AXEL-2023

Introduction to Particle Accelerators

Longitudinal motion:

- *The basic synchrotron equations.*
- *What is Transition ?*
- *RF systems.*
- *Motion of low & high energy particles.*
- *Acceleration.*
- *What are Adiabatic changes?*

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1 March 2023

Motion in longitudinal plane

- What happens when particle momentum increases?
  - ⇒ particles follow longer orbit (fixed B field)
  - ⇒ particles travel faster (initially)
- How does the revolution frequency change with the momentum ?

$$\frac{df}{f} = \frac{dv}{v} - \frac{dr}{r}$$

But

$$\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$$

Change in velocity

Change in orbit length

Momentum compaction factor

Therefore:

$$\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$$

The frequency - momentum relation

$$\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$$

But

$$\frac{dv}{v} = \frac{d\beta}{\beta} \quad \left(\beta = \frac{v}{c}\right)$$

- The relativity theory says: $p = \dfrac{E_0 \beta\gamma}{c}$

$$\frac{dp}{d\beta} = \frac{E_0 \gamma^3}{c}$$

$$\frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta}$$

$$\frac{dv}{v} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$$

$$\frac{df}{f} = \left(\frac{1}{\gamma^2} - \alpha_p\right)\frac{dp}{p}$$

varies with momentum ($\mathbf{E} = \mathbf{E}_0\gamma$)

fixed by the lattice

Transition

- **Lets look at the behaviour of a particle in a constant magnetic field.**
- Low momentum $(\beta << 1, \ \gamma \Rightarrow 1)$ $\longrightarrow$ $\frac{1}{\gamma^2} > \alpha_p$
- The revolution frequency increases as momentum increases
- High momentum $(\beta \approx 1, \gamma >> 1)$ $\longrightarrow$ $\frac{1}{\gamma^2} < \alpha_p$
- The revolution frequency decreases as momentum increases
- For one particular momentum or energy we have:

$$\frac{1}{\gamma^2} = \alpha_p$$

- This particular energy is called the Transition energy

The frequency slip factor

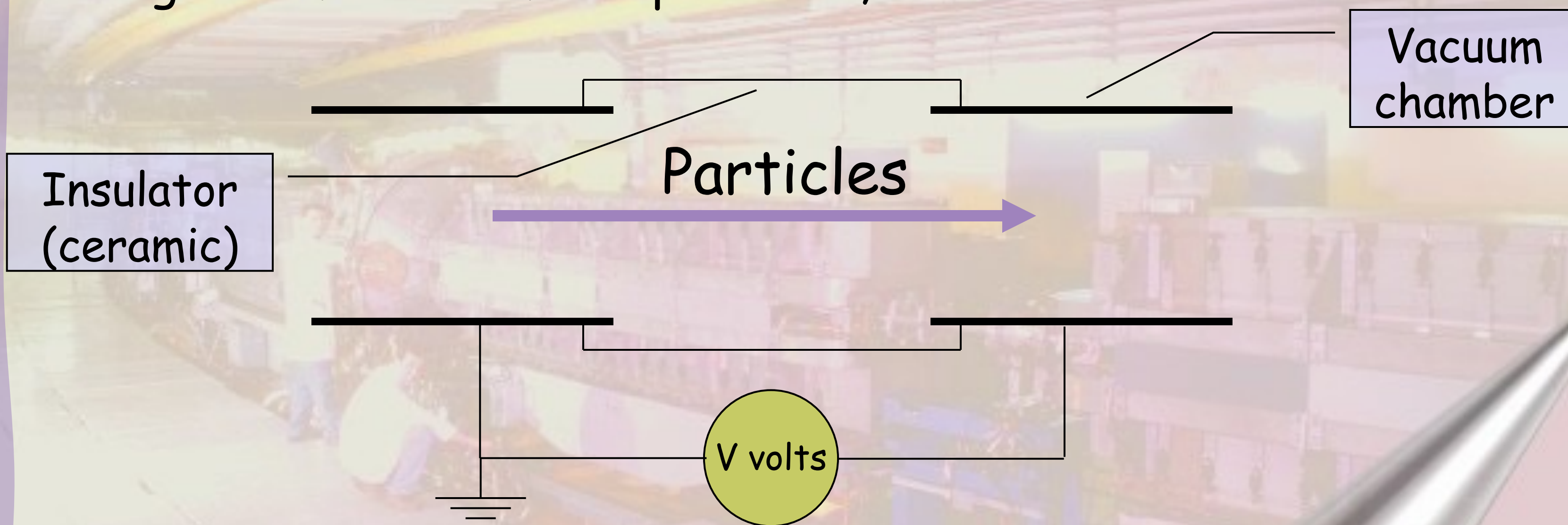
- We found $$\frac{df}{f}=\left(\frac{1}{\gamma^2}-\alpha_p\right)\frac{dp}{p}=\left(\frac{1}{\gamma^2}-\frac{1}{\gamma_{tr}^2}\right)\frac{dp}{p}$$ (η)
- $\frac{1}{\gamma^2}>\alpha_p$ ⟶ **Below transition** ⟶ η = **positive**
- $\frac{1}{\gamma^2}=\alpha_p$ ⟶ **Transition** ⟶ η = **zero**
- $\frac{1}{\gamma^2}<\alpha_p$ ⟶ **Above transition** ⟶ η = **negative**
- Transition is very important in proton machines.
  - A little later we will see why….
- In the PS machine : γtr is at ~6 GeV/c
- In the LHC machine : γtr is at ~55 GeV/c
- Transition does not exist in leptons machines, why?

Radio Frequency System

- Hadron machines:
  - Accelerate / Decelerate beams
  - Beam shaping
  - Beam measurements
  - Increase luminosity in hadron colliders
- Lepton machines:
  - Accelerate beams
  - Compensate for energy loss due to synchrotron radiation.

RF Cavity

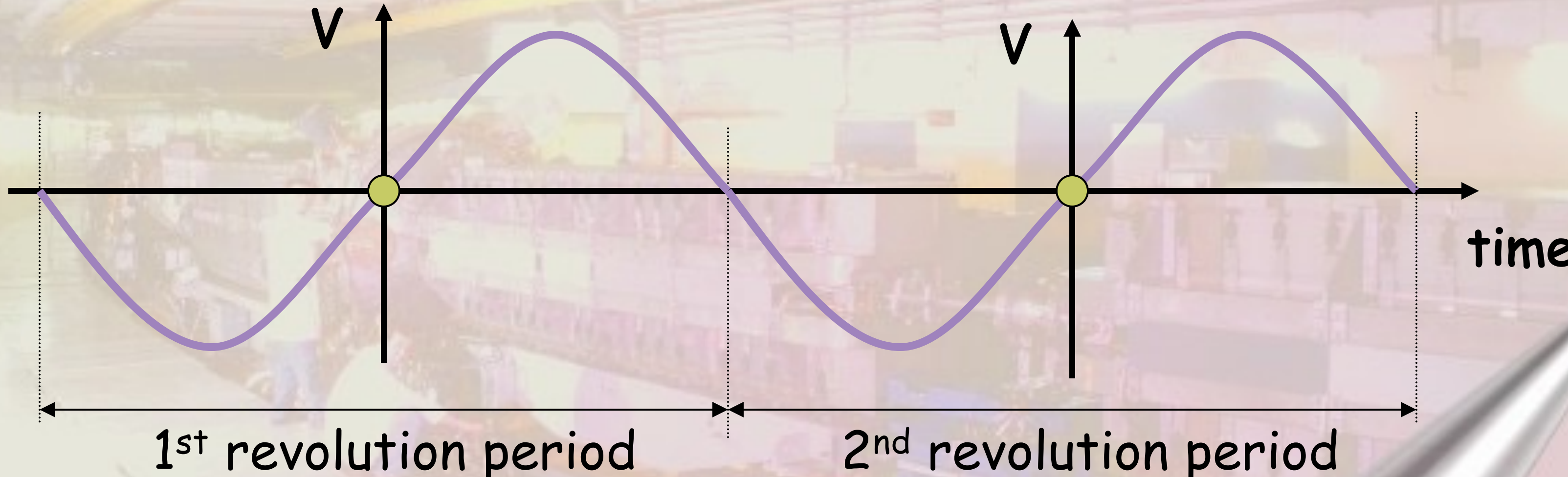
- To accelerate charged particles we need a longitudinal electric field.
- Magnetic fields deflect particles, but do not accelerate them.



- If the voltage is DC then there is no acceleration !
  - The particle will accelerate towards the gap but decelerate after the gap.
- Use an <u>Oscillating Voltage</u> with the right Frequency

A Single particle in a longitudinal electric field

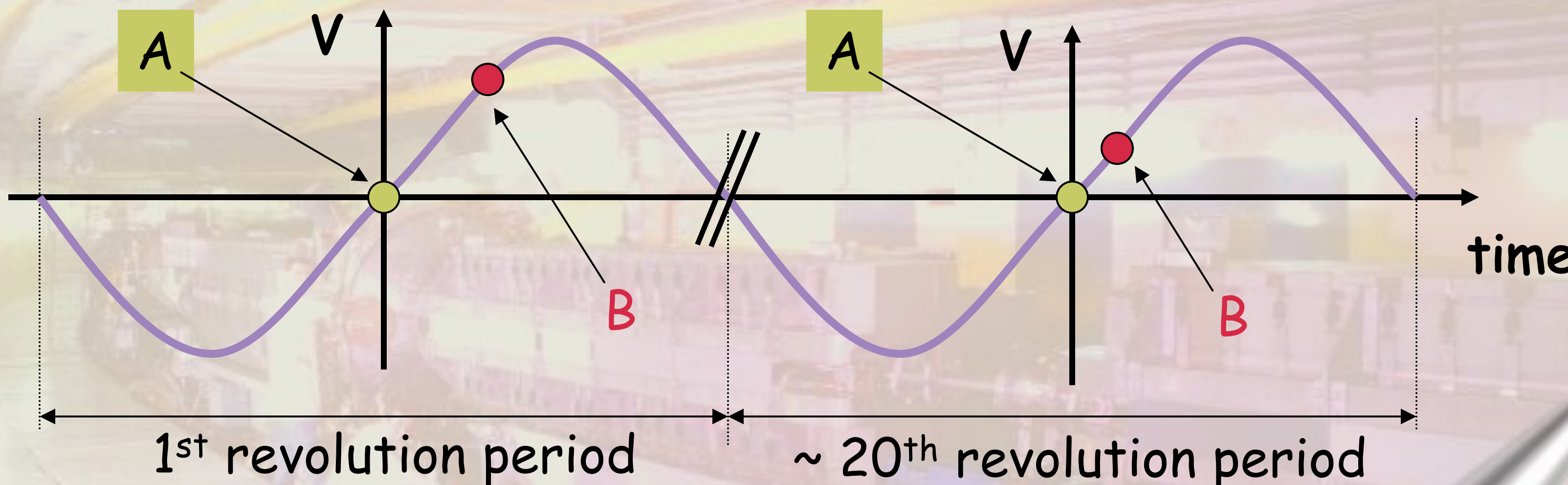
- Lets see what a low energy particle does with this oscillating voltage in the cavity.



- Set the oscillation frequency so that the period is exactly equal to one revolution period of the particle.

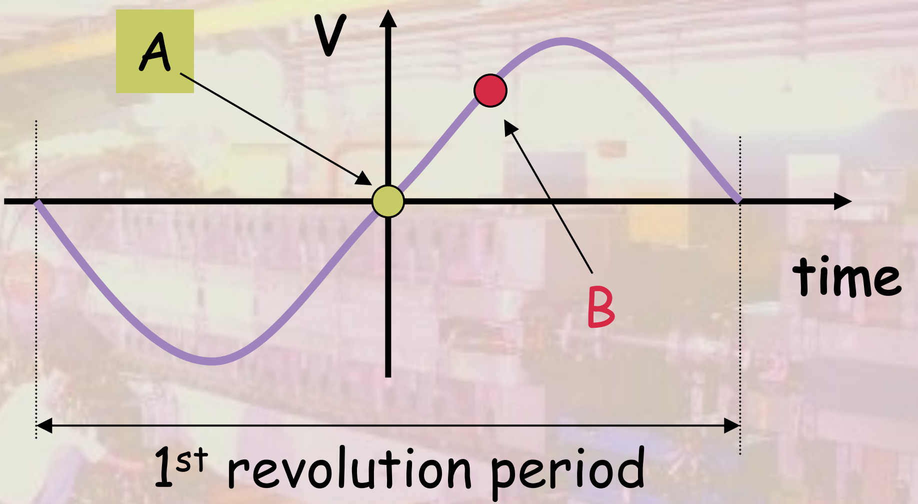
Add a second particle to the first one

- Lets see what a second low energy particle, which arrives later in the cavity, does with respect to our first particle.

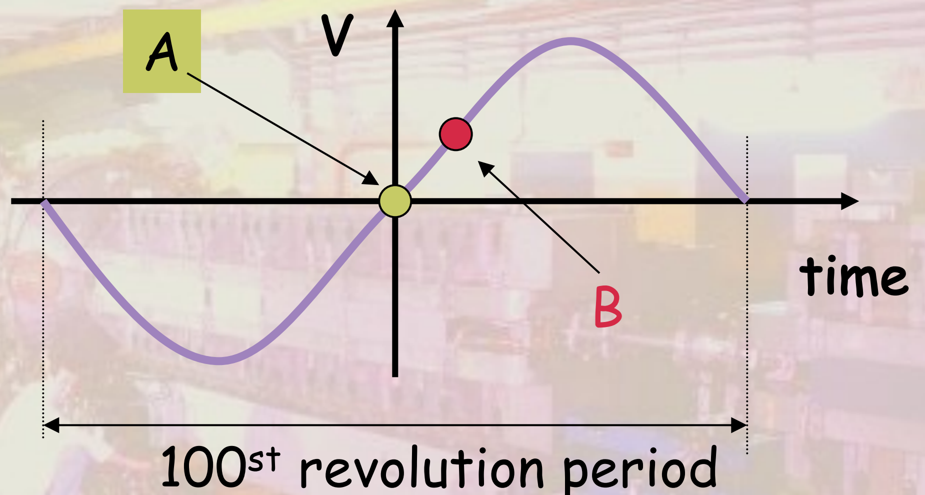


- B arrives late in the cavity w.r.t. A
- B sees a higher voltage than A and will therefore be accelerated
- After many turns B approaches A
- B is still late in the cavity w.r.t. A
- B still sees a higher voltage and is still being accelerated

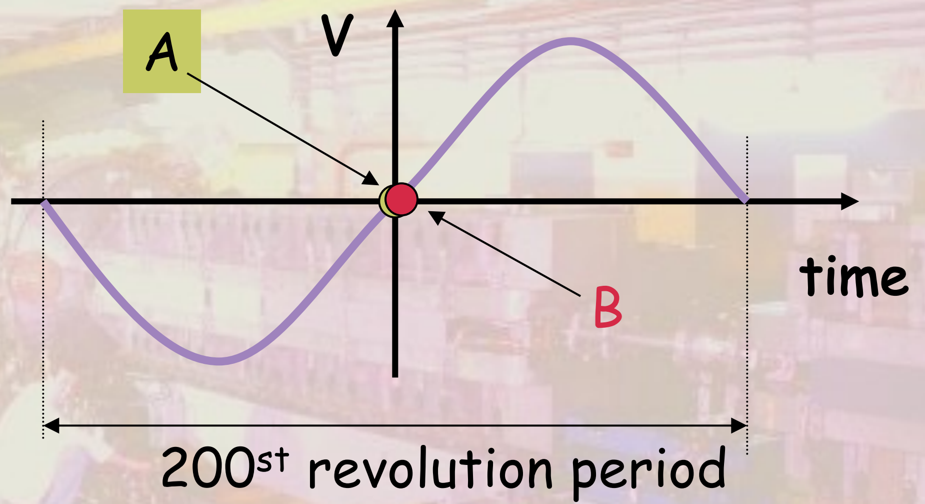
Lets see what happens after many turns



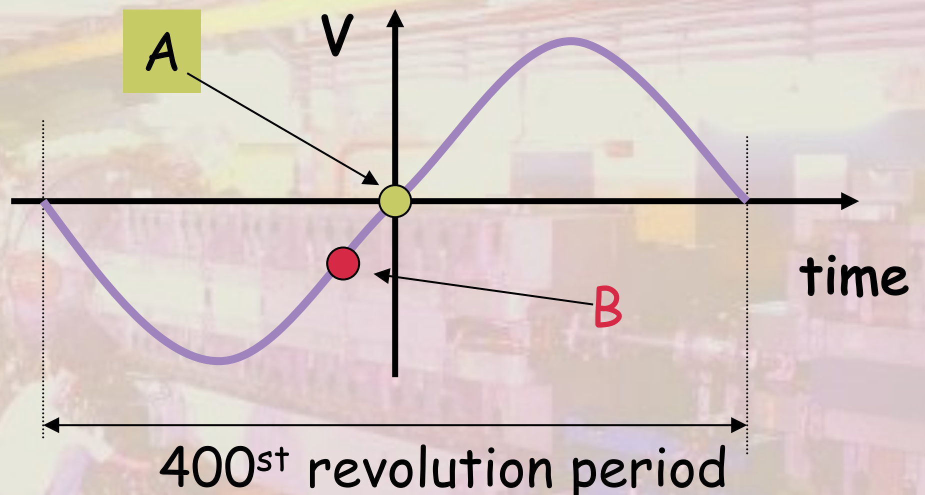
Lets see what happens after many turns



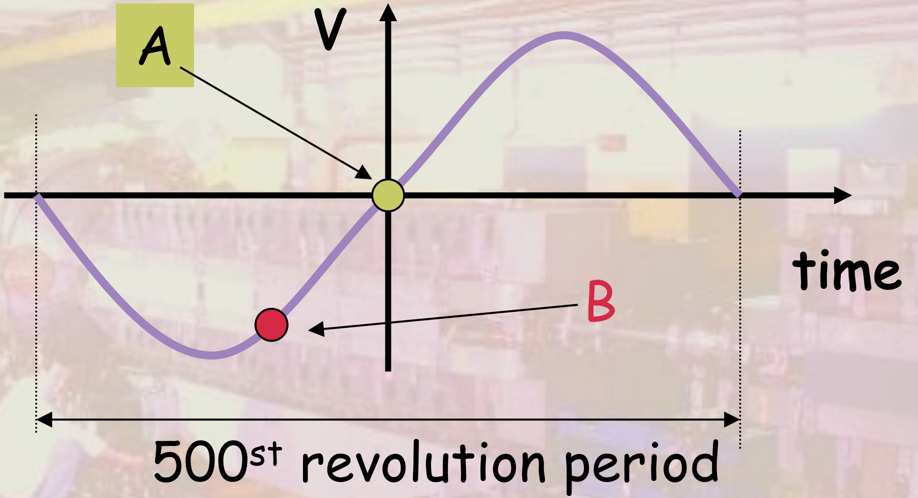
Lets see what happens after many turns



Lets see what happens after many turns



Lets see what happens after many turns



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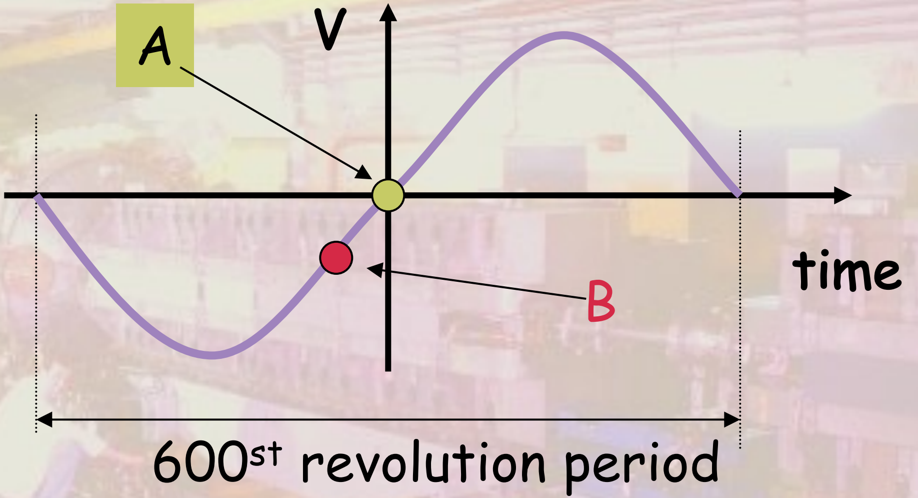

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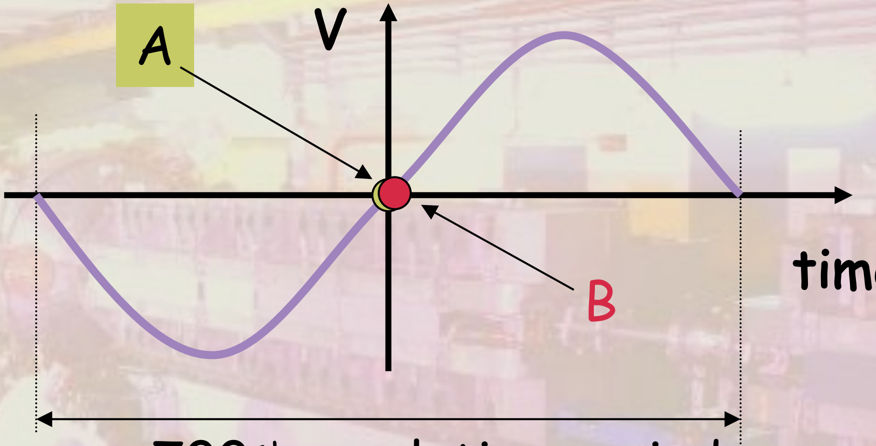
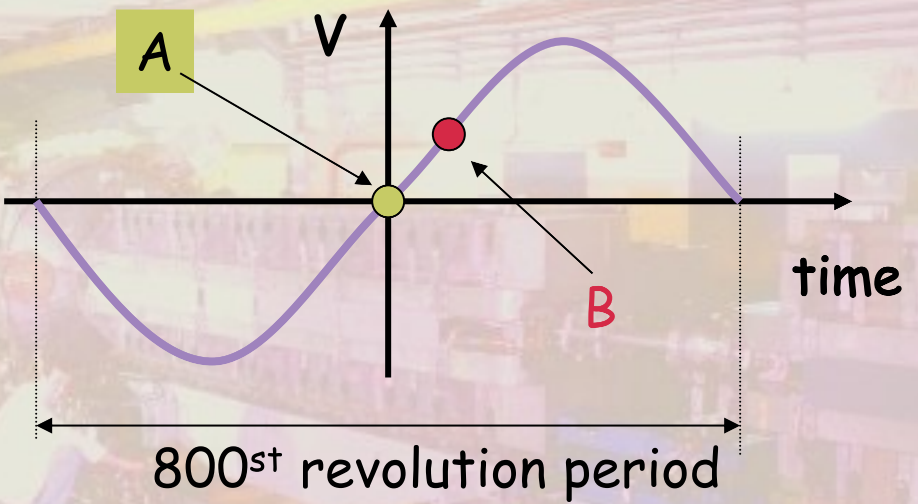
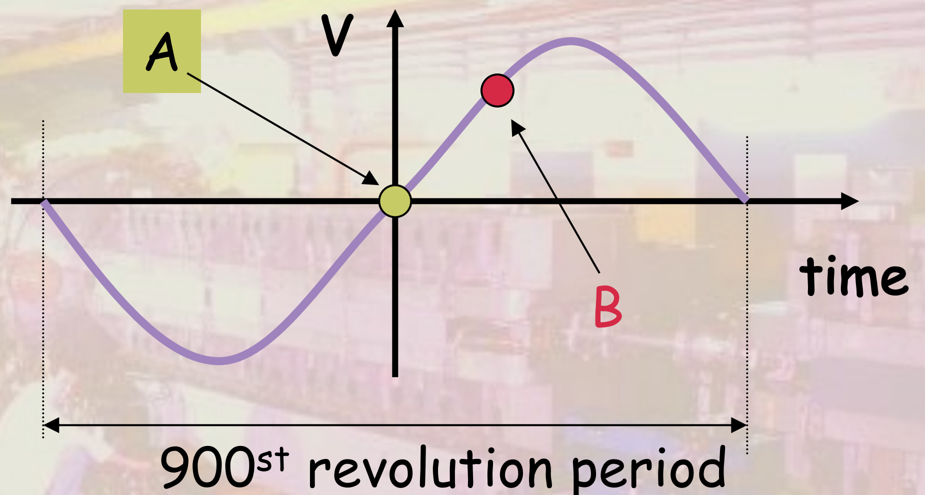


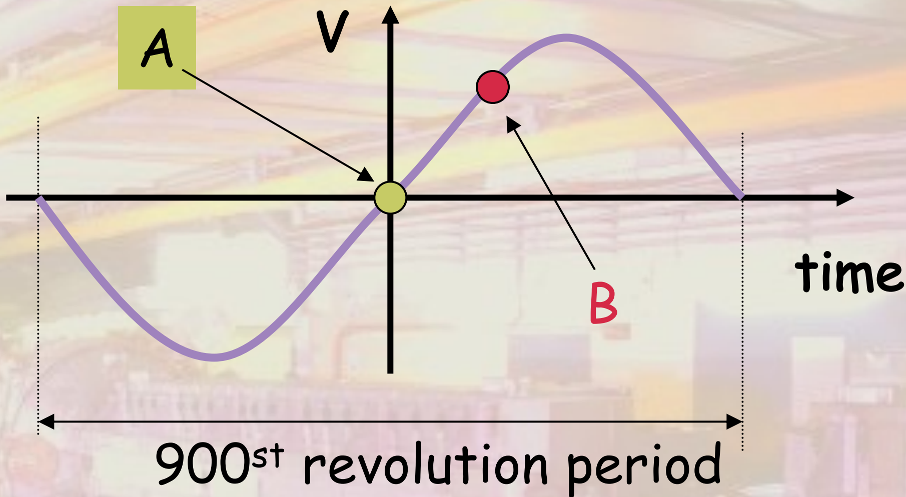
Lets see what happens after many turns



Lets see what happens after many turns



Synchrotron Oscillations



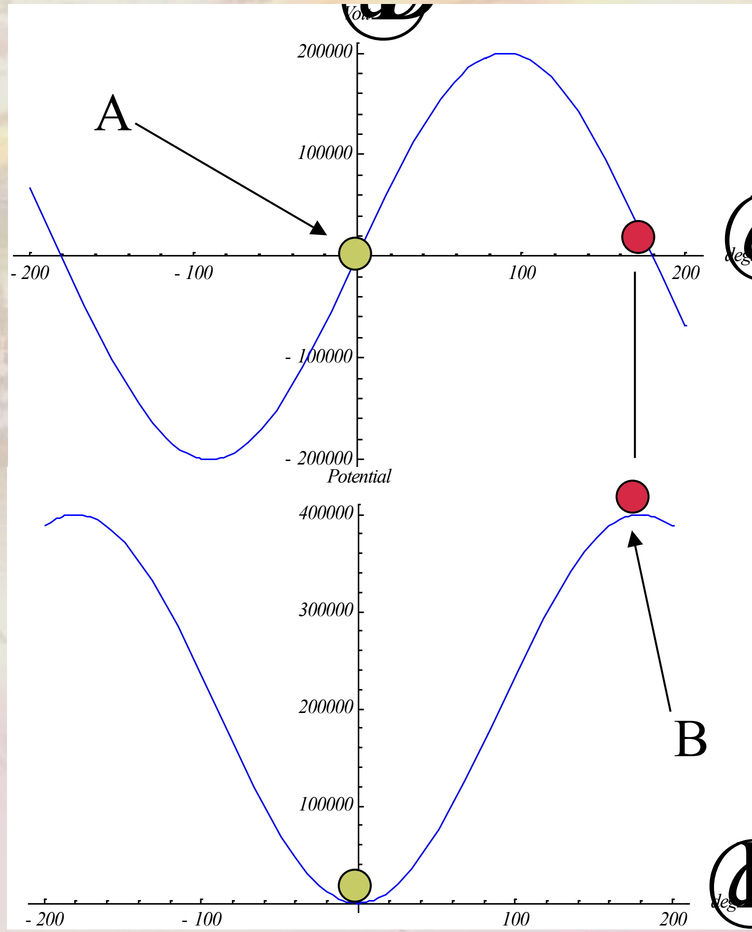
- Particle B has made 1 full oscillation around particle A.
- The amplitude depends on the initial phase.

<u>Exactly like the pendulum</u>

- We call this oscillation:

<u>Synchrotron Oscillation</u>

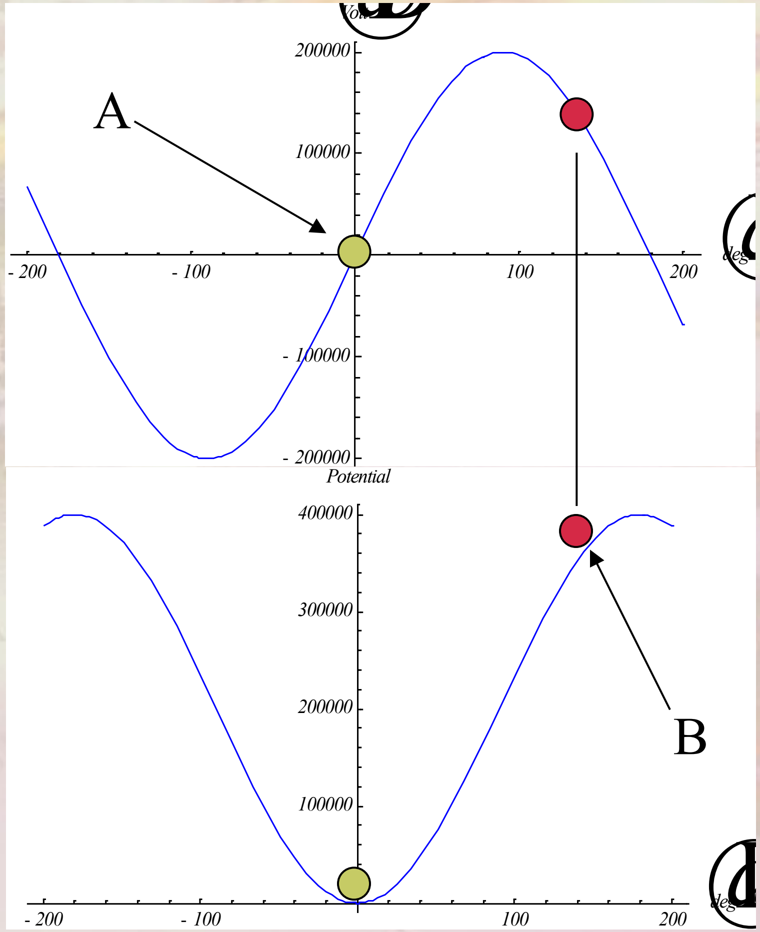
The Potential Well (1)



Cavity voltage

Potential well

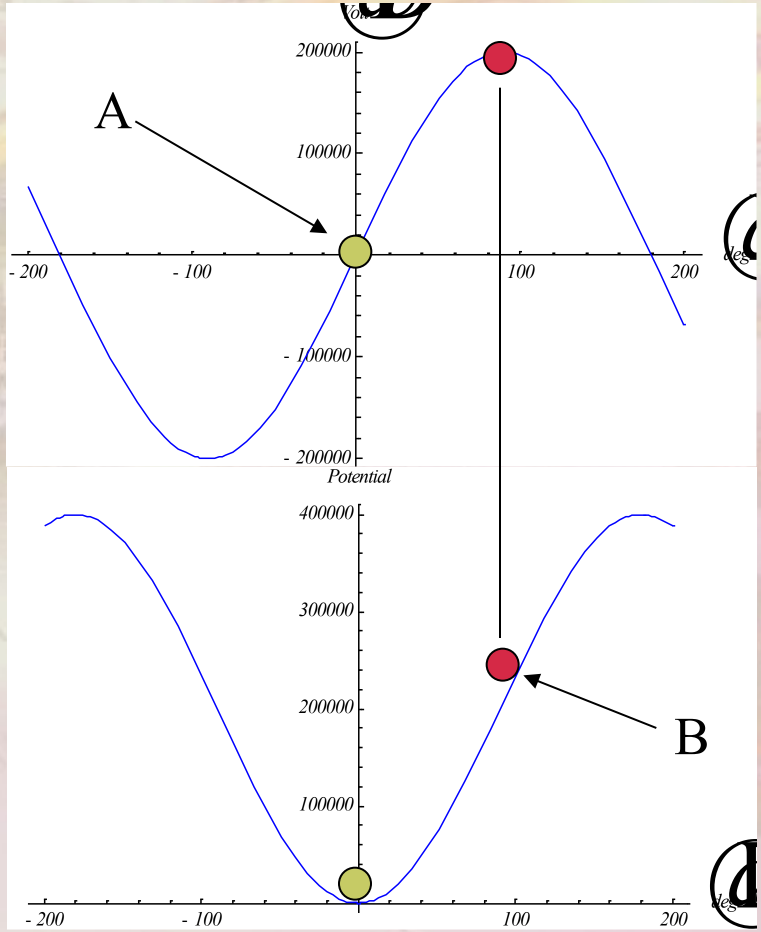
The Potential Well (2)



Cavity voltage

Potential well

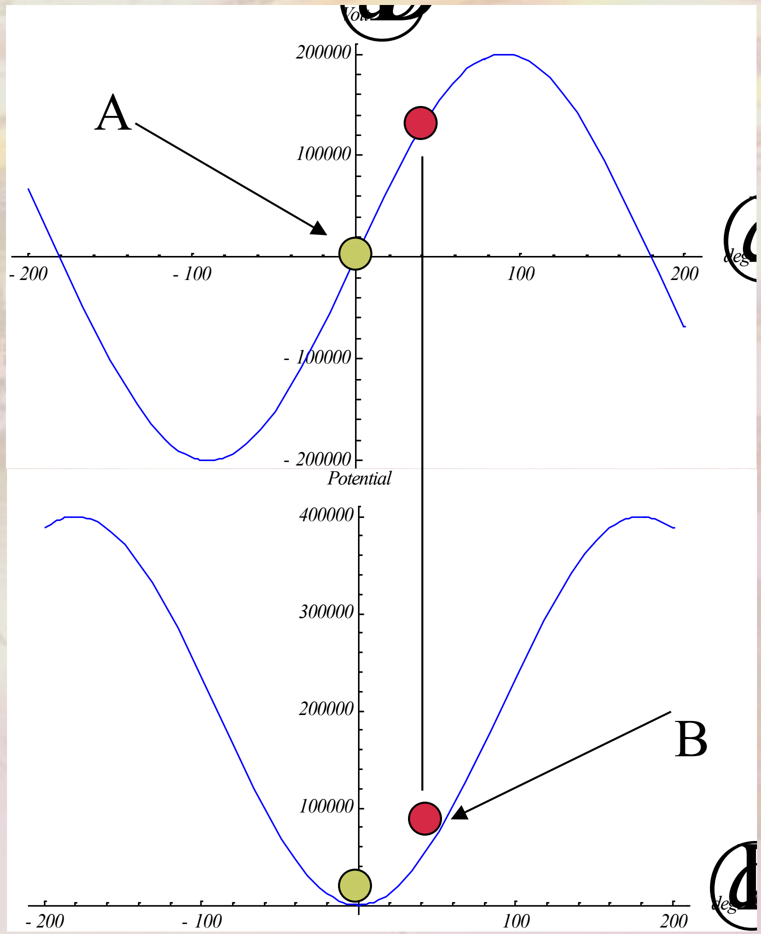
The Potential Well (3)



Cavity voltage

Potential well

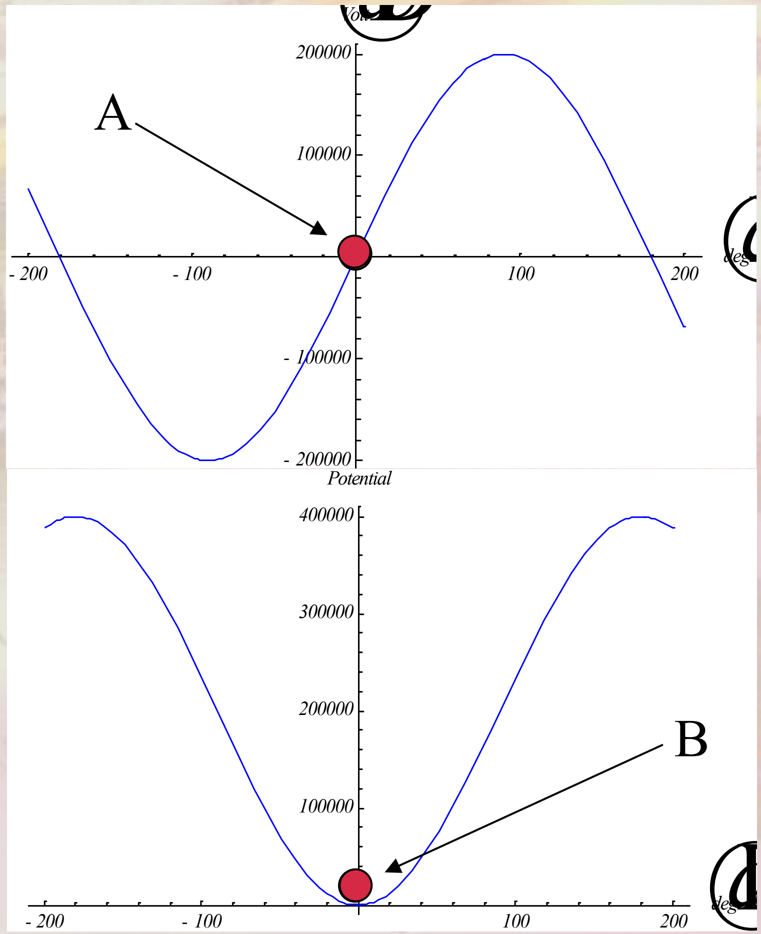
The Potential Well (4)



Cavity voltage

Potential well

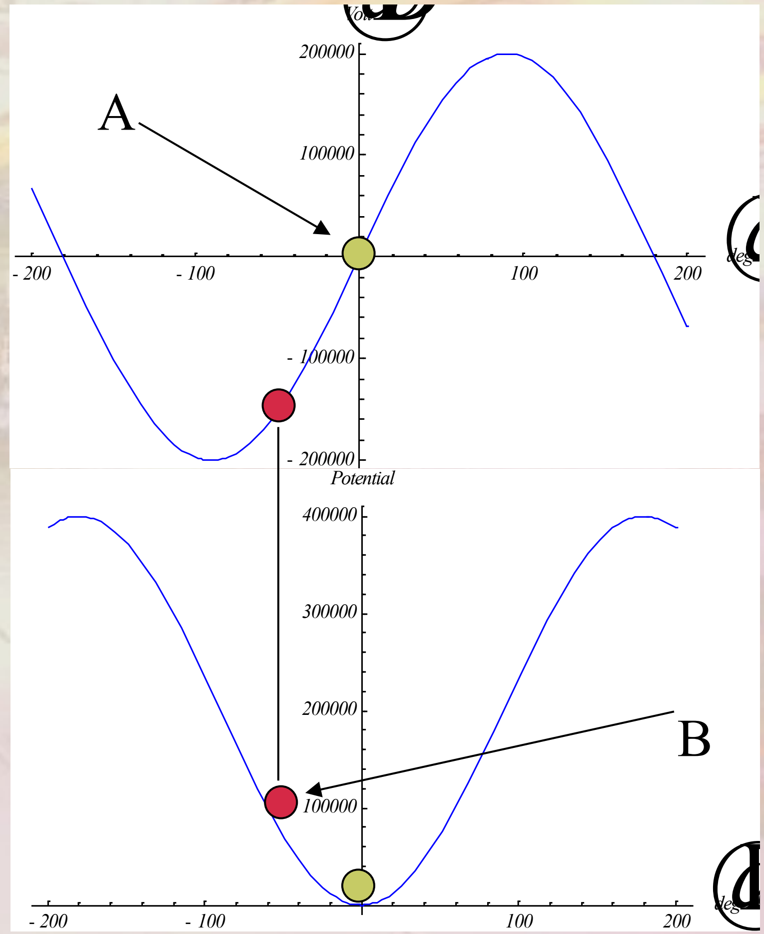
The Potential Well (5)



Cavity voltage

Potential well

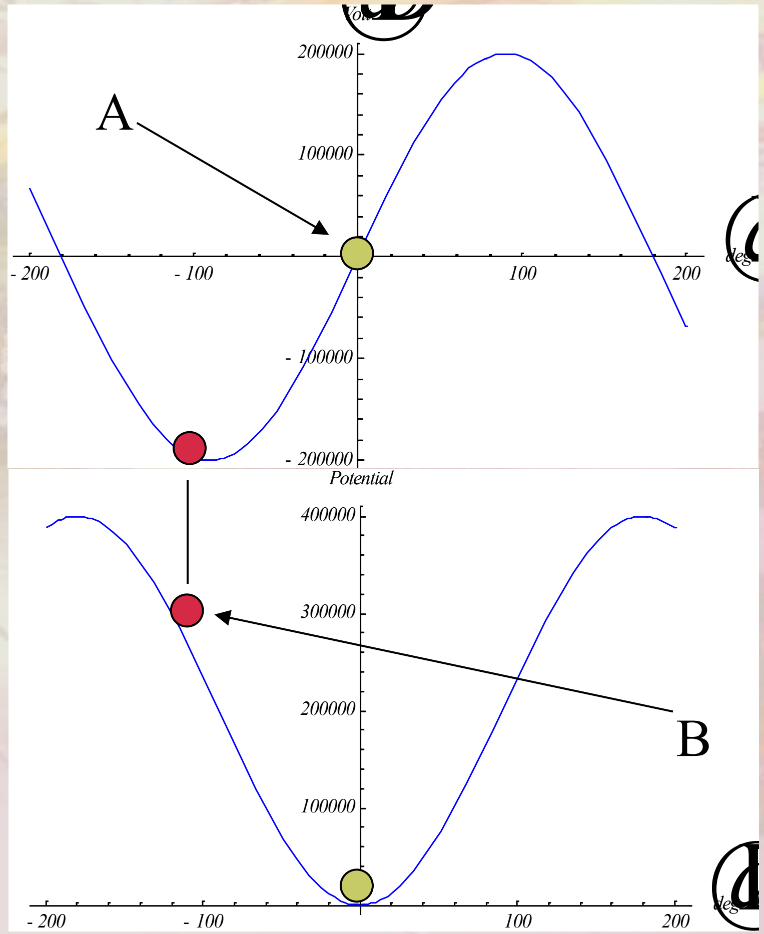
The Potential Well (6)



Cavity voltage

Potential well

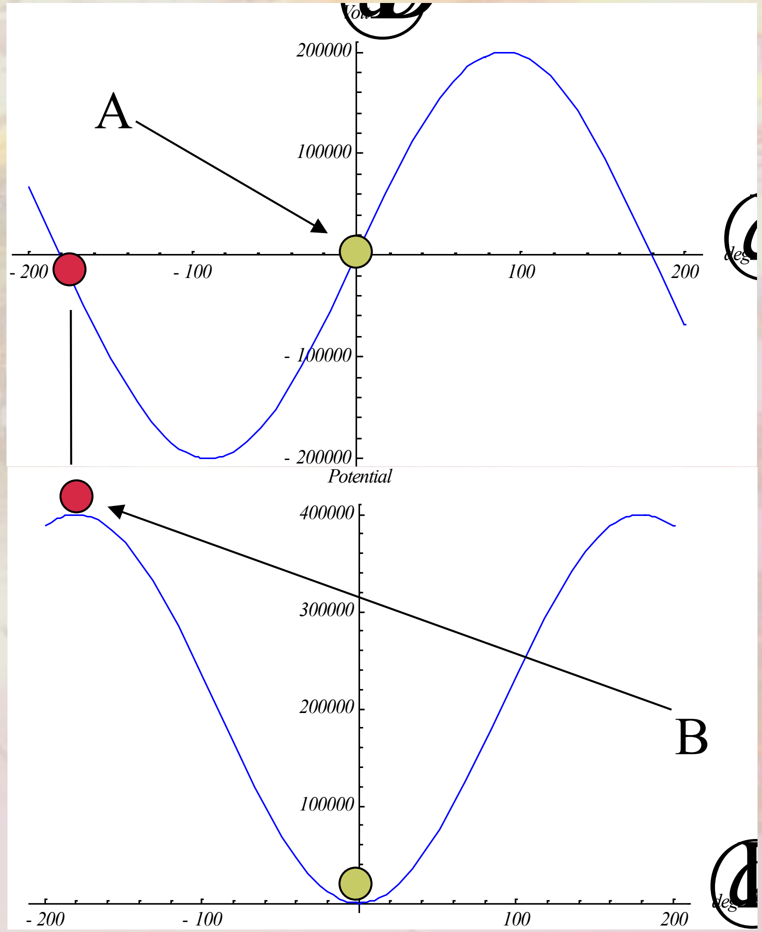
The Potential Well (7)



Cavity voltage

Potential well

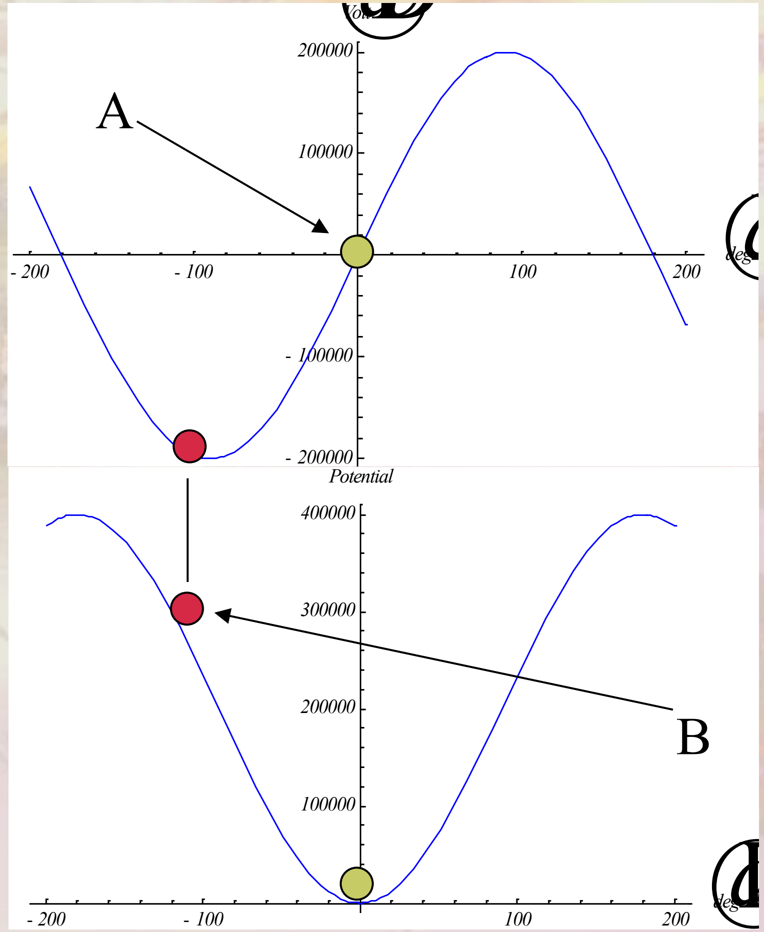
The Potential Well (8)



Cavity voltage

Potential well

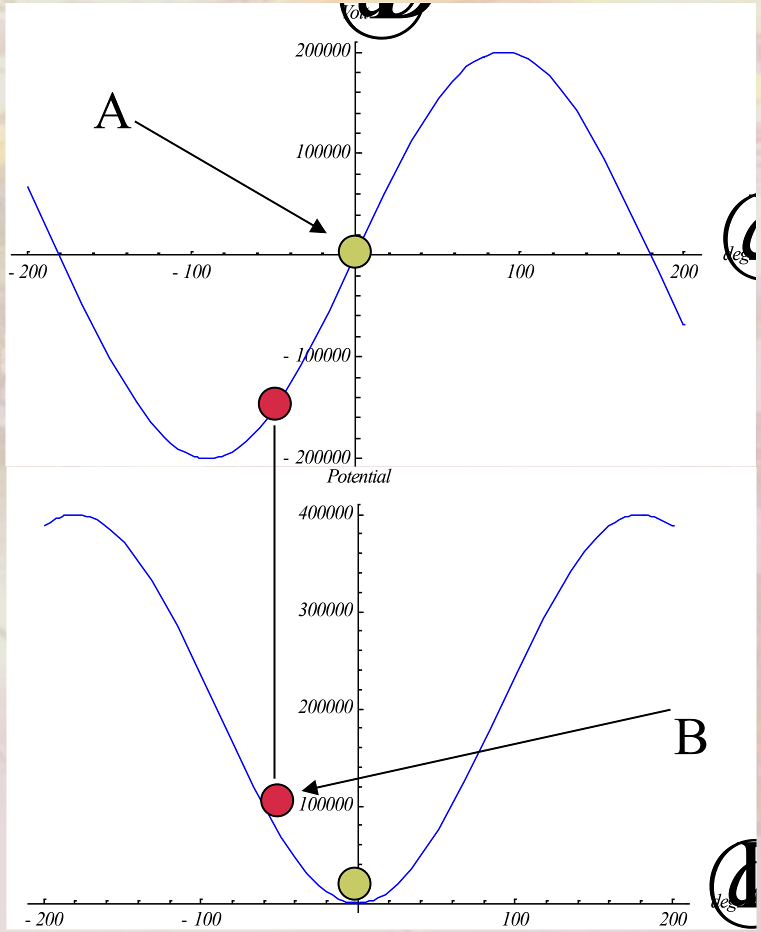
The Potential Well (9)



Cavity voltage

Potential well

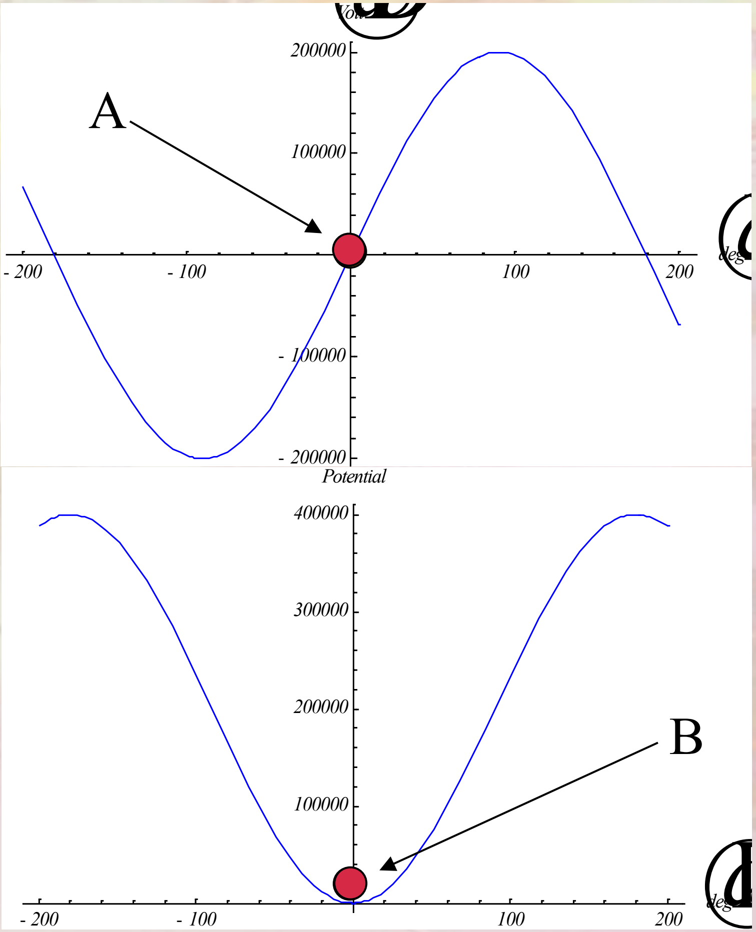
The Potential Well (10)



Cavity voltage

Potential well

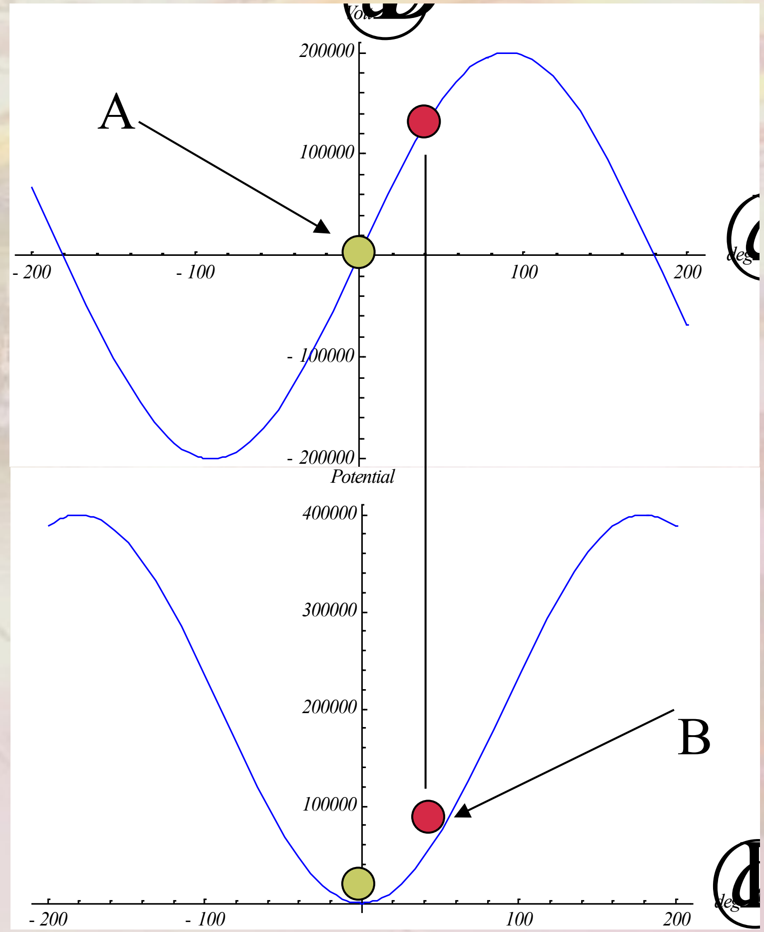
The Potential Well (11)



Cavity voltage

Potential well

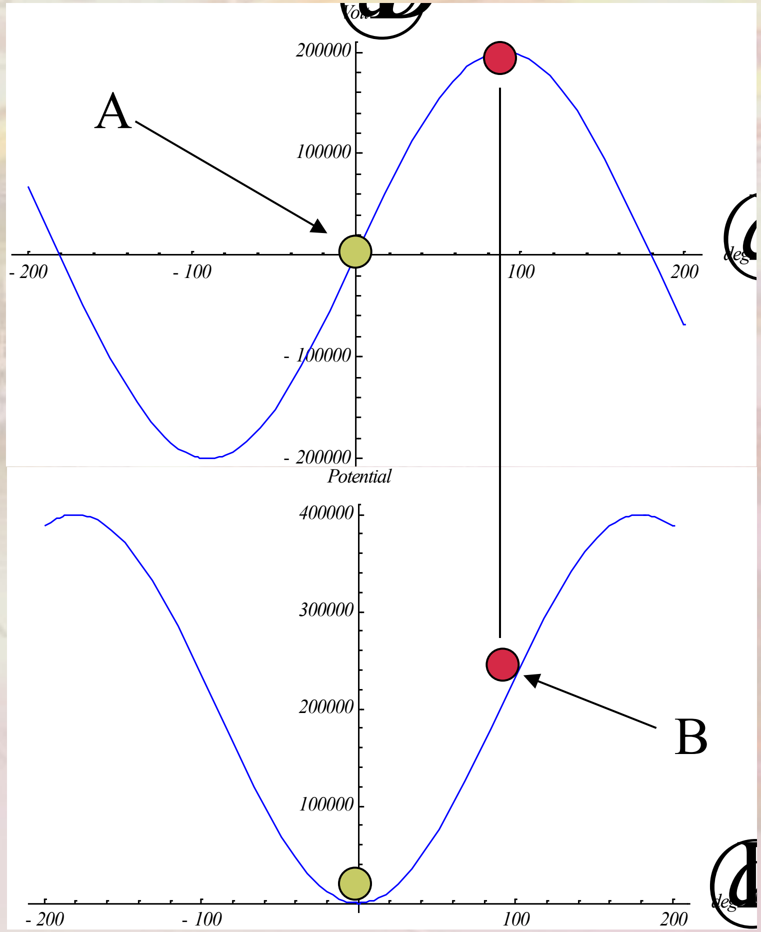
The Potential Well (12)



Cavity voltage

Potential well

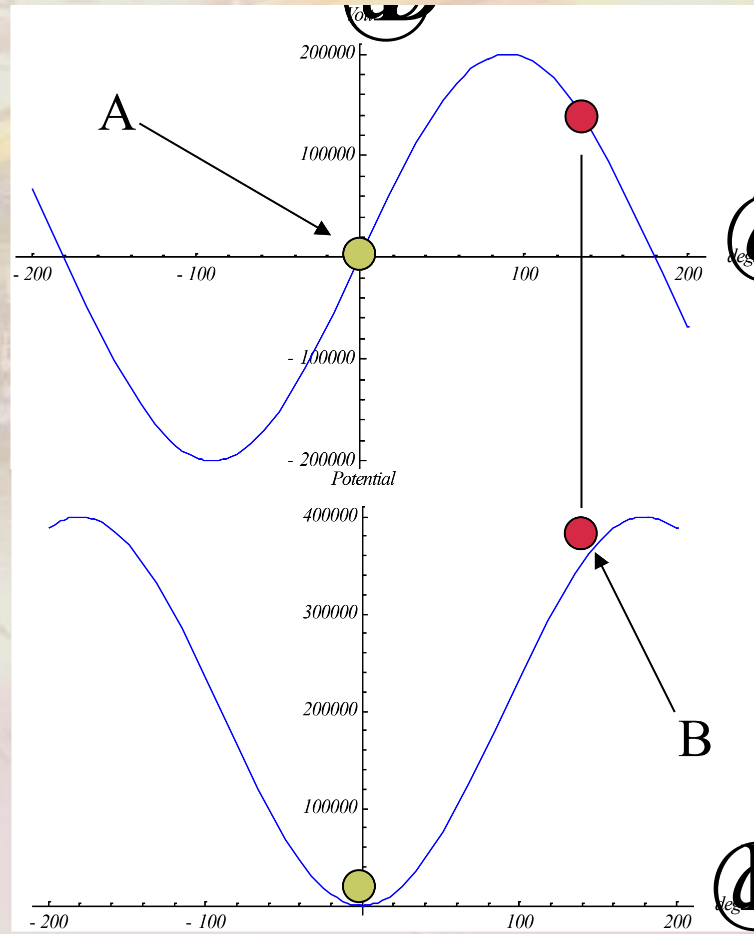
The Potential Well (13)



Cavity voltage

Potential well

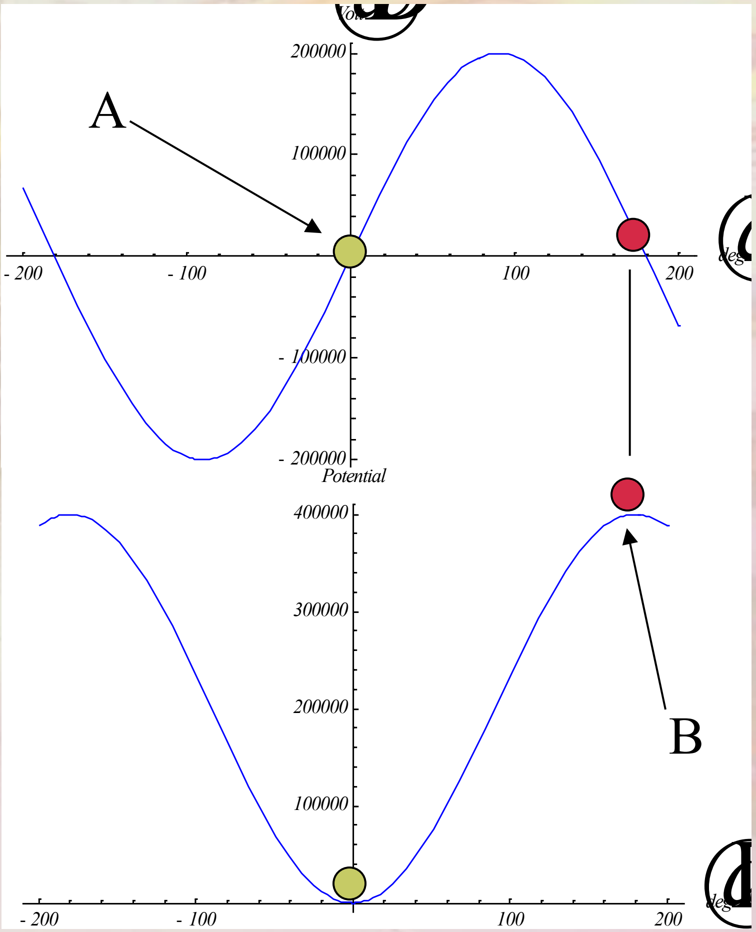
The Potential Well (14)



Cavity voltage

Potential well

The Potential Well (15)

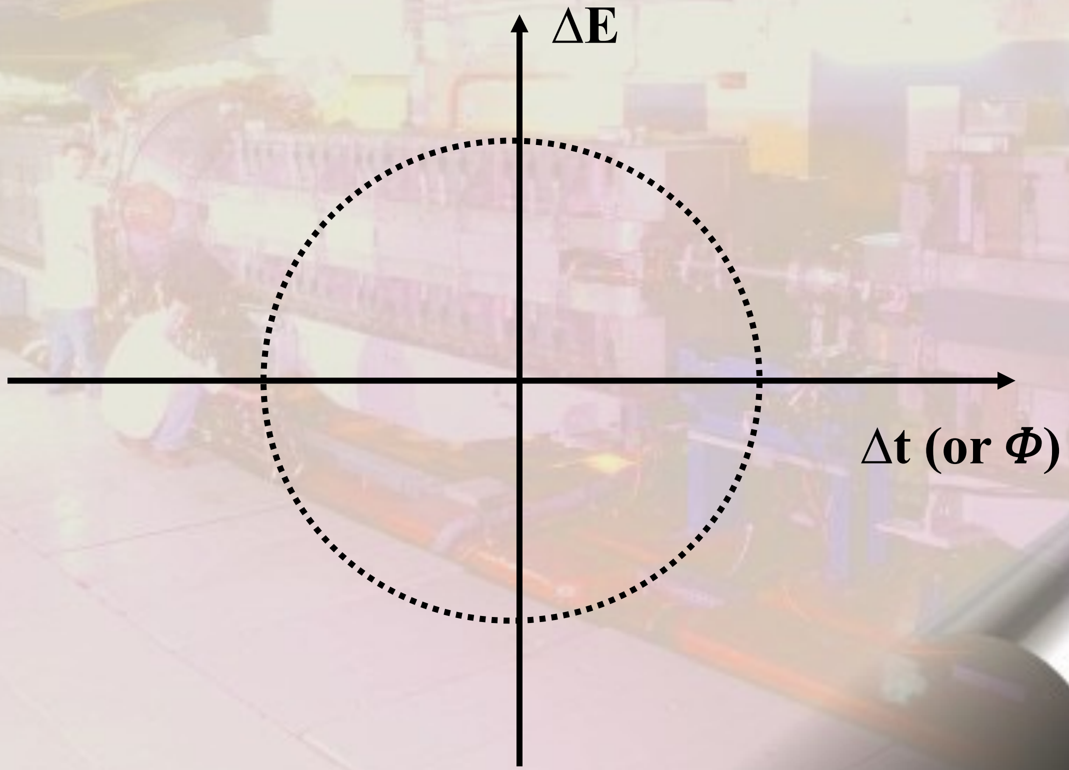


Cavity voltage

Potential well

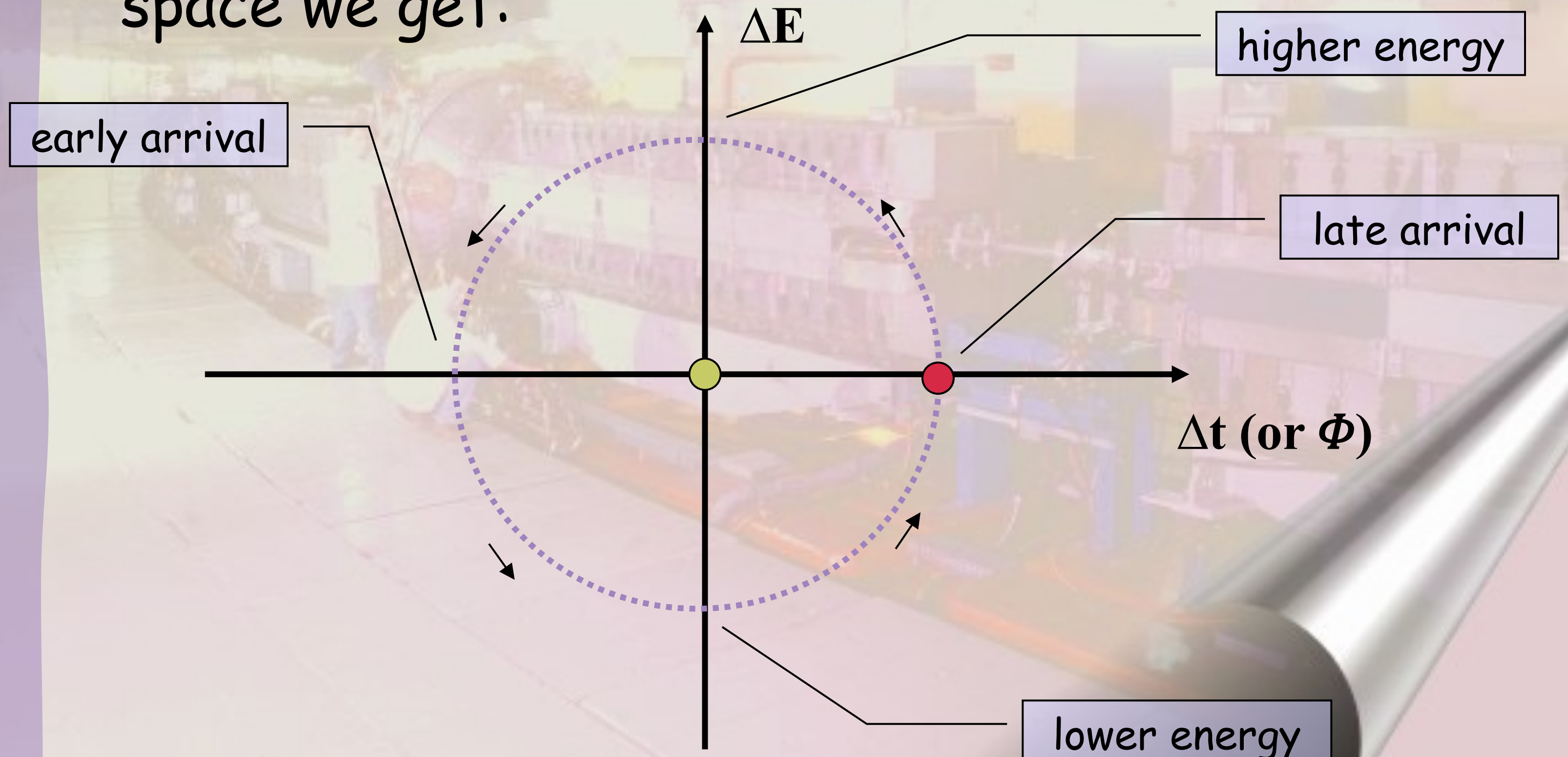
Longitudinal Phase Space

- In order to be able to visualize the motion in the longitudinal plane we define the longitudinal phase space (like we did for the transverse phase space)



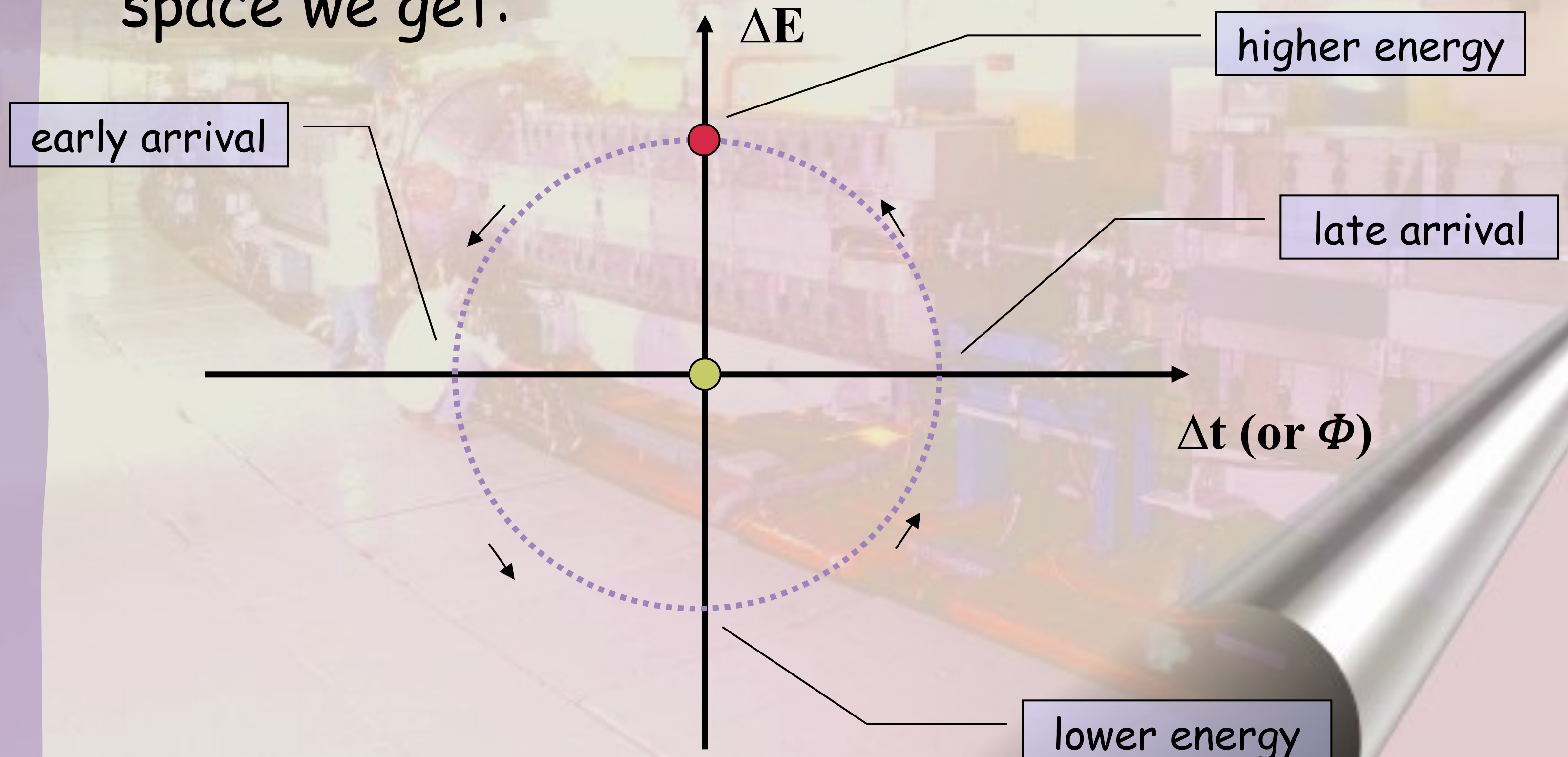
Phase Space motion (1)

- Particle B oscillates around particle A
  - This is synchrotron oscillation
- When we plot this motion in our longitudinal phase space we get:



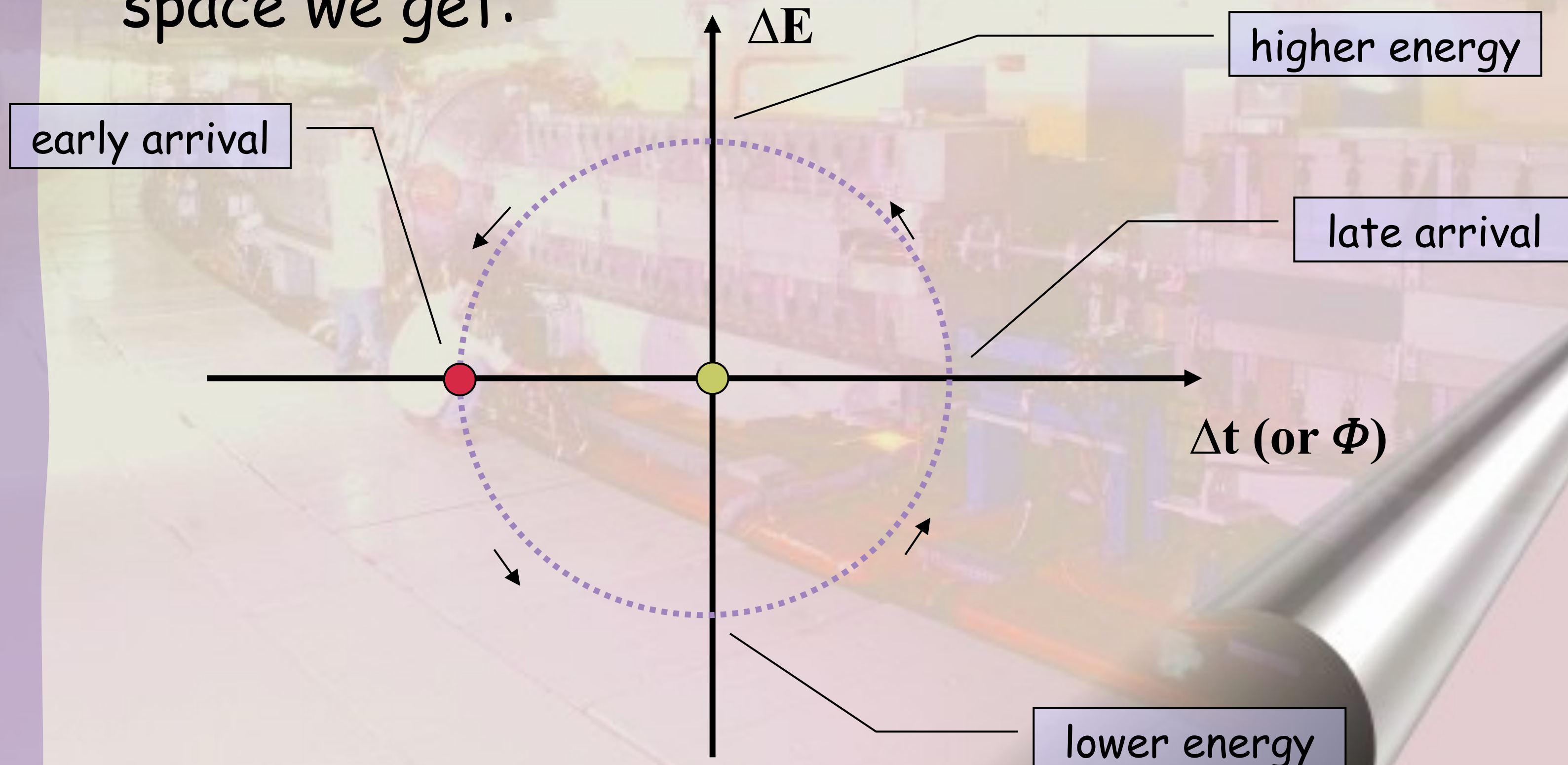
Phase Space motion (2)

- Particle B oscillates around particle A
  - This is synchrotron oscillation
- When we plot this motion in our longitudinal phase space we get:



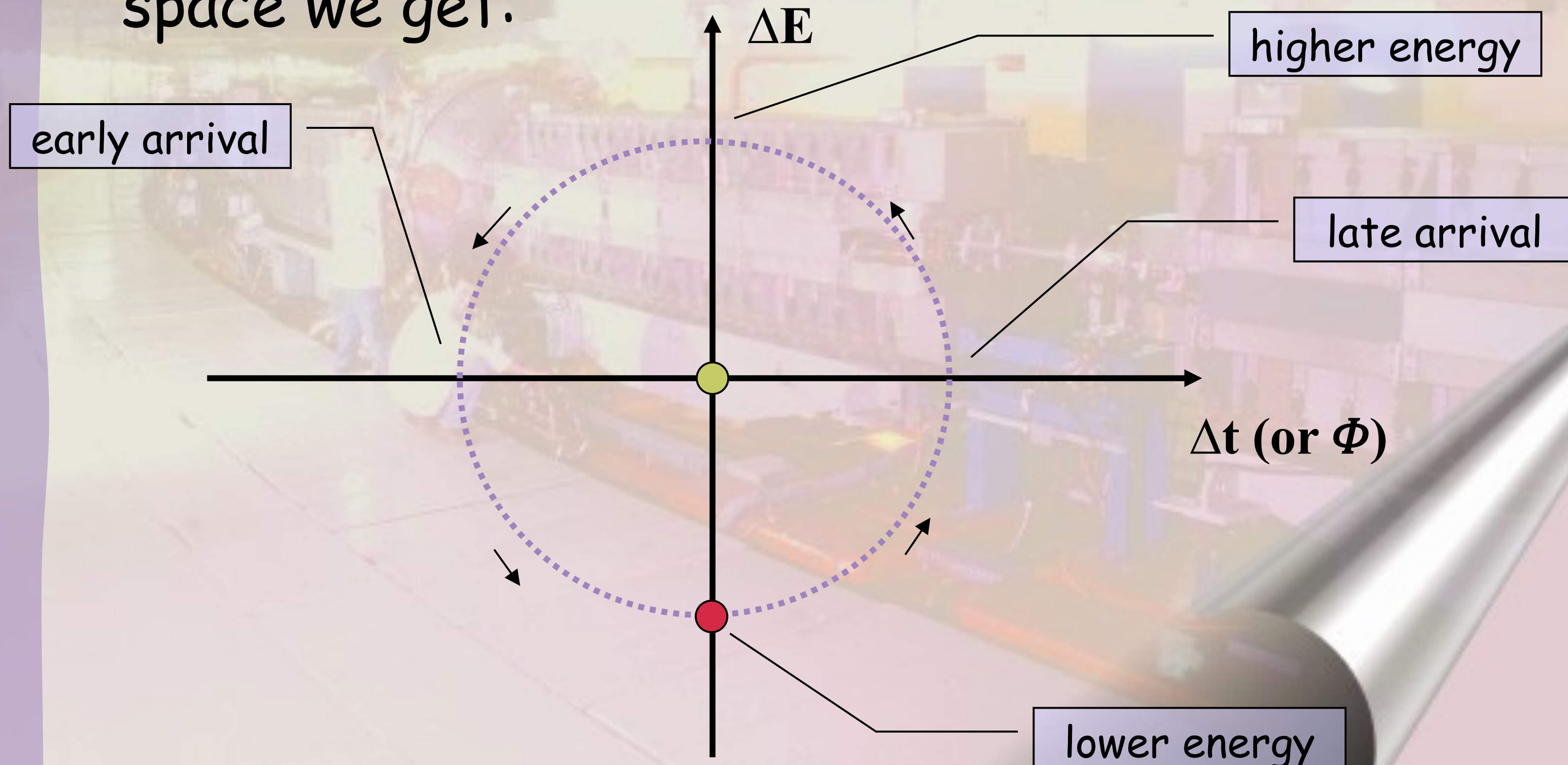
Phase Space motion (3)

- Particle B oscillates around particle A
  - This is synchrotron oscillation
- When we plot this motion in our longitudinal phase space we get:



Phase Space motion (4)

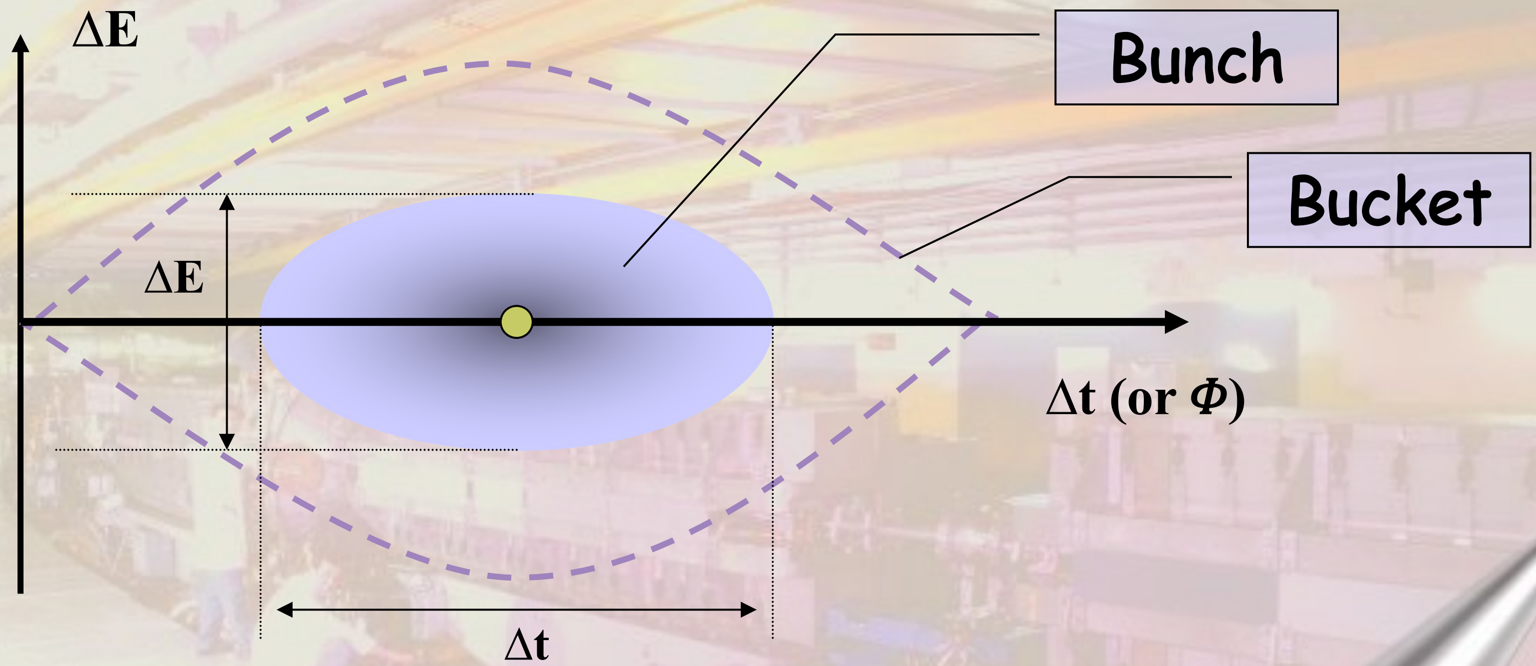
- Particle B oscillates around particle A
  - This is synchrotron oscillation
- When we plot this motion in our longitudinal phase space we get:



Quick intermediate summary…

- We have seen that:
  - The RF system forms a potential well in which the particles oscillate (synchrotron oscillation).
  - We can describe this motion in the longitudinal phase space (energy versus time or phase).
  - This works for particles below transition.
- However,
  - Due to the shape of the potential well, the oscillation is a non-linear motion.
  - The phase space trajectories are therefore no circles nor ellipses.
  - What when our particles are above transition ?

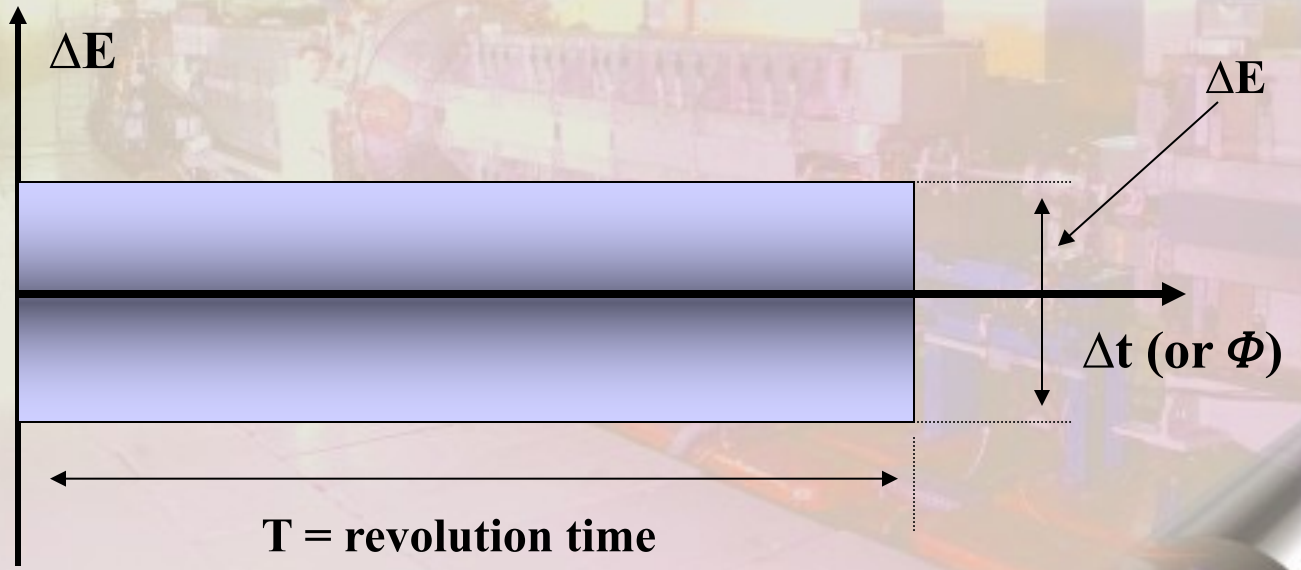
Stationary bunch & bucket



- Bucket area = longitudinal Acceptance [eVs]
- Bunch area = longitudinal beam emittance = $\pi.\Delta E.\Delta t/4$ [eVs]

Unbunched (coasting) beam

- The emittance of an unbunched beam is just ΔET eVs
  - ΔE is the energy spread [eV]
  - T is the revolution time [s]



What happens beyond transition ?

- Until now we have seen how things look like below transition

$\eta =$ **positive**

Higher energy ⇨ faster orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.

Lower energy ⇨ slower orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

- What will happen above transition ?

$\eta =$ **negative**

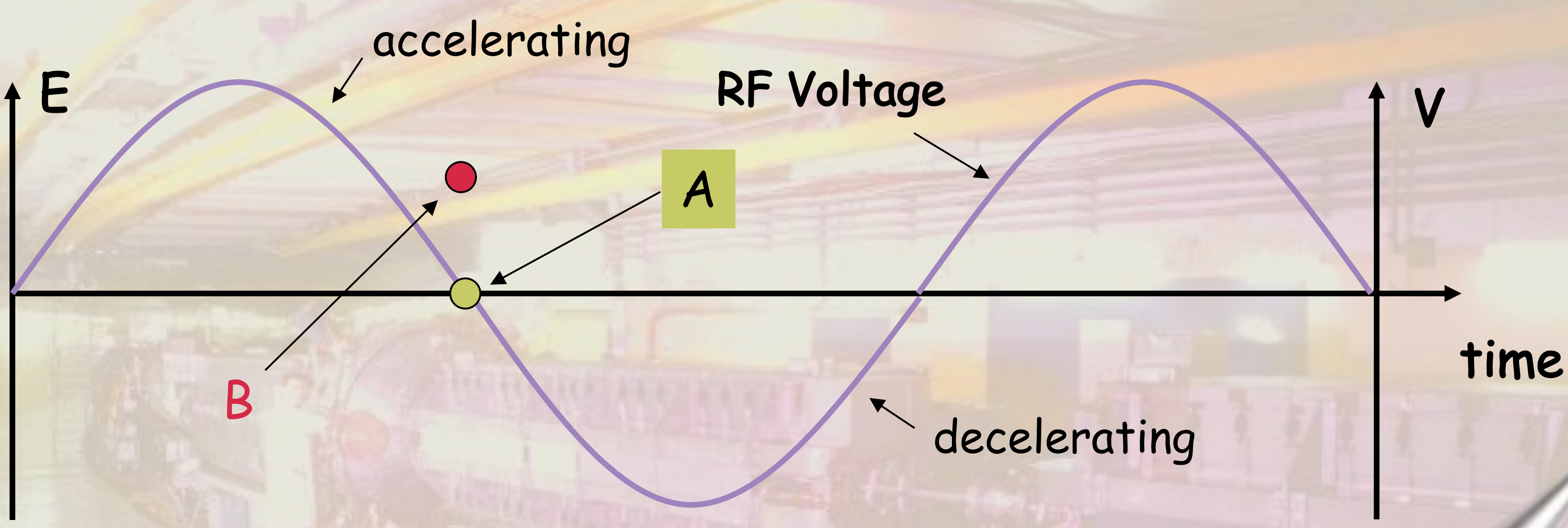
Higher energy ⇨ longer orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

Lower energy ⇨ shorter orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.

What are the implication for the RF ?

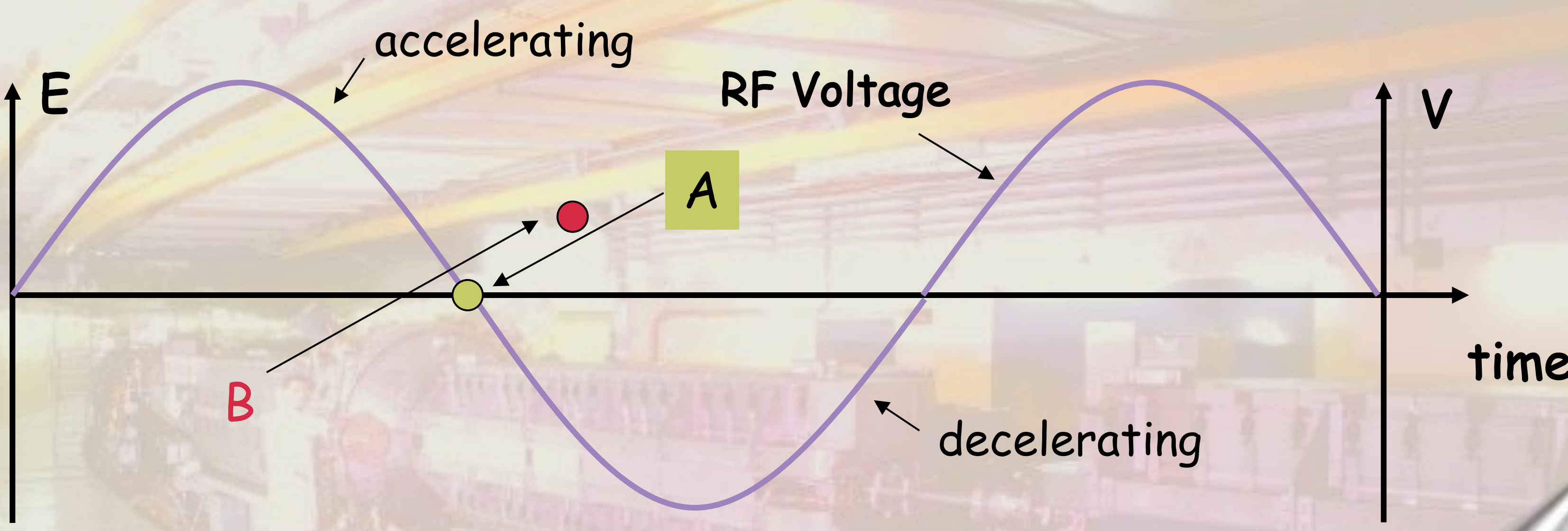
- For particles below transition we worked on the rising edge of the sine wave.
- For Particles above transition we will work on the falling edge of the sine wave.
- We will see why…….

Longitudinal motion beyond transition (1)



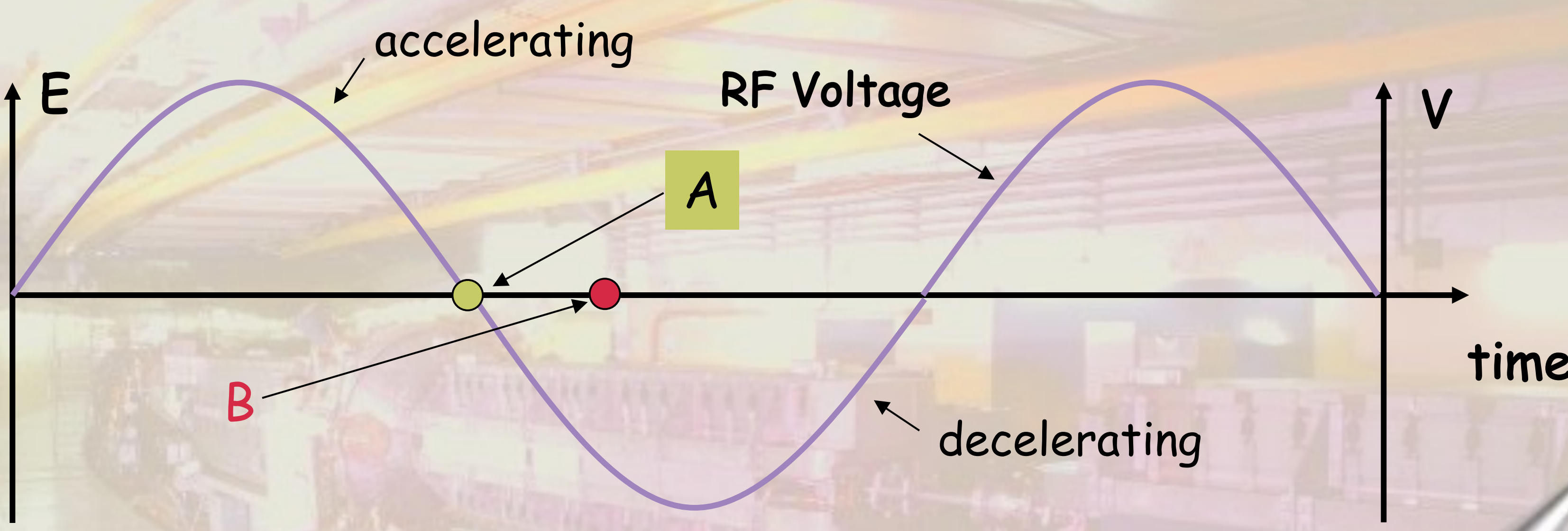
- Imagine two particles A and B, that arrive at the same time in the accelerating cavity (when V_{rf} = 0V)
  - For A the energy is such that $F_{rev\ A} = F_{rf}$.
  - The energy of B is higher → $F_{rev\ B} < F_{rev\ A}$

Longitudinal motion beyond transition (2)



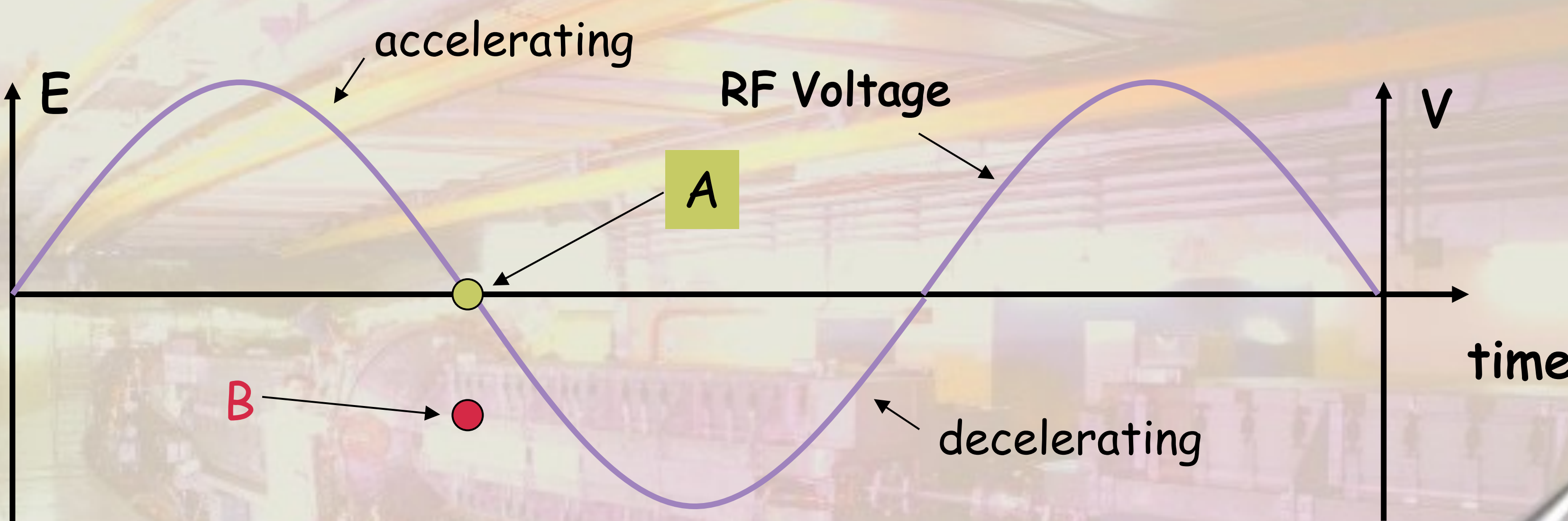
- Particle B arrives after A and experiences a decelerating voltage.
  - The energy of B is still higher, **but less** → $F_{rev\ B} < F_{rev\ A}$

Longitudinal motion beyond transition (3)



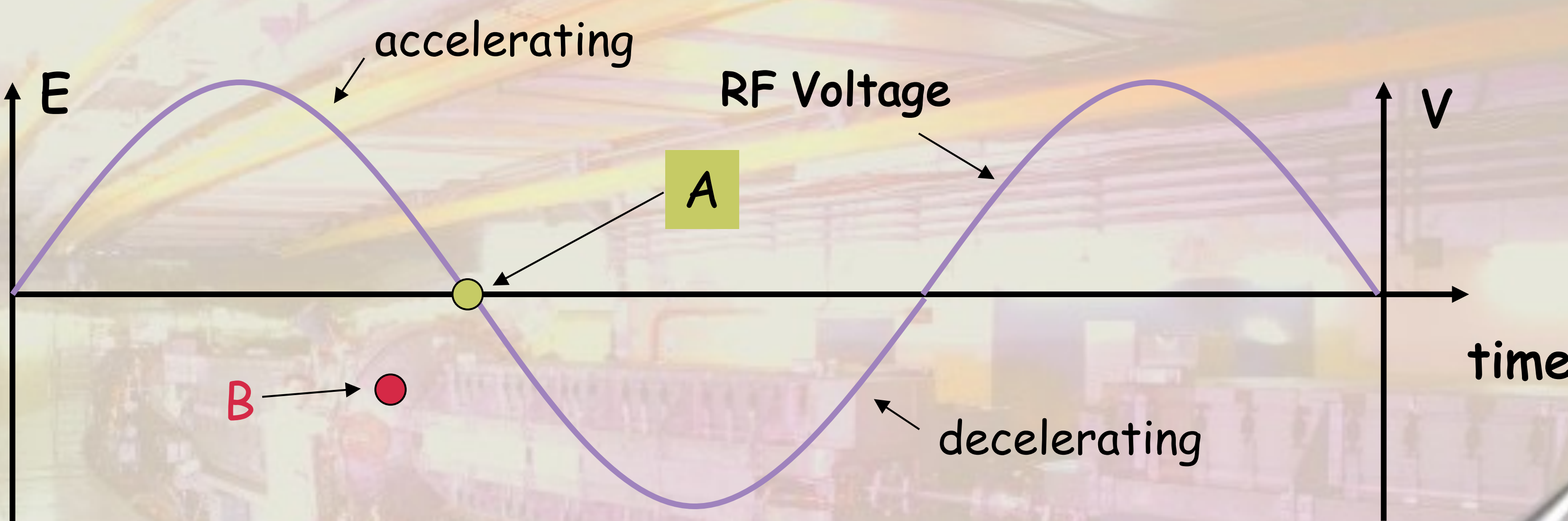
- B has now the same energy as A, but arrives still later and experiences therefore a decelerating voltage.
  - $F_{rev\ B} = F_{rev\ A}$

Longitudinal motion beyond transition (4)



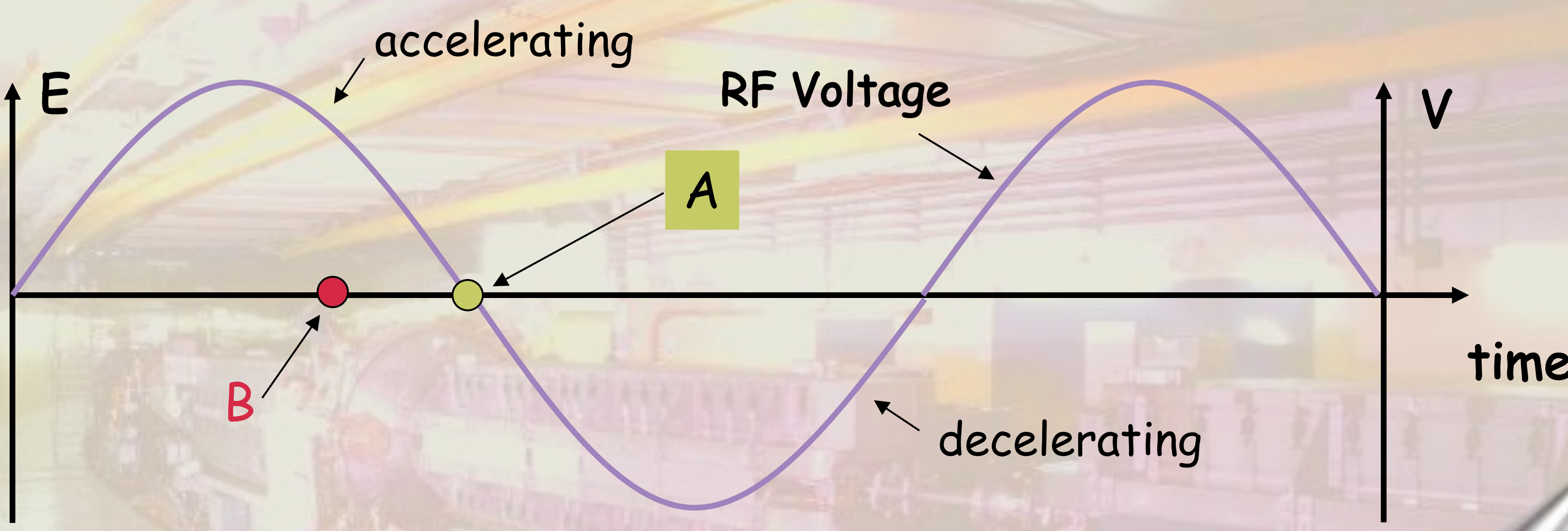
- Particle B has now a lower energy as A, but arrives at the same time
  - $F_{rev\ B} > F_{rev\ A}$

Longitudinal motion beyond transition (5)



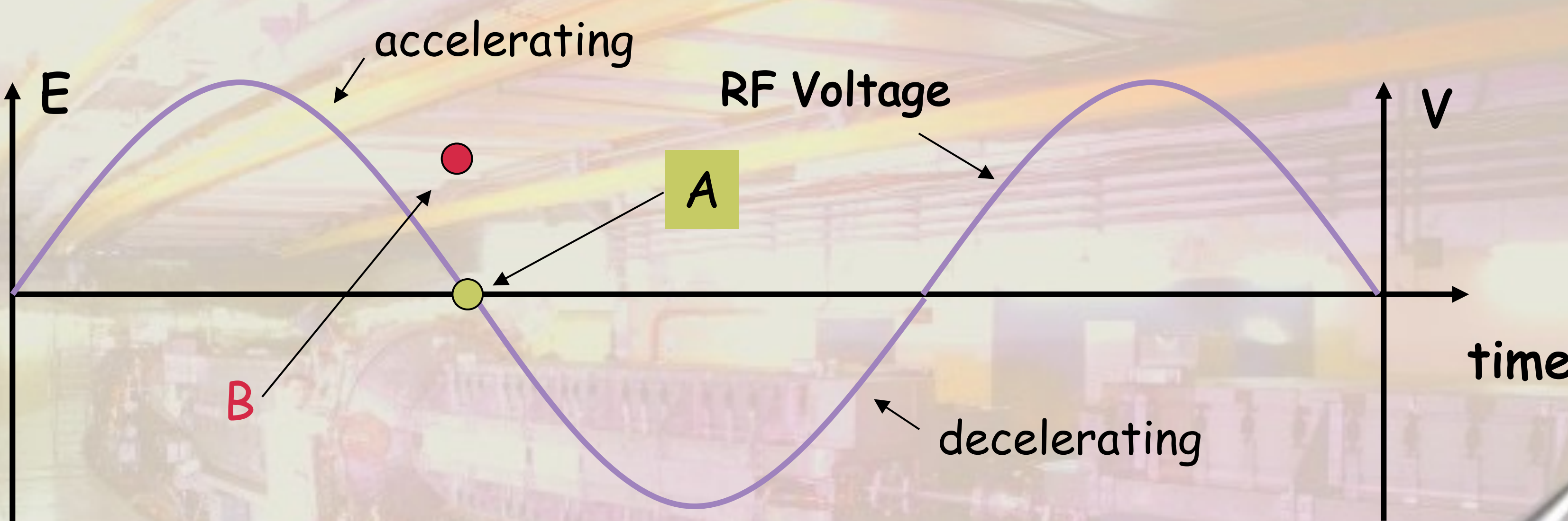
- Particle B has now a lower energy as A, but B arrives before A and experiences an accelerating voltage.
  - $F_{rev\ B} > F_{rev\ A}$

Longitudinal motion beyond transition (6)



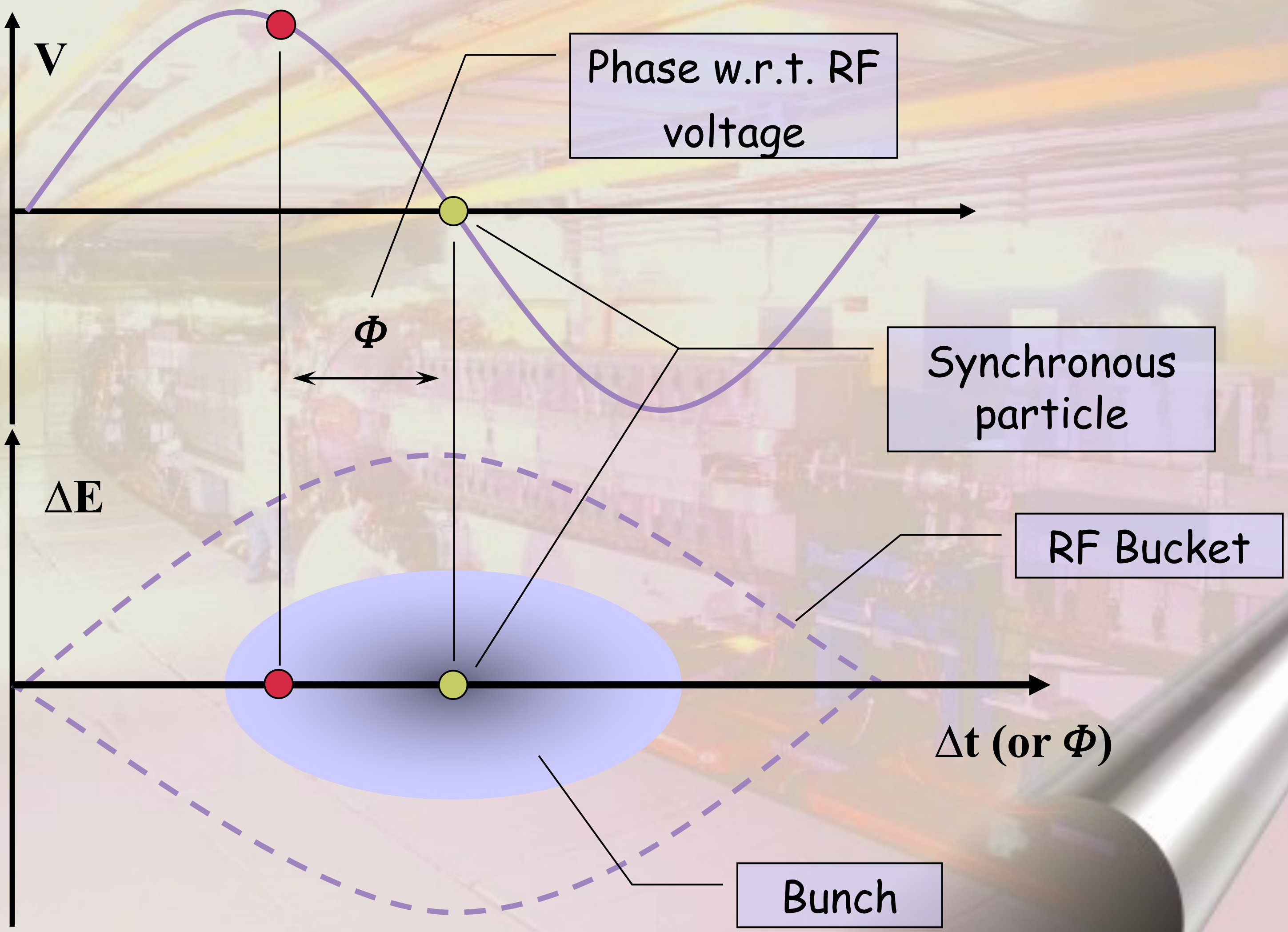
- Particle B has now the same energy as A, but B still arrives before A and experiences an accelerating voltage.
  - $F_{rev\ B} > F_{rev\ A}$

Longitudinal motion beyond transition (7)

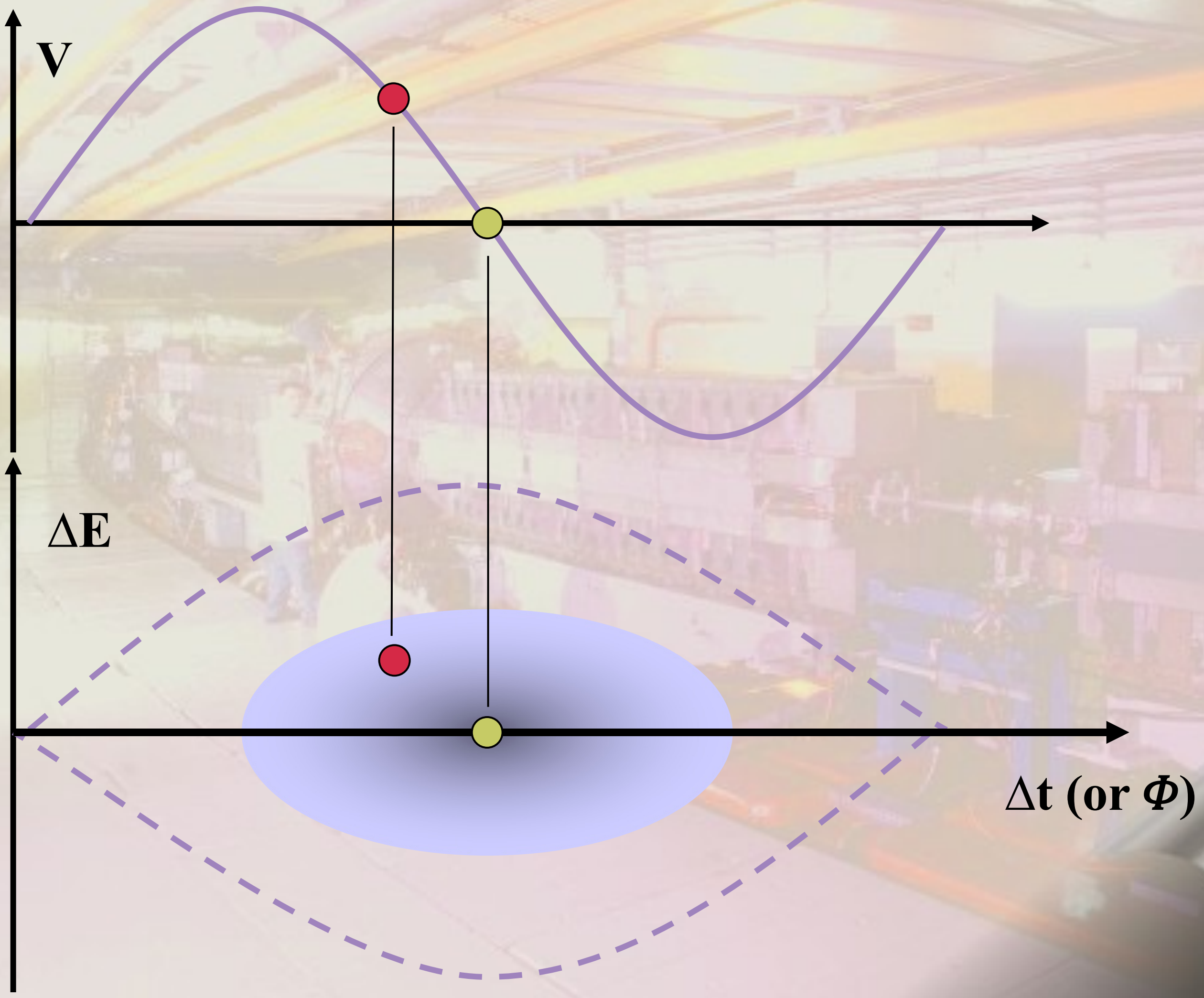


- Particle B has now a higher energy as A and arrives at the same time again....
  - $F_{rev\ B} < F_{rev\ A}$

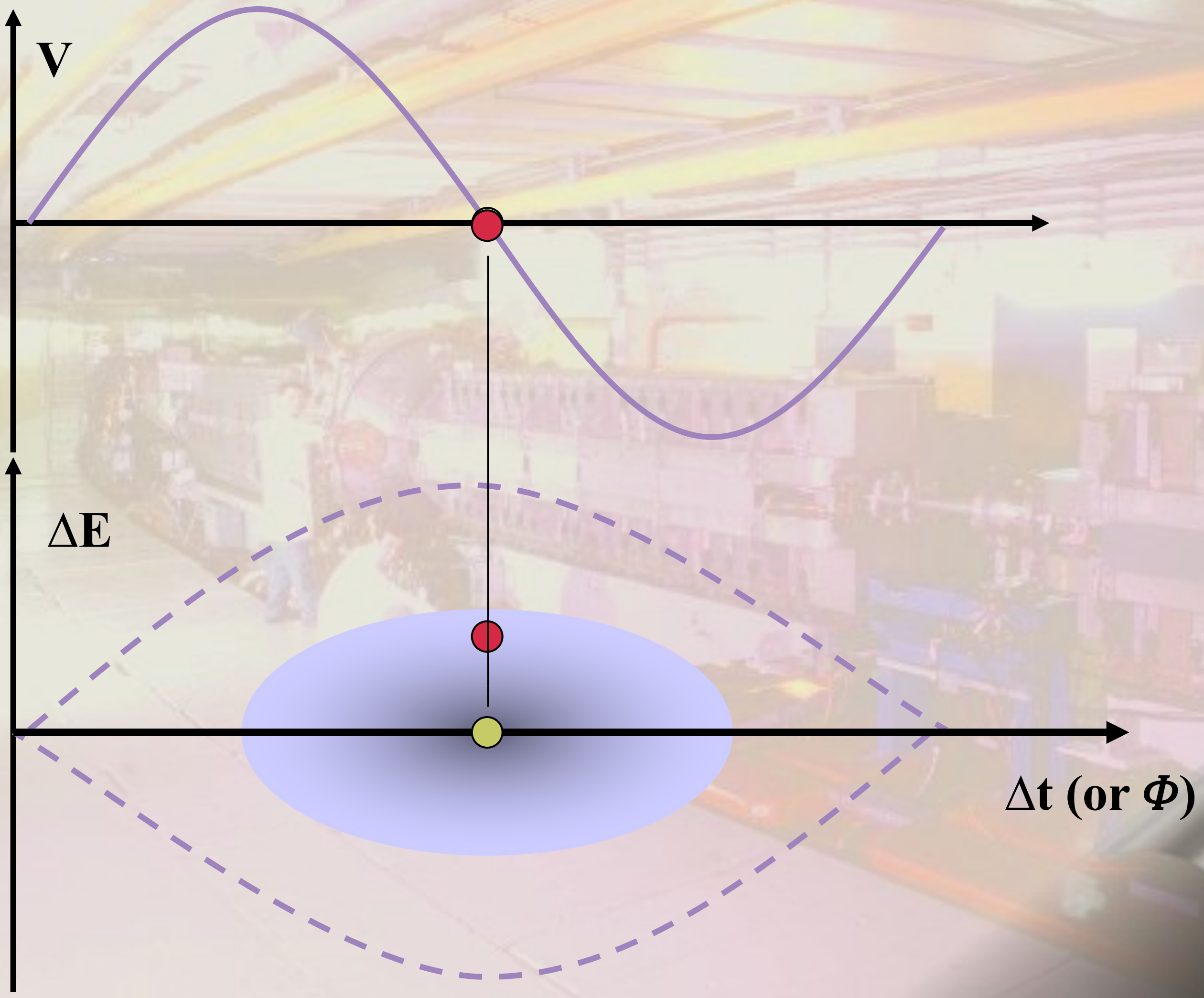
The motion in the bucket (1)



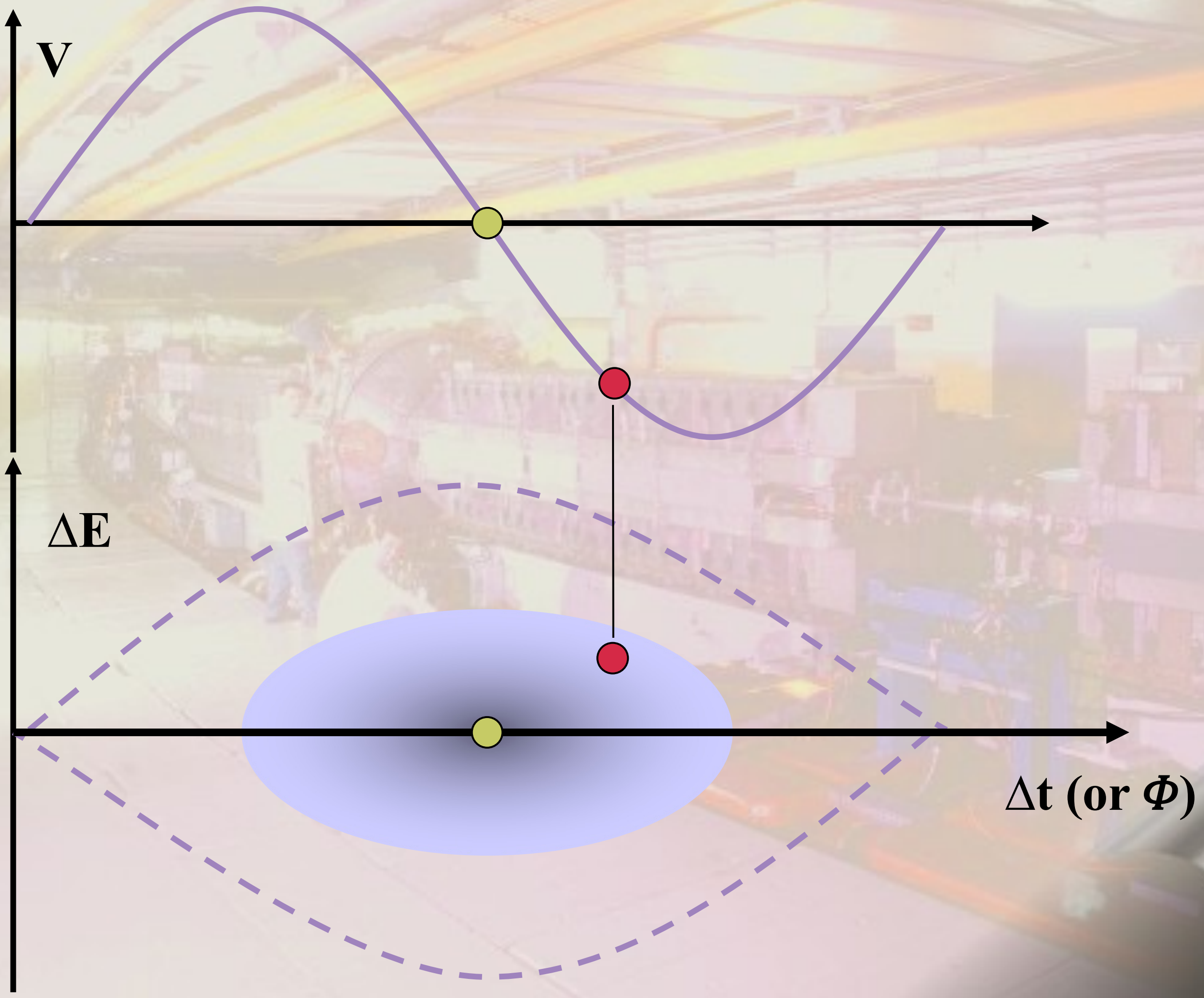
The motion in the bucket (2)



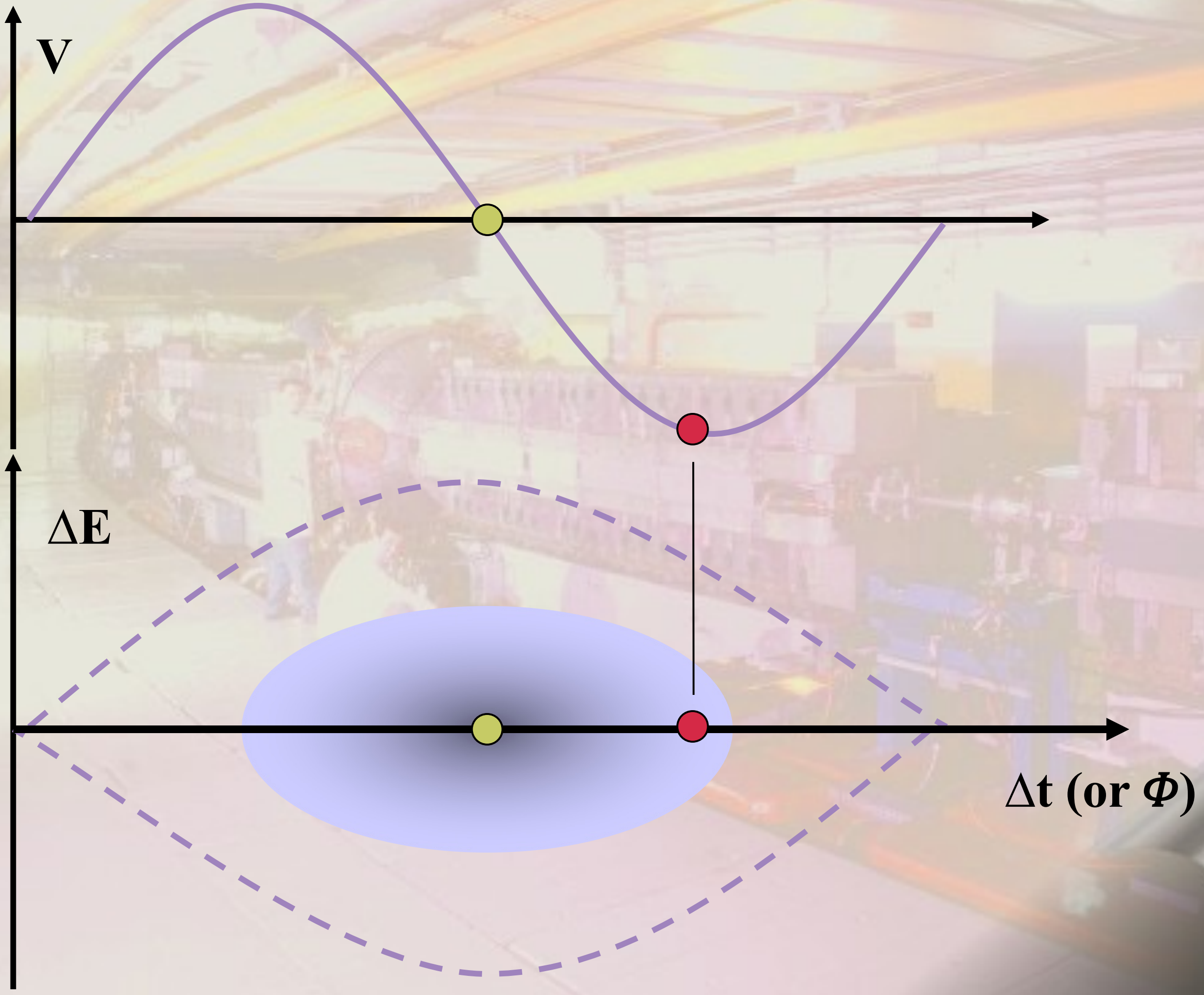
The motion in the bucket (3)



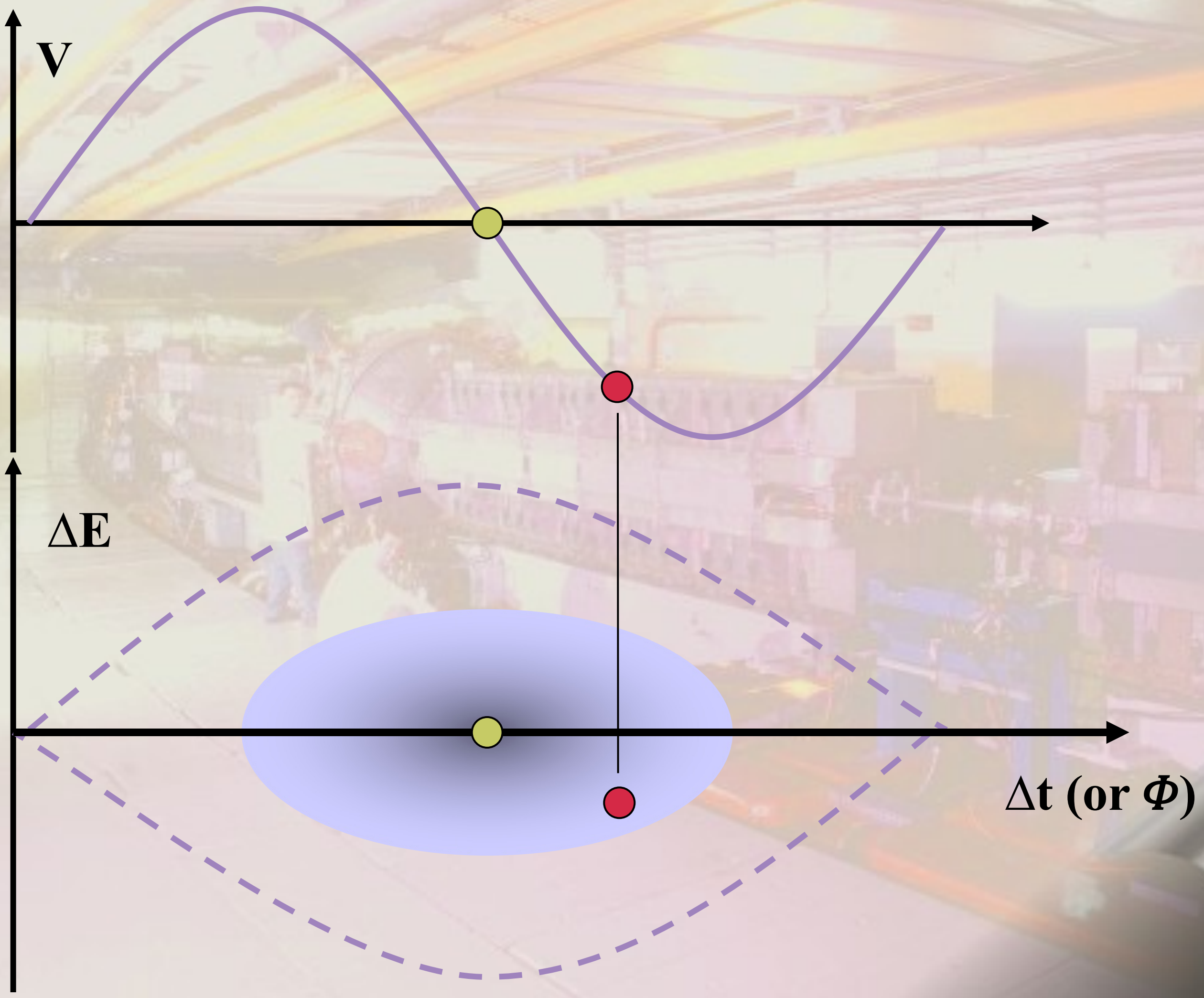
The motion in the bucket (4)



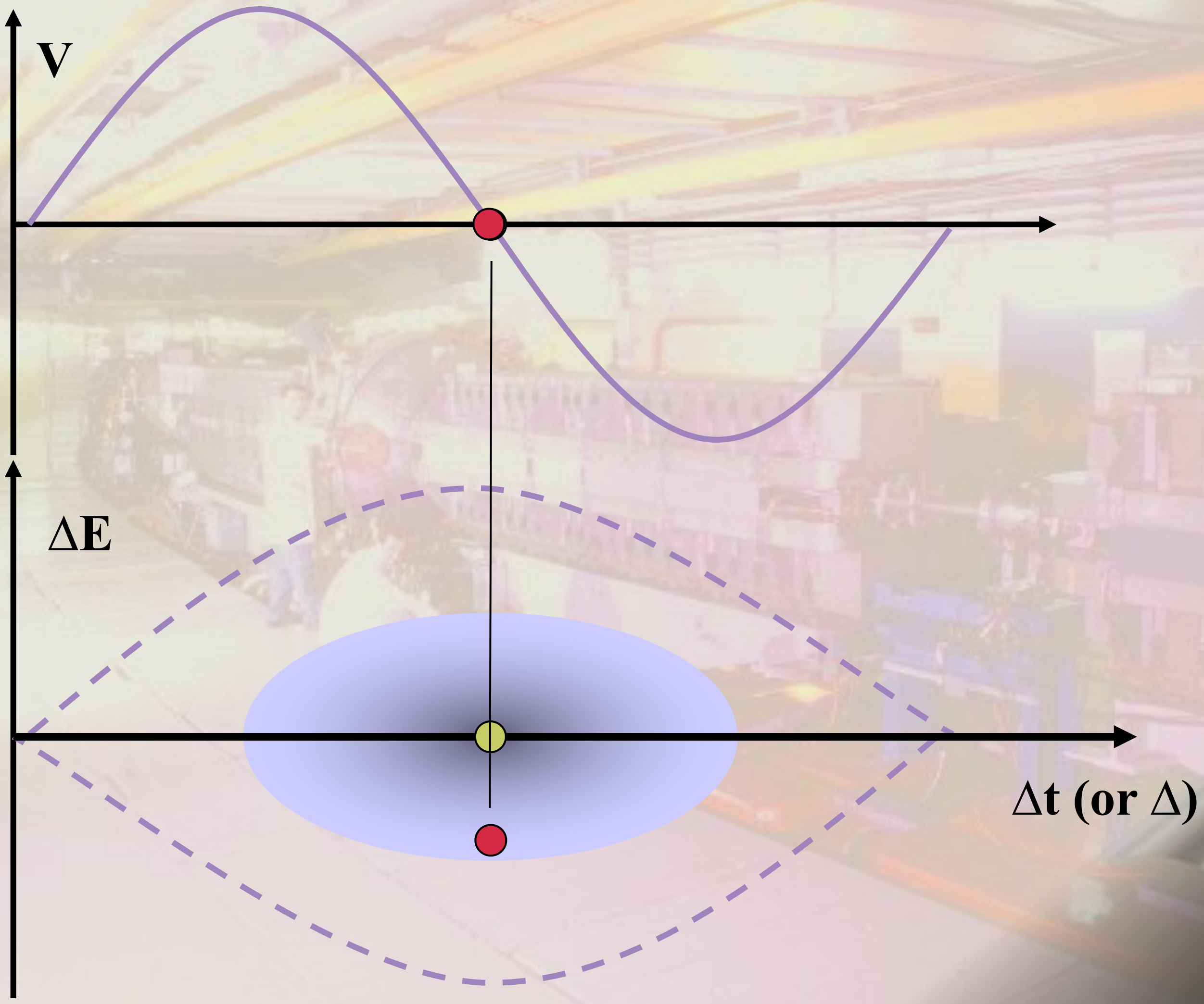
The motion in the bucket (5)



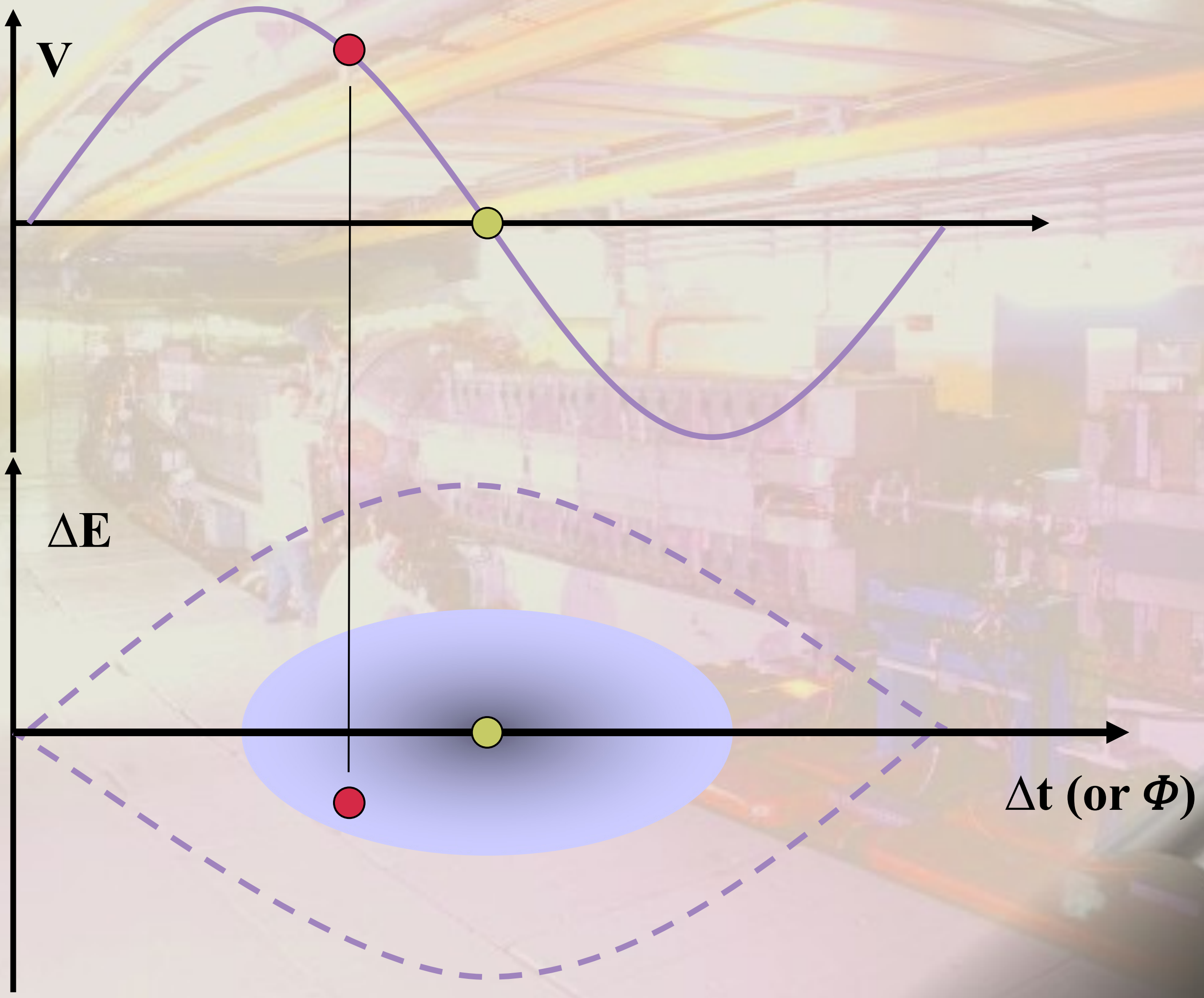
The motion in the bucket (6)



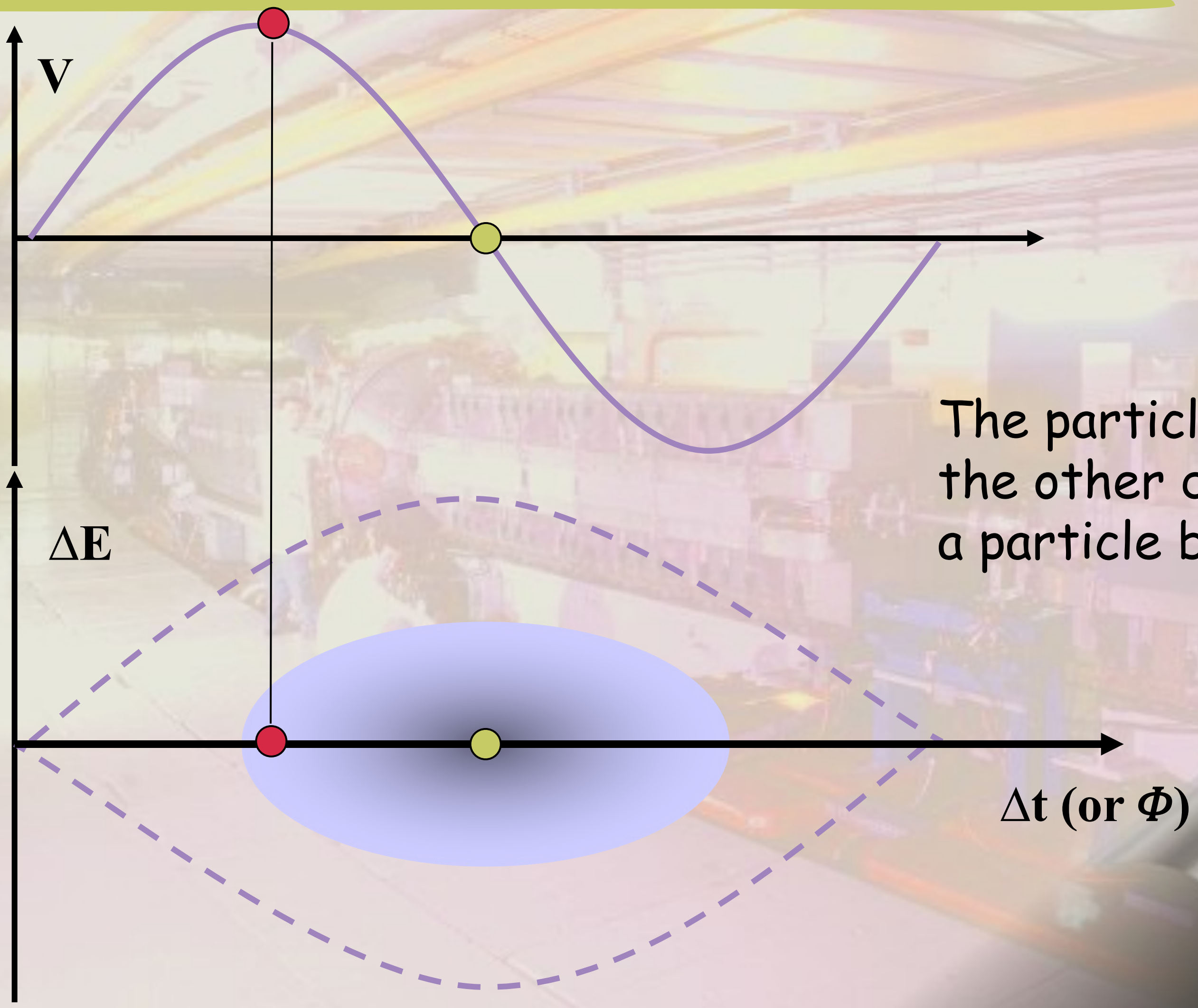
The motion in the bucket (7)



The motion in the bucket (8)

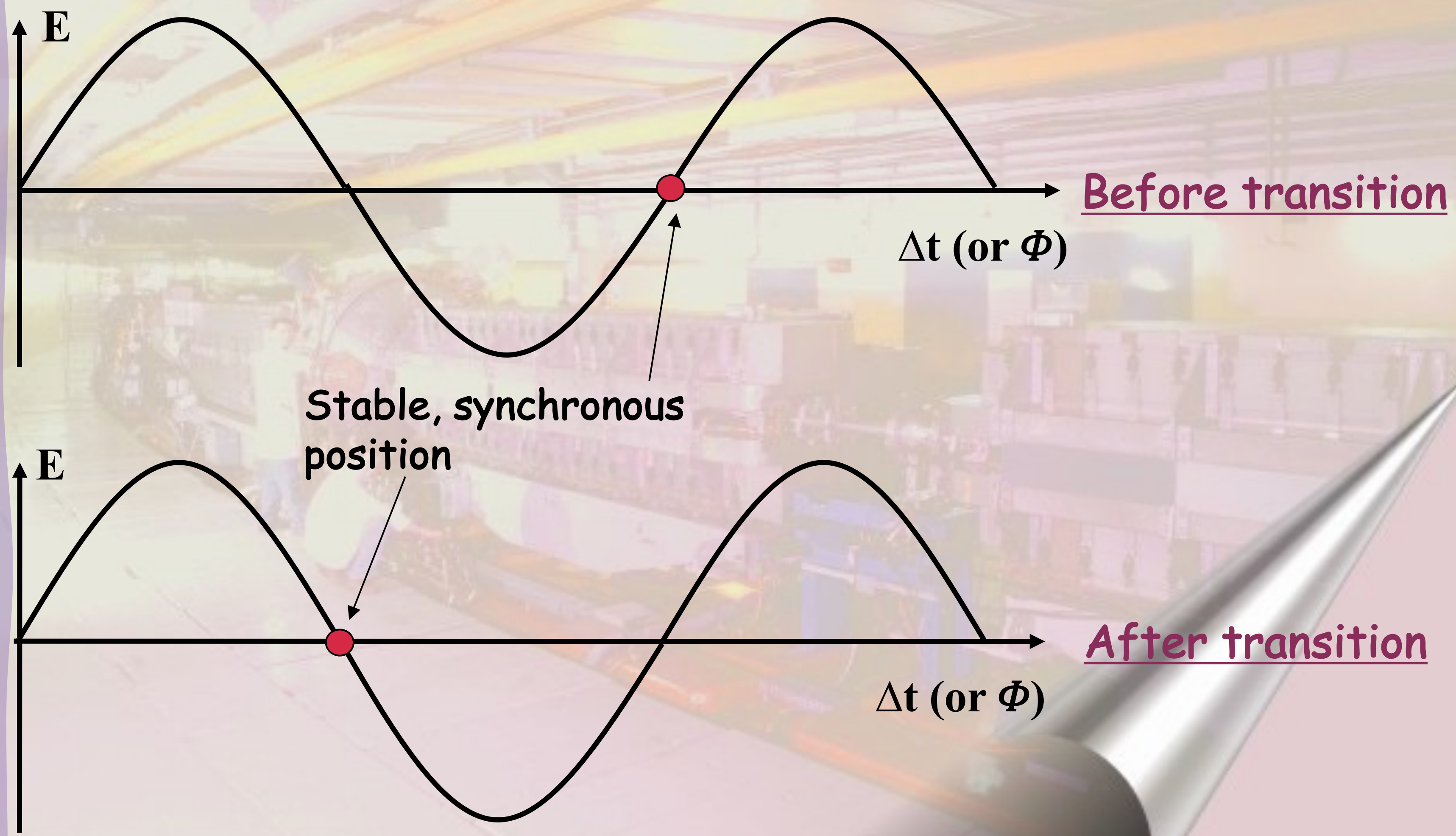


The motion in the bucket (9)



The particle now turns in the other direction w.r.t. a particle below transition

Before and After Transition



Transition crossing in the PS

- Transition in the PS occurs around 6 GeV/c
  - Injection happens at 2.12 GeV/c
  - Ejection can be done at 3.5 GeV/c up to 26 GeV/c

- Therefore the particles in the PS must nearly always cross transition.
- The beam must stay bunched

- Therefore the phase of the RF must "**jump**" by π at transition

Harmonic number (1)

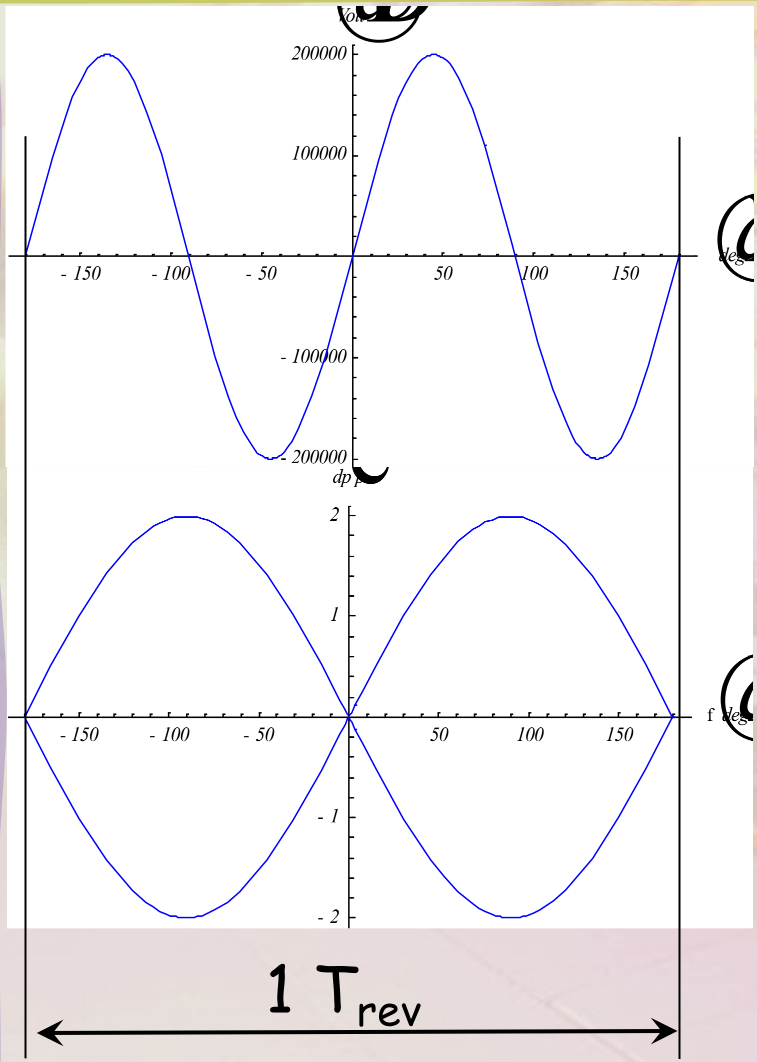
- Until now we have applied an oscillating voltage with a frequency equal to the revolution frequency.

$$F_{rf} = F_{rev}$$

- What will happen when F_{rf} is a multiple of f_{rev} ???

$$F_{rf} = h \times F_{rev}$$

Harmonic number (2)



$$F_{rf} = h \times F_{rev}$$

Variable for $\beta < 1$

Frequency of cavity voltage

Harmonic number

- Then we will have **h buckets**

Frequency of the synchrotron oscillation (1)

- On each turn the phase, Φ, of a particle w.r.t. the RF waveform changes due to the synchrotron oscillations.

$$\frac{d\phi}{dt} = 2\pi h \Delta f_{rev}$$

Harmonic number

Change in revolution frequency

- We know that $\frac{df_{rev}}{f_{rev}} = -\eta \frac{dE}{E}$

- Combining this with the above $\therefore \frac{d\phi}{dt} = \frac{-2\pi h \eta}{E} \cdot dE \cdot f_{rev}$

- This can be written as

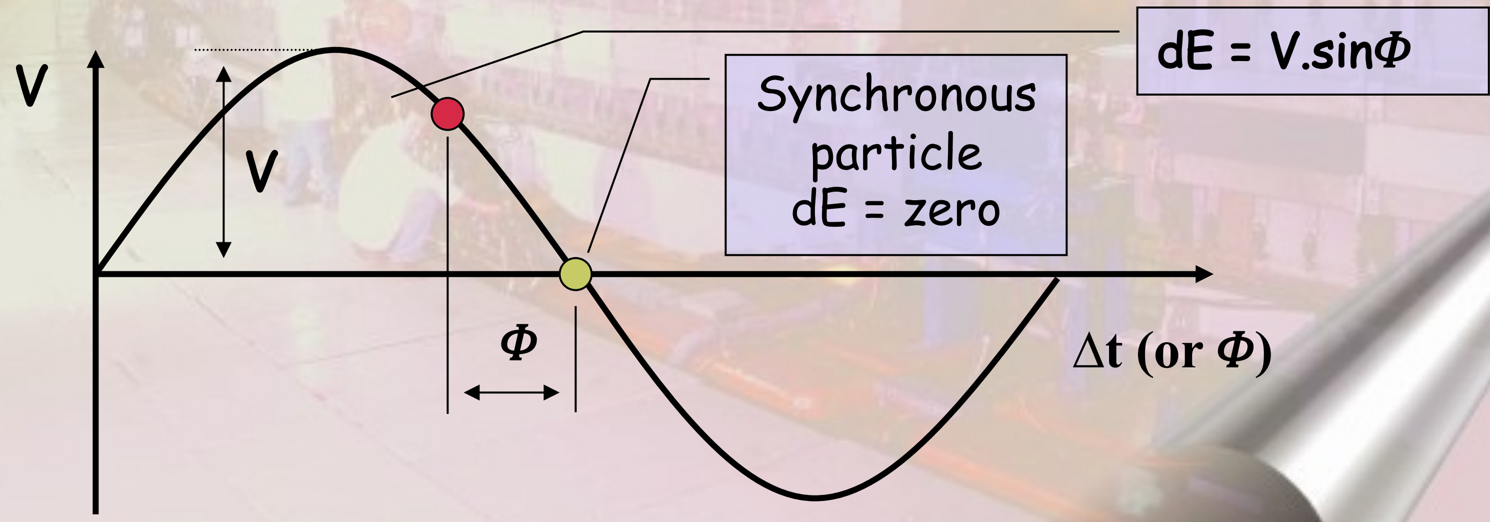
$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h \eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

Change of energy as a function of time

Frequency of the synchrotron oscillation (2)

- So, we have: $$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

- Where dE is just the energy gain or loss due to the RF system during each turn



Frequency of the synchrotron oscillation (3)

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$ and $$dE = V\sin\phi$$ → $$\frac{dE}{dt} = f_{rev} V \sin\phi$$

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev}{}^2 \cdot V.\sin\phi$$

- If Φ is small then **sinΦ=Φ**

$$\frac{d^2\phi}{dt^2} + \left(\frac{2\pi h\eta}{E} \cdot f_{rev}{}^2 \cdot V\right)\phi = 0$$

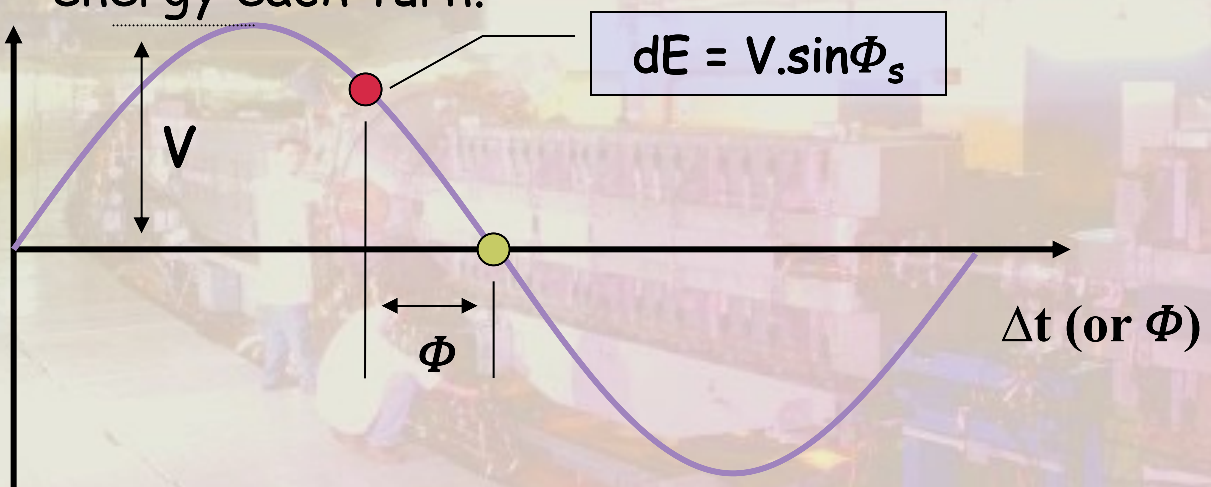
- This is a SHM where the synchrotron oscillation frequency is given by:

$$\left(\sqrt{\frac{2\pi h\eta V}{E}}\right) \cdot f_{rev}$$

Synchrotron tune Qs

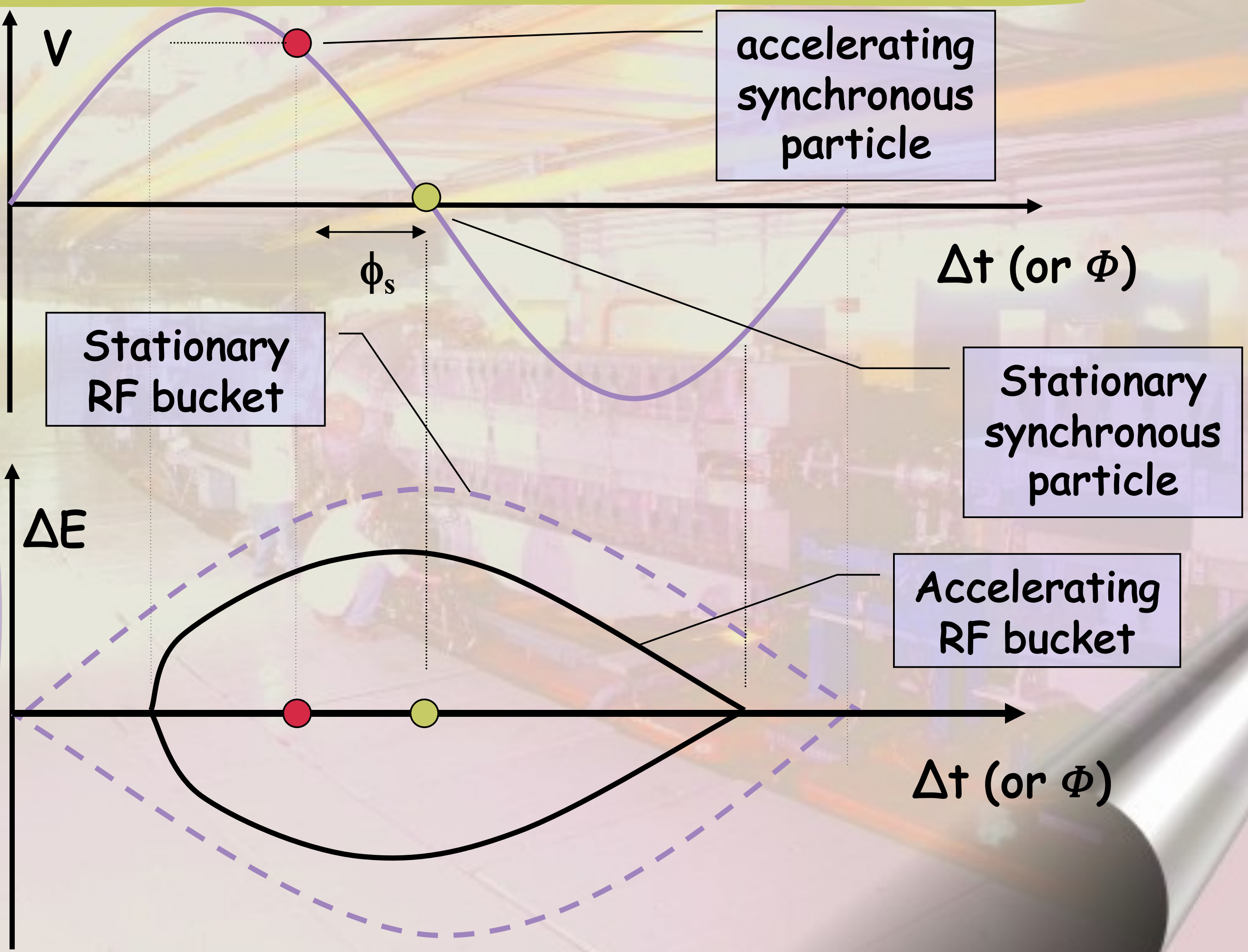
Acceleration

- Increase the magnetic field slightly on each turn.
- The particles will follow a shorter orbit. ($F_{rev} < F_{synch}$)
- Beyond transition, early arrival in the cavity causes a gain in energy each turn.



- We change the phase of the cavity such that the new synchronous particle is at Φ_s and therefore always sees an accelerating voltage
- $V_s = V\sin\Phi_s = V\Gamma$ = energy gain/turn = dE

Acceleration & RF bucket shape (1)

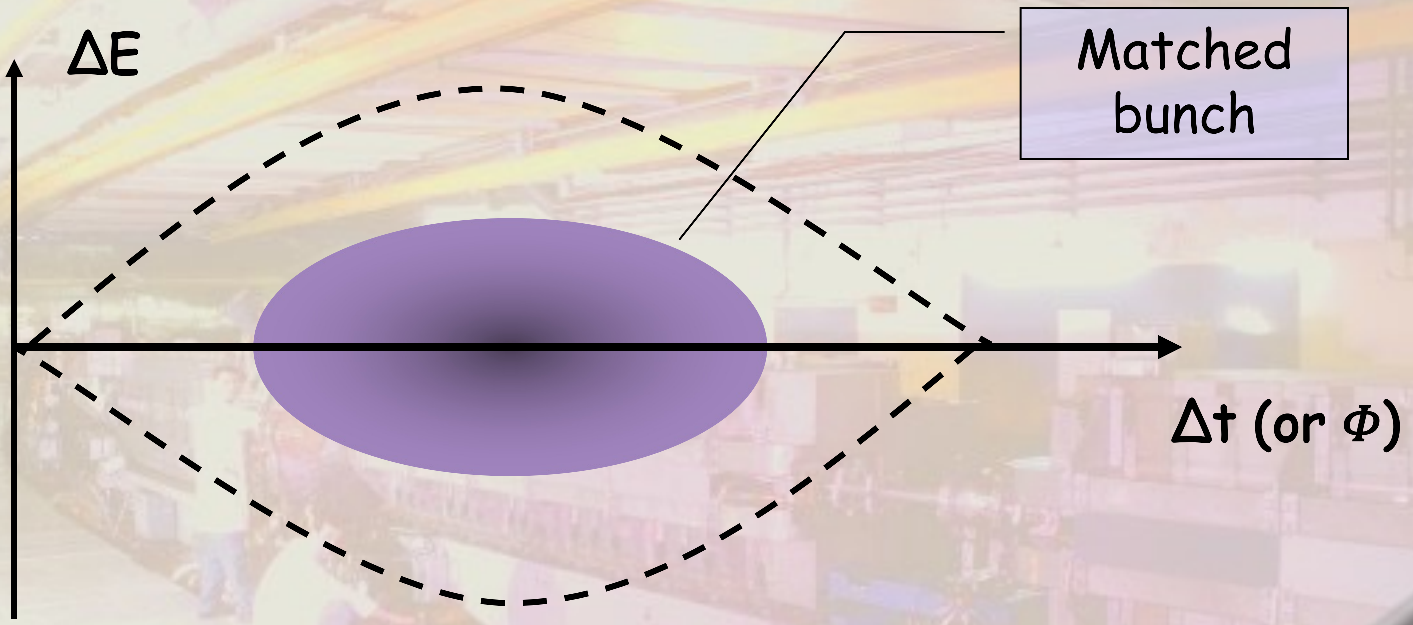


Acceleration & RF bucket shape (2)

- The modification of the RF bucket reduces the acceptance
- The faster we accelerate (increasing sin Φ_s) the smaller the acceptance
- Faster acceleration also modifies the synchrotron tune.
- For a stationary bucket (Φs = 0) we had: $$\left(\sqrt{\frac{2\pi h\eta}{E}}\right)\cdot f_{rev}$$
- For a moving bucket ($\Phi s \neq 0$) this becomes:

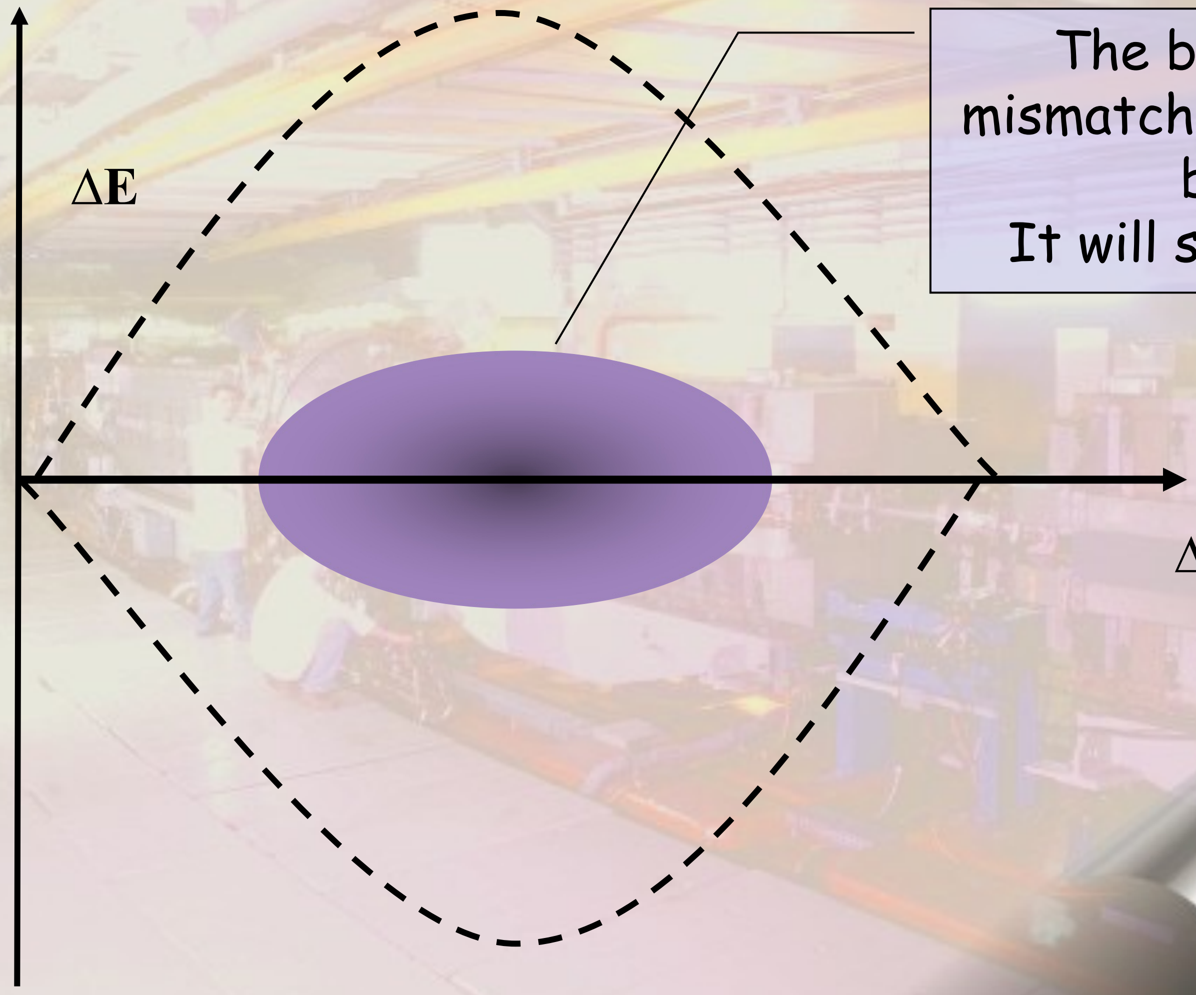
$$\left(\sqrt{\frac{2\pi h\eta}{E}}\right)\cdot f_{rev}\cos\phi_s$$

Non-adiabatic change (1)



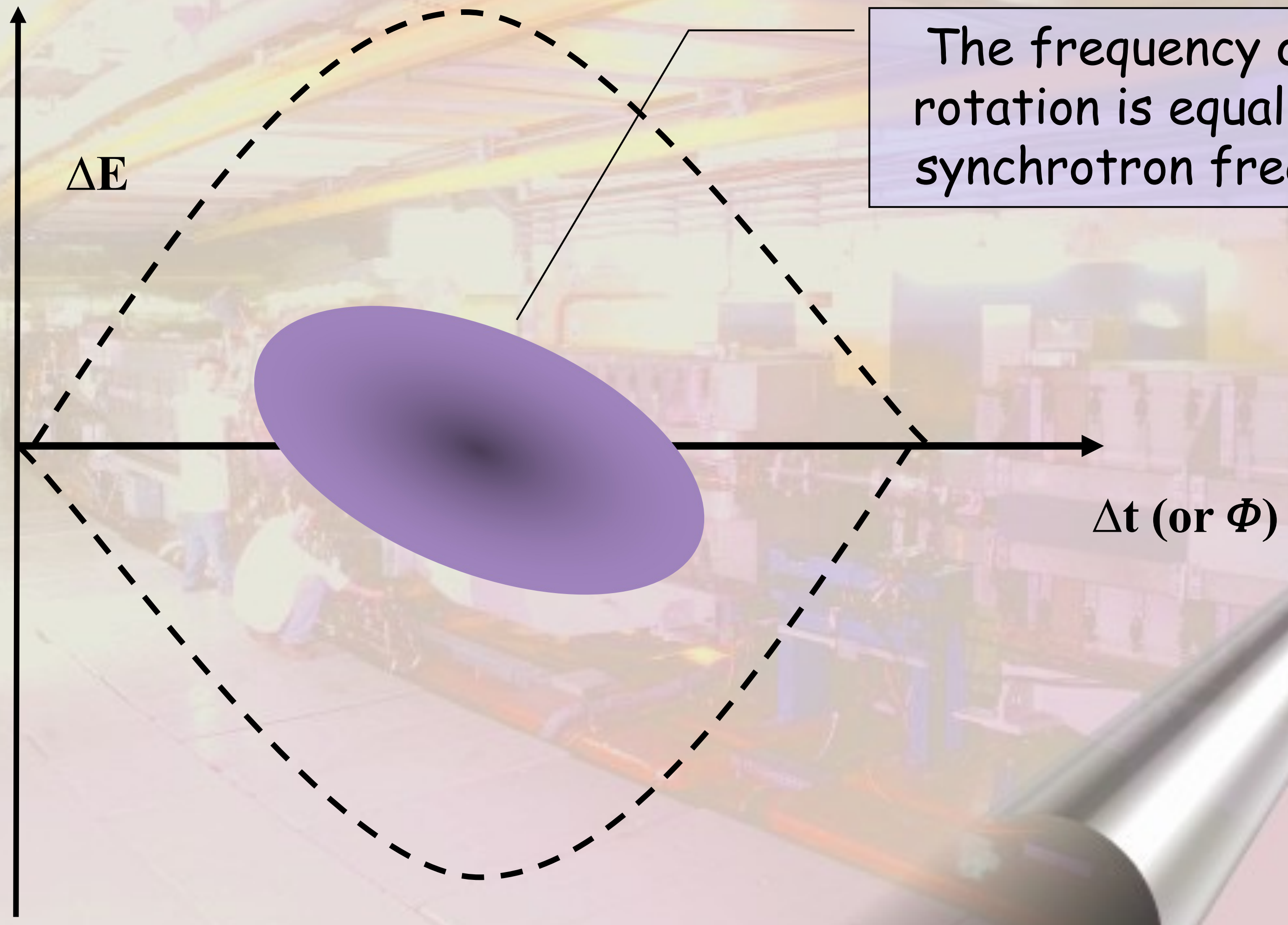
- What will happen when we increase the voltage rapidly ?

Non-adiabatic change (2)



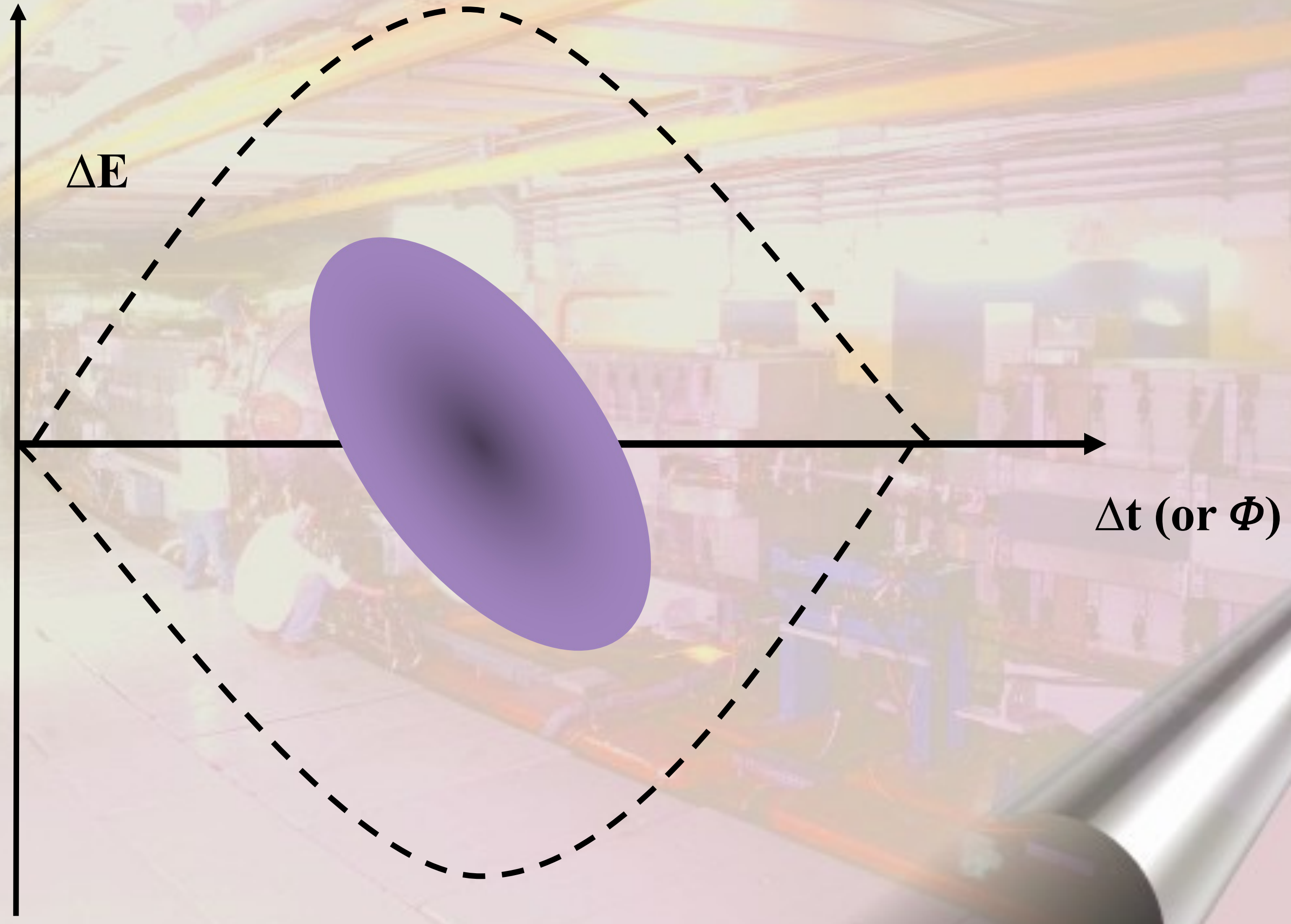
The bunch is now mismatched w.r.t. to the bucket
It will start rotating

Non-adiabatic change (3)

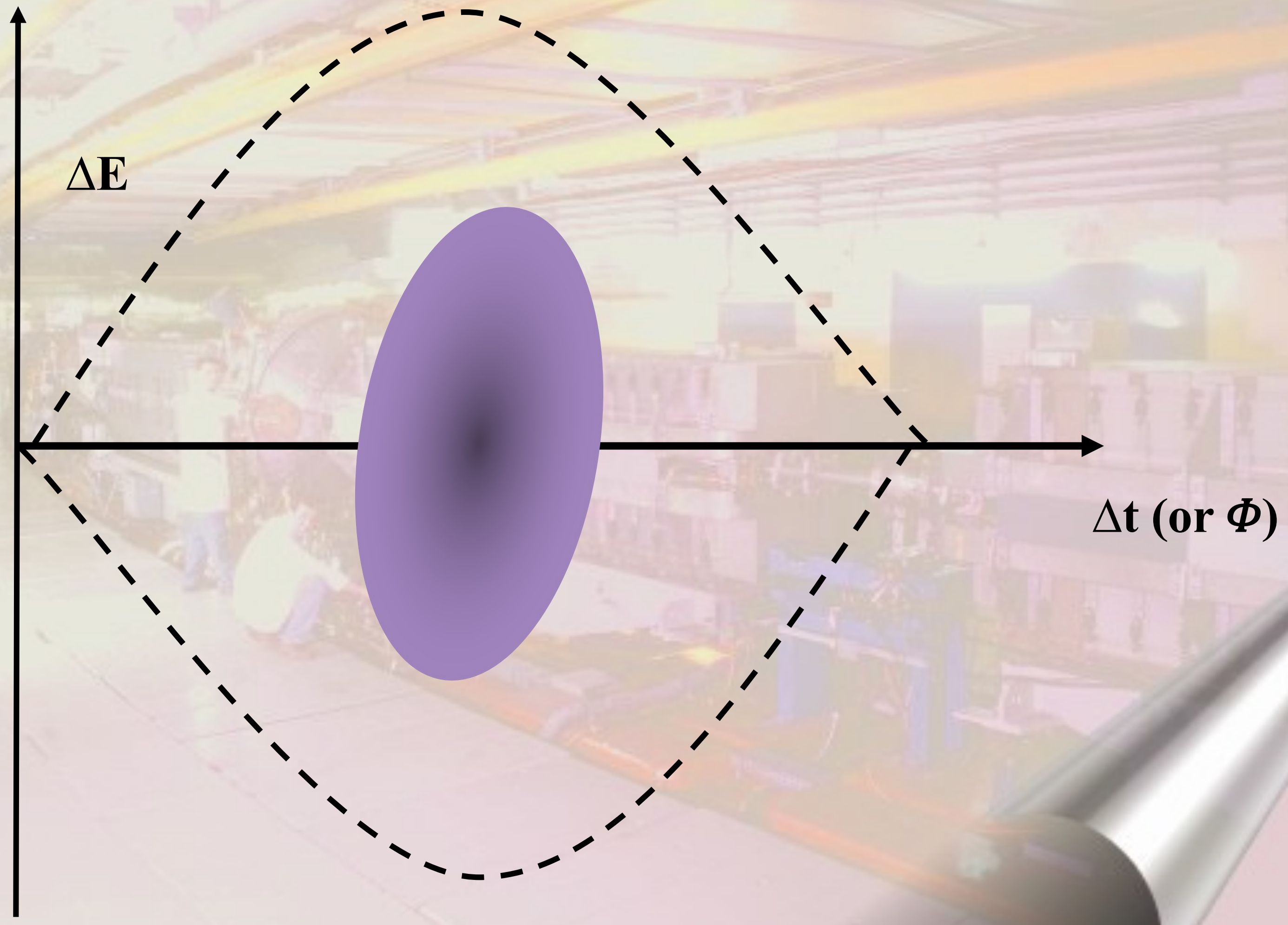


The frequency of this rotation is equal to the synchrotron frequency

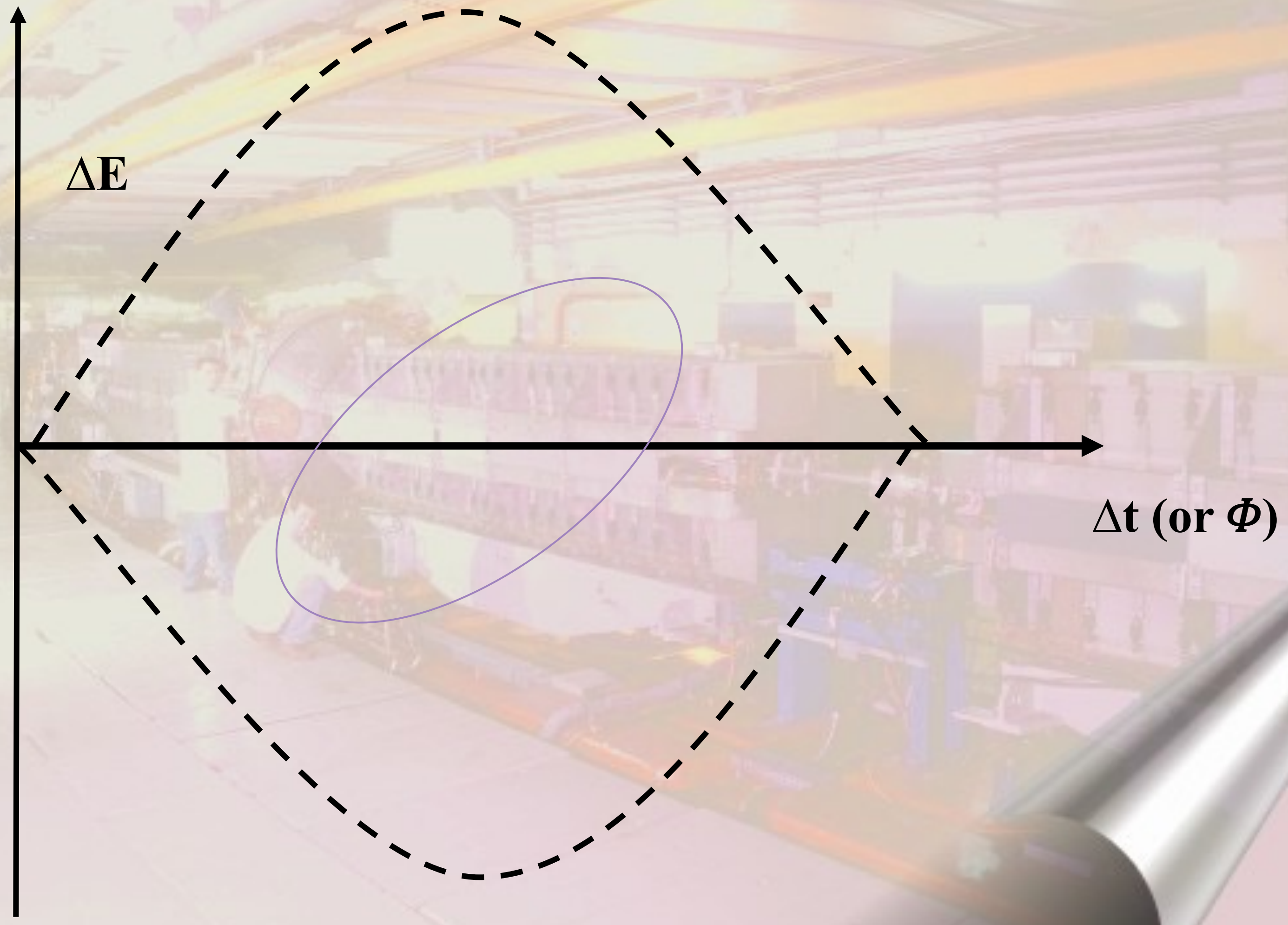
Non-adiabatic change (4)



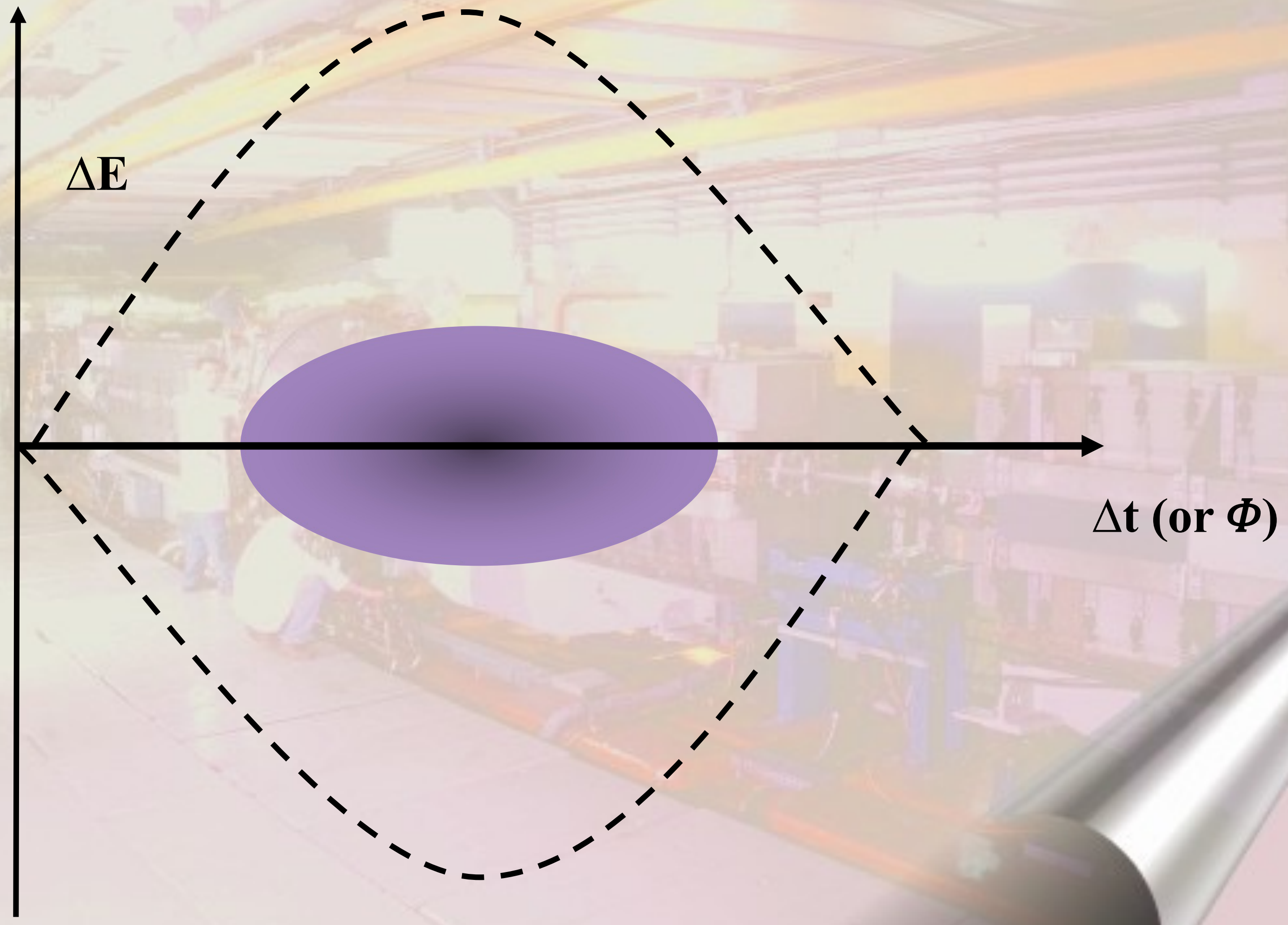
Non-adiabatic change (5)



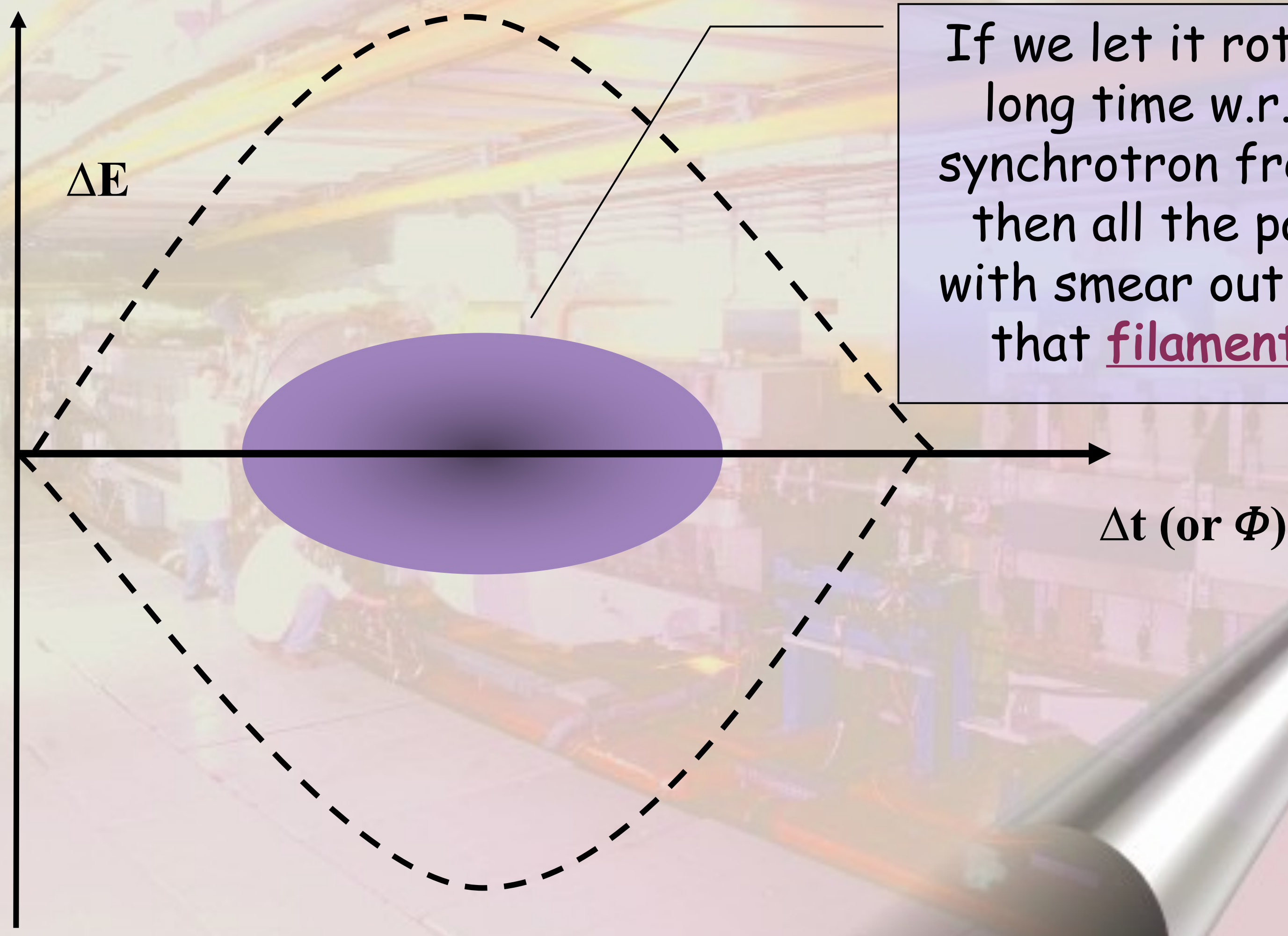
Non-adiabatic change (6)



Non-adiabatic change (7)

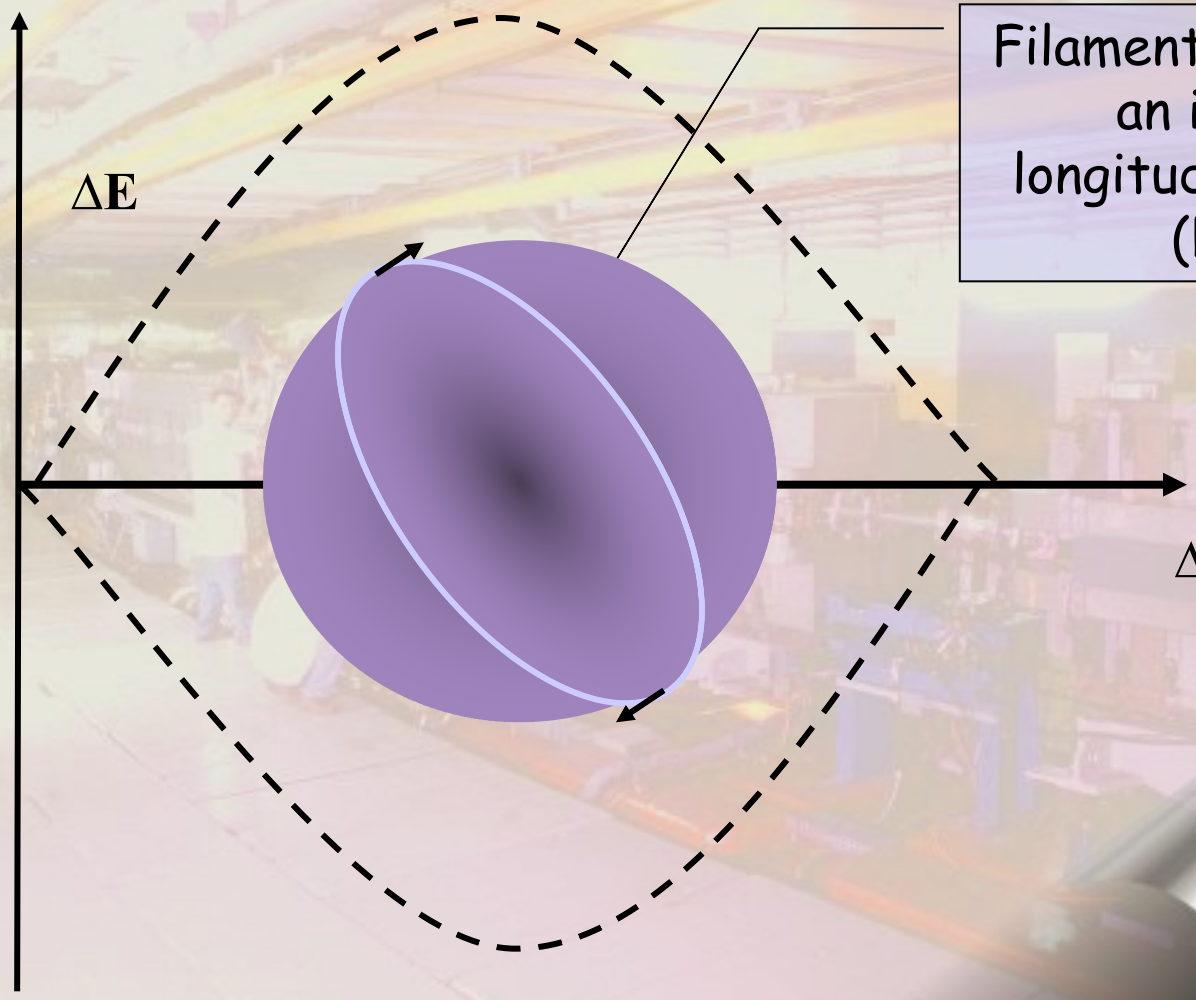


Non-adiabatic change (8)



If we let it rotate for long time w.r.t. the synchrotron frequency then all the particle with smear out, we call that <u>filamentation</u>

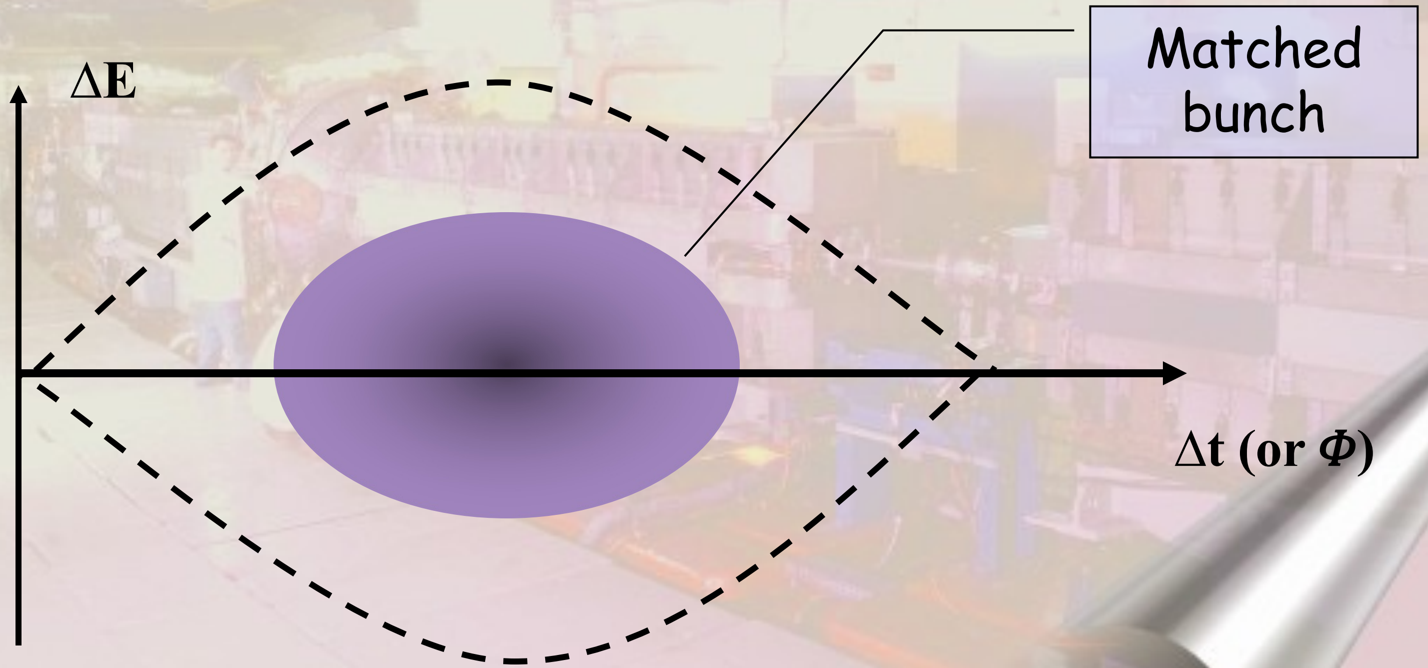
Non-adiabatic change (9)



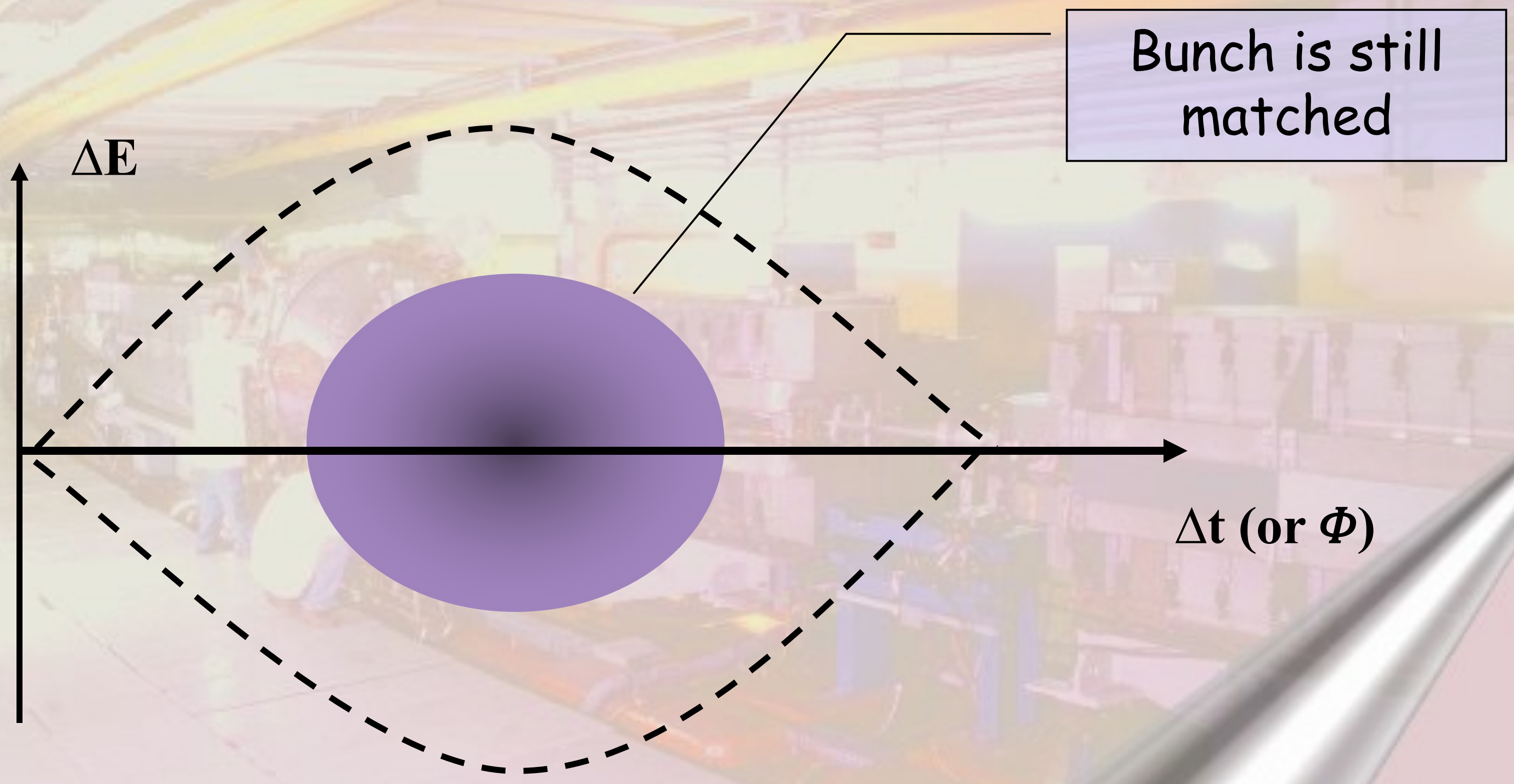
Filamentation will cause an increase in longitudinal emittance (blow-up)

Adiabatic change (1)

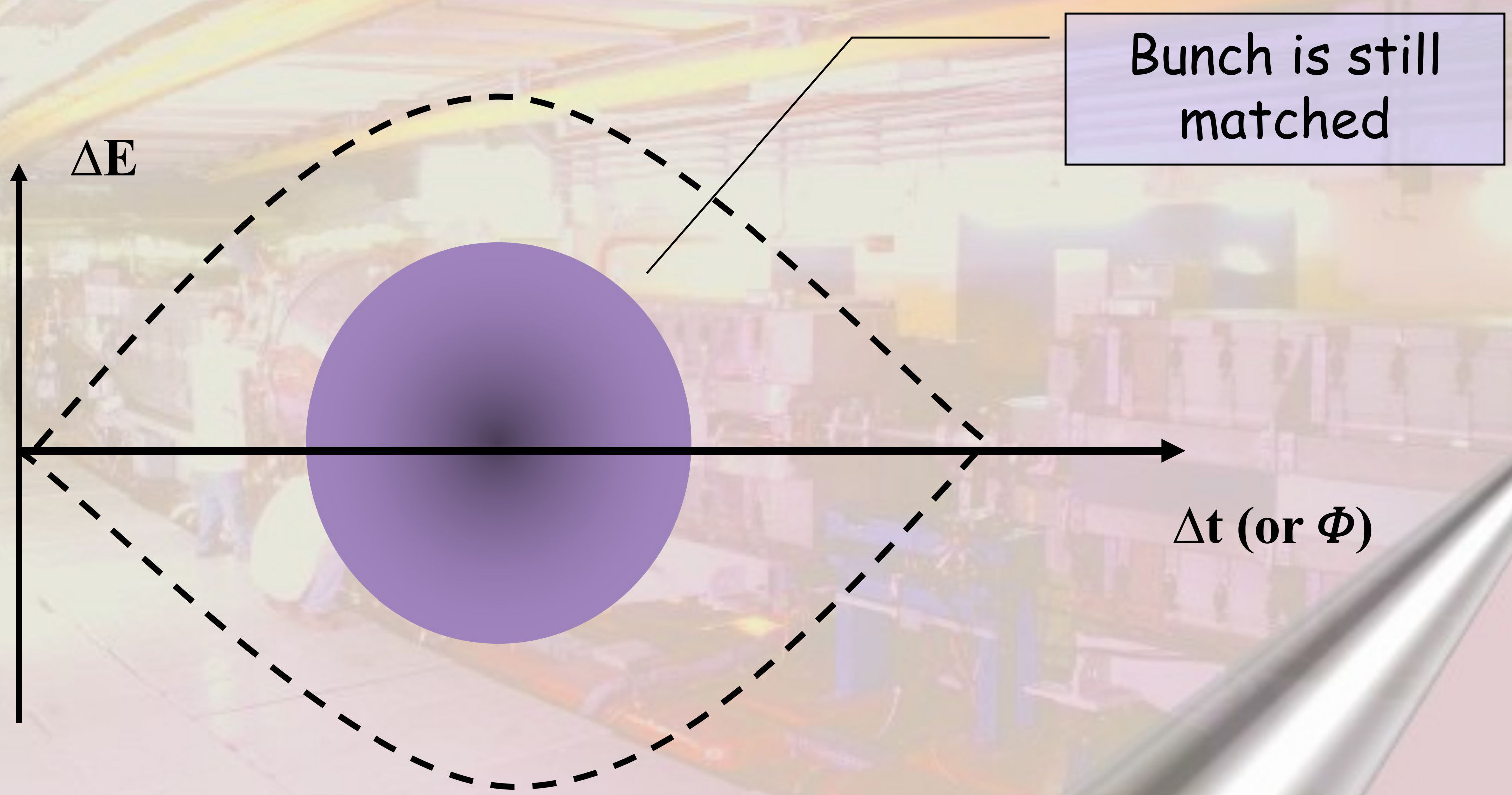
- To avoid this filamentation we have to change slowly w.r.t. the synchrotron frequency.
- This is called 'Adiabatic' change.



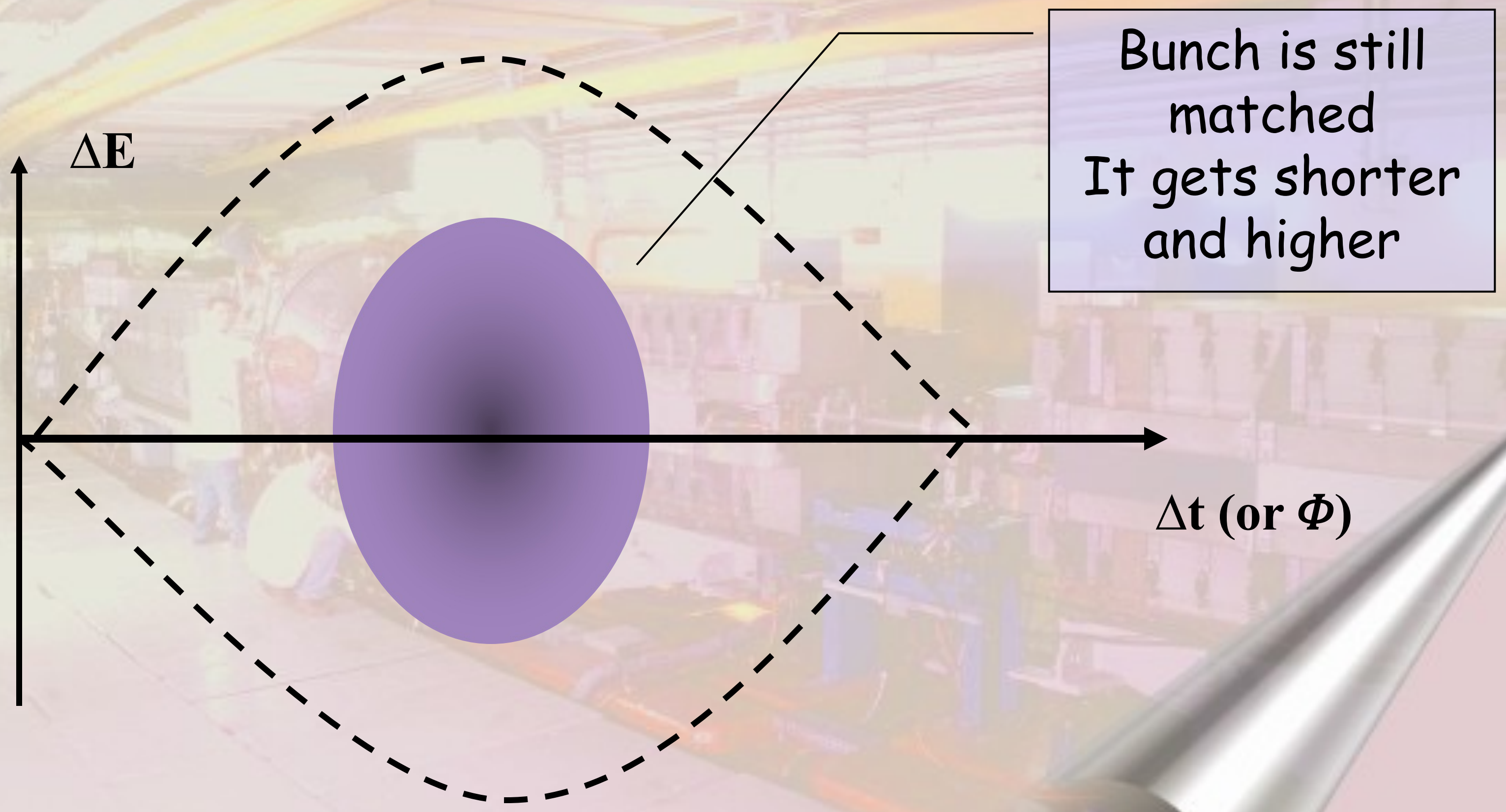
Adiabatic change (2)



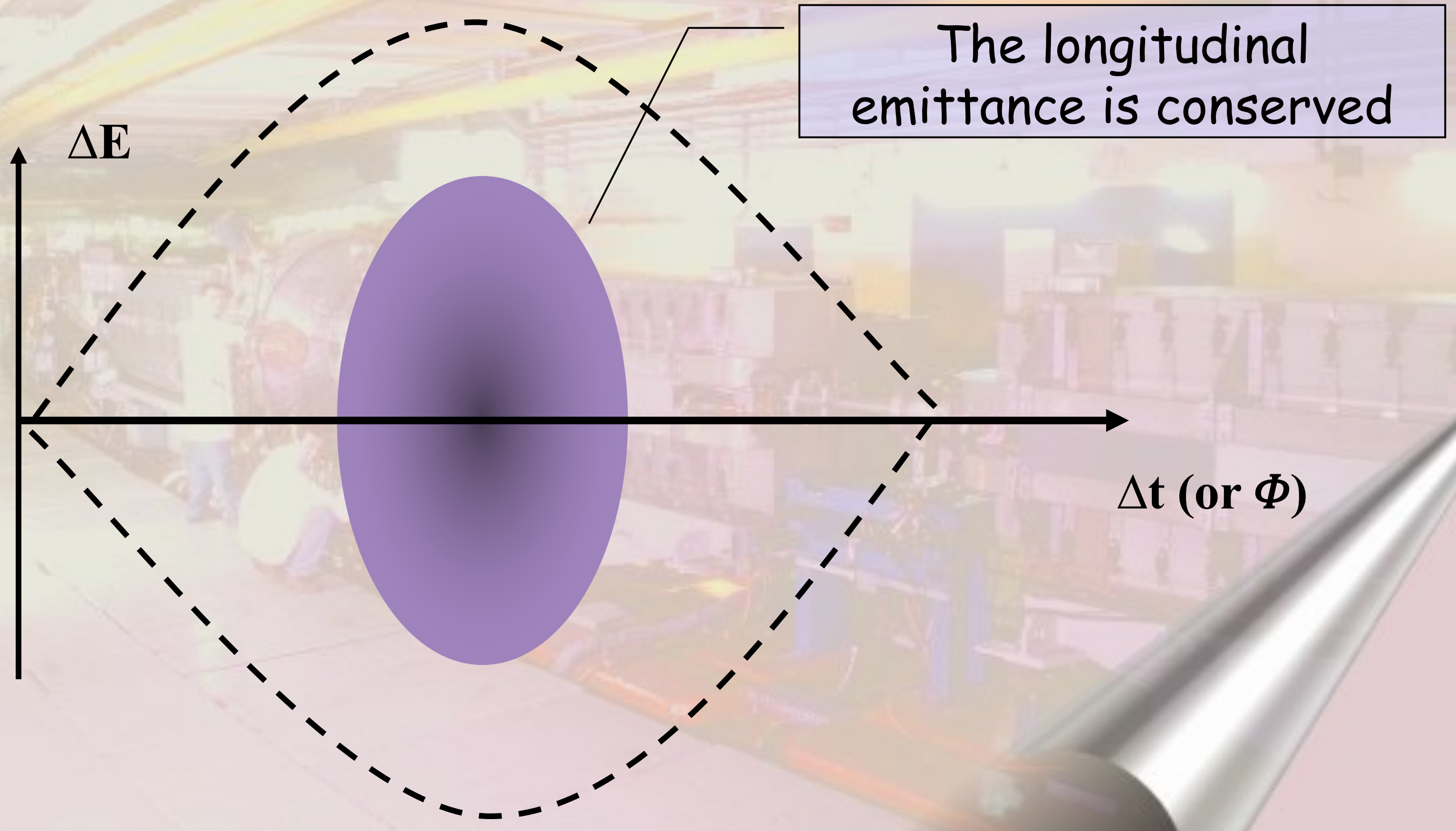
Adiabatic change (3)



Adiabatic change (4)



Adiabatic change (5)



Questions….,Remarks…?