Optimization of muon EDM experimental setup using simulations

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MuEDM Experiment at PSI

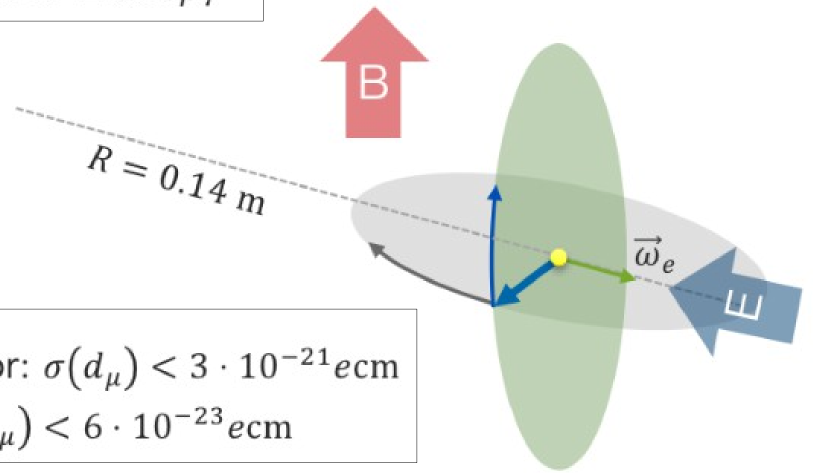
- PSI Muon EDM measurement using Frozen-Spin Technique
- Phase I demonstrates Frozen-Spin technique
- Injecting muons with right experimental design parameters essential for ensuring optimum storage
- Longitudinal asymmetry in positrons decaying from stored muons
  → signal for muon EDM

This talk → simulation studies to optimize initial parameters affecting injection efficiency

$$\vec{\Omega} = \frac{q}{m}\left[a\vec{B} - \left(a + \frac{1}{1-\gamma^2}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] + \frac{\eta q}{2m}\left[\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right]$$

g-2 term EDM term

set $E \cong aBc\beta\gamma^2$

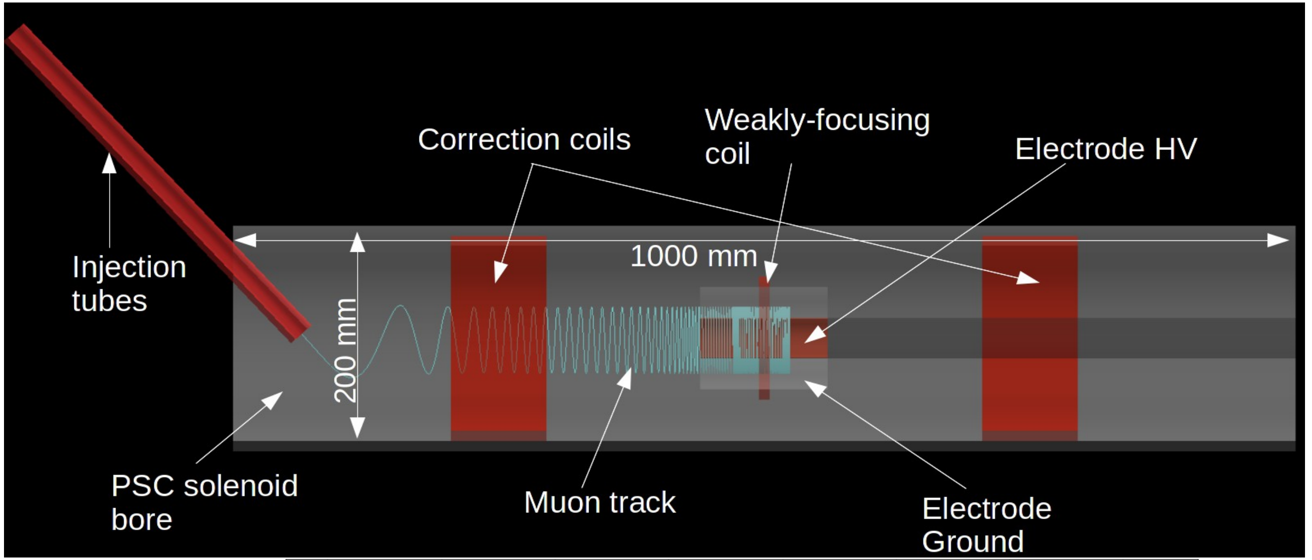


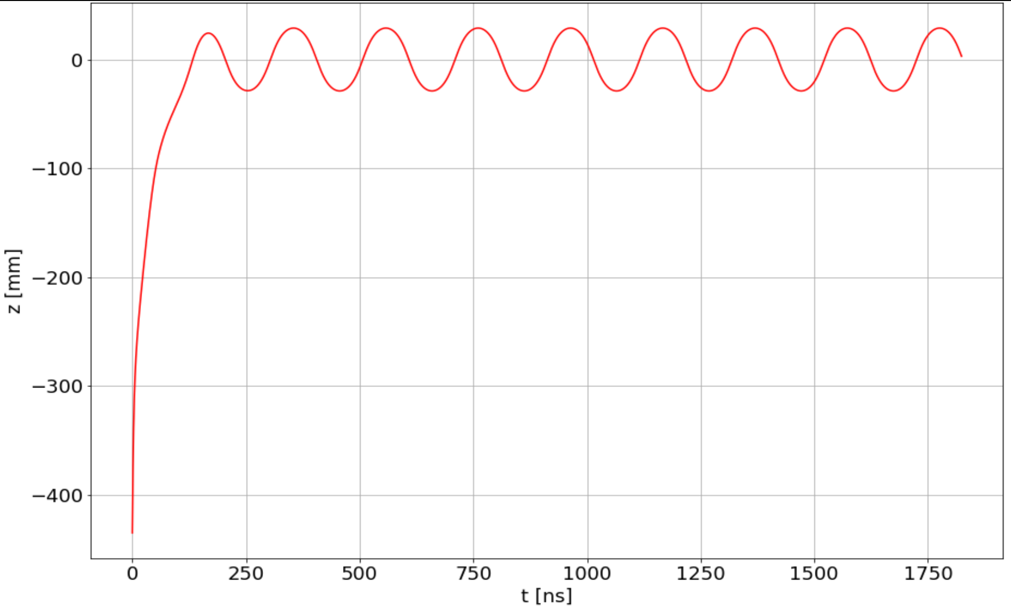
Prescursor: $\sigma(d_\mu) < 3 \cdot 10^{-21} ecm$

Final: $\sigma(d_\mu) < 6 \cdot 10^{-23} ecm$

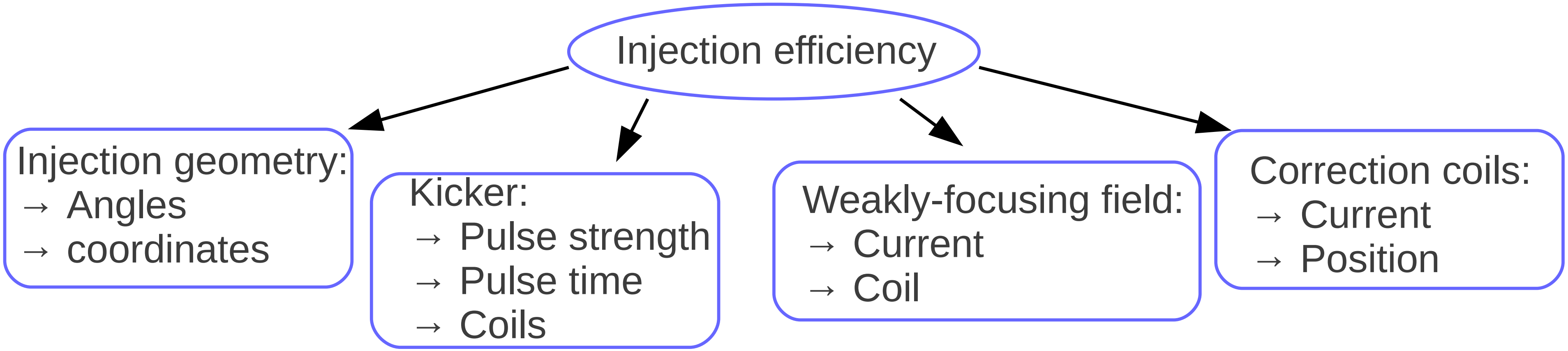
MuEDM Phase I in G4Beamline

- Muons are injected off-axis into 3 T solenoid
- Field gradient at injection corrected by correction coils
- Radial magnetic pulse generated by pair of anti-helmholtz coil
- Muons trapped in weakly-focusing magnetic field at the center generated by thin coils





Multivariate Optimization



- Multivariate optimization → computationally expensive
- G4Beamline simulation run for one configuration of input variables on HPC cluster takes ~6 hrs (for 1M muons)
- High fidelity simulations impossible to do given time constraint

PCE Based Surrogate Modelling

- Replace complex model with approximation → Surrogate Model
- Polynomial Chaos Expansion (PCE) :

$$Y = \sum_{i=0}^{\infty} \alpha_i \Psi_i(\vec{x})$$

Y → Model response (injection efficiency), $\boldsymbol{\Psi_i}$ → Orthogonal polynomials
x → input variables, $\boldsymbol{\alpha_i}$ → expansion coefficients

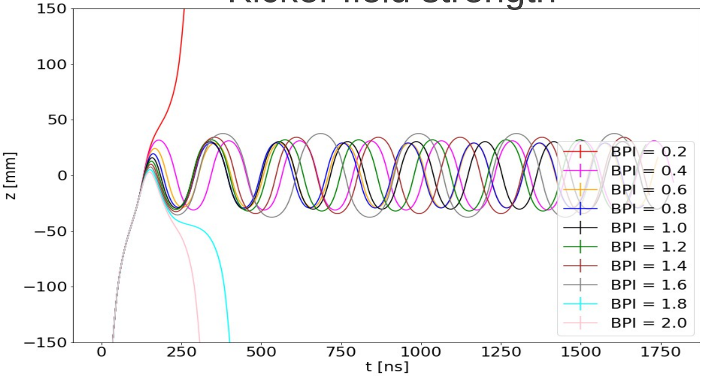
- Polynomial basis based on input variable distribution
- Coefficients determined using regression based methods

$$\vec{\alpha} = \text{Argmin} \frac{1}{N} \sum_{j=1}^{N} \left\{ f(\vec{\xi}^j) - \sum_{i=0}^{P-1} \alpha_i \Psi_i(\vec{x}^j) \right\}^2$$

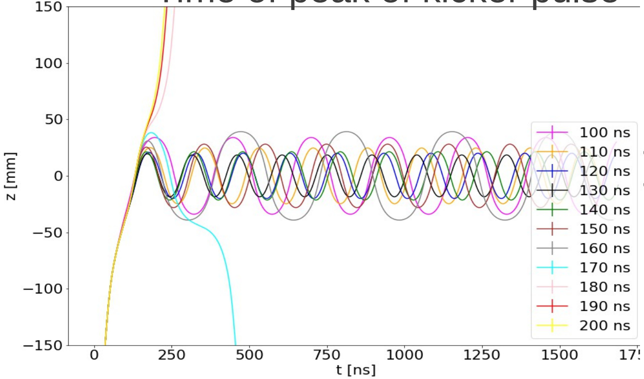
Initial Parameter Bounds

- Preliminary optimization with 8 parameters with range determined using rough scans in G4Beamline with the reference particle

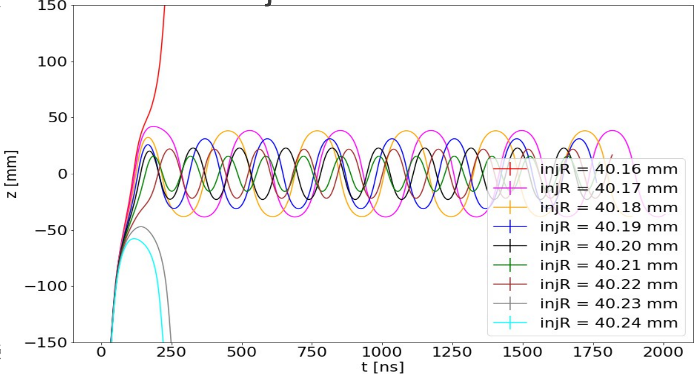
Kicker field strength



Time of peak of kicker pulse



Injection radius

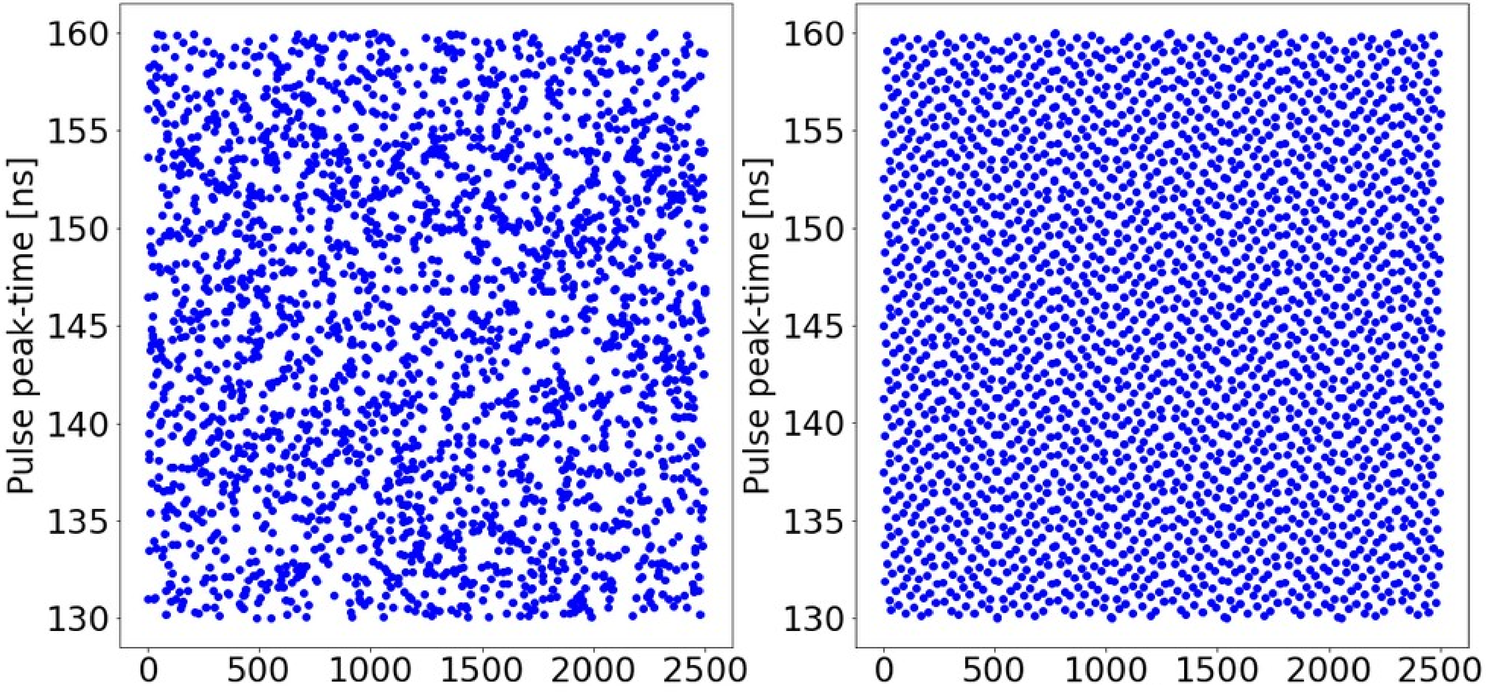


- Best guess input parameter range:

Parameter symbol	Physics Parameter	Lower bound	Upper bound
Theta	Injection angle (degrees)	47.38	47.45
Phi	Transverse angle (degrees)	5.4	5.9
InjR	Injection radius (mm)	40.0	40.4
Z	Longtudinal Injection coordinate (mm)	-437	-433
BPI	Kicker Field Strength (arb. unit)	-1.4	-0.2
KPT	Time of peak of Kicker Pulse (ns)	130	160
W	Width of the Kicker Pulse (ns)	15	55
WC	Weak Current*100 (A)	100	700

Initial Distribution: Sobol Points

- Monte Carlo techniques rely on random distribution of samples
- Prone to clusters and empty spaces, slow convergence, probabilistic error bounds



Random distribution | Sobol distribution

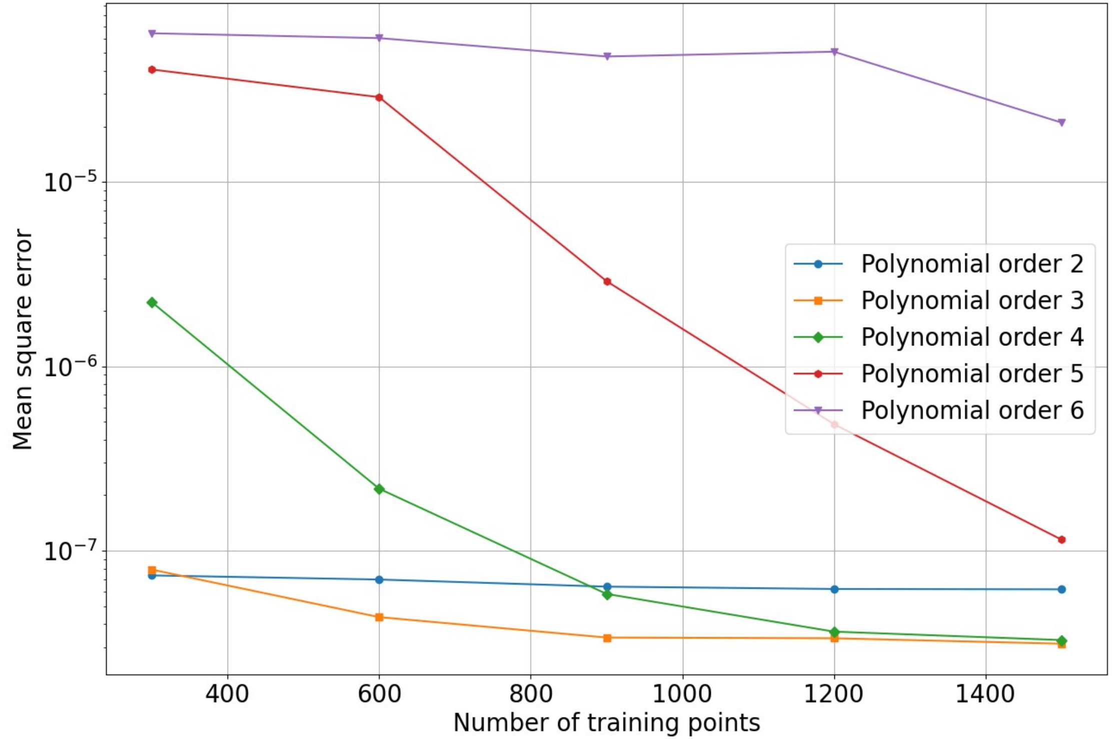
- Low discrepancy distributions like Sobol sequence preferred, deterministic error bounds, uniformly spans the given range

PCE Regression Analysis

- Run G4bl simulations with parameter sample space given by Sobol distribution
- For regression based estimation of coefficients, number of training samples is **N=(d-1)P**;
- P → terms in polynomial expansion: $P = \frac{(p+d)!}{p!d!}$, d → dimension, p → polynomial degree
- Storage criteria:

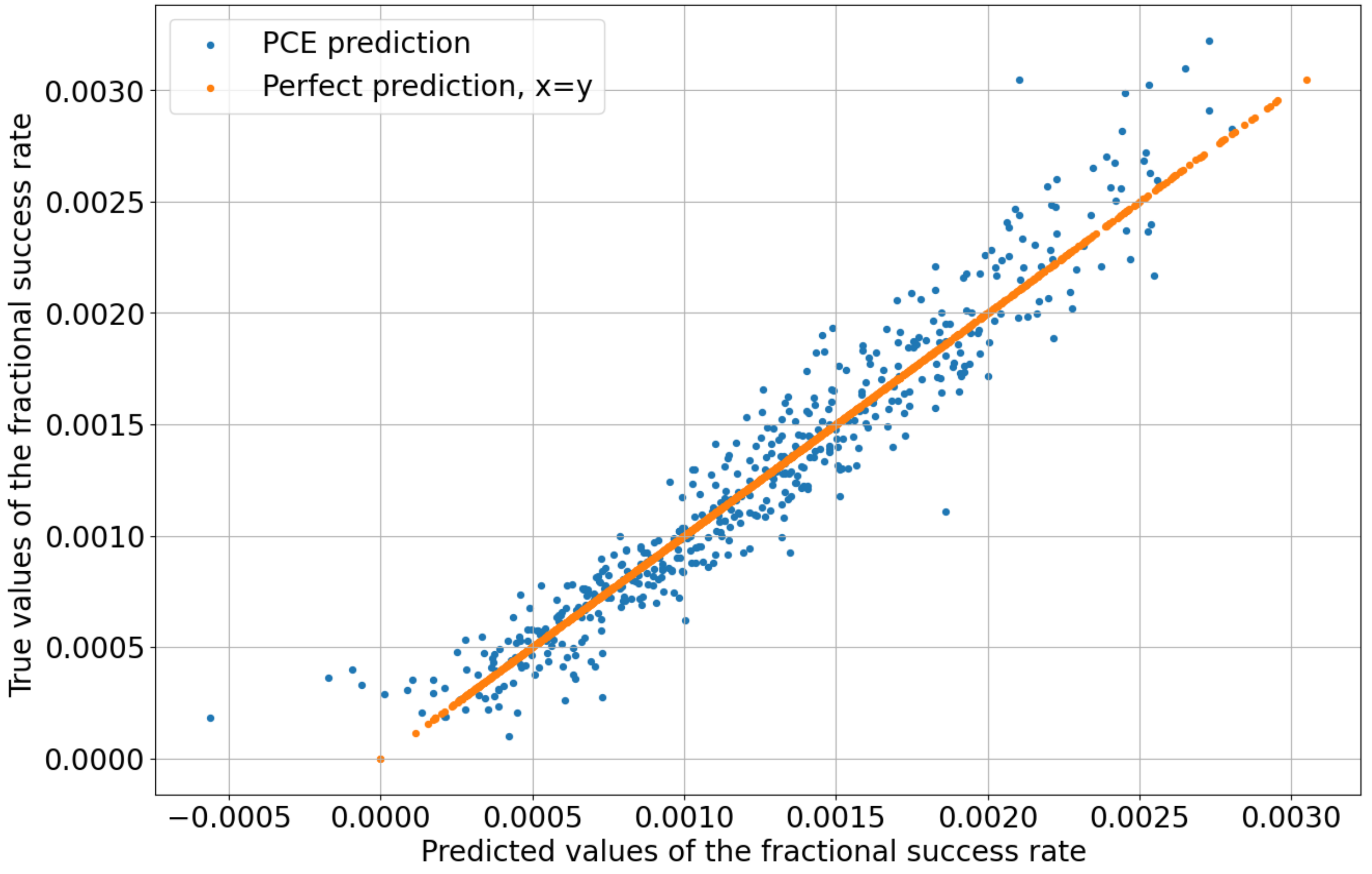
  muon decaying/killed
  with **-40mm<z<40mm**
  and **-0.5MeV<Pz<0.5MeV**

- Python toolbox chaospy for generating expansion and regression fit
- Lowest MSE for poly order 3, poly order 4 converging fast
- ~1100 points needed for poly order 3, ~3500 points needed for poly order 4

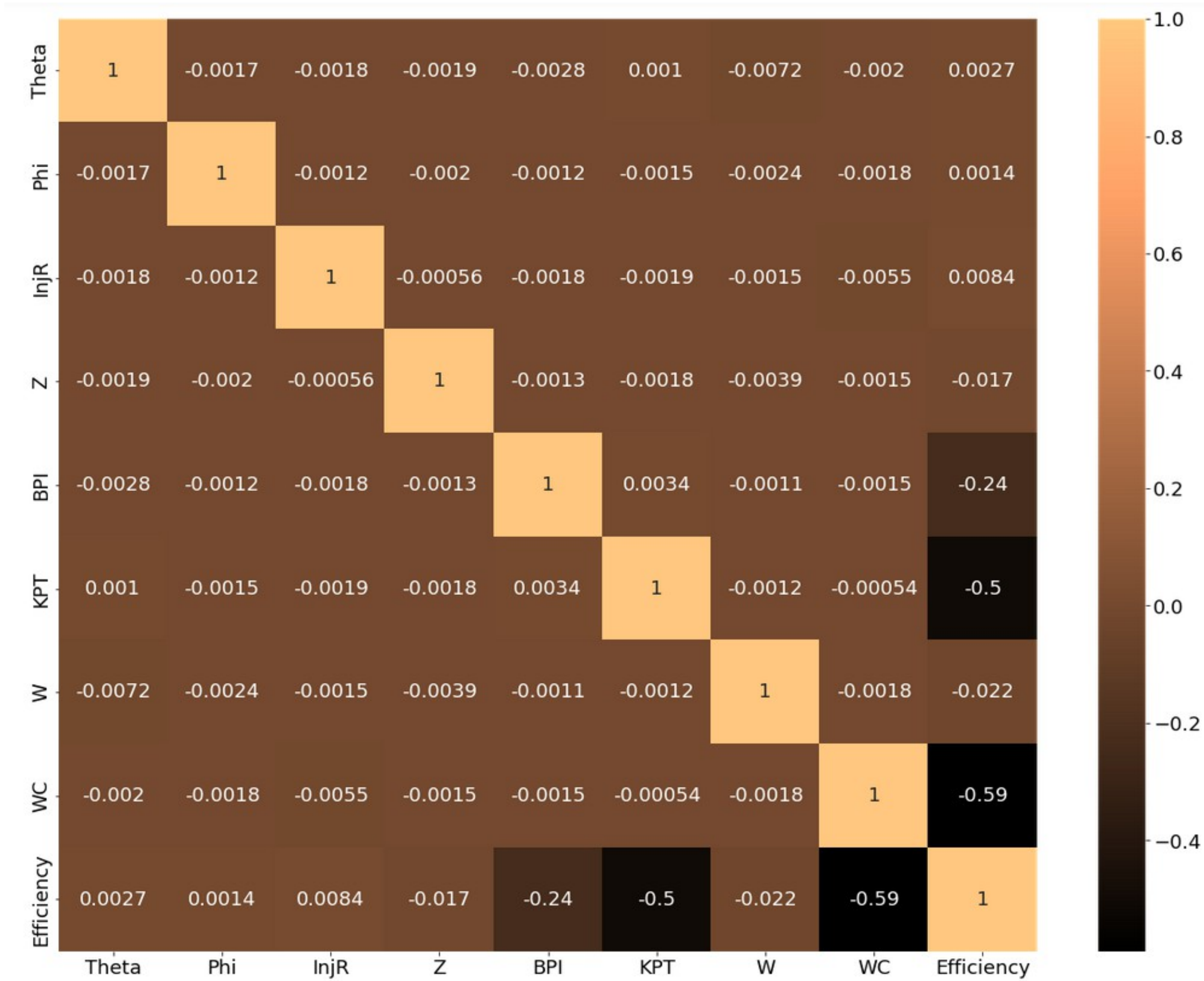


PCE Regression Analysis

- Comparison of true efficiency (orange) and model predicted efficiency (blue) for polynomial order 3

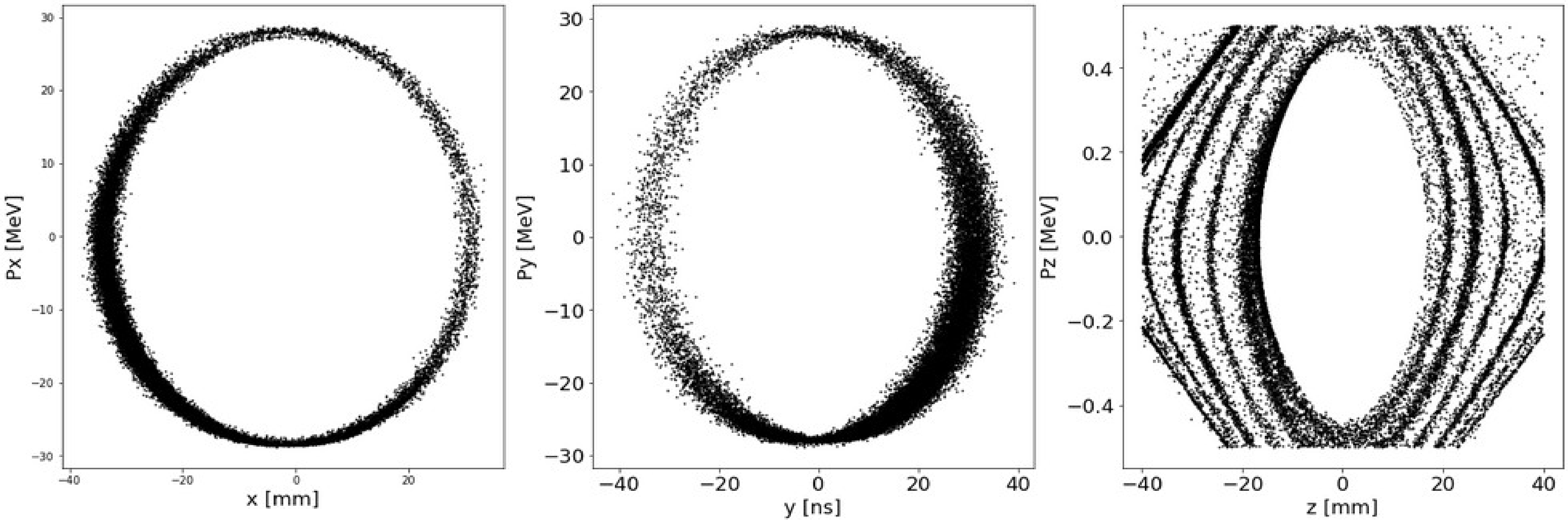


PCE Analysis Correlation Matrix



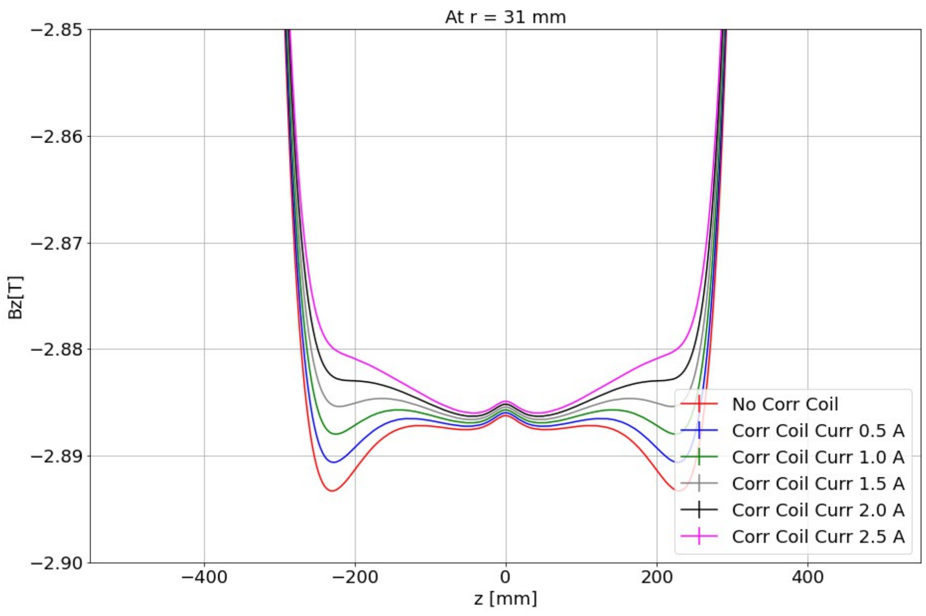
Current Status

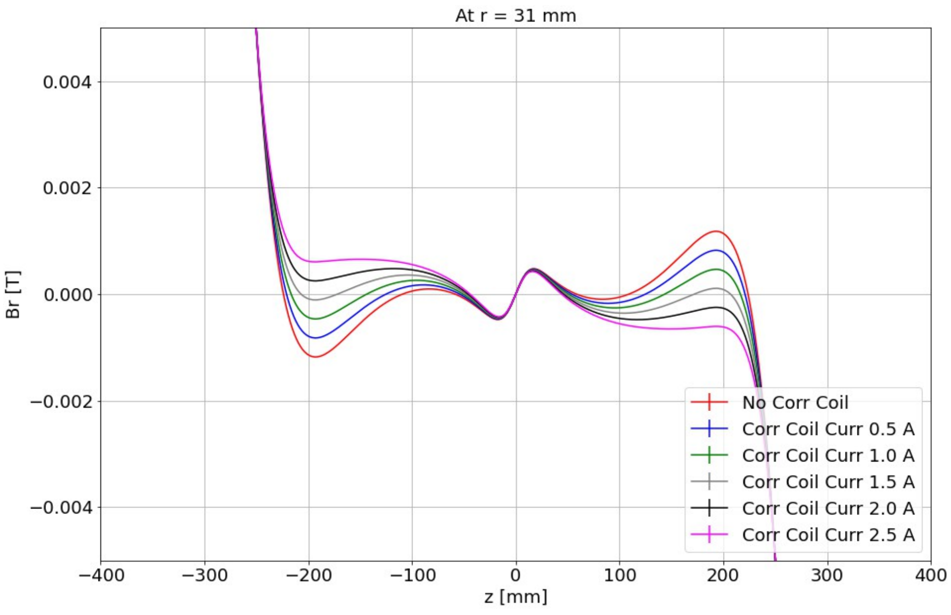
- Need to generate muon distribution in storage region for detector design
- Currently trying to use the maximum efficiency parameters to generate distribution
- 3000 stored from 1M injected muons → Extrapolating to generate larger distributions



Current Status

- Currently running the next iteration of optimization → Excluded geometric parameters
- New parameter → distance between the pair of kicker coils, current in the correction coil
- Correction coils needed to reduce field gradient at injection





- Reduction in CC current desirable to control heat dissipation

Summary

- Injection efficiency depends on a range of design parameter
- Off-axis beam injection simulation in G4Beamline
- Maximize injection efficiency, determine acceptable input variable range
- Multivariate optimization problem → approximated by PCE based surrogate modeling
- Best modeled by polynomial degree 3, less sensitivity to geometric parameters
  → increase in training samples needed to reduce MSE
- Generate muon distribution in storage region based on the maximum efficiency parameters
- Optimize other input variables affecting injection efficiency

* * *

Thank You for Your Attention!

MuEDM Collaboration

ETH · PAUL SCHERRER INSTITUT PSI · UNIVERSITY OF LIVERPOOL · JG|U · UNIVERSITÉ DE GENÈVE · University of Zurich[UZH] · SAPIENZA UNIVERSITÀ DI ROMA · INFN Istituto Nazionale di Fisica Nucleare · UCL · CERN · Cluster of Excellence PRISMA+ · US University of Sussex · tu technische universität dortmund · MANCHESTER 1824 The University of Manchester · 上海交通大学 SHANGHAI JIAO TONG UNIVERSITY · 李政道研究所 Tsung-Dao Lee Institute · TUM · BROOKHAVEN NATIONAL LABORATORY "a passion for discovery" · UNIVERSITÀ DI PISA

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Extra

Sensitivity indices: Example, when d=3

Response function representation:
$Y = f(x_1,x_2,x_3) = f_0 + f_1(x_1) + f_2(x_2) + f_3(x_3) + f_{12}(x_1,x_2) + f_{13}(x_1,x_3) + f_{23}(x_2,x_3) + f_{123}(x_1,x_2,x_3)$

Total varaince:
$D = \int_{Id} f^2(x_1,x_2,x_3)dx_1\, dx_2\, dx_3 - f_0^{\ 2}$

$= D_1 + D_2 + D_3 + D_{12} + D_{13} + D_{23} + D_{123}$

with
$D_1 = \int f_1^{\ 2}(x_1)\, dx_1$, $D_2 = \int f_2^{\ 2}(x_2)\, dx_2$ …....

$D_{12} = \int f_{12}^{\ 2}(x_1,x_2)\, dx_1 dx_2$, $D_{23} = \int f_{23}^{\ 2}(x_2,x_3)\, dx_2 dx_3$,........ and so on

Sensitivity indices: $S_1=D_1/D$, $S_{12}=D_{12}/D$, $S_{123}=D_{123}/D$, …. etc

and

$S_1 + S_2 + S_3 + \ldots + S_{123} = 1$

Sensitivity Analysis in Surrogate Model

PCE expansion: $Y = \sum_{i=0}^{\infty} \alpha_i \Psi_i(\vec{x})$, $(\vec{x}) = (x_1, x_2, \dots, x_d)$, d-dimension

For input variables

$$(x_1, x_2, \dots, x_d) \in I^d := [0,1]^d$$

Model response Y can be represented by

$$Y = f(x_1, x_2, .., x_d) = f_0 + \sum_{i=1}^{d} f_i(x_i) + \sum_{1 \le i < j \le n} f_{ij}(x_i, x_j) + \dots + f_{12\dots d}(x_1, x_2, \dots, x_d),$$

if:

1. $f_0 = \int_{I^d} f(\vec{x}) d\vec{x}$

2. $\int_0^1 f_{i_1, i_2, \dots, i_s}(x_{i_1}, x_{i_2}, \dots, x_{i_s}) dx_k = 0$

where $k = i_i, \dots, i_d$ and $s = 1, 2, \dots, d$

The variance of the function is:

$$D = \int_{I^d} f^2(\vec{x}) d\vec{x} - f_0^2$$

Alternatively $D = \sum_{i=1}^{d} D_i + \sum_{1 \le i < j \le n} D_{ij} + \dots + D_{12\dots d}$

where

$$D_{i_1, i_2, \dots, i_s} = \int_{I^s} f_{i_1, i_2, \dots, i_s}^2 (x_{i_1}, x_{i_2}, \dots, x_{i_s}) dx_{i_i} dx_{i_2} \dots dx_{i_s}$$

with $1 \le i_1 < i_s \le d$

Sobol sensitivity indices are defined as:

$$S_{i_1, i_2, \dots, i_s} = \frac{D_{i_1, i_2, \dots, i_s}}{D}$$

and satisfy the condition

$$\sum_{i=1}^{d} S_i + \sum_{1 \le i < j \le n} S_{ij} + \dots + S_{12\dots d} = 1$$

Sensitivity indices: Results

- Python Chaospy toolbox has built-in functions to calculate sensitivity indices
- First order, second order and total sensitivity indices can be calculated
- The input parameter distribution should be scaled to range [0,1]
- Main Sensitivity indices for poly order 3:

Parameter symbol	Physics Parameter	Main sensitivity index, Polynomial degree 3
Theta	Injection angle (degrees)	3.39230879e-08
Phi	Transverse angle (degrees)	3.86979007e-07
InjR	Injection radius (mm)	1.88544294e-06
Z	Longtudinal Injection coordinate (mm)	4.16966789e-08
BPI	Kicker Field Strength (arb. unit)	1.40754537e-04
KPT	Time of peak of Kicker Pulse (ns)	9.03986321e-04
W	Width of the Kicker Pulse (ns)	4.89511202e-02
WC	Weak Current*100 (A)	9.12374997e-01

Sum = 0.9624

- Relative sensitivities, dependent on initial range provided

Sensitivity indices: From PCE coefficients

$$\hat{S}_{i_1,i_2,\ldots,i_s} = \frac{1}{\hat{D}} \sum_{\boldsymbol{i} \in I_{i_1,i_2,\ldots,i_s}} \alpha_{\boldsymbol{i}}^2 \langle \Psi_{\boldsymbol{i}}^2 \rangle$$

where

$$I_{i_1,i_2,..,i_s} = \left\{ \boldsymbol{i} : \begin{array}{l} i_k > 0 \forall k = 1, \ldots, n, k \in (i_1, \ldots, i_s) \\ i_k = 0 \forall k = 1, \ldots, n, k \notin (i_1, \ldots, i_s) \end{array} \right\}$$

The variance, $\hat{D}$ is given by

$$\hat{D} = \sum_{j=1}^{P-1} \alpha_{\boldsymbol{i}_j}^2 \langle \Psi_{\boldsymbol{i}_j}^2 \rangle.$$

From this, the main and total sensitivity is computed as

$$\hat{S}_j = \frac{1}{\hat{D}} \sum_{\boldsymbol{i} \in I_j} \alpha_{\boldsymbol{i}}^2 \langle \Psi_{\boldsymbol{i}}^2 \rangle$$

where $I_j = \{\boldsymbol{i} = (i_1, i_2, \ldots, i_d) : i_k > 0 \forall k = j \wedge i_k = 0 \forall k \neq j\}$ and

$$\hat{S}_j^T = \frac{1}{\hat{D}} \sum_{\boldsymbol{i} \in I_j} \alpha_{\boldsymbol{i}}^2 \langle \Psi_{\boldsymbol{i}}^2 \rangle$$

where $I_j = \{\boldsymbol{i} = (i_1, i_2, \ldots, i_d) : i_j > 0\}$, respectively.

Neural Network Regression

- No. of neurons: **512**

  Hidden layers: **8**

  Activation function: **Leaky ReLU**

  Scheduler: **ReduceLROnPlateau**

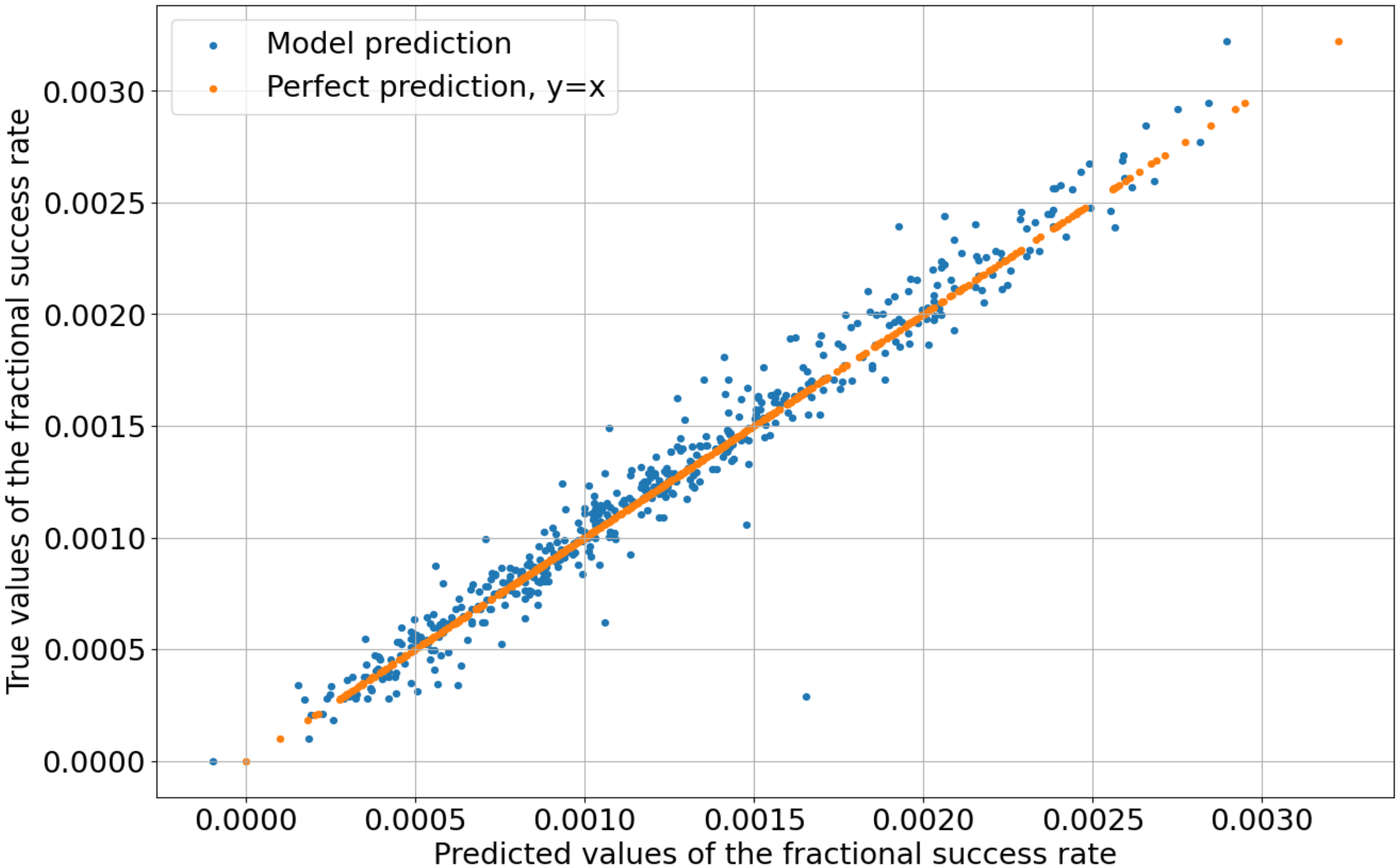
  Batch size: **500**

  Epoch: **400**

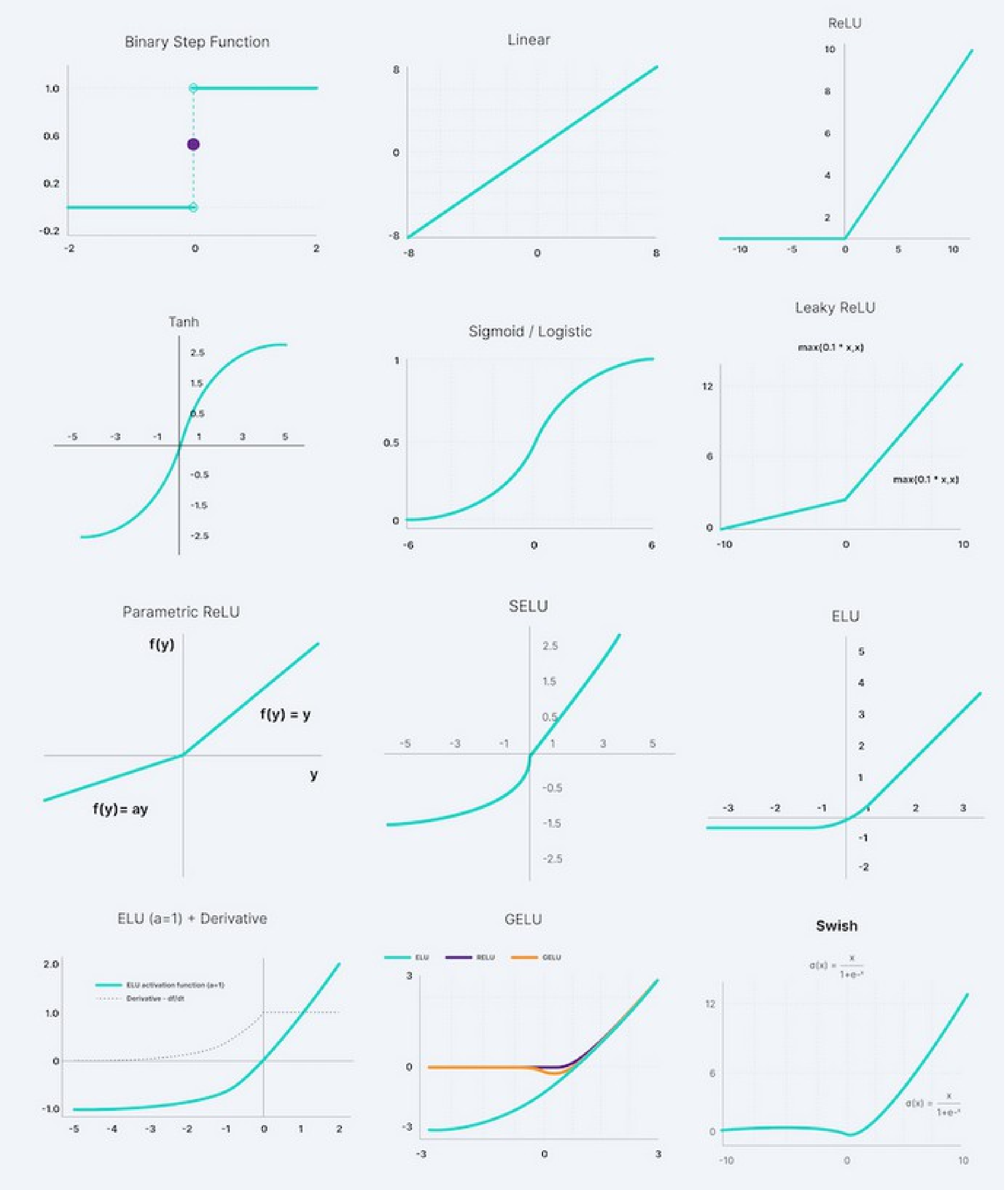
  Optimizer: **Adam**

  Dropout rate: **10%**

- Better agreement between true and predicted



Neural Network activation functions



Number of expansion coefficients, P and Integration points (sample size), N

d	p	P	N
6	3	84	420
6	4	210	1050
6	5	462	2310
7	3	120	720
7	4	330	1980
7	5	792	4752
8	3	165	1155
8	4	495	3465
8	5	1287	9009