AXEL-2023

Introduction to Particle Accelerators

Longitudinal instabilities:

- ✓ *Single bunch longitudinal instabilities*
- ✓ *Multi bunch longitudinal instabilities*
- ✓ *Different modes*
- ✓ *Bunch lengthening*

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Instabilities (1)

- Until now we have only considered independent particle motion.
- We call this incoherent motion.
  - single particle synchrotron/betatron oscillations
  - each particle moves independently of all the others
- Now we have to consider what happens if all particles move in phase, coherently, in response to some excitations

Synchrotron & betatron oscillations

Instabilities (2)

- We cannot ignore interactions between the charged particles
- They interact with each other in two ways:
  - Direct Coulomb interaction between particles
  - Via the vacuum chamber

Space charge effects, intra beam scattering

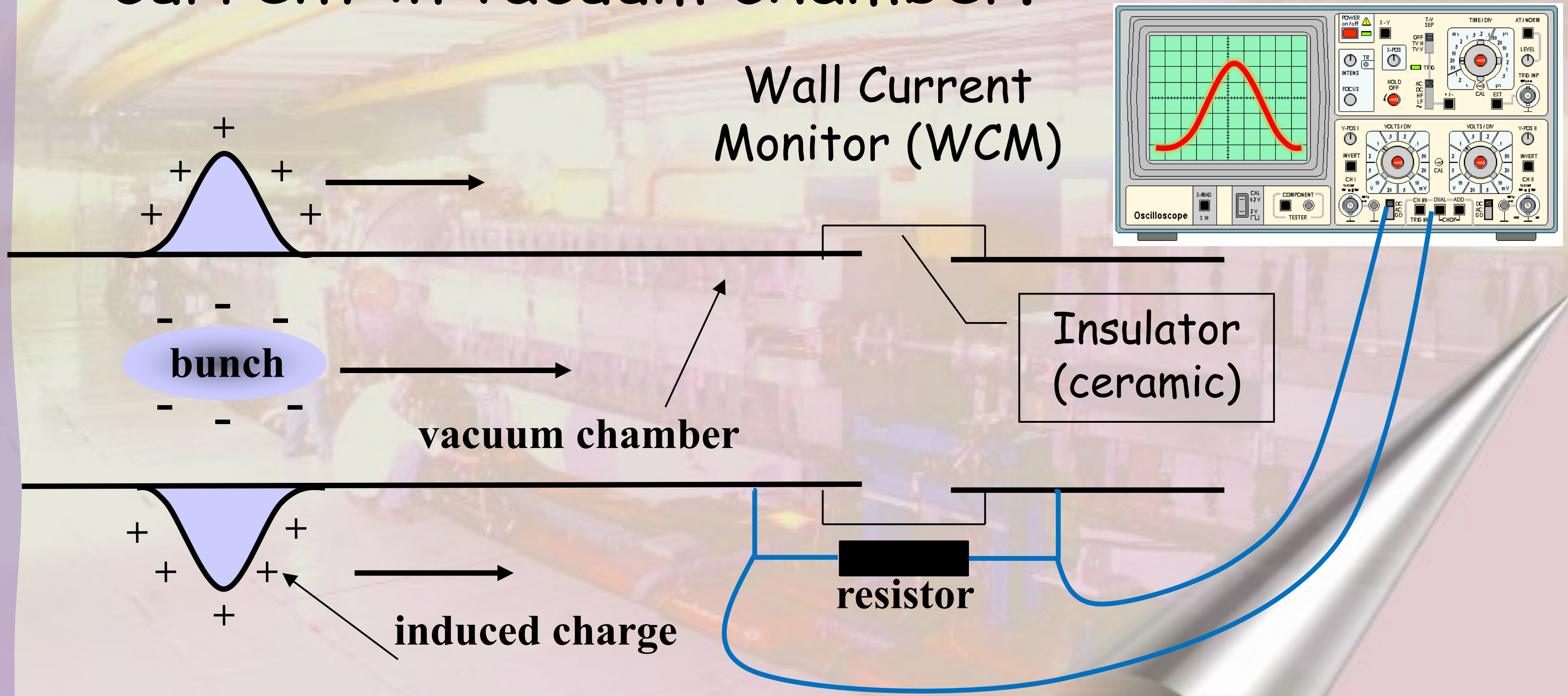
Longitudinal and transverse beam instabilities

Why do Instabilities arise?

- A circulating bunch induces electro magnetic fields in the vacuum chamber
- These fields act back on the particles in the bunch
- Small perturbations to the bunch motion, change the induced EM fields
- If this change amplifies the perturbation then we have an **instability**

Measuring Longitudinal Instabilities

- A circulating bunch creates an image current in vacuum chamber.



- The induced image current is the same size but has the opposite sign to the bunch current.

Impedance and Wall current (1)

- The vacuum chamber presents an impedance to this induced wall current (changes of shape, material etc.)
- The image current combined with this impedance induces a voltage, which in turn affects the charged particles in the bunch

$$\int (Z(\omega) \times I(\omega))\, d\omega = \int_0^s E\, ds$$

Impedance & current ⇒ voltage ⇒ electric field

Resistive, inductive, capacitive

$$Z = Z_r + iZ_i$$

Strong frequency dependence

Real & Imaginary components

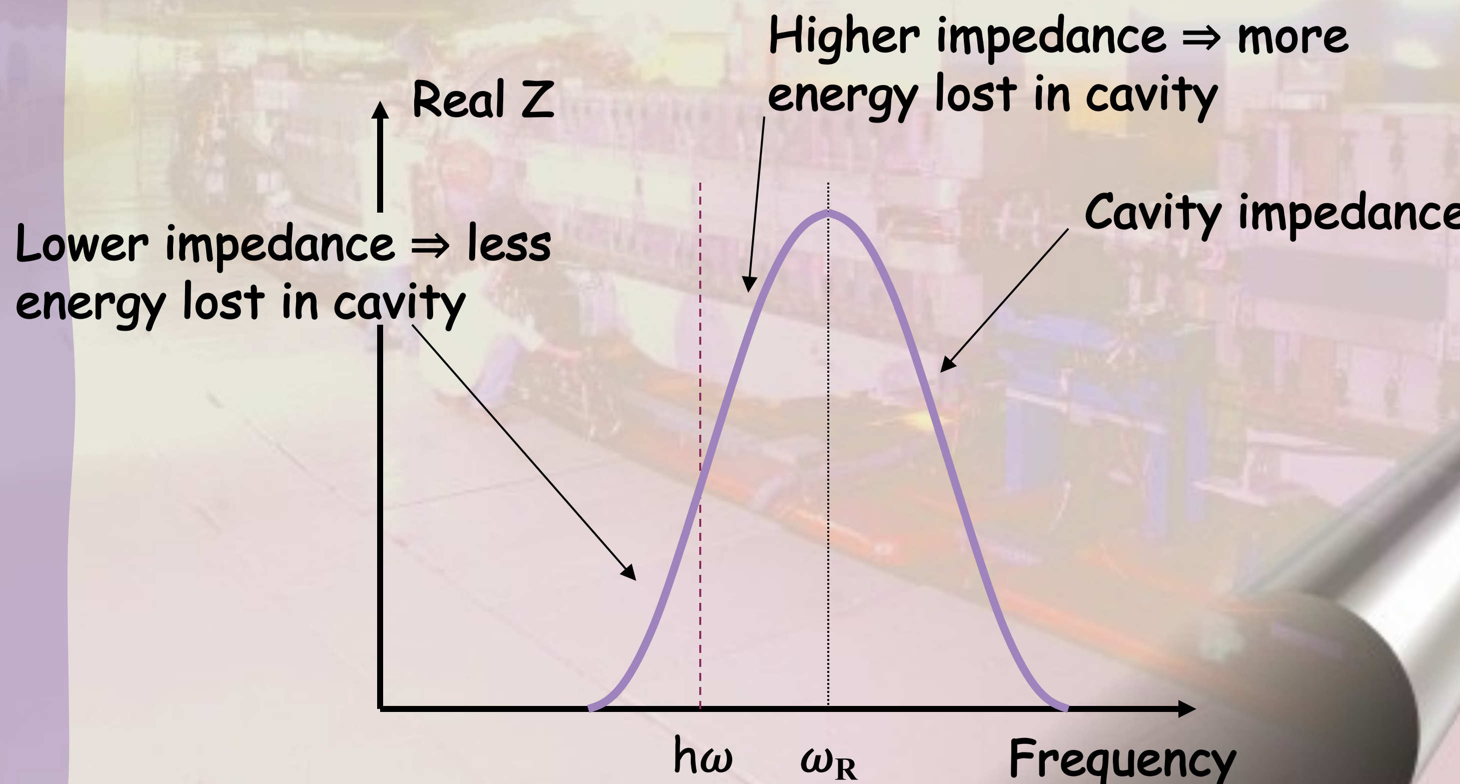
Impedance and Wall current (2)

- Any change of cross section or material leads to a finite impedance
- We can describe the vacuum chamber as a series of cavities
  - Narrow band - High Q resonators - RF Cavities tuned to some harmonic of the revolution frequency
  - Broad band - Low Q resonators - rest of the machine
- For any cavity two frequencies are important:
  - ω = Excitation frequency (bunch frequency)
  - ω_R= Resonant frequency of the cavity
- If $h\omega \approx \omega_R$ then the induced voltage will be large and will build up with repeated passages of the bunch

h is an integer

Single bunch Longitudinal Instabilities (1)

- Lets consider:
  - A single bunch with a revolution frequency = ω
  - That this bunch is not centered in the long. Phase Space
  - A single high-Q cavity which resonates at ω_R ($\omega_R \approx h\omega$)

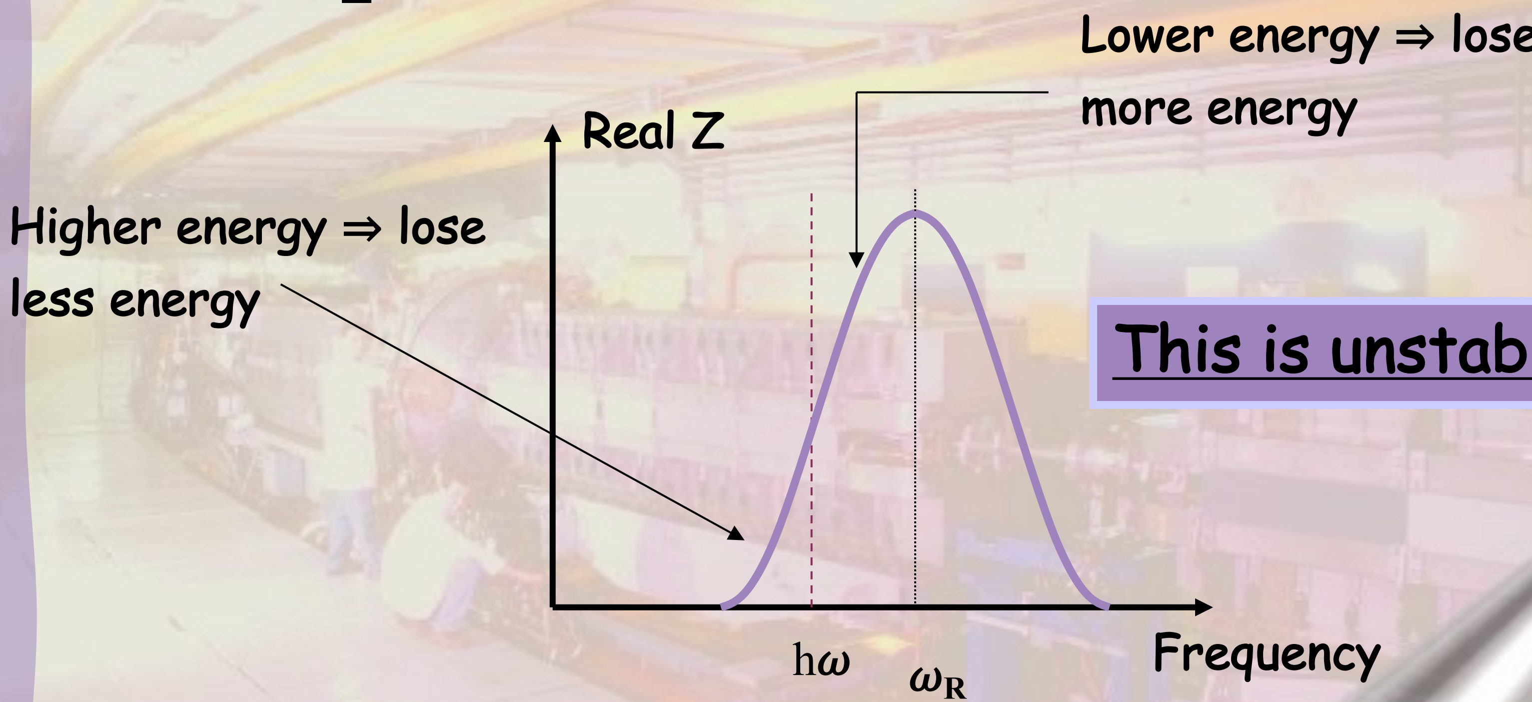


Single bunch Longitudinal Instabilities (2)

- Lets start a coherent synchrotron oscillation (above transition)
- The bunch will gain and loose energy/momentum
- There will be a decrease and increase in revolution frequency
- Therefore the bunch will see changing cavity impedance
- Lets consider two cases:
  - First case, consider $\omega_R > h\omega$
  - Second case, consider $\omega_R < h\omega$

Single bunch Longitudinal Instabilities (3)

- Case: $\omega_R > h\omega$

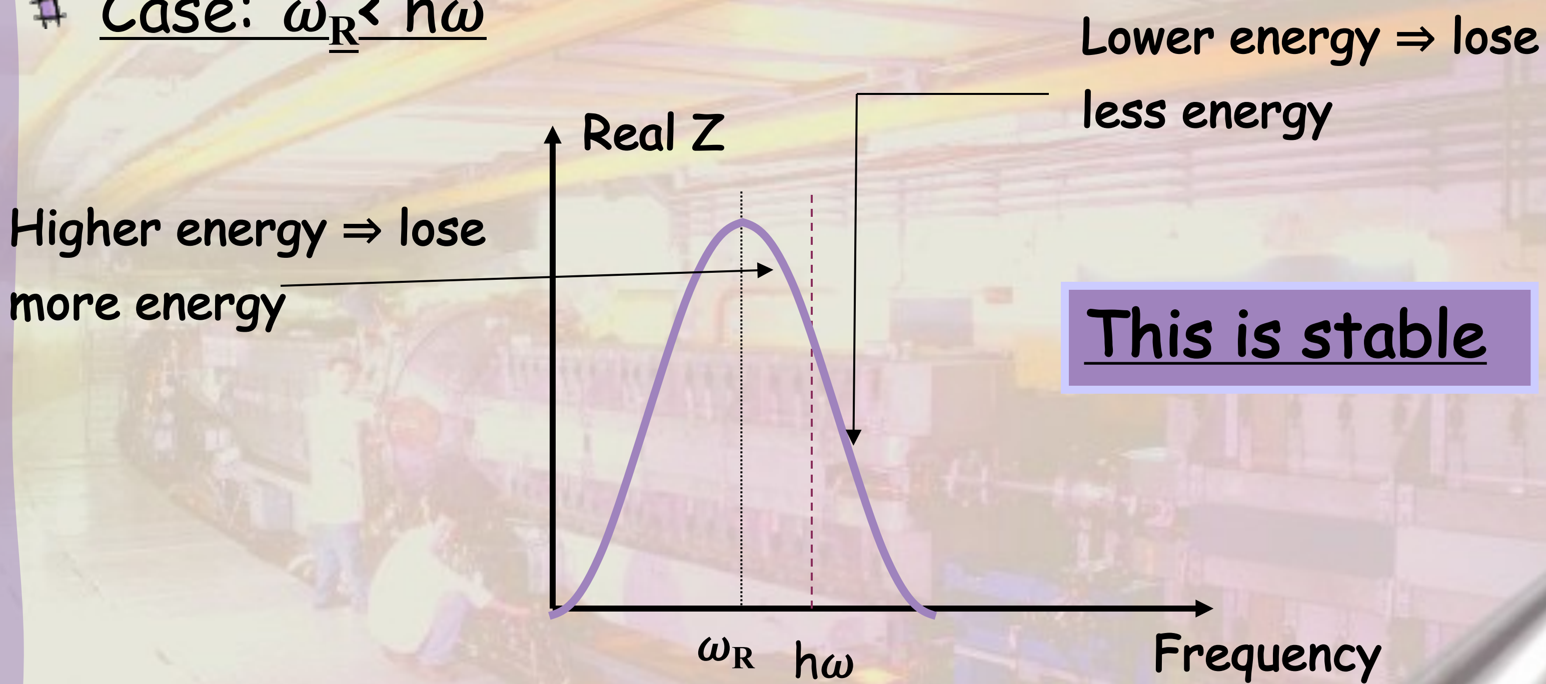


This is unstable

- The cavity tends to increase the energy oscillations
- Now retune cavity so that $\omega_R < h\omega$

Single bunch Longitudinal Instabilities (3)

- Case: $\omega_R < h\omega$

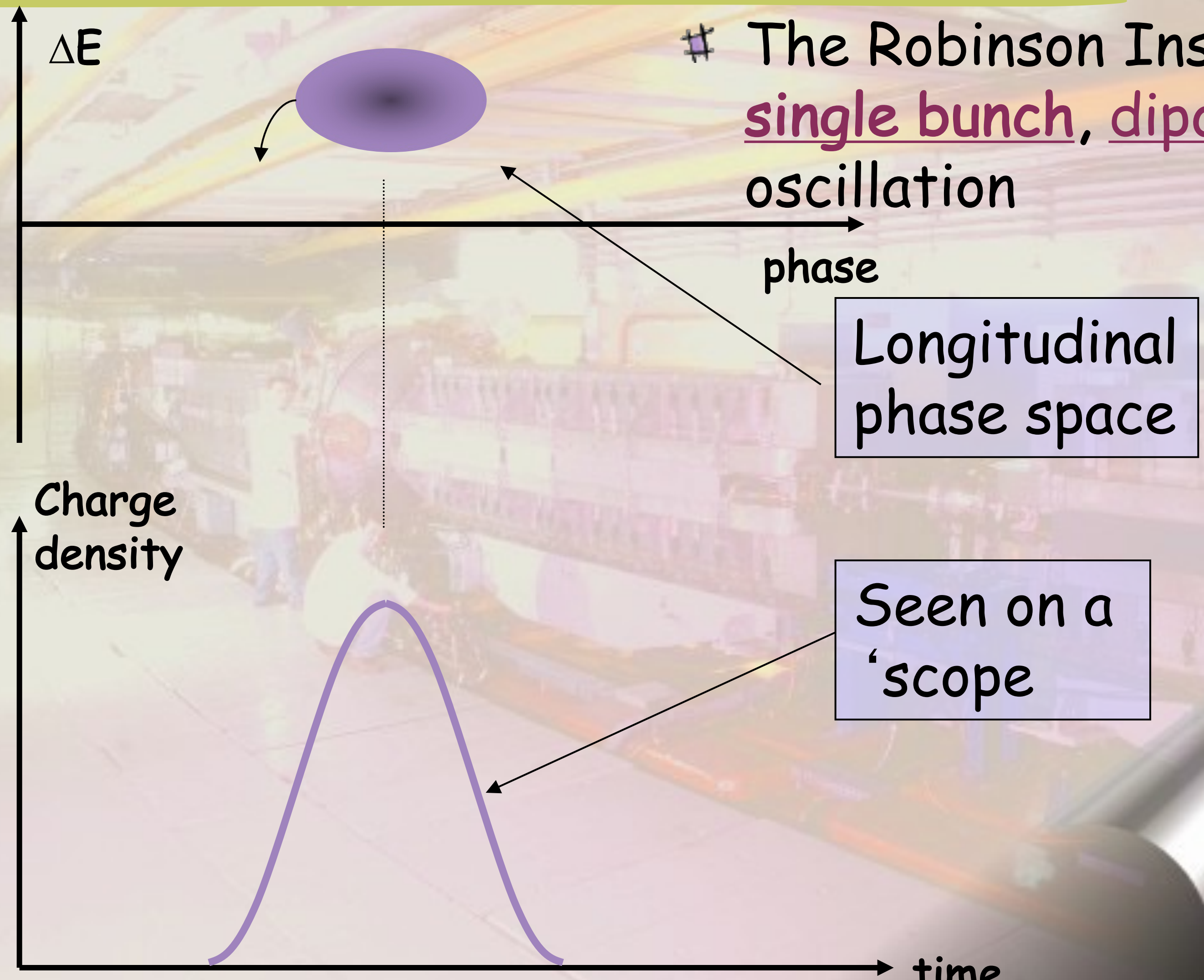


This is stable

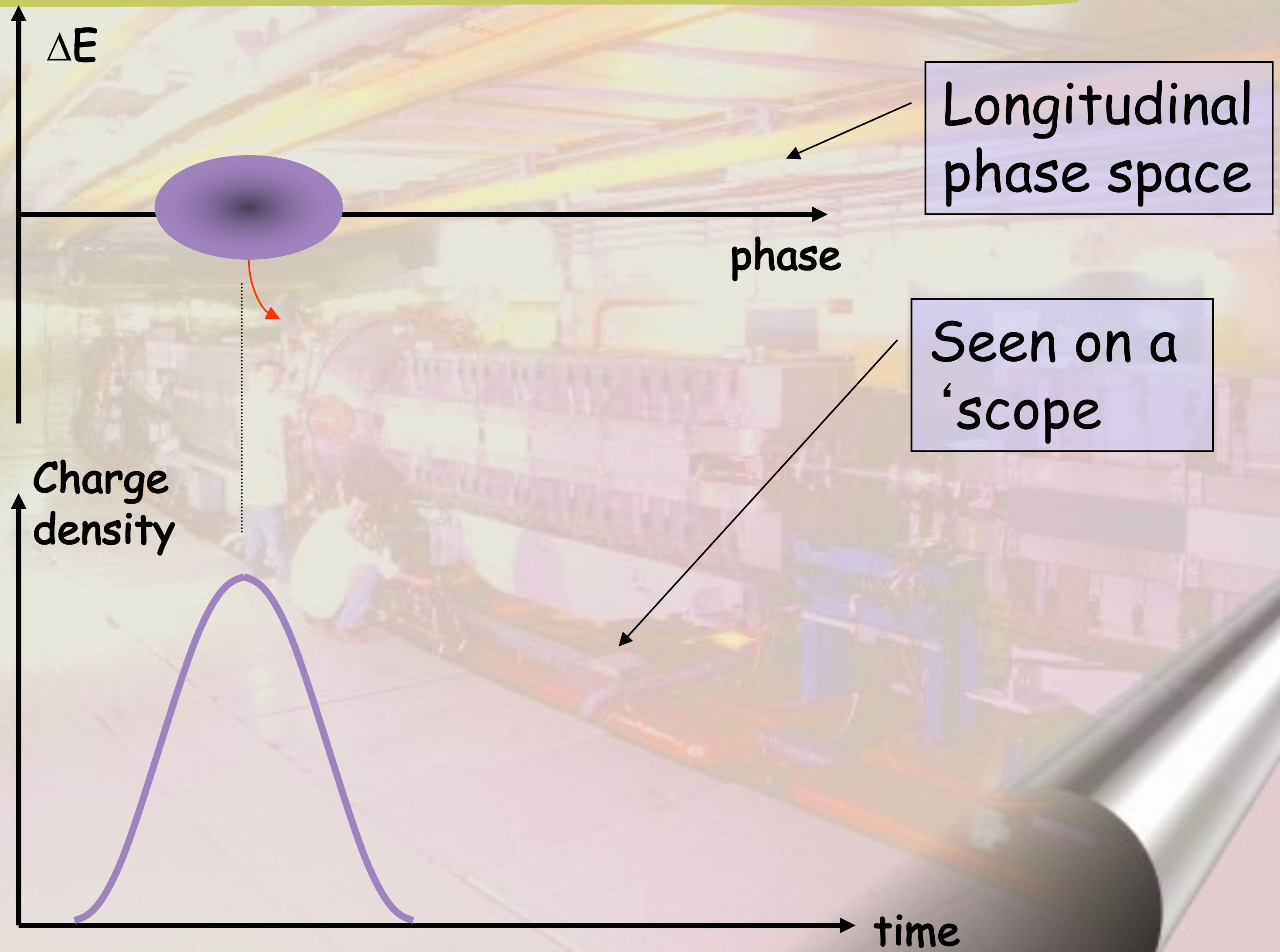
- This is is known as the 'Robinson Instability'
- **To damp this instability one should retune the cavity so that $\omega_R < h\omega$**

Robinson Instability (1)

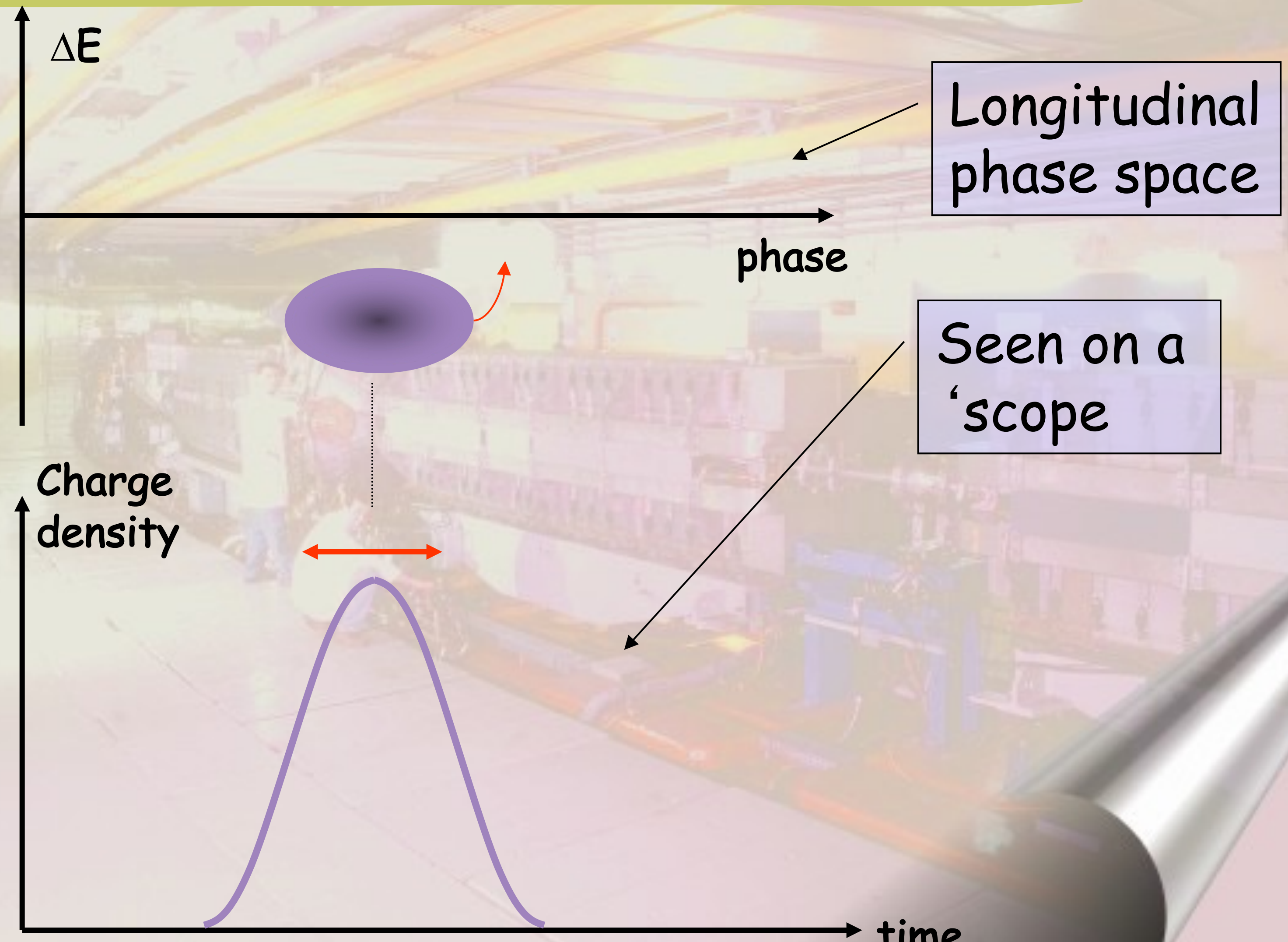
- The Robinson Instability is a single bunch, dipole mode oscillation



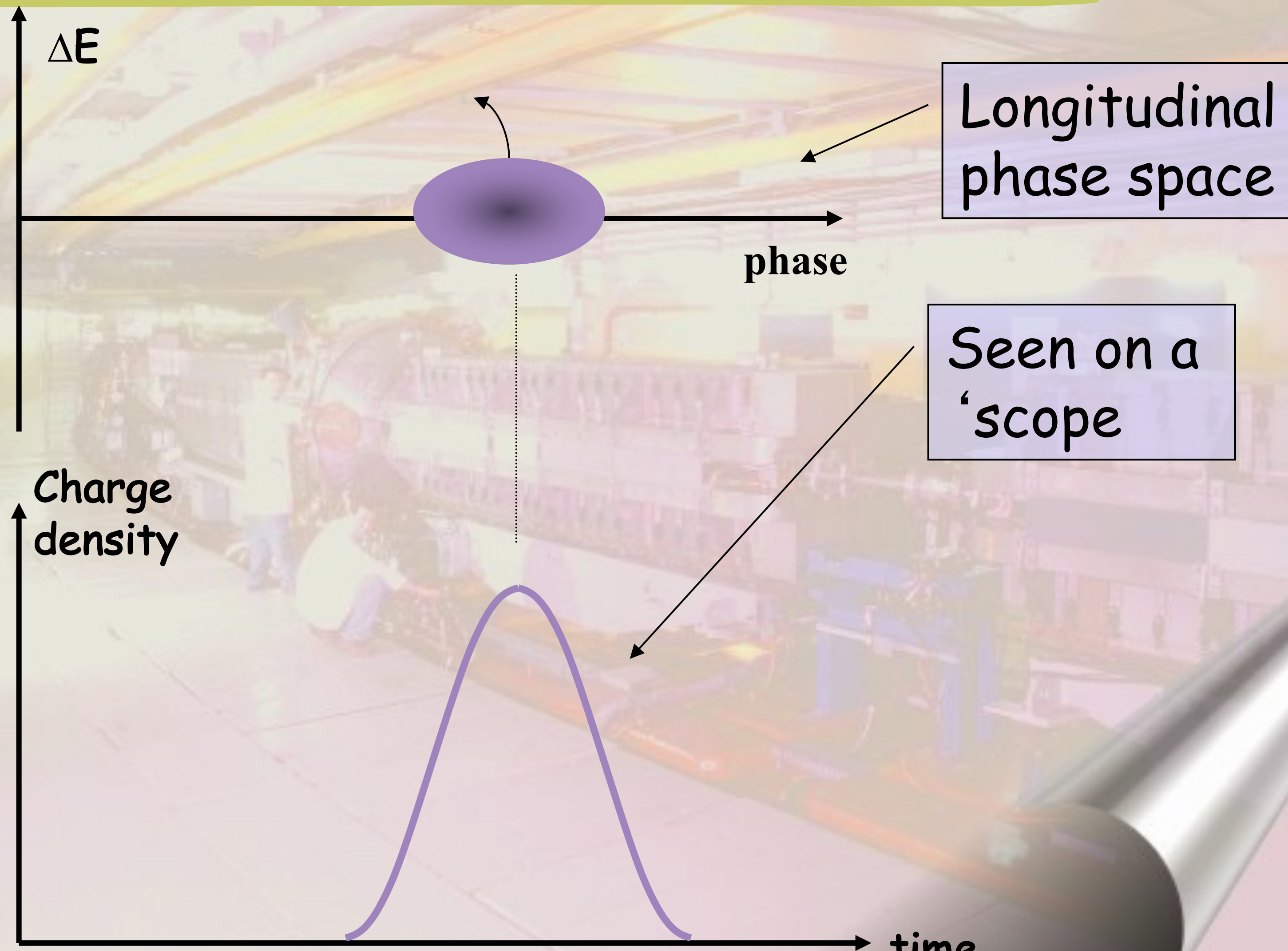
Robinson Instability (2)



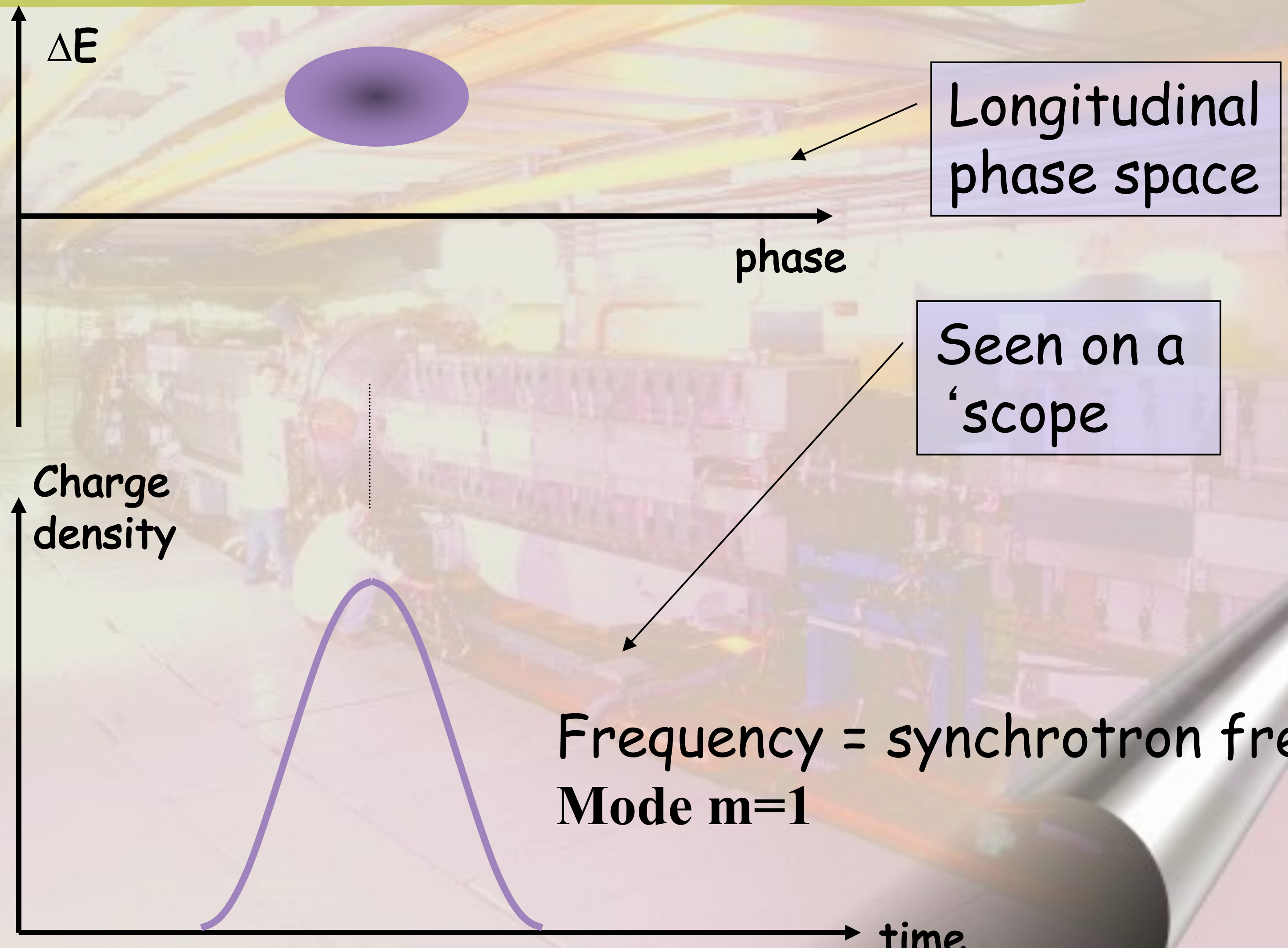
Robinson Instability (3)



Robinson Instability (4)



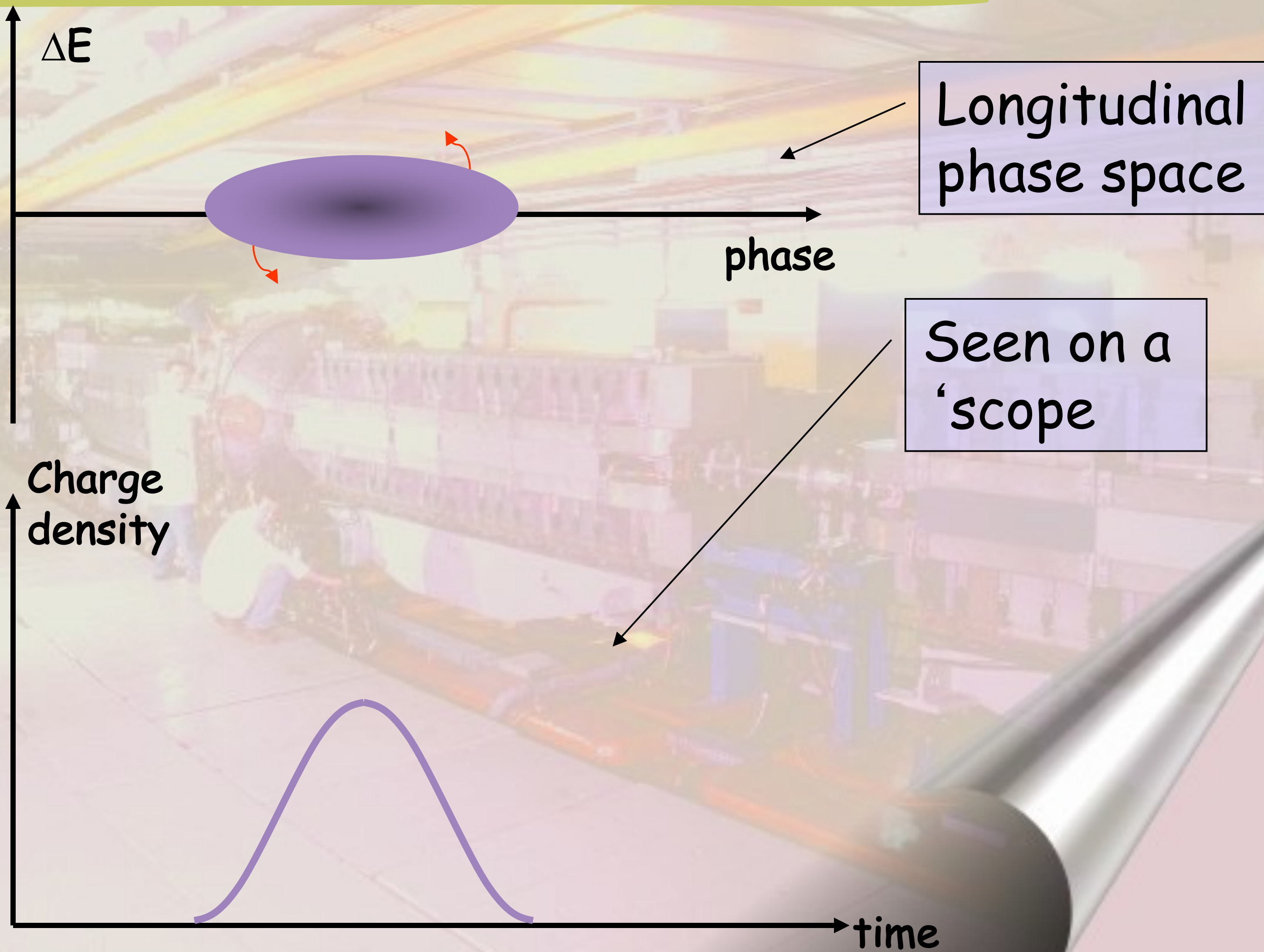
Robinson Instability (5)



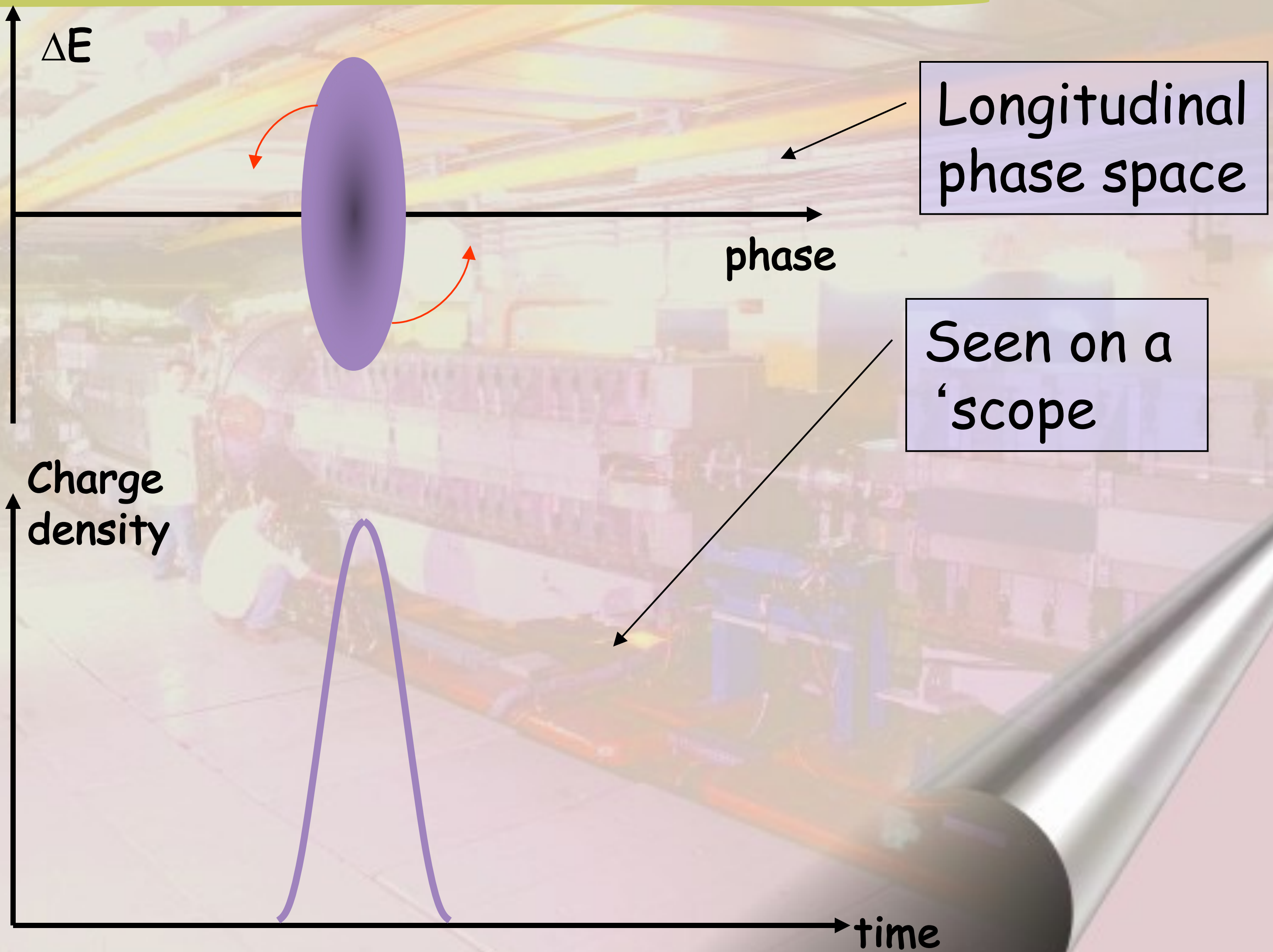
Frequency = synchrotron frequency

Mode m=1

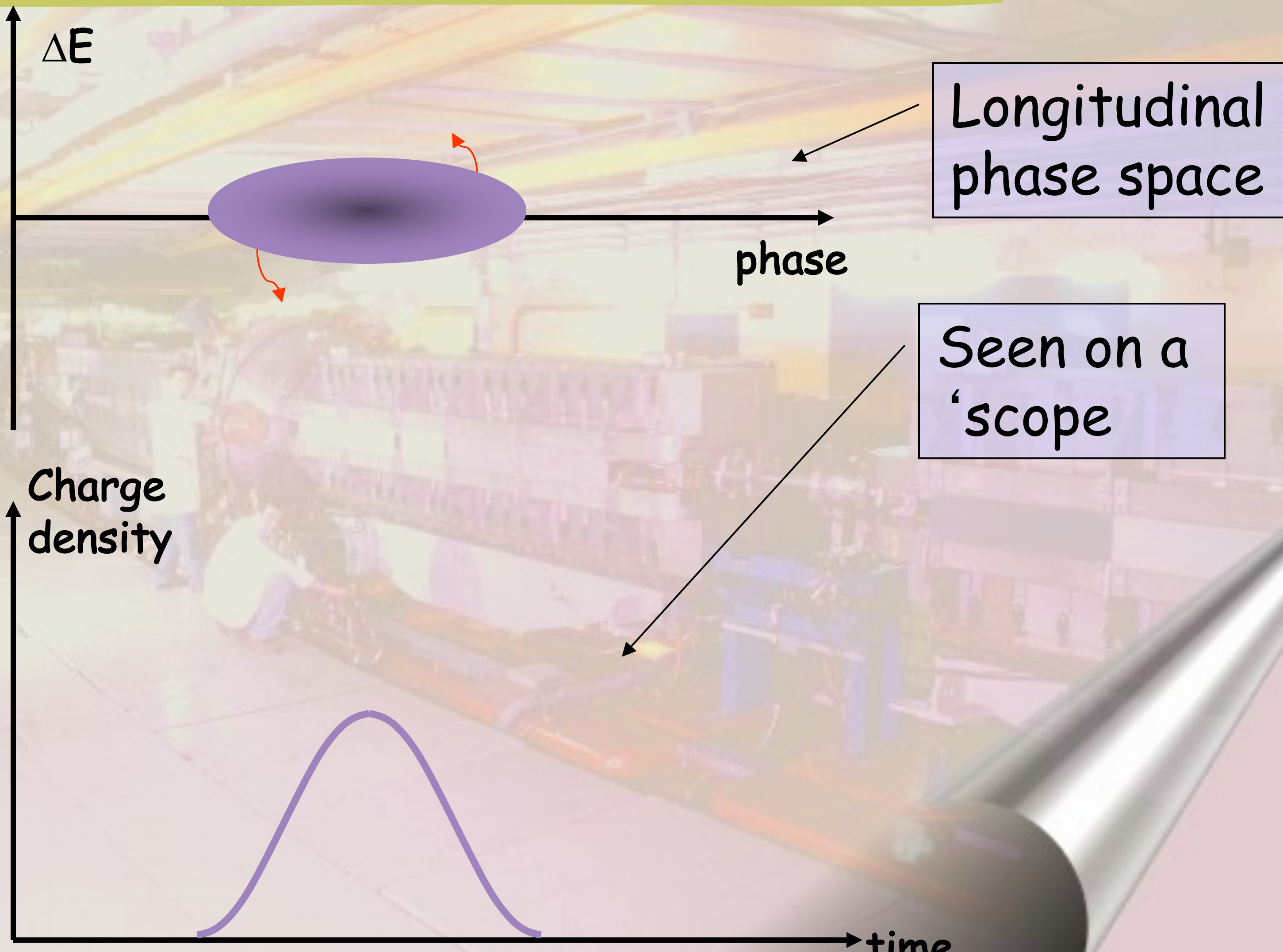
Higher order modes m=2 ….. (1)



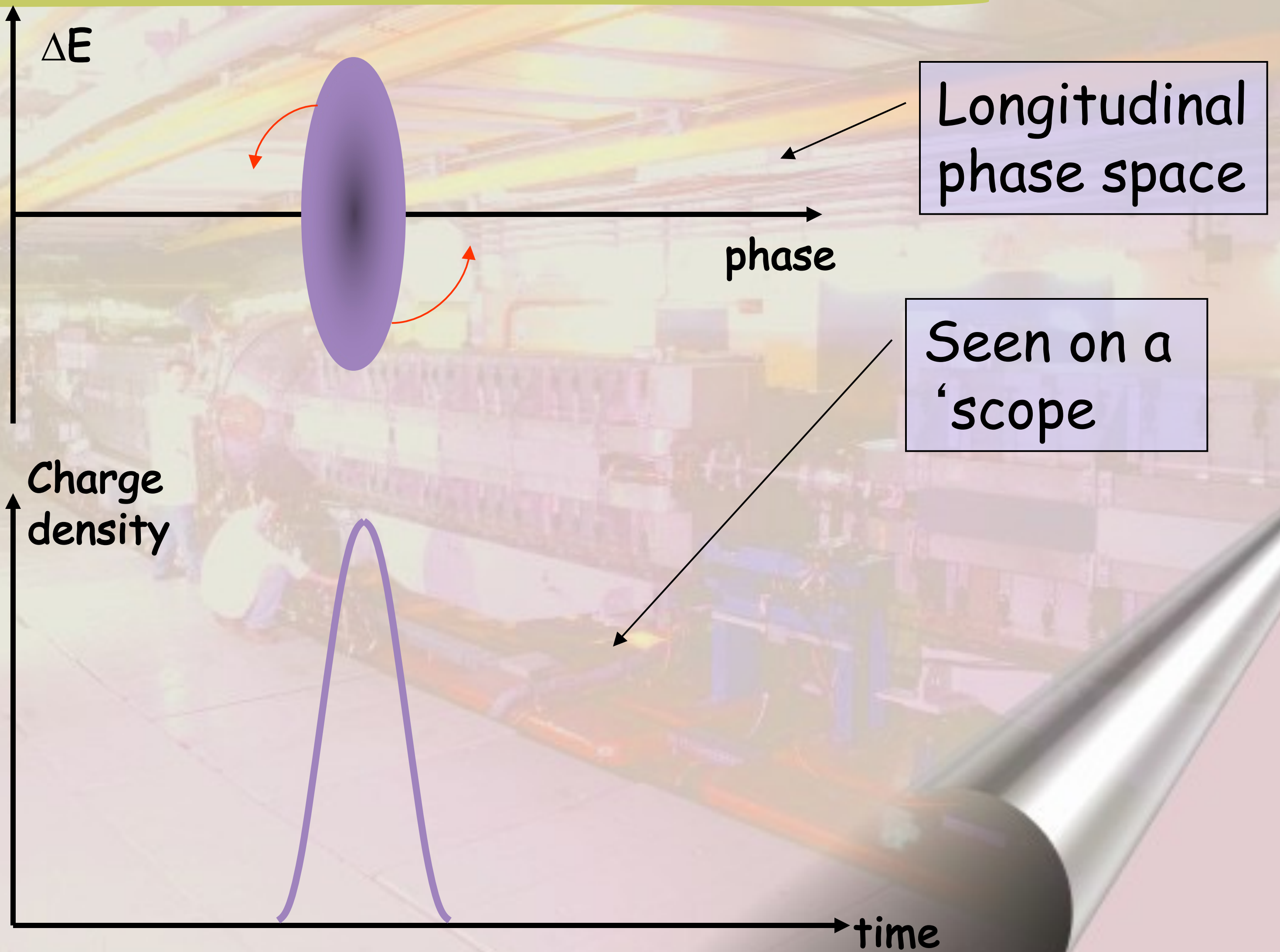
Higher order modes m=2 ….. (2)



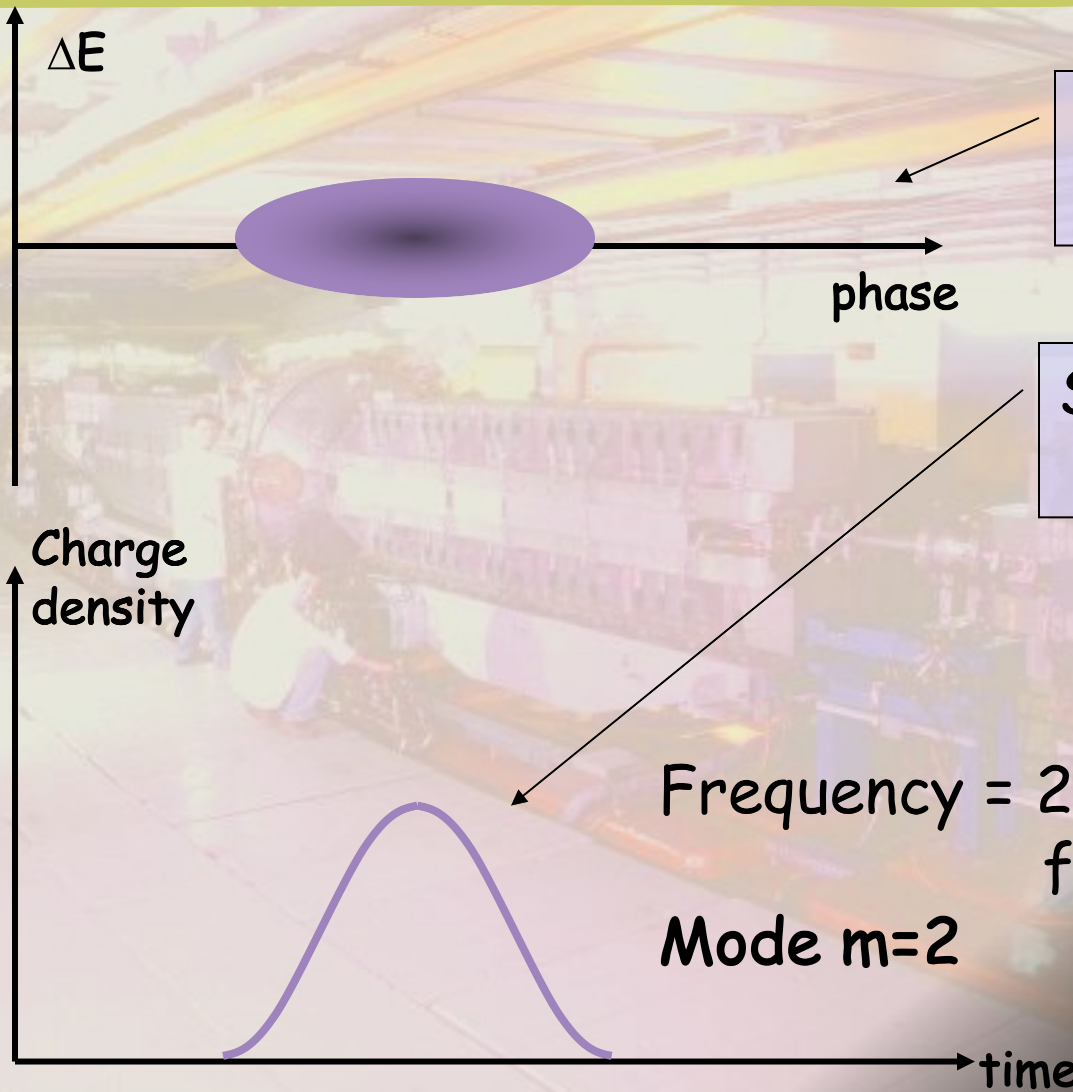
Higher order modes m=2 ….. (3)



Higher order modes m=2 ..... (4)



Higher order modes m=2 ….. (5)



Longitudinal phase space

Seen on a 'scope

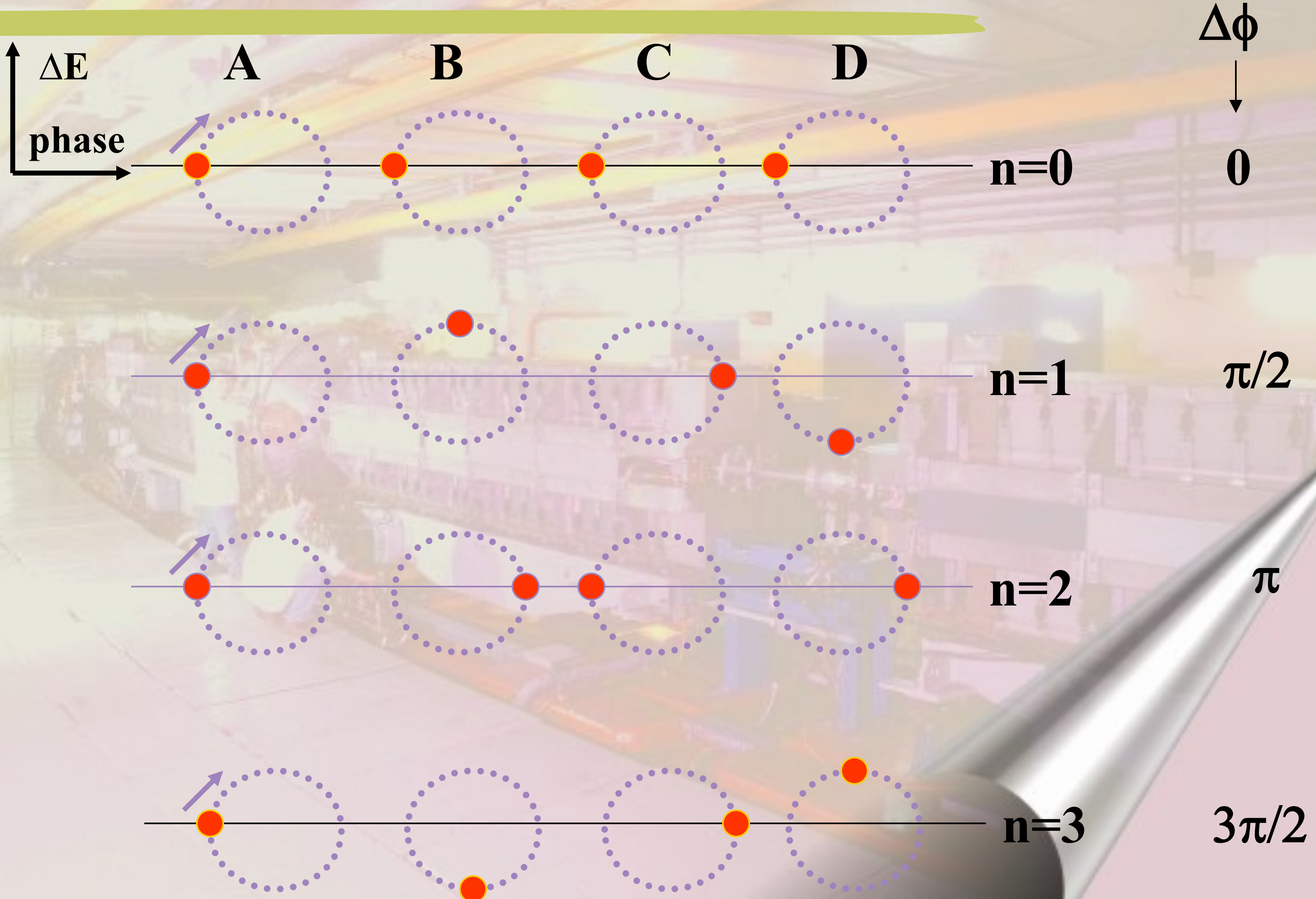
Frequency = 2 × synchrotron frequency

Mode m=2

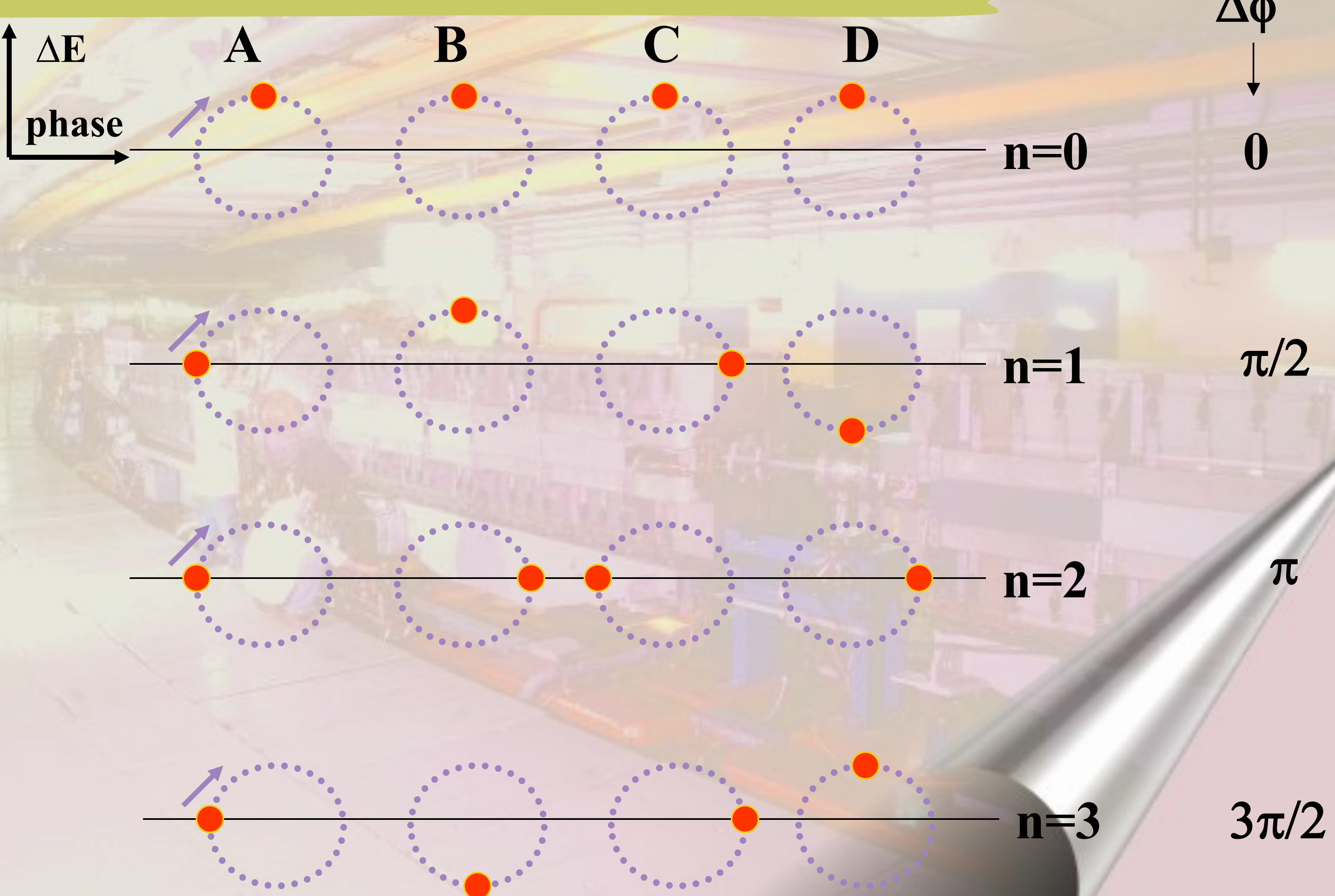
Multi-bunch instabilities (1)

- What if we have more than one bunch in our ring…..?
- Lets take 4 equidistant bunches **A**, **B**, **C** & **D**
- The field left in the cavity by bunch **A** alters the coherent synchrotron oscillation motion of **B**, which changes field left by bunch **B**, which alters bunch **C**……to bunch **D**, etc…etc..
- Until we get back to bunch **A**…..
- For 4 bunches there are 4 possible modes of coupled bunch longitudinal oscillation

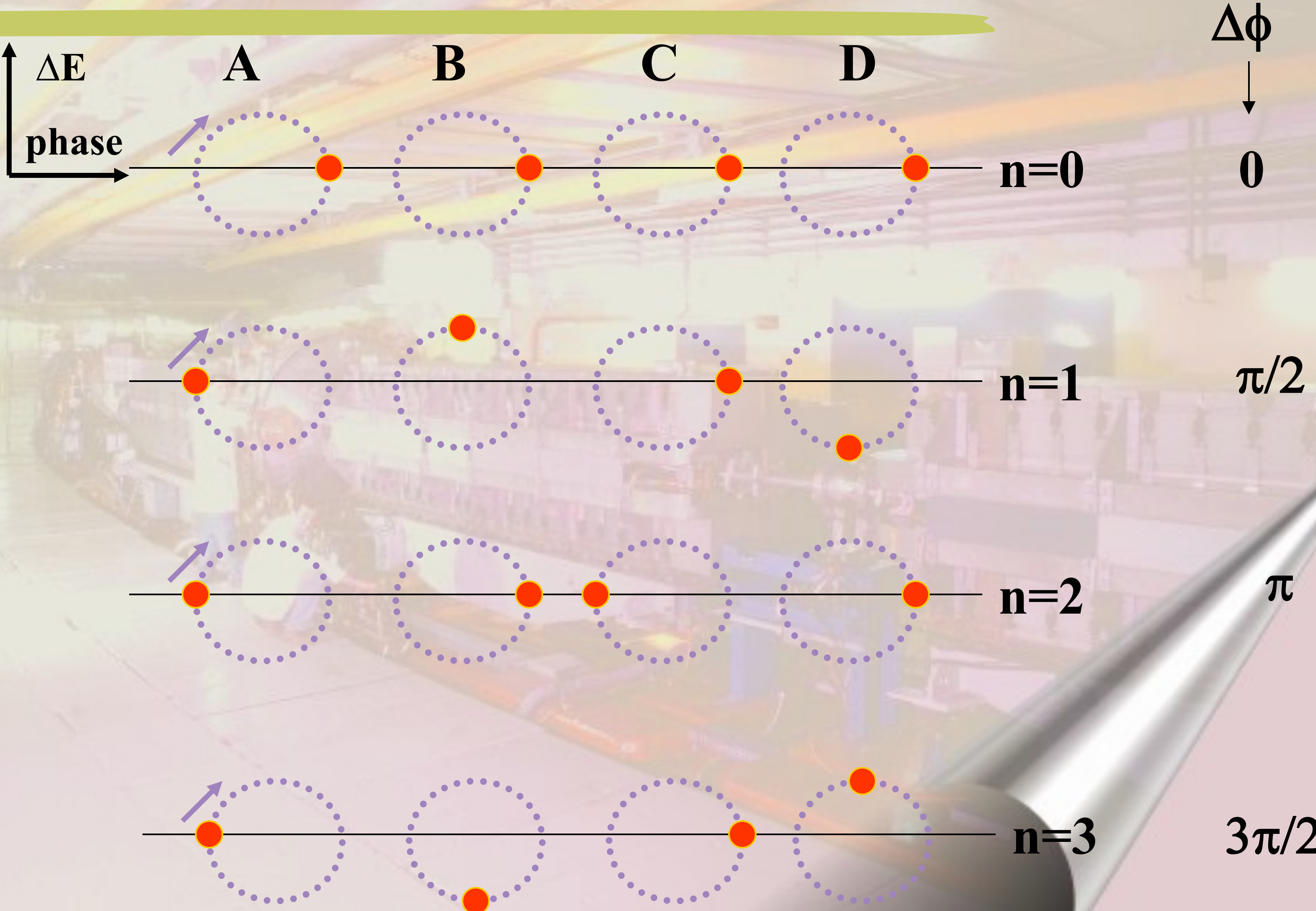
Multi-bunch instabilities (2)



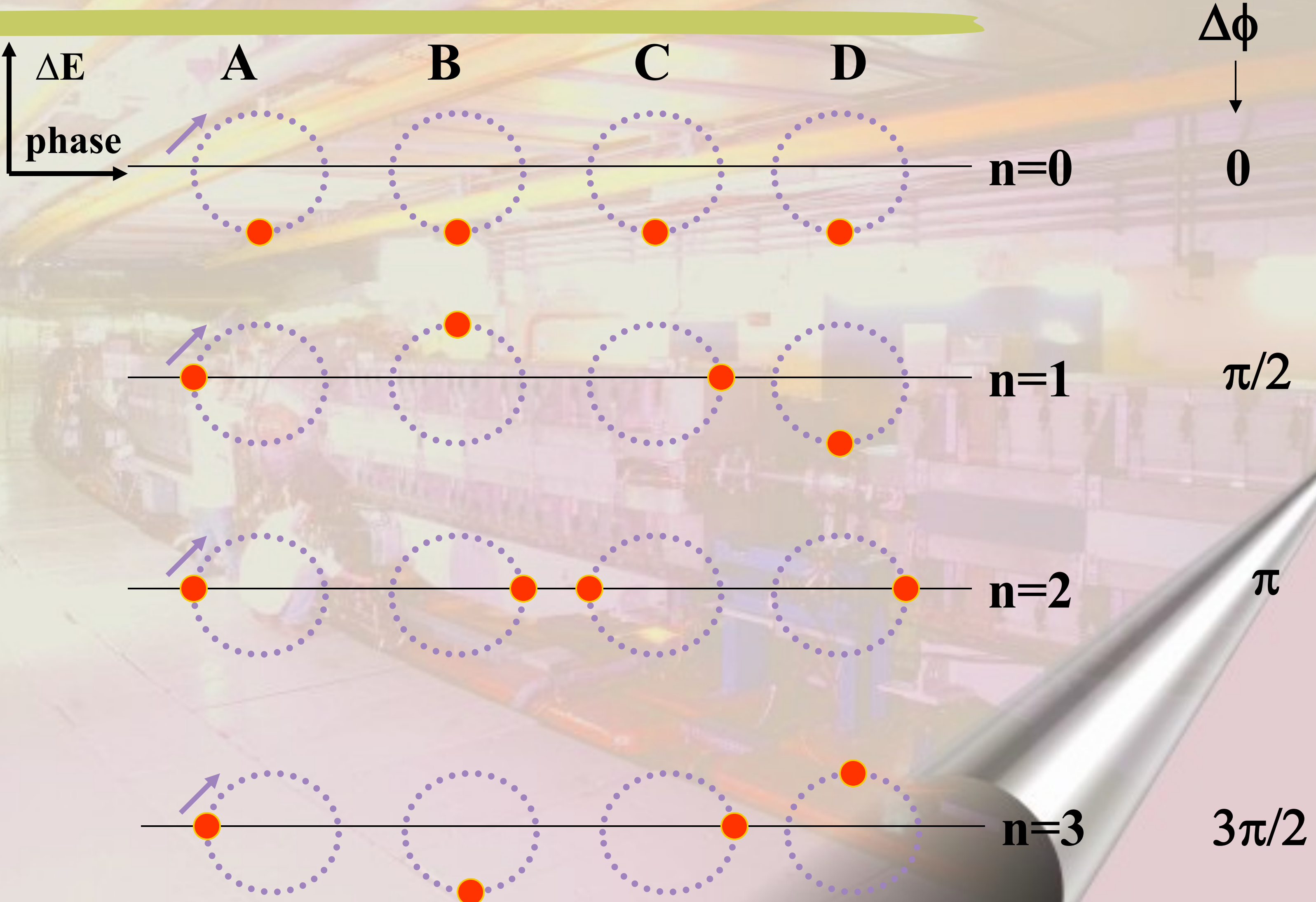
Multi-bunch instabilities (3)



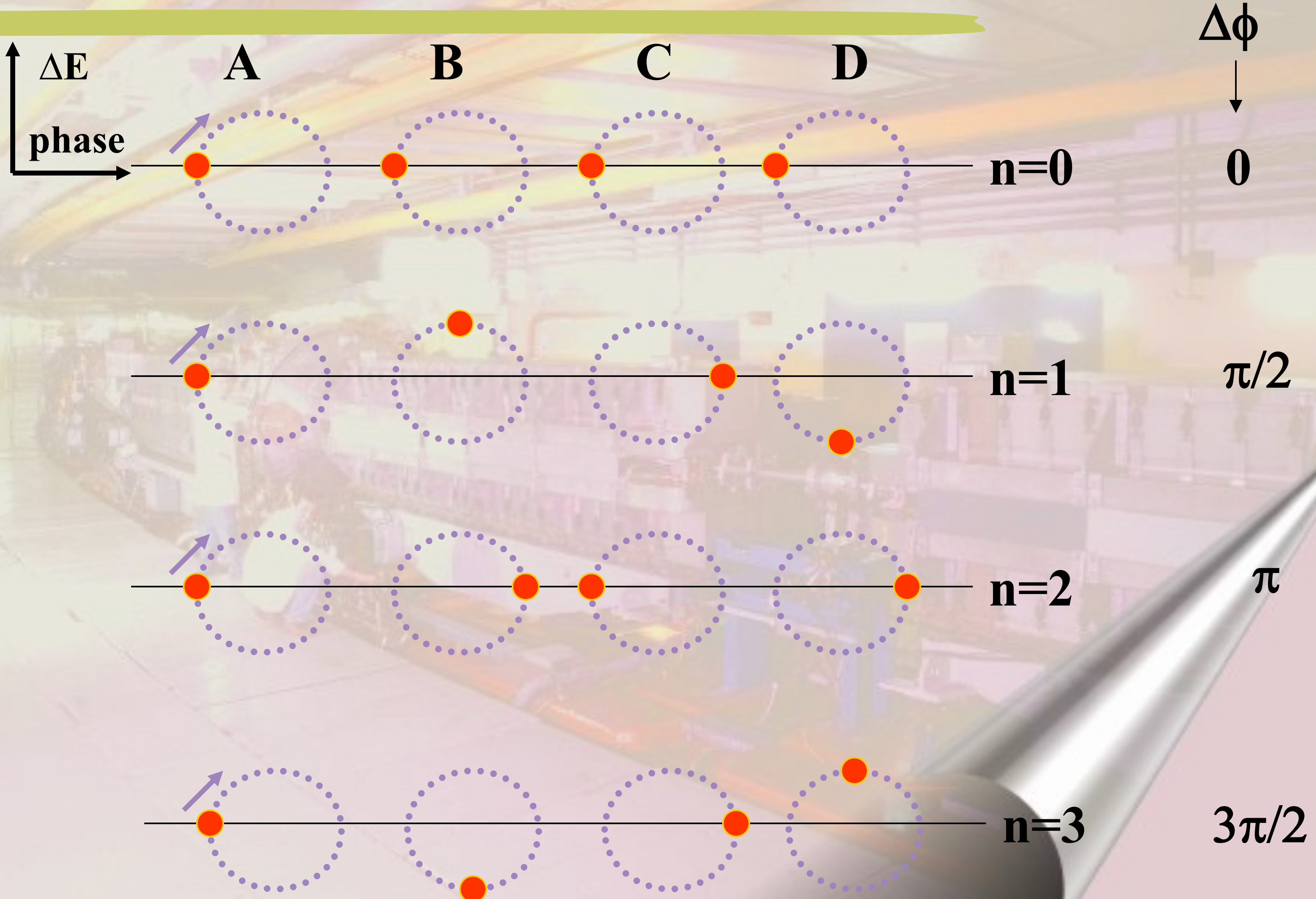
Multi-bunch instabilities (4)



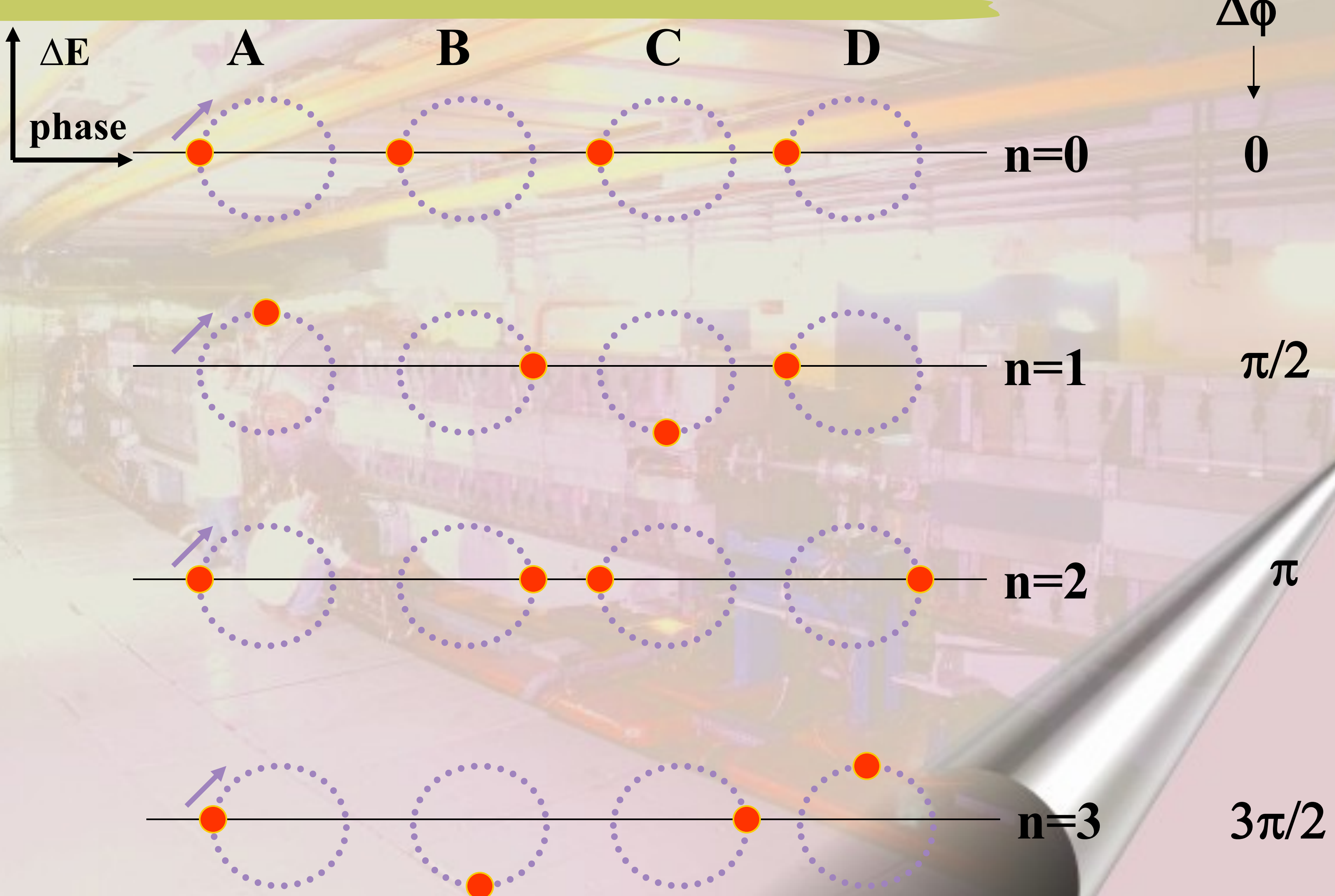
Multi-bunch instabilities (5)



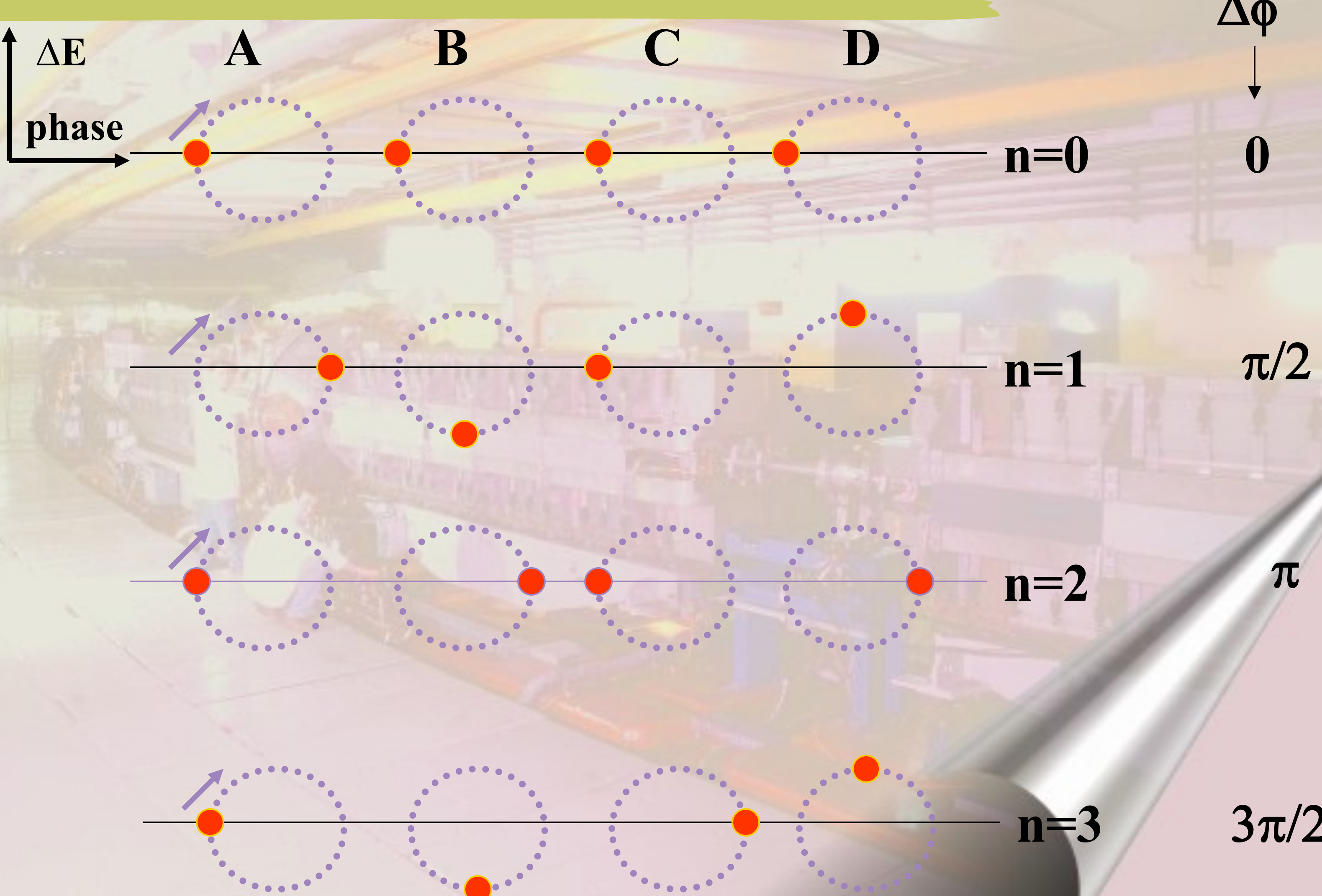
Multi-bunch instabilities (6)



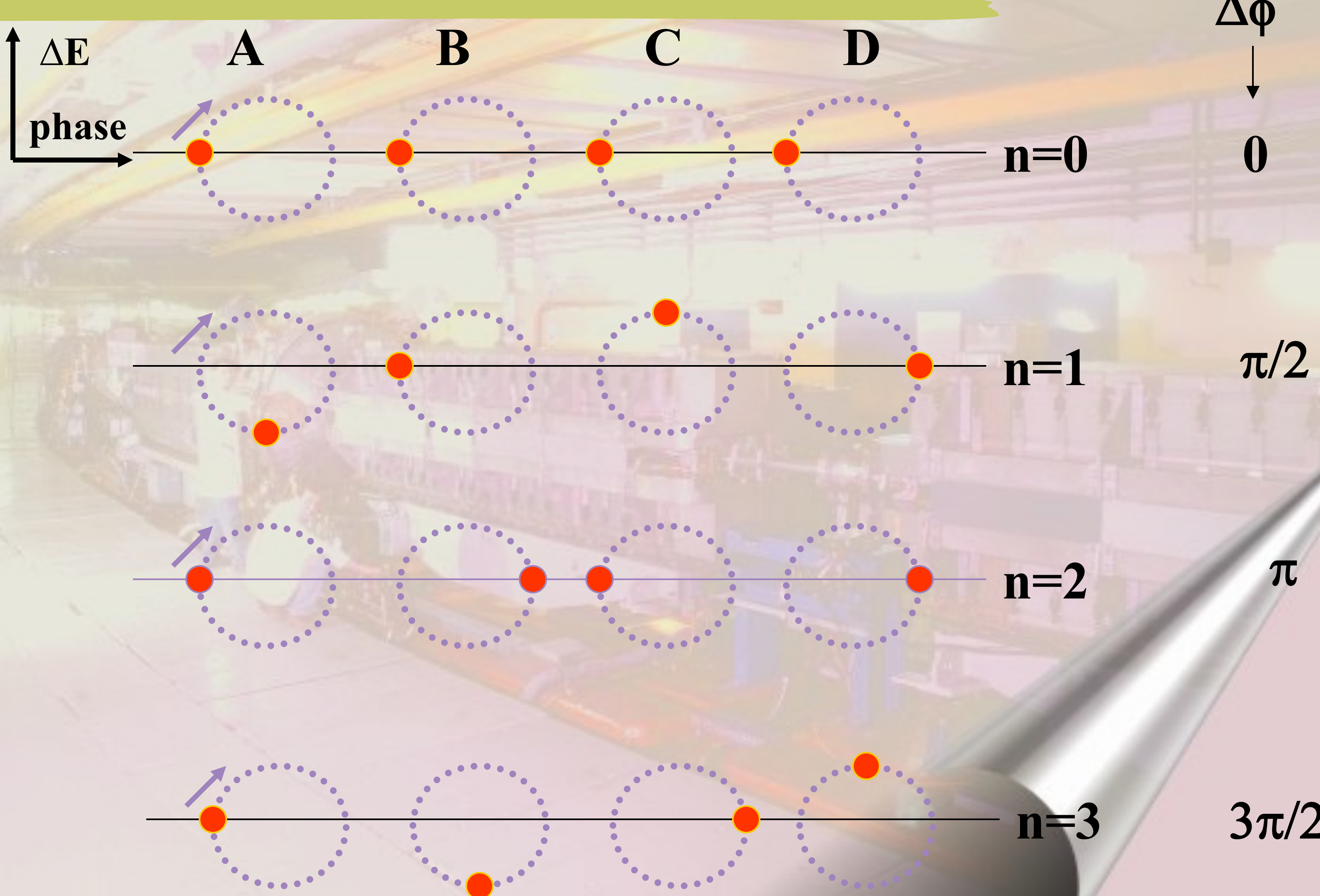
Multi-bunch instabilities (7)



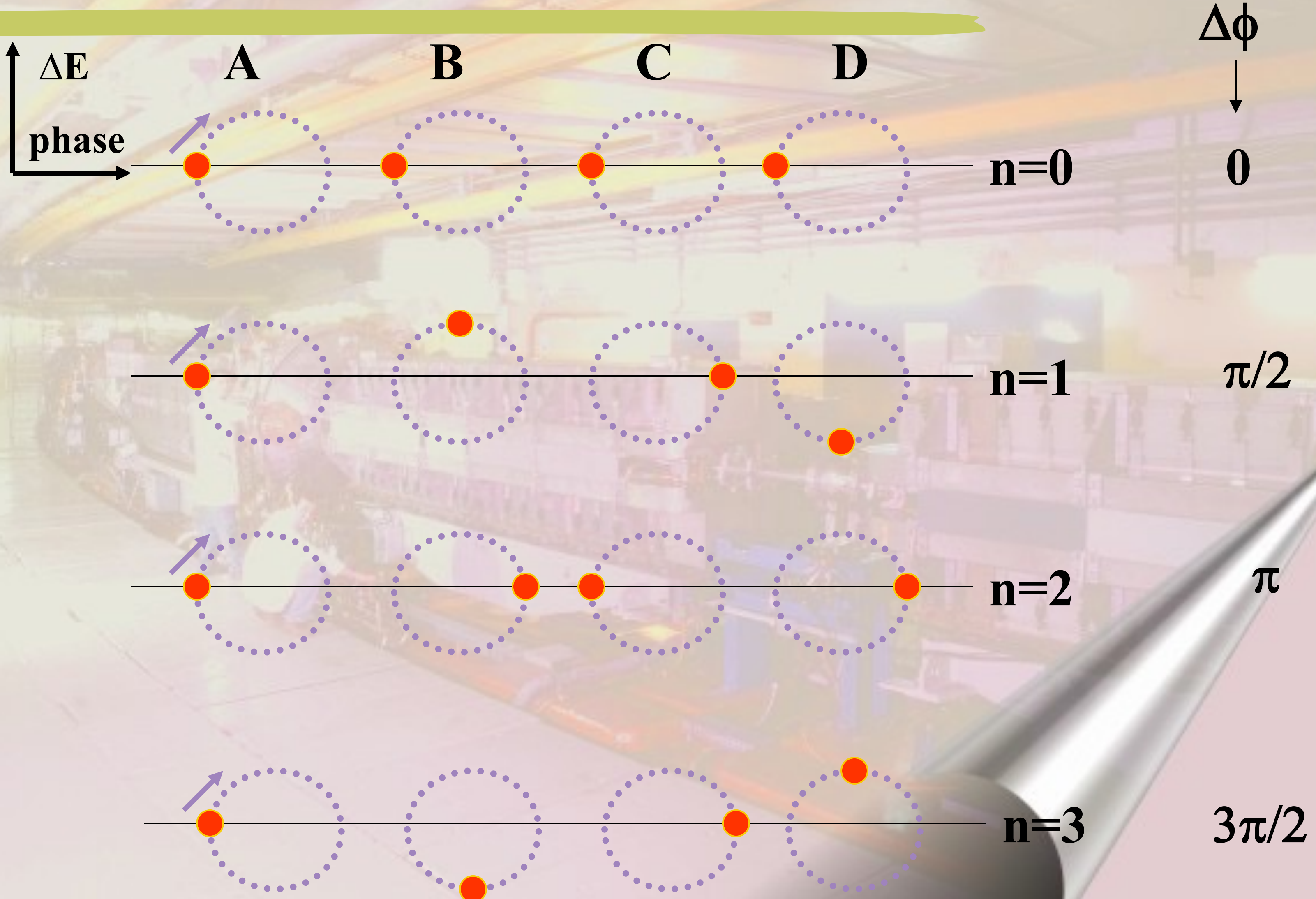
Multi-bunch instabilities (8)



Multi-bunch instabilities (9)

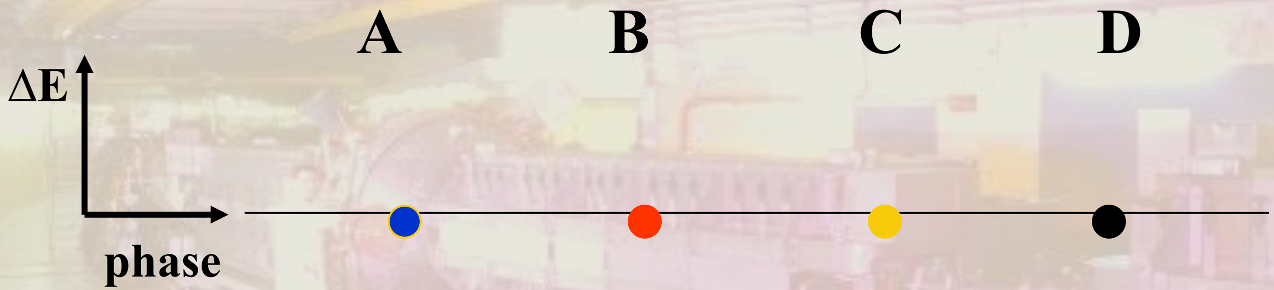


Multi-bunch instabilities (10)



Multi-bunch instabilities (11)

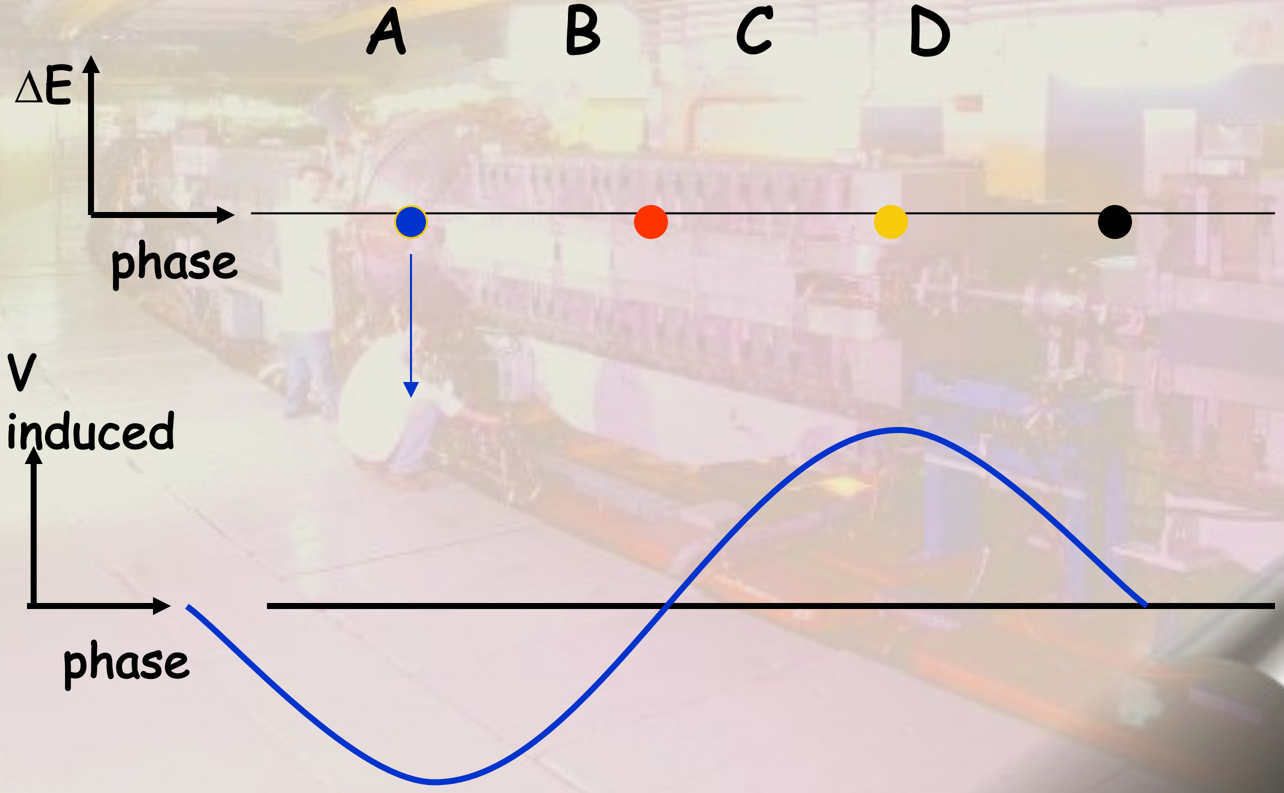
- For simplicity assume we have a single cavity which resonates at the revolution frequency
- With no coherent synchrotron oscillation we have:



- Lets have a look at the voltage induced in a cavity by each bunch

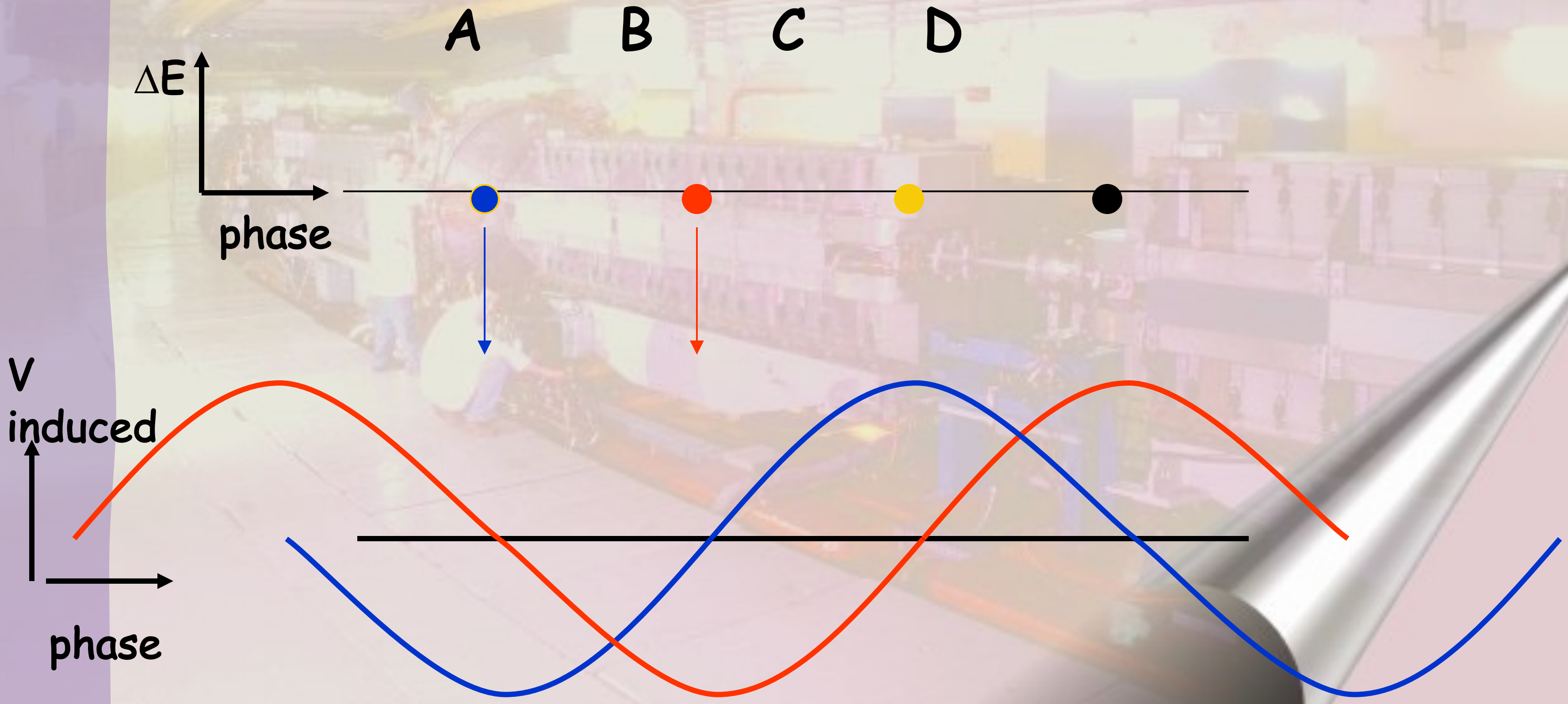
Multi-bunch instabilities (12)

Bunch A



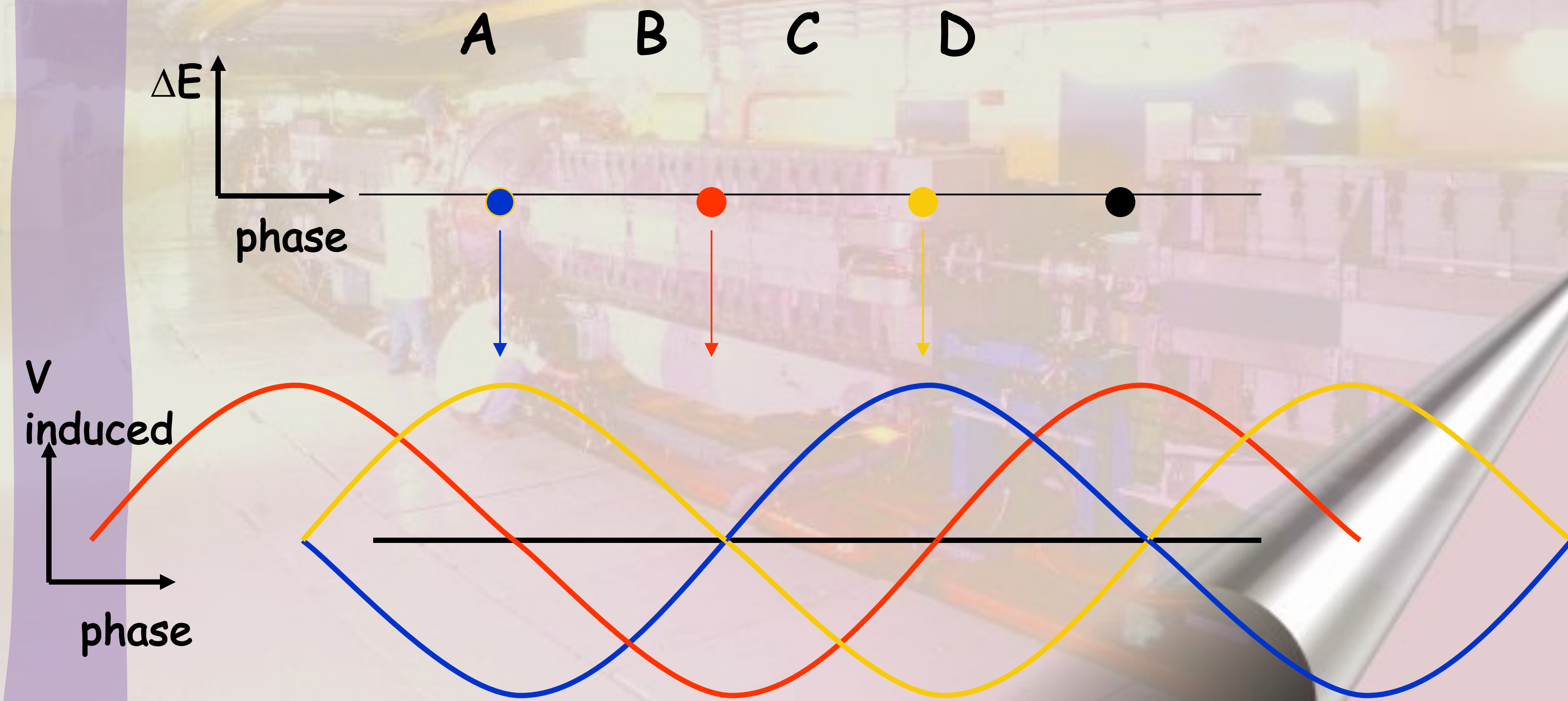
Multi-bunch instabilities (13)

Bunch B



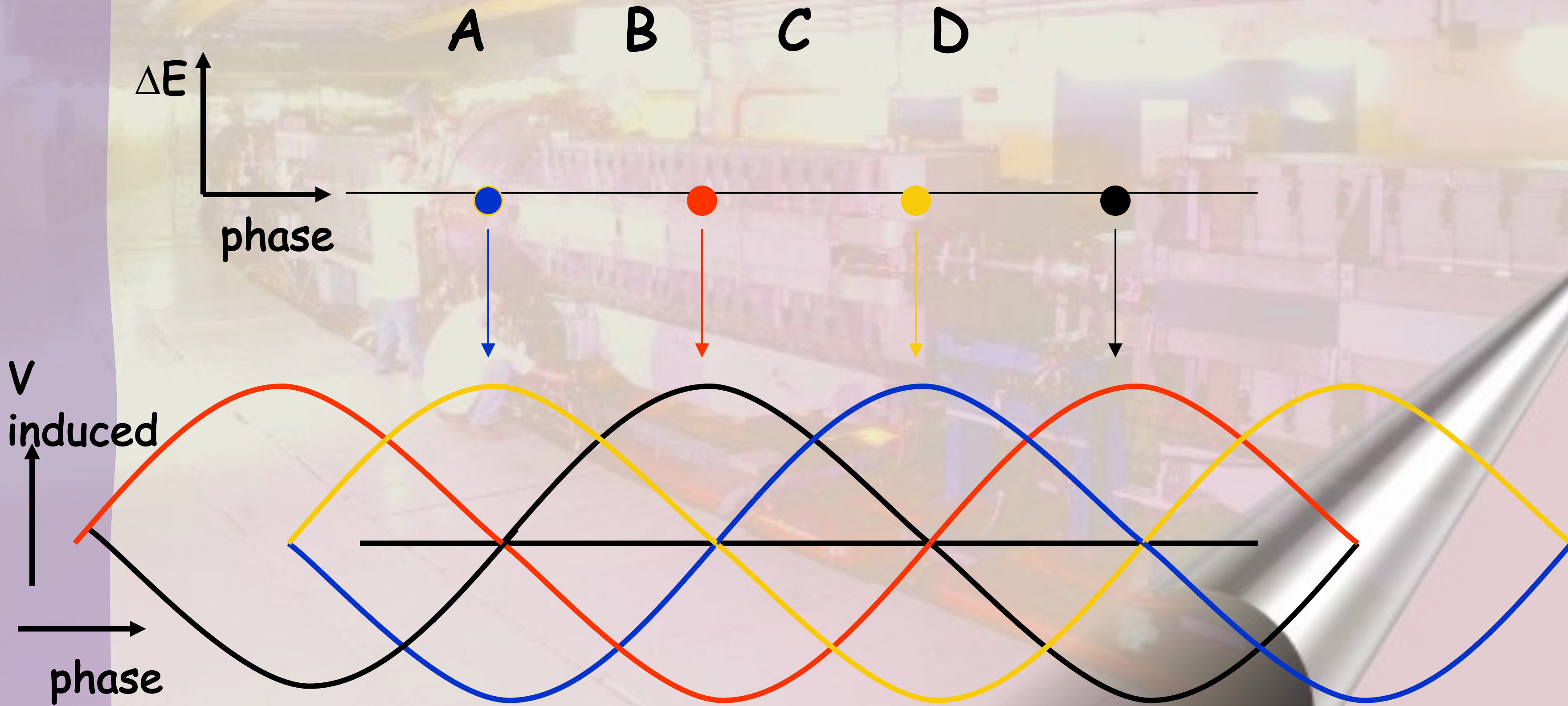
Multi-bunch instabilities (14)

Bunch C



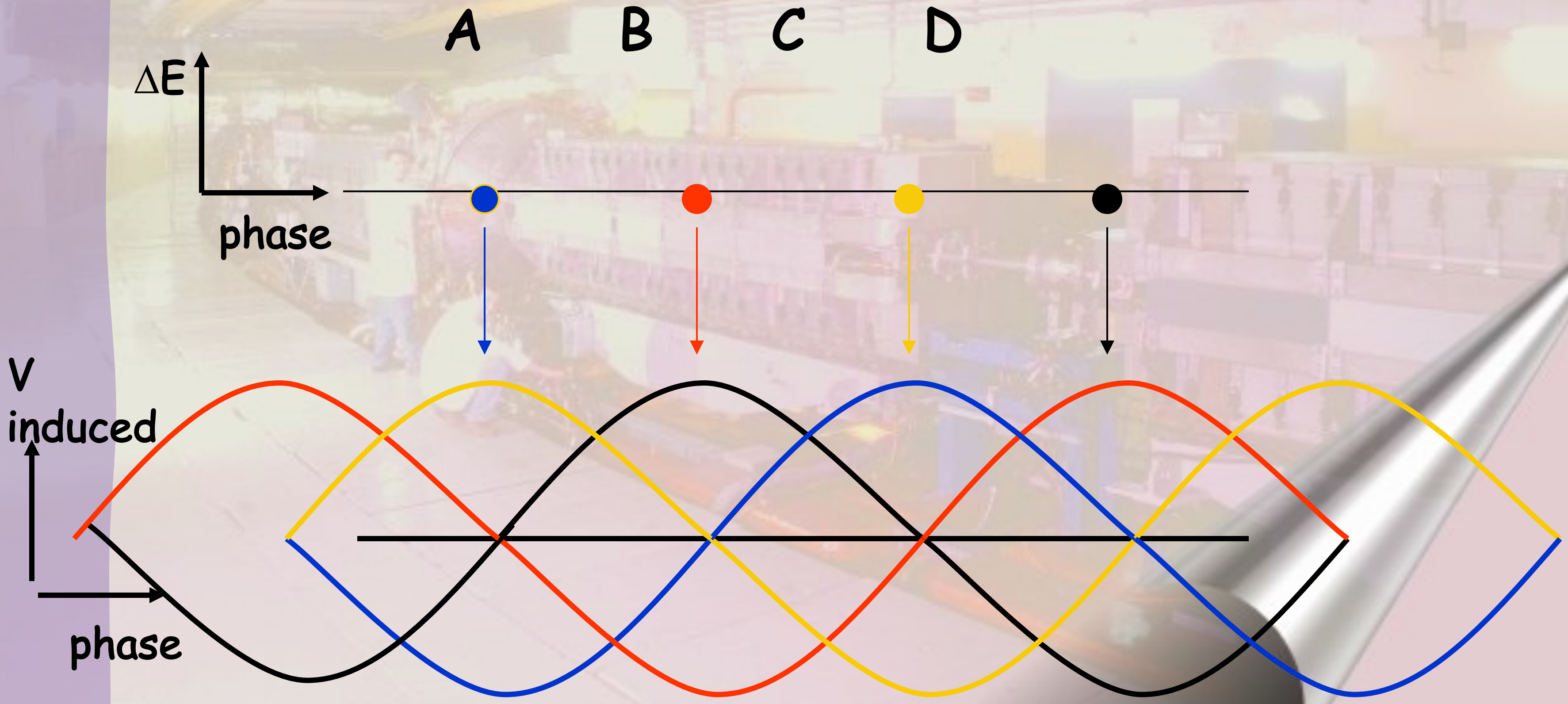
Multi-bunch instabilities (15)

Bunch D



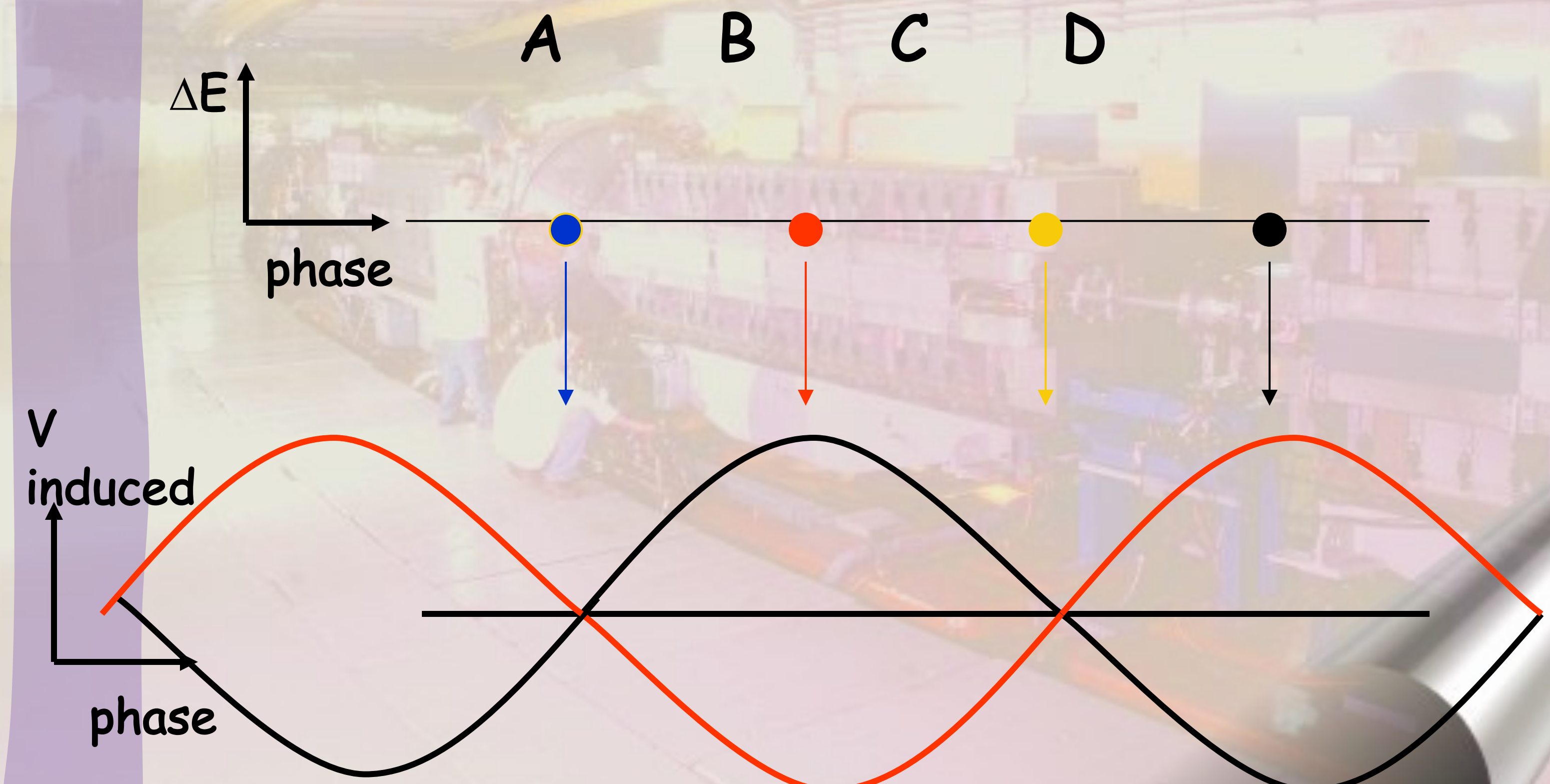
Multi-bunch instabilities (16)

A & C induced voltages cancel



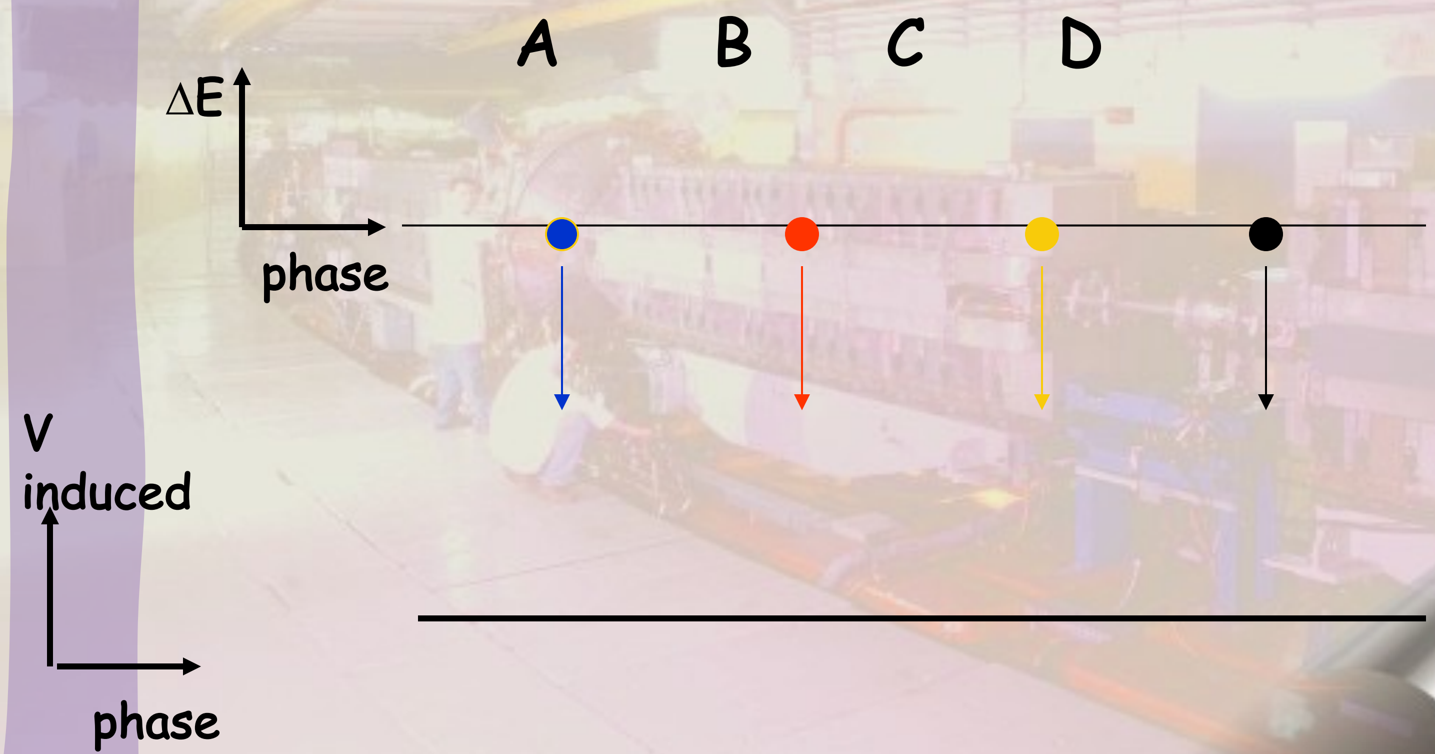
Multi-bunch instabilities (17)

B & D induced voltages cancel

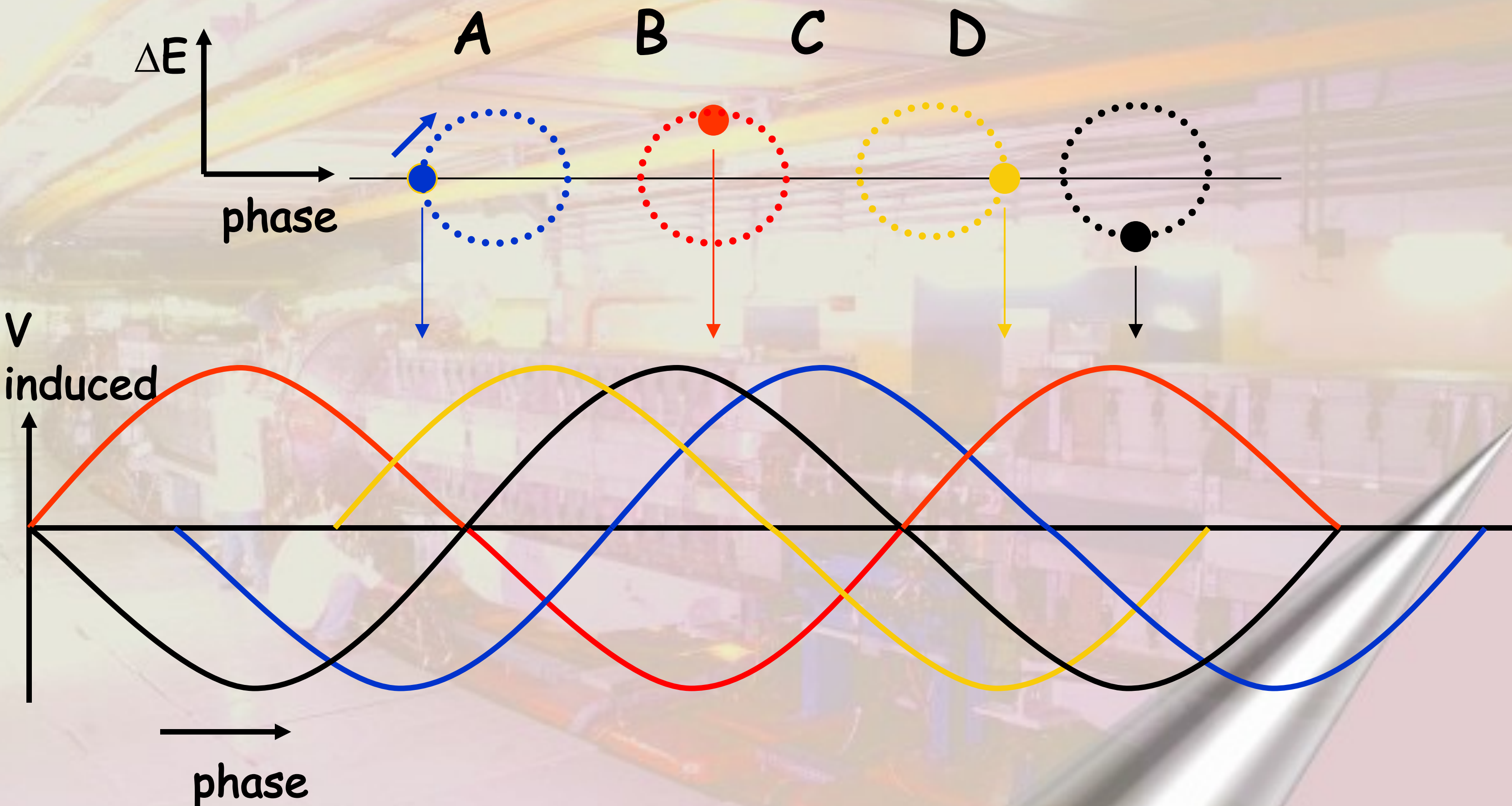


Multi-bunch instabilities (18)

All voltages cancel ⇒ no residual effect



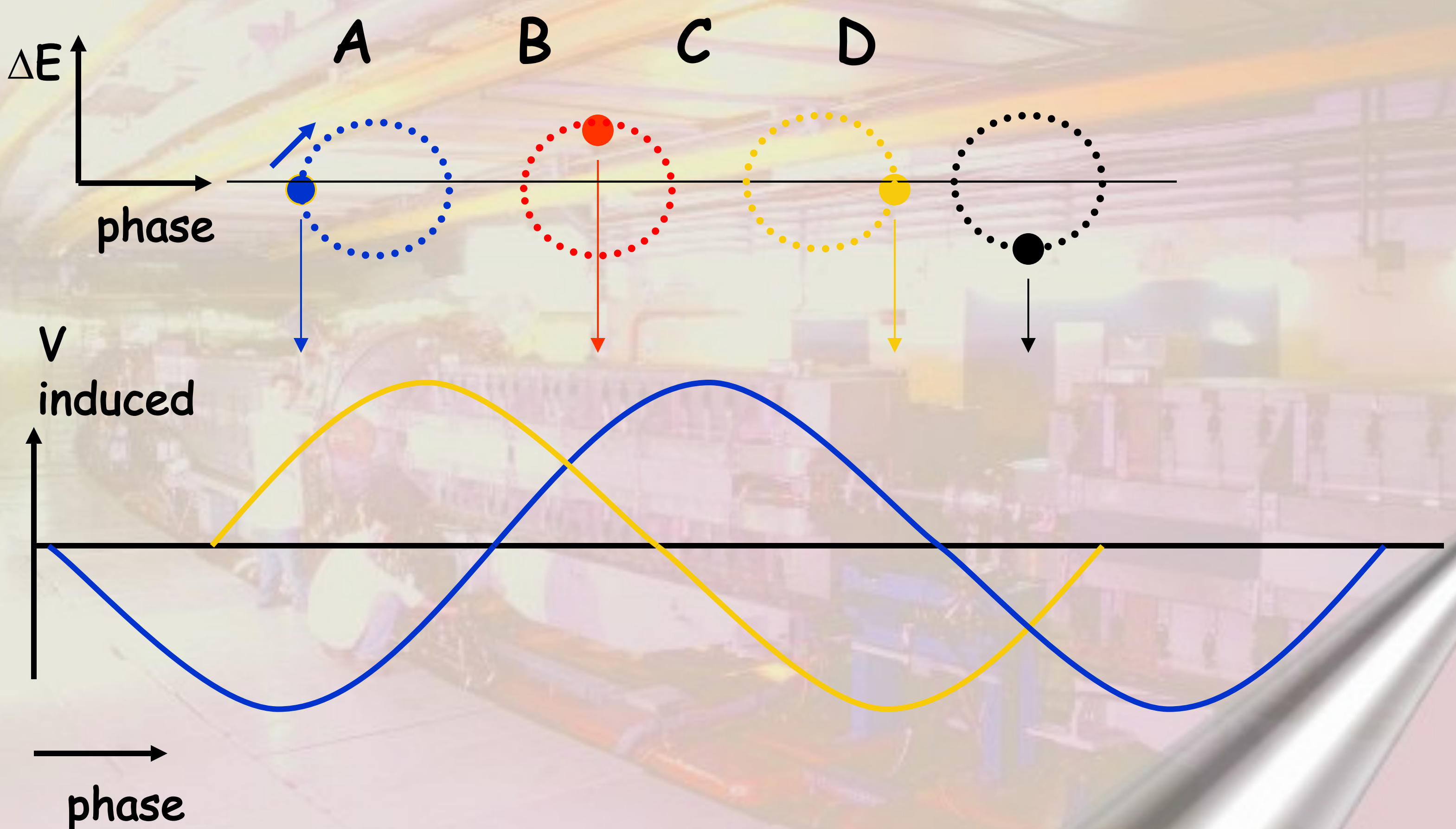
Multi-bunch instabilities (19)



Lets Introduce an **n=1** mode coupled bunch oscillation

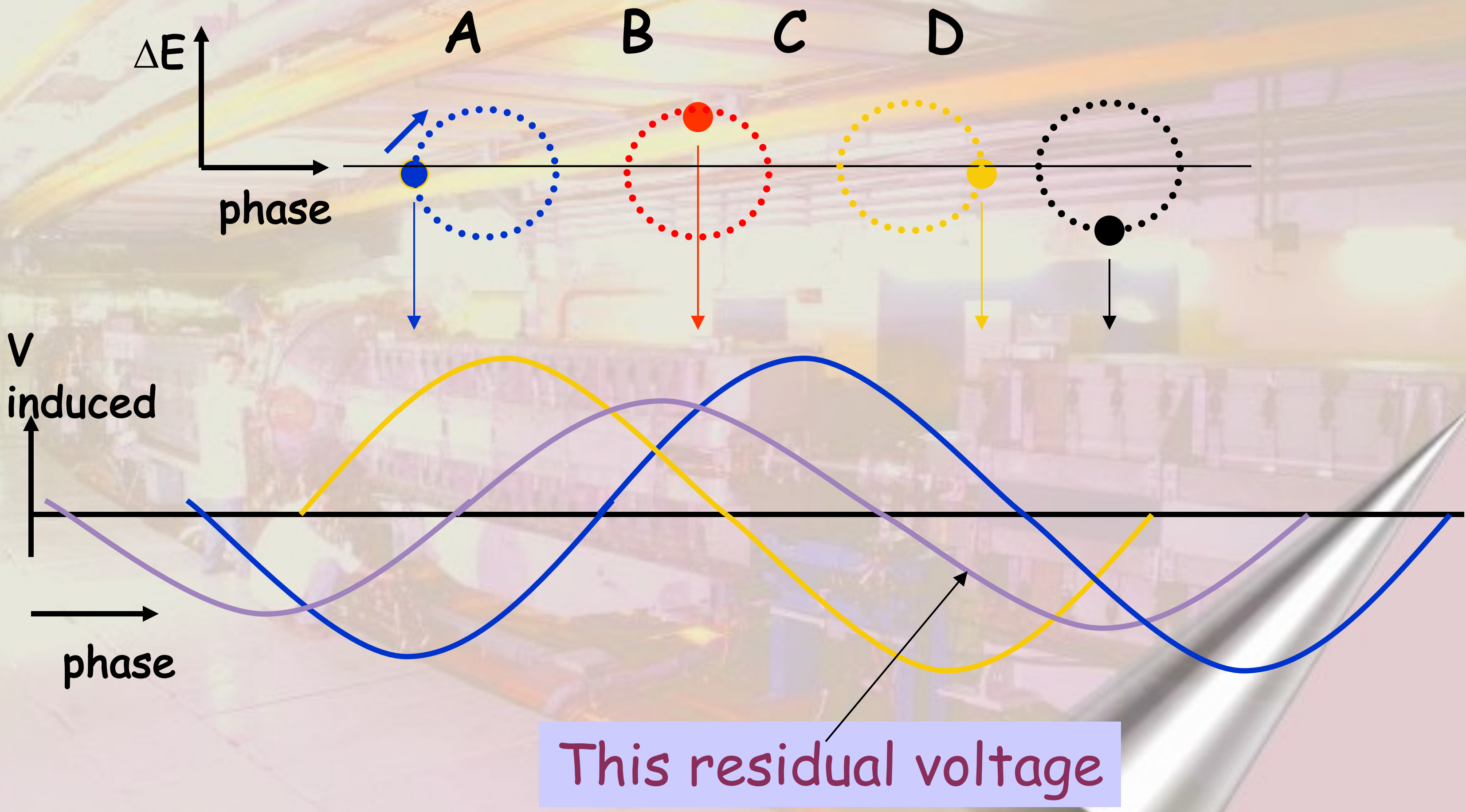
B & D induced voltages cancel

Multi-bunch instabilities (20)

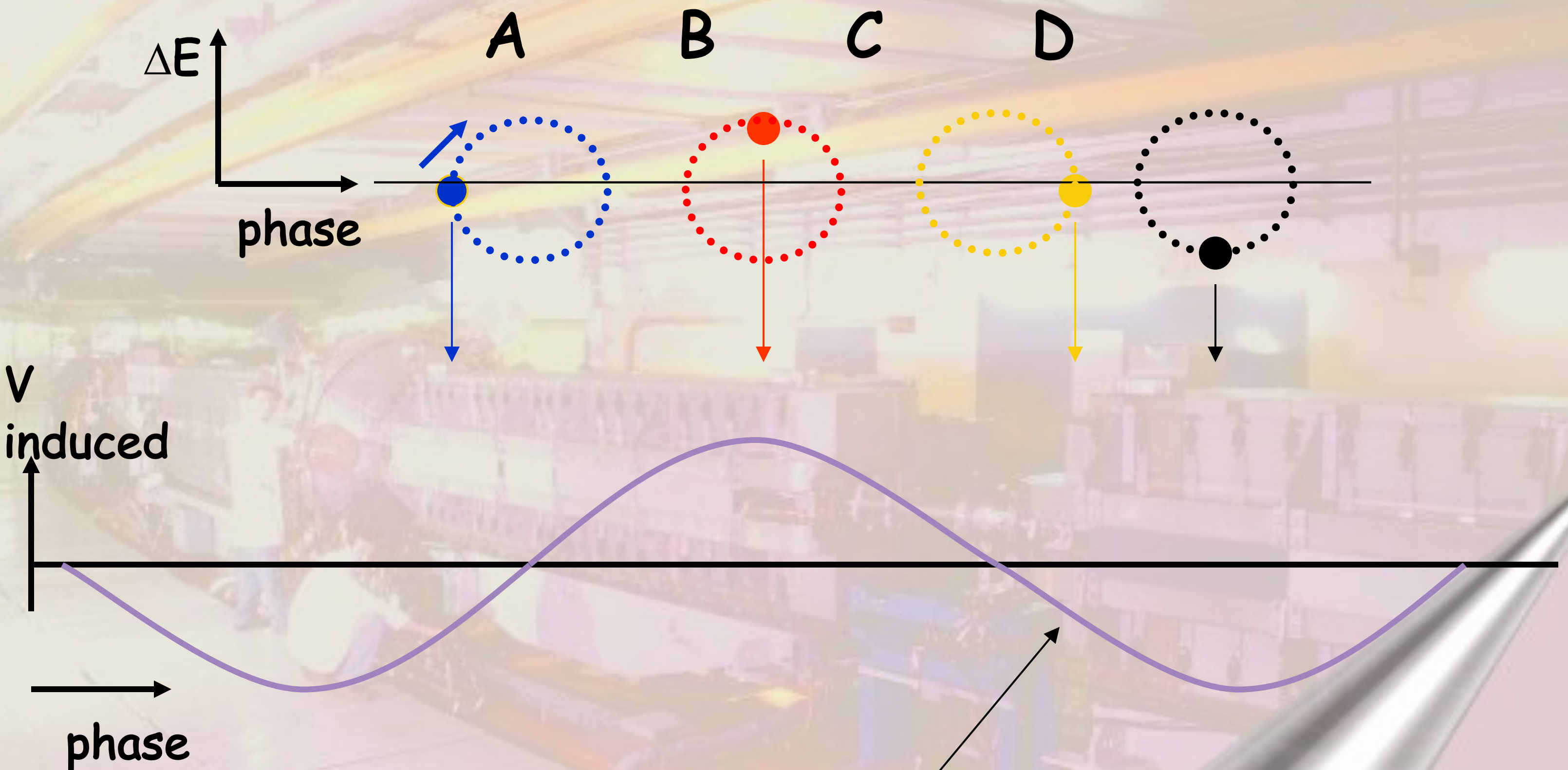


A & C induced voltages do not cancel

Multi-bunch instabilities (21)



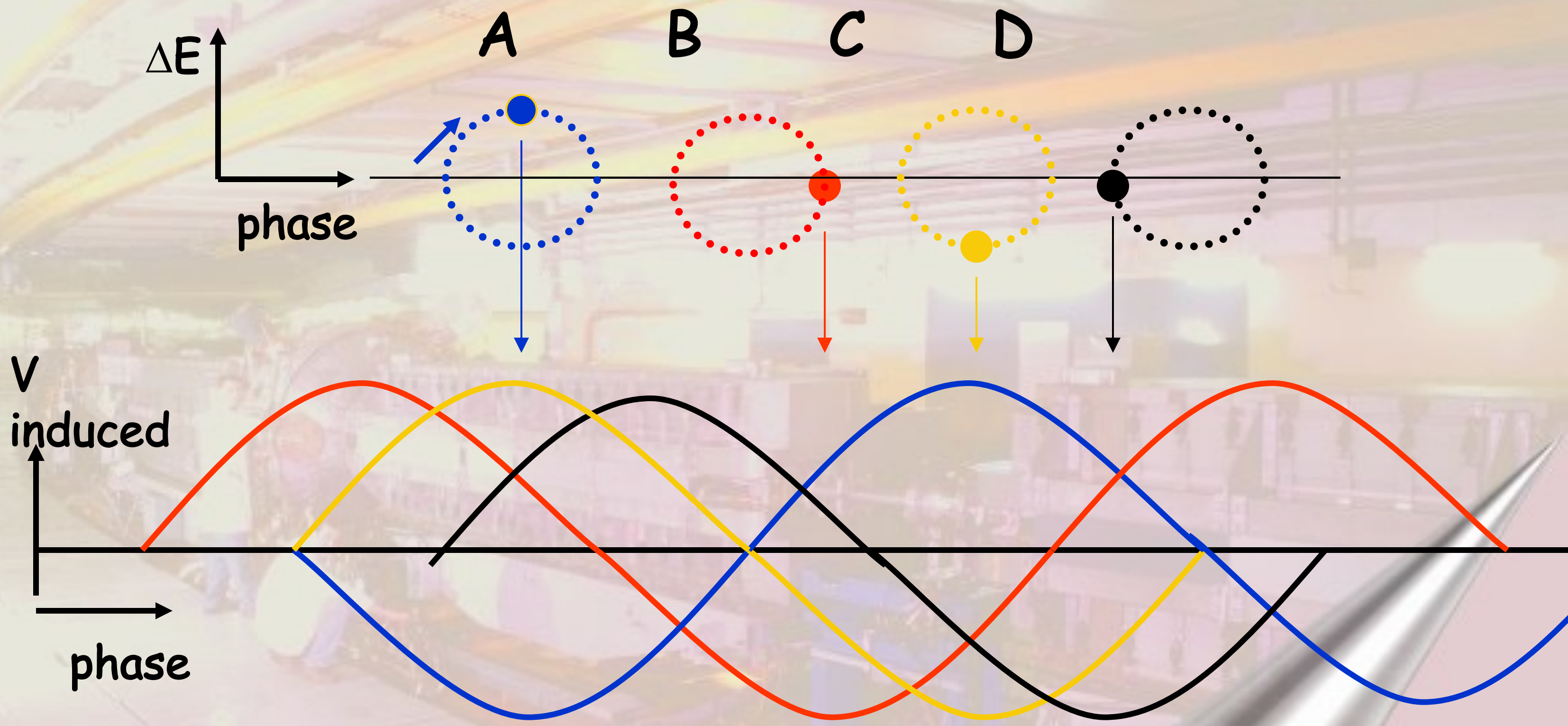
Multi-bunch instabilities (22)



This residual voltage will accelerate B and decelerate D

This increase the oscillation amplitude

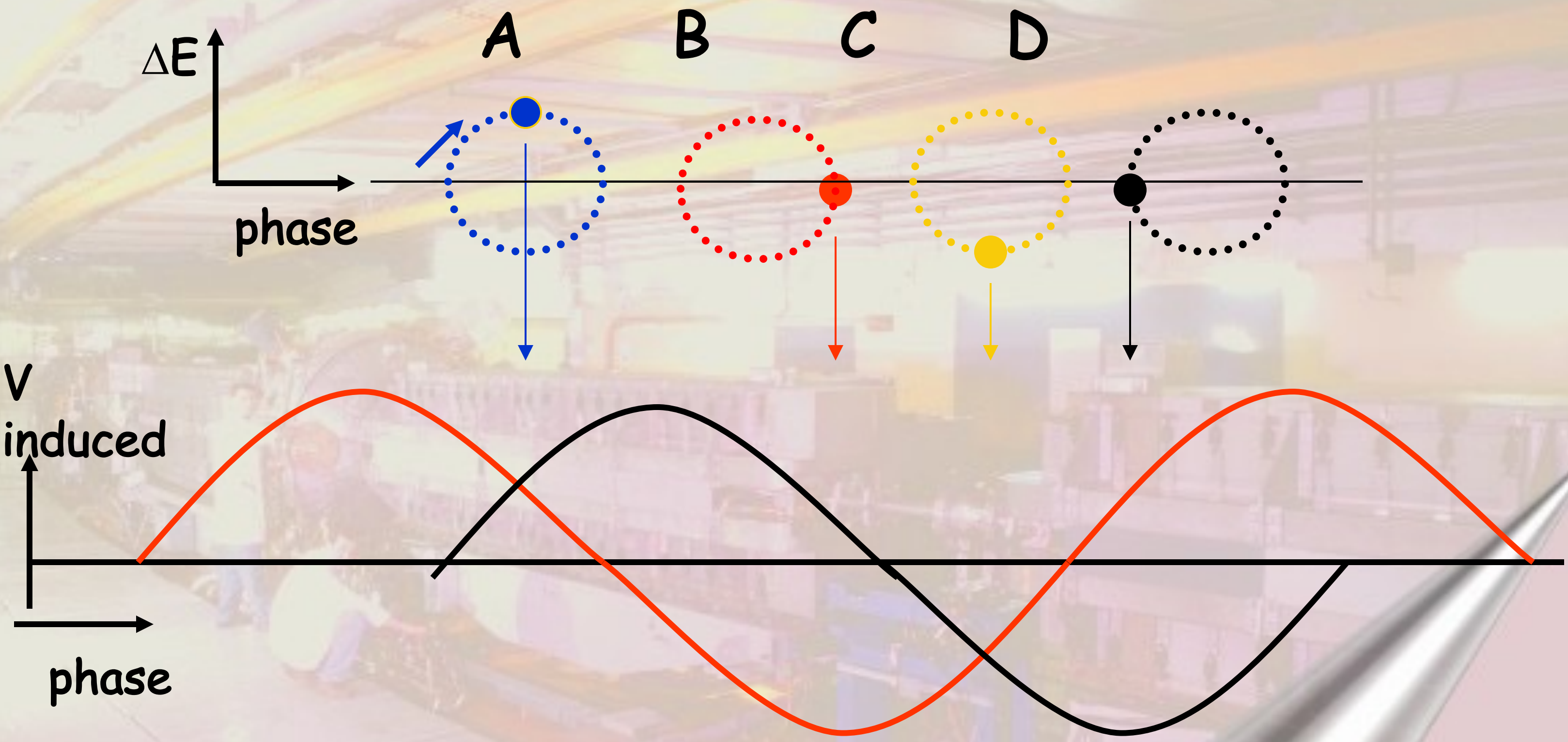
Multi-bunch instabilities (23)



1/4 of a synchrotron period later

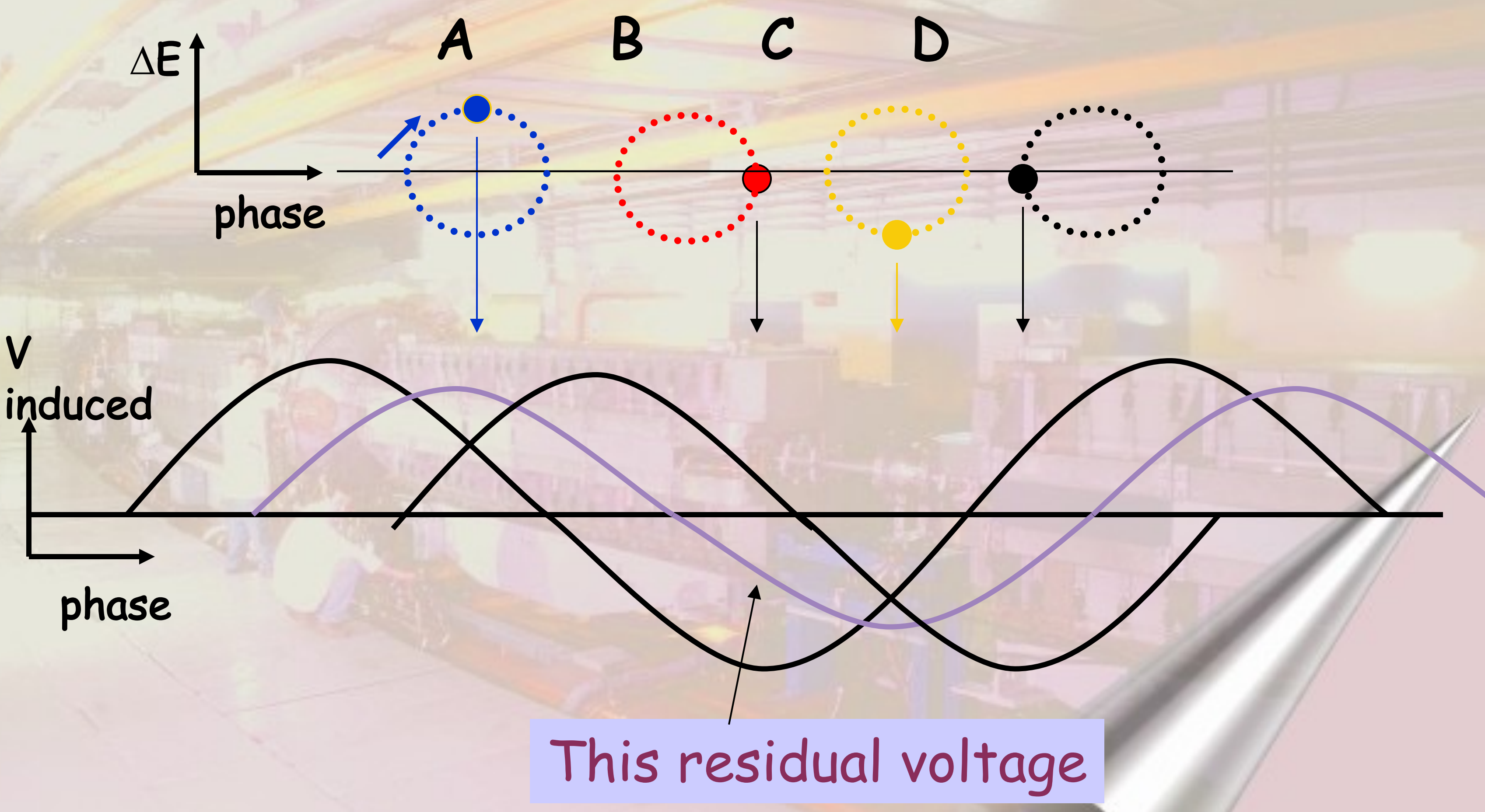
A & C induced voltages now cancel

Multi-bunch instabilities (24)

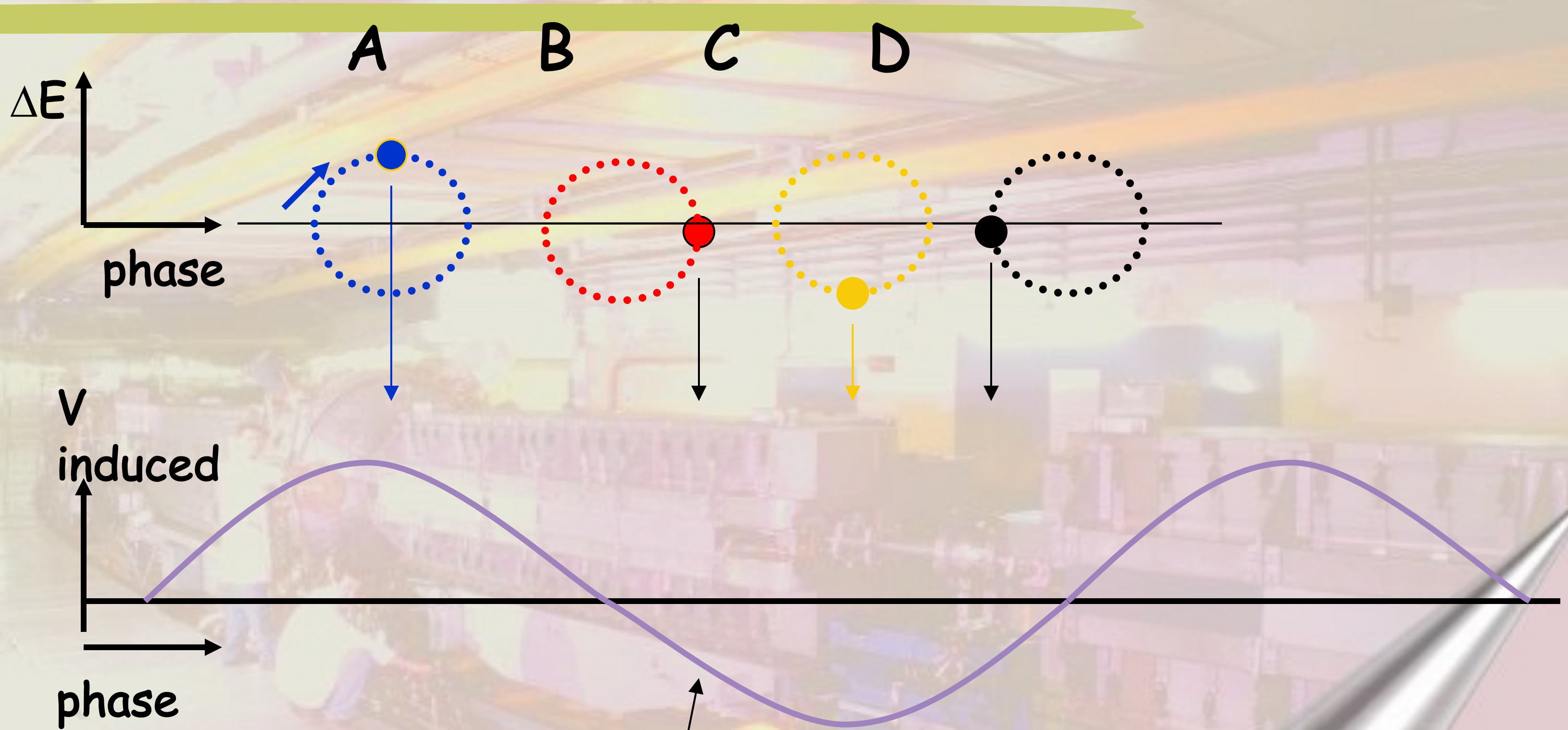


B & D induced voltages do not cancel

Multi-bunch instabilities (25)



Multi-bunch instabilities (26)



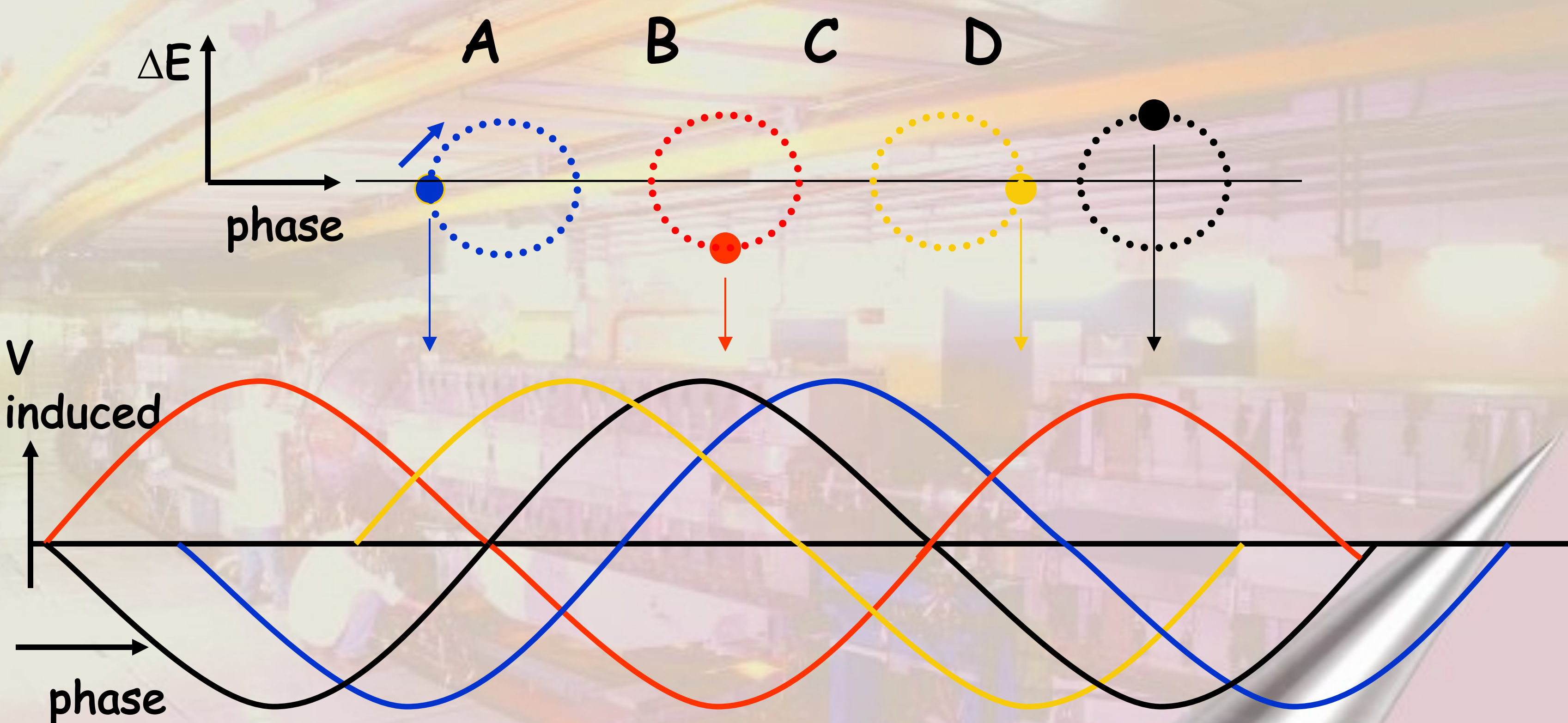
This residual voltage will accelerate A and decelerate C

Again ⇒ increase the oscillation amplitude

Multi-bunch instabilities (27)

- Hence the n=1 mode coupled bunch oscillation is unstable
- Not all modes are unstable look at n=3

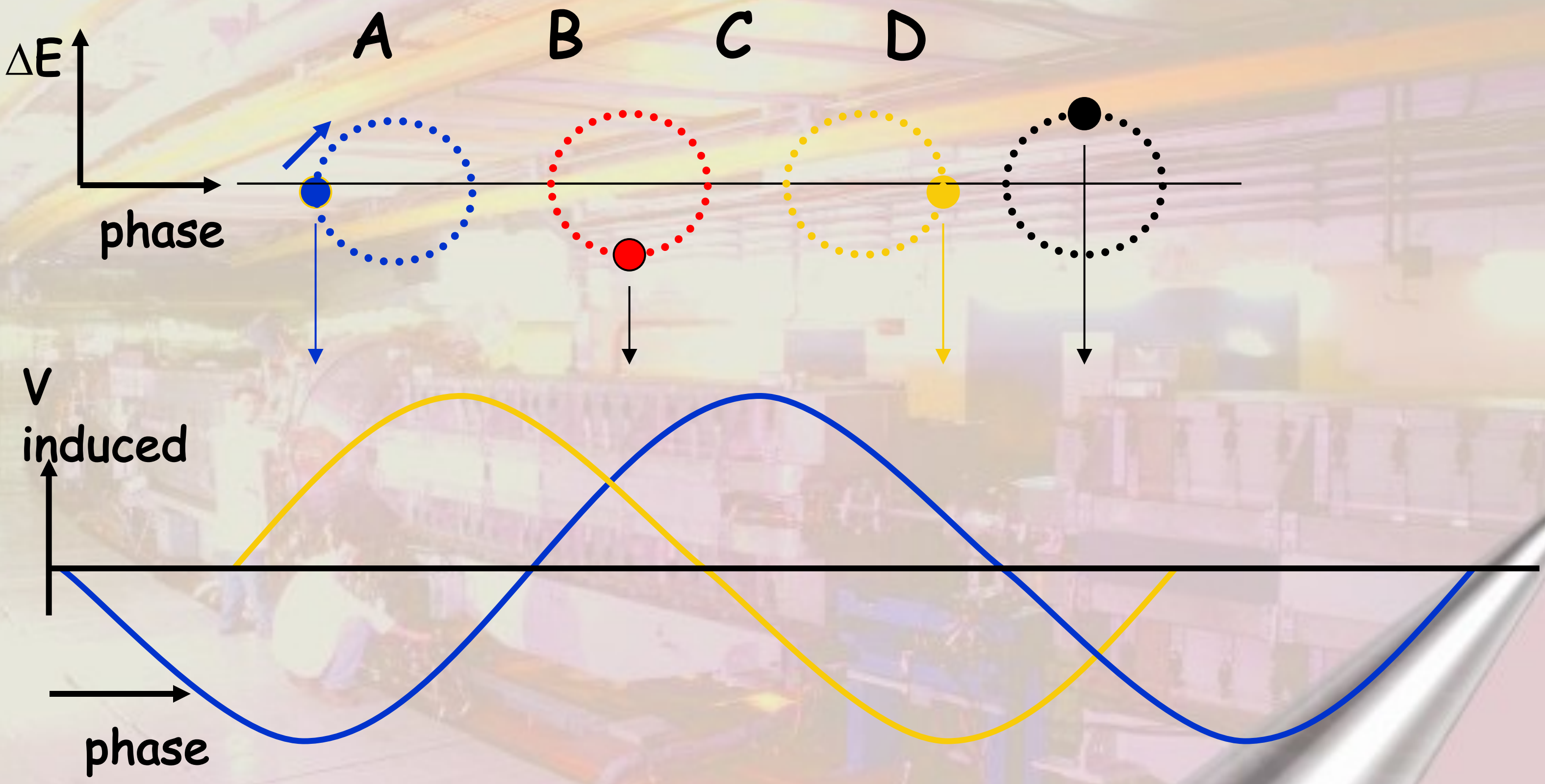
Multi-bunch instabilities (28)



Introduce an **n=3** mode coupled bunch oscillation

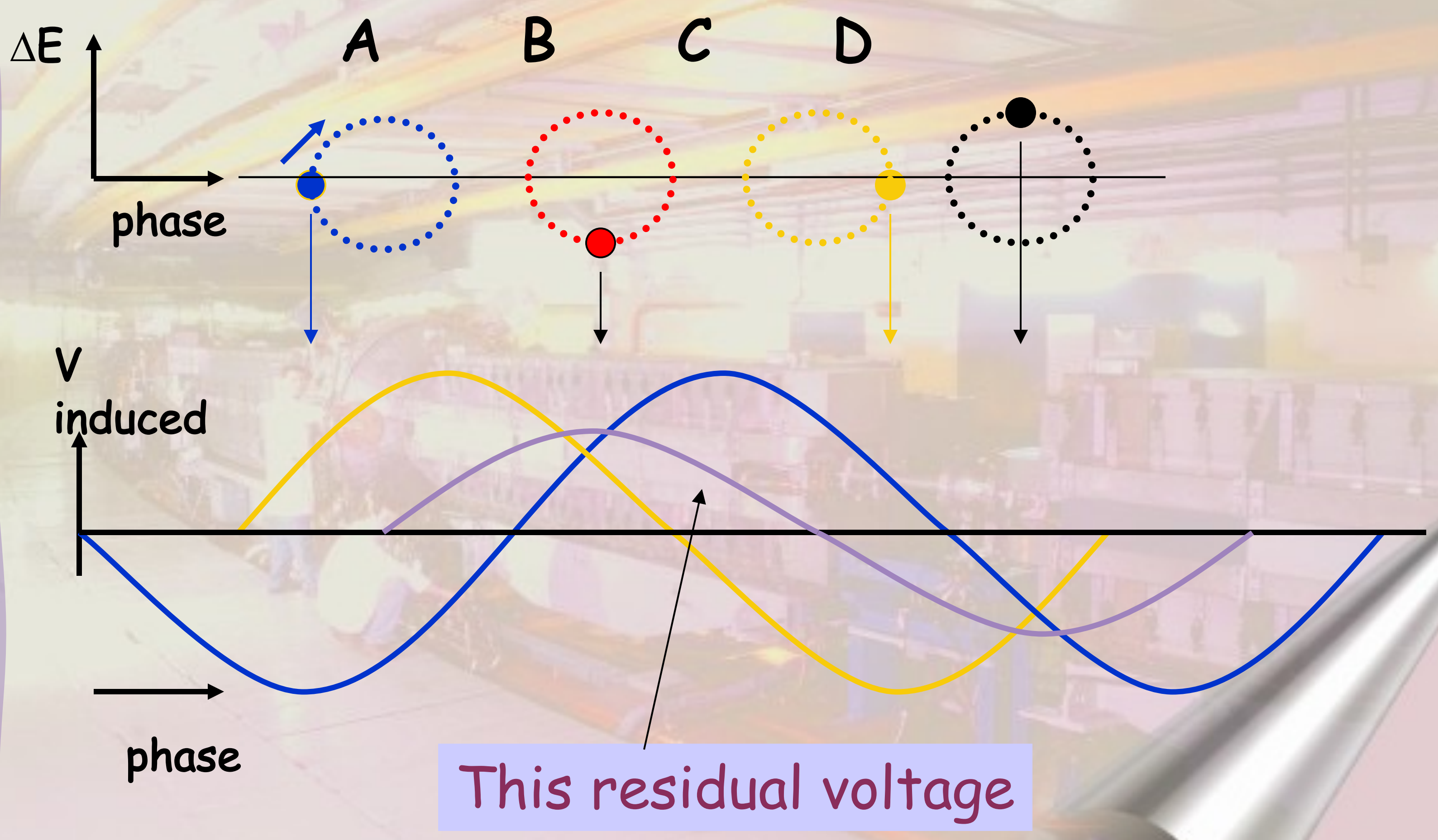
B & D induced voltages cancel

Multi-bunch instabilities (29)

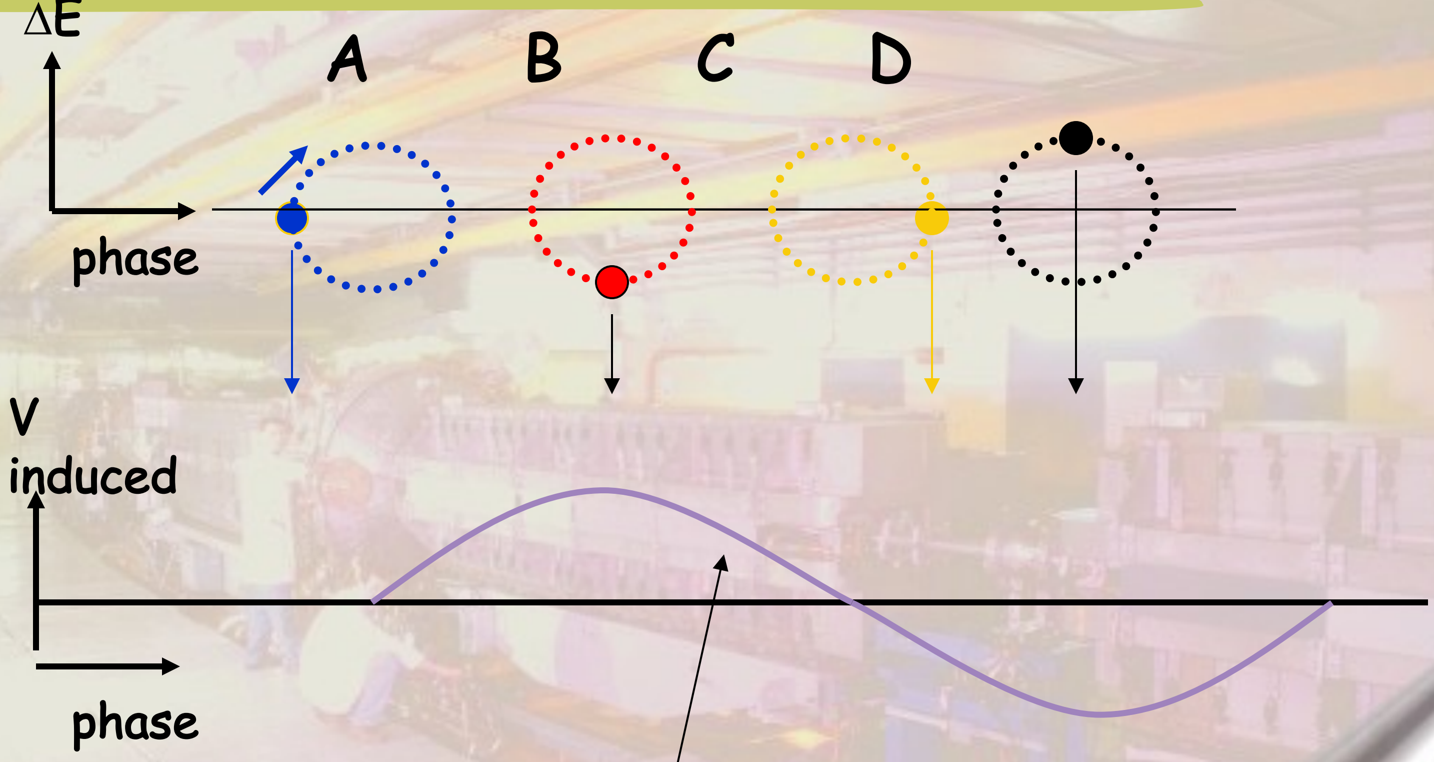


A & C induced voltages do not cancel

Multi-bunch instabilities (30)



Multi-bunch instabilities (31)



This residual voltage will accelerate B and decelerate D
⇒decrease the oscillation amplitude

Multi-bunch instabilities on a ‘scope (1)

Turn “1”

“Mountain range display”

Multi-bunch instabilities on a 'scope (2)

Add snapshot images some turns later

Multi-bunch instabilities on a 'scope (3)

Multi-bunch instabilities on a 'scope (4)

Multi-bunch instabilities on a ‘scope (5)

Multi-bunch instabilities on a 'scope (6)

Multi-bunch instabilities on a 'scope (7)

Multi-bunch instabilities on a 'scope (8)

Multi-bunch instabilities on a 'scope (9)

Multi-bunch instabilities on a 'scope (10)

Multi-bunch instabilities on a 'scope (11)

Multi-bunch instabilities on a ‘scope (12)

Multi-bunch instabilities on a ‘scope (13)

Multi-bunch instabilities on a 'scope (14)

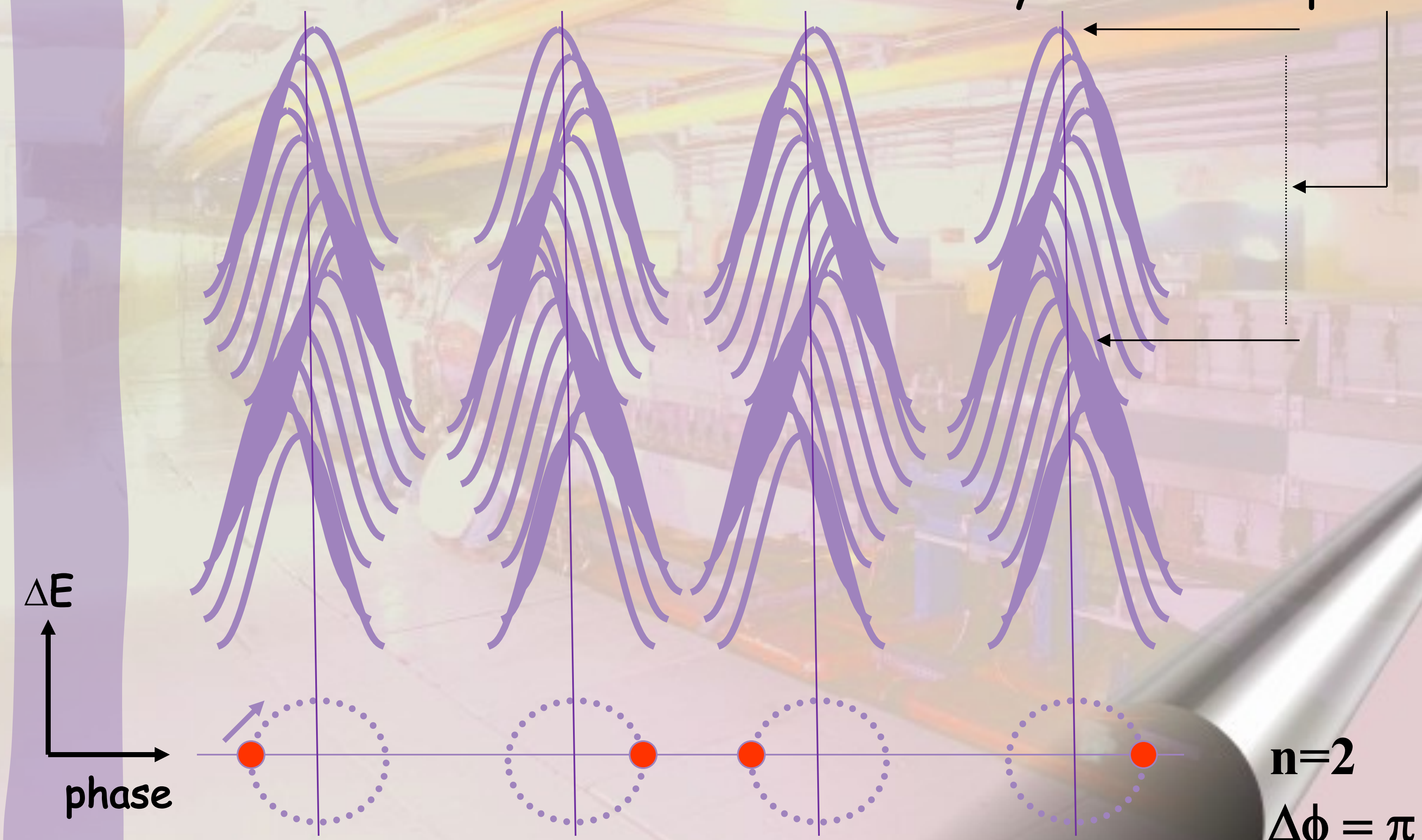
Multi-bunch instabilities on a ‘scope (15)

Multi-bunch instabilities on a ‘scope (16)

- What mode is this ?
- What is the synchrotron period?

Multi-bunch instabilities on a 'scope (17)

- This is Mode n = 2



Possible cures for single bunch modes

- Tune the RF cavities correctly in order to avoid the Robinson Instability
- Have a phase lock system, this is a feedback on phase difference between RF and bunch
- Have correct Longitudinal matching
- Radiation damping (Leptons)
- Damp higher order resonant modes in cavities
- Reduce machine impedance as much as possible

Possible cures for multi-bunch modes

- Reduce machine impedance as far as possible
- Feedback systems - correct bunch phase errors with high frequency RF system
- Radiation damping
- Damp higher order resonant modes in cavities

Bunch lengthening (1)

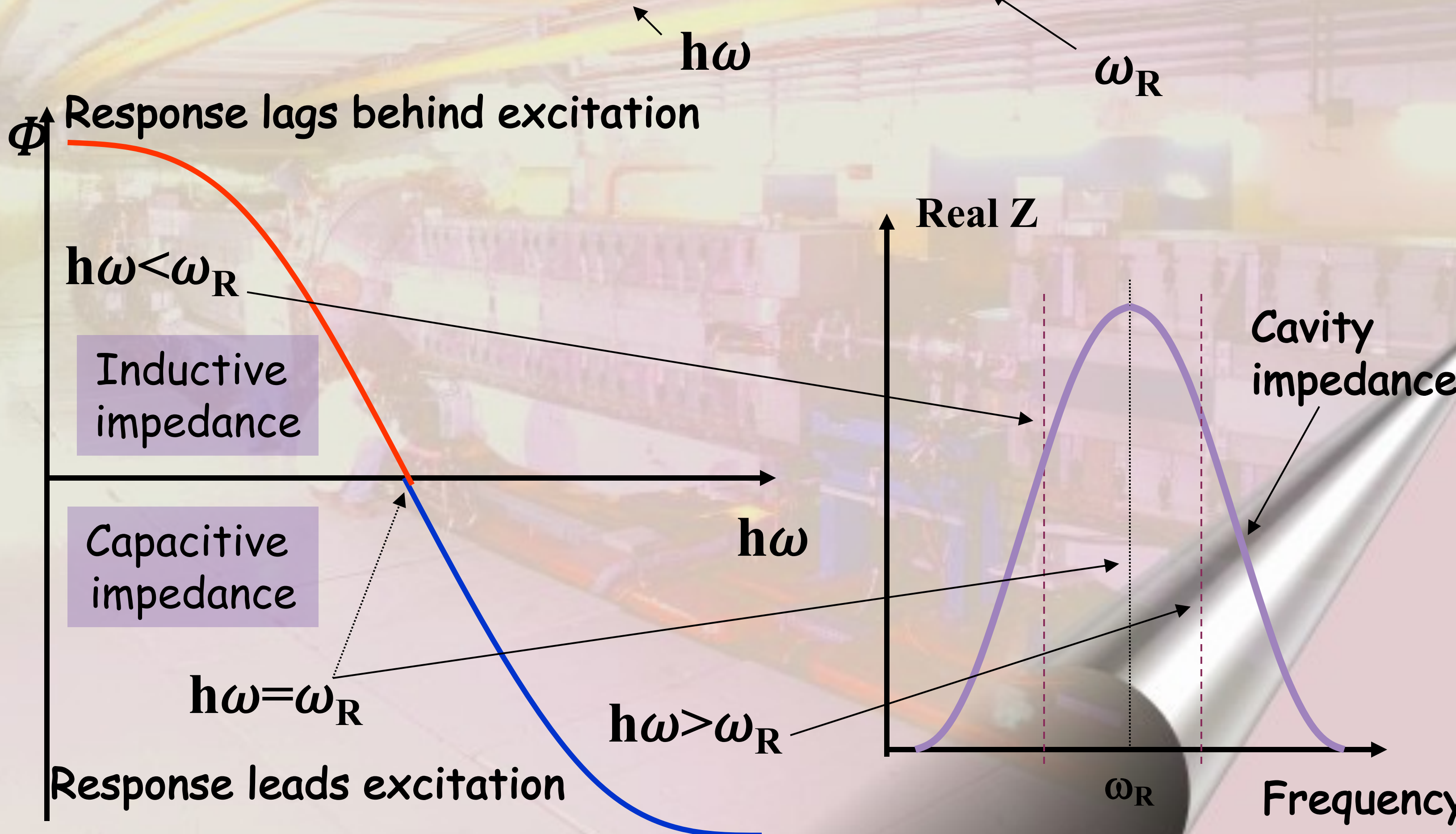
- Now we controlled all longitudinal instabilities, but .....
- It seems that we are unable to increase peak bunch current above a certain level
- The bunch gets longer as we add more particles.

- Why..?
- What happens….?

- Lets look at the behaviour of a cavity resonator as we change the driving frequency.

Bunch lengthening (2)

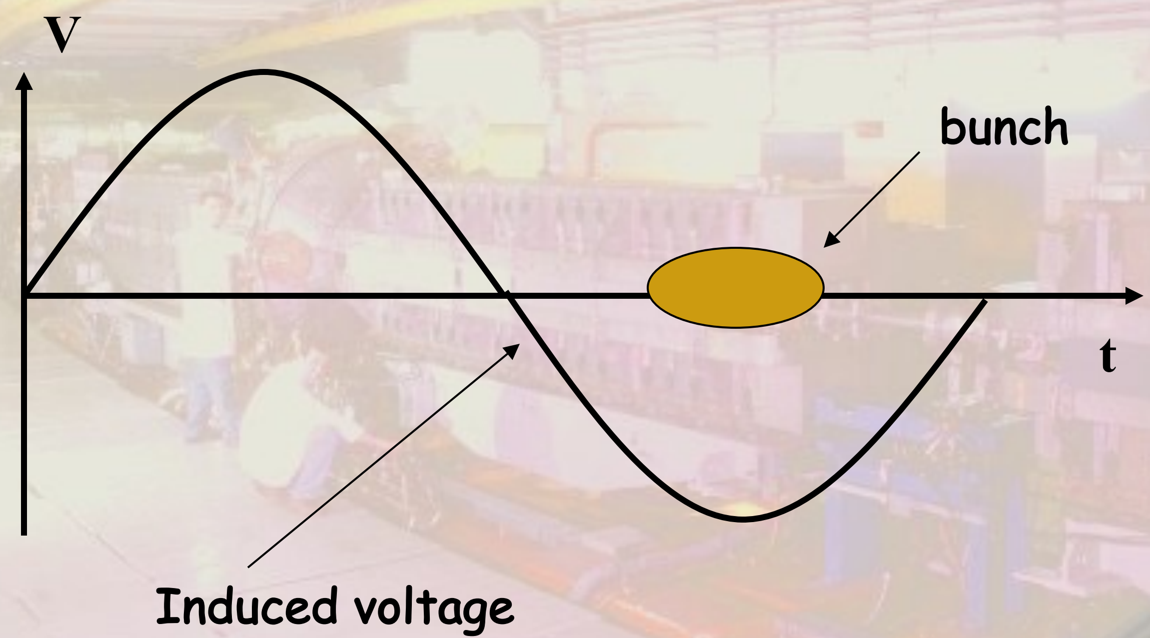
The **phase** of the response of a resonator depends on the difference between the **driving** and the **resonant** frequencies



Bunch lengthening (3)

Cavity driven on resonance

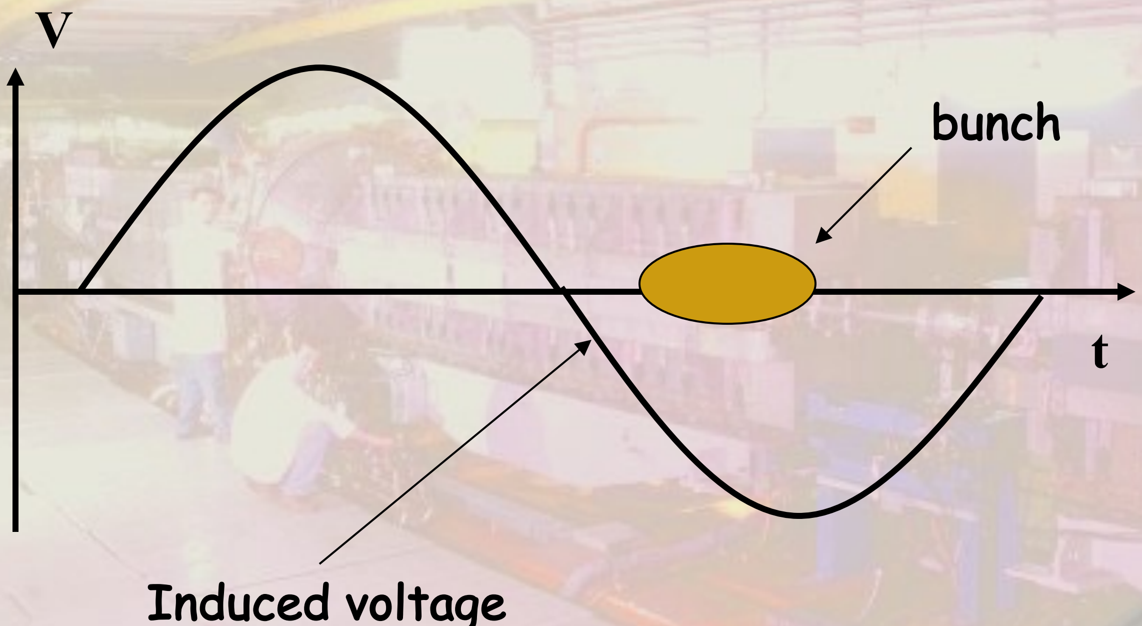
$h\omega = \omega_R \Rightarrow$ resistive impedance



Bunch lengthening (4)

Cavity driven above resonance

$h\omega > \omega_R \Rightarrow$ **capacitive impedance**

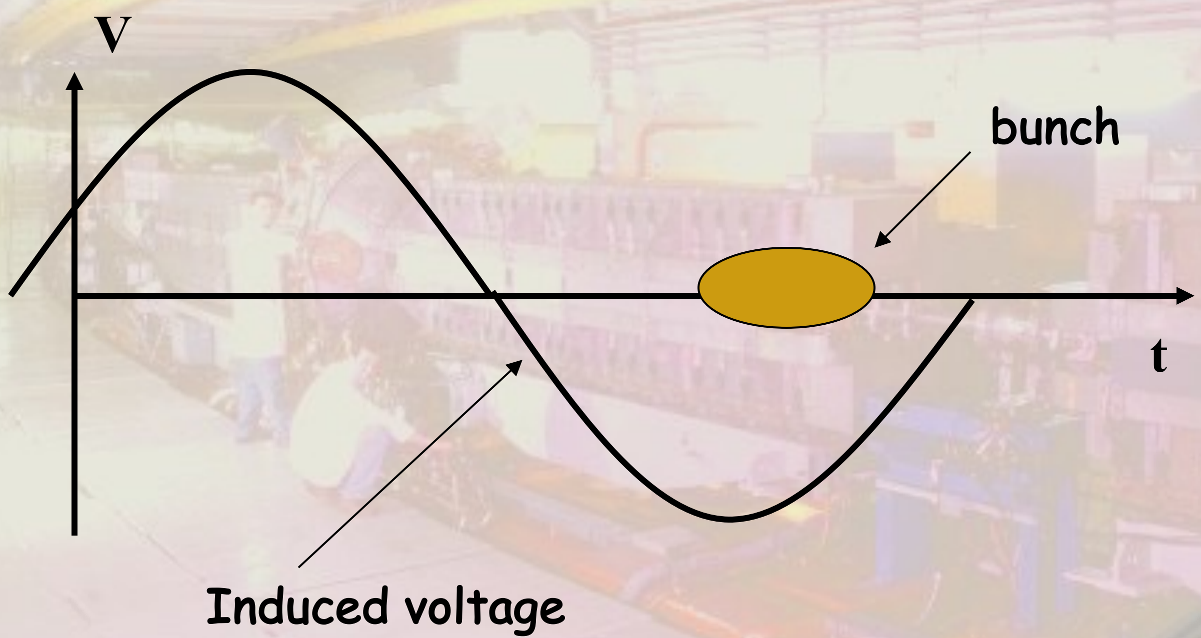


Response leads excitation

Bunch lengthening (5)

Cavity driven below resonance

$h\omega < \omega_R \Rightarrow$ **inductive impedance**



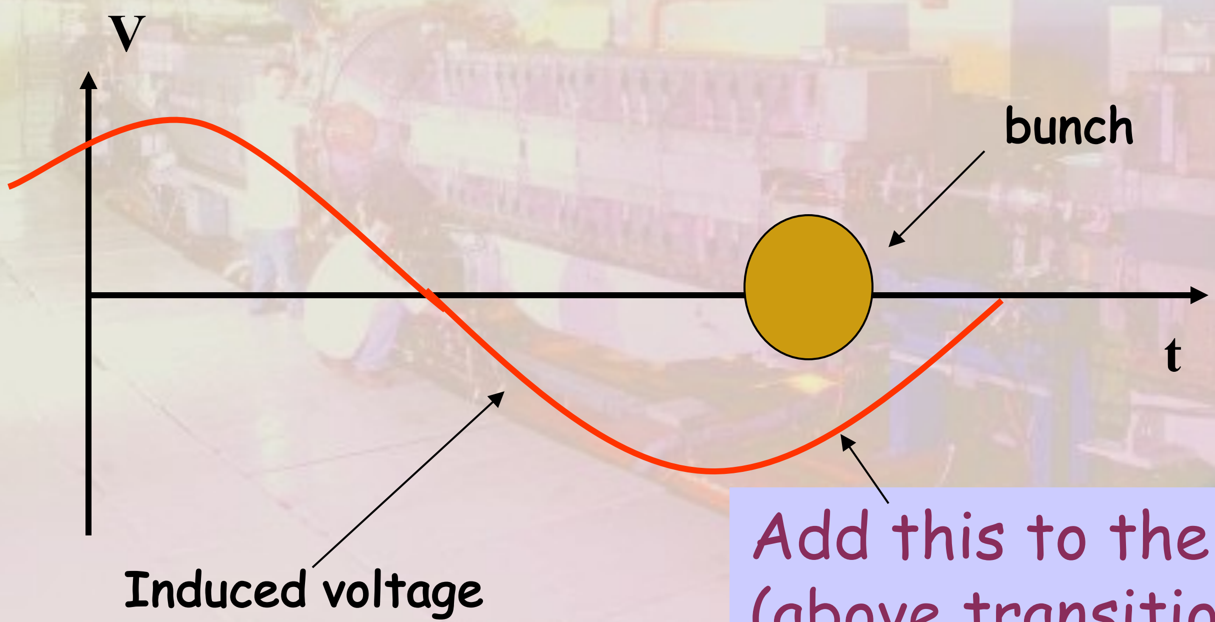
Response lags behind excitation

Bunch lengthening (6)

- In general the Broad Band impedance of the machine, vacuum pipe etc (other than the cavities) is <u>inductive</u>
- The bellows etc. represent very high frequency resonators, which resonate mostly at frequencies above the bunch spectrum

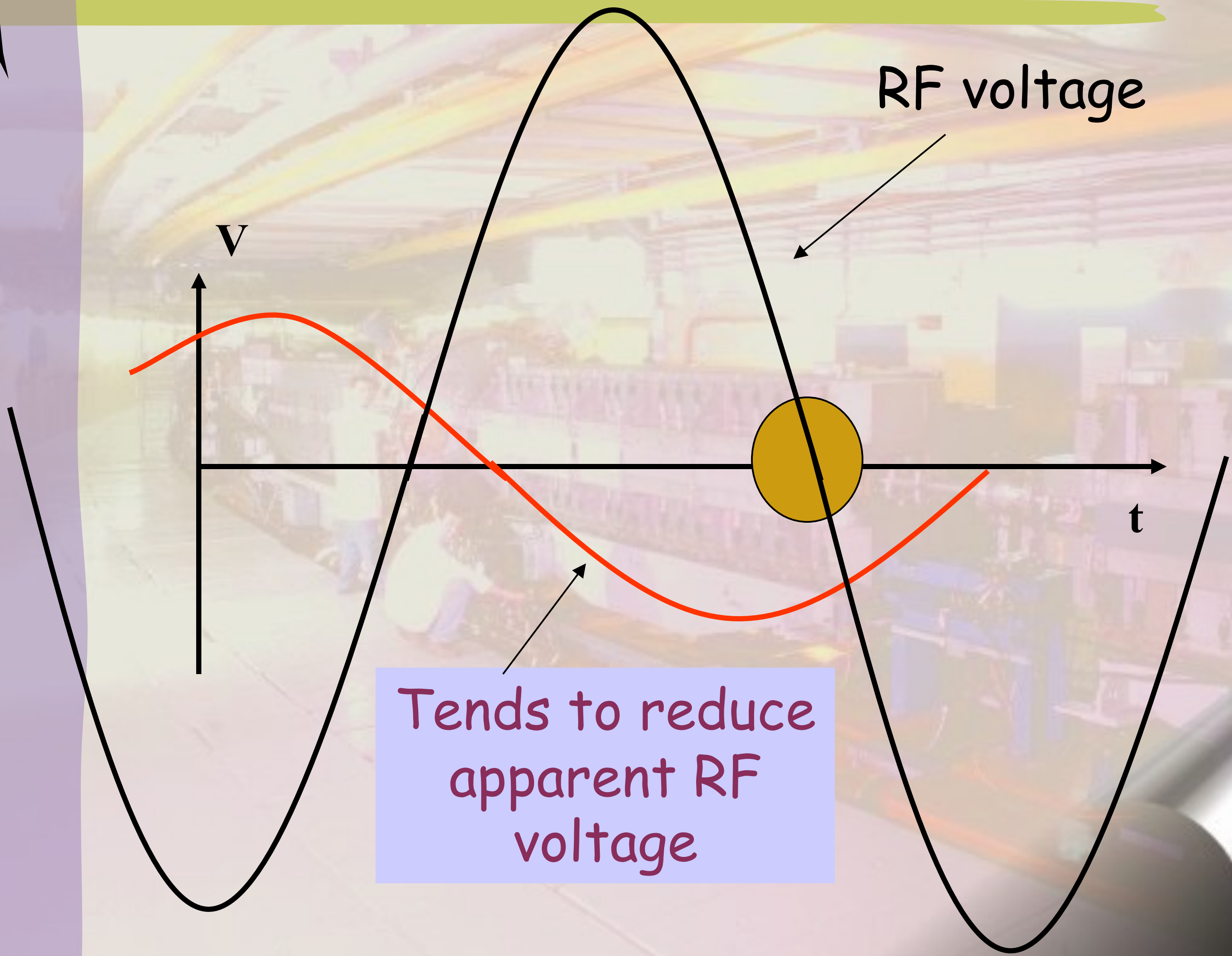
Bunch lengthening (7)

- Since the Broad Band impedance of the machine is predominantly inductive , the **response lags behind excitation**

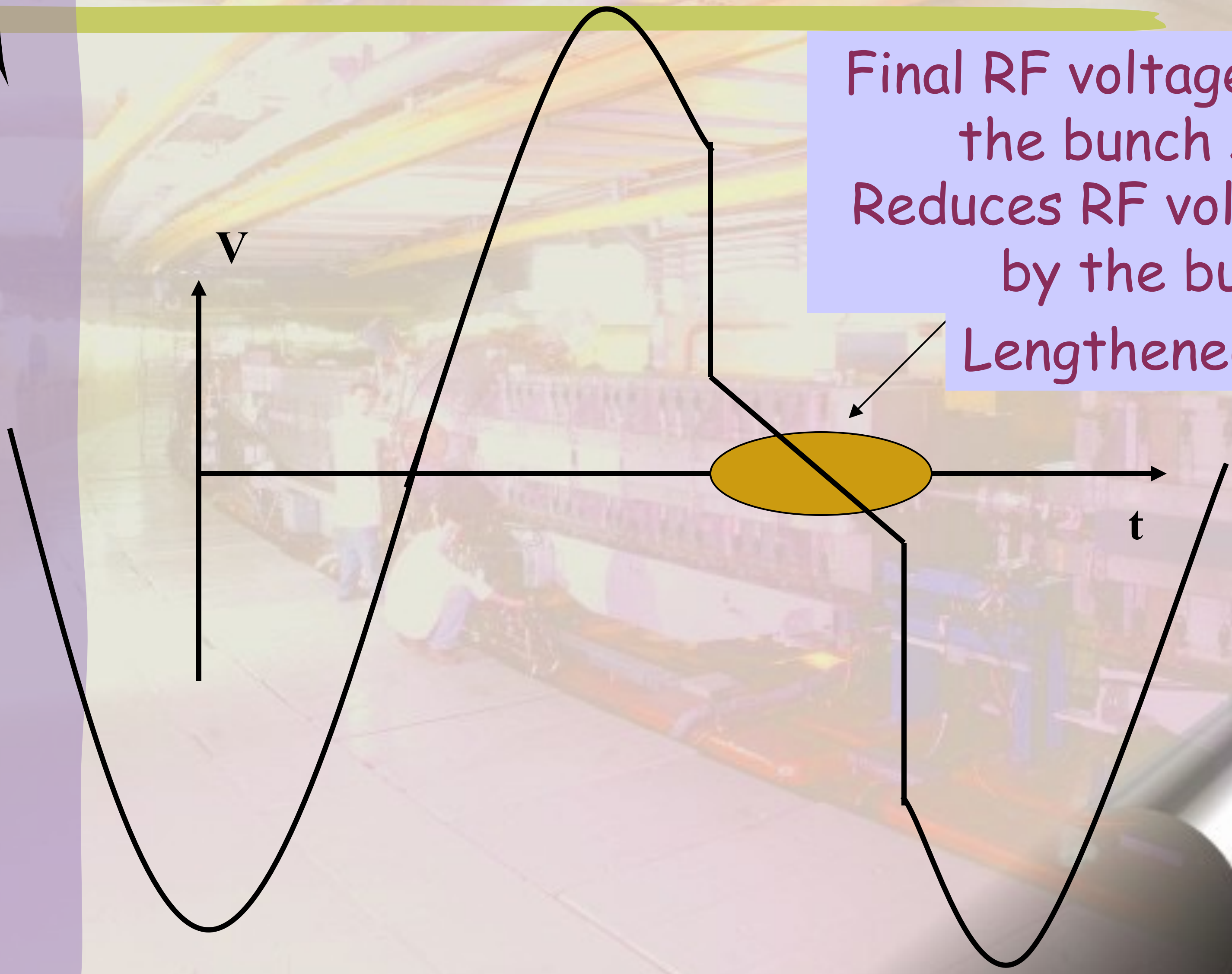


Add this to the RF voltage
(above transition)

Bunch lengthening (8)



Bunch lengthening (10)



Final RF voltage modifies the bunch shape
Reduces RF voltage seen by the bunch

Questions….,Remarks…?