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Doktoratskolleg Particles and Interactions

Vector Glueballs in Holographic QCD

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Outline

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Introduction

- Nonabelian nature of QCD allows for bound states of gauge bosons: Glueballs
- Supported by lattice gauge theory for various quantum numbers J^{PC}
- Mixing with $q\bar{q}$ states makes identification in experiments difficult
- Extraction of glueball couplings, decay rates and mixing from first principle calculations difficult

AdS/CFT correspondence opens up new possibilities to study various processes involving glueballs in an almost parameter free manner

Outline

1. Introduction
2. Holographic principle
3. The Witten-Sakai-Sugimoto model
4. Results
5. Conclusion and outlook

Holographic principle

Different forms of the AdS/CFT correspondence

	4d $\mathcal{N}=4$ Super Yang-Mills (SYM)	IIB String Theory on $AdS_5 \times S^5$
Strongest form	any N and λ	Quantum string theory, $g_s \neq 0$, $\alpha'/L^2 \neq 0$
Strong form	$N \to \infty$, λ fixed but arbitray	Classical string theory, $g_s \to 0$, $\alpha'/L^2 \neq 0$
Weak form	$\boldsymbol{N \to \infty, \ \lambda}$ **large**	**Classical supergravity,** $\boldsymbol{g_s \to 0, \ \alpha'/L^2 \to 0}$

Holographic QCD

Generalization to non-conformal and non-supersymmetric case

Holographic principle

Holographic dictionary

Gauge theory	**Gravity theory**
Degree N of gauge group	Number of branes/curvature radius
Energy scale	Radial coordinate
Renormalization group flow	Movement along radial coordinate
Gauge theory in flat space time	Boundary of gravitational theory
Global symmetry	Gauge symmetry
Particle mass	Eigenvalue of wave equation
Gauge invariant operators	Fields sourcing these operators

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Witten background

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)

Type IIA string theory with large number N_c of $D4$ branes dual to $4+1$ dimensional SYM. Compactification on $\tau = \tau + 2\pi/M_{\mathrm{KK}}$ with antiperiodic boundary conditions for adjoint fermions breaks SUSY.

$\Rightarrow$ dual to large N_c pure-glue $3+1$d YM theory at scales $\ll M_{\mathrm{KK}}$

Near horizon (large N_c) geometry in 10d string frame

$$\mathrm{d}s^2 = \left(\frac{U}{R_{D4}}\right)^{3/2} \left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \boldsymbol{f(U)}\mathrm{d}\tau^2\right) + \left(\frac{R_{D4}}{U}\right)^{3/2} \left(\frac{\mathrm{d}U^2}{\boldsymbol{f(U)}} + U^2 \mathrm{d}\Omega_4^2\right)$$

$$\boldsymbol{f(U) = 1 - \left(\frac{U_{\mathrm{KK}}}{U}\right)^3}, \quad \boldsymbol{e^{\phi} = g_s \left(\frac{U}{R_{D4}}\right)^{3/4}}, \quad F_4 = \mathrm{d}C_3 = \frac{2\pi N_c}{V_4}\epsilon_4$$

Equations of motion

Bosonic closed string action

$$S_{IIA}^{closed} = S_{NS} + S_R + S_{CS}$$

$$S_{NS} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4\nabla_M \phi \nabla^M \phi - \frac{1}{2}|H_3|^2 \right)$$

$$S_R = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g} \left(-\frac{1}{2}|F_2|^2 - \frac{1}{2}|\tilde{F}_4|^2 \right)$$

$$S_{CS} = -\frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \frac{1}{2} B_2 \wedge F_4 \wedge F_4$$

$$F_2 = dC_1, \quad F_4 = dC_3, \quad \tilde{F}_4 = F_4 - C_1 \wedge H_3, \quad H_3 = dB_2$$

Glueball spectrum

$$C_{\mu\nu\tau} = \frac{z}{g_s} M_4(z)\tilde{C}_{\mu\nu}(x^\mu), \;\; B_{\mu z} = -M_4(z)\eta_{\mu\kappa}\epsilon^{\kappa\nu\rho\sigma}\partial_\nu \tilde{C}_{\rho\sigma}(x^\mu), \;\; \tilde{C}_{\mu\nu} = \frac{1}{\sqrt{\Box}}\epsilon_{\mu\nu\rho\sigma}\partial^\rho V^\sigma(x^\mu)$$

$$(1+z^2)M_4''(z) + (1/z + 3z)M_4'(z) + (-3 - 1/z^2 + M^2 M_{\rm KK}^2/(1+z^2)^{1/3})M_4(z) = 0$$

$$S \supset \int d^4x \epsilon^{\mu\nu\rho\sigma} C_{\mu\nu\tau} \text{Sym Tr}(F^{\rho\sigma} W), \;\; W = F^{2n} \implies 1^{--}$$

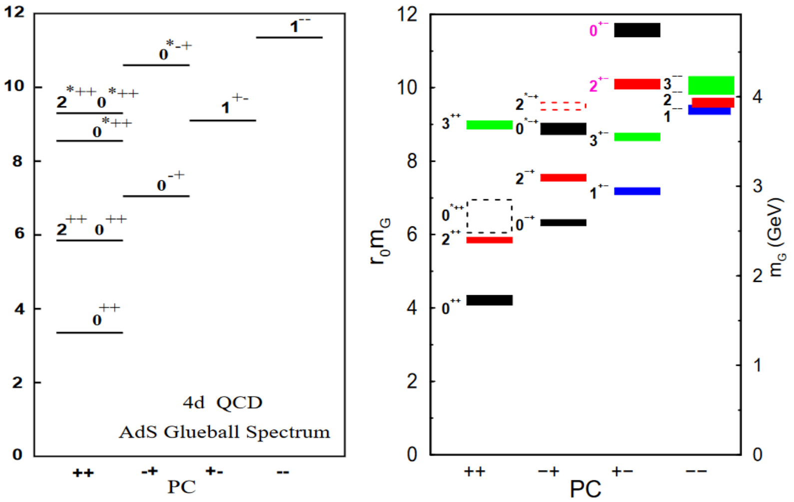
G_{MN}				A_{MNO}		
G_{mn}	$G_{m,11}$	$G_{11,11}$	$\frac{M}{M_{\rm KK}}$	$A_{mn,11}$	A_{mno}	$\frac{M}{M_{\rm KK}}$
$G_{\mu\nu}$ 2^{++}	C_μ $1^{++}_{(-)}$	ϕ 0^{++}	1.567	$B_{\mu\nu}$ 1^{+-}	$C_{\mu\nu\rho}$ $0^{+-}_{(-)}$	2.435
$G_{\mu\tau}$ $1^{-+}_{(-)}$	C_τ 0^{-+}		1.886	$B_{\mu\tau}$ $1^{--}_{(-)}$	$C_{\mu\nu\tau}$ 1^{--}	3.037
$G_{\tau\tau}$ 0^{++}			0.901	$G^{\alpha}_{\ \alpha}$ 0^{++}		3.575

Glueball spectrum

G_{MN}				A_{MNO}		
G_{mn}	$G_{m,11}$	$G_{11,11}$	$\frac{M}{M_{\text{KK}}}$	$A_{mn,11}$	A_{mno}	$\frac{M}{M_{\text{KK}}}$
$G_{\mu\nu}$ 2^{++}	C_{μ} $1^{++}_{(-)}$	ϕ 0^{++}	1.567	$B_{\mu\nu}$ 1^{+-}	$C_{\mu\nu\rho}$ $0^{+-}_{(-)}$	2.435
$G_{\mu\tau}$ $1^{-+}_{(-)}$	C_{τ} 0^{-+}		1.886	$B_{\mu\tau}$ $1^{--}_{(-)}$	$C_{\mu\nu\tau}$ 1^{--}	3.037
$G_{\tau\tau}$ 0^{++}			0.901	$G^{\alpha}_{\ \alpha}$ 0^{++}		3.575

Glueball spectrum

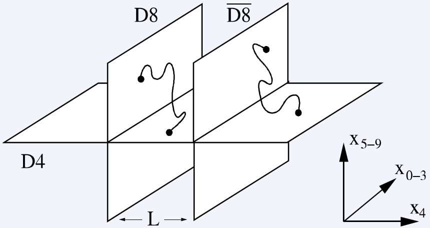
R. Brower, S. Mathur, C. Tan, Nucl. Phys. B 587 (2000)

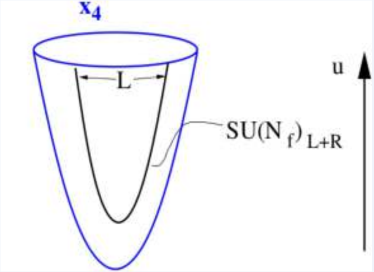


T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

		0	1	2	3	4	5	6	7	8	9
N_c	$D4$	x	x	x	x	x					
N_f	$D8 - \overline{D8}$	x	x	x	x		x	x	x	x	x

Chiral quarks and symmetry breaking





Sakai-Sugimoto model: Adding flavor

D8 brane action in the Witten background...

$$S_{D8} = -T_8 \int_{D8} \mathrm{d}^9 x e^{-\phi} \mathrm{STr}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN} + B_{MN})} + T_8 \sum_p \int_{D8} C_p \wedge \mathrm{Tr}\left[\exp\left\{2\pi\alpha' F_2 + B_2\right\}\right]$$

$$S_{DBI} \supset \kappa \int \mathrm{d}^4 x \mathrm{d}z \mathrm{Tr}\left[\frac{1}{2} K^{-1/3} F_{\mu\nu}^2 + K M_{\mathrm{KK}}^2 F_{\mu z}^2\right], \; \kappa = \frac{\lambda N_c}{216\pi^3}, \; K(z) = 1 + z^2 = \frac{U^3}{U_{\mathrm{KK}}^3}$$

...gives an effective action

$$A_\mu(x^\mu, z) = \textstyle\sum_{n=1}^{\infty} A_\mu^n(x^\mu)\psi_n(z), \; A_z(x^\mu, z) = \pi(x^\mu)\frac{K^{-1}}{\sqrt{\kappa\pi} M_{\mathrm{KK}}}, -K^{-1/3}\partial_z(K\psi_n')) = \lambda_n \psi_n$$

$$S_{DBI} = -\int \mathrm{d}^4 x \left[\frac{1}{2}(\partial_\mu \pi)^2 + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}\lambda_1 M_{\mathrm{KK}}^2 \rho_\mu^2 + \dots\right]$$

Comparison to experiment

Matching...

$$m_\rho \approx 776\mathrm{MeV} \to M_{\mathsf{KK}} \approx 949\mathrm{MeV}$$

$$f_\pi \approx \frac{\lambda N_c}{54\pi^4} M_{\mathsf{KK}}^2 \to \lambda = g_{YM}^2 N_c \approx 16.63$$

(or matching instead $m_\rho/\sqrt{\sigma}$ to large N_c lattice result $\to \lambda \approx 12.55$)

...yields for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$

- $m_{a_1}^2/m_\rho^2 \approx 2.4$ (2.5)
- $m_{\eta'} \approx 967 \ldots 730$ MeV (958 MeV)
- $\Gamma_{\rho\to 2\pi}/m_\rho = 0.1535 \ldots 0.2034$ (0.191(1))
- $\Gamma_{\omega\to 3\pi}/m_\omega = 0.0033 \ldots 0.0102$ (0.0097(1))

Vector meson dominance

T. Sakai, S. Suimoto, Prog.Theor.Phys. 114 1083 (2005)

$$A_{L\mu}(x^\mu) = A_{R_\mu}(x^\mu) = eQA^{em}_\mu(x^\mu), \quad Q = \frac{1}{3}\mathrm{diag}(2,-1,-1)$$

$$\mathcal{V}_\mu(x^\mu) = \frac{1}{2}\left(A_{L\mu}(x^\mu) + A_{R\mu}(x^\mu)\right), \quad v^n_\mu(x^\mu) = B^{(2n-1)}_\mu(x^\mu), \quad a^n_\mu(x^\mu) = B^{(2n)}_\mu(x^\mu)$$

$$\boldsymbol{A_\mu(x^\mu, z) = \mathcal{V}_\mu(x^\mu) + \sum_{n=1}^{\infty} v^n_\mu(x^\mu)\psi_{2n-1}(z) + \sum_{n=1}^{\infty} a^n_\mu(x^\mu)\psi_{2n}(z)}$$

$$\frac{\kappa}{2}\int \mathrm{d}z\, K^{-1/3}F^2_{\mu\nu} = a_{\mathcal{V}v^1}\mathrm{Tr}\left(\left(\partial_\mu\mathcal{V}_\nu - \partial_\nu\mathcal{V}_\mu\right)\left(\partial_\mu v^1_\nu - \partial_\nu v^1_\mu\right)\right) + \ldots$$

with $a_{\mathcal{V}v^1} = \kappa\int \mathrm{d}z\, K^{-1/3}\psi_1 = 0.0385\sqrt{N_c\lambda}$

Outline

Glueball-meson mixing

Mixing with vector mesons and mass correction (probe)

$$S_{DBI} = -T_8 \mathrm{Tr} \int \mathrm{d}^9 x e^{-\phi} \sqrt{-g_{MN} + 2\pi\alpha' F_{MN} + B_{MN}} \supset -\mathrm{Tr} \int \mathrm{d}^4 x \xi_1 \eta^{\mu\nu} v_\mu V_\nu + \frac{1}{2} \delta M_V^2 \eta^{\mu\nu} V_\mu V_\nu,$$

$$\xi_1 = -0.0125 M_{KK}^2 \frac{\lambda}{\sqrt{N_c}}, \qquad \delta M_V^2 = 7 \cdot 10^{-5} M_{KK}^2 \frac{\lambda^2}{N_c}$$

$\rho\pi$ puzzle

Suppressed $\psi(2S) \to \rho\pi$ decay can be explained if J/ψ mixes with vector glueball, provided $|\theta| < 2^\circ$.

$$|\theta| = (0.33 \ldots 0.25)^\circ, \quad |\theta^{ex}| = (0.32 \ldots 0.24)^\circ$$

Including backreacted mass corrections

- For vector mesons $\delta m = -(48.5 \ldots 36.9)$MeV
- Glueball mass almost unchanged.

Decay rates (preliminary)

	$\Gamma_{G_{PV}}(M_G = 2311\text{MeV})[\text{MeV}]$	$\Gamma_{G_V}(M_G = 2882\text{MeV})[\text{MeV}]$
$G \to a_1\rho$	$206\ldots273$	$295\ldots390$
$G \to \rho\pi$	$585\ldots775$	$137\ldots182$
$G \to K^*K$	$259\ldots338$	$151\ldots200$
$G \to \eta\omega$	$83.2\ldots141$	$32.1\ldots39.2$
$G \to \pi\rho\rho$	$465\ldots817$	$335\ldots589$
$G \to \pi K^*K^*$	$24.9\ldots43.8$	$90.8\ldots160$
$G \to \pi\rho\gamma$	$0.97\ldots1.28$	$0.45\ldots0.78$
$G \to KK^*\gamma$	$0.25\ldots0.33$	$0.24\ldots0.42$
$G \to a_1\gamma$	0.03	0.18
$G \to \pi^0\gamma$	$0.01 \cdot 10^{-3}$	5.37

The Vector Glueball and the Odderon

- C=-1 glueballs contained in form fields of the bulk theory: $C_1,\ B_2,\ C_3$

$$1^{+-}: B_{\mu\nu},\ C_{\mu\tau z},\ m \approx 2.44 M_{\mathrm{KK}}$$
$$1^{--}: B_{\mu z},\ C_{\mu\nu\tau},\ m \approx 3.04 M_{\mathrm{KK}}$$

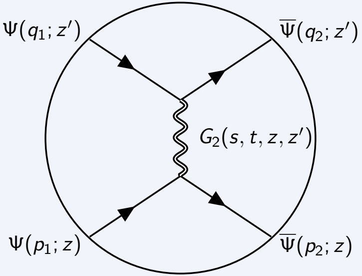
- WSS construction incapable of capturing full Regge-behaviour. But can be a strong guiding principle for constructing a viable bottom-up model (soft-wall) to study TOTEM and DØ data (in preparation, w/ I. Zahed).
- $J = 1$ glueballs only have anomalous couplings to leading order in α'

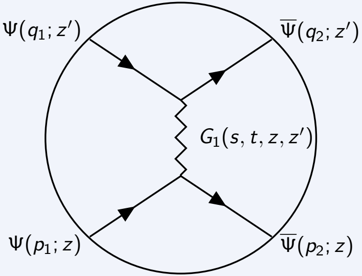
$$S_{CS}^{D8} = T_8 \sum_p \int_{D8} \sqrt{\hat{\mathcal{A}}(\mathcal{R})} \mathrm{Tr} \exp\left(2\pi\alpha' F + B\right) \wedge C_p$$
$$\supset T_8 \int_{D8} \frac{(2\pi\alpha')^2}{2!} \mathrm{Tr} F \wedge F \wedge B_2 \wedge C_3 + \frac{(2\pi\alpha')^2}{2!} \mathrm{Tr} F \wedge F \wedge C_5$$

$\Longrightarrow$ Couples to baryon density

Holographic Odderon at TOTEM?

$$ds^2 = (R/z)^2(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \qquad \phi = (2\kappa)^2 z^2$$





$$G_{j_\pm}(s, t, z, z') = \oint \frac{dj}{4\pi i} \frac{(\alpha' s)^{j-j_\pm} \pm (-\alpha' s)^{j-j_\pm}}{\sin \pi (j - j_\pm)} (\alpha' z z')^{j-j_\pm}\, G_0(j, t, z, z')$$

$$j_+ = 2, \qquad j_- = 1$$

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Conclusion and outlook

Conclusion

- Resulting decay rates suggest a relatively narrow 1^{--} but broad 1^{+-}
- Mixing between J/ψ meson and vector glueball small, as suggested for a possible resolution of the long-standing $\rho\pi$-puzzle.

Outlook

Various applications to study glueball physics relevant for experimental studies

- Top-down construction can be used to estimate low energy couplings for models that capture the correct Regge behaviour $\Longrightarrow$ Pomeron and Odderon physics.
- Low-x physics (DIS, DVCS...) and photoproduction through $\mathbb{P}/\mathbb{O}$ exchange

For questions, comments, suggestions: florian.hechenberger@tuwien.ac.at

Backup Slides

Going beyond the probe limit (preliminary)

Backreaction of $D8$ branes

- Treatment in terms of massive type IIA SUGRA ($M_R \sim N_f/N_c$)
- Smeared approximation to preserve isometries ($\Leftrightarrow$ truncate at lowest τ KK-mode)
- Glueball-meson mixing (beyond $\sqrt{N_f/N_c}$)

Selected results

- (Some) Mass ratios improved and degeneracies lifted:

$$\frac{m^2_{\rho(1450)}}{m^2_\rho} = 3.575\ (3.573), \qquad \frac{m^2_{2^{++}}}{m^2_{0^{++}}} = 1.12\ (1.46)$$

- ρ mass eigenvalue slightly corrected downward $\rightarrow$ refit M_{KK} to compensate!

Vector Glueball Lagrangians

$$\mathcal{L}_{G_V \Pi v} = -\frac{1}{M_V} g_1^m \mathrm{Tr}\left(\Pi \partial_\mu v_\nu^{(m)} + v_\mu^{(m)} \partial_\nu \Pi\right) \star F_{\mu\nu}^V \quad g_1^m = \frac{\{15.04, 12.13, 7.88\}}{\sqrt{\lambda} N_c}.$$

$$\mathcal{L}_{G_V \to va} = \frac{1}{M_V} f_{1/2}^{mn} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left(v_\mu^m \partial_\nu a_\rho^n \pm a_\mu^n \partial_\nu v_\rho^m\right) V_\sigma$$

$$f_1^{mn} = \frac{\{177.83, 58.91, 51.79\} M_{\mathrm{KK}}}{N_c \sqrt{\lambda}}, \qquad f_2^{mn} = \frac{\{16.60, 24.58, 37.79\} M_{\mathrm{KK}}}{N_c \sqrt{\lambda}}$$

$$\mathcal{L}_{G_V \to \Pi vv} = \frac{i}{M_V} g_1^{mn} \mathrm{Tr}\left(\Pi \left[v_\mu^{(m)}, v_\nu^{(n)}\right]\right) \star F_{\mu\nu}^V, \quad g_1^{mm} = \frac{\{1061, 618, 451\}}{\lambda N_c^{3/2}}$$

Interaction Lagrangians

Vector Glueball Lagrangians

$$\mathcal{L}_{G_V \Pi \mathcal{V}} = \frac{1}{M_V} g_1^{\mathcal{V}} \mathrm{Tr}\left(\Pi \partial_\mu \mathcal{V}_\nu + \mathcal{V}_\mu \partial_\nu \Pi\right) \star F_{\mu\nu}^V, \quad g_1^{\mathcal{V}} = \frac{0.31}{\sqrt{N_c}}.$$

$$\mathcal{L}_{G_V a \mathcal{V}} = \frac{1}{M_V} f_{1/2}^{\mathcal{V}n} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left(\mathcal{V}_\mu \partial_\nu a_\rho^n \pm a_\mu^n \partial_\nu \mathcal{V}_\rho\right) V_\sigma$$

$$f_1^{\mathcal{V}n} = \frac{\{5.53, 2.81, 0.27\} M_{\mathrm{KK}}}{\sqrt{N_c}}, \qquad f_2^{\mathcal{V}n} = \frac{\{0.72, 0.92, 0.53\} M_{\mathrm{KK}}}{\sqrt{N_c}},$$

$$\mathcal{L}_{G_V \to \Pi v \mathcal{V}} = \frac{i}{M_V} g_1^{m\mathcal{V}} 2\mathrm{Tr}\left(\Pi \left[\mathcal{V}_\mu, v_\nu^{(m)}\right]\right) \star F_{\mu\nu}^V, \quad g_1^{m\mathcal{V}} = \frac{\{22.6, 18.2, 11.8\}}{\sqrt{\lambda} N_c}.$$

Interaction Lagrangians

Pseudovector Glueball Lagrangians

$$\mathcal{L}_{G_{PV}\to\Pi v} = -\frac{1}{M_{PV}} b_1^m \mathrm{Tr}\left(v_\mu^{(m)}\partial_\nu\Pi + \Pi\partial_\mu v_\nu^{(m)}\right) F_{\mu\nu}^{\tilde{V}}, \quad b_1^m = \frac{\{93.4, 49.3, 18.7\}}{\sqrt{\lambda}N_c}$$

$$\mathcal{L}_{G_{PV}\to va} = -\frac{1}{M_{PV}} b_3^{mn} \mathrm{Tr}\left(v_\mu^{(m)} a_\nu^{(m)}\right) F_{\mu\nu}^{\tilde{V}}, \quad b_3^{mm} = \frac{\{98.9, 164.4, 237.6\} M_{KK}}{\sqrt{\lambda}N_c}$$

$$\mathcal{L}_{G_{PV}\Pi vv} = \frac{i}{M_{PV}} b_2 \mathrm{Tr}\left(\Pi\left[v_\mu, v_\nu\right]\right) F_{\mu\nu}^{\tilde{V}}, \quad b_2 = \frac{\{6048, 3188, 2763\}}{\lambda N_c^{3/2}}$$

$$\mathcal{L}_{G_{PV}\to\Pi\mathcal{V}} = -\frac{1}{M_{PV}} b_1^{\mathcal{V}} \mathrm{Tr}\left(\mathcal{V}_\mu\partial_\nu\Pi + \Pi\partial_\mu\mathcal{V}_\nu\right) F_{\mu\nu}^{\tilde{V}}, \quad b_1^{\mathcal{V}} = \frac{2.25}{\sqrt{N_c}}$$

$$\mathcal{L}_{G_{PV}\Pi v\mathcal{V}} = 2i b_2^{\mathcal{V}} \mathrm{Tr}\left(\Pi\left[v_\mu, \mathcal{V}_\nu\right]\right) F_{\mu\nu}^{\tilde{V}}, \quad b_2^{\mathcal{V}} = \frac{\{140, 73.9, 28.0\}}{\sqrt{\lambda}N_c}$$

$$\mathcal{L}_{G_{PV}\to\mathcal{V}a} = -\frac{1}{M_{PV}} b_3^{m\mathcal{V}} \mathrm{Tr}\left(\mathcal{V}_\mu a_\nu^{(m)}\right) F_{\mu\nu}^{\tilde{V}}, \quad b_3^{m\mathcal{V}} = \frac{\{1.46, 2.0, 2.87\} M_{KK}}{\sqrt{N_c}}$$