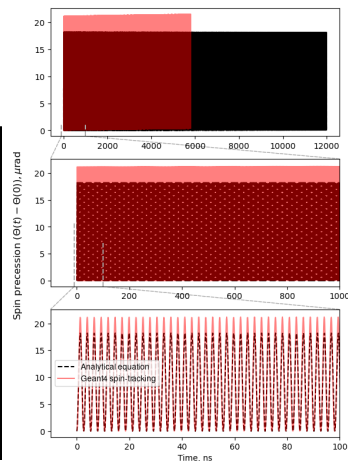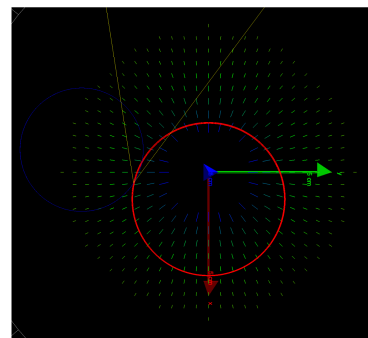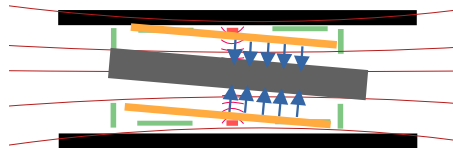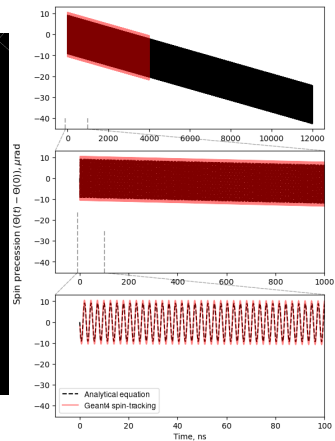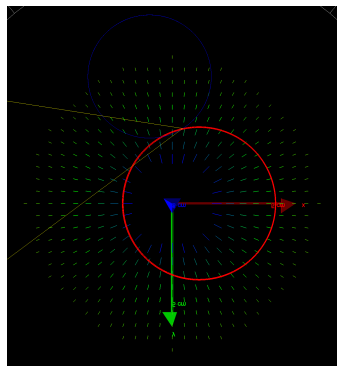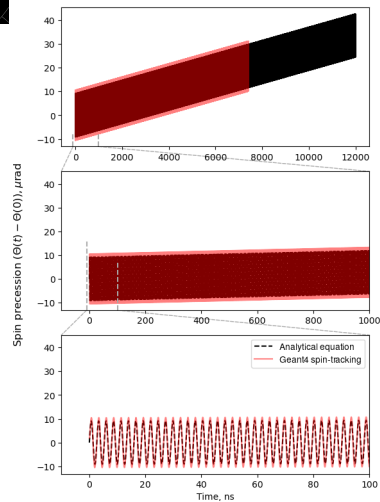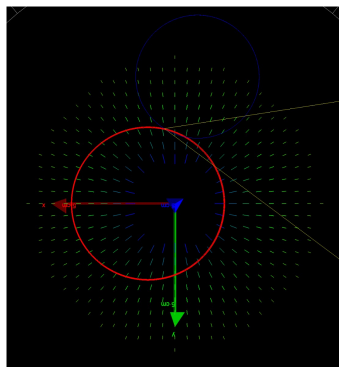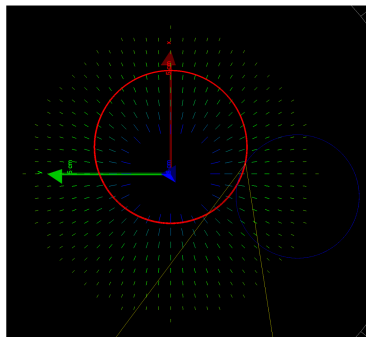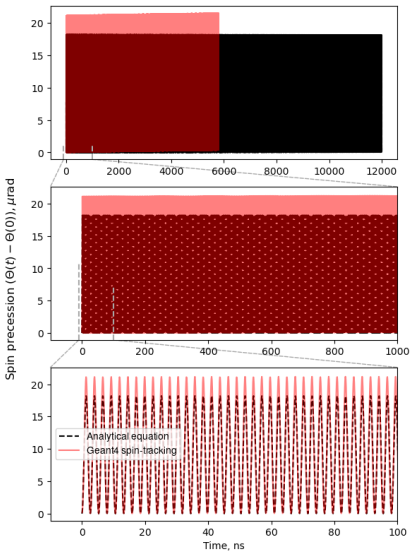
$$\alpha(t; \omega, \beta_0) = -\frac{1}{2\omega} \Omega_x \Omega_y t \sin(\beta_0)$$

Calculation of geometric phases

- Spin precesses around axis **x** with amplitude C_1 and frequency Ω_x, and around **y** with amplitude C_2 and frequency Ω_y. Phase difference between the two β_0.

$$x = C_1 \sin(\Omega_x t), \quad y = C_2 \sin(\Omega_y t + \beta_0)$$

- The movement of the spin encloses an area A on some abstract surface. The area can be calculated from Green's theorem:

$$A = \frac{1}{2} \int_{t_0}^{t_1} (xy' - yx') dt$$

- The geometric phase as a function time is then:

$$\alpha(t; \omega_x, \omega_y, \beta_0) = \frac{1}{2} \frac{\Omega_x \Omega_y}{\omega_x \omega_y} \int \left(\omega_y \cos(\omega_y t + \beta_0) \sin(\omega_x t) - \omega_x \cos(\omega_x t) \sin(\omega_y t + \beta_0) \right) dt =$$

$$= \frac{1}{4} \frac{\Omega_x \Omega_y}{\omega_x \omega_y} \left[\frac{\omega_x - \omega_y}{\omega_x + \omega_y} \cos((\omega_x + \omega_y)t + \beta_0) - \frac{\omega_x + \omega_y}{\omega_x - \omega_y} \cos((\omega_y - \omega_x)t + \beta_0) \right]$$

Calculation of geometric phases

- In the case where the two oscillations have the same frequency the geometric phase is:

$$\alpha(t; \omega, \beta_0) = \frac{1}{2} \frac{\Omega_x \Omega_y}{\omega^2} \int (\omega \cos(\omega t + \beta_0) \sin(\omega t) - \omega \cos(\omega t) \sin(\omega t + \beta_0)) \, dt = -\frac{1}{2\omega} \Omega_x \Omega_y t \sin(\beta_0)$$
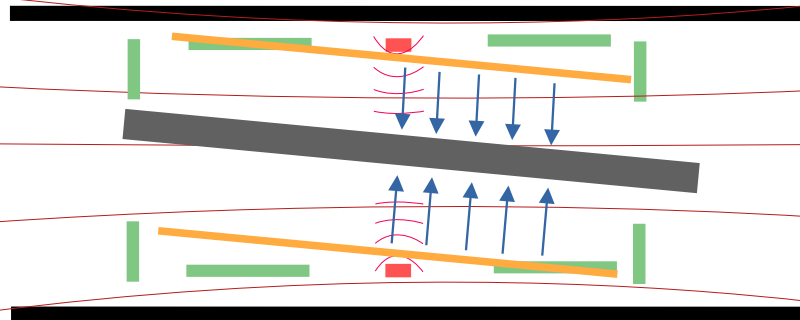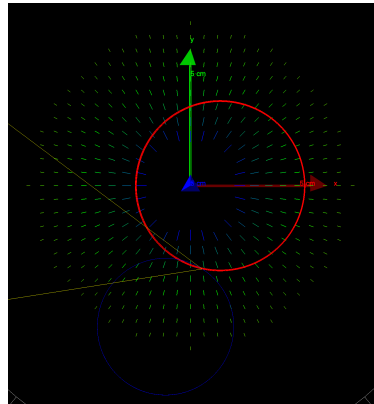
- The motion of the spin in this case is an ellipse with eccentricity defined by the phase difference between oscillations

  - no phase difference: ellipse looks like a line – no geometric phase

  - **π/2 phase difference: ellipse is a circle and maximum area**
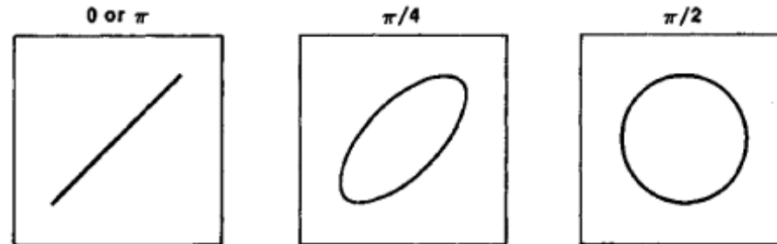
Example of a geometric phase accumulation

$$\alpha(t; \omega, \beta_0) = -\frac{1}{2\omega}\Omega_x\Omega_y t \sin(\beta_0)$$

- Spin precession due to misalignment of E-field:

  - longitudinal oscillations due to stronger and weaker freeze field *(cyclotron frequency)*

  - radial oscillations due to longitudinal E-field oscillating between upstream and downstream directions *(cyclotron frequency)*

Tilted and shifted E-field with respect to the center of the muon orbit