Holographic QCD and the anomalous magnetic moment of the muon

Josef Leutgeb

w/ Jonas Mager, Anton Rebhan

Institute for Theoretical Physics
TU Wien, Vienna, Austria

Joint Annual Meeting of the ÖPG and SPS
September 6, 2023

$\int dk\,\Pi$

Doktoratskolleg
Particles and Interactions

TU WIEN

Outline of the talk

- Current status of the anomalous magnetic moment of the muon
- Introduction to holographic QCD
- Predictions from holographic QCD

Anomalous magnetic moment

Angular momentum L of charged particles produces magnetic moment

$$\vec{\mu} = \mu_B \vec{L} \qquad \text{with} \quad \mu_B = \frac{e}{2m} \quad (\hbar = 1, c = 1)$$

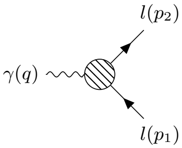
Fundamental point particles with spin S have intrinsic magnetic moment with anomalous g-factor

$$\vec{\mu} = g \mu_B \vec{S}$$

Special relativity plus quantum mechanics (Dirac equation): $g = 2$

QFT corrections parametrized by $a = \frac{1}{2}(g - 2)$

Field theory definition:



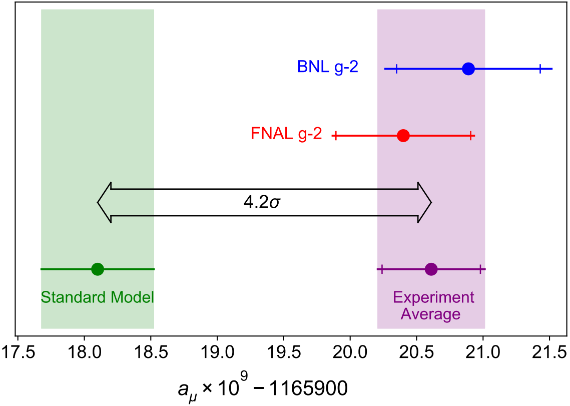
$$= (-ie)\bar{u}(p_2)\left[\gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_M(q^2)\right]u(p_1)$$

with $q = p_2 - p_1$ and $a = F_M(0)$

Status of anomalous magnetic moments

- Perfect agreement for the electron to $\mathcal{O}(\alpha^5)$
- However 4.2σ discrepancy for muon [BNL 2004 and FNL 2021 vs. Aoyama et al. 2020]

$$a_\mu^{\text{exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$$



- New result consistent and uncertainties halved (5.1σ) [FNL 2023 Preprint]

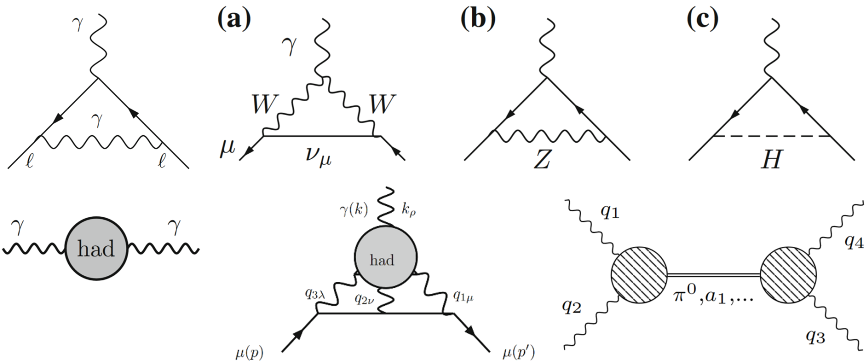
Standard Model prediction

[Aoyama et al., 2006.04822]

Muon 200 times heavier than electron $\Rightarrow$ more sensitive to non-QED and BSM physics

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

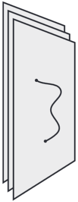
$$\begin{aligned}
a_\mu^{\text{QED}} &= (116\,584\,718.931 \pm 0.104) \times 10^{-11} \\
a_\mu^{\text{EW}} &= (153.6 \pm 1.0) \times 10^{-11} \\
a_\mu^{\text{HVP}} &= (6845 \pm 40) \times 10^{-11} \quad (0.6\% \text{ uncertainty}) \\
a_\mu^{\text{HLbL}} &= (92 \pm 19) \times 10^{-11} \quad (20\% \text{ uncertainty!})
\end{aligned}$$

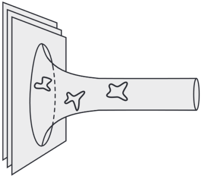


Holographic QCD

Strongly coupled gauge theory in D dimensions at large N
is dual to a weakly coupled theory of gravity in D+1 dimensions

gauge theory	**gravity dual**
degree N of the gauge group	number of branes, curvature radius
flat space time on which the gauge theory lives	boundary of higher-dimensional geometry
global symmetry	gauge symmetry
gauge invariant operators	fields acting as sources to these operators
particle mass	eigenvalue in wave equation
energy scale	radial coordinate in the AdS-space
renormalisation group flow	movement along the radial coordinate





Hard wall model (HW1)

In bottom-up hQCD models, pions & (axial) vector mesons described by 5d-YM fields $\mathcal{F}^{L,R}_{MN} = \mathcal{F}^{V}_{MN} \mp \mathcal{F}^{A}_{MN}$ in AdS$_5$ space with metric

$$ds^2 = z^{-2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

To make the theory confining a finite (hard wall) cutoff is introduced at $z = z_0$

The 5-dimensional Yang-Mills action reads

$$S_{\rm YM} = -\frac{1}{4g_5^2} \int d^4x \int_0^{z_0} dz\, \sqrt{-g}\, g^{PR} g^{QS}\, \mathrm{tr}\left(\mathcal{F}^{\rm L}_{PQ}\mathcal{F}^{\rm L}_{RS} + \mathcal{F}^{\rm R}_{PQ}\mathcal{F}^{\rm R}_{RS}\right)$$

with $P, Q, R, S = 0, \dots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

In the HW1 model we additionally have a minimally coupled scalar field

$$S_X = \int d^4x \int_0^{z_0} dz\, \sqrt{-g}\, \mathrm{tr}\left(|DX|^2 + 3|X|^2\right)$$

where $DX = \partial X - i\mathcal{B}^{\rm L} X + iX\mathcal{B}^{\rm R}$ and $X = e^{i\pi^a(x,z)t^a}[\frac{1}{2}v(z)]e^{i\pi^a(x,z)t^a}$, with $v(z) = m_q z + \sigma z^3$, where m_q is the quark mass and σ the quark condensate.

Anomalous TFFs from holographic QCD

Anomalies follow uniquely from 5-dimensional *Chern-Simons* term:

$$S_{\rm CS}^L - S_{\rm CS}^R, \quad S_{\rm CS} = \frac{N_c}{24\pi^2}\int \mathrm{tr}\left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5\right)$$

- The pion transition form factor is given by

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} dz\, \mathcal{J}(Q_1,z)\mathcal{J}(Q_2,z)\Psi(z) + \text{b.t.}$$

  with *bulk-to-boundary* propagator $\mathcal{J}$ and holographic pion profile Ψ

- The amplitude for axial-vector mesons $a_\mu^{(n)}$ decaying into two virtual photons following from the Chern-Simons action has the form

$$\mathcal{M}^a = i\frac{N_c}{4\pi^2}\mathrm{tr}(\mathcal{Q}^2 t^a)\,\epsilon^\mu_{(1)}\epsilon^\nu_{(2)}\epsilon^{*\rho}_A\epsilon_{\mu\nu\rho\sigma}\left[q^\sigma_{(2)}Q_1^2 A_n(Q_1^2,Q_2^2) - q^\sigma_{(1)}Q_2^2 A_n(Q_2^2,Q_1^2)\right]$$

  where

$$A_n(Q_1^2,Q_2^2) \quad = \tfrac{2g_5}{Q_1^2}\int_0^{z_0} dz\left[\tfrac{d}{dz}\mathcal{J}(Q_1,z)\right]\mathcal{J}(Q_2,z)\psi_n^A(z)$$

- Landau-Yang theorem (AV$\to\gamma\gamma$ is forbidden) realized by $\mathcal{J}'(Q,z)=0$ for $Q^2=0$

Short-distance constraints on TFFs

Amazingly, bottom-up models with asymptotic AdS_5 geometry reproduce **asymptotic momentum dependence of pQCD** [Brodsky & Lepage 1979-81] for PS and AV

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]

$$F^{\mathrm{HW1}}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \quad\rightarrow\quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right]$$

with $Q^2=\frac{1}{2}(Q_1^2+Q_2^2)$, $w=(Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponds to the asymptotic behavior

$$F^\infty(Q^2,0)=\frac{2f_\pi}{Q^2},\qquad F^\infty(Q^2,Q^2)=\frac{2f_\pi}{3Q^2}$$

- **Axial-vector mesons** [JL & Rebhan, 1912.01596]
(agreeing with later pQCD result [Hoferichter & Stoffer 2004.06127]):

$$A_n(Q_1^2,Q_2^2)\rightarrow\frac{12\pi^2F_n^A}{N_cQ^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Other hQCD models

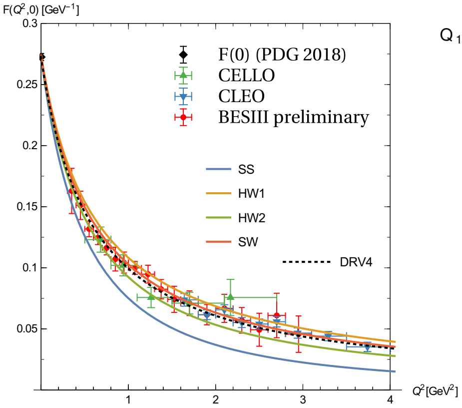
- Hirn-Sanz model (HW2)
  - No scalar field X
  - Chiral symmetry breaking through boundary conditions
- Soft-wall model (SW)
  - No scalar field X
  - No Hard-wall cutoff ($z_0 \to \infty$)
  - Confinement through non-trivial dilaton
- Sakai-Sugimoto model (SS)
  - No scalar field X
  - Top-down model (from 10d string-theory/supergravity)
  - Not asymptotically AdS$_5$
  - Confinement and chiral symmetry breaking through brane construction

Holographic TFFs and experimental data

Single-virtual pion TFF:

[JL, J. Mager & A. Rebhan, 1906.11795]

(data from Danilkin et al., Prog.Part.Nucl.Phys. 107 (2019) 20)



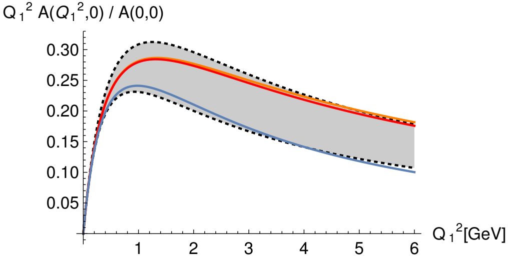
Danilkin et al. (DRV) fit below 4 GeV^2

bracketed by HW1 and HW2!

Single-virtual axial TFF:

[JL & A. Rebhan, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:



$$A(0,0)^{\mathrm{L3\,exp.}}_{f_1(1285)} = 16.6(1.5)\,\mathrm{GeV}^{-2}$$

Roig & Sanchez-Puertas, 1910.02881:

$$A(0,0)_{a_1(1230)} = 19.3(5.0)\,\mathrm{GeV}^{-2}$$

hQCD results:	HW1	HW2
$\lvert A(0,0)\rvert$ [GeV^{-2}]	21.04	16.63

Hadronic light-by-light scattering

[Colangelo et al. 1506.01386]

Lorentz- and gauge invariance: interaction of four electromagnetic currents described by 12 scalar functions $\bar{\Pi}_i$

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$

PS only contribute to $\bar{\Pi}_1$ (and $\bar{\Pi}_2$, $\bar{\Pi}_3$ through crossing symmetry)

$$\bar{\Pi}_1^{PS} = -\sum_{n=1}^\infty \frac{F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2)F_{\pi^0\gamma^*\gamma}(Q_3^2,0)}{Q_3^2+m_\pi^2}$$

AV contribute to all 12. $\bar{\Pi}_1$ has contribution from longitudinal component

$$\bar{\Pi}_1^{AV} = -\frac{g_5^2}{2\pi^4}\sum_{n=1}^\infty \int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q,z)\right]\mathcal{J}(Q,z)\psi_n^A(z)\frac{1}{(M_n^A Q_3)^2}\int_0^{z_0} dz' \left[\frac{d}{dz'}\mathcal{J}(Q_3,z')\right]\psi_n^A(z')$$

Melnikov-Vainshtein short-distance constraint

- Asymptotic (high-energy) regime not expected to give large contribution to HLbL but important for error estimate
- Short-distance constraint (SDC) derived from OPE by Melnikov & Vainshtein

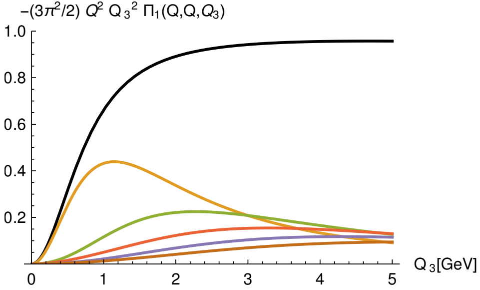
$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

  can be used to see if a particular set of intermediate states is sufficient
- Only 3 known ways to satisfy with hadronic degrees of freedom:
  - Replacing the single-virtual TFF by hand (MV)
  - Summing an infinite number of excited PS mesons in a Regge model (Colangelo et al.)
  - Summing an infinite number of AV mesons in holographic models (LR, Cappiello et al.)

Axial-vector contributions to SDC

Infinite tower of axial-vector mesons responsible for satisfying the longitudinal SDC

- MV-SDC $\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$: 100% for HW1 and HW2(UV-fit)



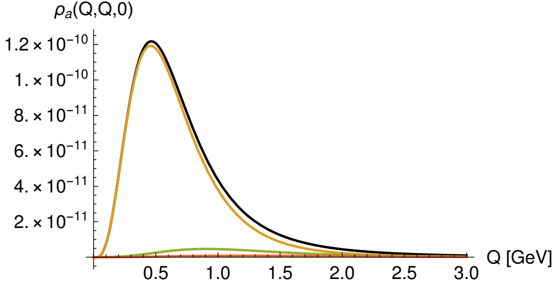
black line: infinite sum
colored lines: first 5 axial-vector modes

- SDC for symmetric limit $Q_1^2 = Q_2^2 = Q_3^2 \to \infty$ satisfied qualitatively, but quantitatively only at max. 80% level (for HW1 and HW2(UV-fit))

Contributions to muon $g-2$

$a_\mu^{\rm AV} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, \rho_a(Q_1,Q_2,\tau)$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible (25%) contribution from higher modes

	(z_0 s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; degenerate a_1, f_1, f_1')		
	HW1 (100% LSDC)	HW2 (62% LSDC)	SM
$a_\mu^{\rm PS}[\pi^0+\eta+\eta'] \times 10^{11}$	99 [65+18+16]	84 [57+15+12]	93.8(4.0)
$a_\mu^{\rm AV}[L+T] \times 10^{11}$	41 [23+18]	29 [17+12]	21(16) [15(10)+6(6)]
$a_\mu^{\rm PS+AV} \times 10^{11}$	140	112	115(20)

(compare with MV model: longitudinal contribution estimated $\sim$ 38 $\times 10^{-11}$)

Additional holographic predictions

- Effect of finite quark masses (in HW1 model) and perturbative corrections (estimated by reducing g_5^2 by 15%) [JL & A. Rebhan, 2108.12345]

$$a_\mu^{\mathrm{P}^*} = (3.2\ldots7.2)\times10^{-11}$$
$$a_{\mu(L)}^{\mathrm{A}} = (20.8\ldots25.0)\times10^{-11}$$
$$a_\mu^{\mathrm{A}} = (36.6\ldots43.3)\times10^{-11}$$
$$a_\mu^{\mathrm{A}+\mathrm{P}^*} = (39.8\ldots50.5)\times10^{-11}$$

  MV-SDC is still satisfied through tower of axial-vector mesons; massive pions only have subleading contribution $\propto \log(Q_3^2)/Q_3^4Q^2$

- Estimate of glueball contribution (in SS model) [JL, Dissertation]
  - Glueballs are dual to fluctuations of the background geometry
  - Brane-embedding determines glueball-meson interaction
  - Combined with the model's VMD this leads to surprisingly large radiative glueball decays (decay rates in keV instead of eV)
  - However glueball contribution is negligible

$$a_\mu^G \lesssim 0.16\times10^{-11}$$

Conclusions

- hQCD is not QCD, but sophisticated toy model that can give clues on
  - how short-distance constraints can be implemented at the hadronic level
    - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly
  - a semi-quantitative estimates of the ballparks to be expected (HW1–HW2 brackets experimental results for pion TFF!)
    - axial-vector contributions more important numerically than estimated previously

$$a_\mu^{\mathrm{AV}}[L+T] = \mathbf{35(6)}\,[20(3)+15(3)]\times 10^{-11} \qquad \text{for HW1–HW2}$$

vs. WP: $a_\mu^{\mathrm{SDC+axials}} = \mathbf{21(16)}\,[15(10)+6(6)]\times 10^{-11}$

    - with quark masses MV-SDC still completely satisfied through tower of axial-vector mesons; massive pions have subleading contribution
  - an estimate of the glueball contribution
    - although glueballs have surprisingly large radiative decays their contribution is negligible at the current uncertainty