AXEL-2023

Introduction to Particle Accelerators

Transverse optics 1:

- ✓ *Relativity, Energy & Units*
- ✓ *Accelerator co-ordinates*
- ✓ *Magnets and their configurations*
- ✓ *Hill's equation*

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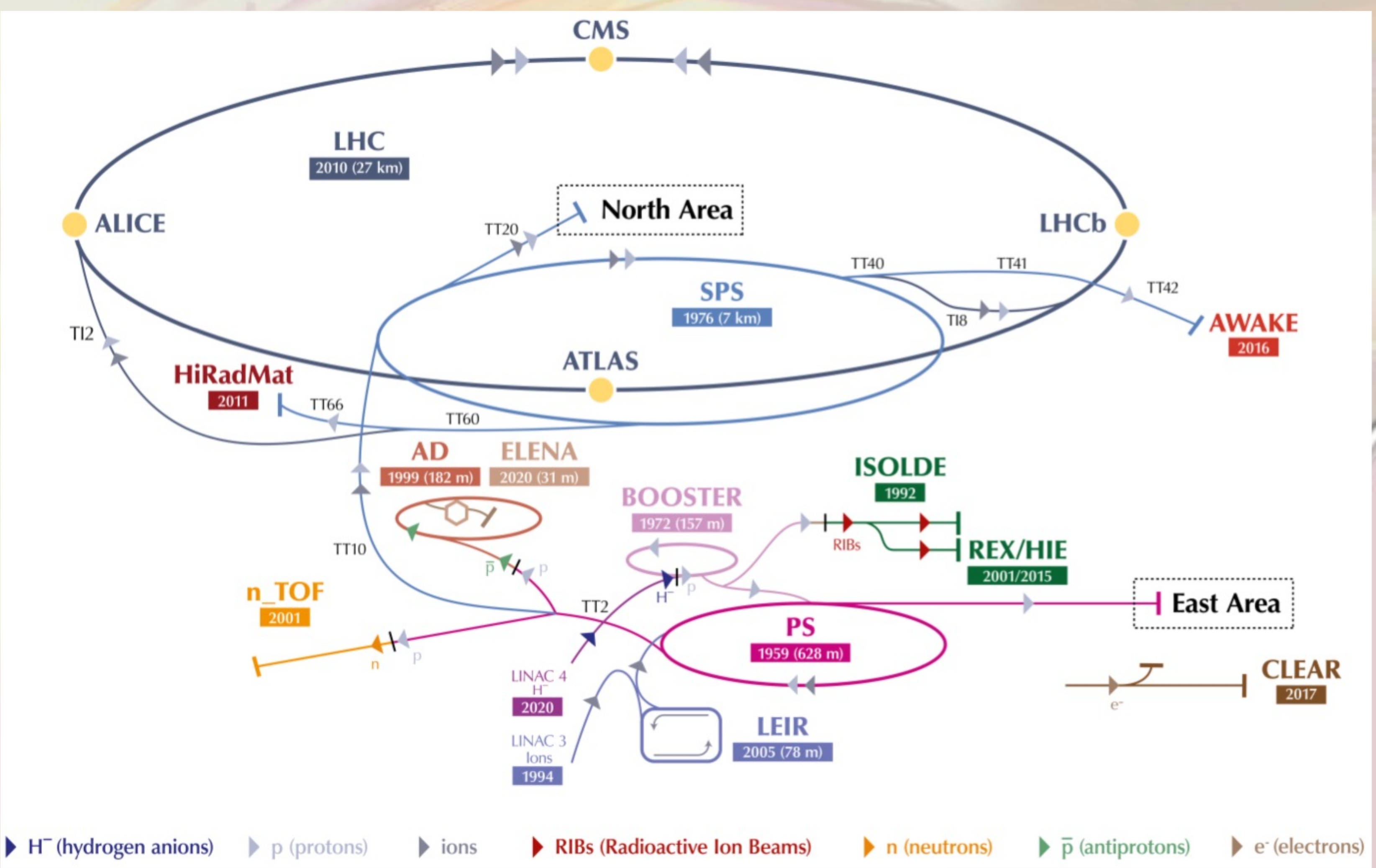
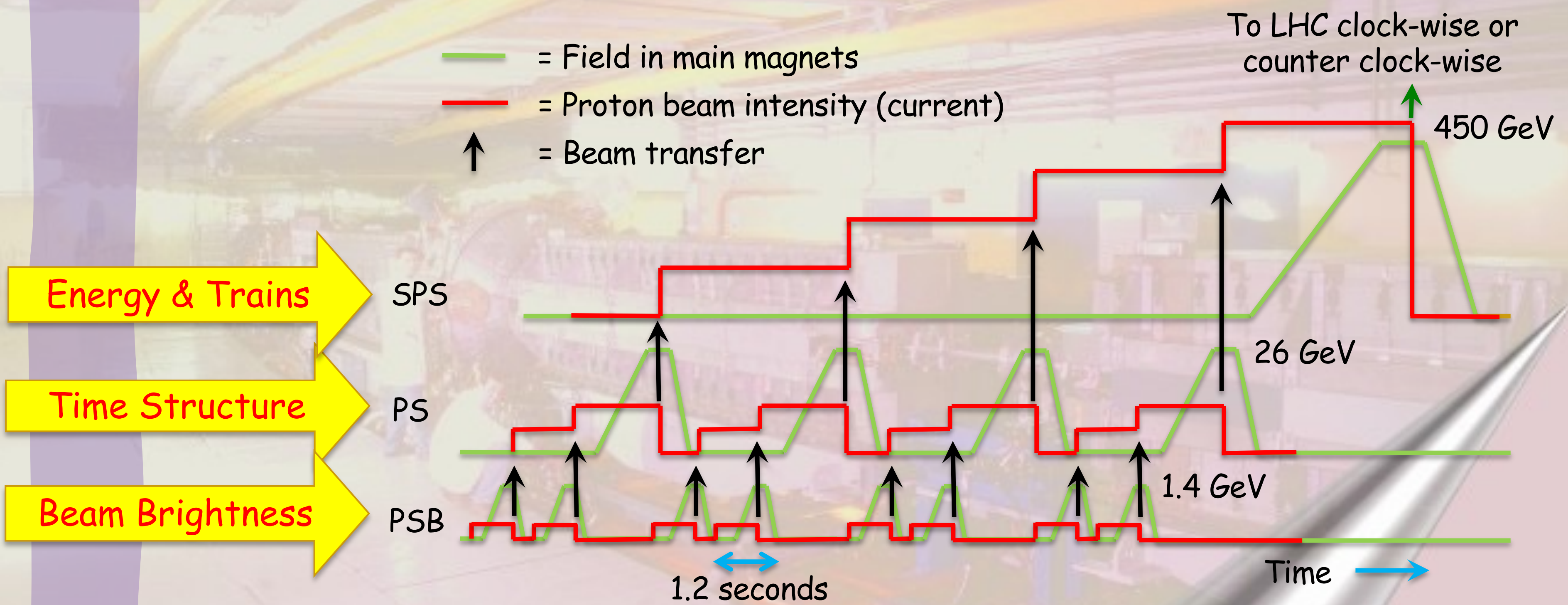
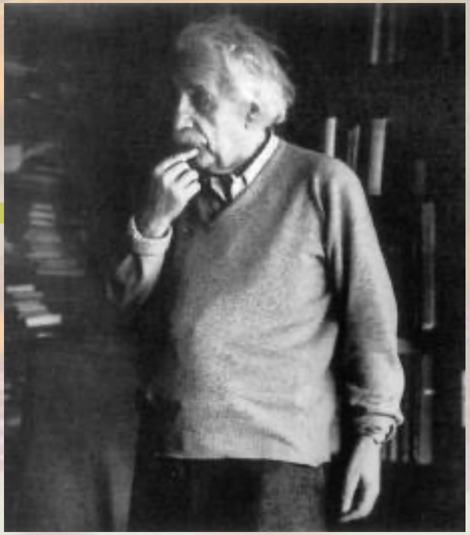
CERN Accelerators

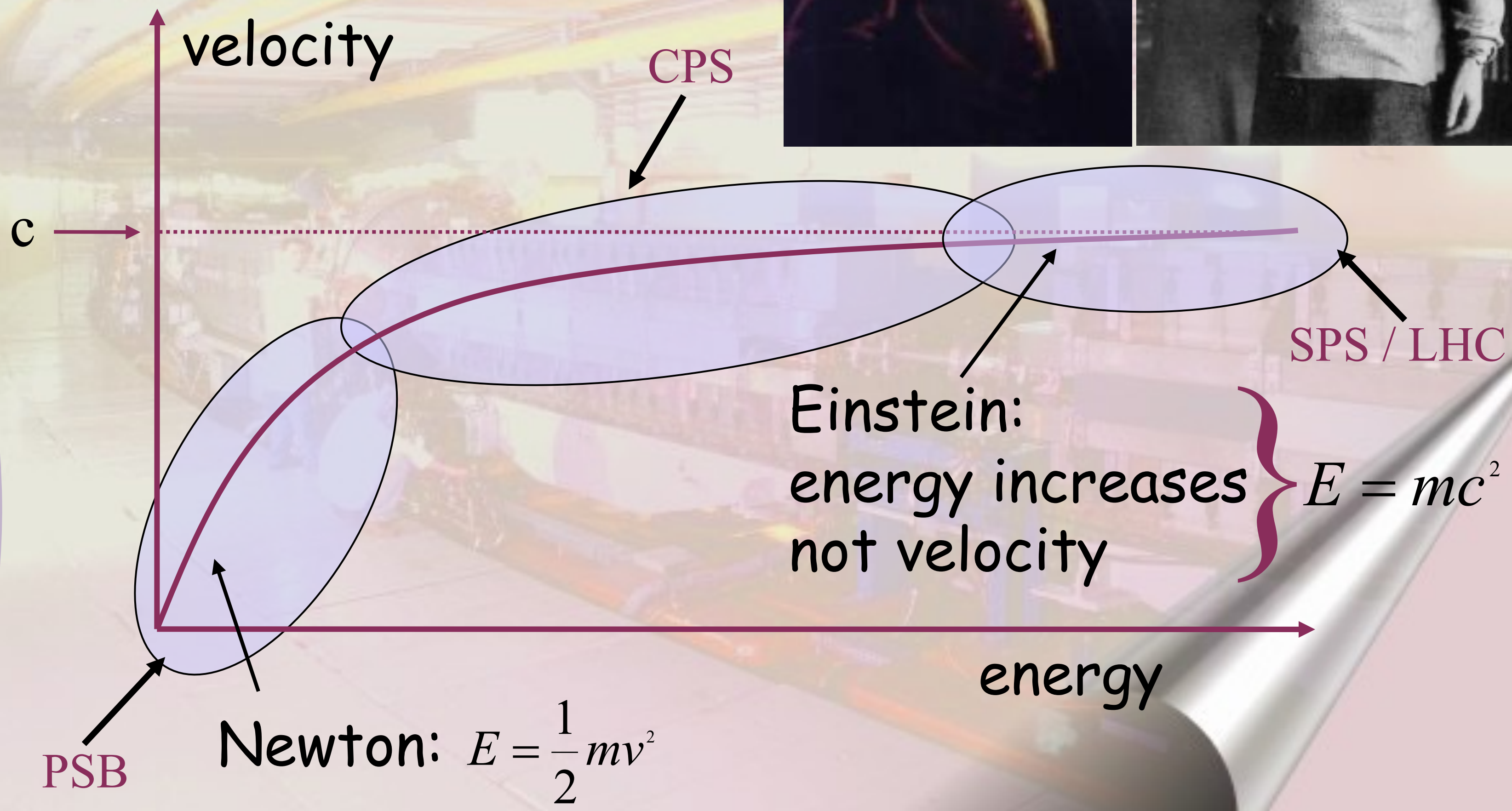


Classical Filling of the LHC with Protons



Relativity




velocity
c
CPS
SPS / LHC
Einstein:
energy increases
not velocity
$E = mc^2$
energy
PSB
Newton: $E = \frac{1}{2}mv^2$

Energy & Momentum

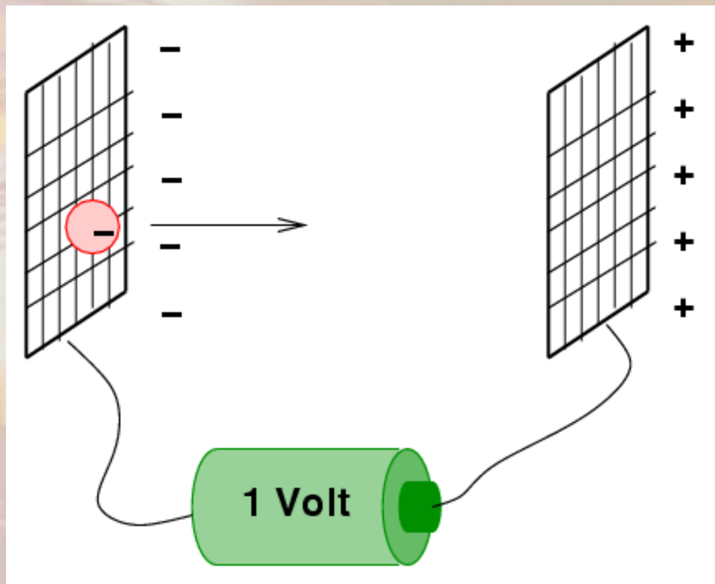
- Einstein's relativity formula: $E = mc^2$
- For a mass at rest this will be: $E_0 = m_0 c^2$

  Rest mass

  Rest energy
- Define: $\gamma = \frac{E}{E_0}$ As being the ratio between the total energy and the rest energy
- Then the mass of a moving particle is: $m = \gamma m_0$
- Define: $\beta = \frac{v}{c}$,then we can write: $\beta = \frac{mvc}{mc^2}$
- $p = mv$,which is always true and gives: $\beta = \frac{pc}{E}$ or $p = \frac{E\beta}{c}$

The Units we use for Energy

- The energy acquired by an electron in a potential of 1 Volts is defined as being 1 eV



- The unit eV is too small to be used today, we use:

  1 KeV = 10^3, MeV = 10^6, GeV = 10^9, TeV = 10^{12}

Energy: eV versus Joules

- The unit most commonly used for **Energy** is **Joules [J]**
- In accelerator and particle physics we talk about **eV**…!?
- The **energy** acquired by an **electron** in a potential of **1 Volt** is defined as being **1 eV**
- **1 eV** is **1 elementary charge** 'pushed' by **1 Volt**.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$$

Units: Energy & Momentum (2)

- However:

Momentum

$$p = \frac{E\beta}{c}$$

Energy

- Therefore the **units** for **momentum** are GeV/c…etc.

Attention:

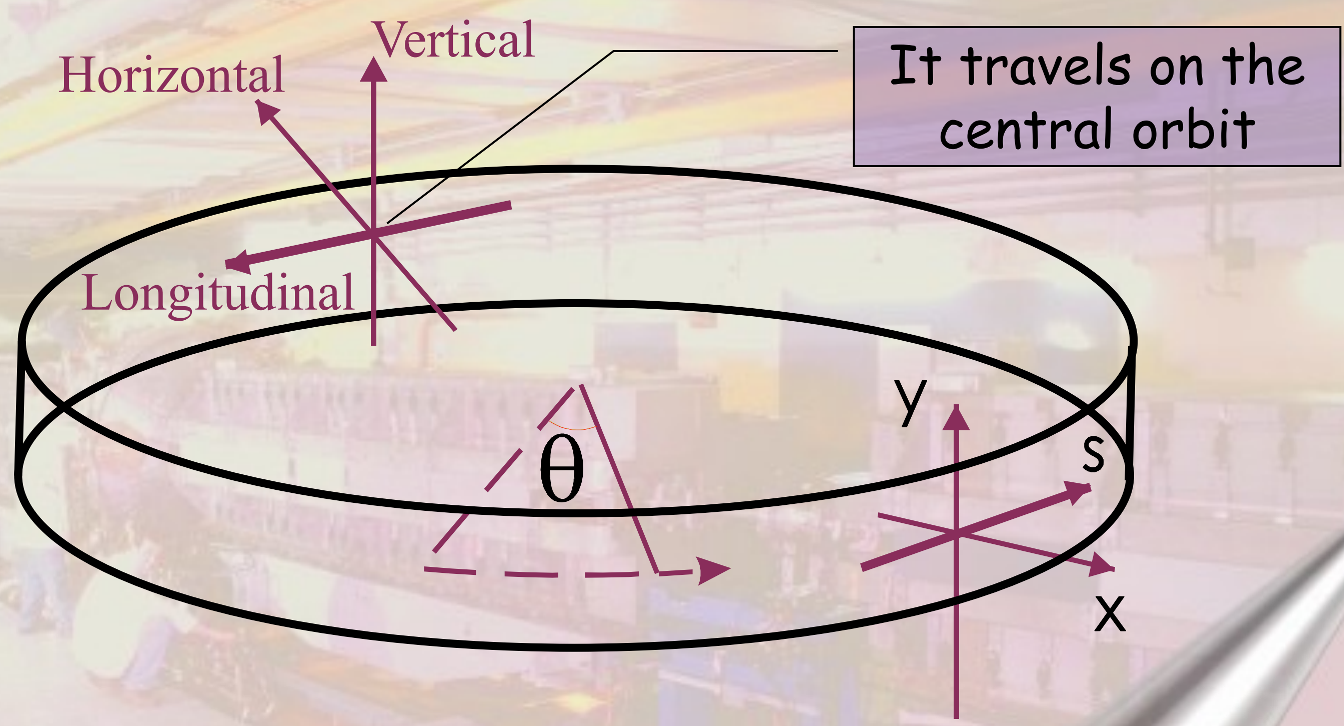
when β=1 energy and momentum are equal

when β<1 the energy and momentum are not equal

Units: Example PS injection

- ✓ Kinetic energy at injection $E_{kinetic}$ = 1.4 GeV
- ✓ Proton rest energy E_0=938.27 MeV
- ✓ The total energy is then: $E = E_{kinetic} + E_0$ =2.34 GeV
- ✓ We know that $\gamma = \frac{E}{E_0}$,which gives γ = 2.4921
- ✓ We can derive $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$,which gives β = 0.91597
- ✓ Using $p = \frac{E\beta}{c}$ we get p = 2.14 GeV/c
- ✓ In this case: Energy ≠ Momentum

Accelerator co-ordinates



✓ We can speak about a:

<u>**Rotating Cartesian Co-ordinate System**</u>

Magnetic rigidity

- ✓ The force evB on a charged particle moving with velocity v in a dipole field of strength B is equal to its mass multiplied by its acceleration towards the centre of its circular path.
- ✓ As a formula this is: $F = evB = \frac{mv^2}{\rho}$

Radius of curvature

Like for a stone attached to a rotating rope

- ✓ Which can be written as: $B\rho = \frac{mv}{e} = \frac{p}{e}$

Momentum p=mv

- ✓ Bρ is called the magnetic rigidity, and if we put in all the correct units we get:

Bρ = 33.356·p [KG·m] = 3.3356·p [T·m] (if p is in [GeV/c])

Some LHC figures

- ✓ LHC circumference = 26658.883 m
  - ✓ Therefore the radius r = 4242.9 m
- ✓ There are 1232 main dipoles to make 360˚
  - ✓ This means that each dipole deviates the beam by only 0.29 ˚
- ✓ The dipole length = 14.3 m
  - ✓ The total dipole length is thus 17617.6 m, which occupies 66.09 % of the total circumference
- ✓ The bending radius ρ is therefore
  - ✓ $\rho = 0.6609 \times 4242.9$ m → $\rho = 2804$ m

Dipole magnet

- ✓ A dipole with a uniform dipolar field deviates a particle by an angle θ.
- ✓ The deviation angle θ depends on the length L and the magnetic field B.
- ✓ The angle θ can be calculated:

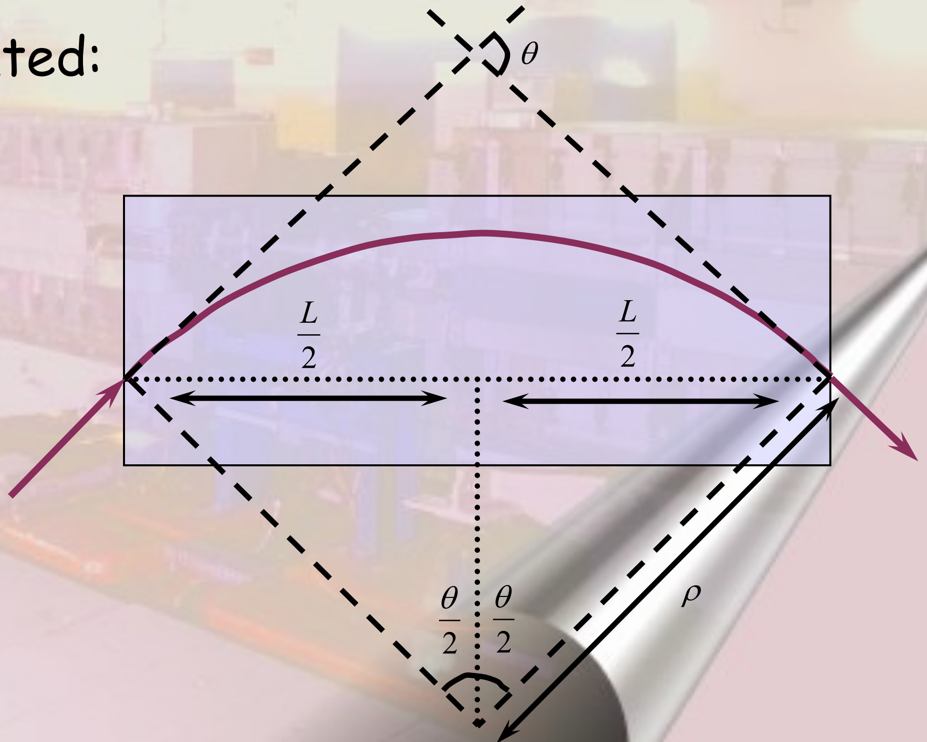
$$\sin\left(\frac{\theta}{2}\right) = \frac{L}{2\rho} = \frac{1}{2}\frac{LB}{(B\rho)}$$

- ✓ If θ is small:

$$\sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2}$$

- ✓ So we can write:

$$\theta = \frac{LB}{(B\rho)}$$



A Real Dipole Magent

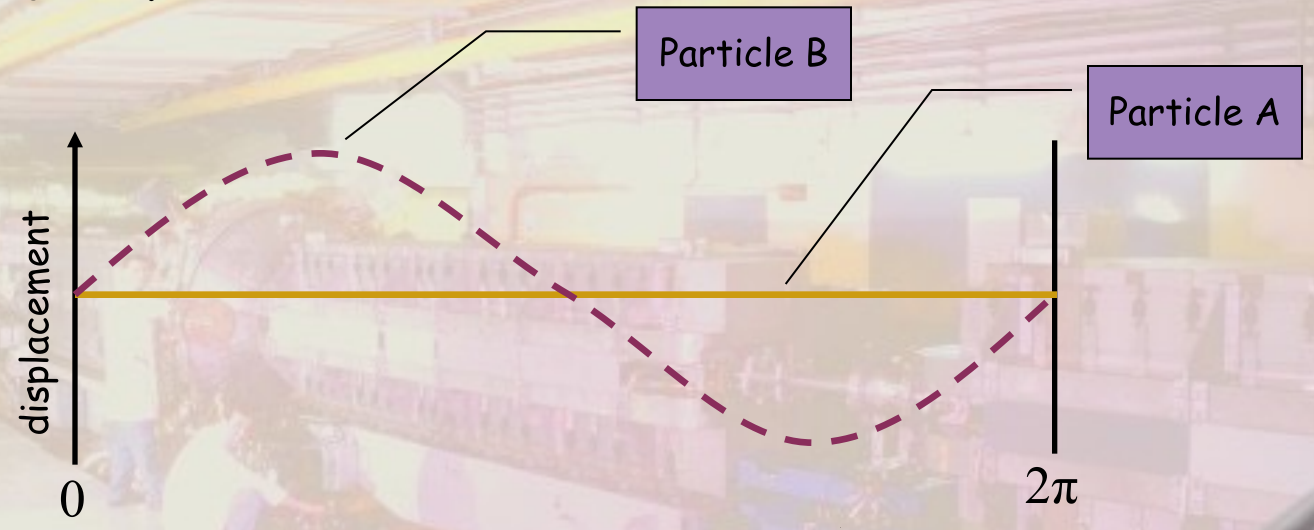
Two particles in a dipole field

- What happens with two particles that travel in a dipole field with different initial angles, but with equal initial position and equal momentum ?

- Assume that Bρ is the same for both particles.
- Lets unfold these circles……

The 2 trajectories unfolded

- ✓ The horizontal displacement of particle B with respect to particle A.



- ✓ Particle B oscillates around particle A.
- ✓ This type of oscillation forms the basis of all transverse motion in an accelerator.
- ✓ It is called <u>**‘Betatron Oscillation’**</u>

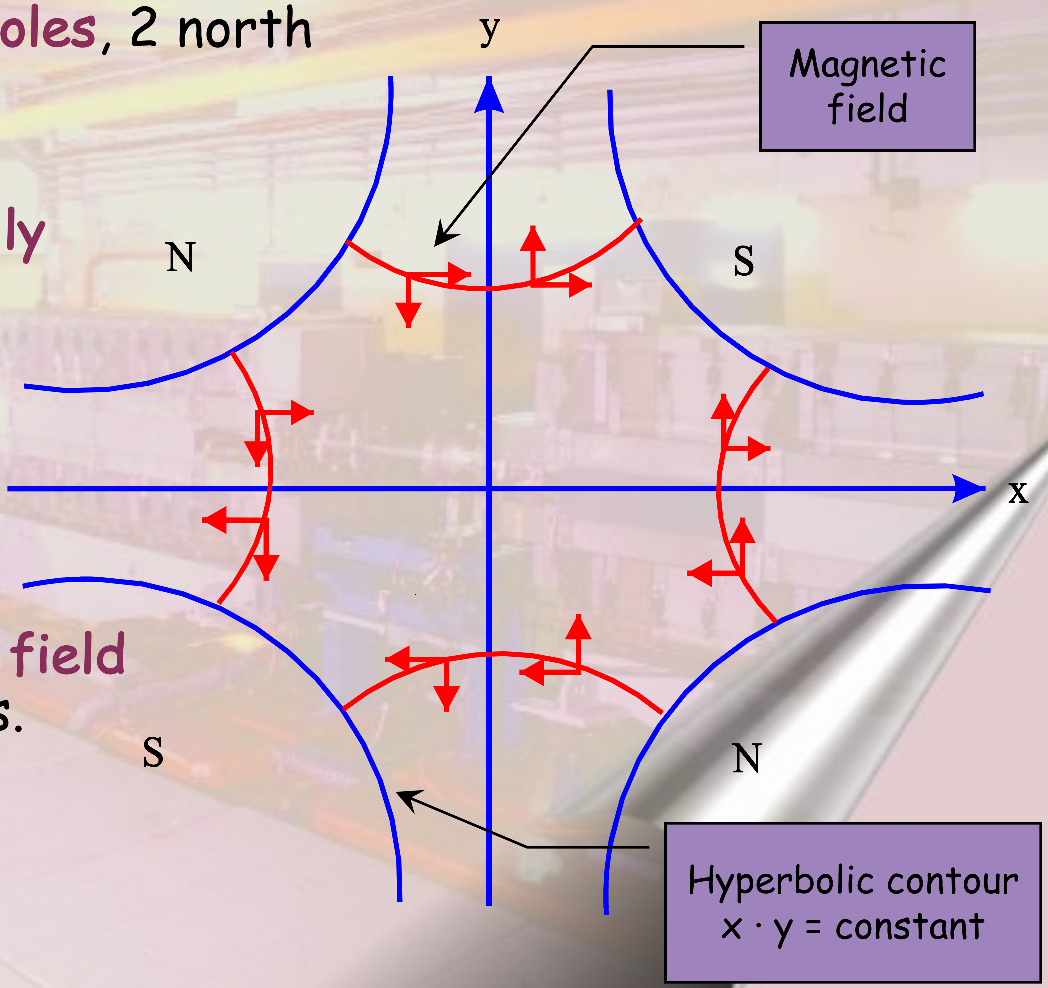
'Stable' or 'unstable' motion ?

- ✓ Since the horizontal trajectories close we can say that the horizontal motion in our simplified accelerator with only a horizontal dipole field is <u>'stable'</u>
- ✓ What can we say about the vertical motion in the same simplified accelerator ? Is it <u>'stable'</u> or <u>'unstable'</u> and why ?
- ✓ What can we do to make this motion stable ?
- ✓ We need some element that 'focuses' the particles back to the reference trajectory.
- ✓ This extra focusing can be done using:

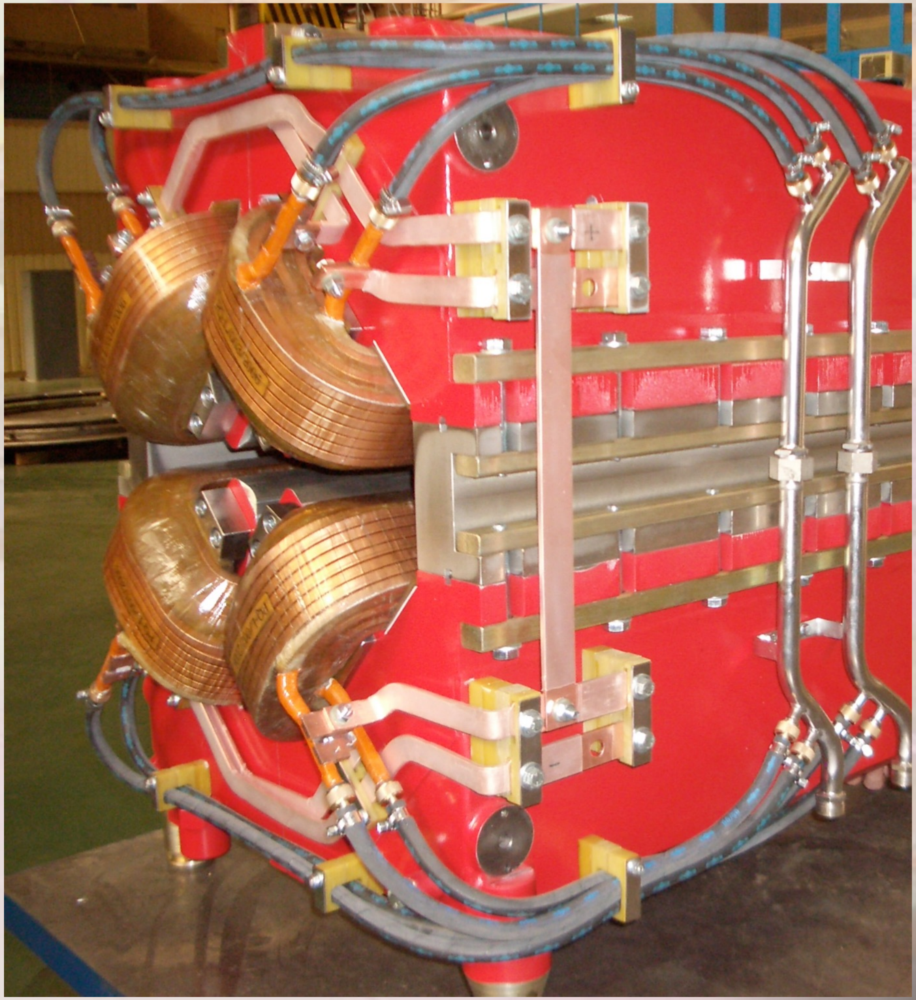
<u>**Quadrupole magnets**</u>

Quadrupole Magnet

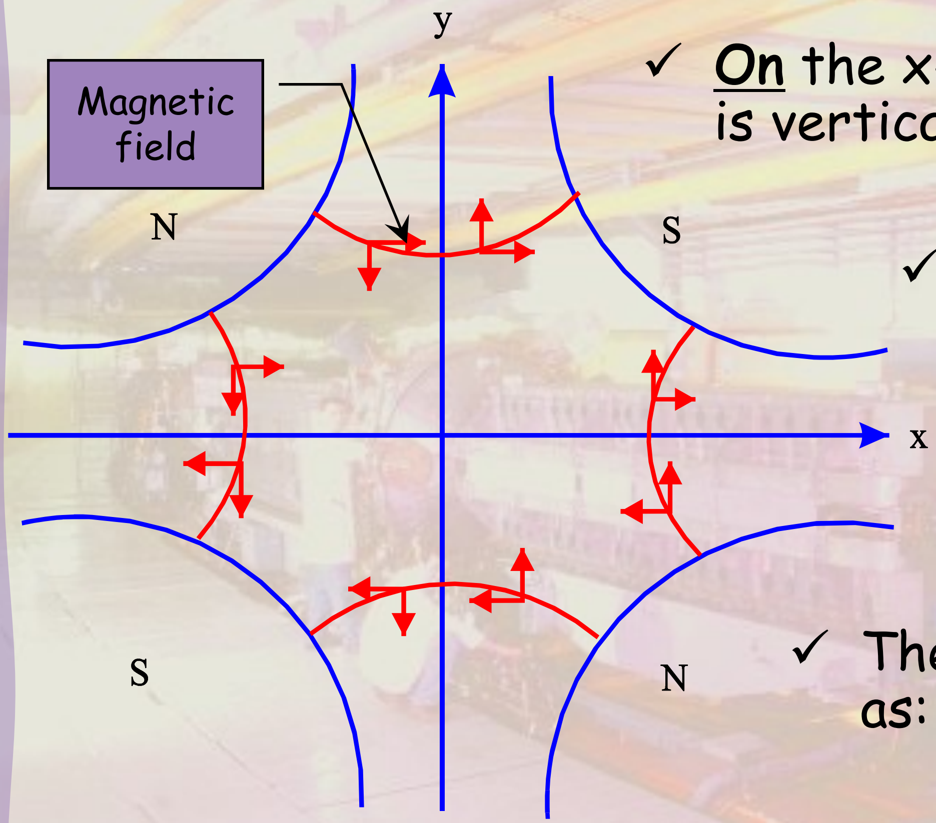
- A Quadrupole has 4 poles, 2 north and 2 south
- They are symmetrically arranged around the centre of the magnet
- There is no magnetic field along the central axis.



A Real Quadrupole Magnet

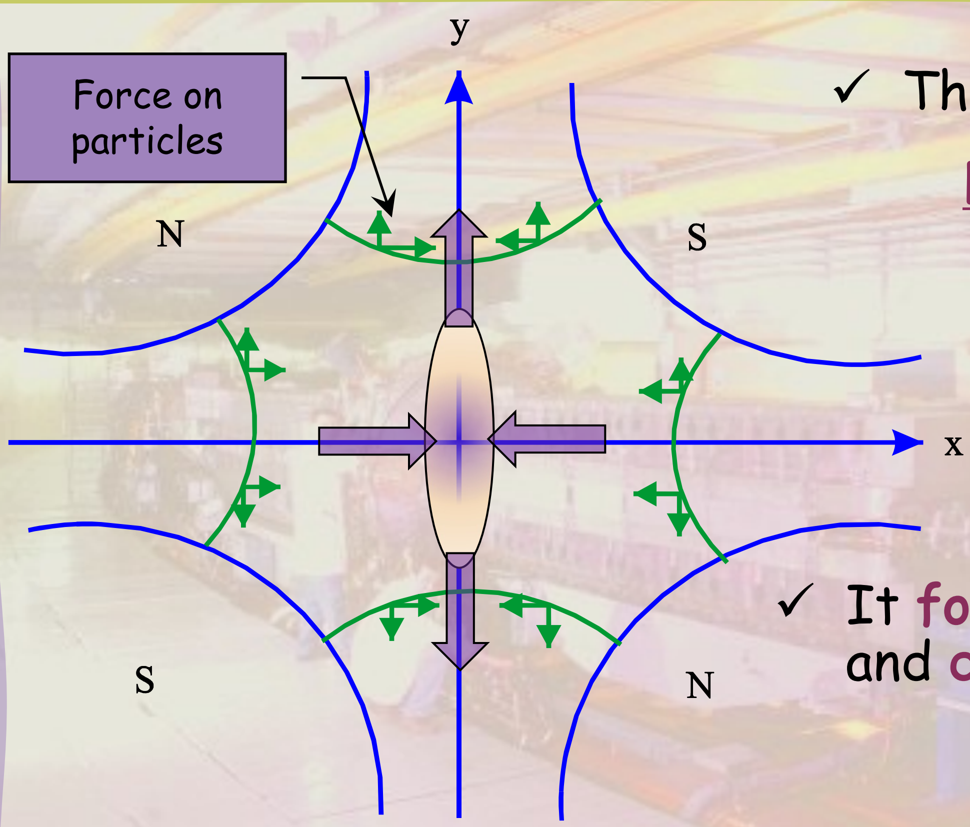


Quadrupole fields



- ✓ **On** the x-axis (horizontal) the field is vertical and given by: $B_y \propto x$
- ✓ **On** the y-axis (vertical) the field is horizontal and given by: $B_x \propto y$
- ✓ The field gradient, K is defined as: $\frac{d(By)}{dx}(Tm^{-1})$
- ✓ The 'normalised gradient', k is defined as: $\frac{K}{(B\rho)}(m^{-2})$

Types of quadrupoles



- This is a:

  Focusing Quadrupole (QF)

- It focuses the beam horizontally and defocuses the beam vertically.

- Rotating this magnet by 90° will give a:

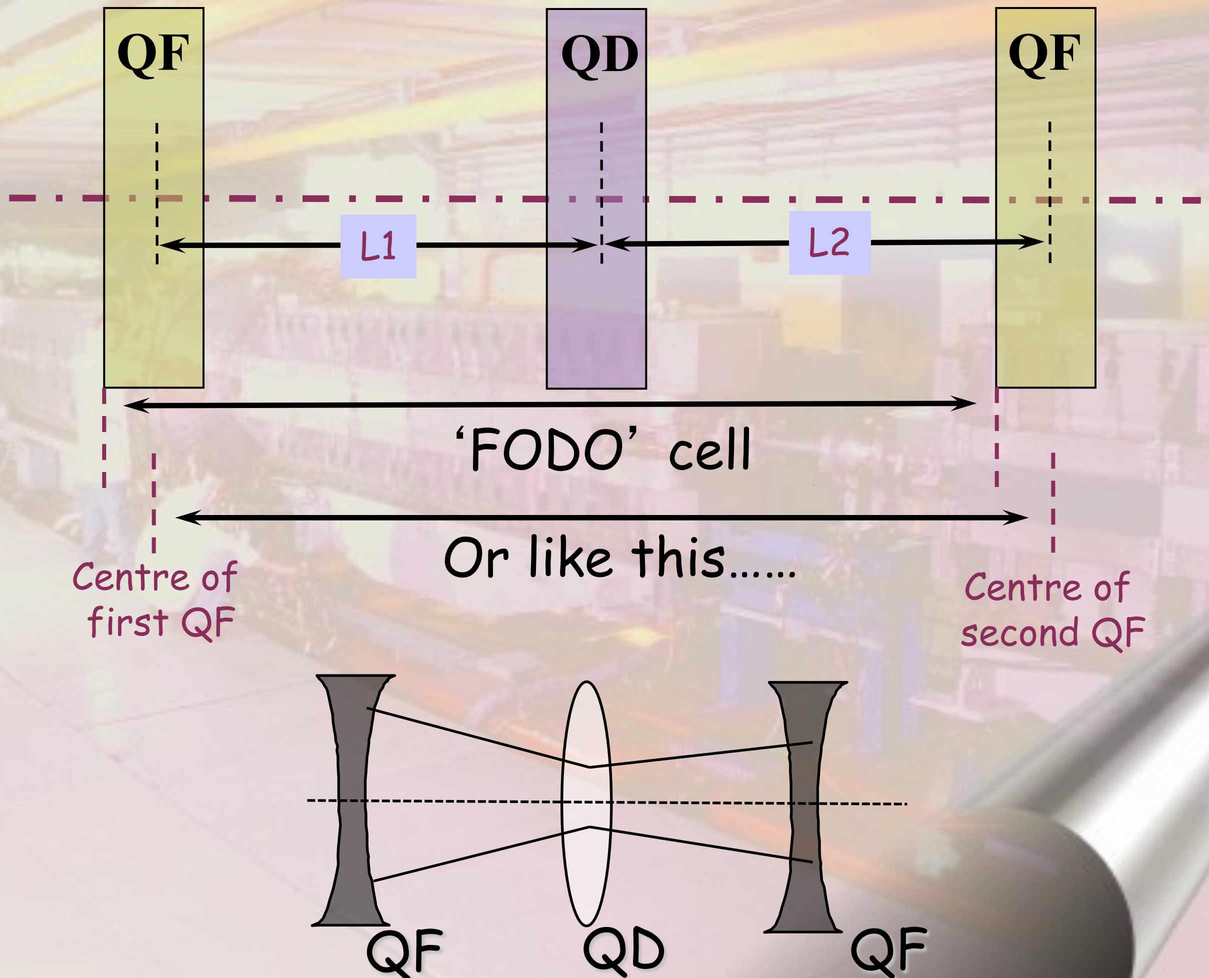
  Defocusing Quadrupole (QD)

Focusing and Stable motion

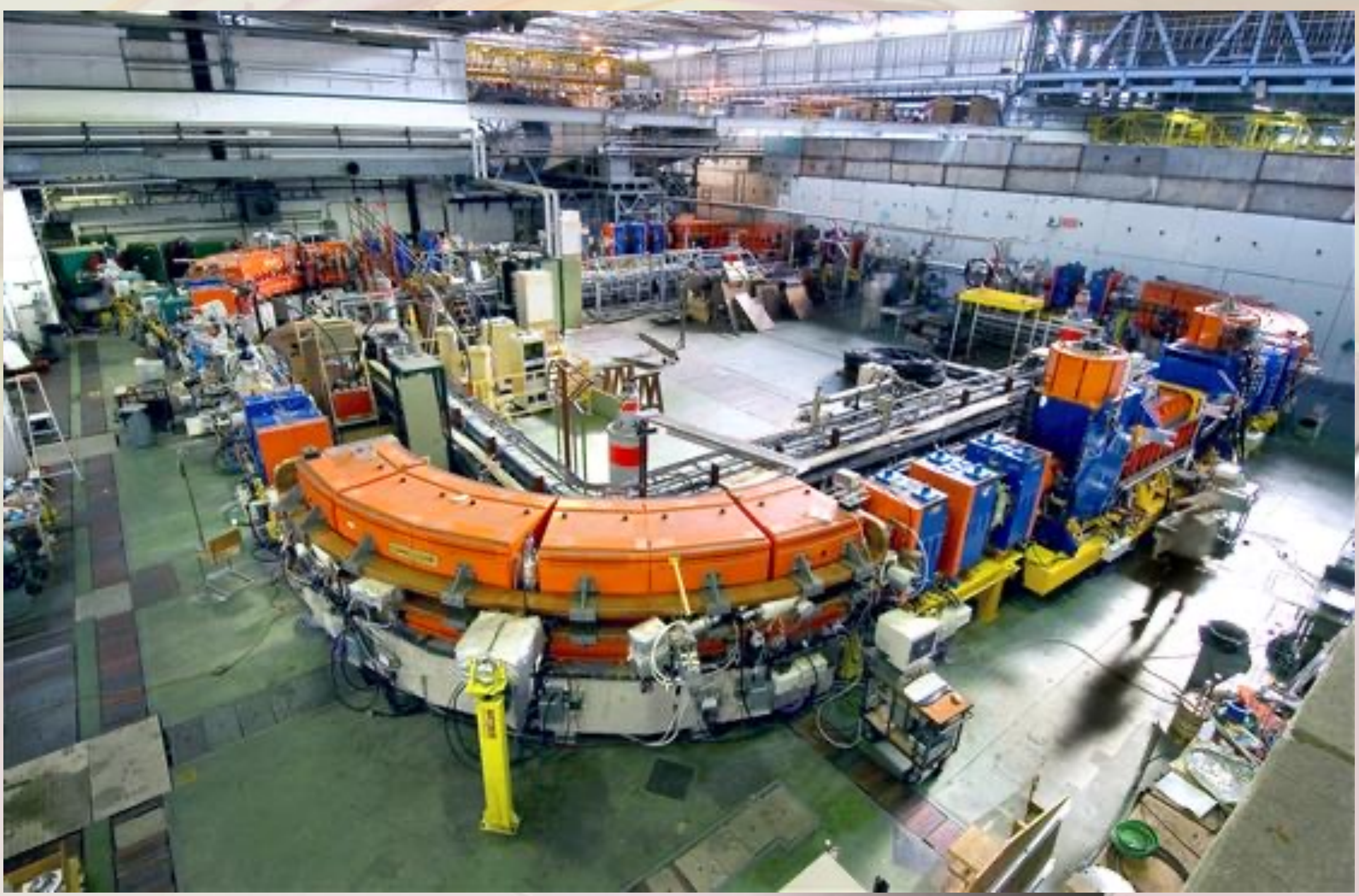
- Using a combination of focusing (QF) and defocusing (QD) quadrupoles solves our problem of 'unstable' vertical motion.
- It will keep the beams focused in **both planes** when the position in the accelerator, type and strength of the quadrupoles are well chosen.
- By now our accelerator is composed of:
  - Dipoles, constrain the beam to some closed path (orbit).
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions.
- A combination of focusing and defocusing sections that is very often used is the so called: FODO lattice.
- This is a configuration of magnets where focusing and defocusing magnets alternate and are separated by non-focusing drift spaces.

FODO cell

✓ The 'FODO' cell is defined as follows:

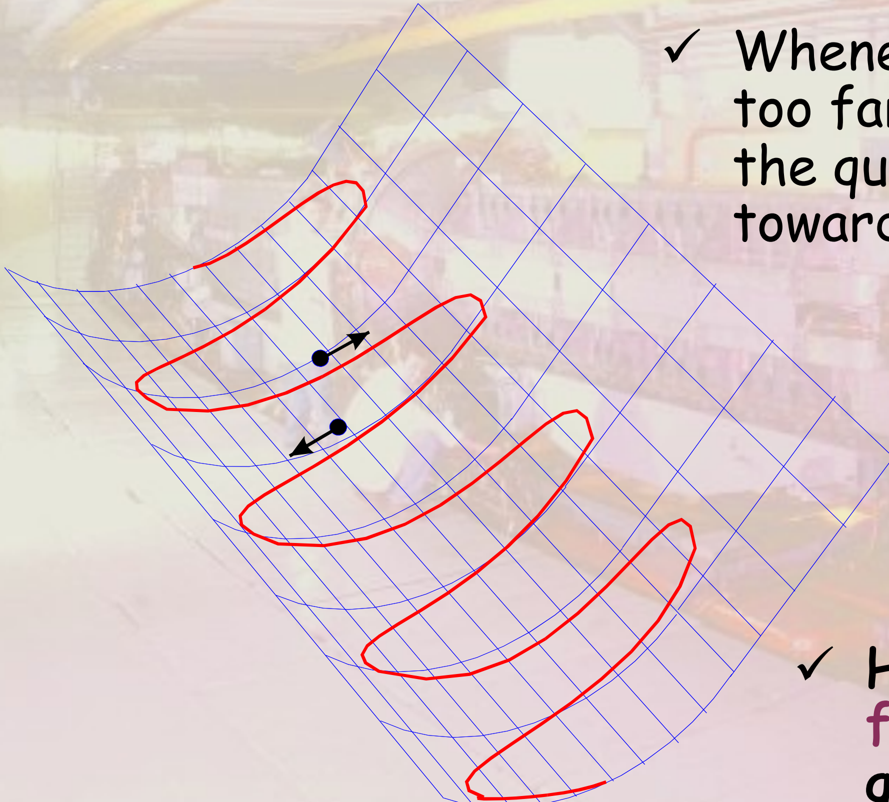


A Real Machine



The mechanical equivalent

- ✓ The gutter below illustrates how the particles in our accelerator behave due to the quadrupolar fields.

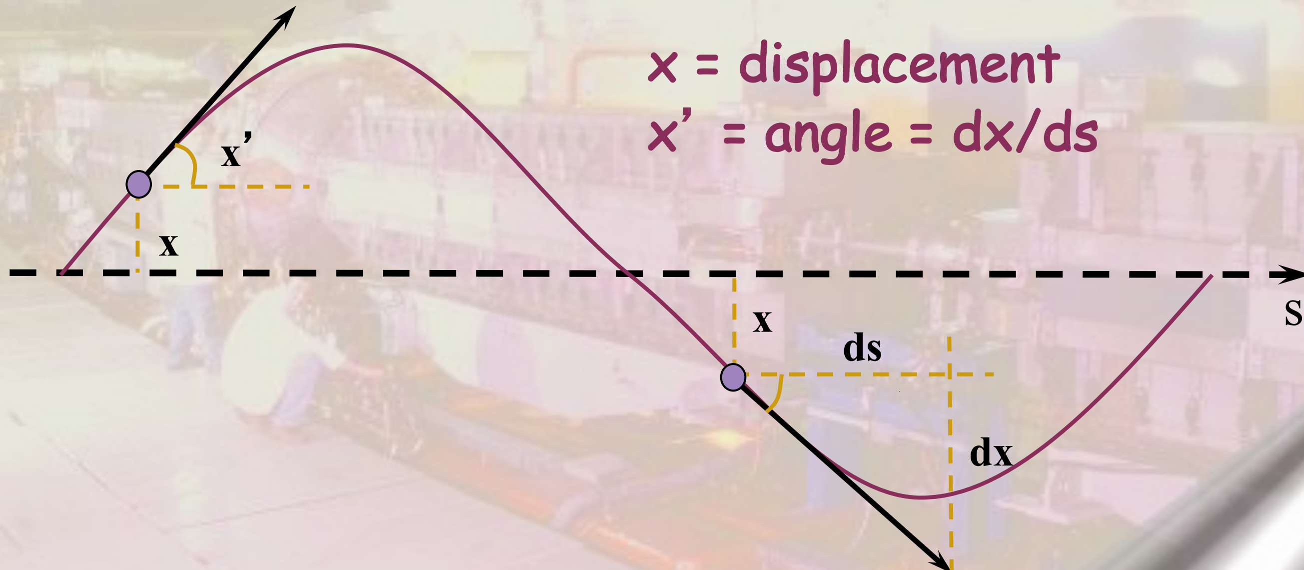


- ✓ Whenever a particle beam diverges too far away from the central orbit the quadrupoles focus them back towards the central orbit.

- ✓ How can we **represent** the **focusing gradient** of a quadrupole in this **mechanical equivalent** ?

The particle characterized

- A particle during its transverse motion in our accelerator is characterized by:
  - Position or displacement from the central orbit.
  - Angle with respect to the central orbit.



x = displacement

x' = angle = dx/ds

- This is a motion with a constant restoring force, like in the first lecture on differential equations, with the pendulum

Hill's equation

- ✓ These **betatron oscillations** exist in both **horizontal** and **vertical** planes.
- ✓ The **number of betatron oscillations per turn** is called the **betatron tune** and is defined as Qx and Qy.
- ✓ Hill's equation describes this motion mathematically

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- ✓ If the restoring force, K is constant in 's' then this is just a **Simple Harmonic Motion**.
- ✓ 's' is the longitudinal displacement around the accelerator.

Hill's equation (2)

- ✓ In a real accelerator K varies strongly with 's'.
- ✓ Therefore we need to solve Hill's equation for K varying as a function of 's'

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- ✓ What did we conclude on the mechanical equivalent concerning the shape of the gutter……?
- ✓ How is this related to Hill's equation……?

Questions….,Remarks…?