AXEL-2023

Introduction to Particle Accelerators

Transverse instabilities:

- ✓ *How do they arise*
- ✓ *Single-bunch effects ("head-tail" instability)*
- ✓ *Multi-bunch modes (very brief)*
- ✓ *Possible cures*
- ✓ *Space charge effects*

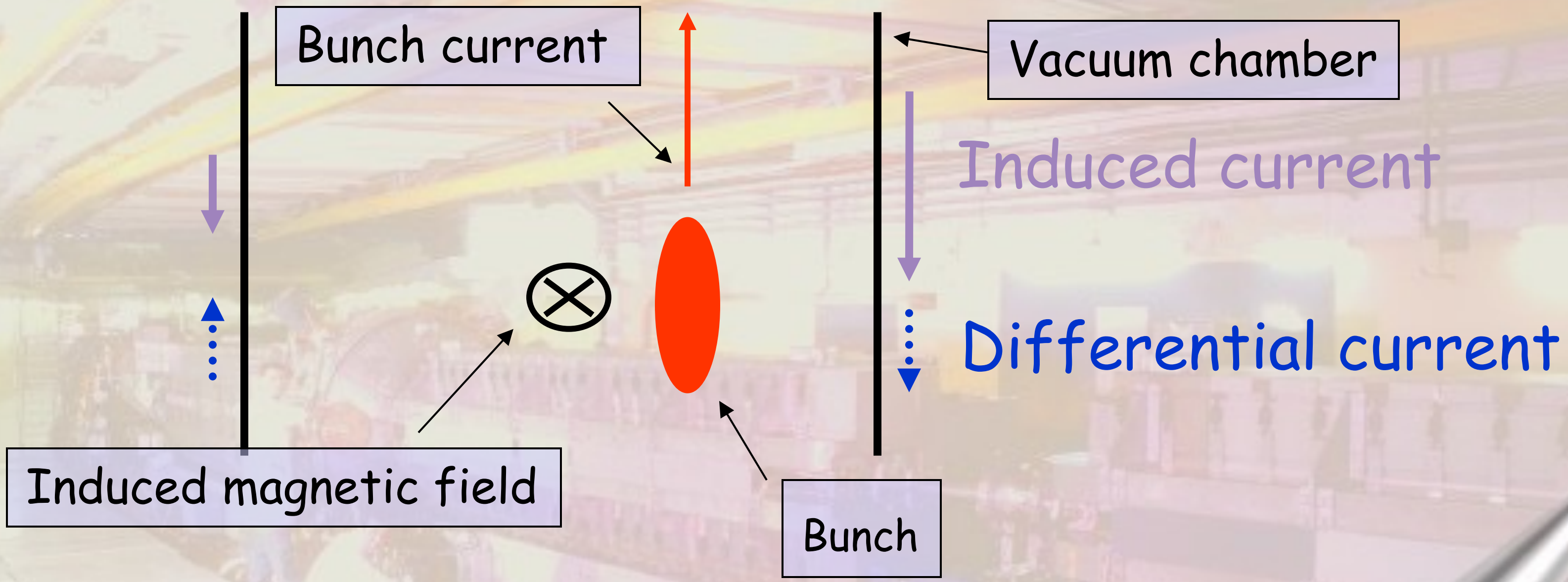
Rende Steerenberg (BE/OP)

3 March 2023

Coherent Transverse Oscillation (1)

- The complete bunch is displaced form side to side (or up and down)
- A bunch of charged particles induces a charge in the vacuum chamber
- This creates an image current in the vacuum chamber walls
- How can these currents affect transverse motion?

Coherent Transverse Oscillation (2)



Bunch current

Vacuum chamber

Induced current

Differential current

Induced magnetic field

Bunch

- If the bunch is displaced form the centre of the vacuum chamber it will drive a differential wall current
- This leads to a magnetic field, which deflects the bunch

Transverse coupling impedance (1)

- We characterize the electromagnetic response to the bunch by a "transverse coupling impedance" (as for longitudinal case)

$$\int (Z_{\perp}(\omega)\times I(\omega))\,d\omega = \int_{0}^{S} (E+v\times B)\,ds$$

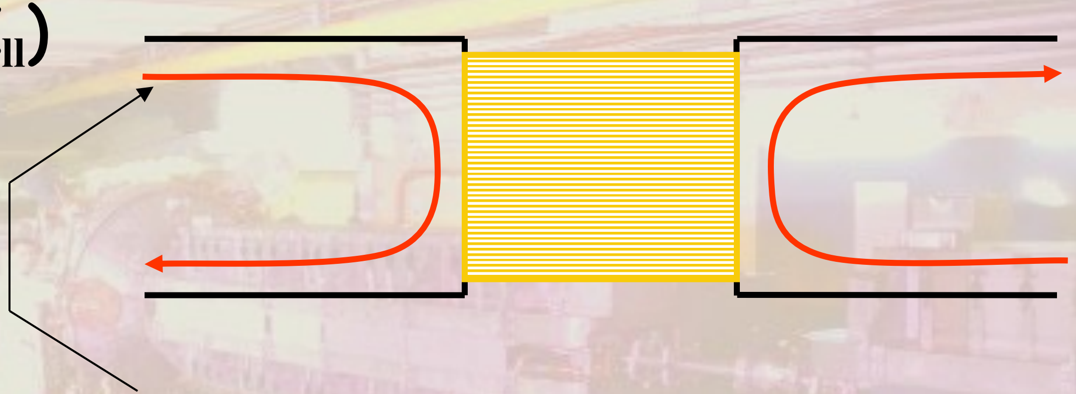
Frequency spectrum of bunch current

Transverse E & B fields summed around the machine

- $Z_{\perp}$ (exactly as $Z_{\parallel}$) is also a function of frequency
- $Z_{\perp}$ also has resistive, capacitive and inductive components
- However, there is one big difference between $Z_{\perp}$ & $Z_{\parallel}$

Transverse coupling impedance (2)

- For a vacuum chamber with a short non-conduction section the direct image current sees a high impedance (large Z_{ll})



- For The differential current (current loops) is not greatly affected so $Z_{\perp}$ is unchanged by the non-conducting section
- Thus:
  - Any interruption to a smooth vacuum chamber increases Z_{ll}
  - Any structure that will support current loops increases $Z_{\perp}$

Relationship with the longitudinal plane

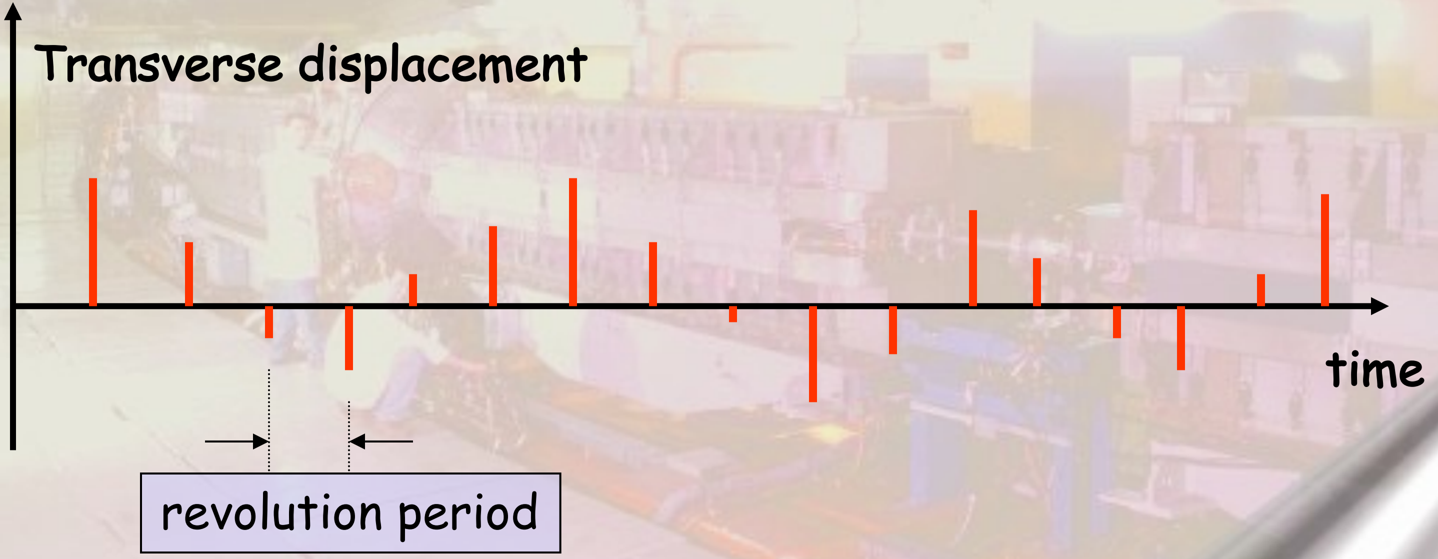
- **Longitudinal instabilities** are related to **synchrotron** oscillations
- **Transverse instabilities** are related to **synchrotron and betatron** oscillations
- Why….?….
- Particles move around the machine and execute synchrotron and betatron oscillations
- If the chromaticity $\left(\xi = \frac{\Delta Q}{Q} \bigg/ \frac{\Delta p}{p}\right)$ is non zero
- Then the changing energy, due to synchrotron oscillations will also change the betatron oscillation frequency (Q)

Single bunch modes

- As for longitudinal oscillation there are different **modes** for single bunch transverse oscillations
- We can observe the transverse bunch motion from the difference signal on a position monitor

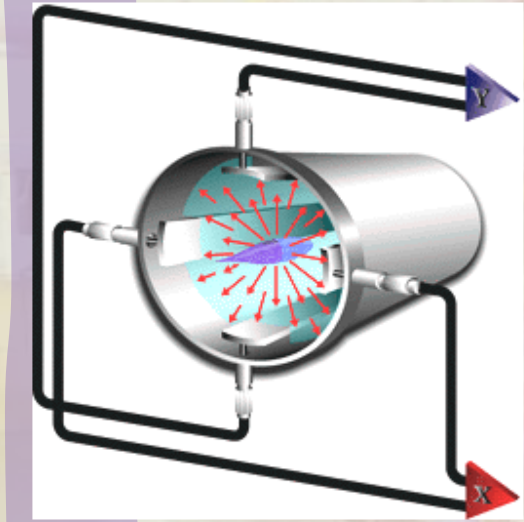
Rigid bunch mode (1)

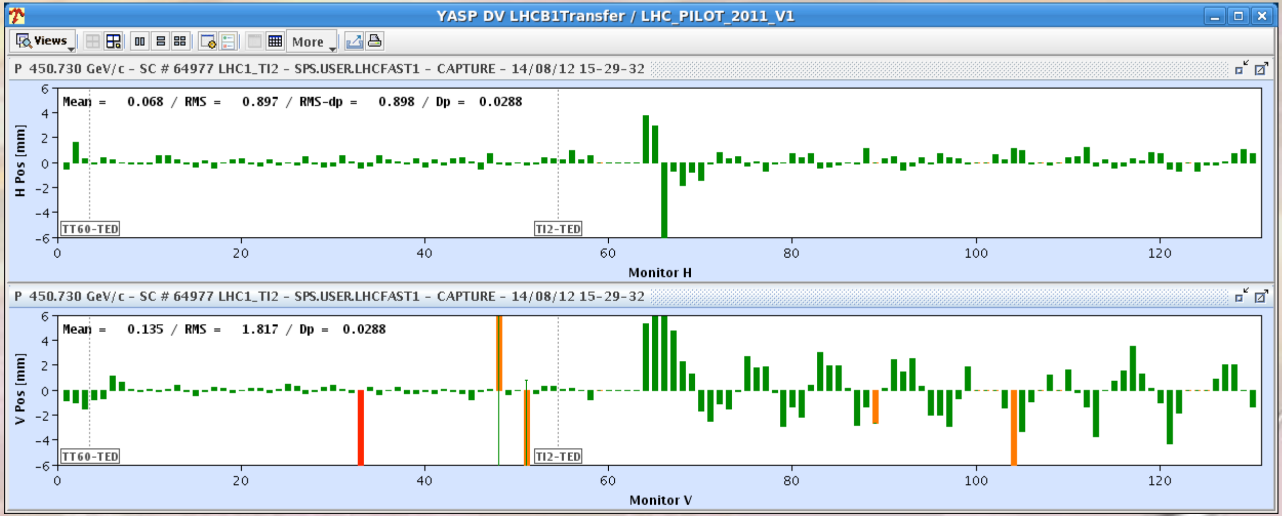
- The bunch oscillates transversely as a rigid unit
- On a single position sensitive pick-up we can observe the following:



Change in position/turn ⇒ betatron phase advance/turn

Beam Position Measurement





P 450.730 GeV/c - SC # 64977 LHC1_TI2 - SPS.USER.LHCFAST1 - CAPTURE - 14/08/12 15-29-32

Mean = 0.068 / RMS = 0.897 / RMS-dp = 0.898 / Dp = 0.0288

P 450.730 GeV/c - SC # 64977 LHC1_TI2 - SPS.USER.LHCFAST1 - CAPTURE - 14/08/12 15-29-32

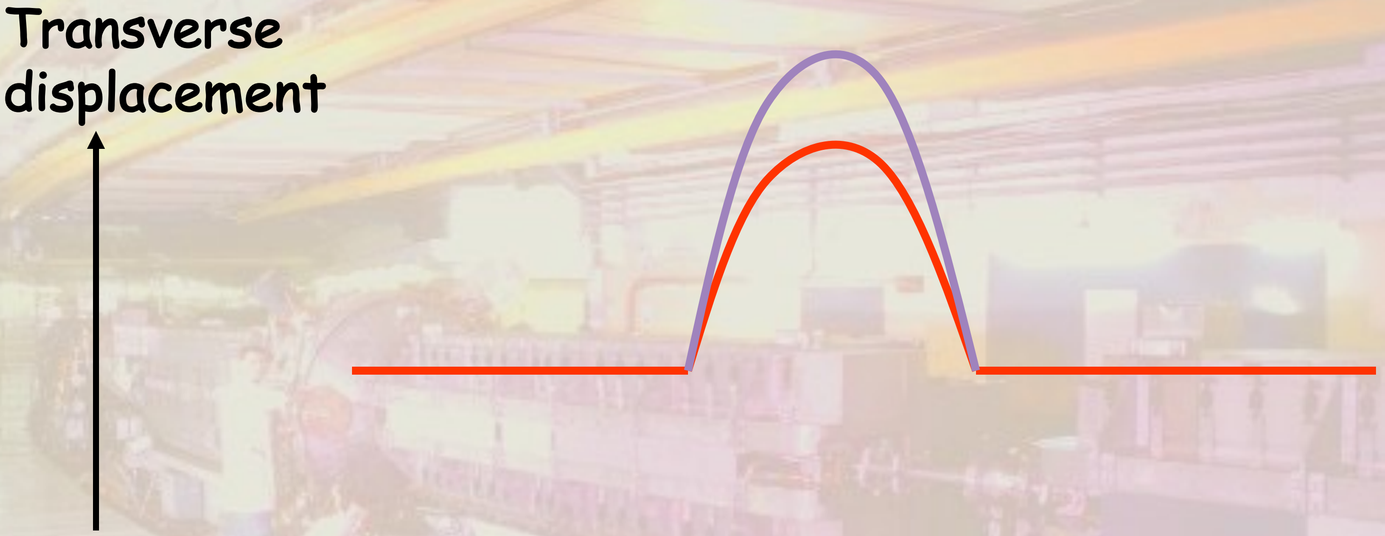
Mean = 0.135 / RMS = 1.817 / Dp = 0.0288

$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta}{\Sigma}$$

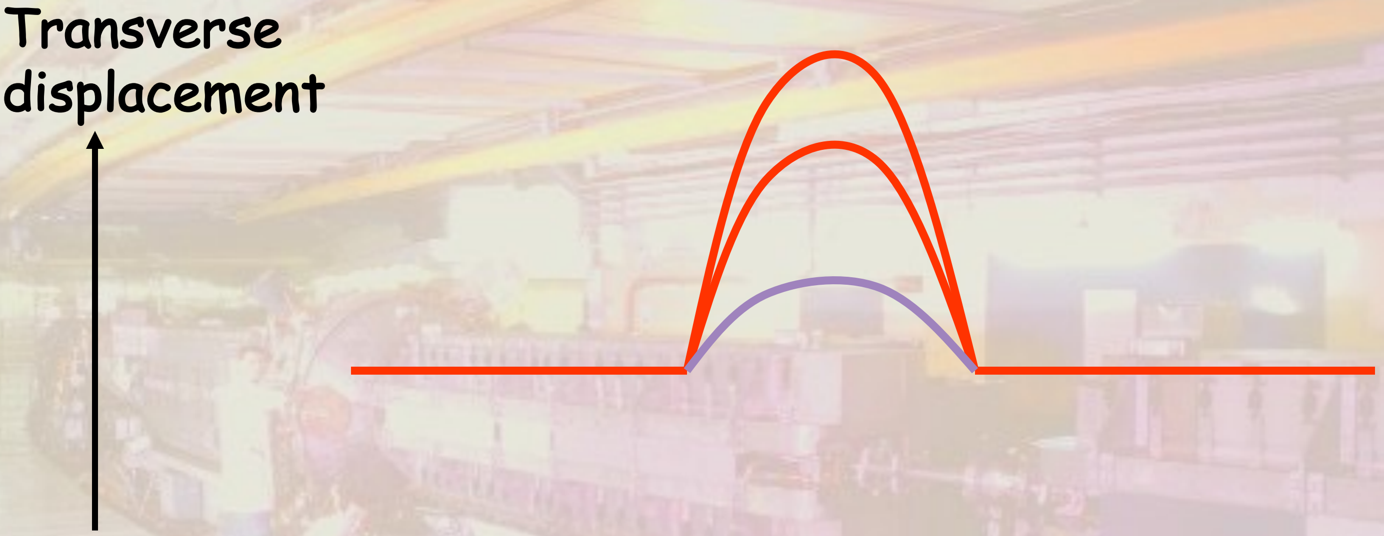
Rigid bunch mode (2)

Lets superimpose successive turns

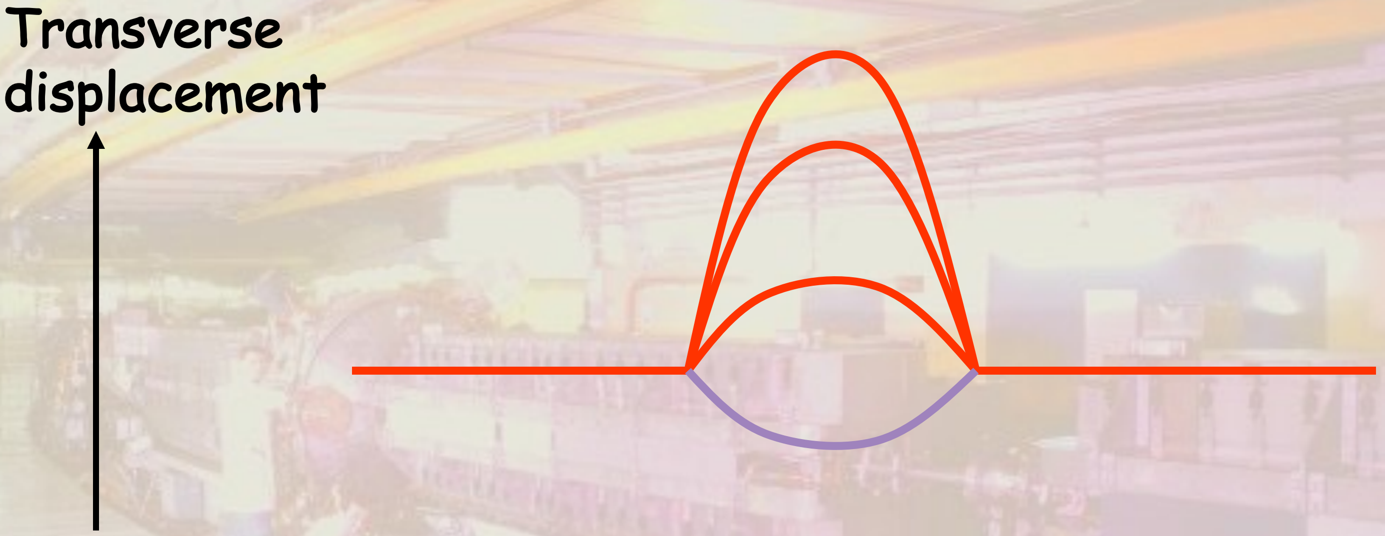
Rigid bunch mode (3)



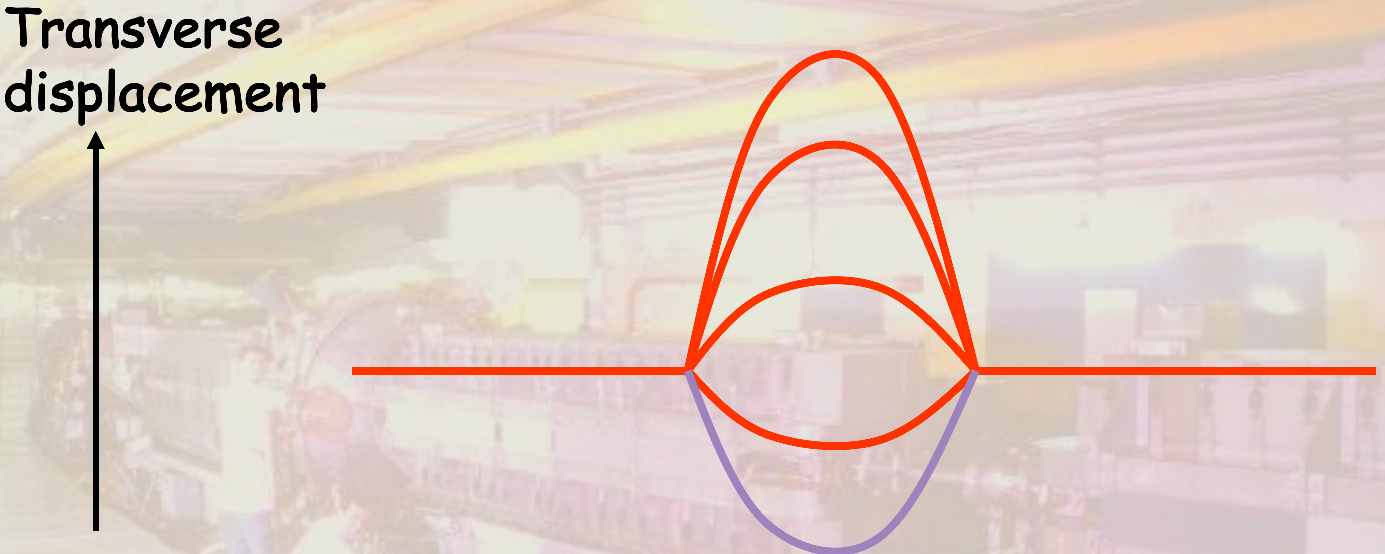
Rigid bunch mode (4)



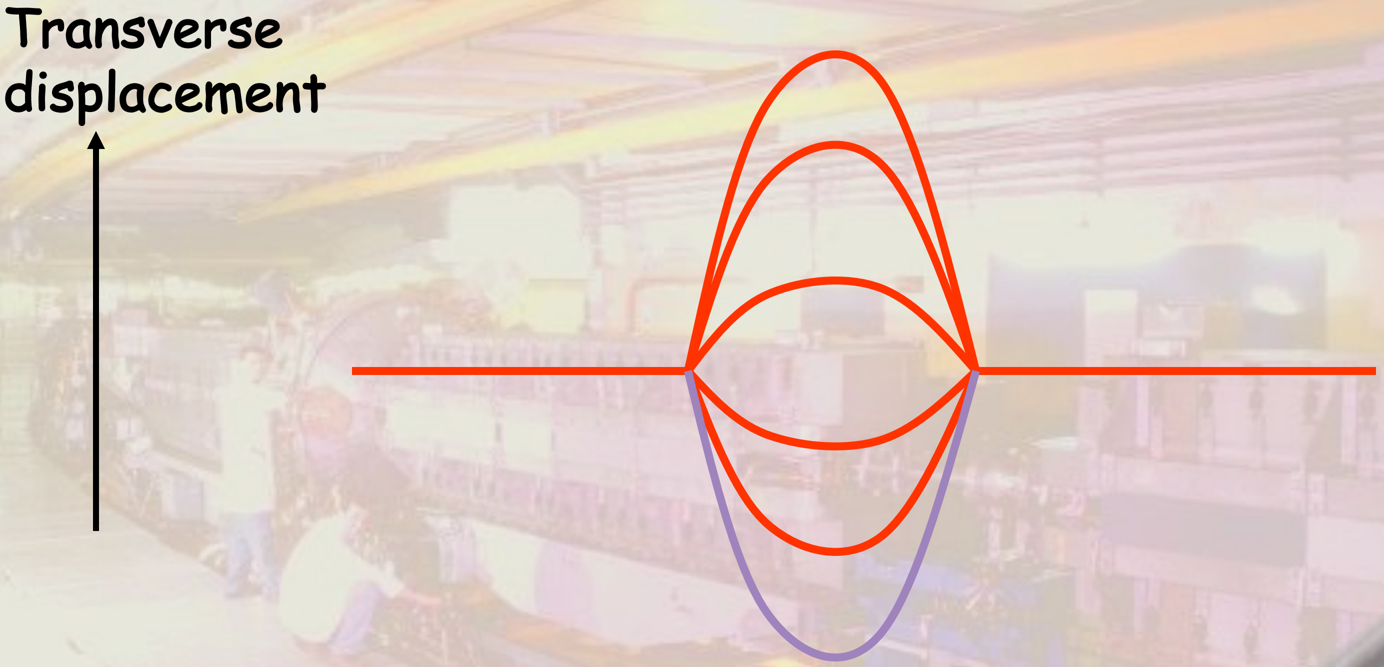
Rigid bunch mode (5)



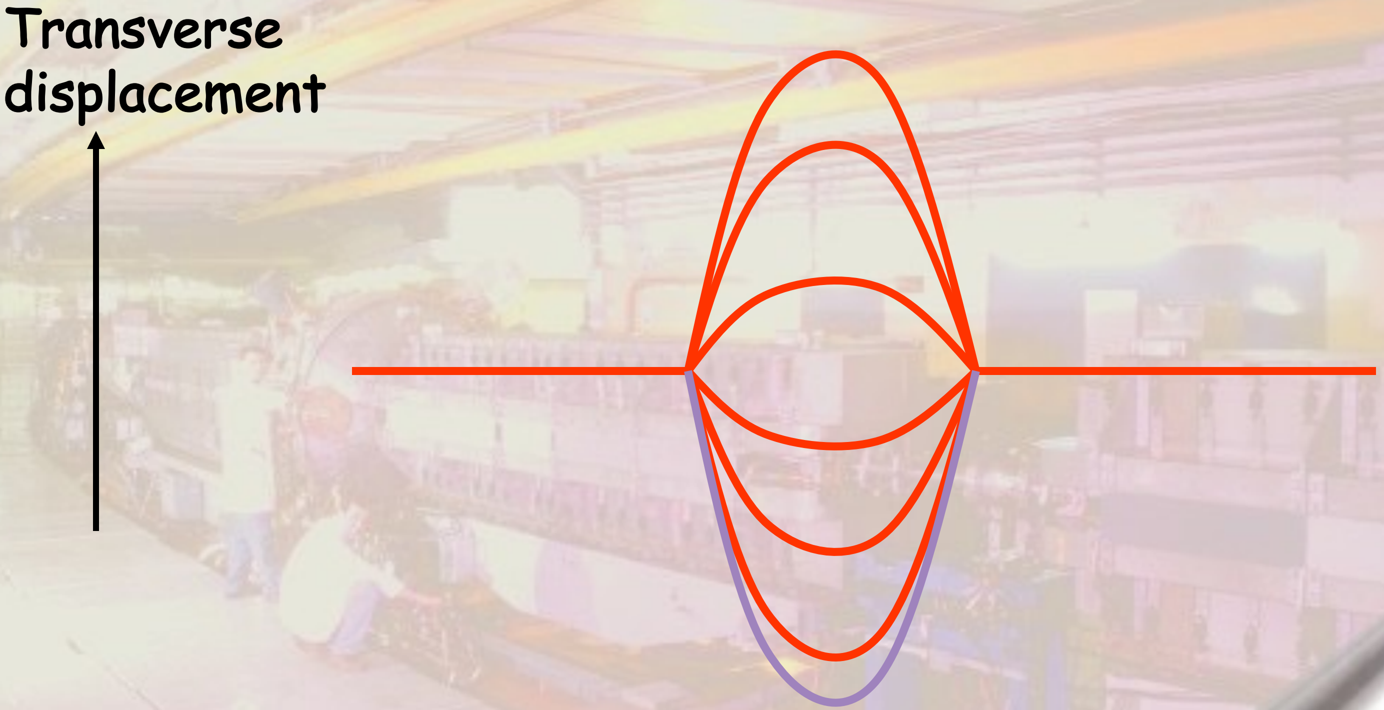
Rigid bunch mode (6)



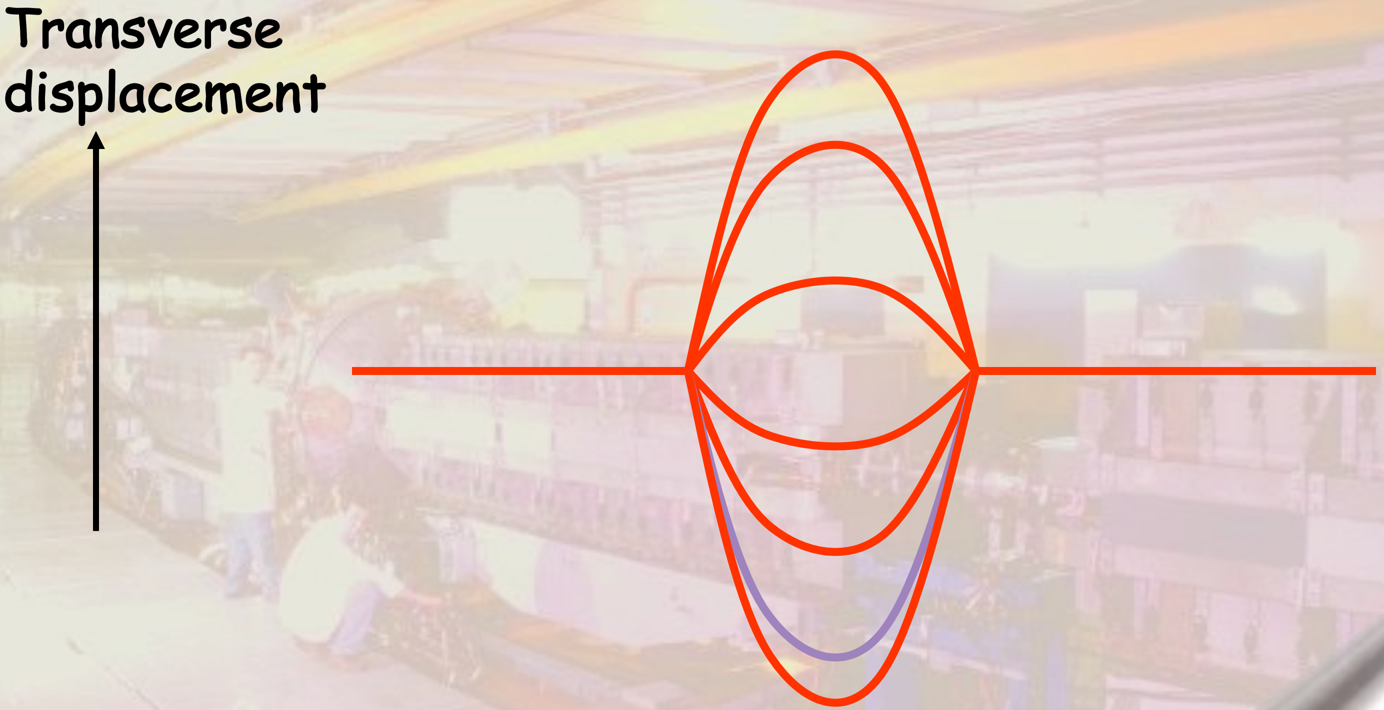
Rigid bunch mode (7)



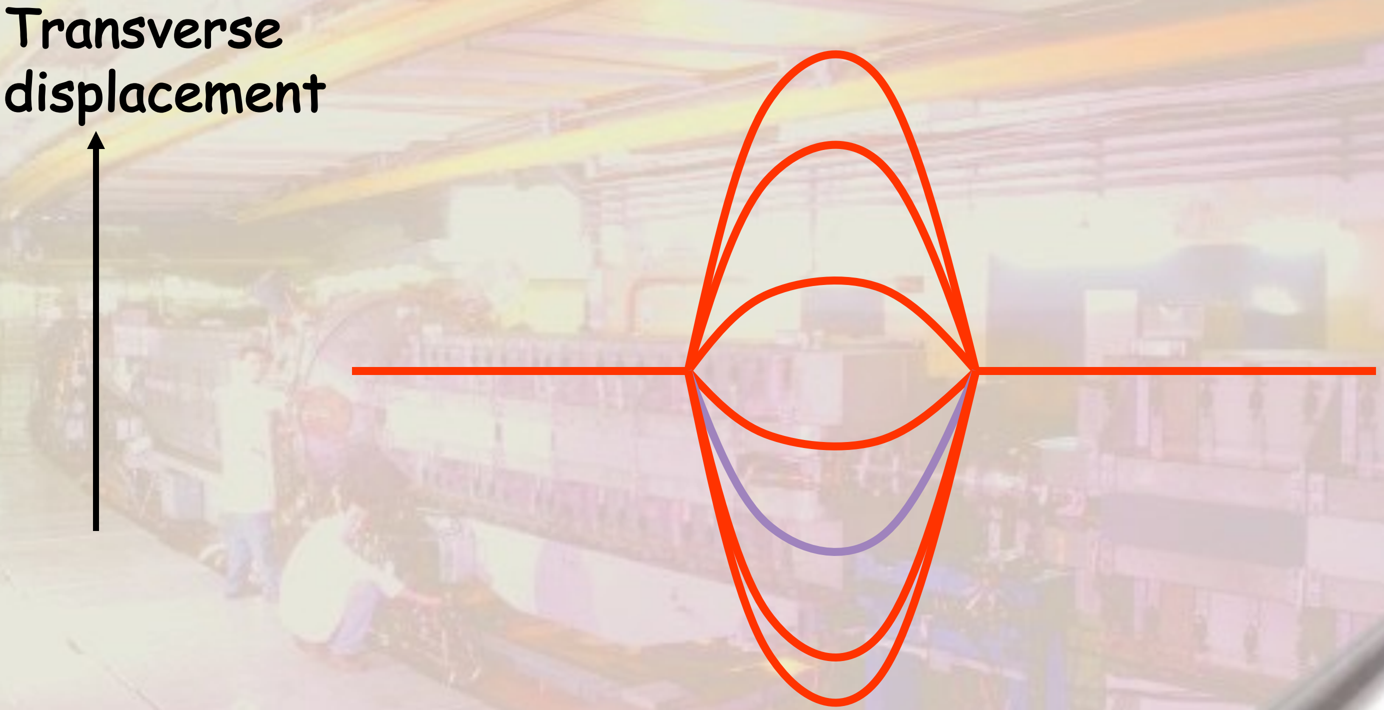
Rigid bunch mode (8)



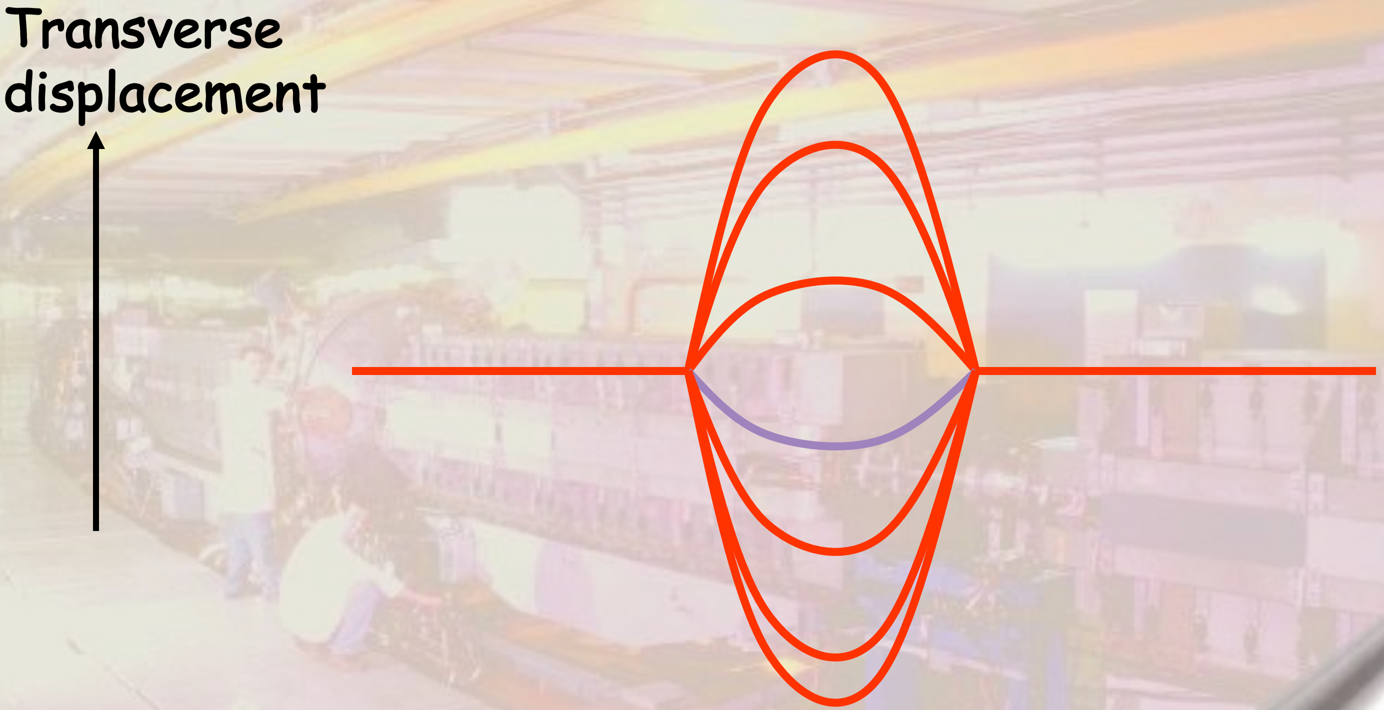
Rigid bunch mode (9)



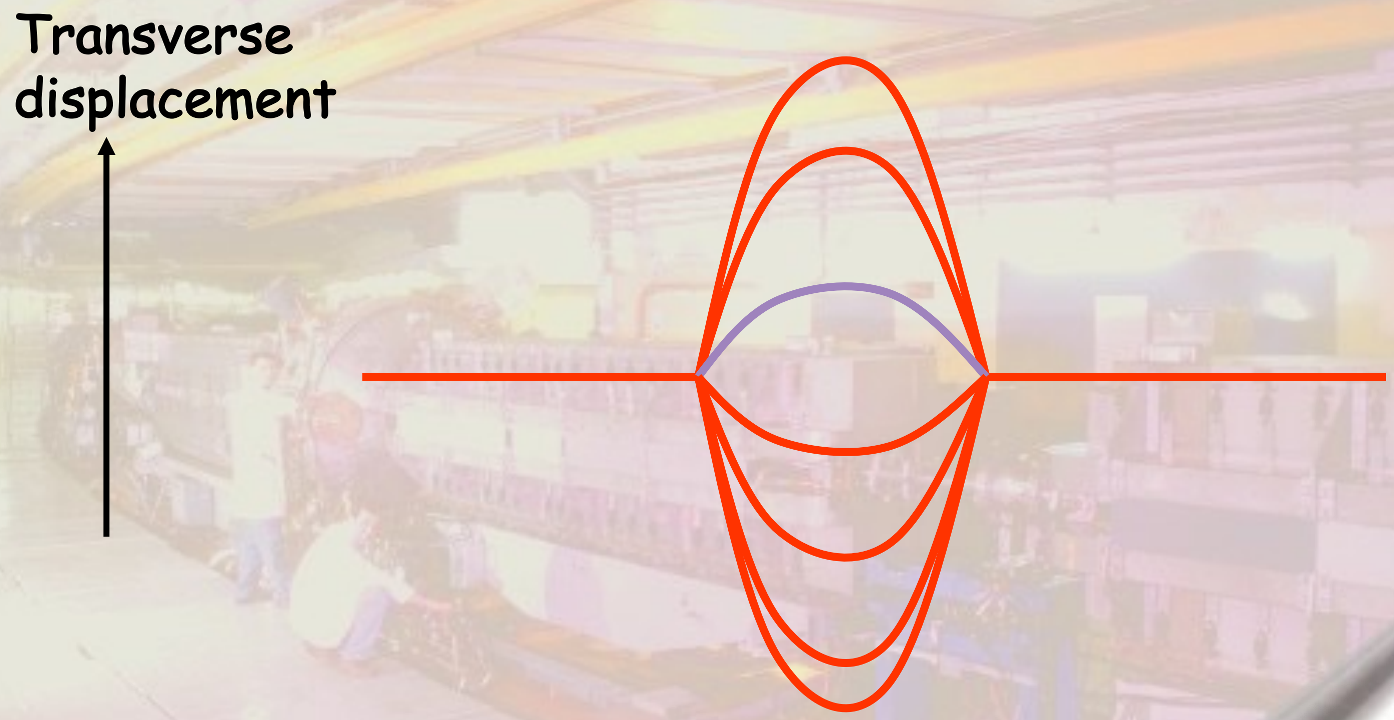
Rigid bunch mode (10)



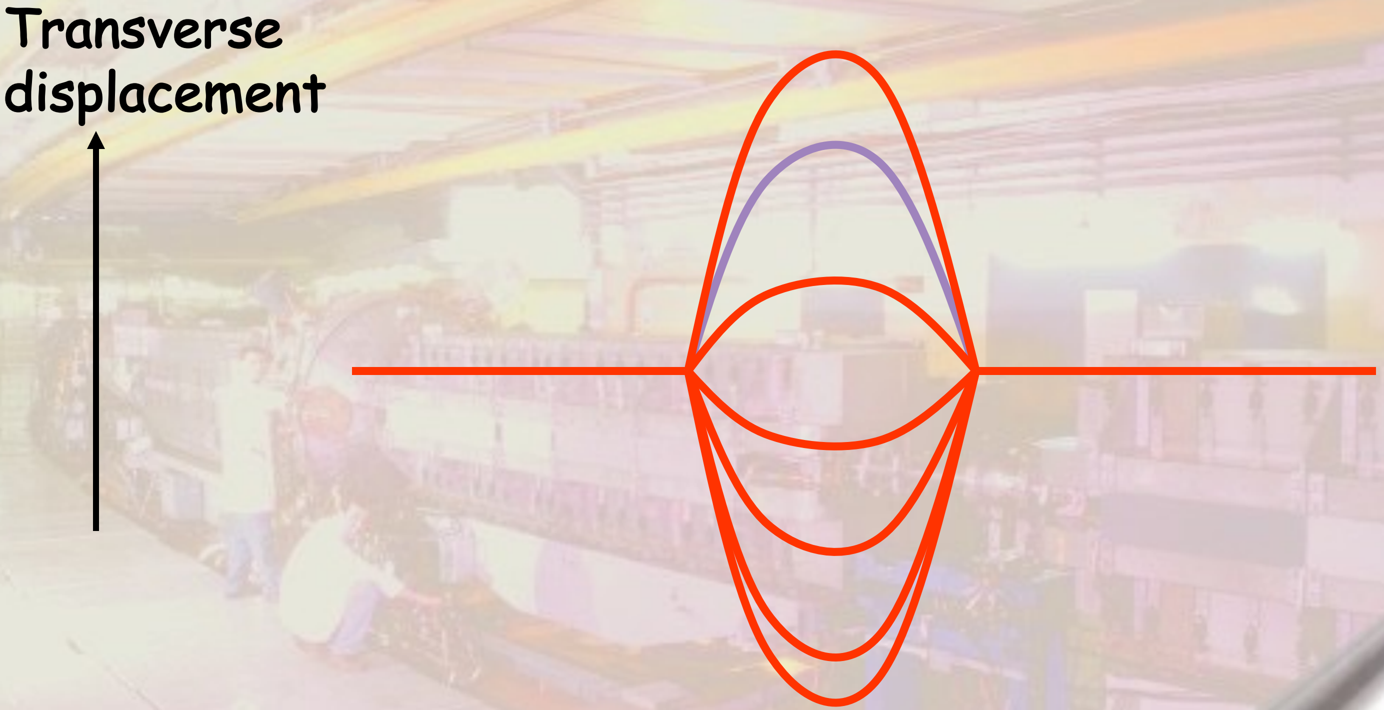
Rigid bunch mode (11)



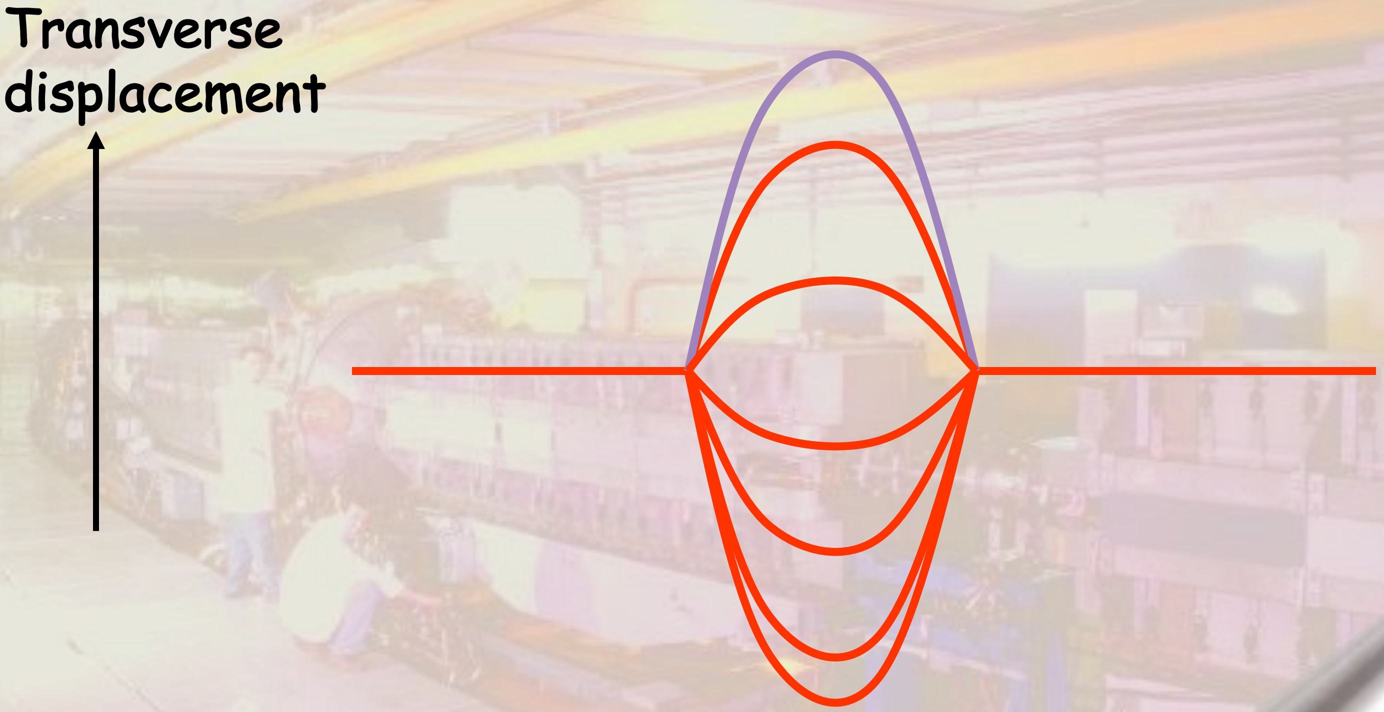
Rigid bunch mode (12)



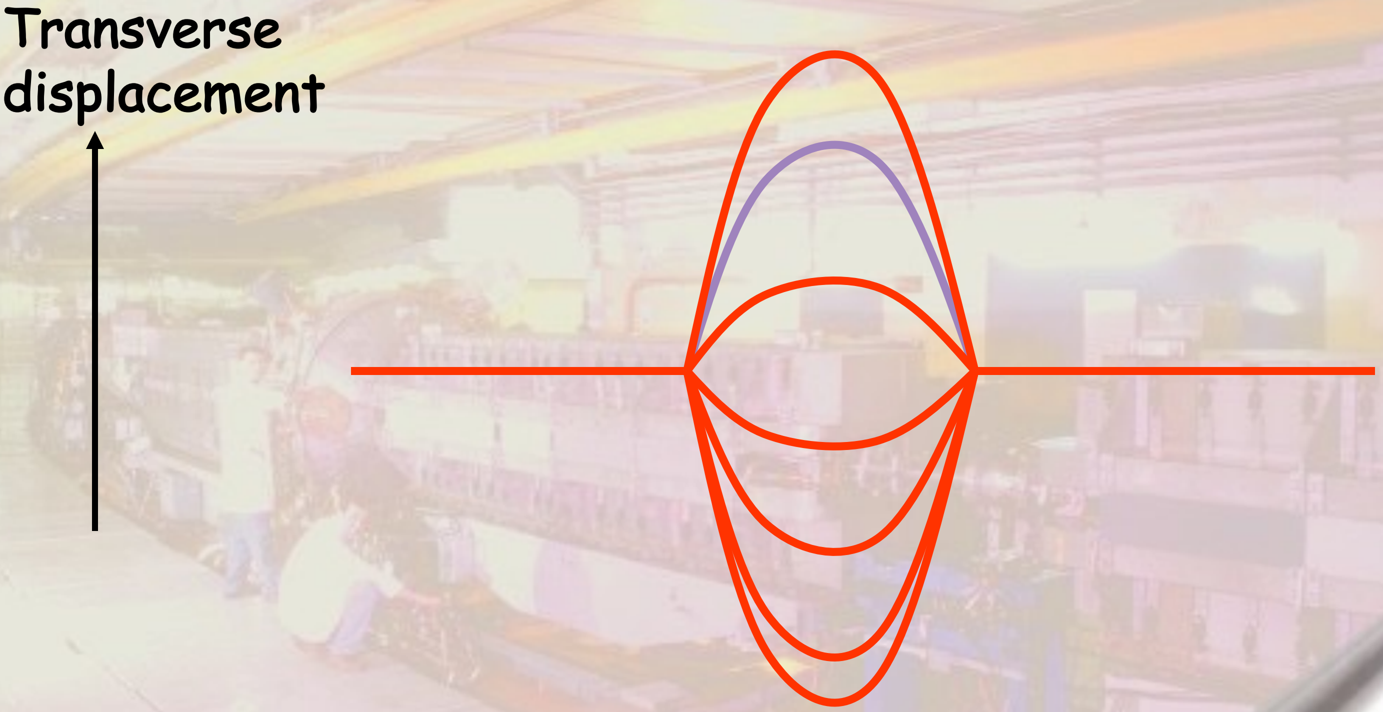
Rigid bunch mode (13)



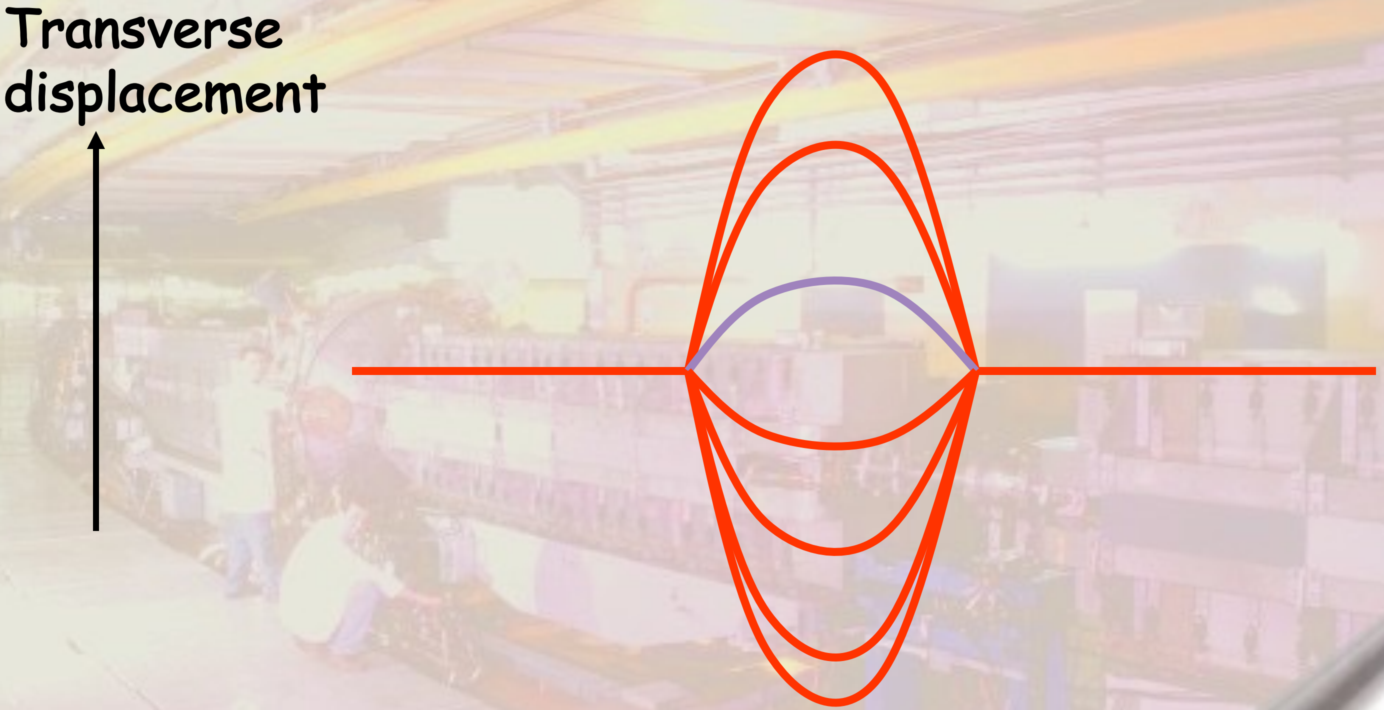
Rigid bunch mode (14)



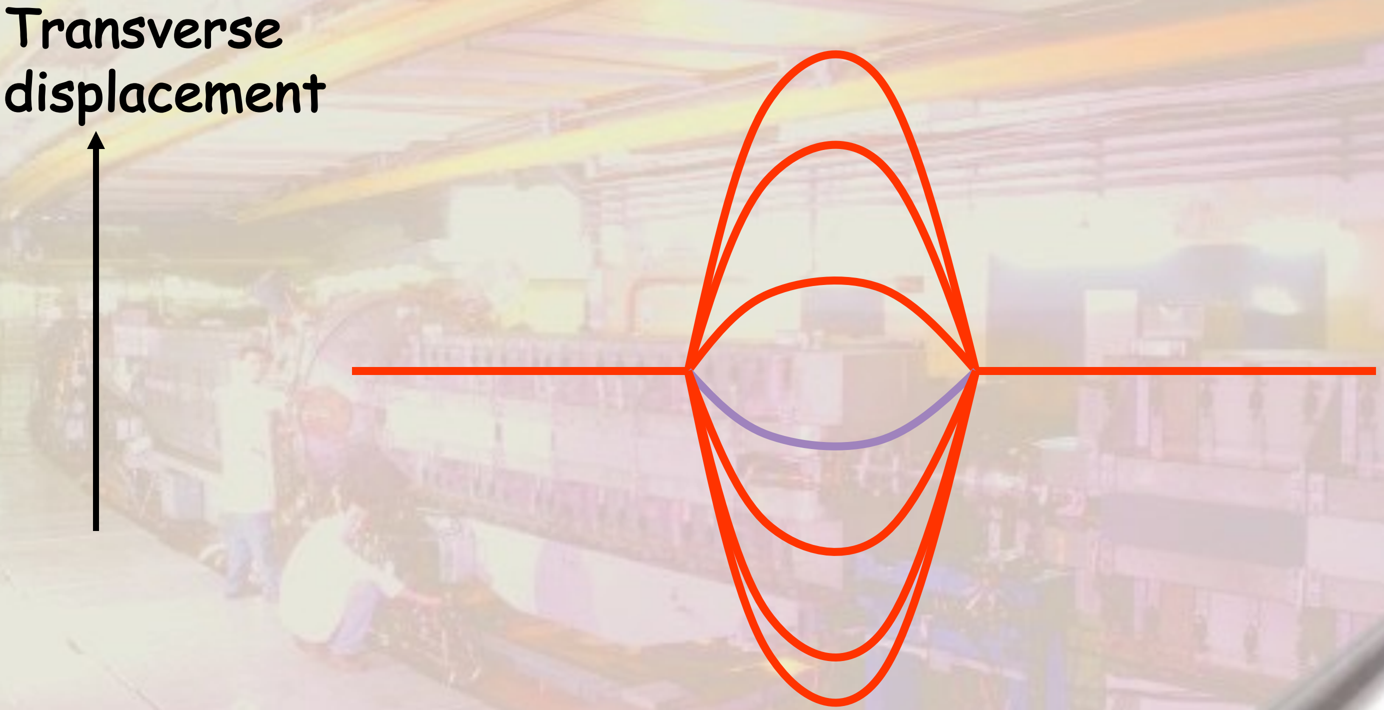
Rigid bunch mode (15)



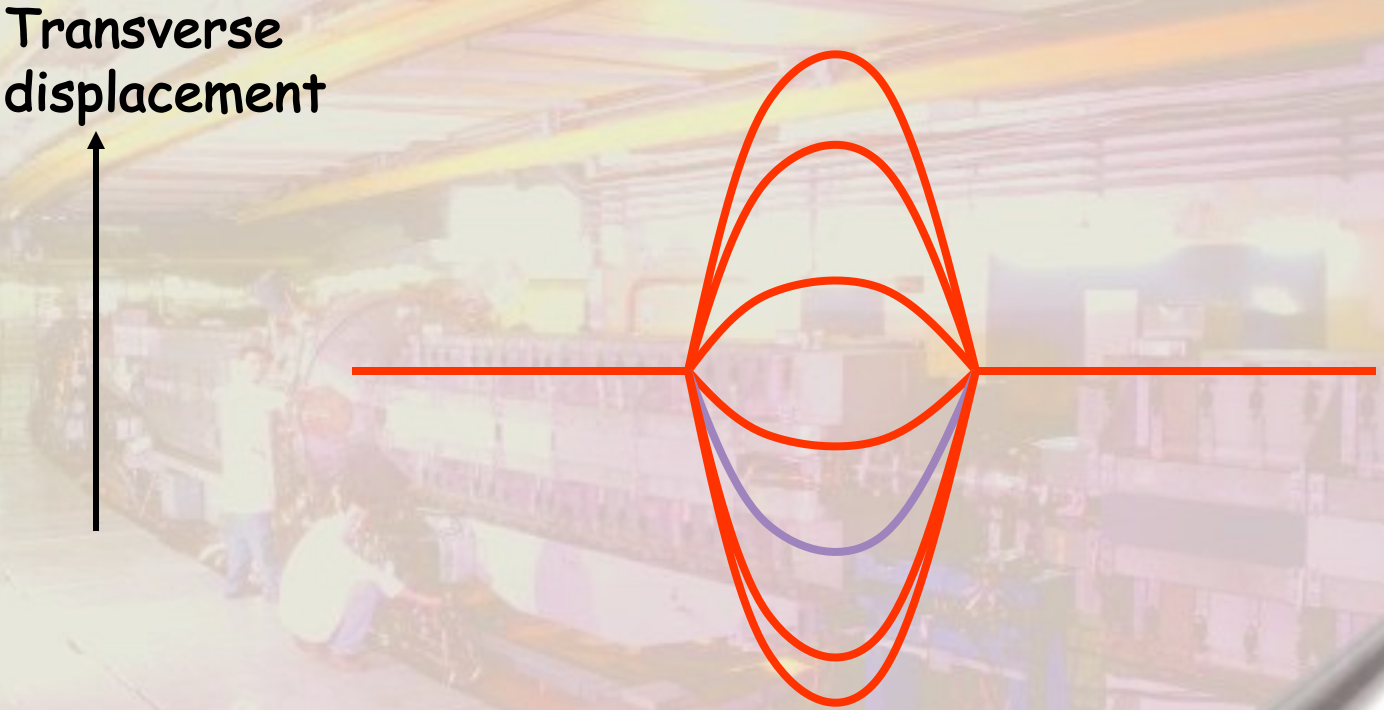
Rigid bunch mode (16)



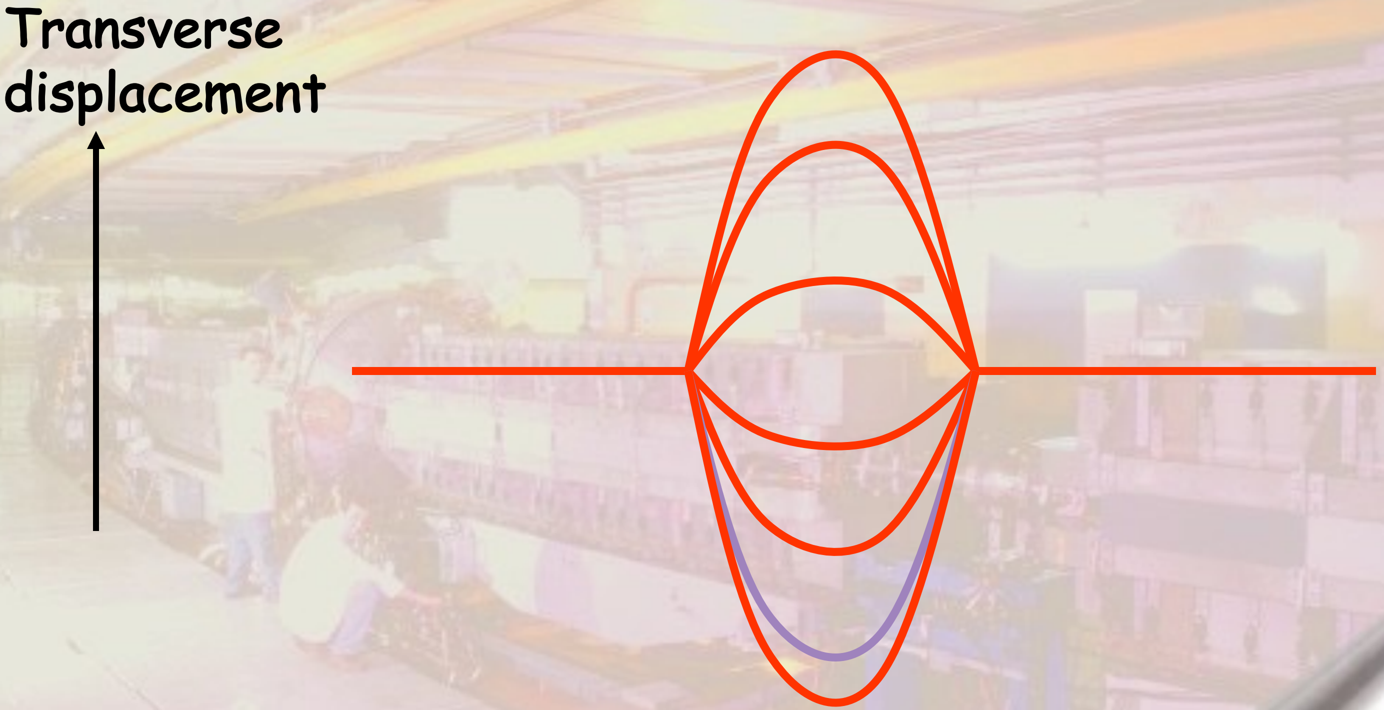
Rigid bunch mode (17)



Rigid bunch mode (18)



Rigid bunch mode (19)



Standing wave without node ⇒ Mode M=0

Cure for rigid bunch mode instability

- To help avoid this instability we need a **non-zero chromaticity**

$$\left(\xi = \frac{\Delta Q}{Q} \Big/ \frac{\Delta p}{p}\right)$$

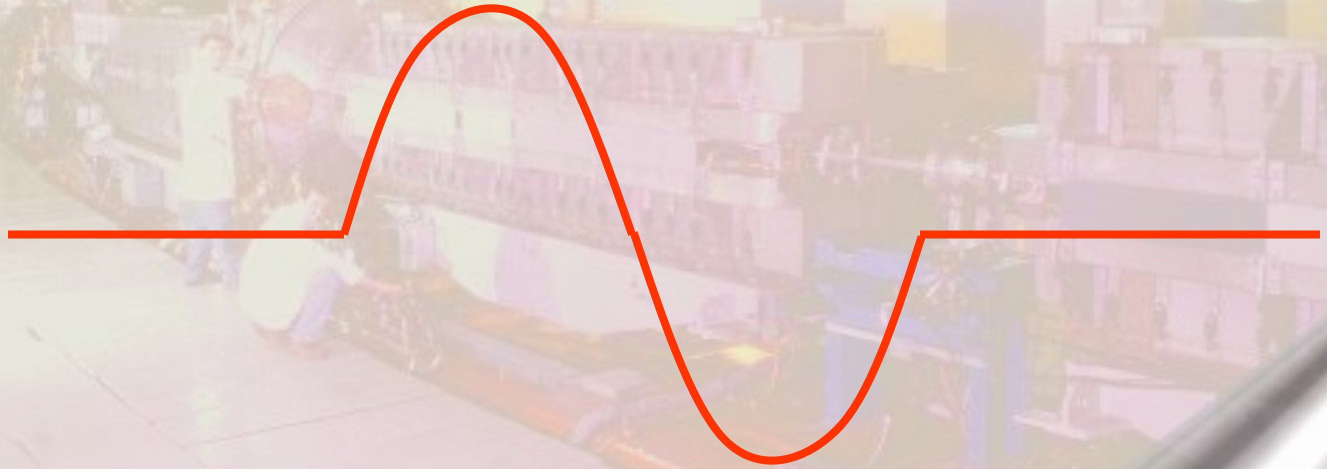
- The bunch has an **energy/momentum spread**
- The Particles will have a **spread in betatron frequencies**
- A **spread in betatron frequencies** will mean that any coherent transverse oscillation (all particles moving together) will very quickly become **incoherent** again.

Higher order bunch modes

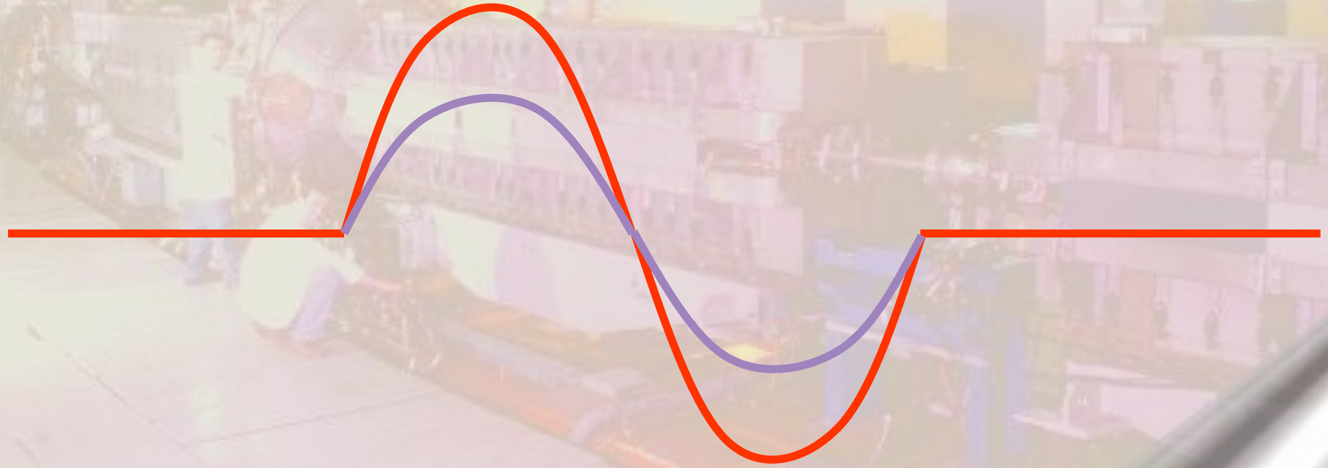
- Higher order modes are called "Head-tail" modes as the electro-magnetic fields induced by the head of the bunch excite oscillation of the tail
- However, these modes may be harder to observe as the centre of gravity on the bunch may not move.....
- Nevertheless, they are very important and cannot be neglected

Head-tail modes (1)

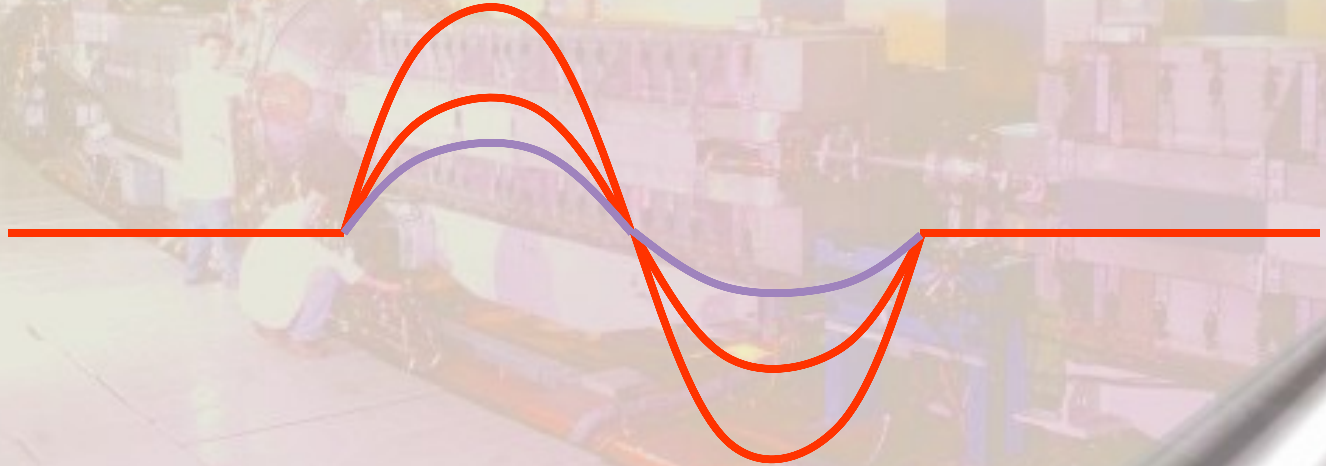
- Head & Tail of bunch move π out of phase with each other
- Again, lets superimpose successive turns



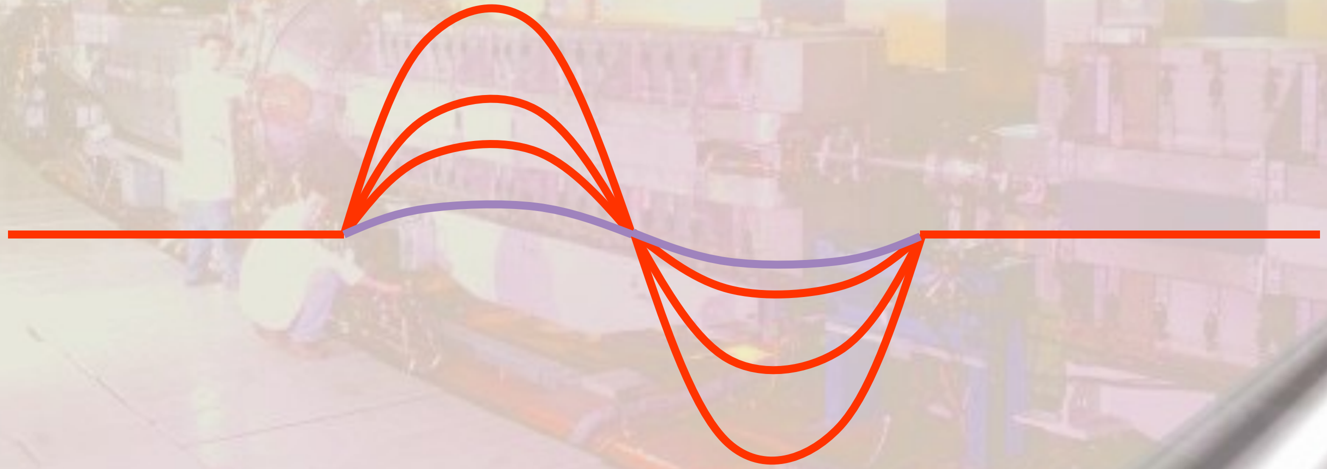
Head-tail modes (2)



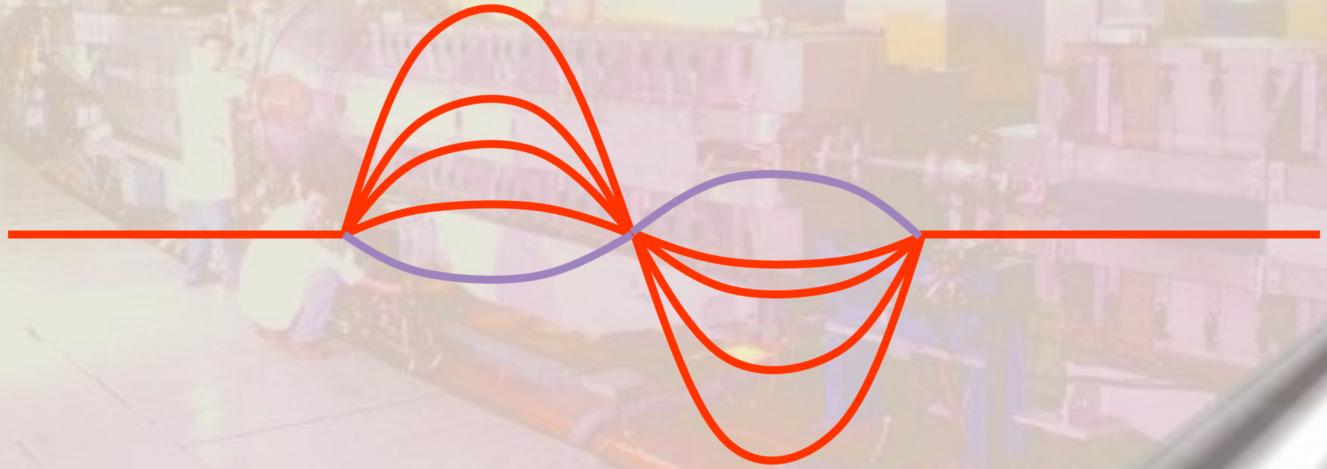
Head-tail modes (3)



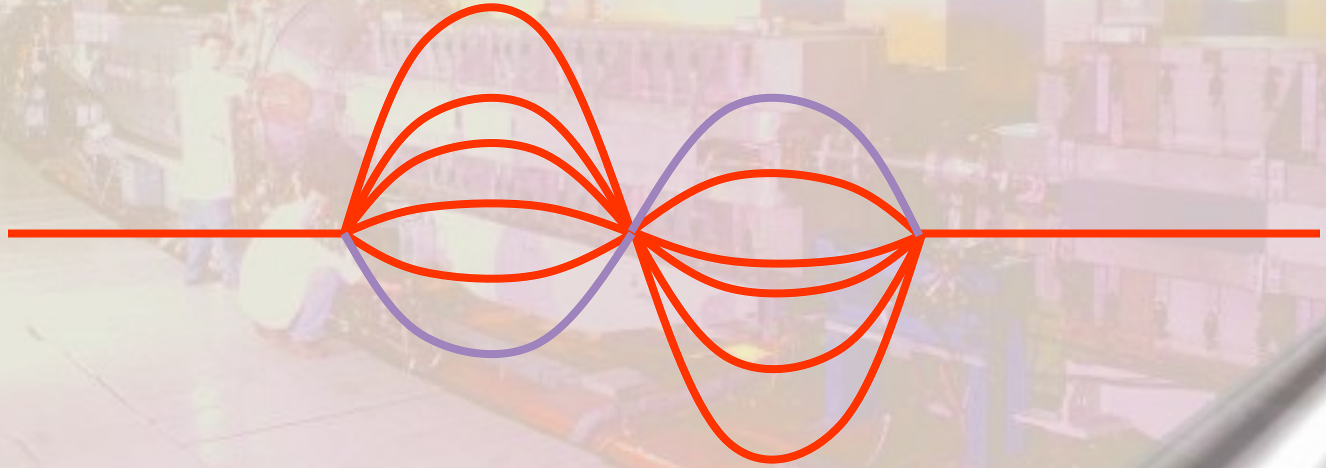
Head-tail modes (4)



Head-tail modes (5)

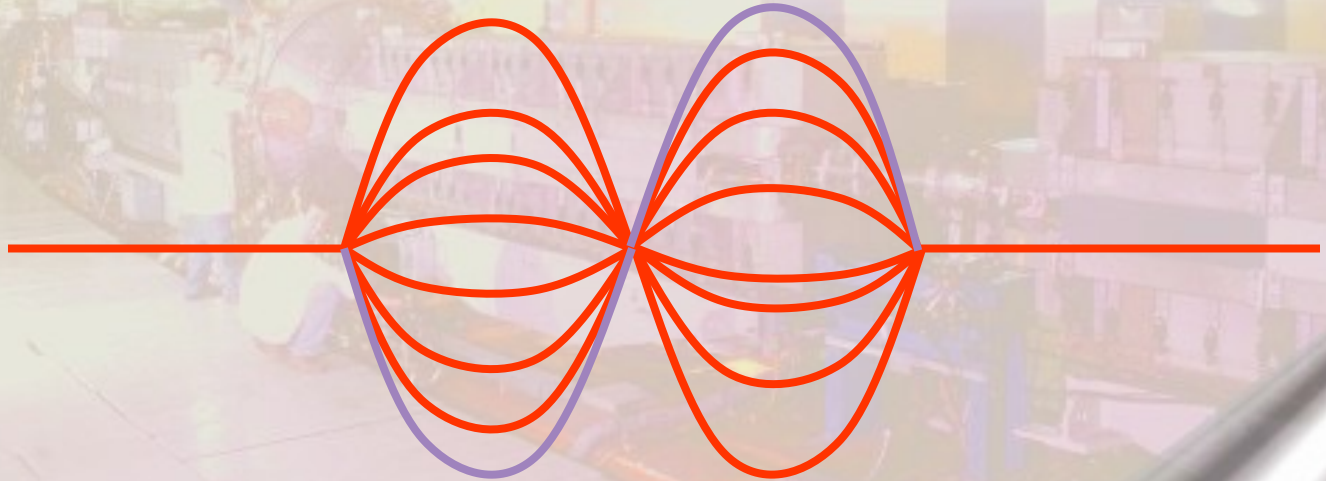


Head-tail modes (6)



Head-tail modes (7)

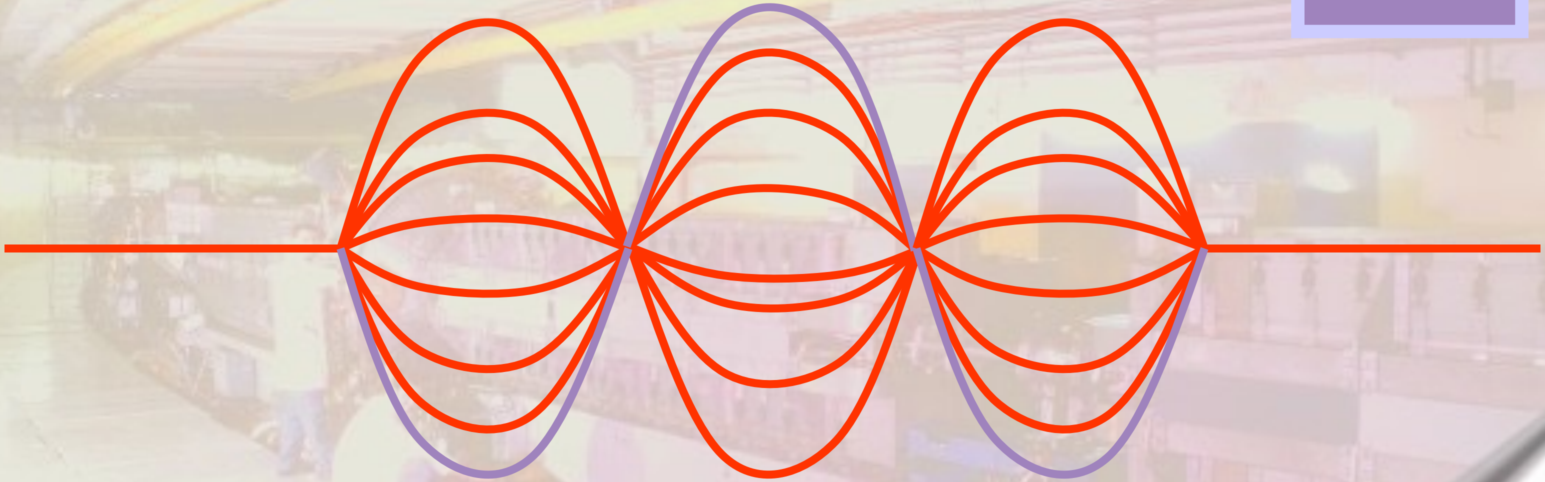
- This is a standing wave with one node
- Thus: **Mode M=1**



Head-tail modes (8)

- This is (obviously!) **Mode:**

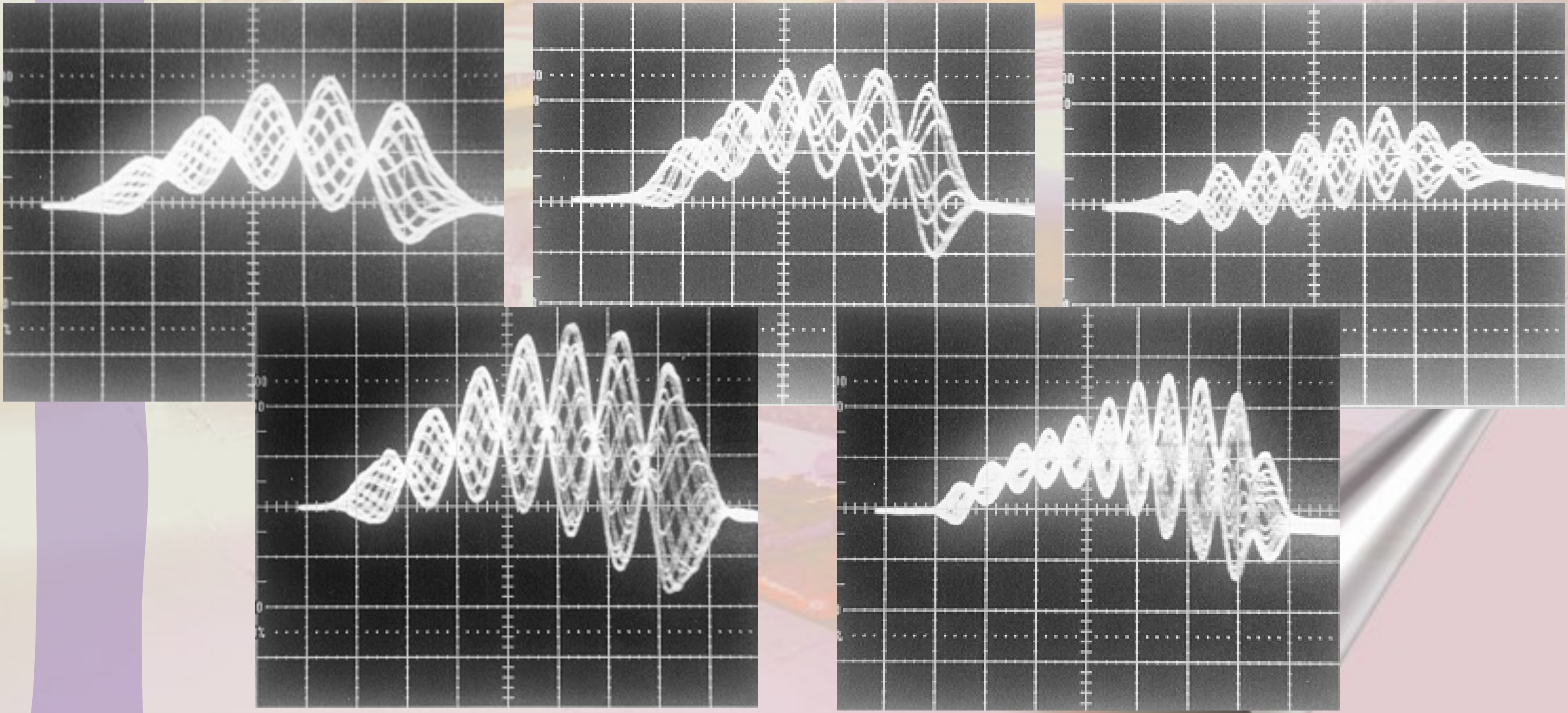
M=2



- Let's look more in detail at the **M=1 "head-tail"** mode
- But first some real life examples……

Head-tail modes (8)

- Some real life examples:

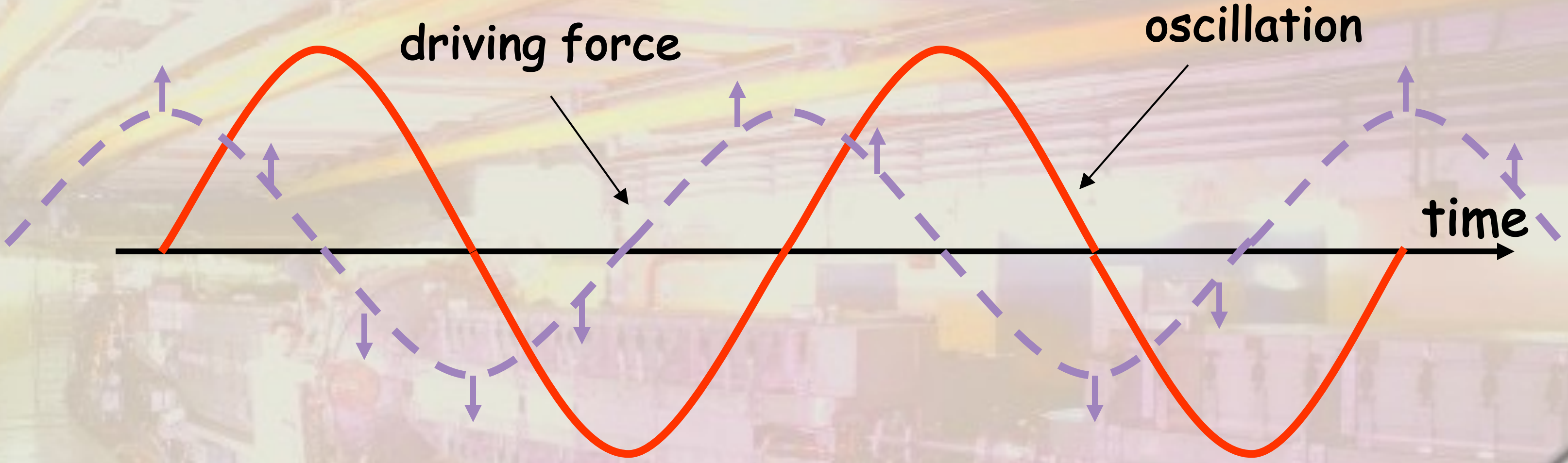


Oscillation and the driving force (1)

- Before continuing, first a memory refresher....

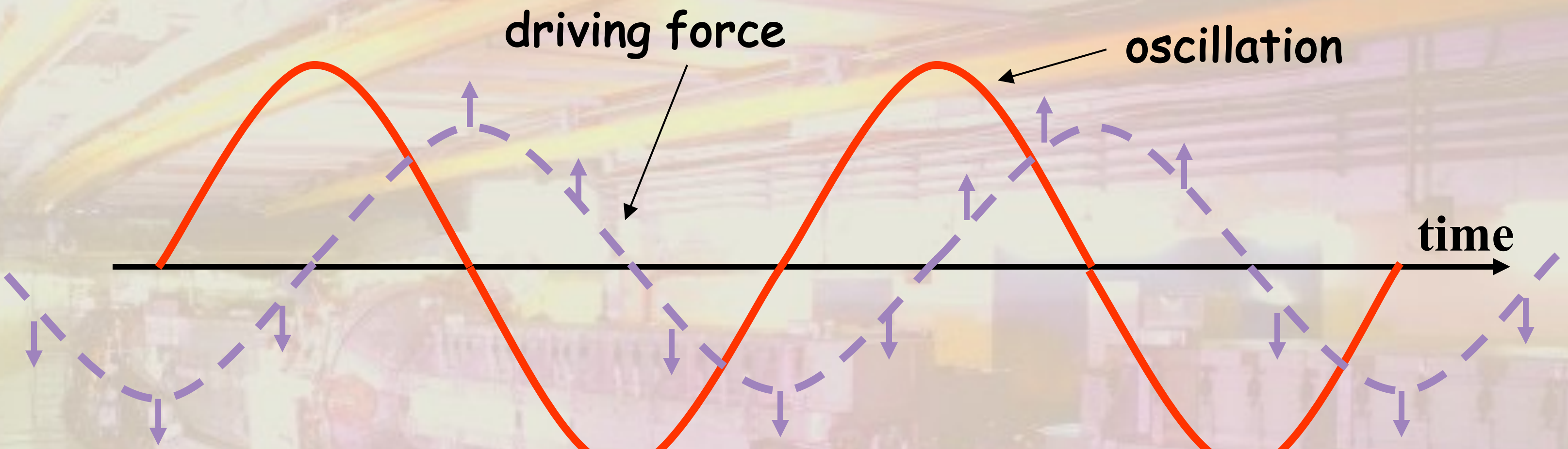
- In order to increase the amplitude of a driven oscillator the driving force must be ahead (in phase) of the motion
- Anyone who has pushed a child on a swing will know this.....

Oscillation and the driving force (2)



Driving force ahead of oscillation ⇒ increasing amplitude
Makes children happy but the beam unstable
INSTABILITY

Oscillation and the driving force (3)

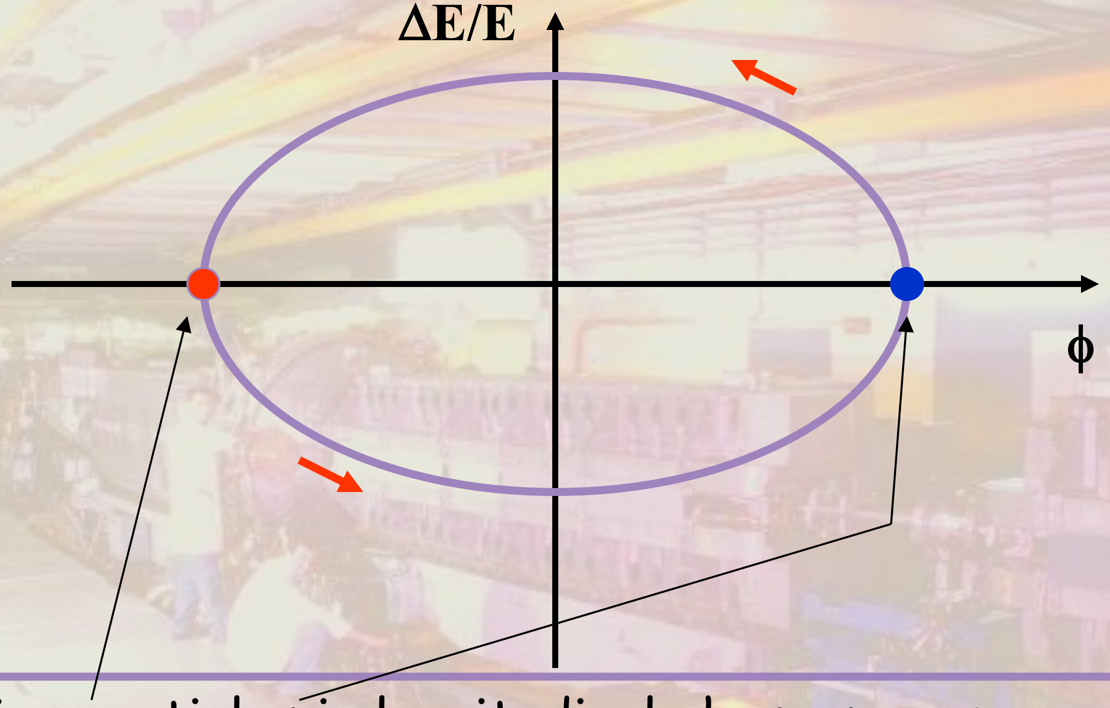


Driving force behind the oscillation ⇒ decreasing amplitude
Makes children unhappy but the beam stable
DAMPING

M=1 Head-tail mode (1)

- The M=1 head tail mode includes both **betatron and synchrotron** oscillations
- There are many betatron oscillations during one synchrotron oscillation
- Thus: **Qs « Qh** and also **Qs « Qv**
- Lets set up an M=1 mode transverse bunch oscillation
- This means that the particles in the tail of the bunch are deflected by the electro-magnetic field left behind by the head of the bunch

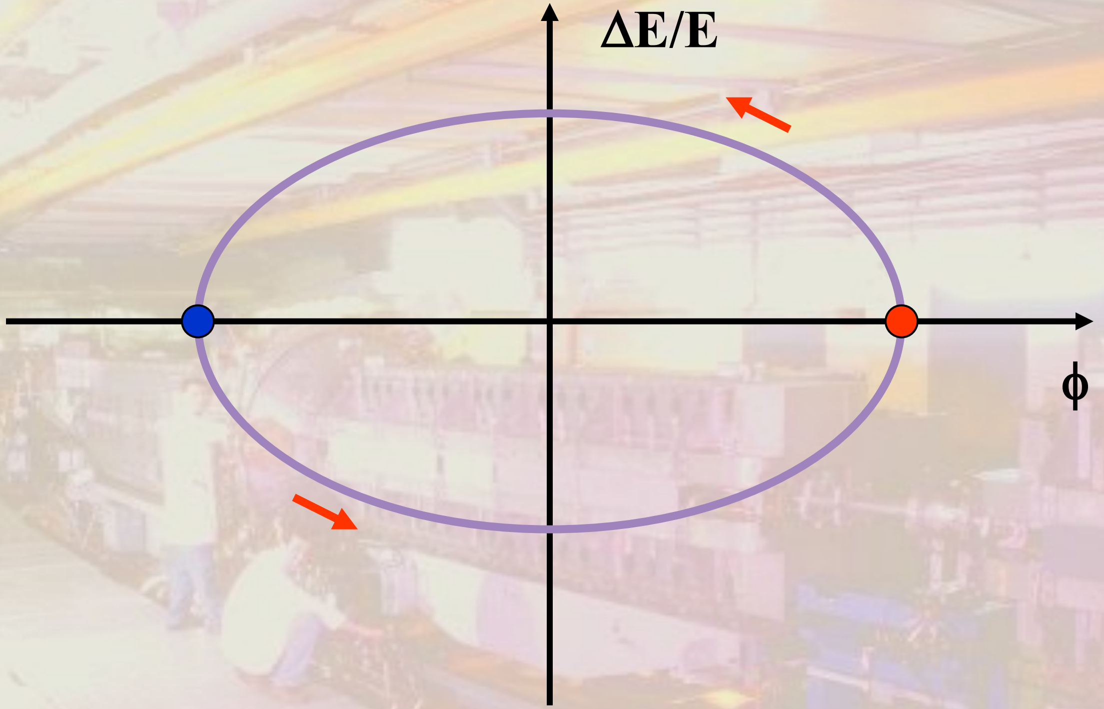
M=1 Head-tail mode (2)



Two particles in longitudinal phase space:
Transverse oscillation of the blue particle is exactly out of phase with red one ⇒ red particle is exactly out of phase with the field left by the blue particle

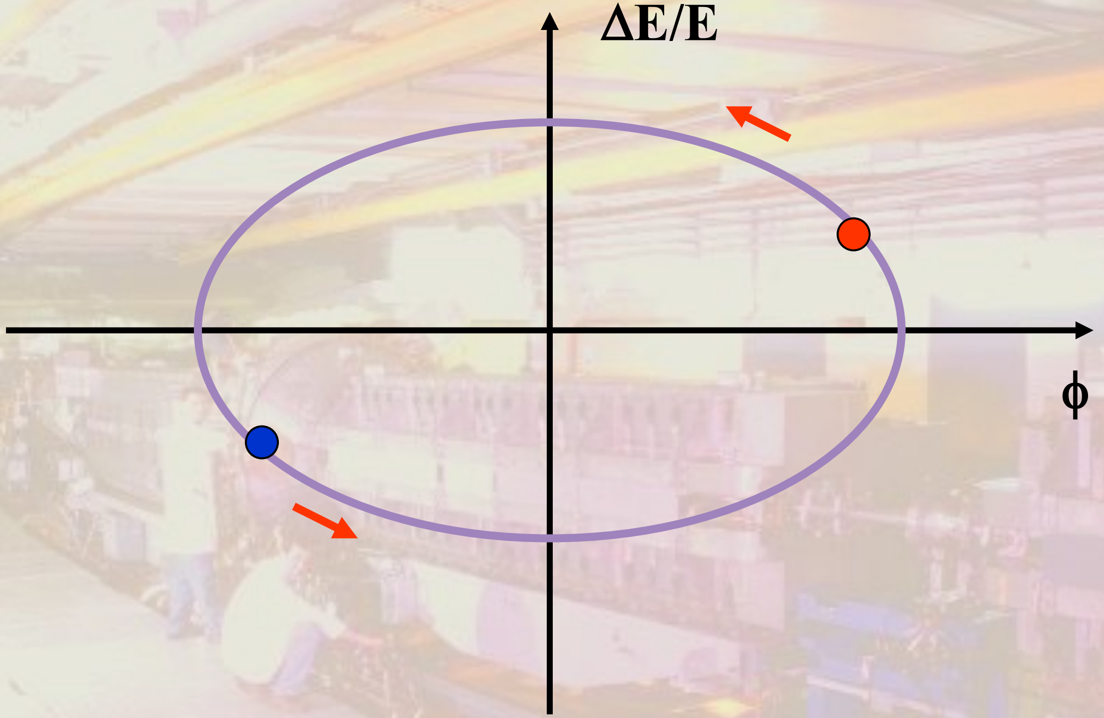
NO EXCITATION

M=1 Head-tail mode (3)

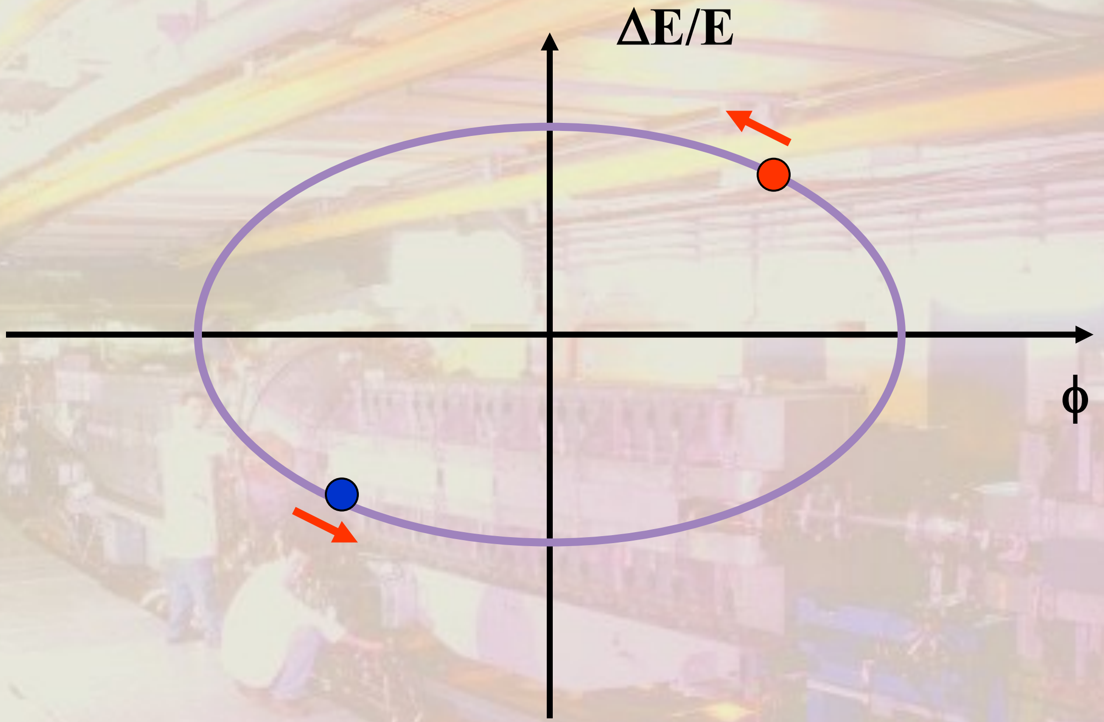


However in 1/2 of a synchrotron period the particles will change places

M=1 Head-tail mode (4)

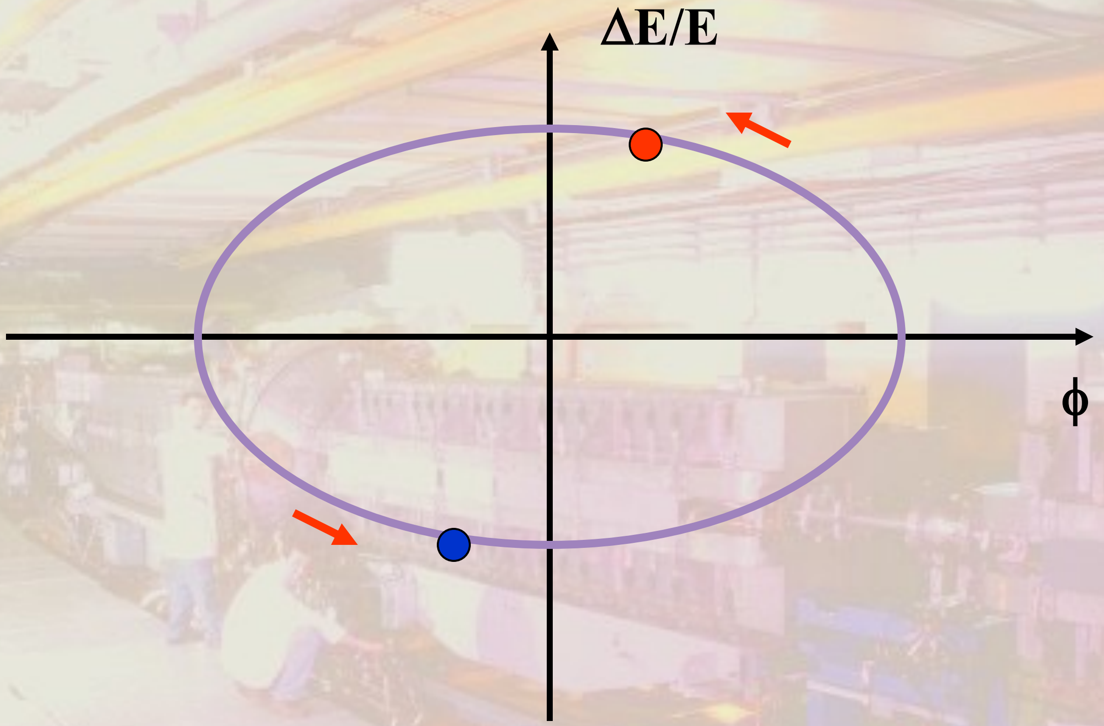


M=1 Head-tail mode (5)

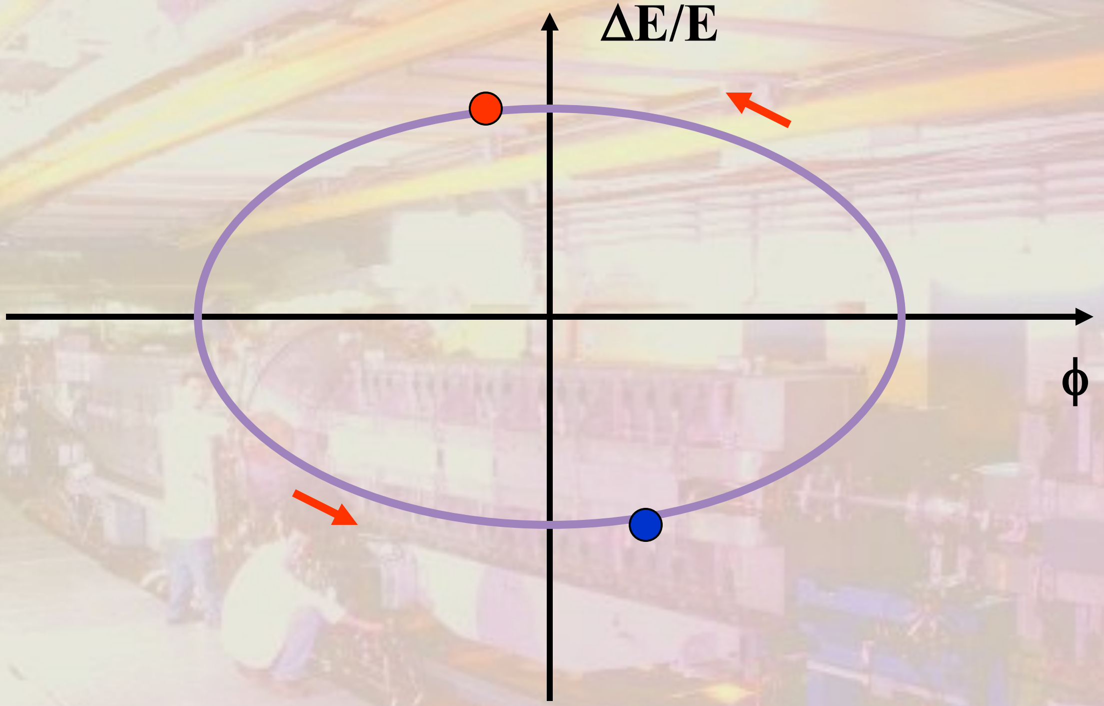


The energy of red particle is increasing
The energy of blue particle is decreasing

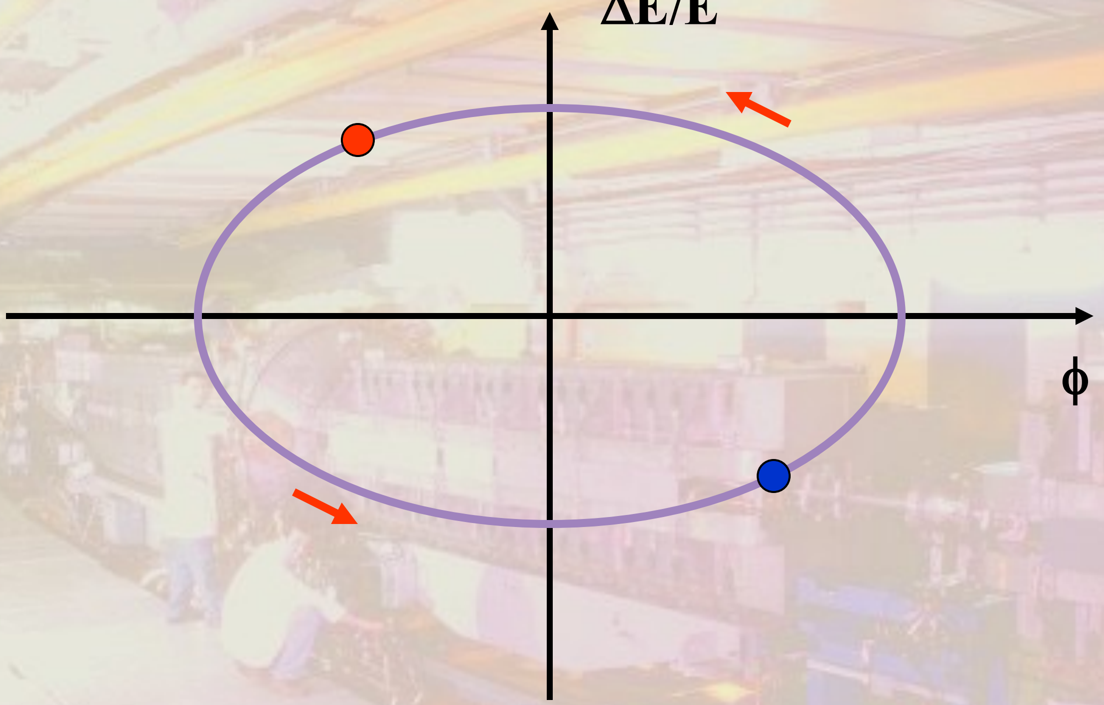
M=1 Head-tail mode (6)



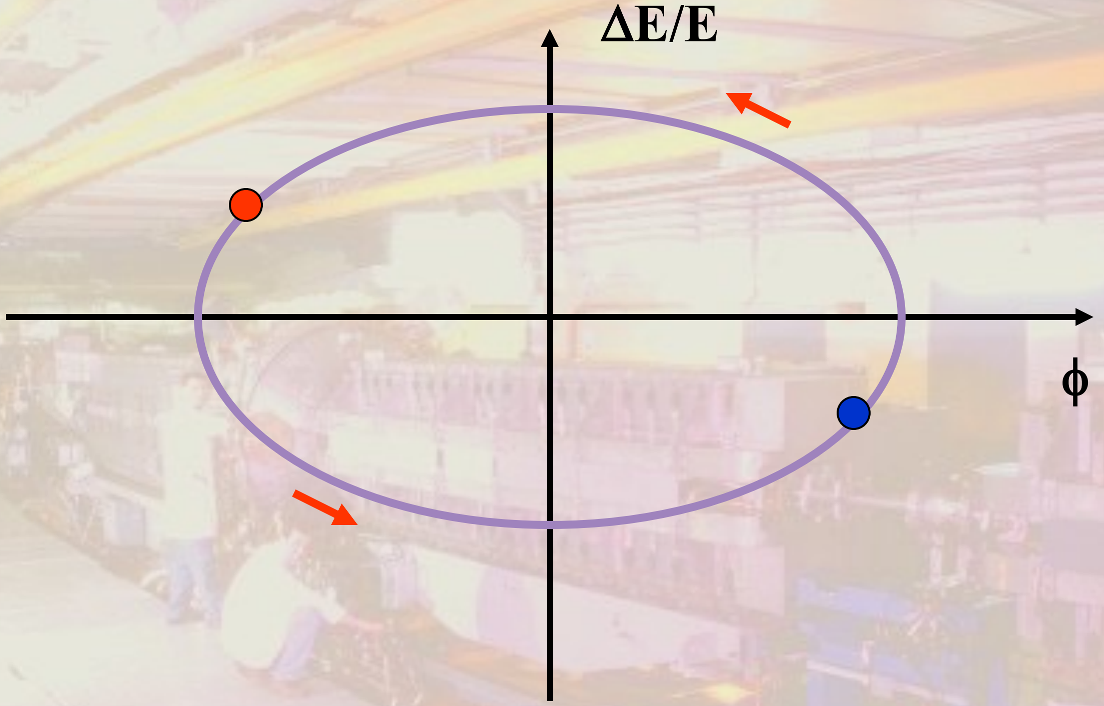
M=1 Head-tail mode (7)



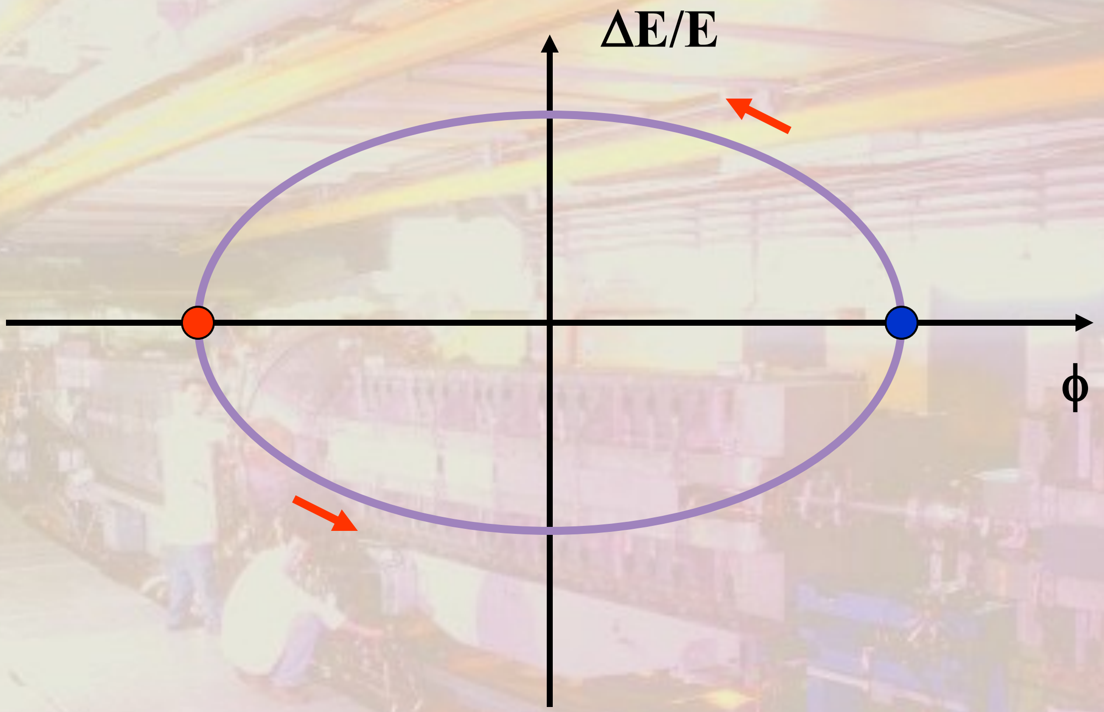
M=1 Head-tail mode (8)



M=1 Head-tail mode (9)

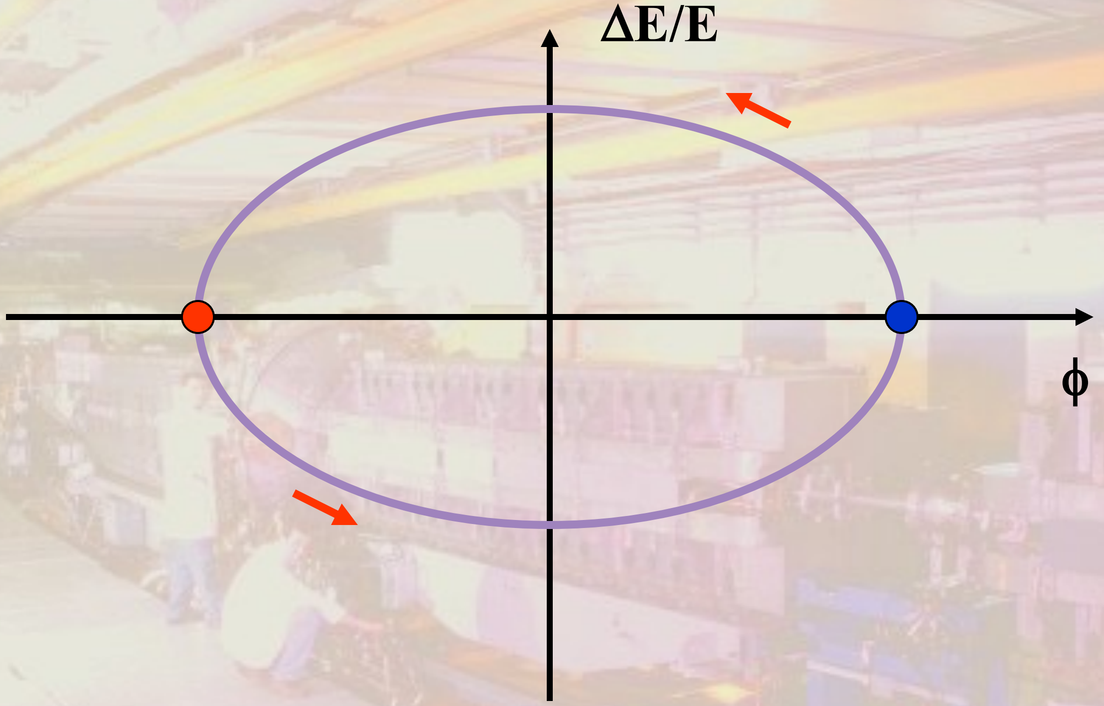


M=1 Head-tail mode (10)



Now they have changed places and have returned to their original energies

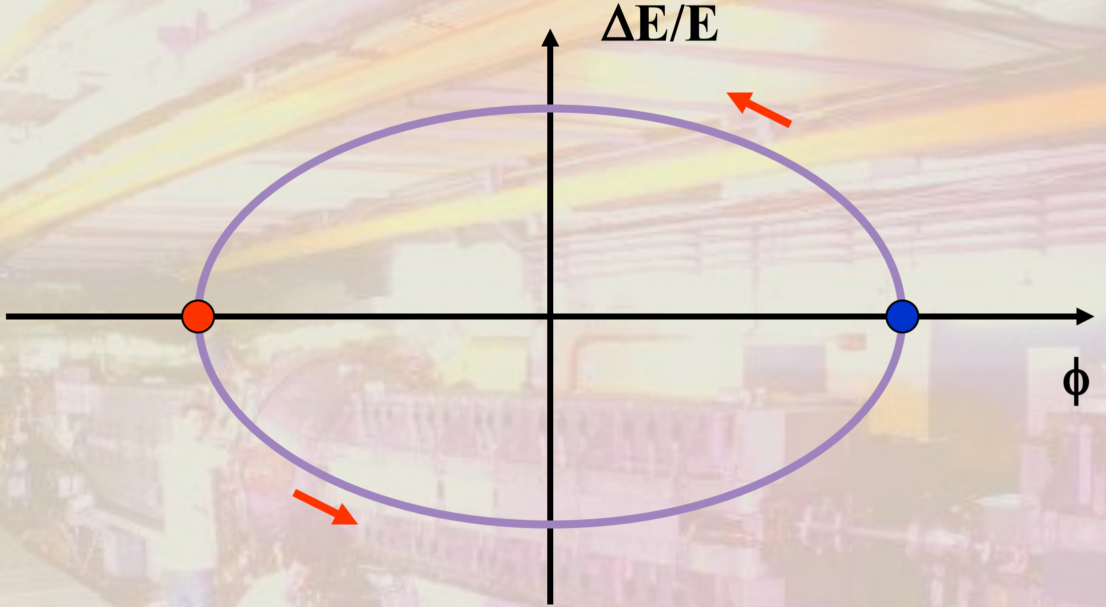
M=1 Head-tail mode (11)



If the chromaticity is zero red will still be exactly out of phase with the wake field left behind by blue

STABLE CONDITION

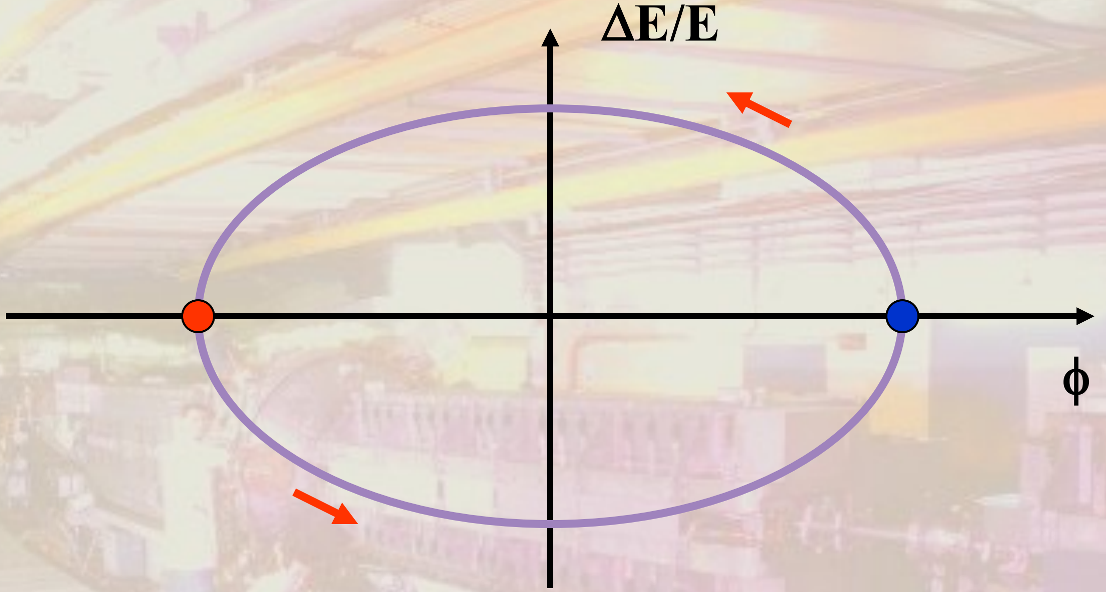
M=1 Head-tail mode (12)



If Chromaticity is negative red would have made slightly less betatron oscillations than blue
Then red's transverse oscillation would lag slightly behind the wake field left by blue

INSTABLE

M=1 Head-tail mode (13)



If Chromaticity is positive red would have made slightly more betatron oscillations than blue

Then red's transverse oscillation would be **slightly ahead of the wake field** left by blue

STABLE

M=1 Head-tail mode (14)

- Conclusion:
  - **Above transition we must have a positive chromaticity to avoid the M=1 mode Head-Tail instability.**
  - **Below transition we must have a negative chromaticity.**

- The natural chromaticity of the machine without sextupoles is normally negative (E⇧ → Q⇩)

- We therefore we need sextupoles to be able to correct the chromaticity.

Transverse multi-bunch modes

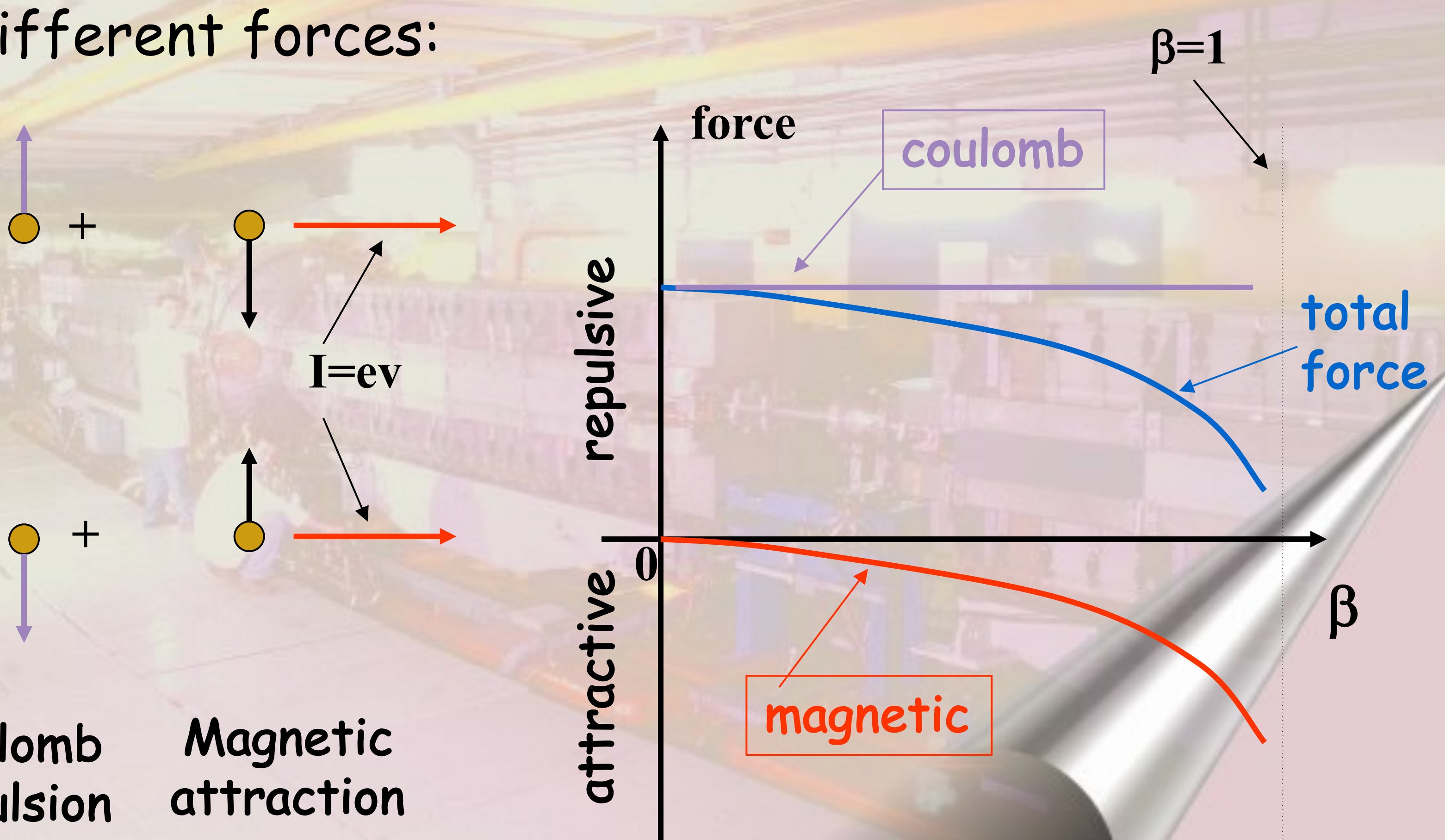
- Longitudinal multi-bunch instabilities limit the bunch intensity before the transverse modes become a problem
- However, once a longitudinal feed back system has been built, one may need to consider a transverse feed back system too.....

Cures

- Correct the natural chromaticity of the machine (set chromaticity negative below transition and positive above transition, but not zero)
- Install a feed-back system.
  - Detect a coherent oscillation and damp it using a transverse kicker
- Damp transverse modes in cavities, where they will remain longest, using a damping antenna

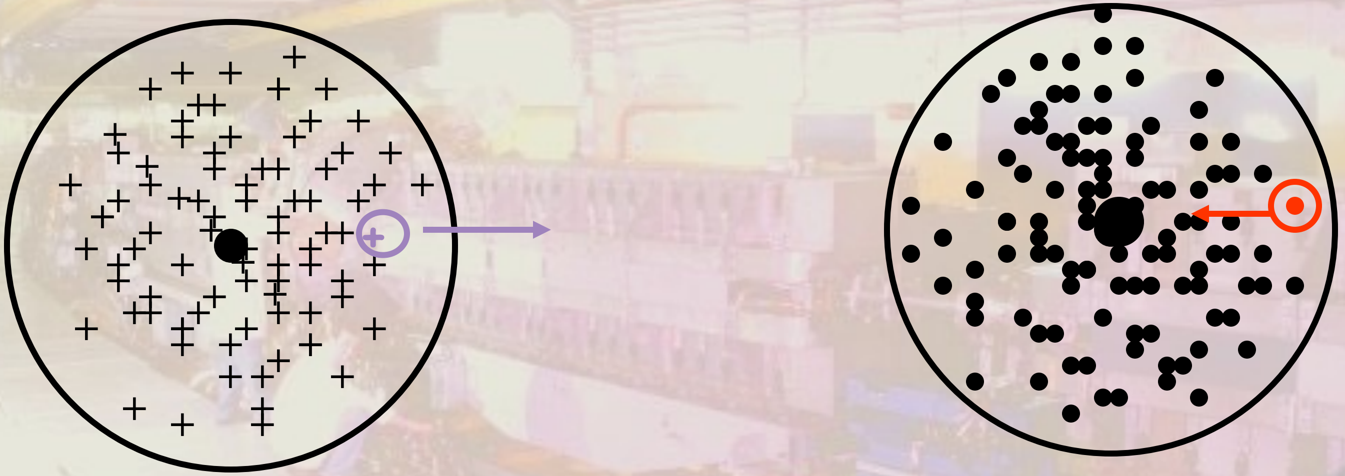
Space Charge effects (1)

- Between two charged particles in a beam we have different forces:



Space Charge effects (2)

- For many particles in a beam we can represent it as following:

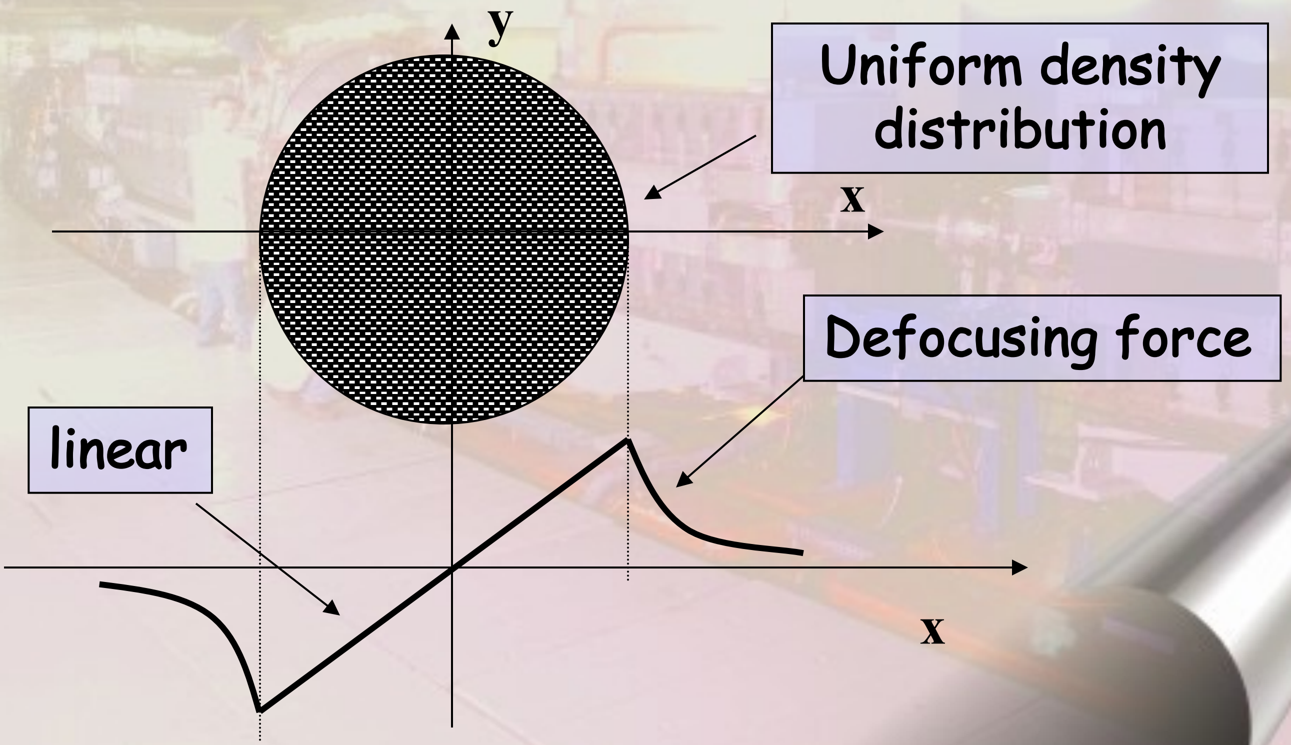


Charges ⇒ repulsion

Parallel currents ⇒ attraction

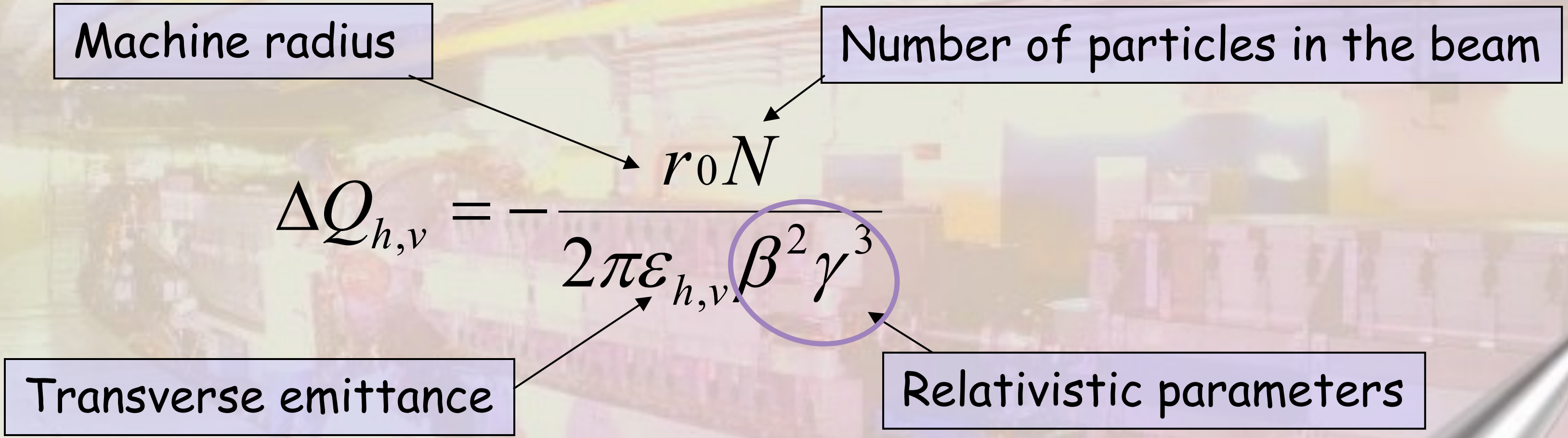
Space Charge effects (2)

- At low energies, which means β<<1, the force is mainly repulsive ⇒ defocusing
- It is zero at the centre of the beam and maximum at the edge of the beam



Space Charge effects (3)

- For the uniform beam distribution, this linear defocusing leads to a tune shift given by:



Machine radius

Number of particles in the beam

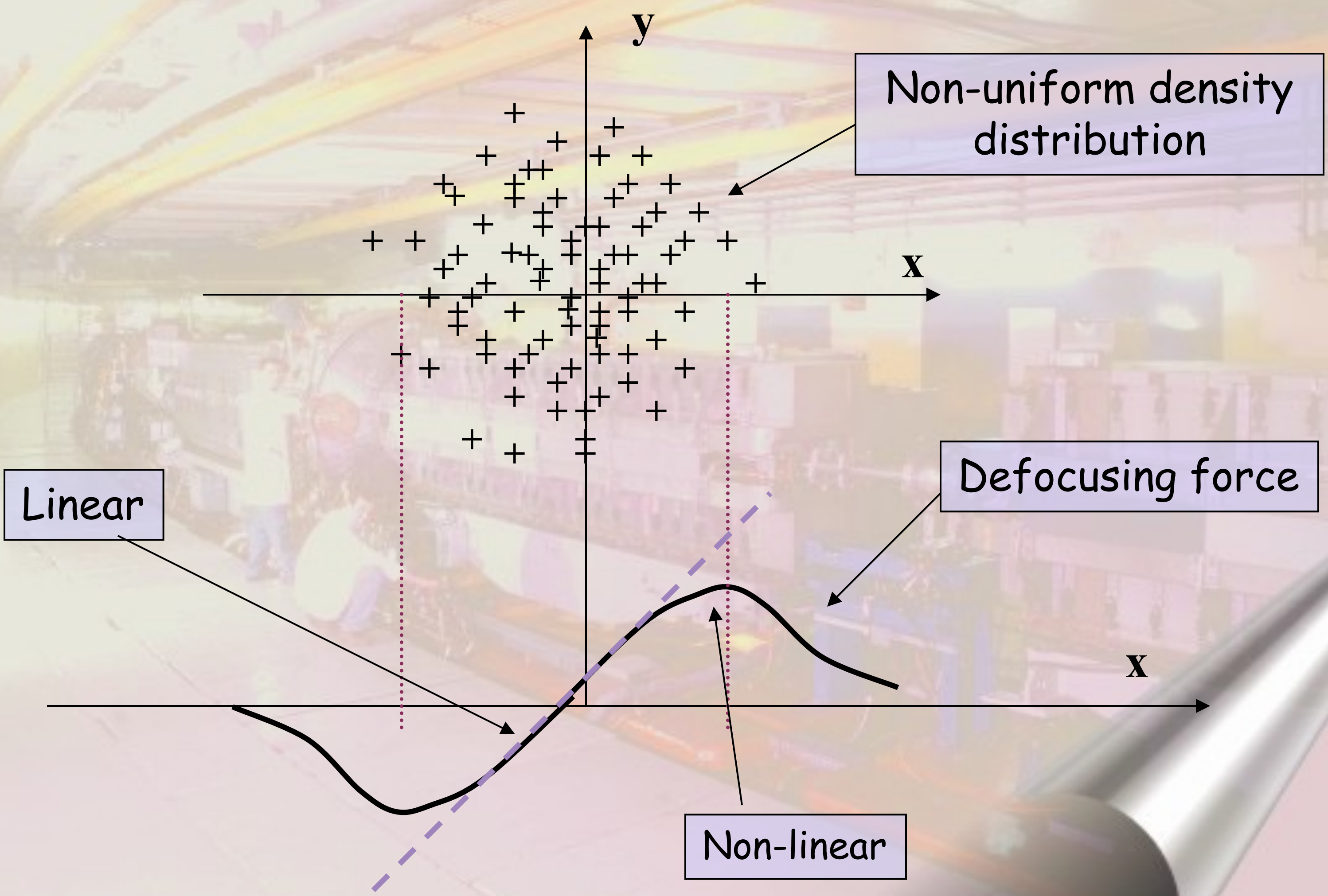
$$\Delta Q_{h,v} = -\frac{r_0 N}{2\pi\varepsilon_{h,v}\beta^2\gamma^3}$$

Transverse emittance

Relativistic parameters

- This tune shift is the **same for all particles** and vanishes at high momenta **(β=1, γ>>1)**
- However in reality the beam distribution is not uniform….

Space charge effects (4)



Non-uniform density distribution

Defocusing force

Linear

Non-linear

Laslett tune shift (1)

- For the non-uniform beam distribution, this non-linear defocusing means the ΔQ is a function of **x** (transverse position)
- This leads to a spread of tune shift across the beam
- This tune shift is called the '**LASLETT tune shift**'

$$\boxed{\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi\varepsilon_{h,v}\beta^2\gamma^3}}$$

half of the uniform tune shift

- This tune spread cannot be corrected and does get very large at high intensity and low momentum

Laslett tune shift (2)

Tune Shift

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi \varepsilon_{h,v} \beta^2 \gamma^3}$$

Large neck tie in tune diagram

- At injection into the PS Booster
  - E = 0.988 GeV, γ = 1.053, β = 0.313 $\Rightarrow$ **ΔQ ≈ 0.3**
- For the same beam at injection into the PS
  - E = 2.3826 GeV, γ = 2.475, β = 0.915 $\Rightarrow$ **ΔQ ≈ 0.005**
- For the same beam at injection into the SPS
  - E = 14 GeV, γ = 14.93, β = 0.998 $\Rightarrow$ **ΔQ ≈ 0.00001**
- We accelerate the beam in the PSB as quickly as possible to avoid problems of blow-up due to betatron resonances

Questions….,Remarks…?

Single bunch modes

Head-tail modes

Space charge

Tune shift

Beam Break-up around transition….