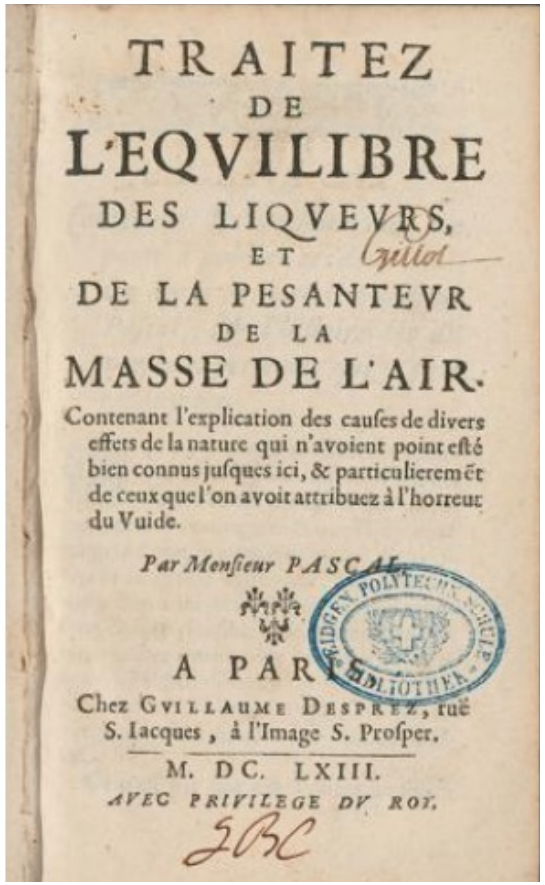
Pascal's law and the Pascal unit in material science and engineering



TRAITEZ
DE
L'EQVILIBRE
DES LIQVEVRS,
ET
DE LA PESANTEVR
DE LA
MASSE DE L'AIR.

Contenant l'explication des causes de divers effets de la nature qui n'avoient point esté bien connus jusques ici, & particulierem ēt de ceux que l'on avoit attribuez à l'horreur du Vuide.

Par Monsieur PASCAL.

A PARIS,
Chez GVILLAUME DESPREZ, ruë S. Iacques, à l'Image S. Prosper.

M. DC. LXIII.
AVEC PRIVILEGE DV ROY.

Helena Van Swygenhoven-Moens

Prof. em. EPFL & Paul Scherrer Institute

Blaise Pascal (1623 – 1662)

Published first in 1663,
1 year after his dead, by his nephew Etienne Périer

Outline

Pascal's discoveries in hydrostatics

- Pressure in a vessel, Pascal's paradox
- Pascal's law and Pascal's barrel
- The SI unit for pressure is Pascal, comparison with other units

Applications

- in engineering
- in medicine

Pascal as unit for stress

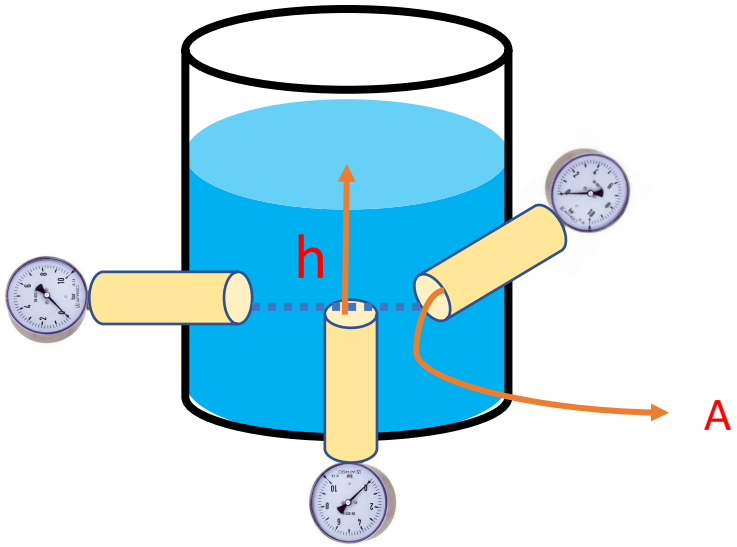
- Stress ⇔ pressure
- Materials selection and GPa stresses

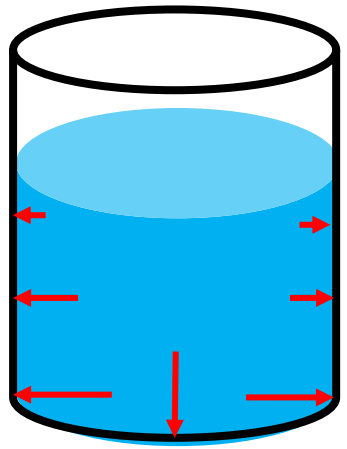
Basics of hydrostatics: the pressure

In a fluid at rest, the *pressure* $P = \frac{Force}{Area} = \frac{F}{A}$

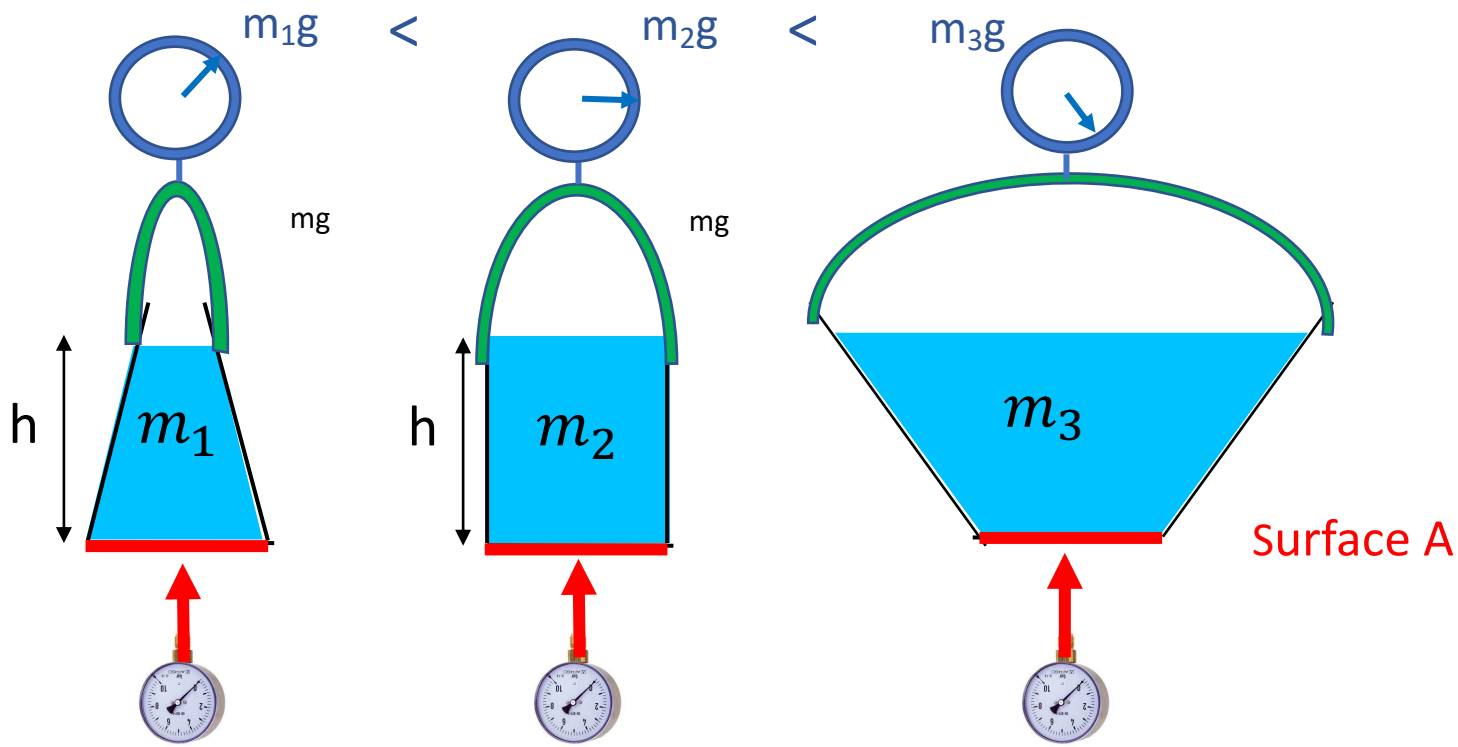
is independent of the orientation of the surface used to measure

The *force* is always perpendicular to the surface A





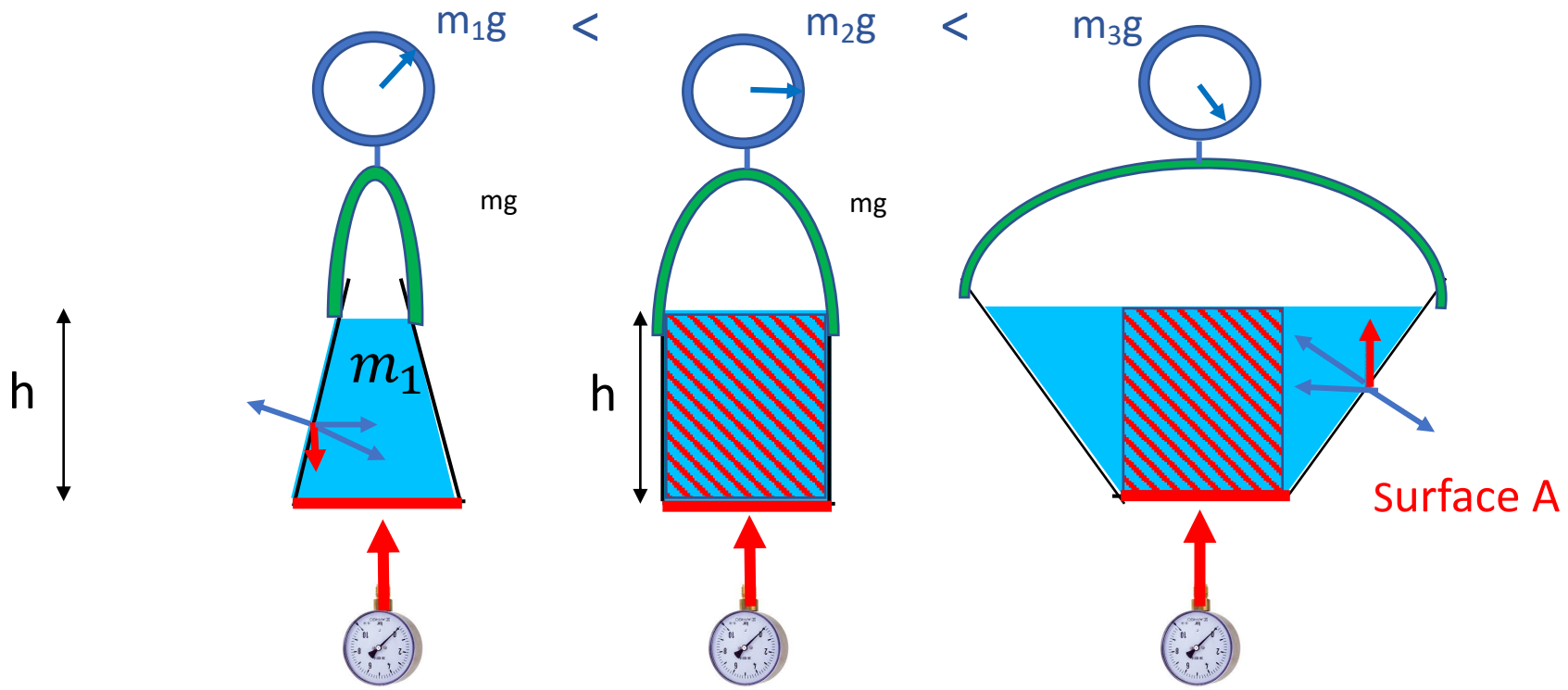
Basics of hydrostatics: Pascal's paradox



$F_1 = F_2 = F_3$

$P_1 = P_2 = P_3$

Basics of hydrostatics: Pascal's paradox



in analogy to pressure of air
Evangelista Torricelli(1608-1647)

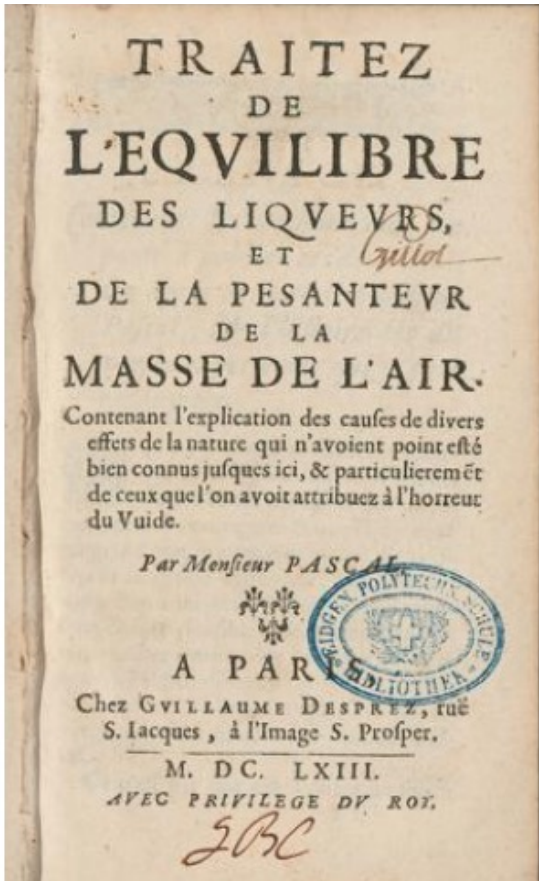
$$P = \frac{mg}{A} = \frac{\rho A h g}{A} = \rho g h$$

$F_1 = F_2 = F_3$

$P_1 = P_2 = P_3$

The only important parameter is the height h!

Basics of hydrostatics: Pascal's paradox



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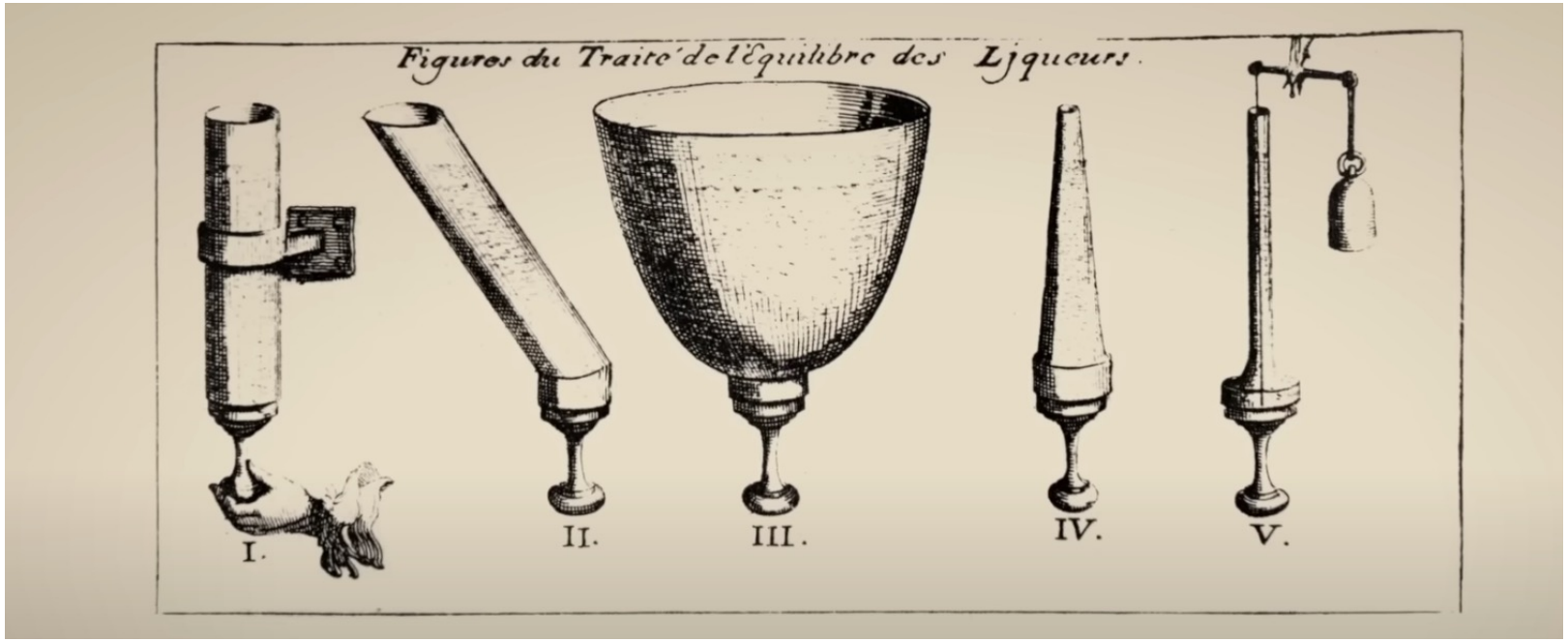
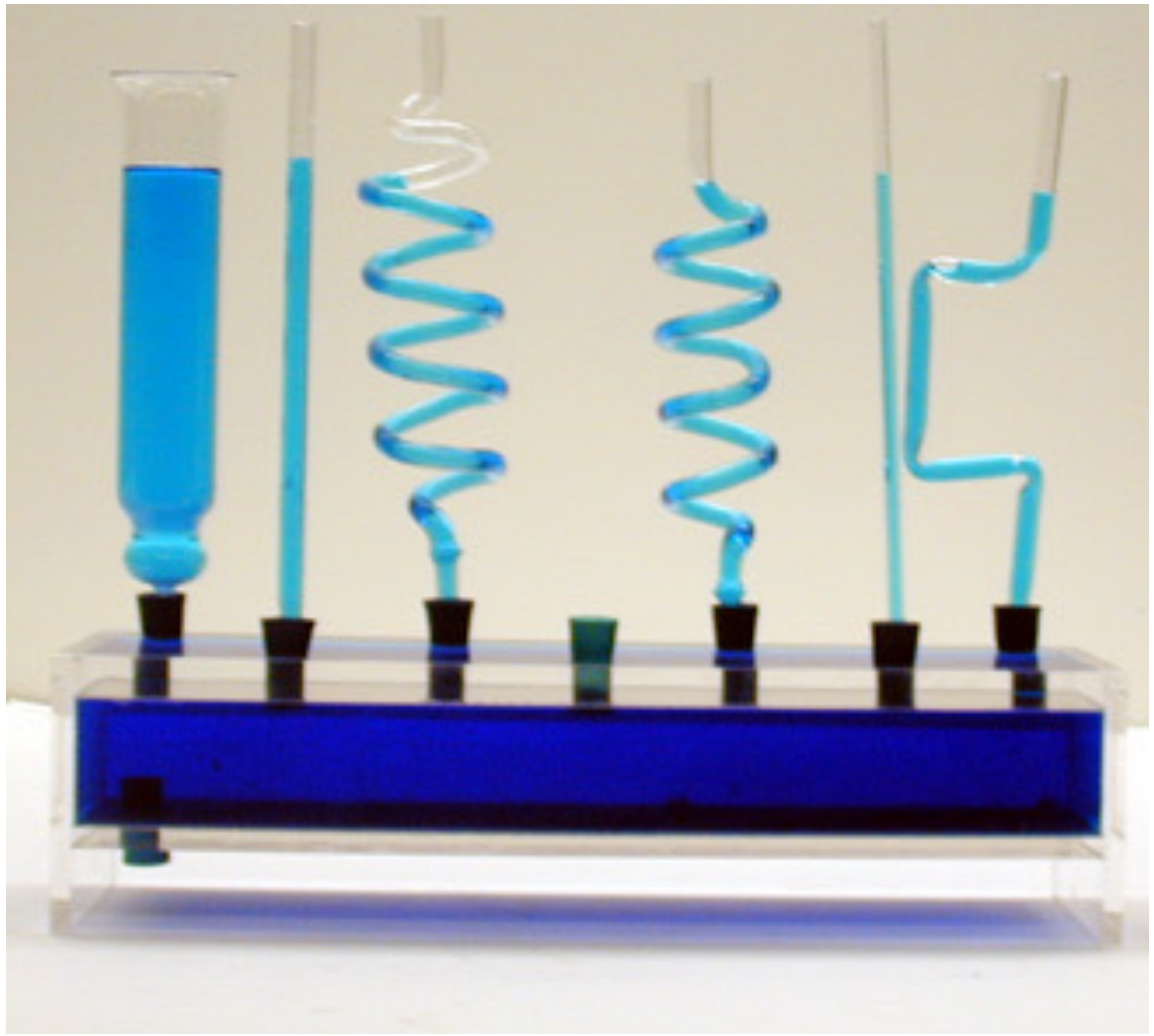


https://fr.wikisource.org/wiki/Trait%C3%A9_de_l%E2%80%99%C3%A9quilibre_des_liqueurs/Chapitre_I

Basics of hydrostatics: Pascal's vases



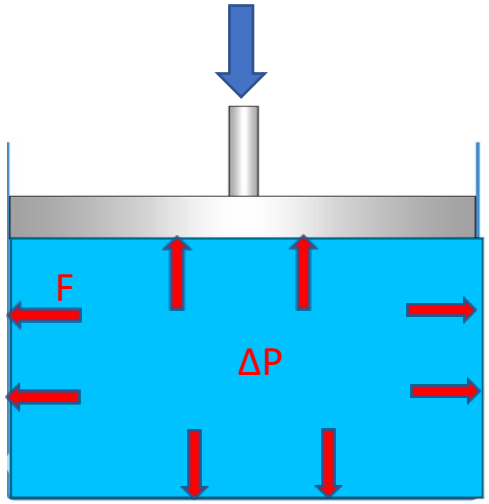
Pascal's vases

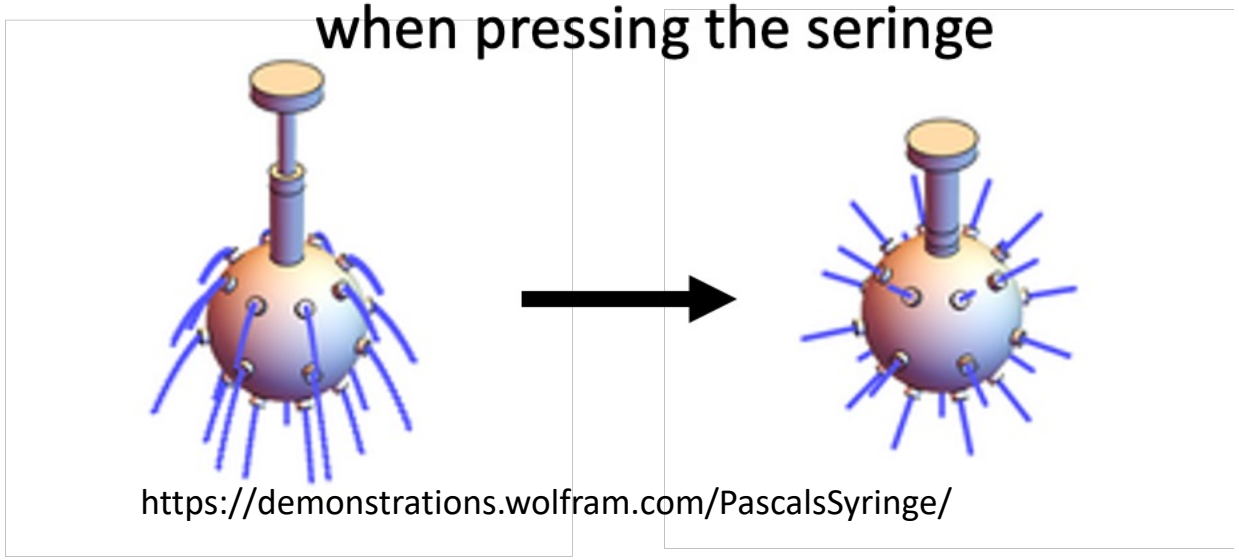
Pascal's law (the principle of fluid-pressure transmission)

In a fluid at rest in a closed container,
a pressure change in one part is transmitted without loss to every portion of the fluid and to the walls of the container.

Pascal demonstrated this with a syringe in 1653. It consisted of a piston plunger connected to a sphere with holes, filled with water. When the plunger is depressed, the water squirts out with equal force in all directions.

$\Delta P = \frac{F}{A}$

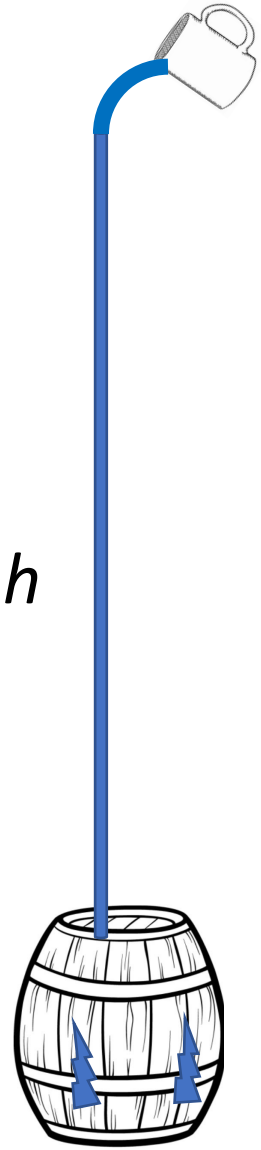




Pascal's barrel

With the same amount of water
one can break the barrel !!

$$\Delta P = \rho\, g\, \Delta h$$



International System of Units (SI) accepted the unit Pascal in 1971

$$\frac{\text{Force (F)}}{\text{Area (A)}} = \text{Pressure (P)} \qquad \frac{\text{Newton}}{\text{m}^2} = 1 \text{ Pascal (Pa)}$$

1 Pa is a small amount of pressure!

Blood pressure ≈ 120/80 mmHg = 16.0 /10.6 kPa
Freeze drying coffee ≈ 1-50 Pa

MBE systems ≈ 10^{-8} Pa

Air pressure at sea level ≈ 100 kPa
Matterhorn ≈ 84 kPa
Mount Everest ≈ 34 kPa

Pressure in bike tires ≈ 550 kPa

The unit Pascal and other units

1 bar = 100 000Pa

1 atm = 101 325 Pa. (average air pressure at sea level at 15C)

1 Psi = 6 895 Pa (pound-force applied to an area of one square inch)

1 mmHg = 133.322387415 Pa (pressure exerted by a column of mercury 1 millimeter high at 0°C at standard gravity) .

1 torr = $\frac{1}{760}$ of an atmosphere (101 325/760 Pa) (after Evangelista Torricelli, an Italian physicist and mathematician who discovered the principle of the barometer in 1644)

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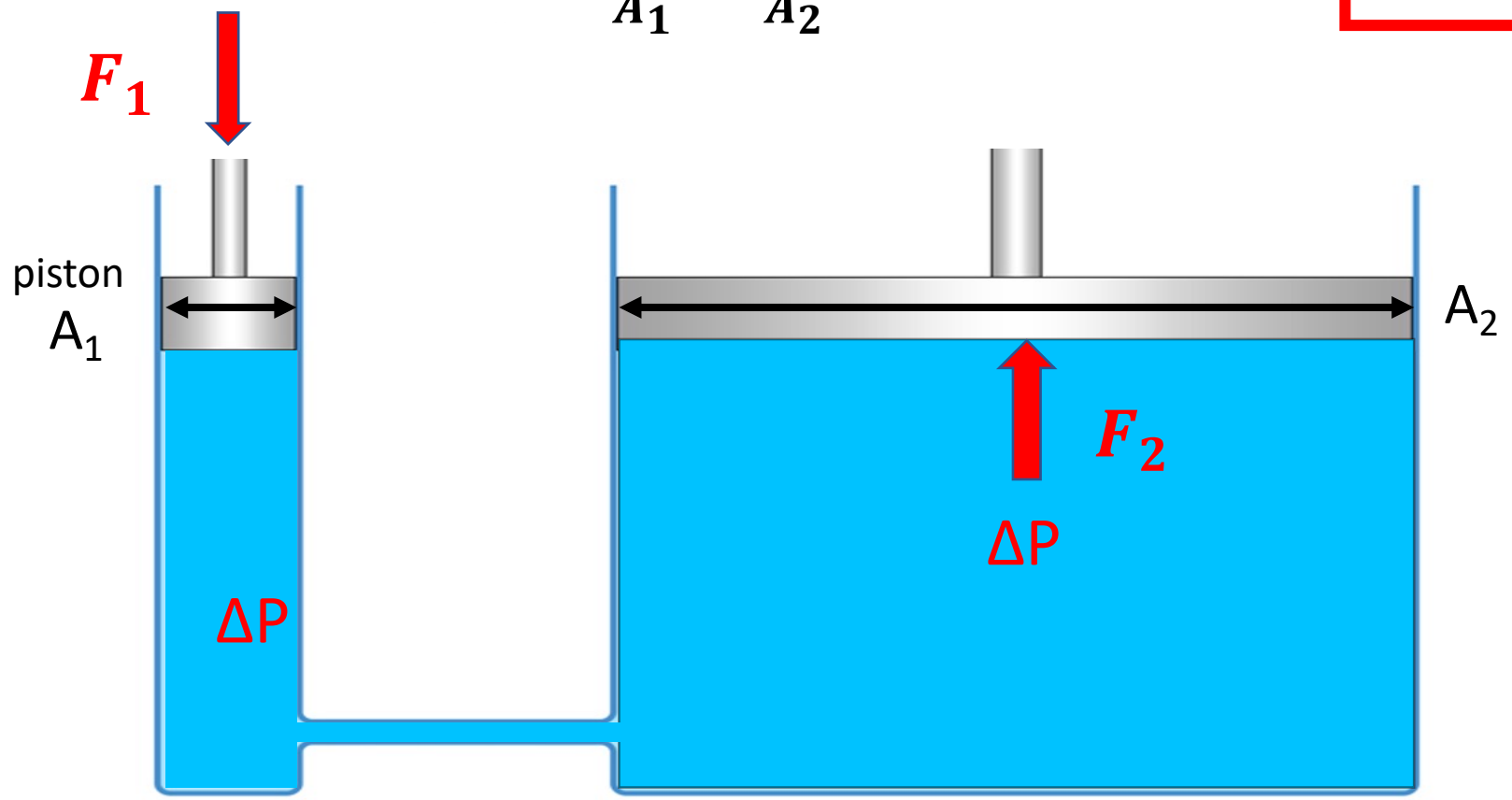
Applications of Pascal's law in engineering: principle of force multiplication

$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 > F_1$$



Pascal'law

Applications of Pascal's law in engineering: principle of force multiplication

1738. Daniel Bernoulli, a Swiss mathematician, who in 1738 has put theory into practice by using pressurised water in mills and pumps

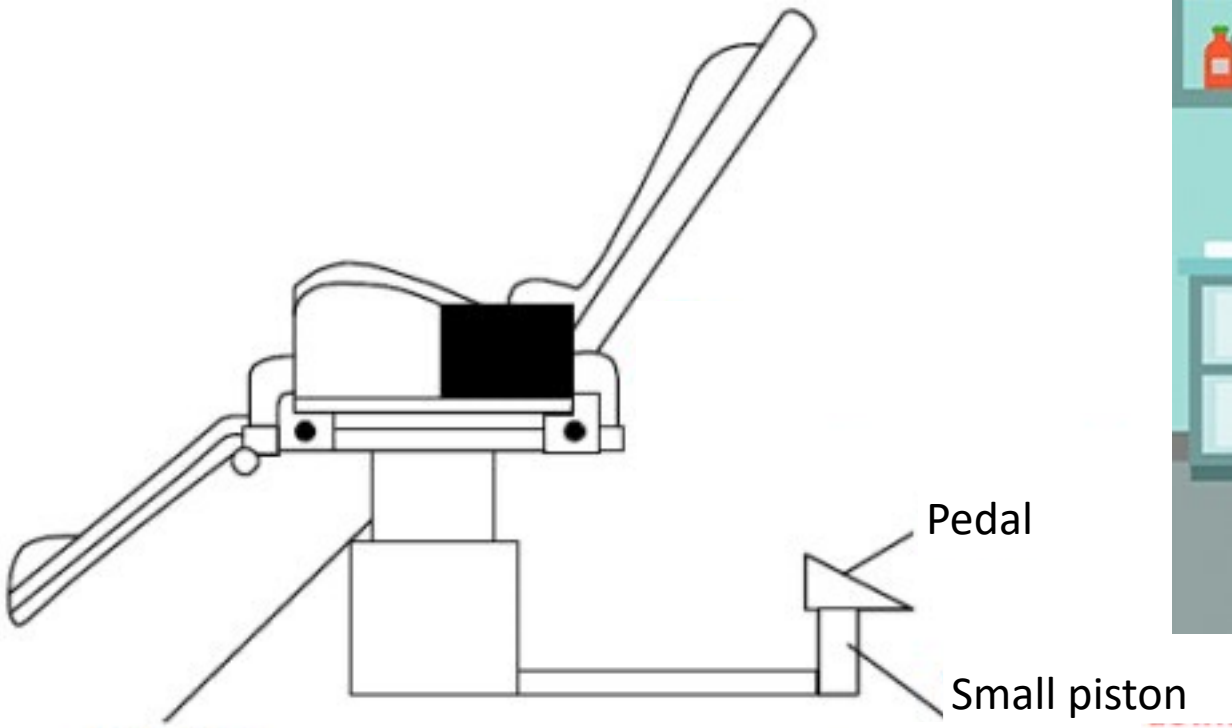
1795. Joseph Bramah, an English inventor, patented the first hydraulic press, forming the foundations of an industrial revolution.

Applications of Pascal's law in engineering

Hydrolic lift

https://www.maxjax.com/maxjax/

Applications of Pascal's law in engineering

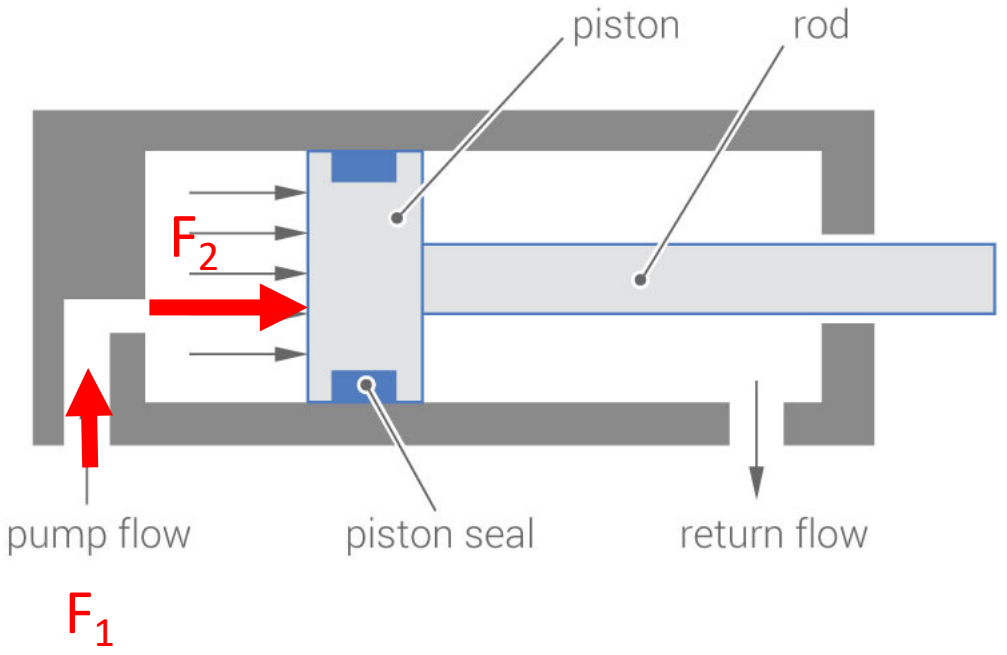


https://www.galaxyeduworld.com/blogs/all-you-need-to-know-about-mds

Applications of Pascal's law in engineering

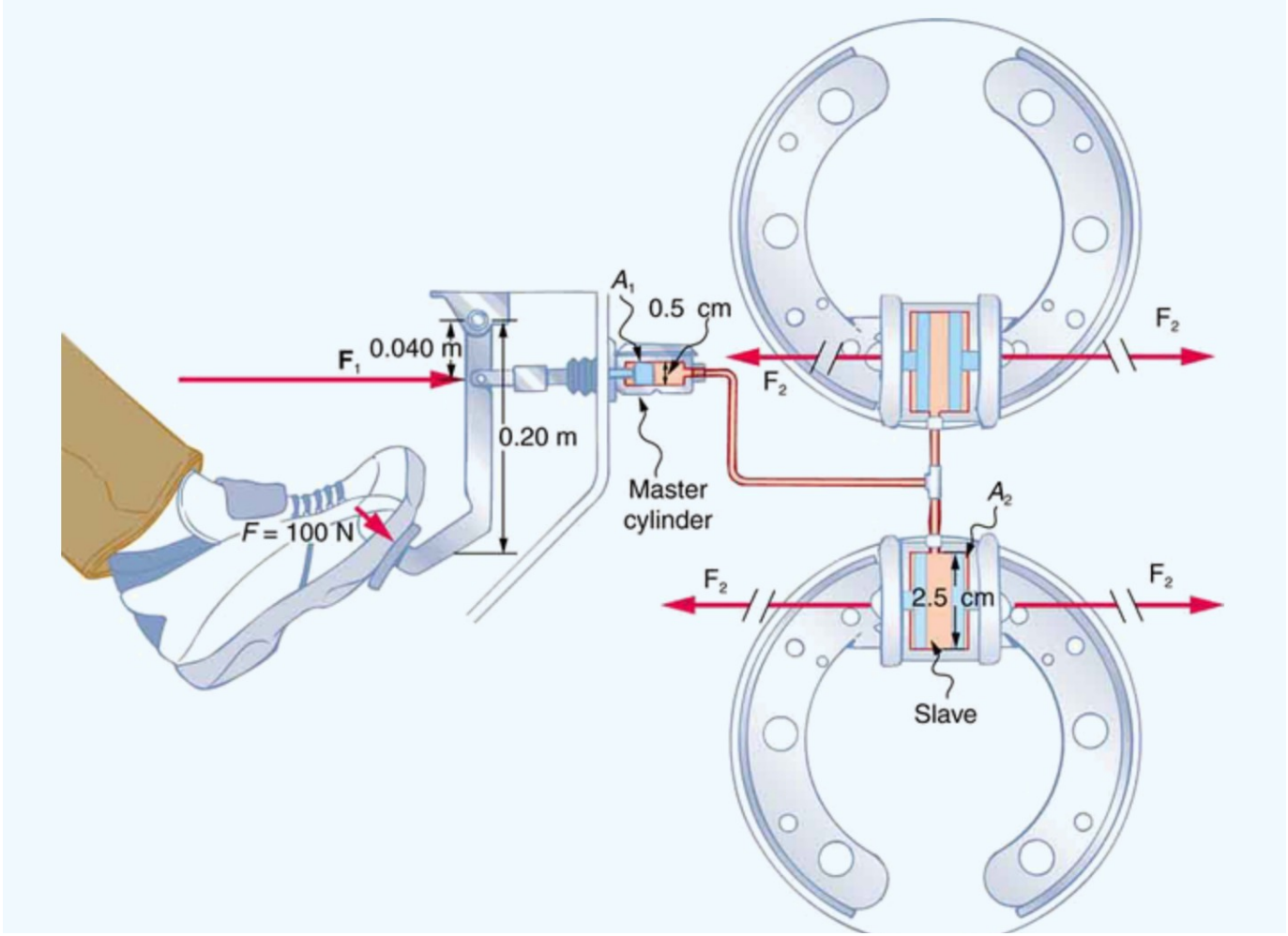
https://whyps.com/
hydraulics-in-mobile-heavy-equipment

Hydrolic actuator



https://www.engineeringclicks.com/
hydraulic-actuators/

Applications of Pascal's law in engineering

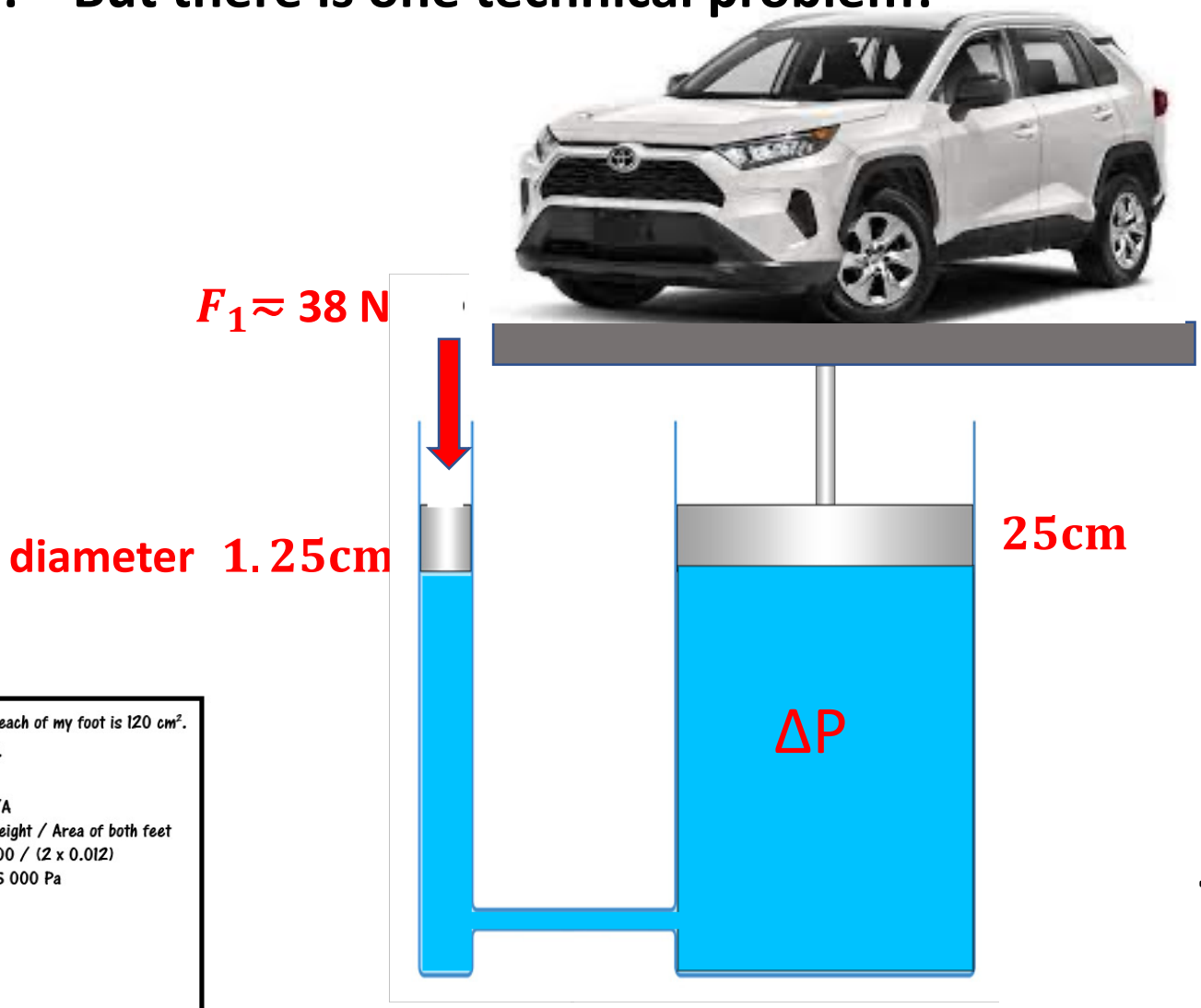


Hydrolic breaks

https://phys.libretexts.org/

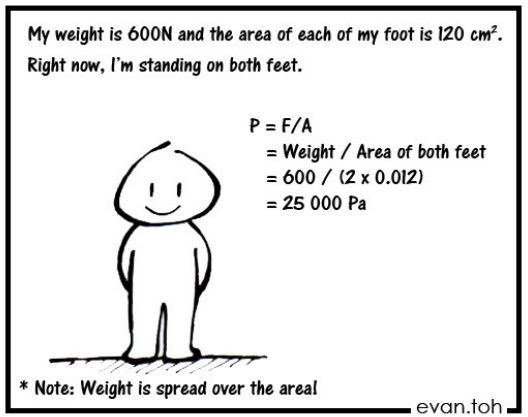
Applications of Pascal's law in engineering

GREAT!!! But there is one technical problem!



1500kg → $F_2 \approx$ 15 000 N

ΔP = 240 000 Pa

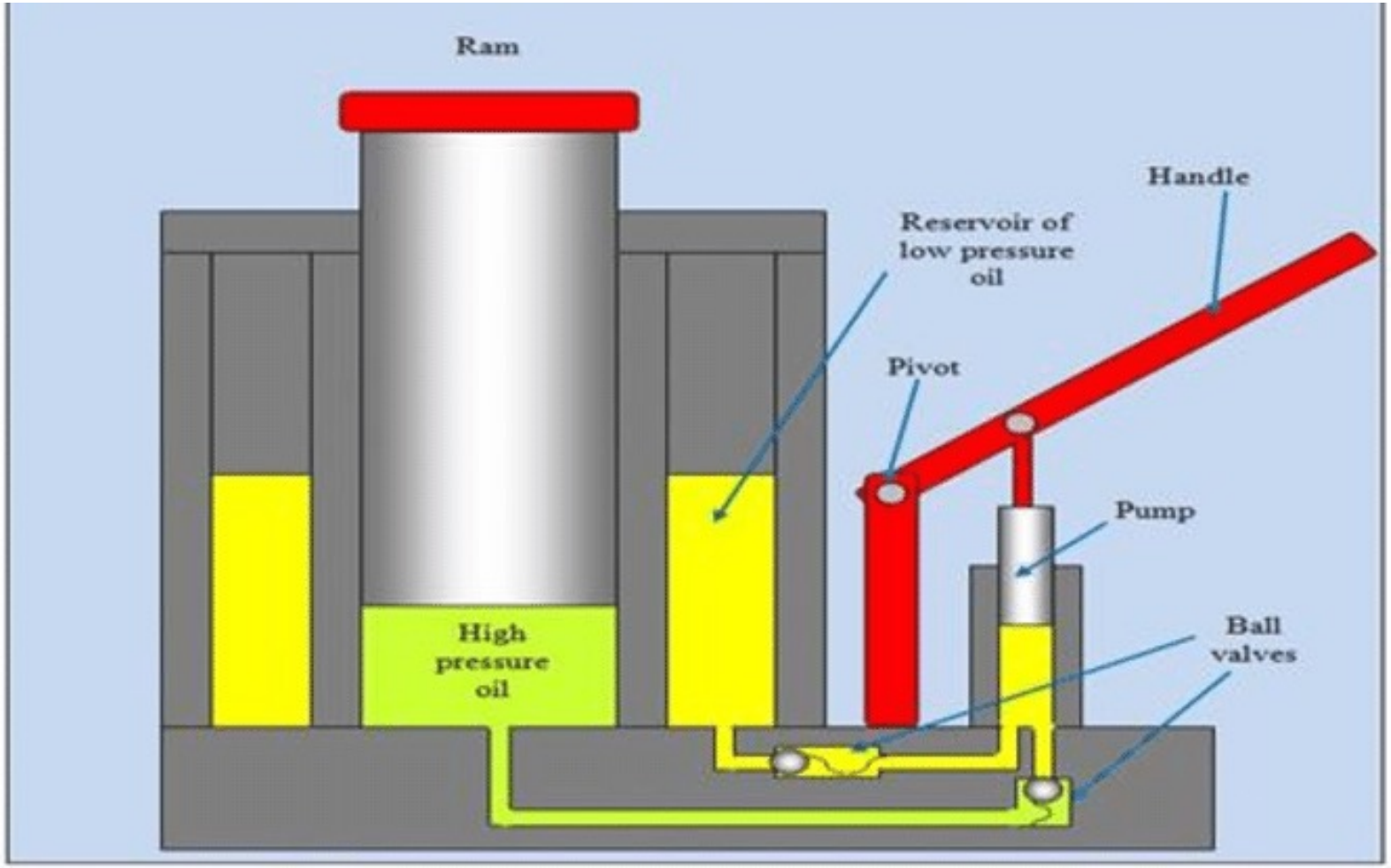


Conservation of energy!!

$$F_1 d_1 = F_2 d_2$$

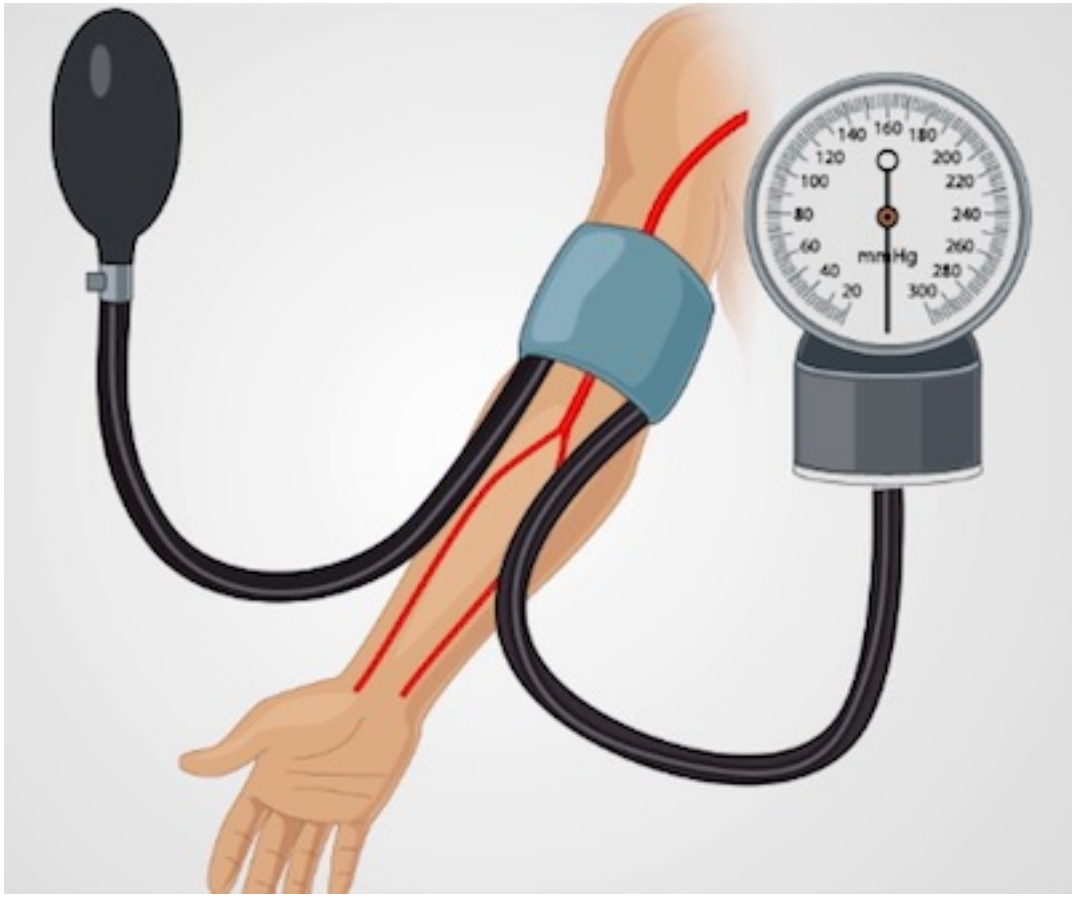
To lift the car 0.5m
→ piston must be moved 200m !!

Applications of Pascal's law in engineering

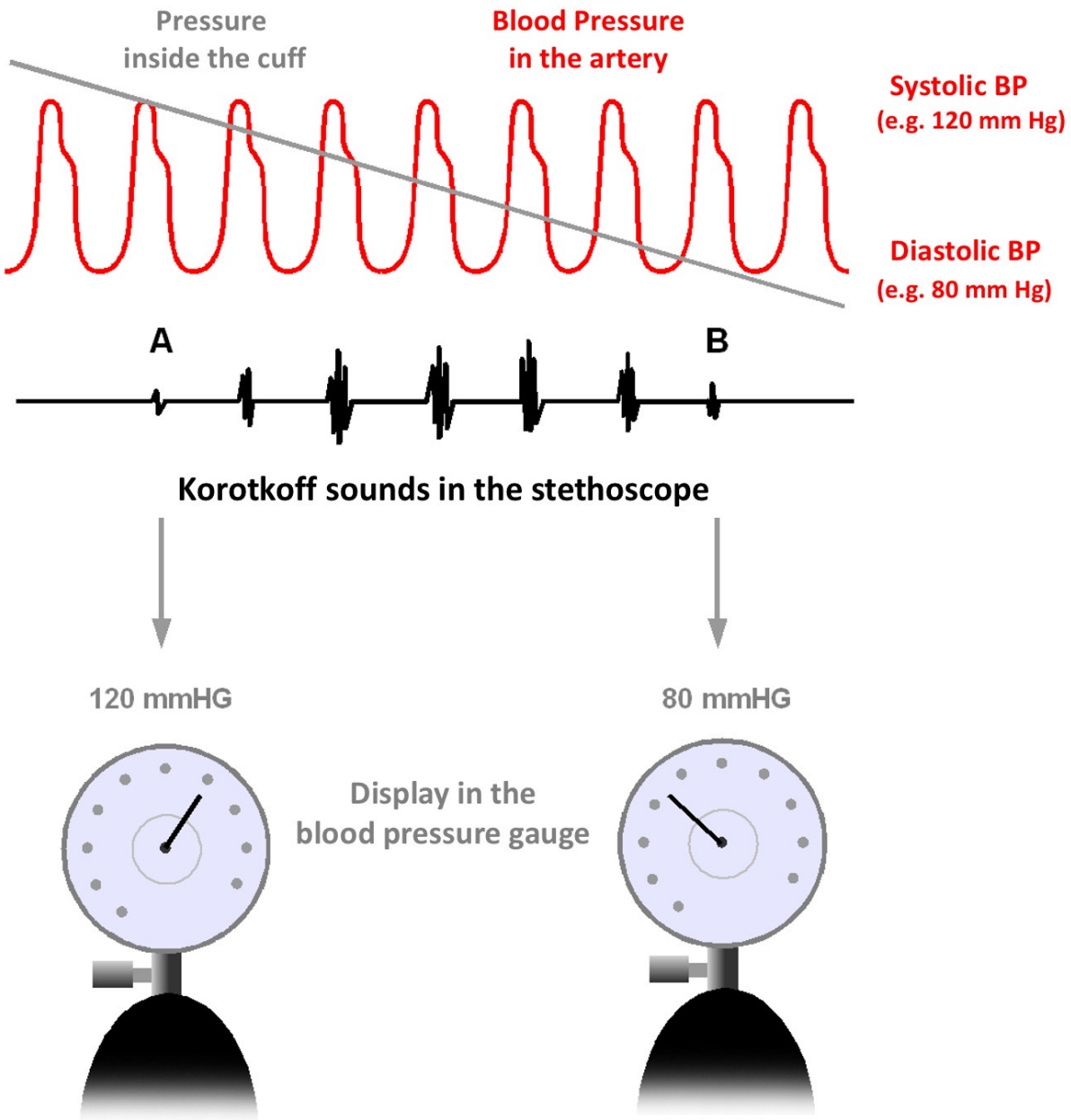


https://www.researchgate.net/figure/Principle-of-hydraulic-jack_fig1_309235120

Applications Pascal's law in medicine
Blood pressure measurement



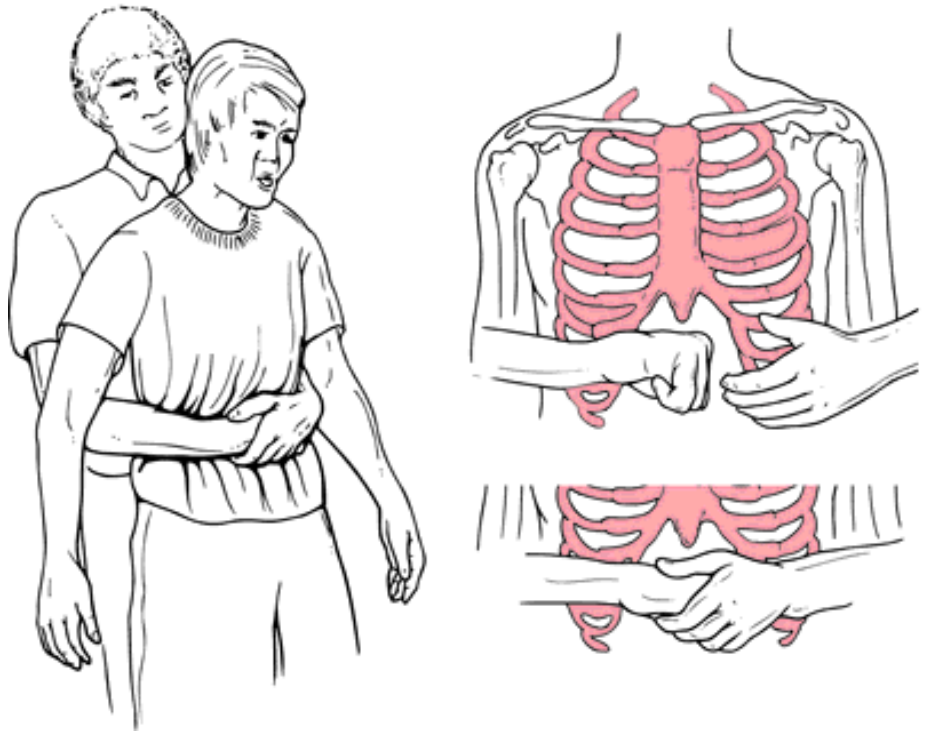
https://en.wikipedia.org/wiki/Sphygmomanometer



Applications Pascal's law in medicine: The Heimlich maneuver

In 1974 by Dr. Henry Heimlich suggested an emergency procedure for removing a foreign object lodged in the airway that is preventing a person from breathing

by applying a sudden force on the abdomen above the navel,
→ sudden change in pressure in the lung
→ Pascal's law → equal change of pressure in airway that dislodges the object



± 50 cm H_2O → 4900 Pa

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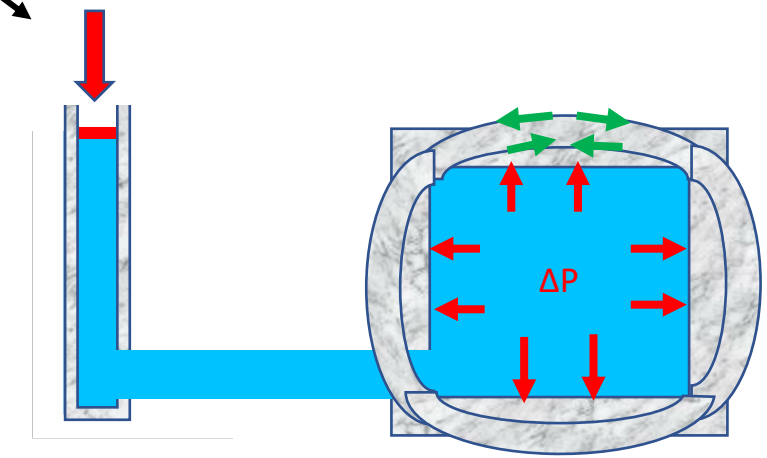
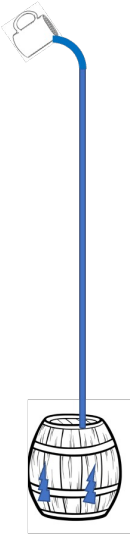
The Pascal (Pa) unit to quantify stress

Pressure (Pa) ← ? → **Stress (Pa)**

$\Delta P = \frac{F_1}{A_1}$



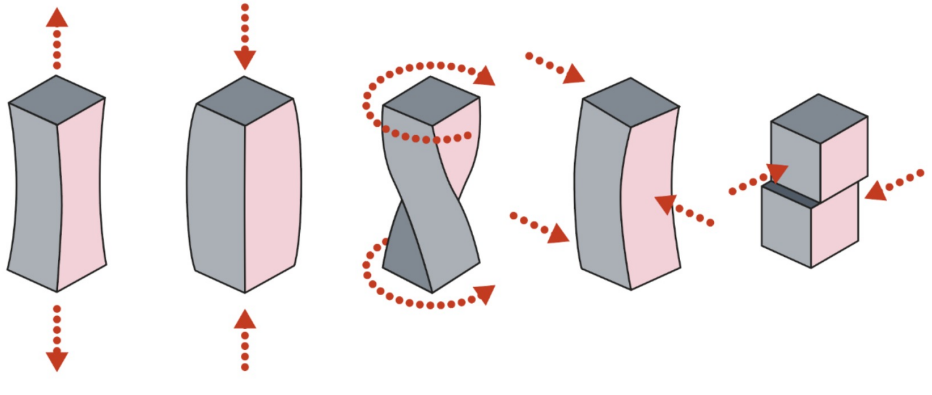
Pascal's barrel



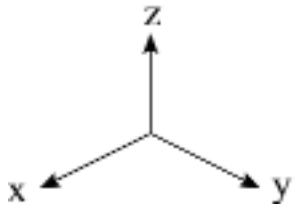
Pressure
=
External force to per unit area on gas, liquid
Force is normal to plane
⟹ a scalar

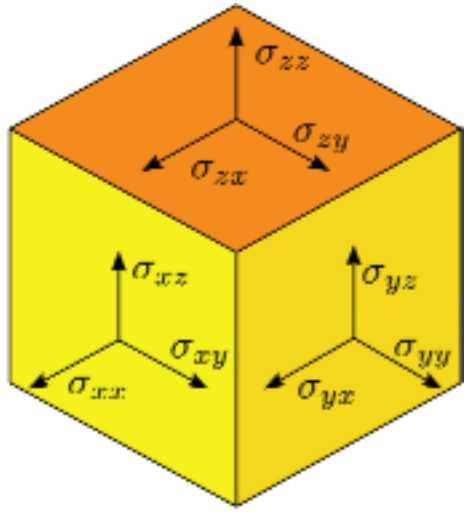
Stress
=
Internal resistive force to deformation per unit area in a solid
Angle between force and plane is important
⟹ a tensor

The Pascal (Pa) unit to quantify stress
In material science and engineering. … and reach GPa



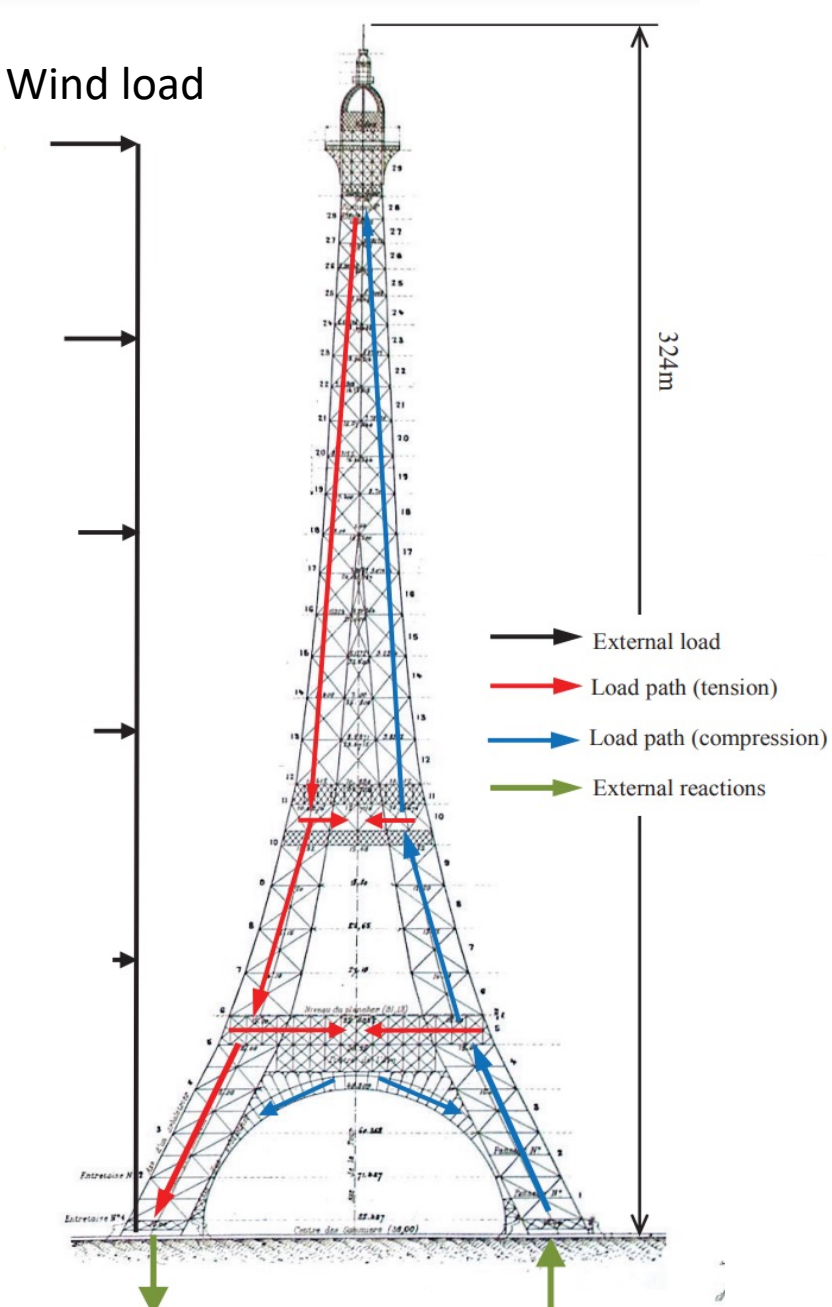
$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & \tau_{12} & 0 \\ \tau_{12} & 0 & \tau_{12} \\ 0 & \tau_{12} & 0 \end{pmatrix}$$

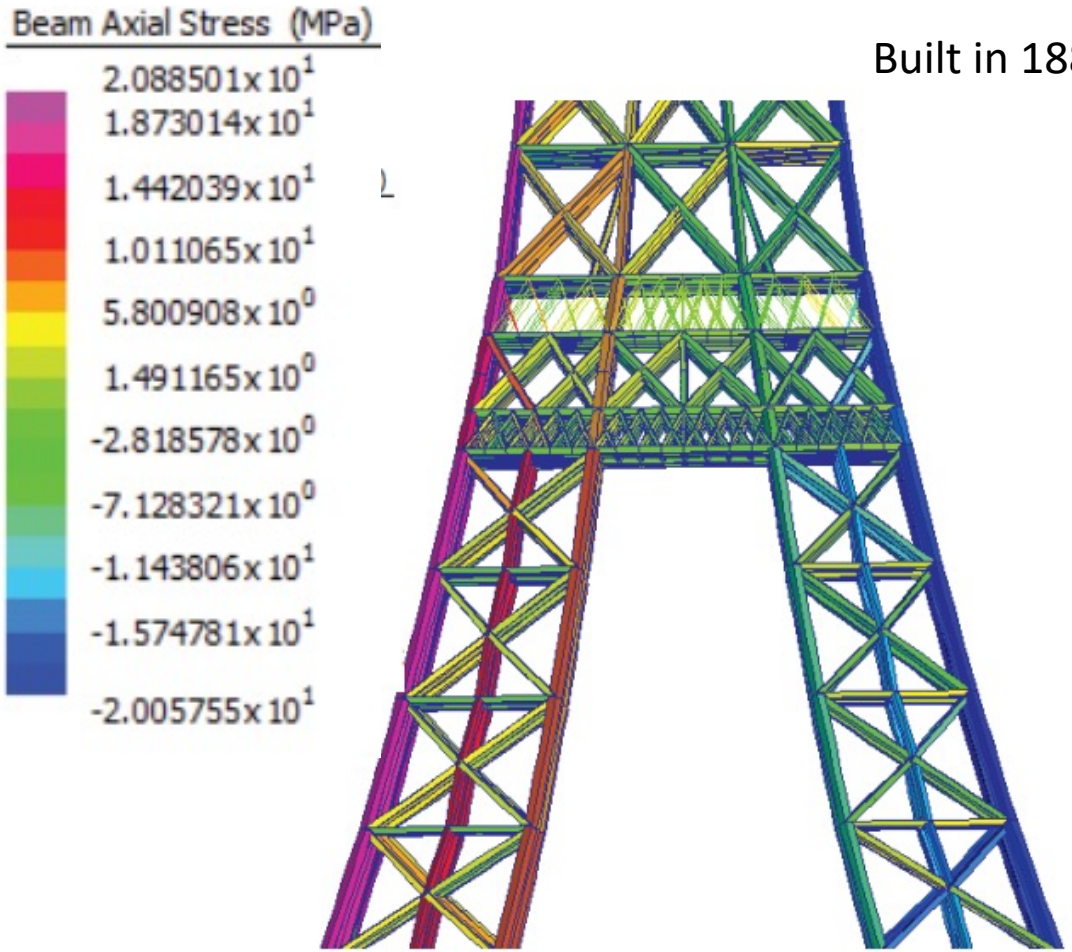




$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{xz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Material selection

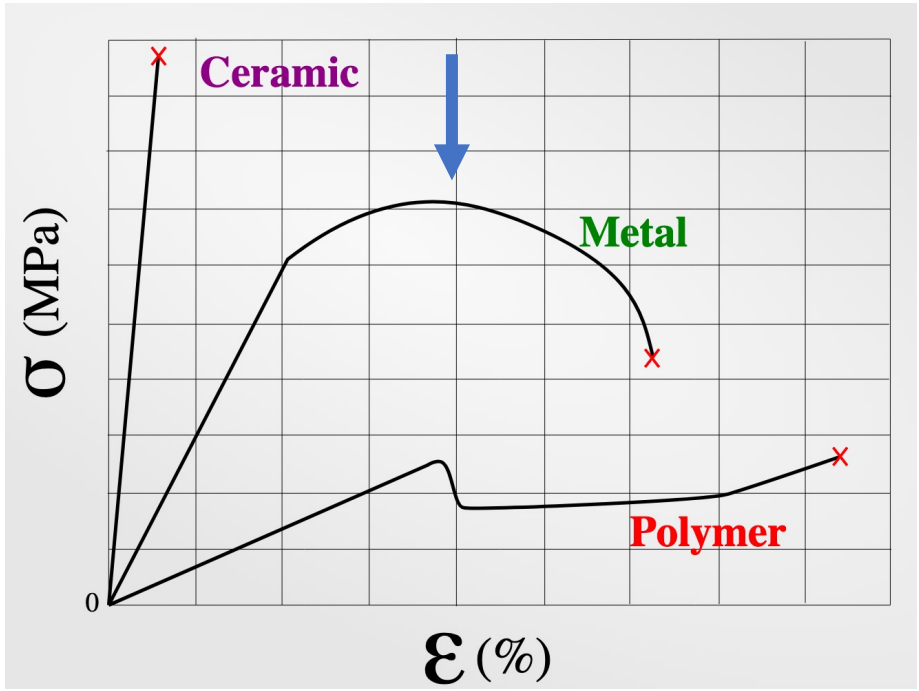


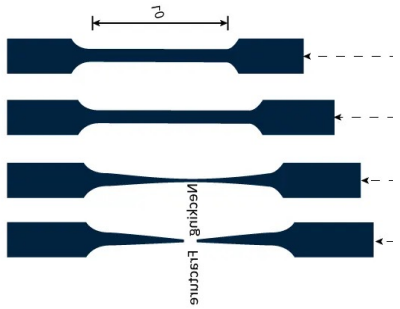


Built in 1889

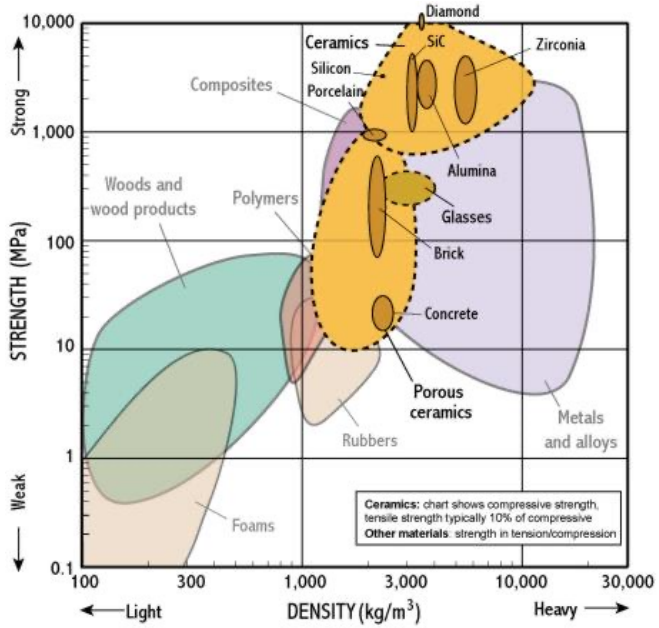
L. Horlick et al.
AIP Conference Proceedings 1762, 020003 (2016)

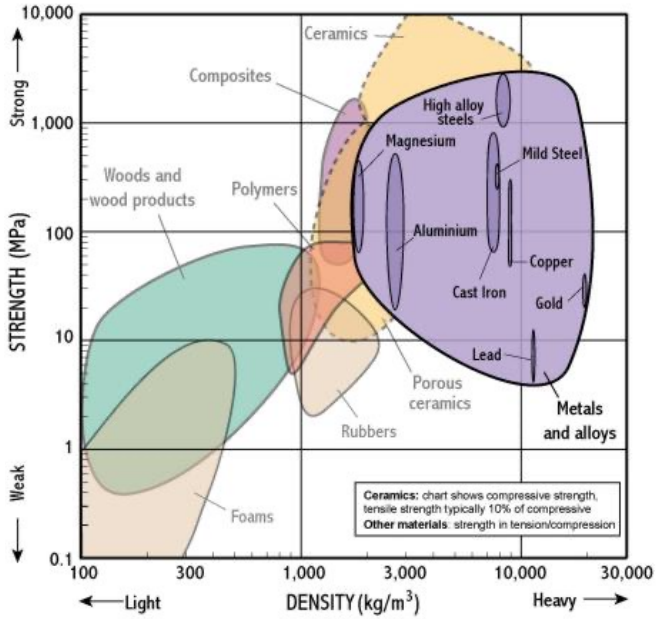
Material selection

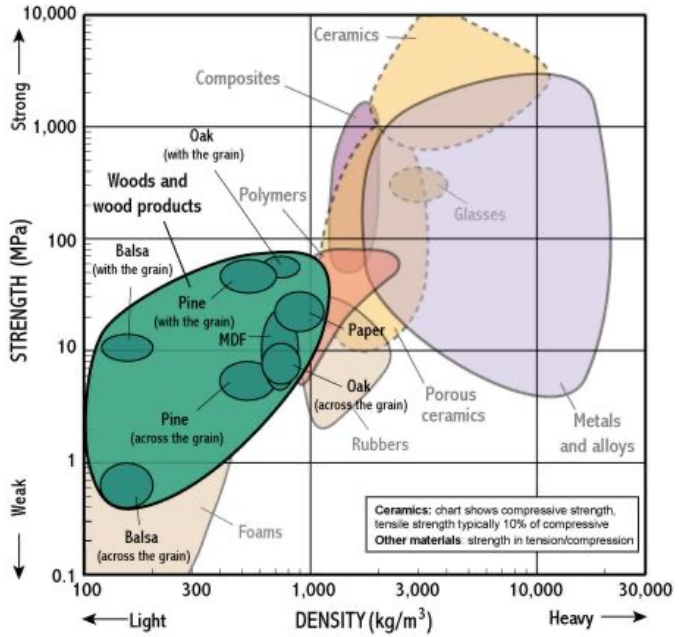




Ashby charts http://www-g.eng.cam.ac.uk/125/noflash/now/mfs/tutorial/non_IE/charts.html







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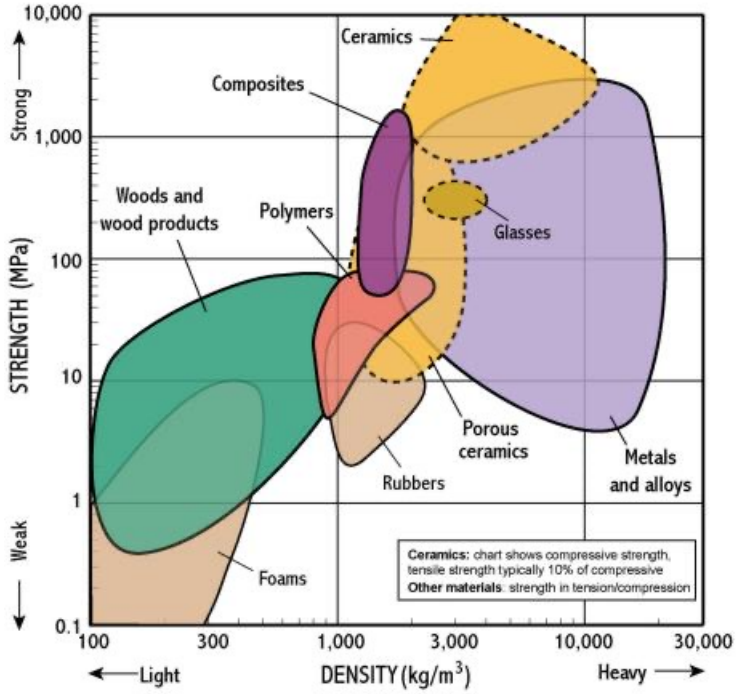
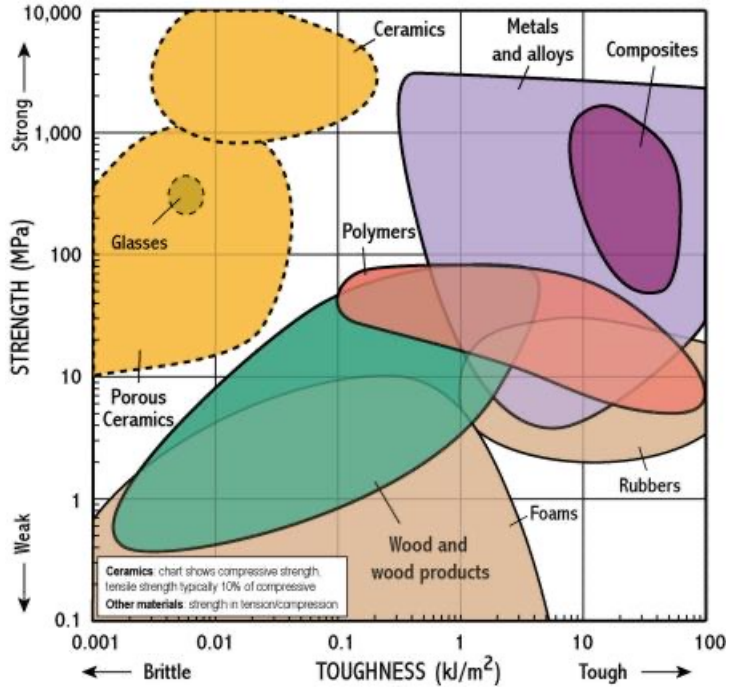
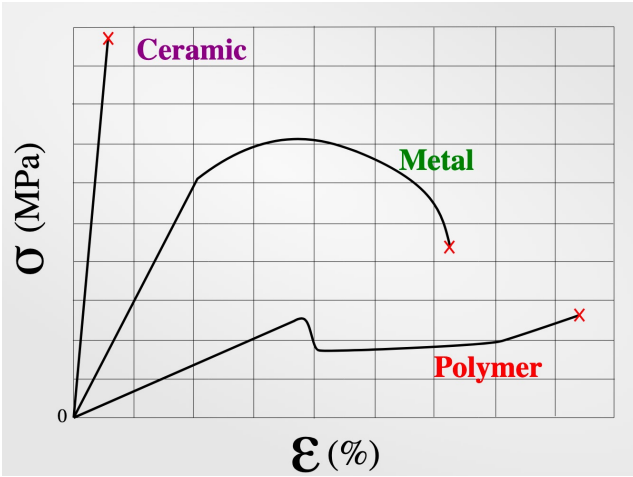
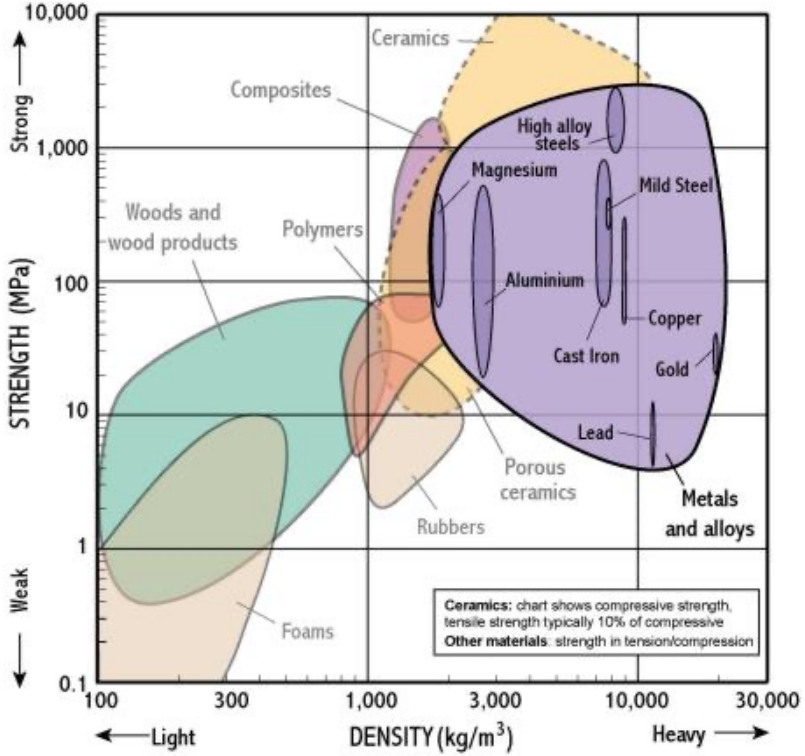


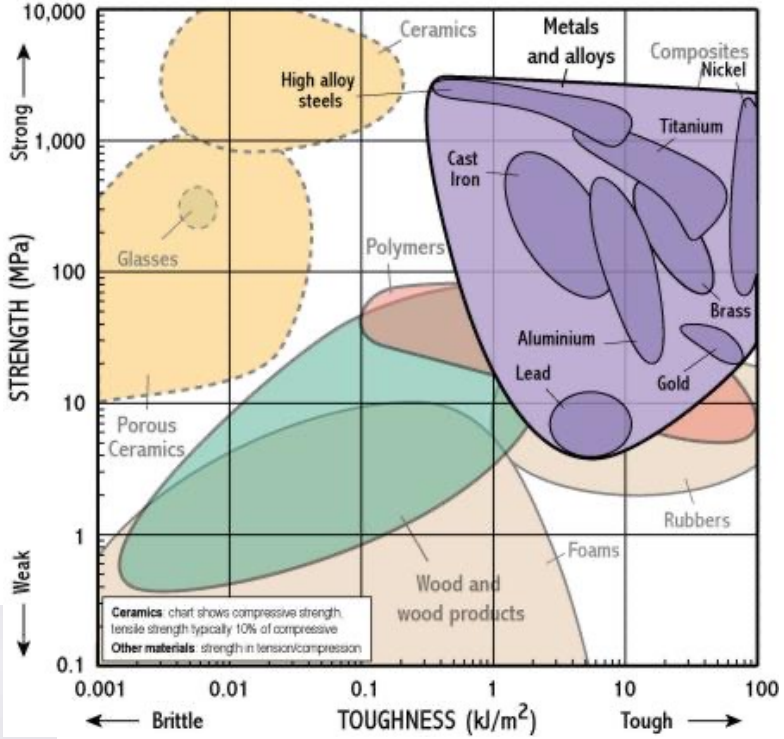


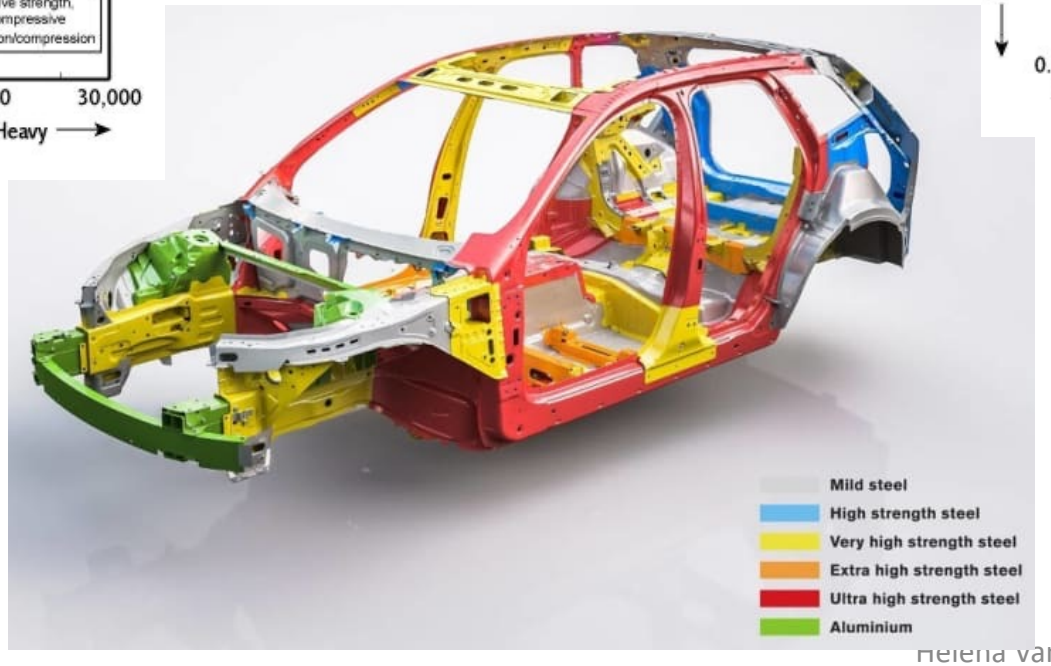
http://www-materials.eng.cam.ac.uk/mpsite/interactive_charts/strength-toughness/NS6Chart.html



Material selection

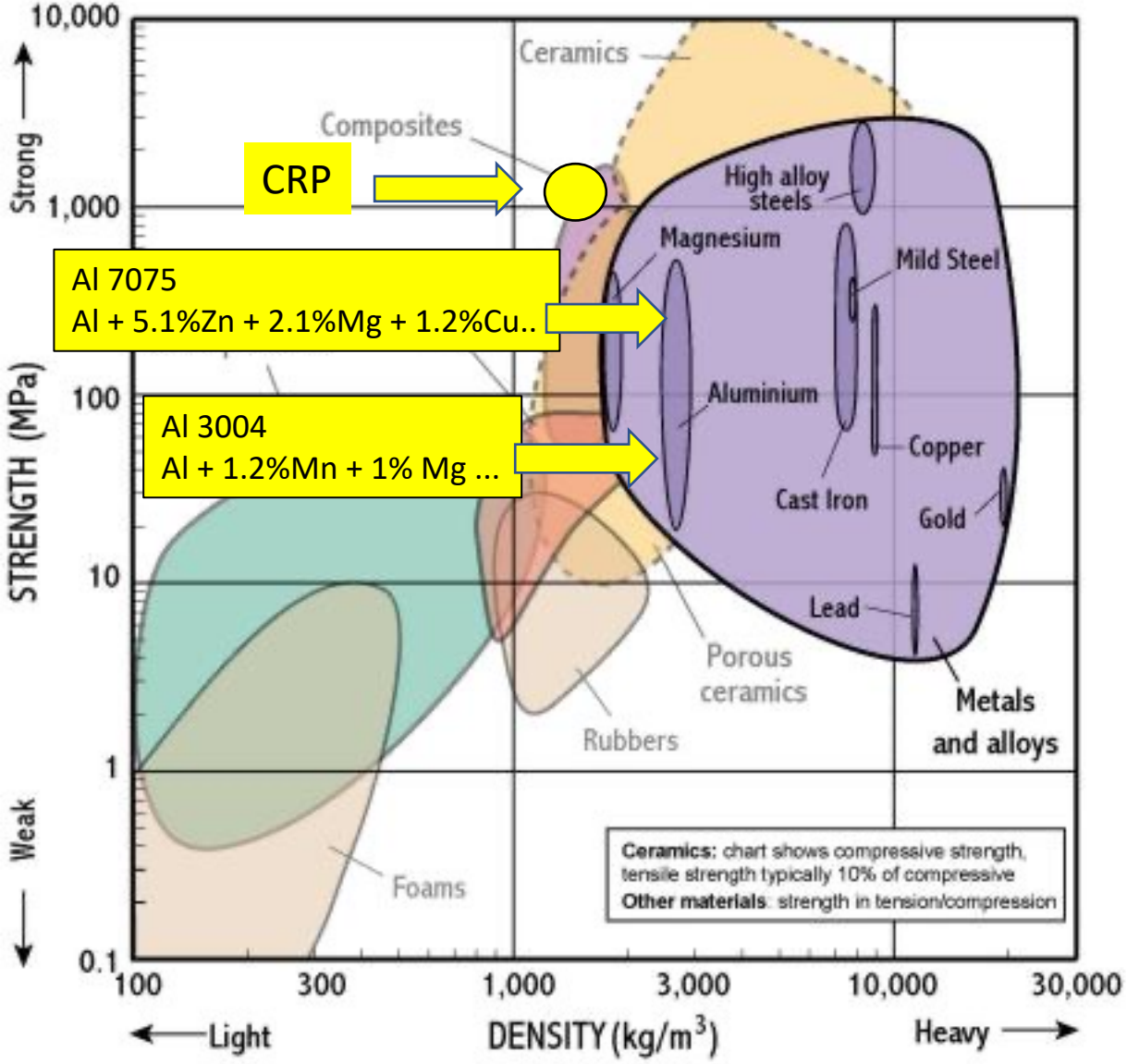






https://www.carsguide.com.au/oversteer/materials-in-motion-metal-56860

Material selection



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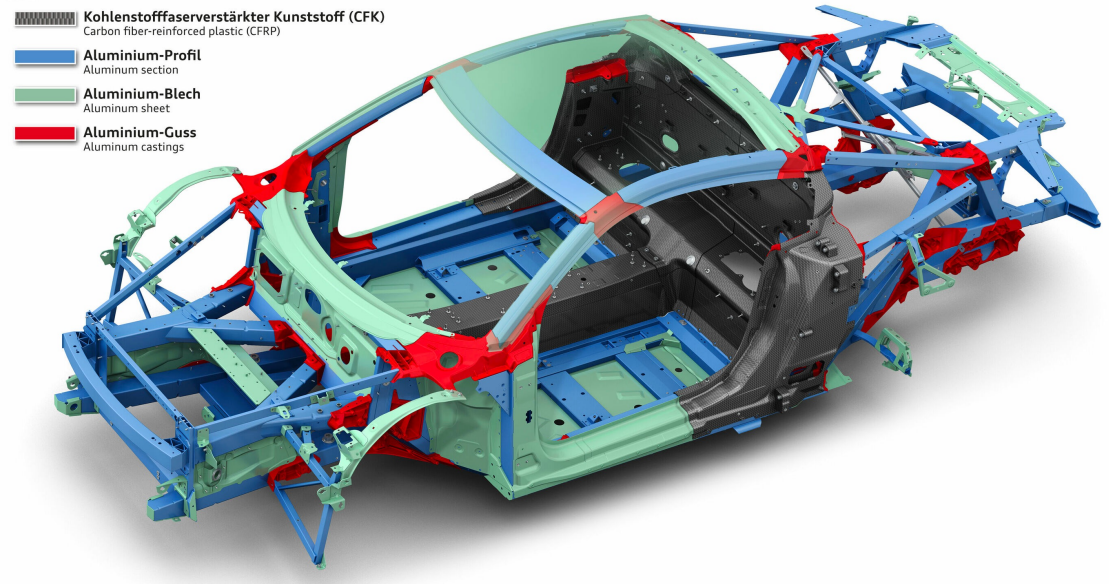
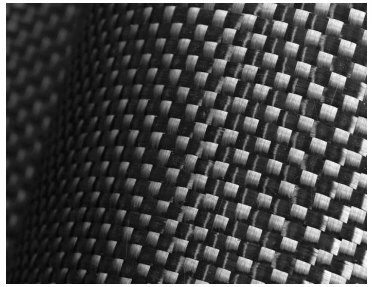


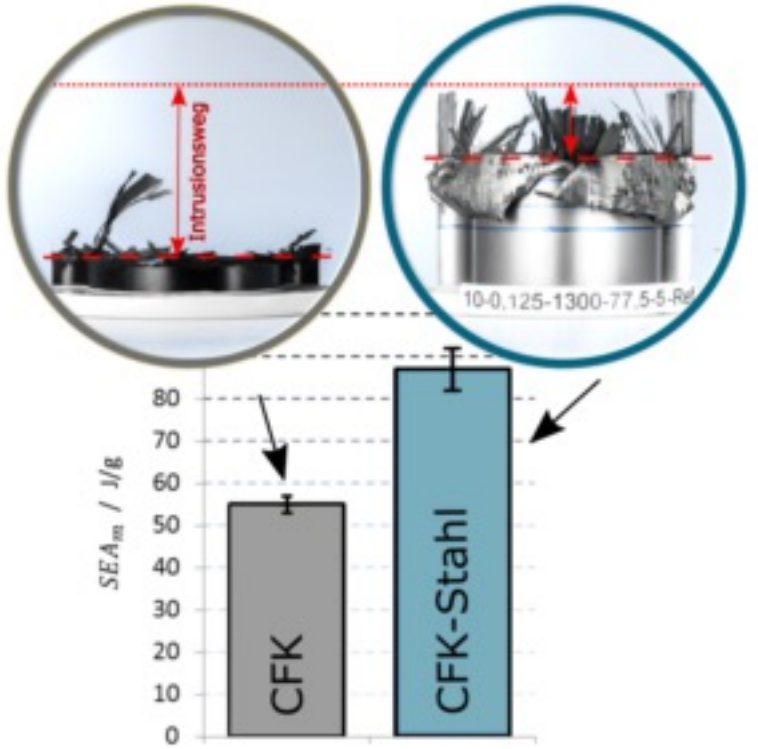


Strength 3-7GPa

Young's mod. 200-500GPa

Material selection: toughness and strength at impact!

https://www.ibf.rwth-aachen.de/cms/IBF/



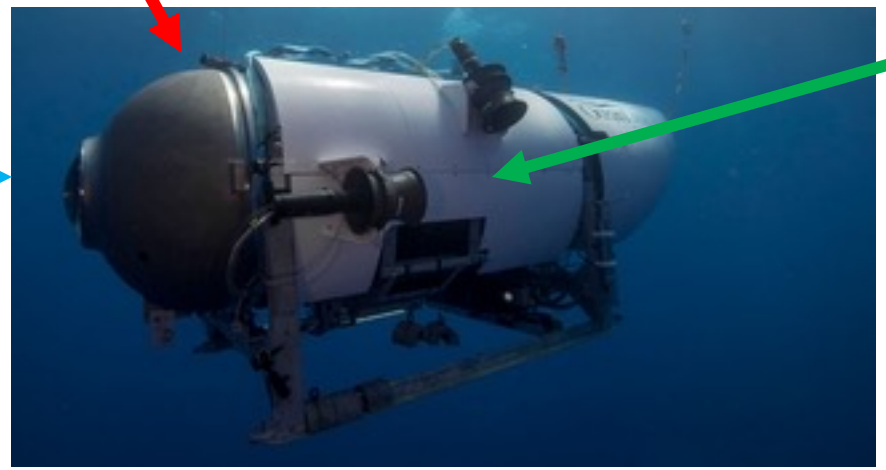
https://leichtbau.dlr.de/efficient-crash-structures-made-of-fiber-metal-laminates

Material selection: Pascal law, design!

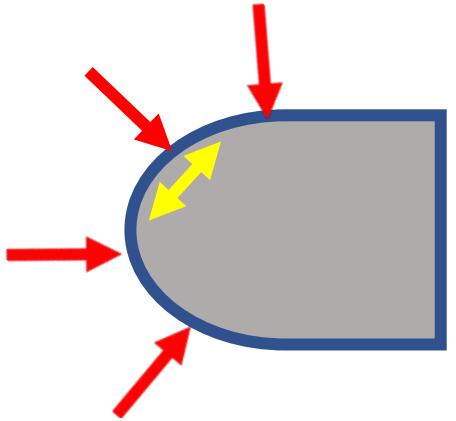
Titan submersible at 3500m under sea leval experience a pressure of **35-40 MPa**

Titanium (250-1250 MPa)

Acryl window



CFC 1500-3500 MPa
Carbon Fibre Composite



Pascal: pressure force is always perpendicular to surface

➔ Creates shear, compressive and buckling stresses in the material

Combination of different materials + CFC is very sensitive for failure under buckling ⇒ design!

Conclusion

Blaise Pascal's research on liquids at rest had an enormous footprint in engineering

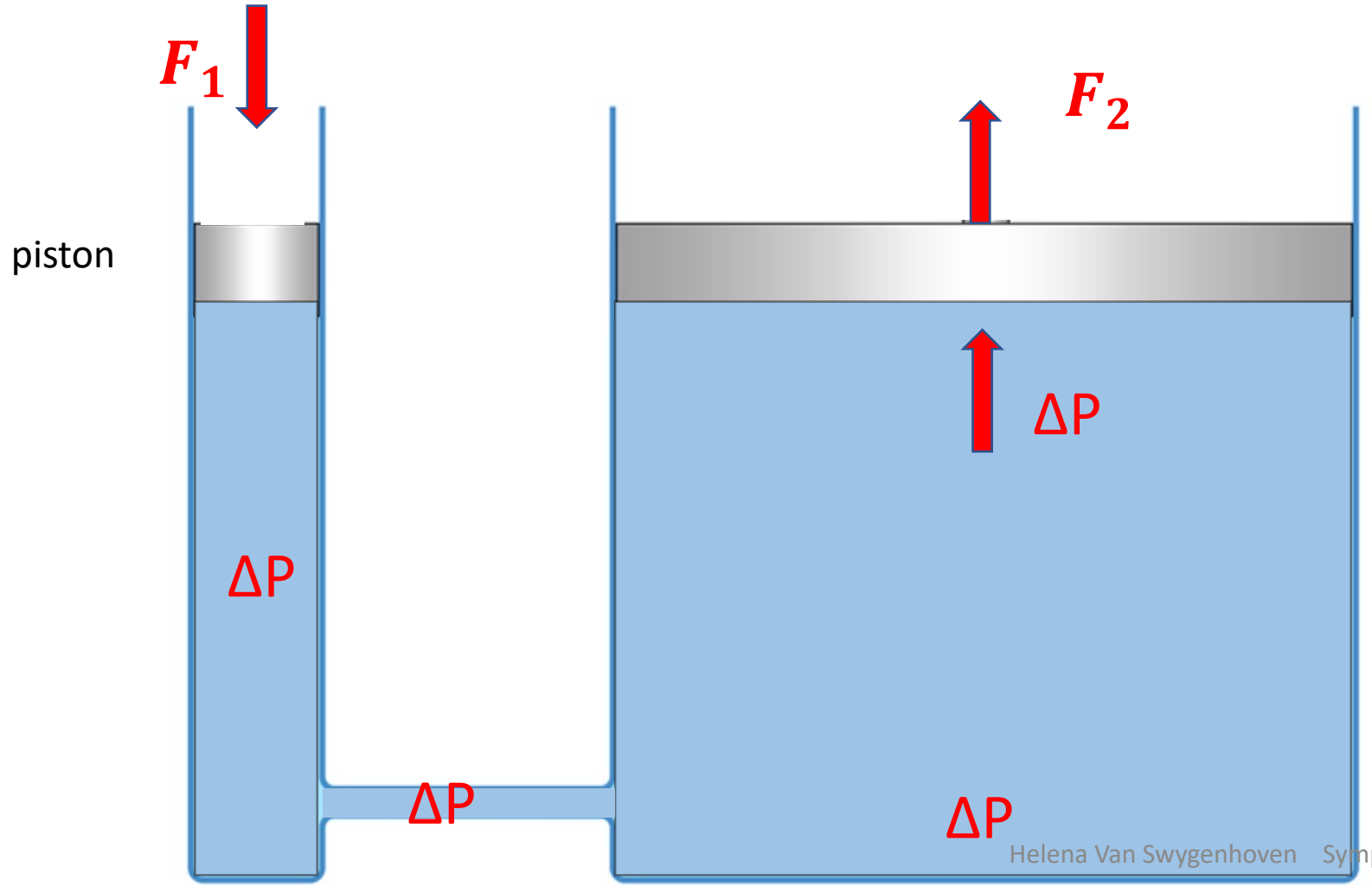
The unit Pascal named to honour his work is used on a daily basis in material science and engineering

Thank you Blaise Pascal !

Applications of Pascal's law in engineering:

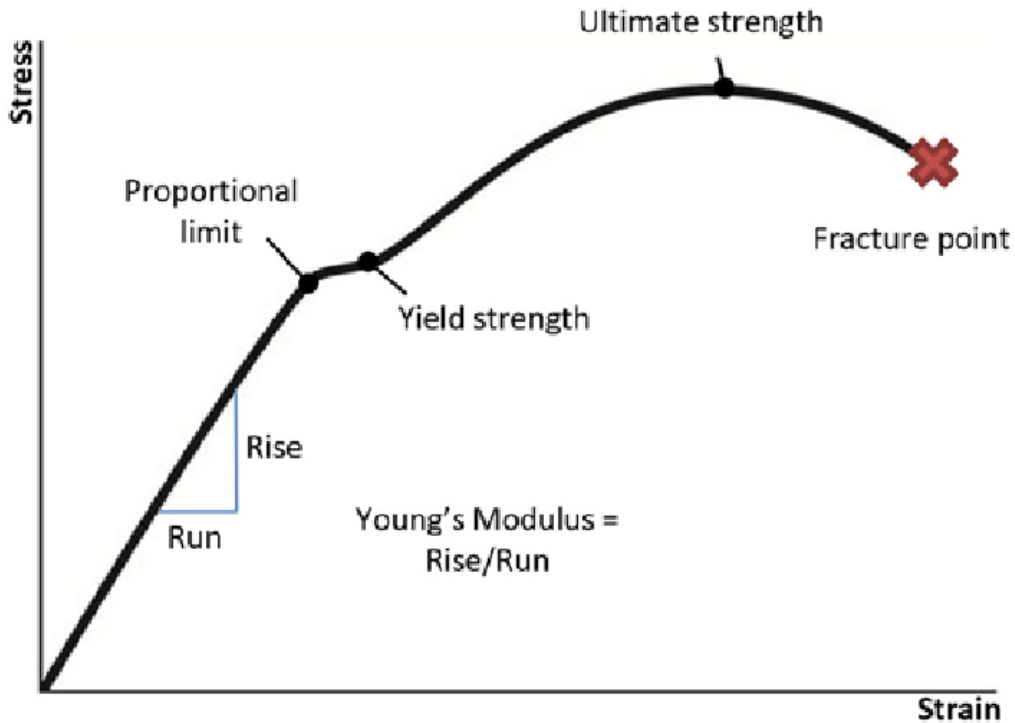
$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 > F_1$$



Pascal'law

Stress is a very important concept in materials science and eng



So not force but stress in Pa!!

$$E_c = V_m E_m + V_f E_f$$

Very good slides material selection

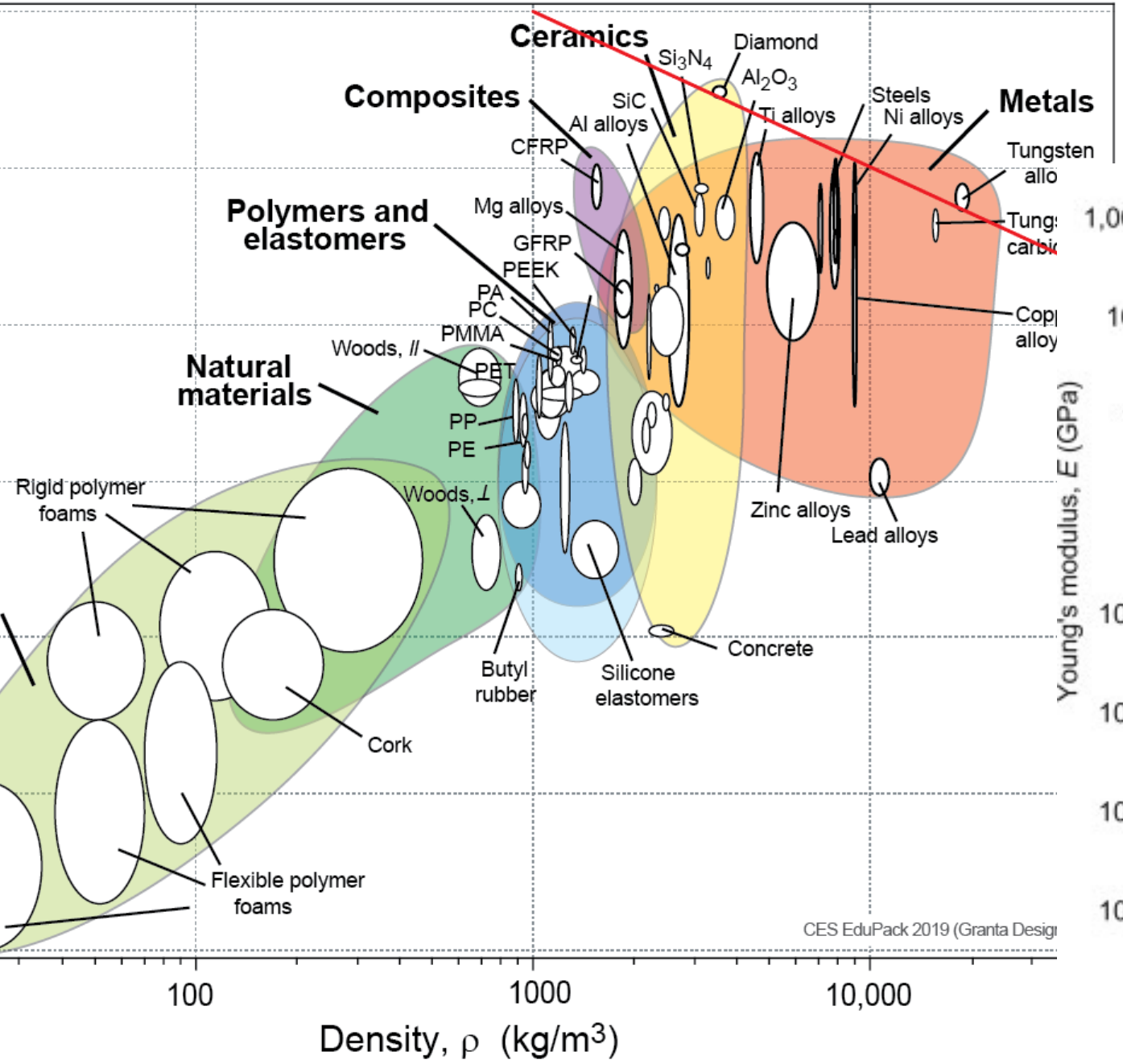
CFRP

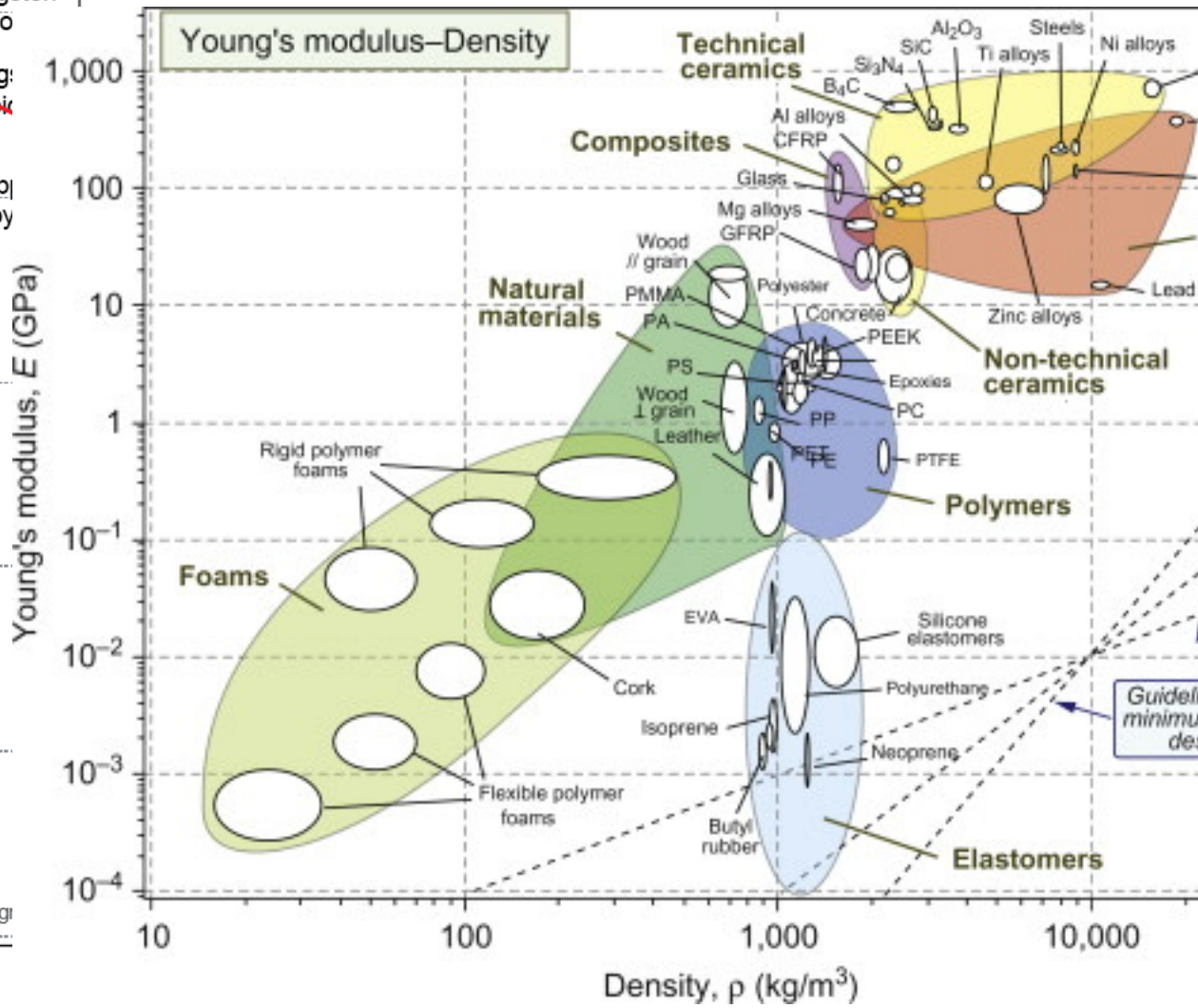
http://www.metaux.ulg.ac.be/Fichierpourtelech/Selection/Case%20Study%201.pdf

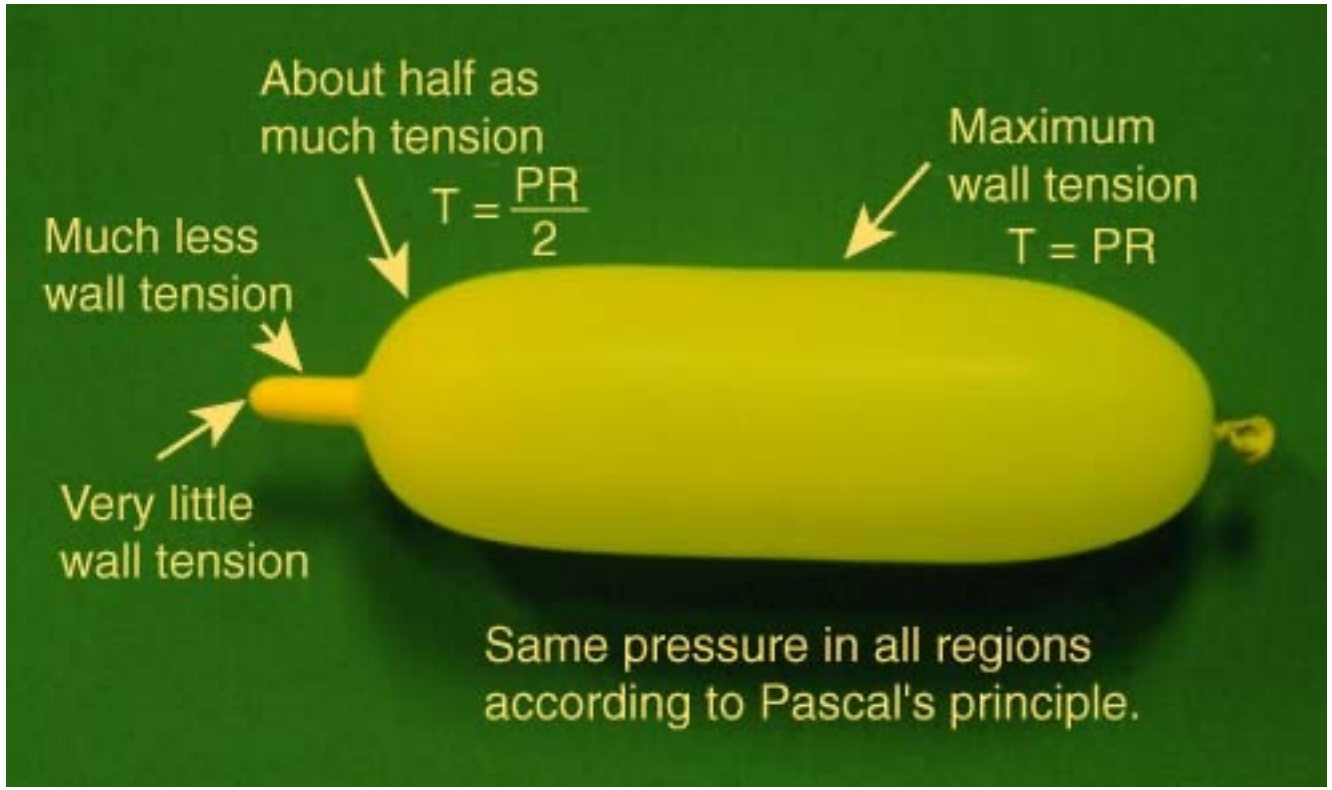
The binding polymer is often a thermoset resin such as epoxy, but other thermoset or thermoplastic polymers, such as polyester, vinyl ester, or nylon, are sometimes used

Commercial carbon fibers possess high tensile strength (ranging from 3 to 7 GPa) and high Young's modulus (ranging from 200 to 500 GPa).

https://www.grantadesign.com/education/students/charts/


Ceramics
Composites
Metals
Polymers and elastomers
Natural materials
Si3N4
Diamond
Al2O3
SiC
Al alloys
Ti alloys
Steels
Ni alloys
CFRP
Tungsten allo
Tung carbi
Mg alloys
GFRP
PEEK
PA
PC
PMMA
Woods, //
PET
PP
PE
Cop alloy
Rigid polymer foams
Woods, ⊥
Zinc alloys
Lead alloys
Concrete
Butyl rubber
Silicone elastomers
Cork
Flexible polymer foams
CES EduPack 2019 (Granta Desig
100
1000
10,000
Density, ρ (kg/m3)


Young's modulus–Density
Young's modulus, E (GPa)
1,000
100
10
1
10−1
10−2
10−3
10−4
Technical ceramics
Si3N4
B4C
SiC
Al2O3
Steels
Ni alloys
Ti alloys
Al alloys
CFRP
Composites
Glass
Mg alloys
GFRP
Wood // grain
Natural materials
PMMA
PA
Polyester
Concrete
PEEK
Zinc alloys
Lead
Non-technical ceramics
PS
Epoxies
Wood ⊥ grain
PC
Leather
PP
PET
PE
PTFE
Polymers
Rigid polymer foams
Foams
EVA
Silicone elastomers
Cork
Polyurethane
Isoprene
Neoprene
Flexible polymer foams
Butyl rubber
Elastomers
10
100
1,000
10,000
Density, ρ (kg/m3)


About half as
much tension
$T = \frac{PR}{2}$
Maximum
wall tension
$T = PR$
Much less
wall tension
Very little
wall tension
Same pressure in all regions
according to Pascal's principle.