Quantum algorithms for particle physics

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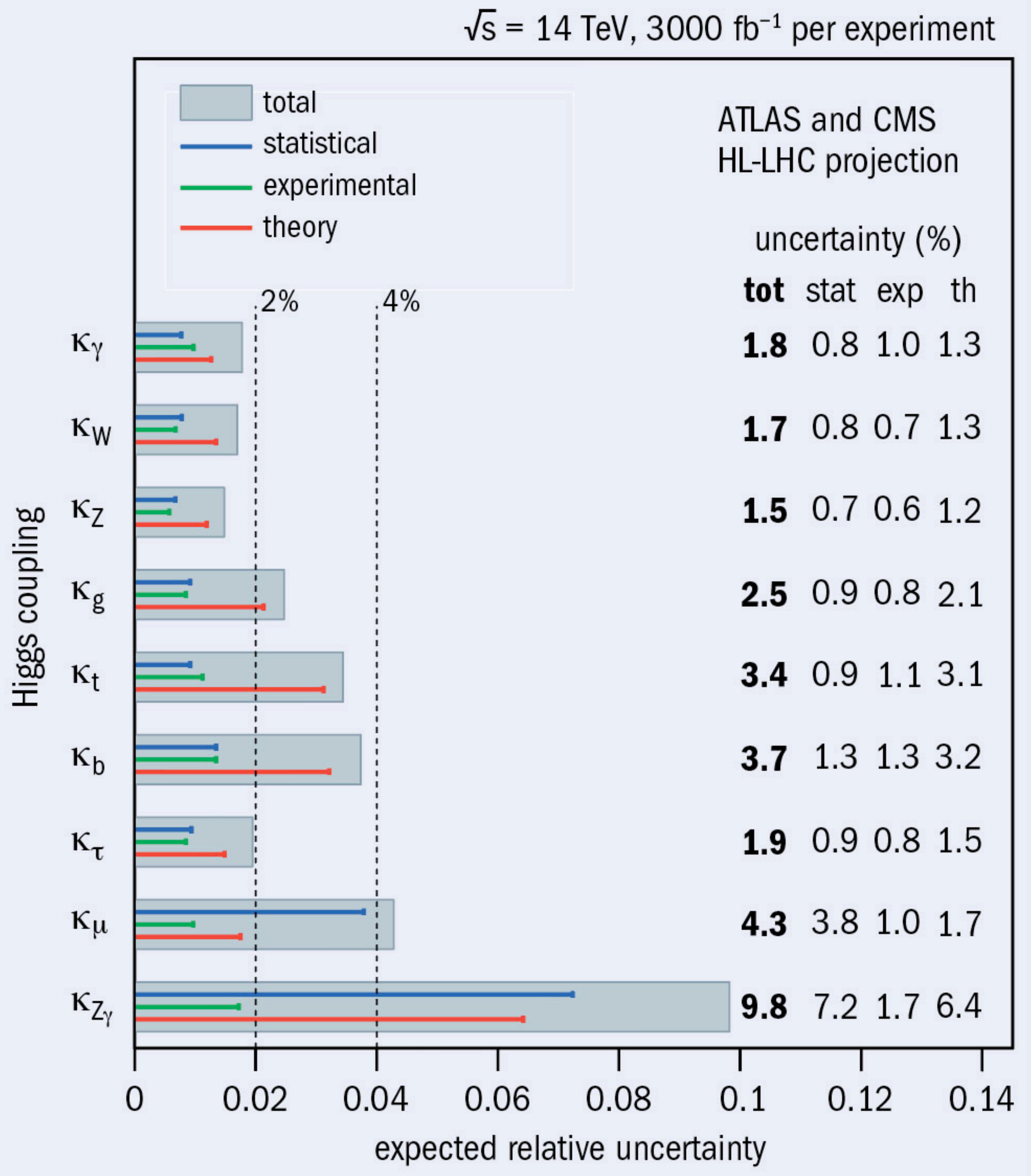
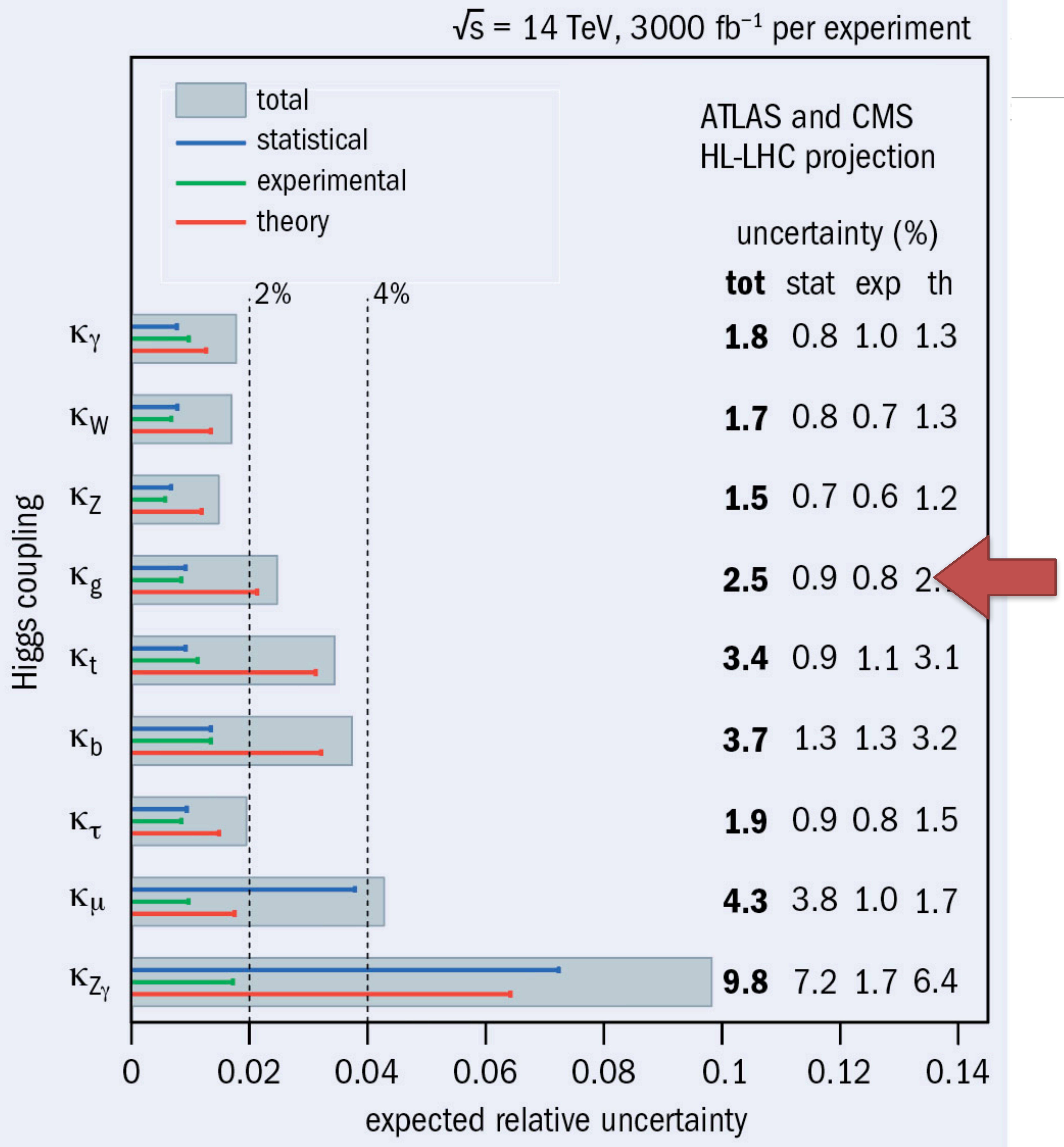
7th Red LHC Workshop
10-12 May 2023



- Need to go beyond the current state of the art in theoretical predictions at colliders
- Theory is the main limiting factor to achieve precision measurements



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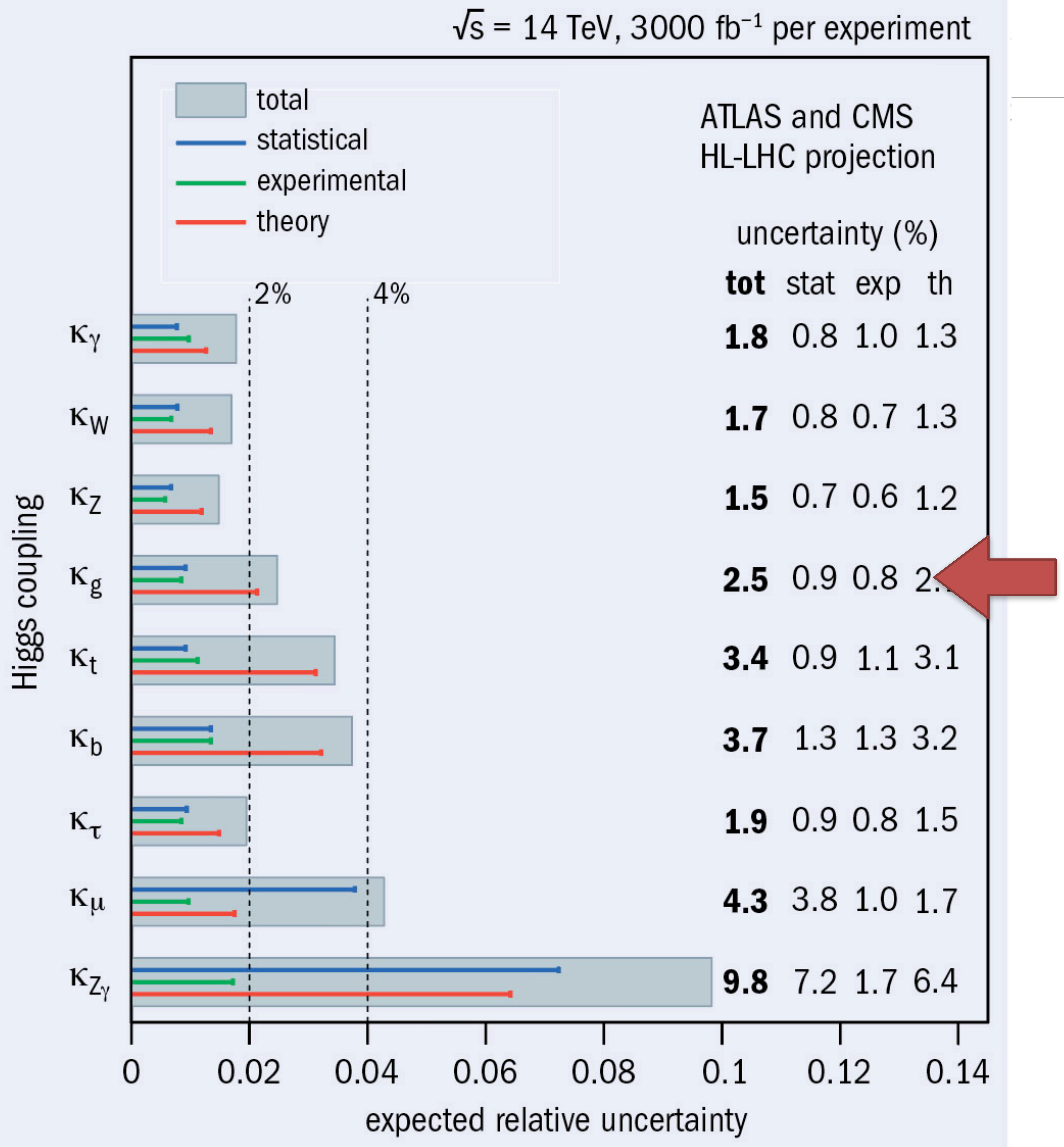

- Need to go beyond the current state of the art in theoretical predictions at colliders
- Theory is the main limiting factor to achieve precision measurements
- Very optimistic projections that today are unreachable
- Years of CPU in large-scale clusters and huge energy consumption in the best scenario


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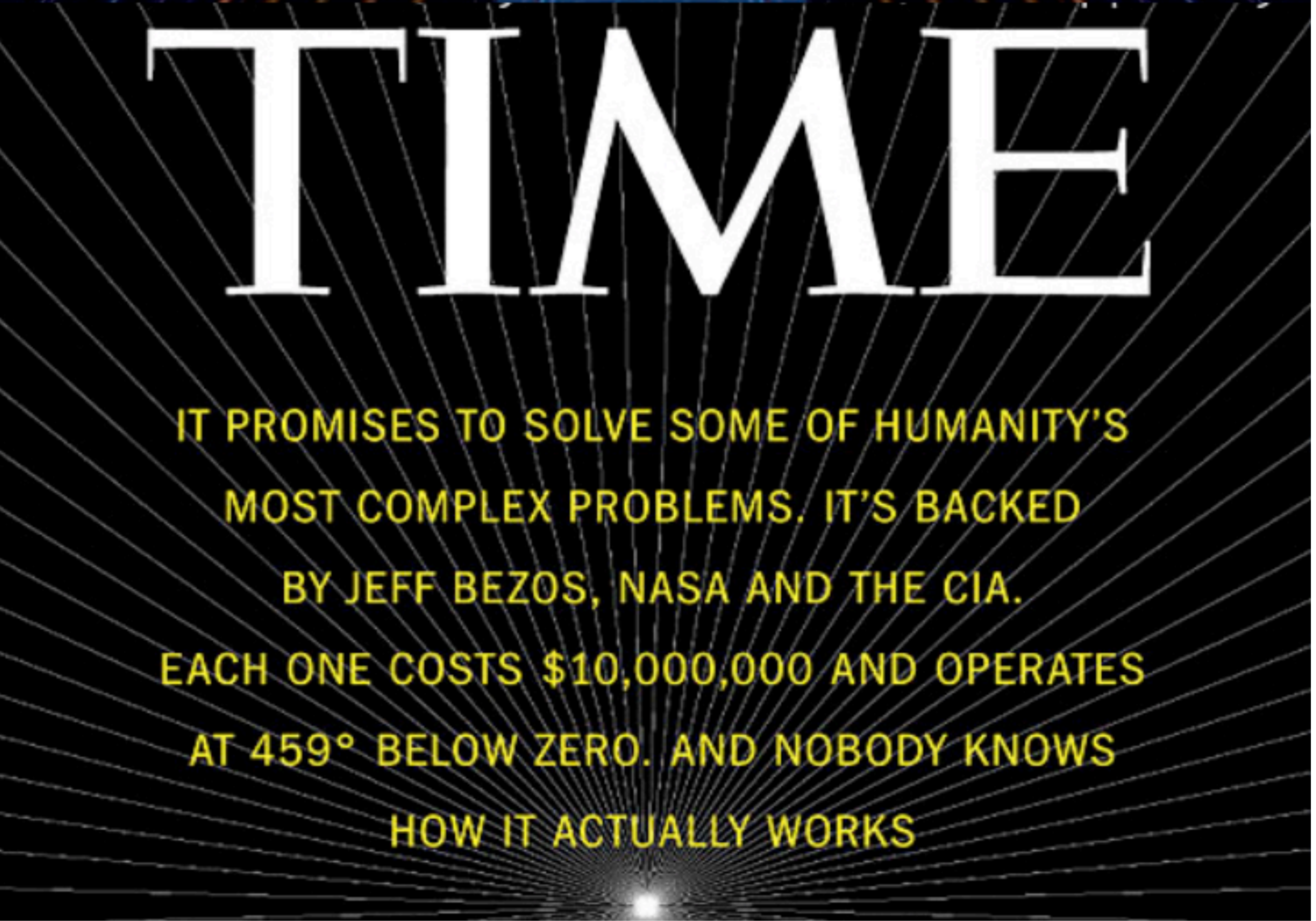
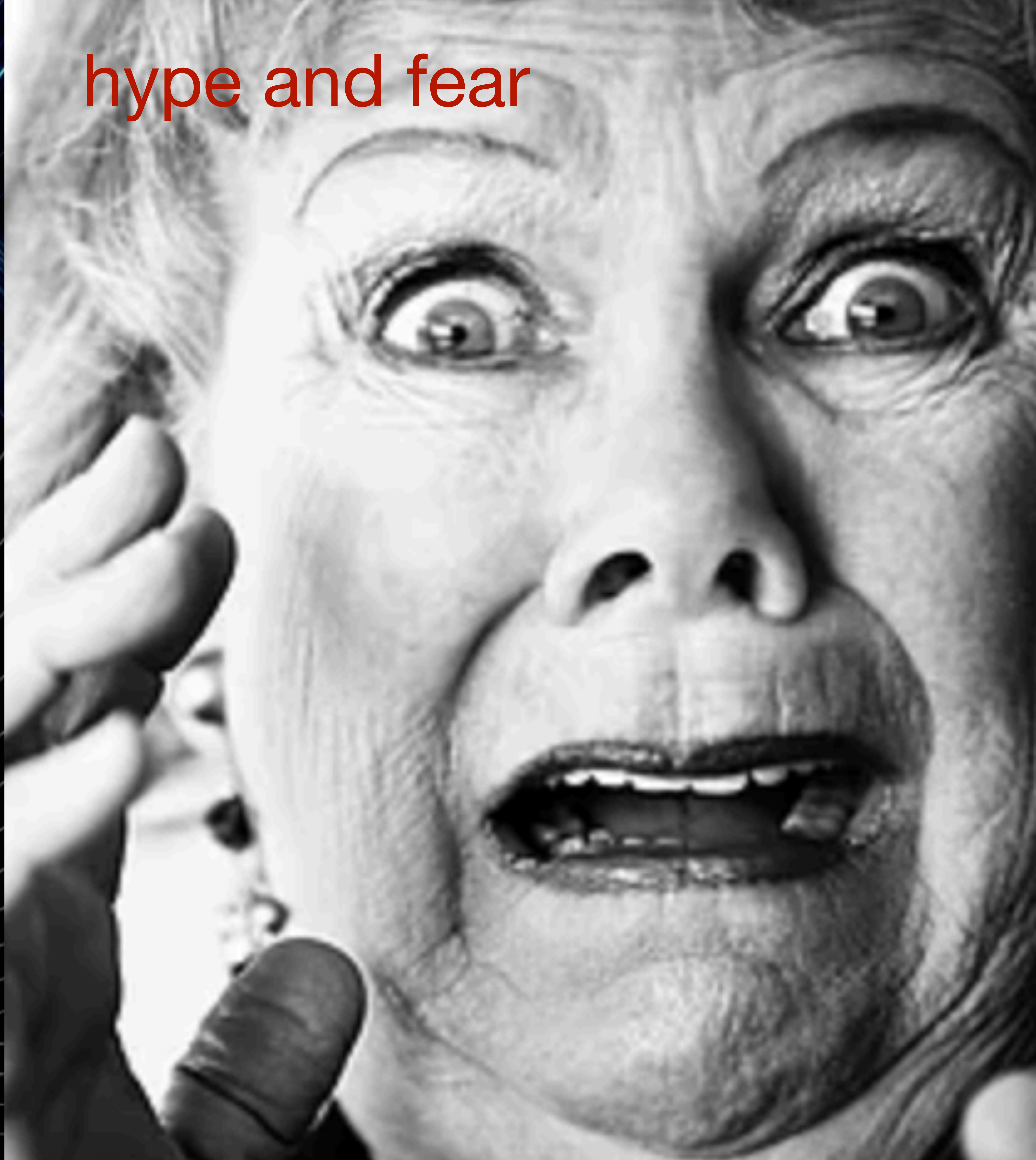
Quantum technologies
On the cusp of a revolution

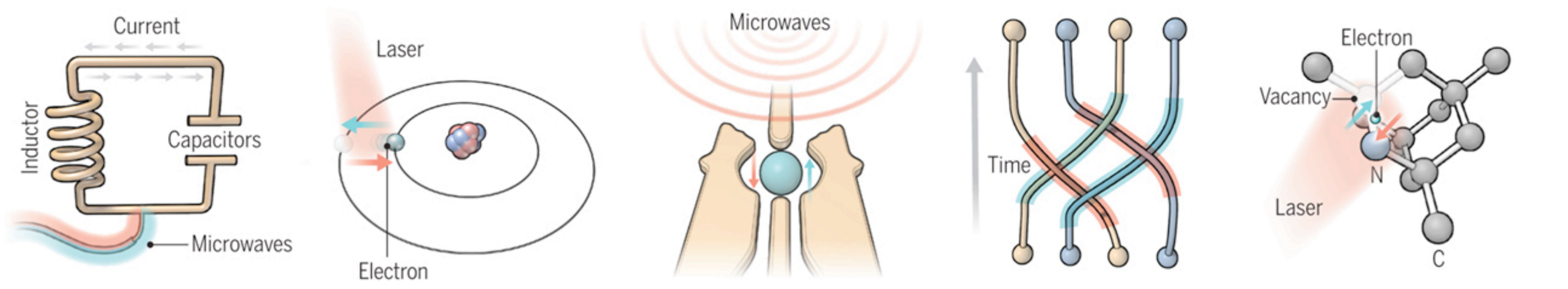

TIME
IT PROMISES TO SOLVE SOME OF HUMANITY'S
MOST COMPLEX PROBLEMS. IT'S BACKED
BY JEFF BEZOS, NASA AND THE CIA.
EACH ONE COSTS $10,000,000 AND OPERATES
AT 459° BELOW ZERO. AND NOBODY KNOWS
HOW IT ACTUALLY WORKS

hype and fear



A bit of the action

In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.

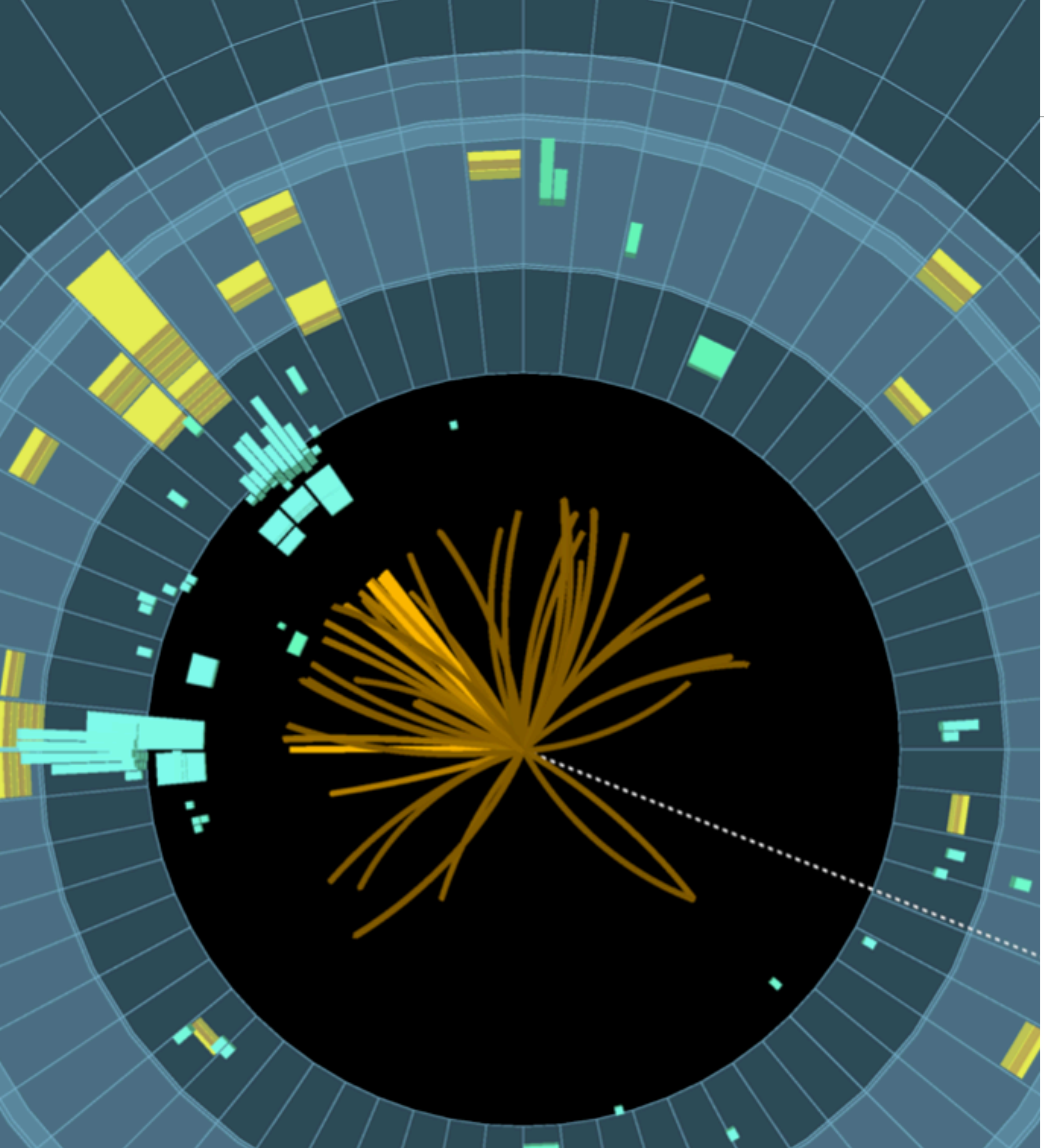


	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into superposition states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

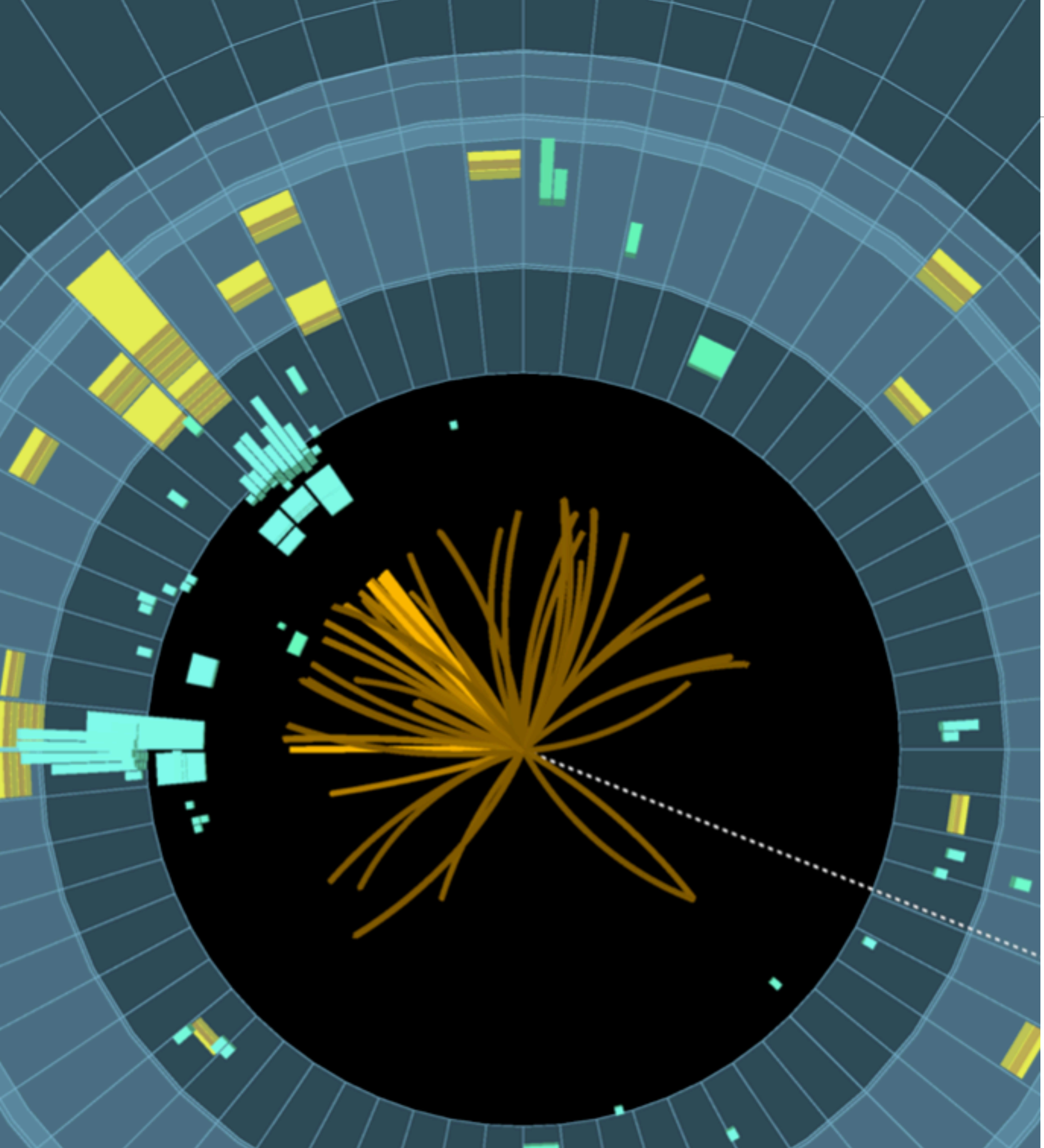
Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

https://www.science.org/doi/10.1126/science.354.6316.1090

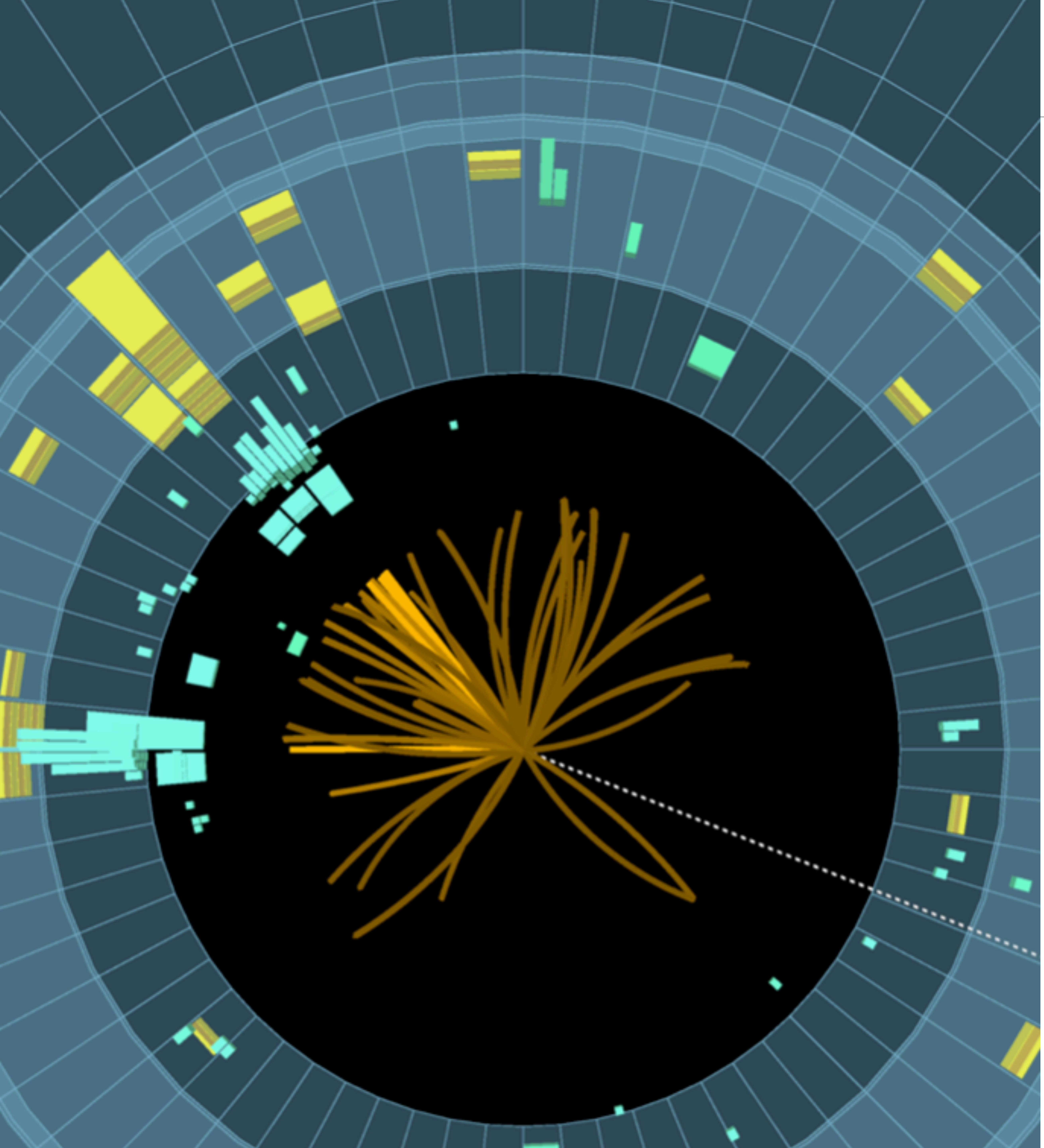
- **Noisy Intermediate Scale Quantum (NISQ)** era [Preskill 2018]: $\mathcal{O}(100)$ qubits with $\mathcal{O}(\mu s)$ coherence time
- Limited, yet promising applications



- Quantum computing in collider physics? QFT is quantum



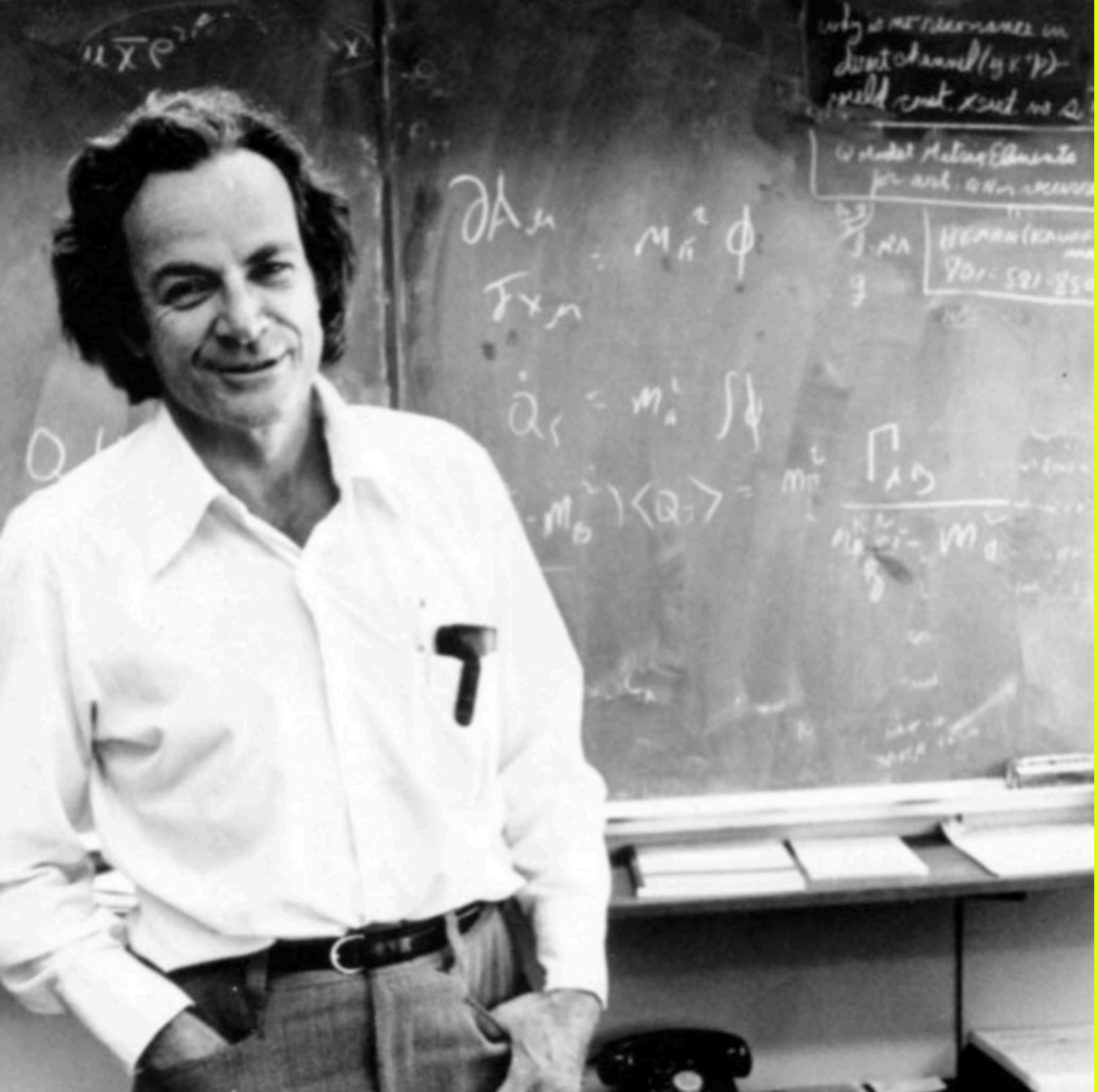
- Quantum computing in collider physics? QFT is quantum
- Quantum computers are expected to use less energy



- Quantum computing in collider physics? QFT is quantum

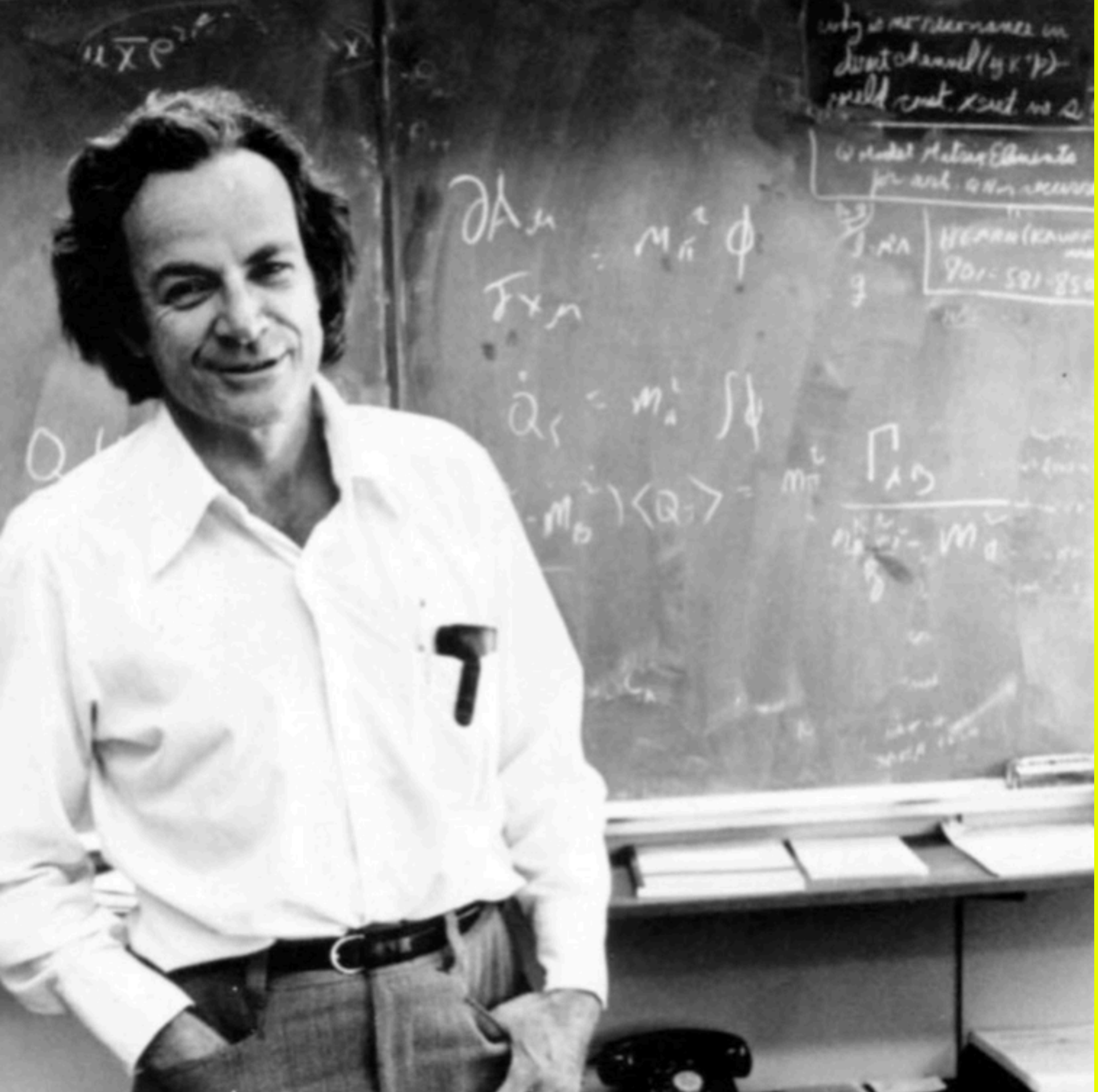
- Quantum computers are expected to use less energy

- Apart from a potential speedup: a quantum way of doing calculations



"Nature isn't classical, dammit, and if you want to make a simulation of nature, you better make it quantum"

- Richard P. Feynman



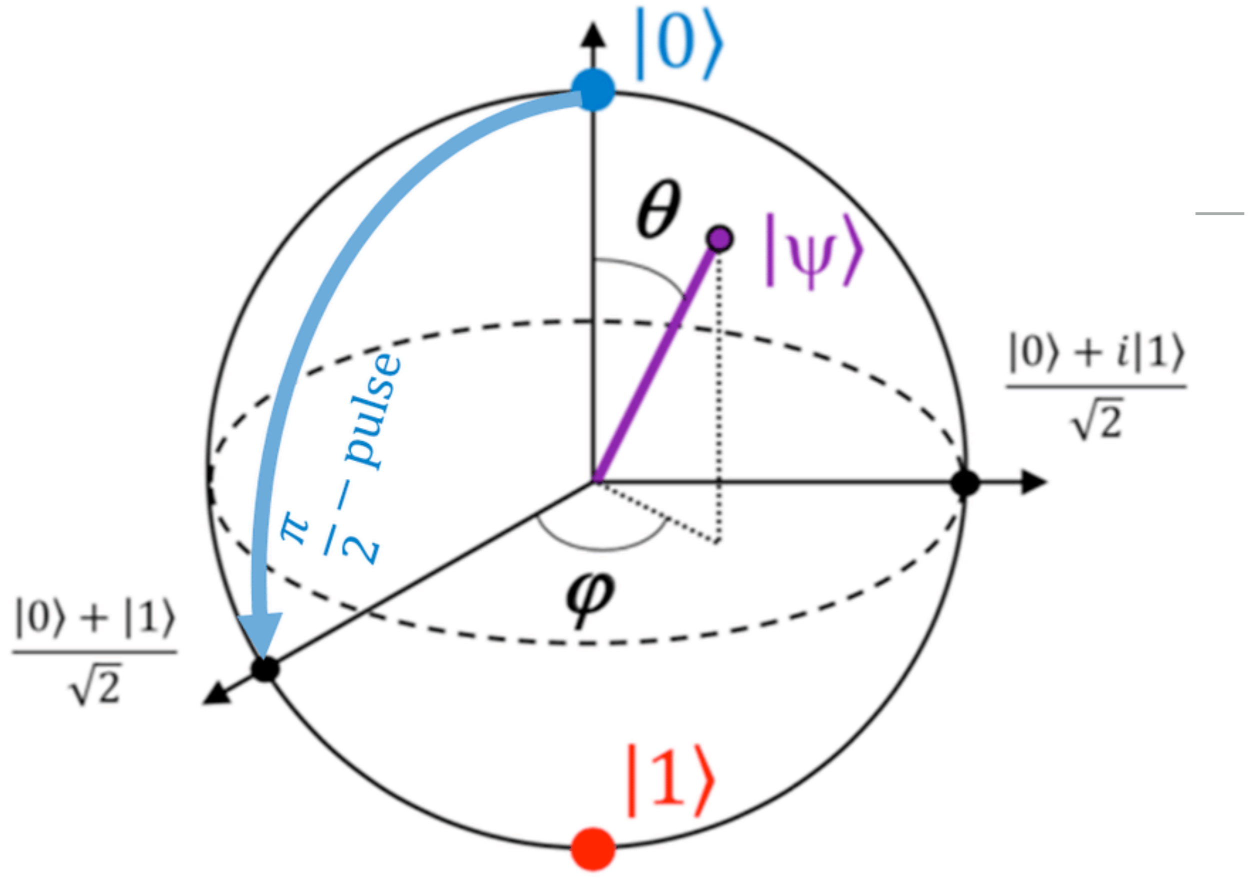
"Nature isn't classical, dammit, and if you want to make a simulation of nature, you better make it quantum"

quantum entanglement session on Thursday

- Richard P. Feynman

QUANTUM COMPUTING ADVANTAGE

- Promising avenue for solving specific problems that become too complex or even intractable for classical computers because they **scale either exponentially or superpolynomially**: e.g. factoring integers into primes (Shor's algorithm), database querying (Grover's algorithm), optimisation, finding minima …



$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi} sin\frac{\theta}{2}|1\rangle$$

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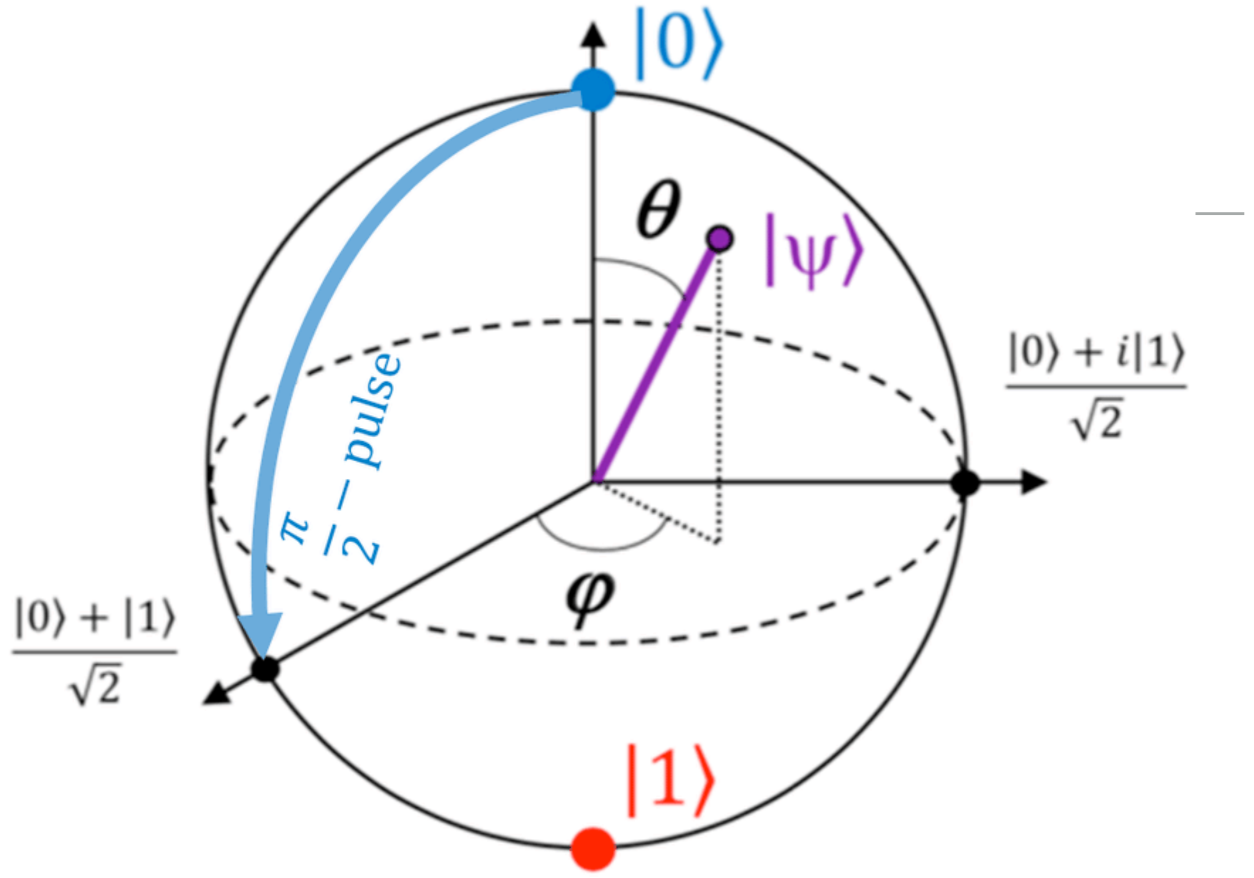

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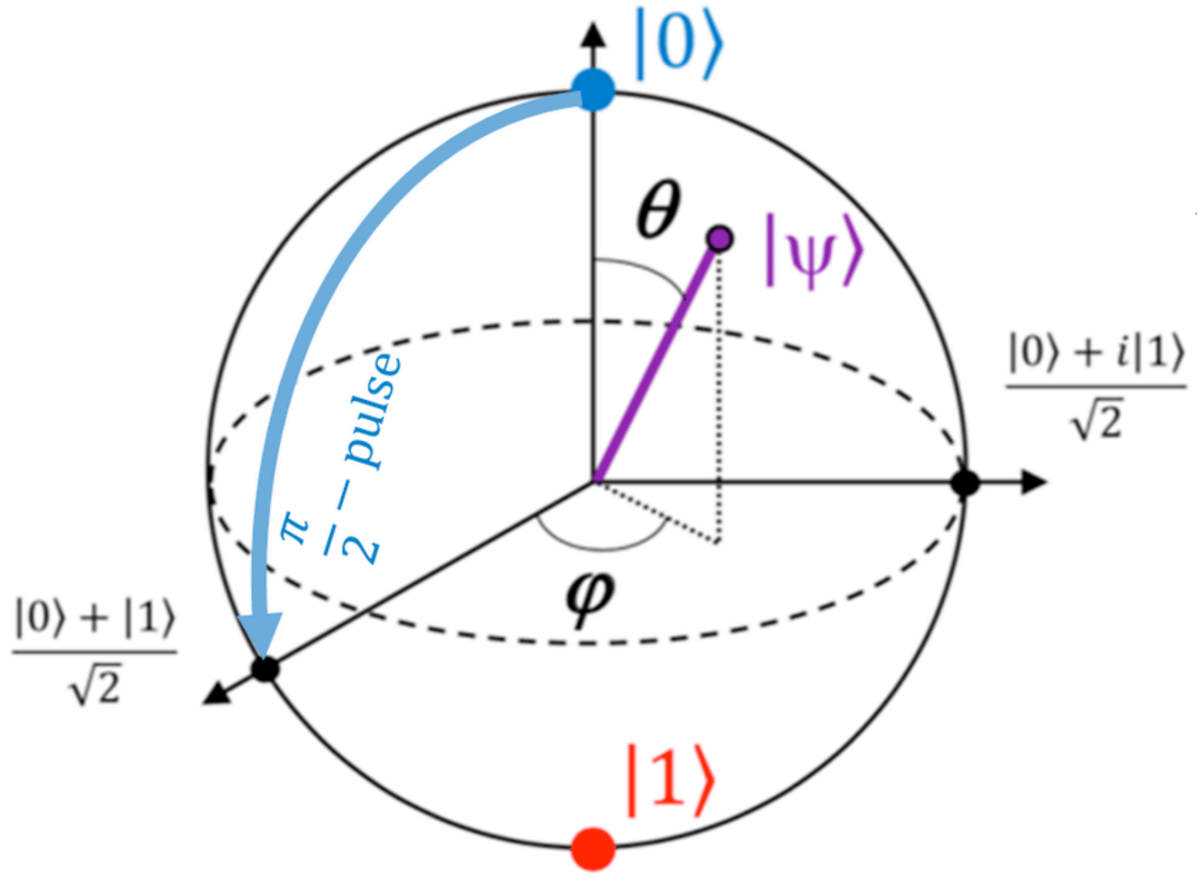
- Need to **exploit the quantum mechanic principles** of

  - **Superposition:** $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$
  - **Entanglement:** $|\psi_1\psi_2\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$

QUANTUM COMPUTING ADVANTAGE



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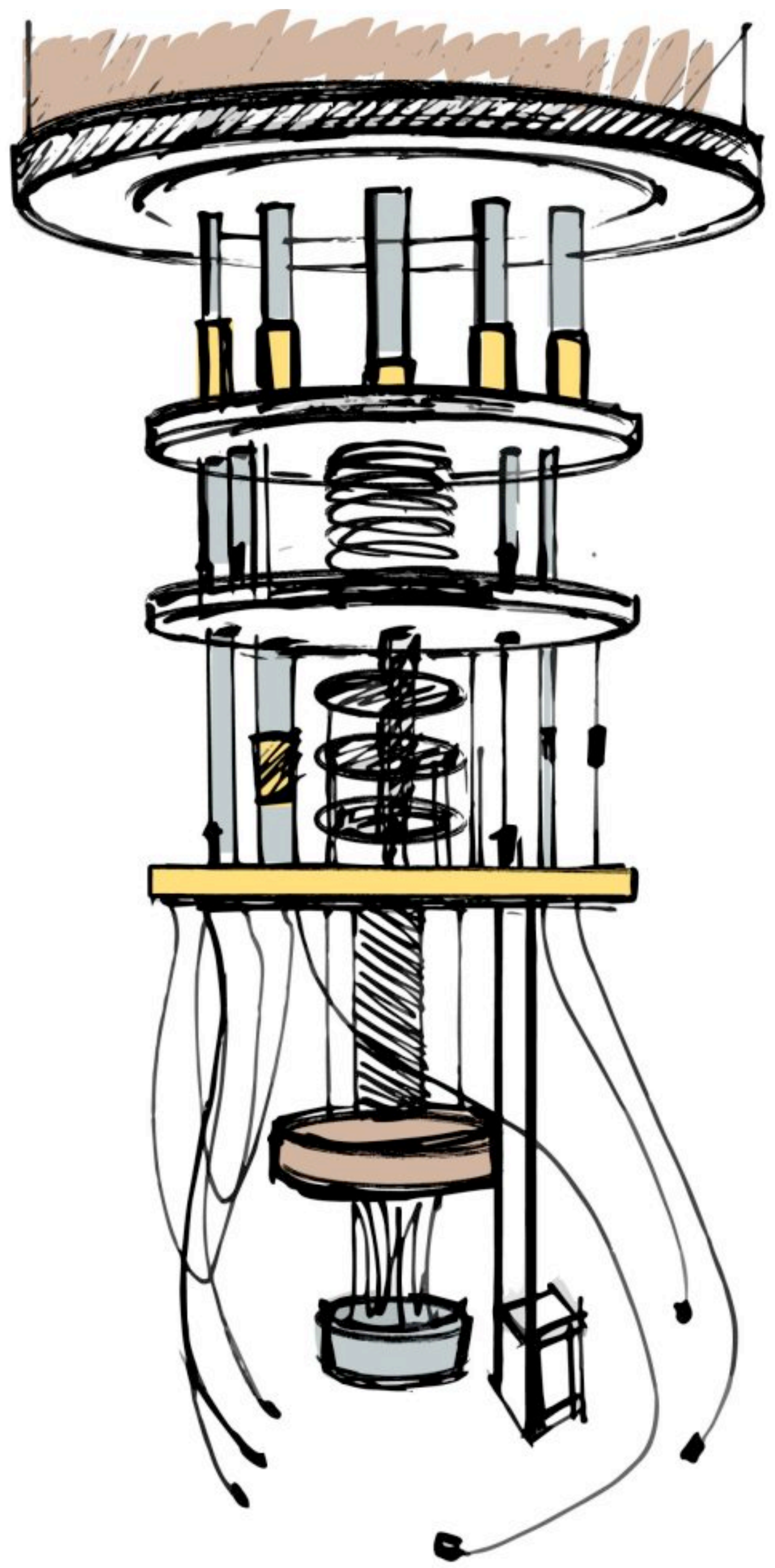
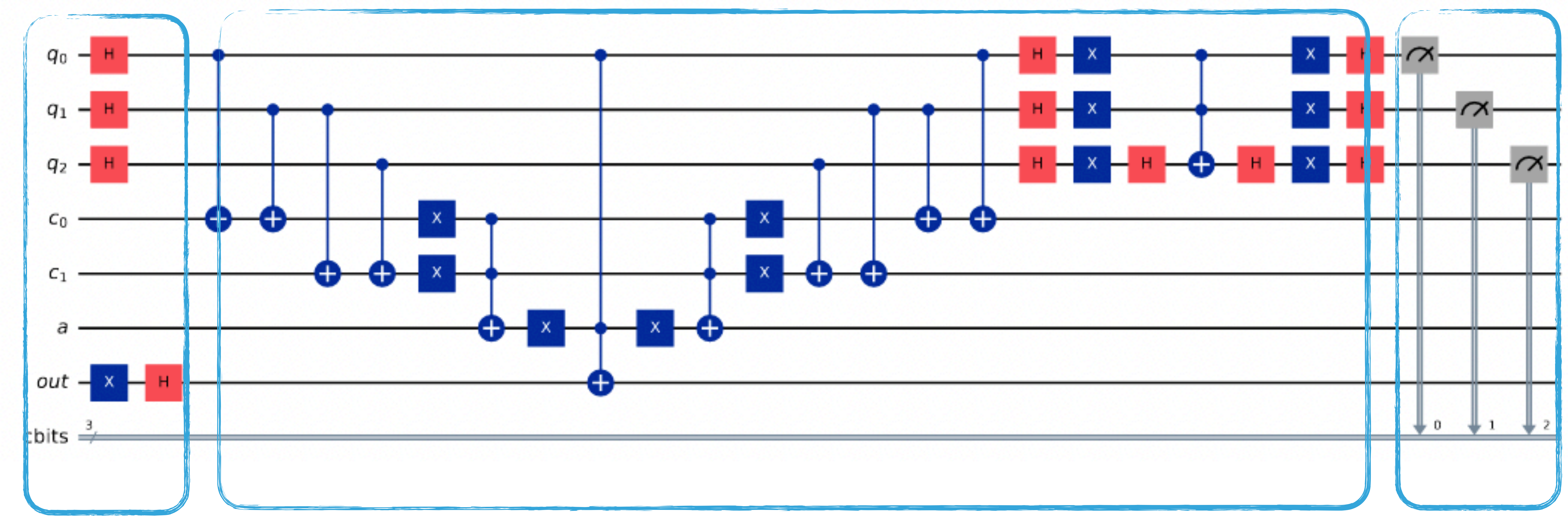
- Difficult to obtain quality results in current quantum devices (superconducting qubits, cold atoms in a lattice, photonic devices) due to **decoherence (noise)**, still interesting to probe in **quantum simulators**



QUANTUM CIRCUITS

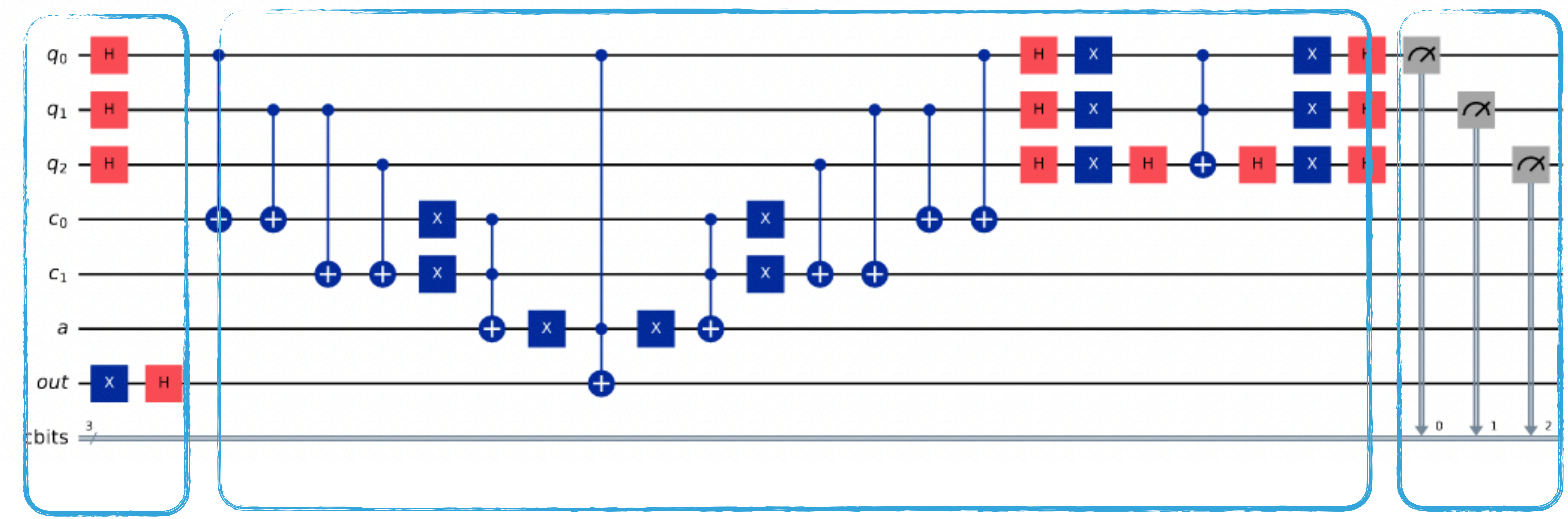


initialisation and superposition

Unitary transformations through logic quantum gates (H, NOT, CNOT, …)

measurement

QUANTUM CIRCUITS



initialisation and superposition

Unitary transformations through logic quantum gates (H, NOT, CNOT, …)

measurement

- Quantum computing ≠ **parallelisation:** the superposition colapse after each measurement

RECENT QUANTUM APPLICATIONS

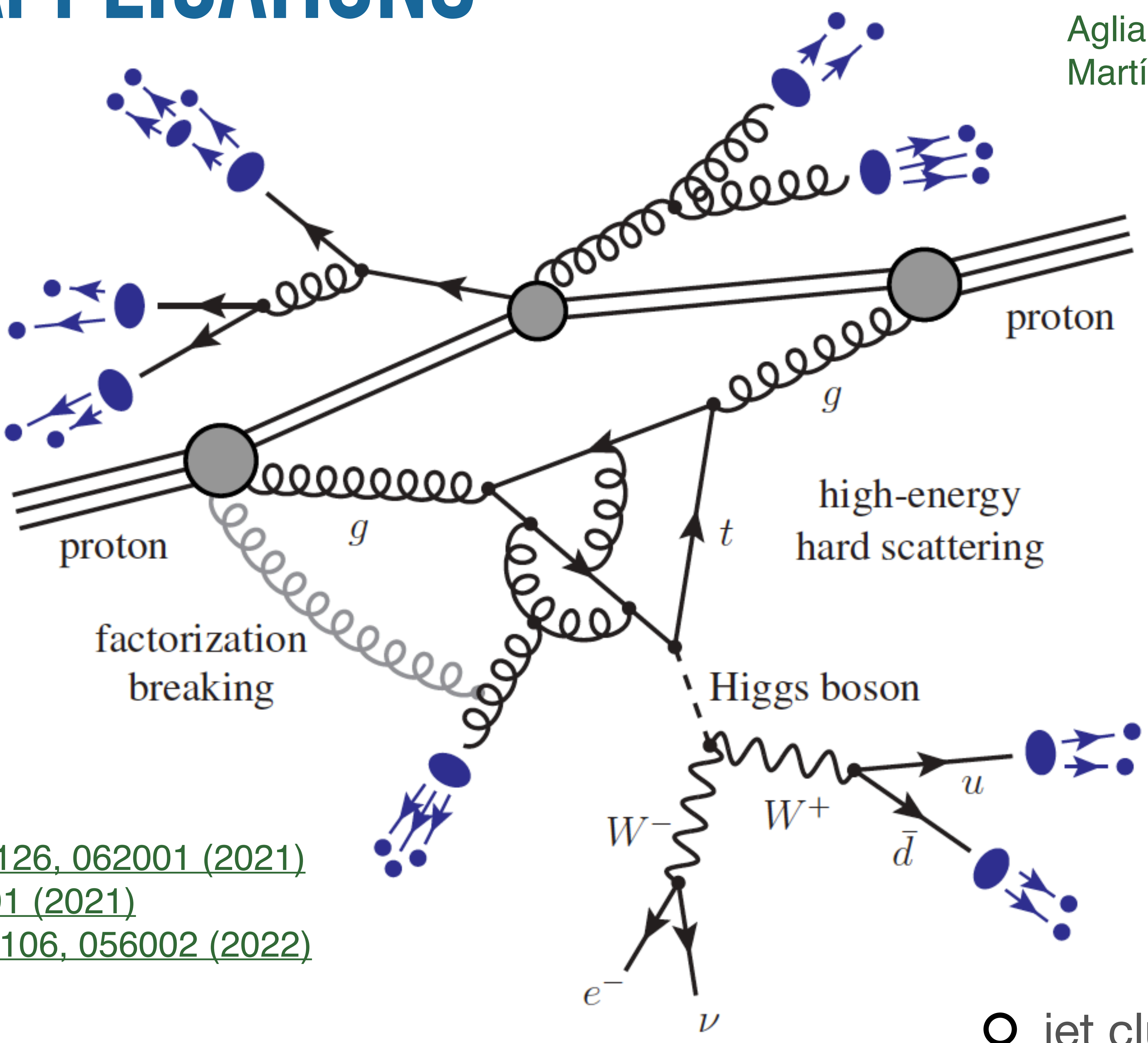
- Monte Carlo integration:
  Herbert, Q6, 823 (2022)
  Agliardi, Grossi, Pellen, Prati, PLB 832, 137228 (2022)
  Martínez de Lejarza, Grossi, Cieri, GR, 2305.01686

- track reconstruction:
  Mangano et al., PRD 105, 076012 (2022)
  Duckett, Facini, Jastrzebski, Malik, Scanlon, Rettie, 2212.07279
  Schwägerl, Issever, Jansen, Khoo, Kühn, Tüysüz, Weber, 2303.13249

- tree-level helicity amplitudes:
  Bepari, Malik, Spannowsky, Williams, PRD103, 076020 (2021)

- multiloop scattering amplitudes:
  Ramírez, Rentería, GR, Sborlini, Vale Silva, JHEP 2205, 100 (2022)
  Clemente, Crippa, Jansen, Ramírez, Rentería, GR, Sborlini, Vale Silva, 2210.13240

- parton densities:
  Pérez-Salinas, Cruz-Martínez, Alhajri, Carrazza, PRD 103, 034027 (2021)

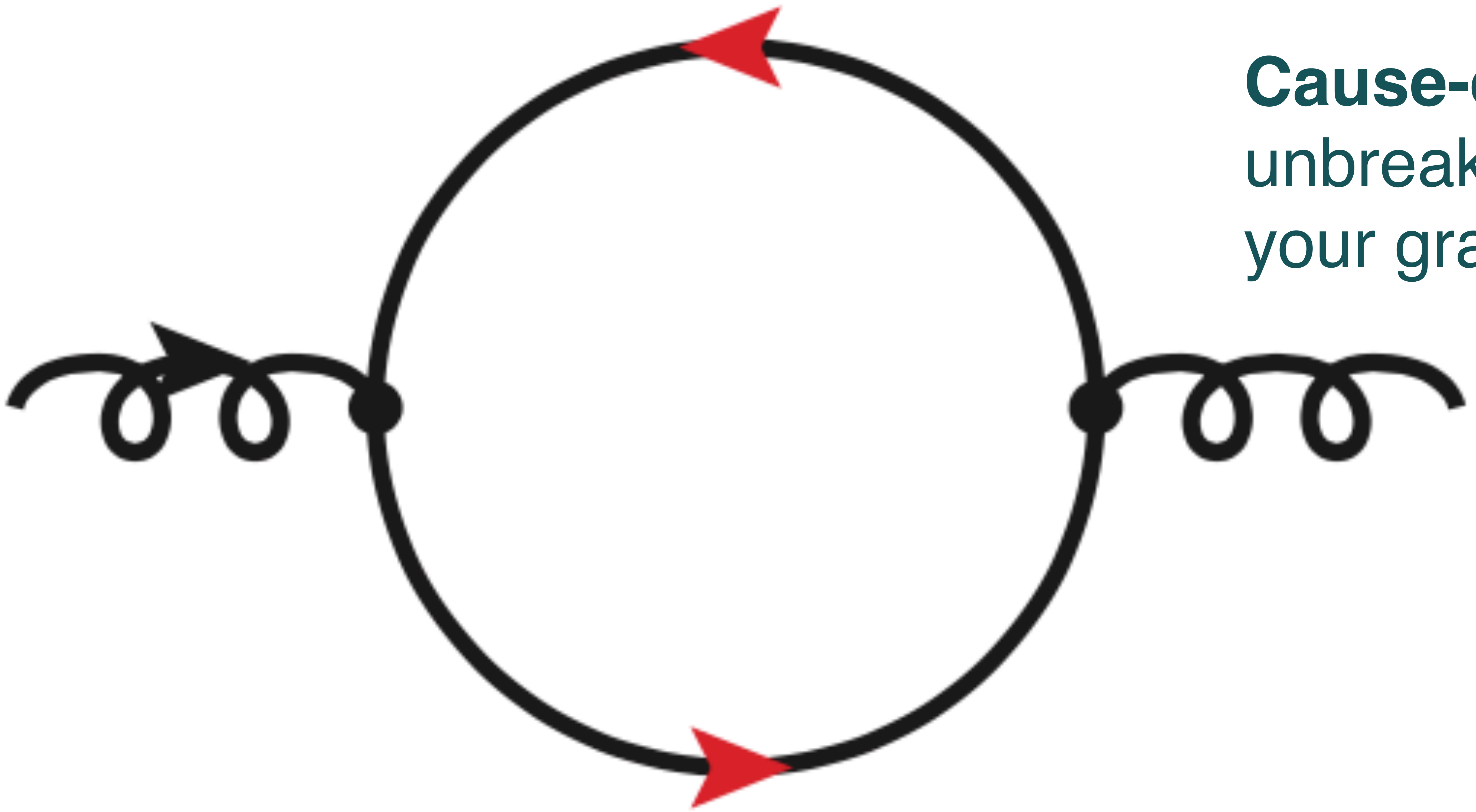


- jets in a medium:
  Barata, Du, Li, Qian, Salgado, PRD 106, 074013 (2022)
  Barata, Salgado, EPJC 81, 862 (2021)

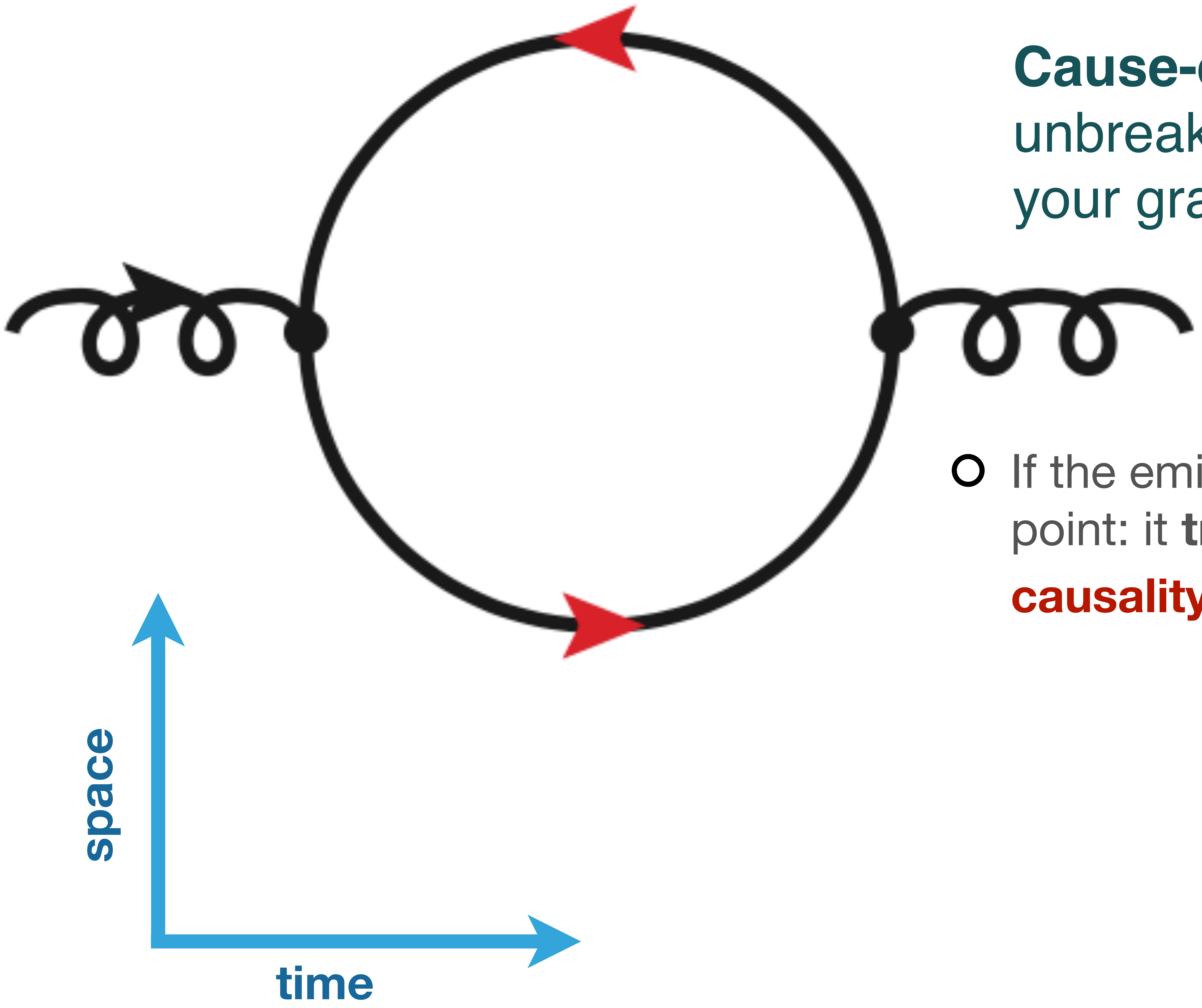
- parton showers:
  Bauer, de Jong, Nachman, Provasoli, PRL 126, 062001 (2021)
  Bauer, Freytsis, Nachman, PRL 127, 212001 (2021)
  Bepari, Malik, Spannowsky, Williams, PRD 106, 056002 (2022)

- jet clustering:
  Wei, Naik, Harrow, Thaler, PRD 101, 094015 (2020)
  Pires, Bargassa, Seixas, Omar, 2101.05618
  Pires, Omar, Seixas, 2012.14514
  Martinez de Lejarza, Cieri, GR, PRD 106, 036021 (2022)

- quantum machine learning:
  Guan, Perdue, Pesah, Schuld, Terashi, Vallecorsa, Vlimant, MLST 2, 011003 (2021)
  Wu et al., JPG 48, 125003 (2021)
  Felser, Trenti, Sestini, Gianelle, Zuliani, Lucchesi, Montangero, npjQI 7, 111 (2021)



Cause-effect relationships are unbreakable: You cannot be born before your grandfather [Symmetry Magazine]



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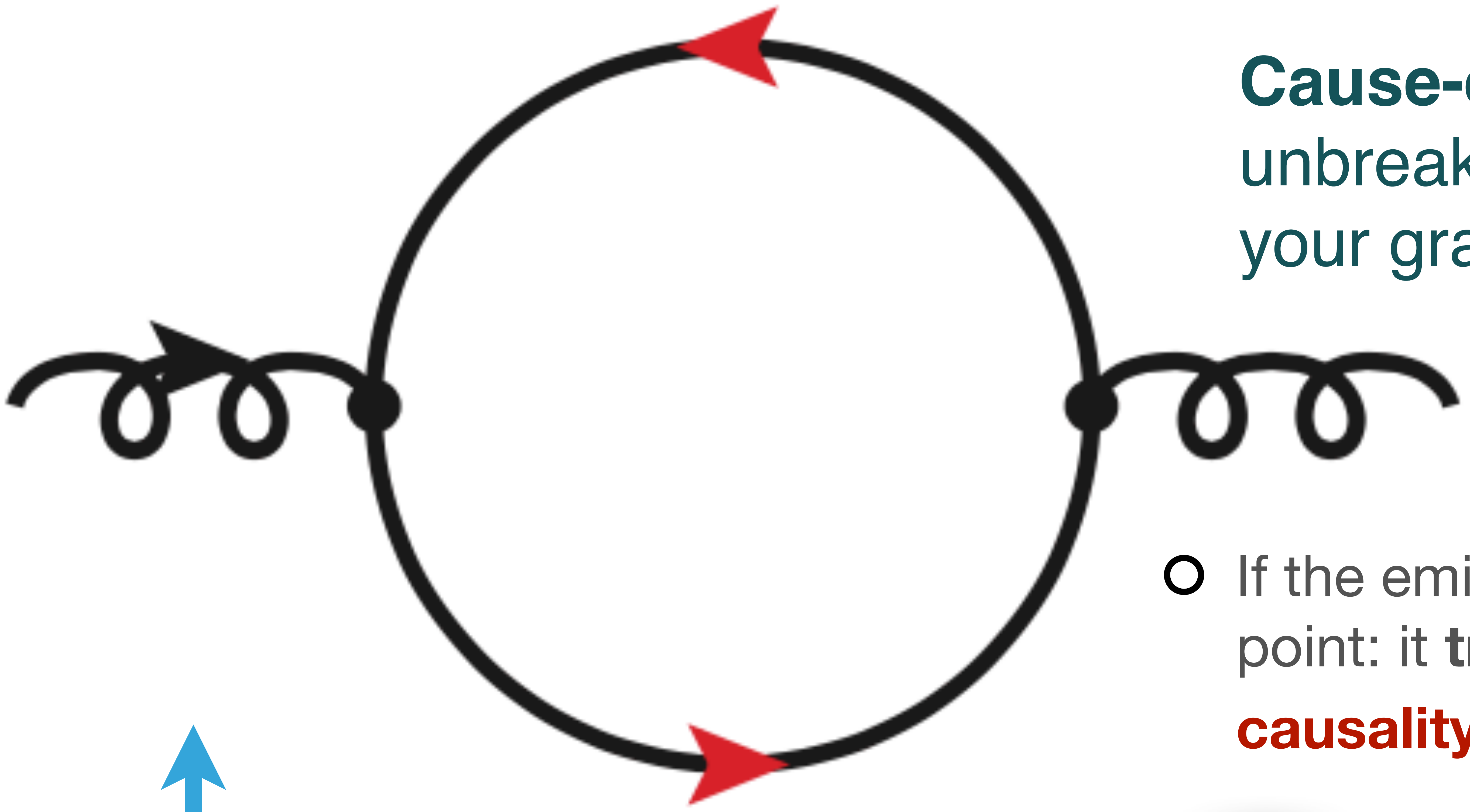
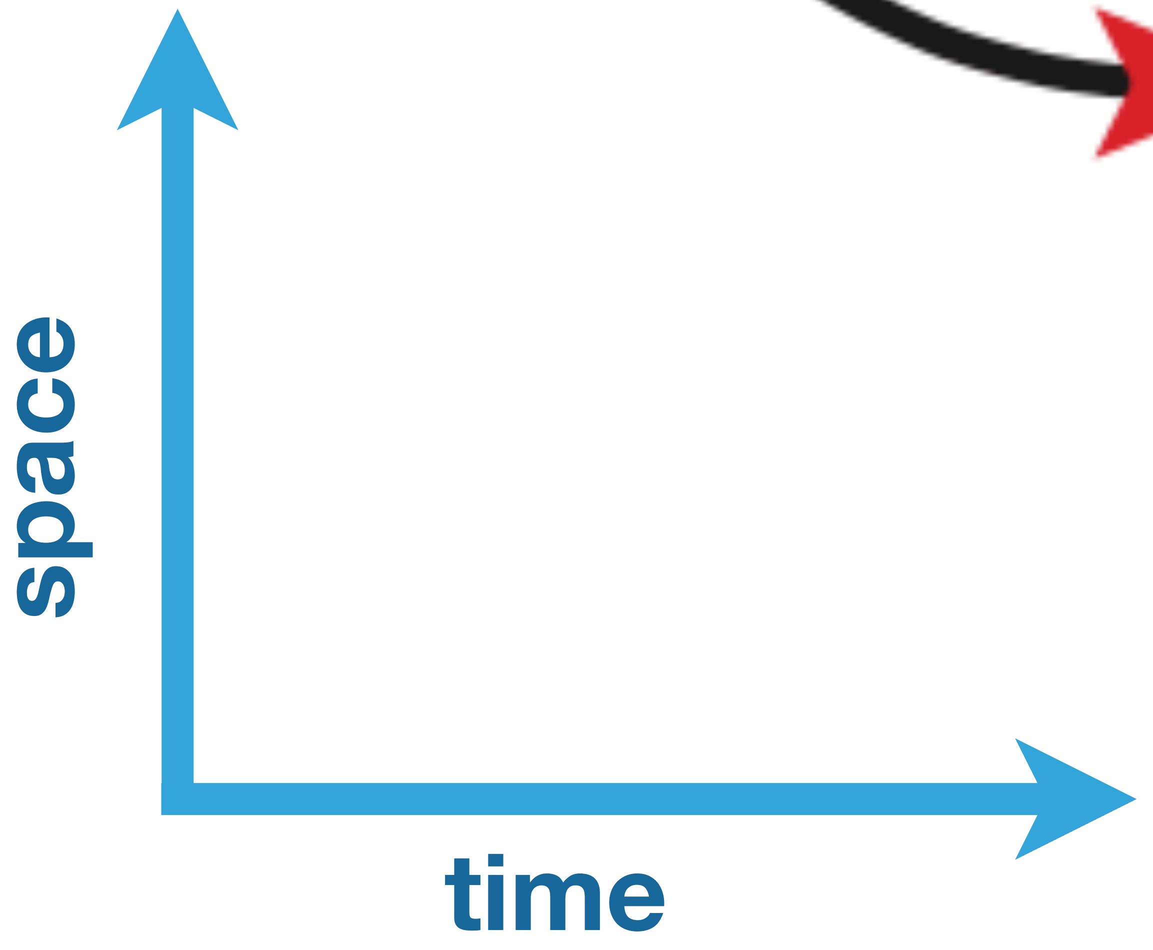
- If the emitted particle returns to the starting point: it **travels back in time** and thus **breaks causality** ≡ cyclic configurations are forbidden





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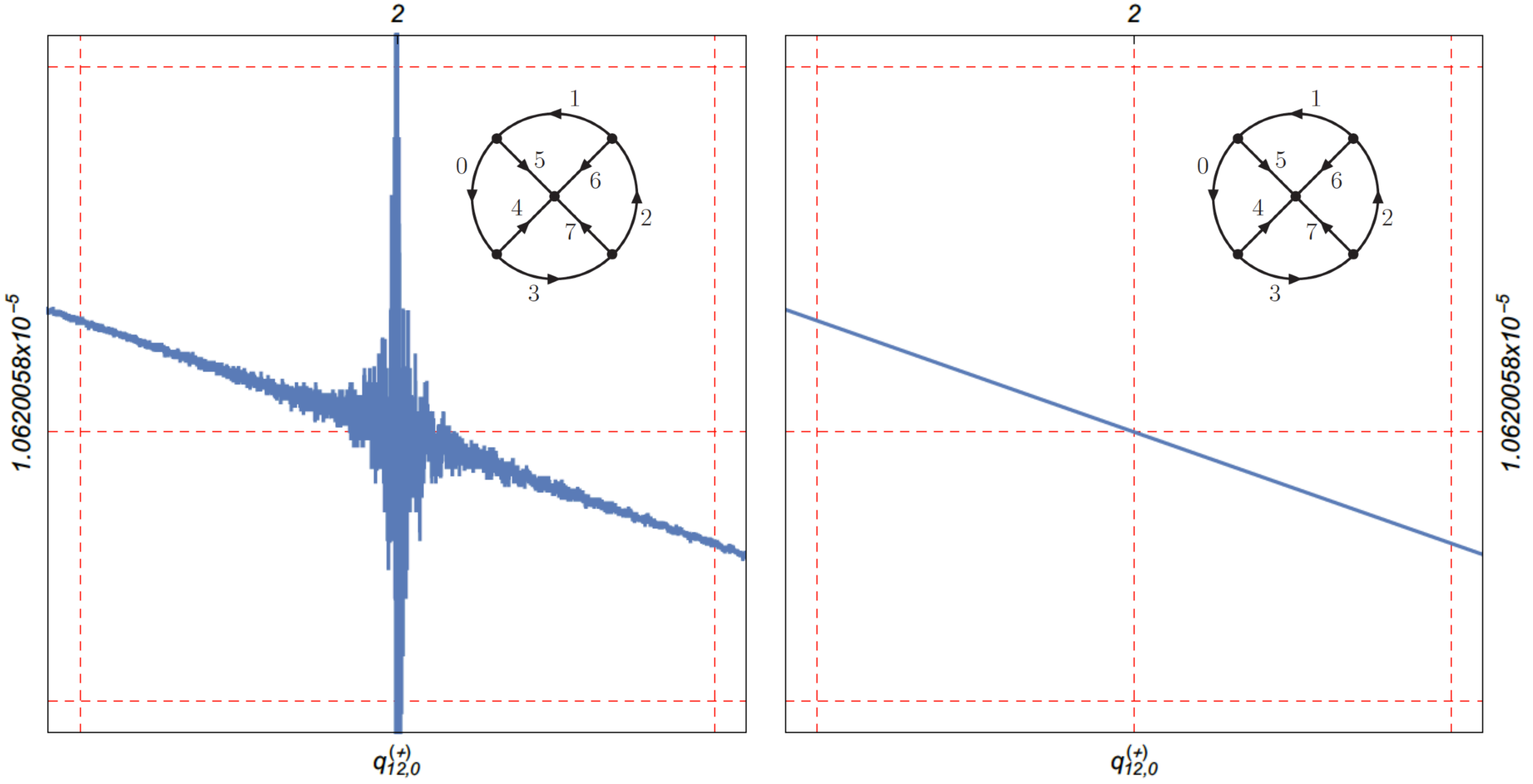
- Feynman propagators describe a quantum superposition of propagation in both directions

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124, 211602 (2020)

JHEP **1912**, 063 | JHEP **2101**, 069 | JHEP **2102**, 112 | JHEP **2104**, 129

NUMERICAL STABILITY OF CAUSAL LTD

- integrands in the Feynman representation have singularities that are nonphysical ≡ not related to the optical theorem
- LTD leads to **manifestly causal representations** (free of non-causal singularities): more stable numerically



- **Integrand numerical instabilities** across a noncausal threshold
- manifestly causal LTD representation

Ramírez, Rentería, Rodrigo, Sborlini, Vale Silva, JHEP 2205, 100
Clemente, Crippa, Jansen, Ramírez, Rentería, Rodrigo, Sborlini, Vale Silva, 2210.13240

LOOPS IN A QUANTUM COMPUTER

- Each Feynman propagator has **two on-shell states**, one with positive energy $|1\rangle$, and another with negative energy $|0\rangle$ ≡ momentum flow in one direction or the opposite

- Objective: identify those configurations that are **causal ≡ acyclic** momentum flows
- Bootstrapping the integrand representation in the **Loop-Tree duality**

Grover's algorithm
Amplitude amplification

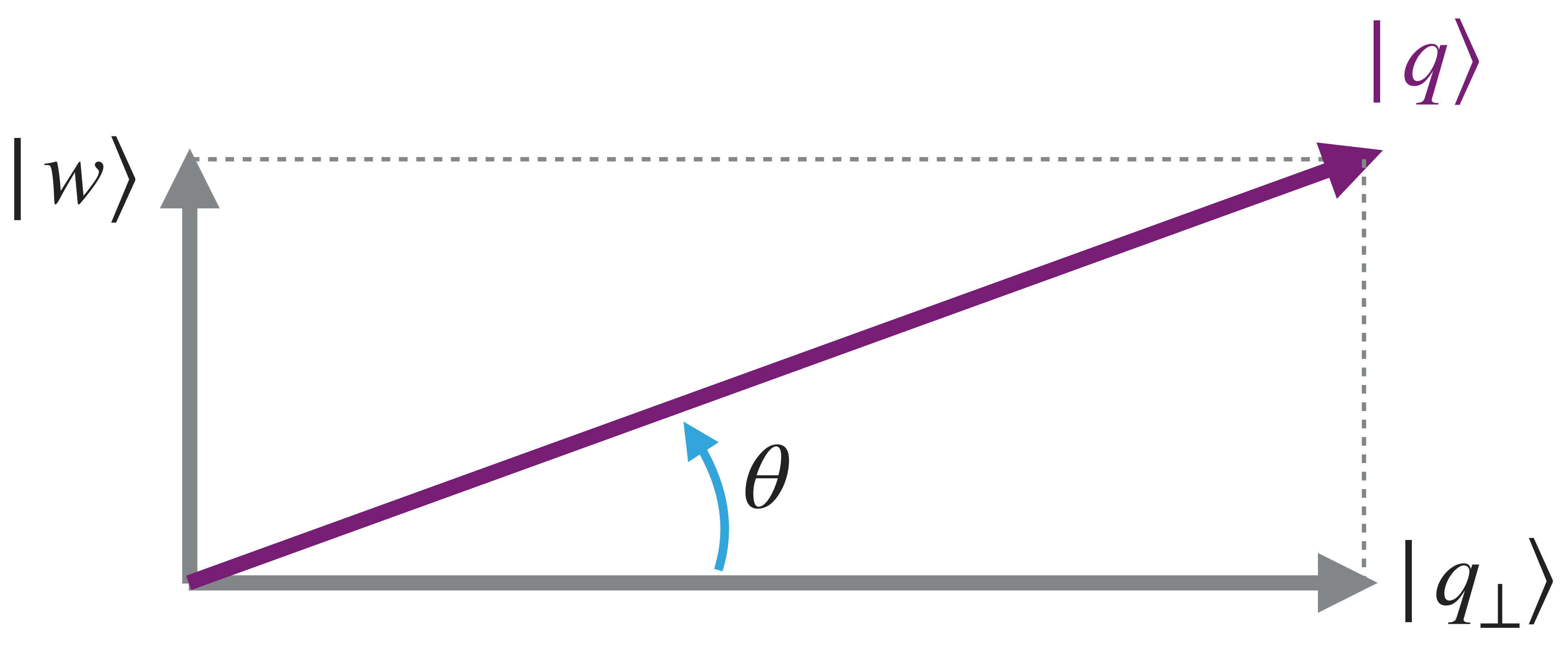
Variational Quantum Eigensolver
Minimization of a Hamiltonian

GROVER'S ALGORITHM

1 Starting from a uniform superposition of $N = 2^n$ states

$$|q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

GROVER'S ALGORITHM



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$$|q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

- which is a superposition of the winning state $|w\rangle$ encoding the r **causal states**, and the orthogonal state $|q_\perp\rangle$ collecting the noncausal states

$$|q\rangle = \sin\theta |w\rangle + \cos\theta |q_\perp\rangle$$

$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x\in w} |x\rangle \qquad |q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x\notin w} |x\rangle$$

- With mixing angle is

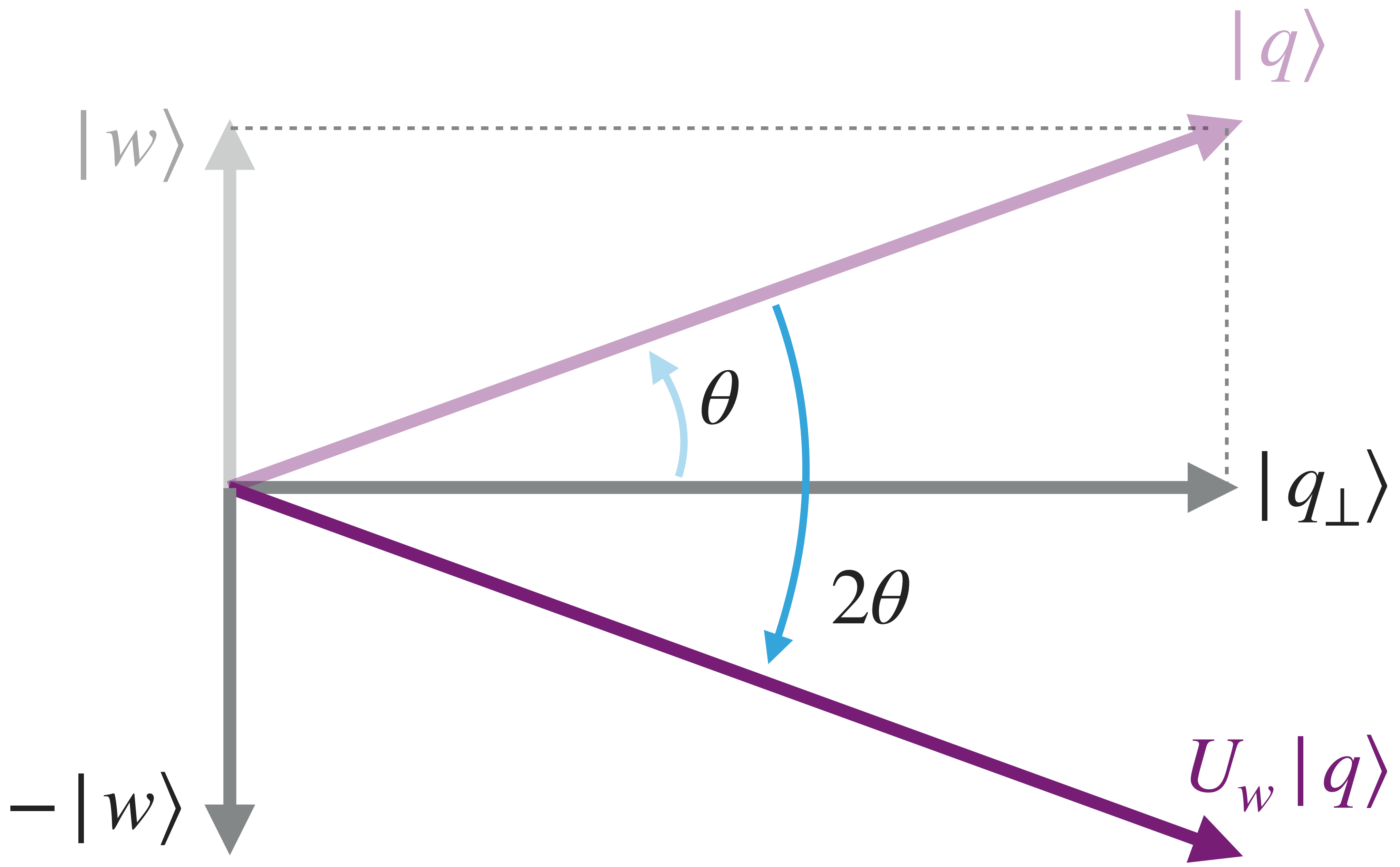
$$\theta = \arcsin \sqrt{r/N}$$

GROVER'S ALGORITHM

2 The **oracle operator**

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|$$

- **flips** the state $|x\rangle$ if $x \in w : U_w|x\rangle = -|x\rangle$
- leaves it unchanged otherwise: $U_w|x\rangle = |x\rangle$ if $x \notin w$



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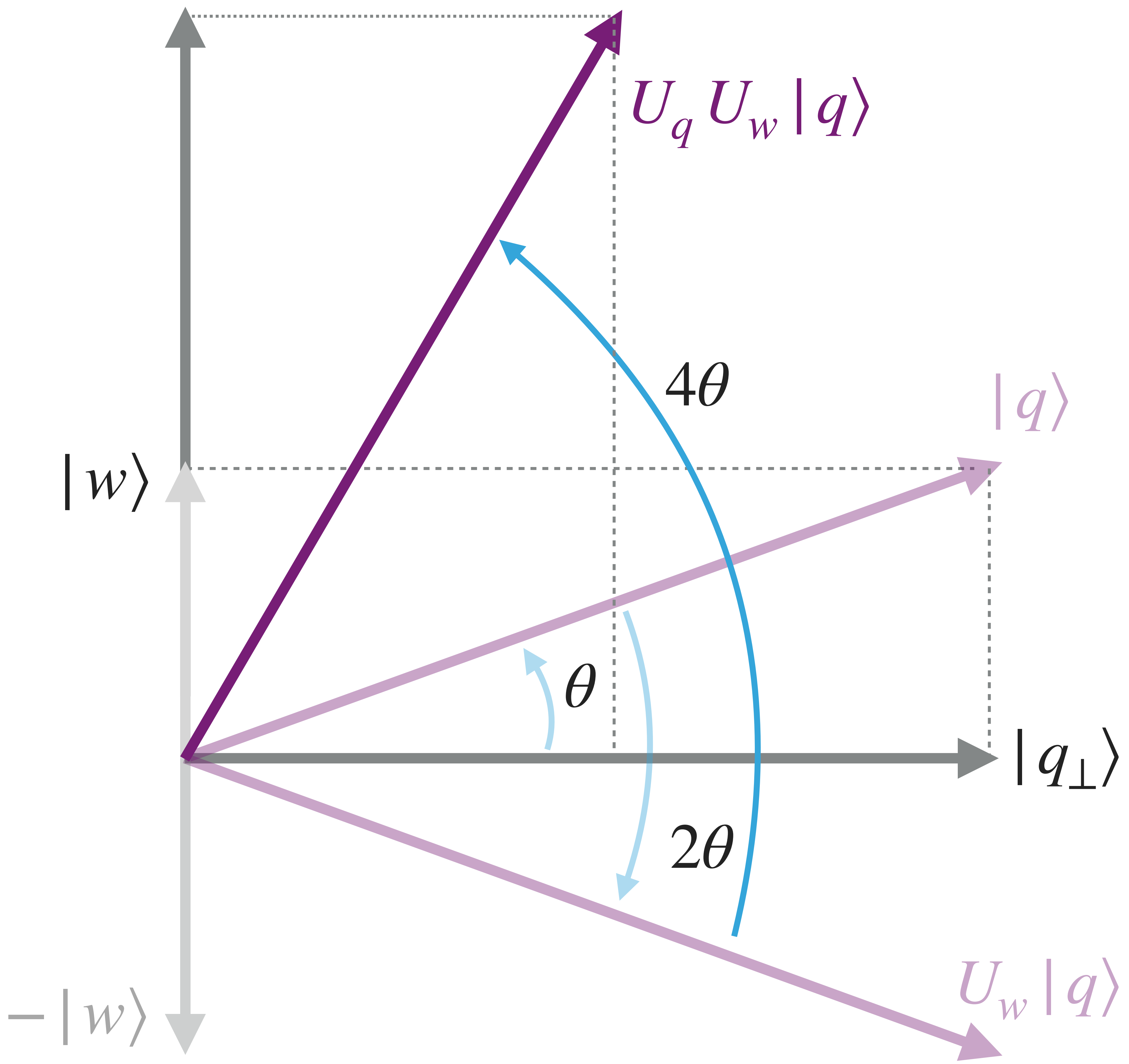

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3 The **diffusion operator**

$$U_q = 2|q\rangle\langle q| - \boldsymbol{I}$$

- performs a **reflection** around the initial state.

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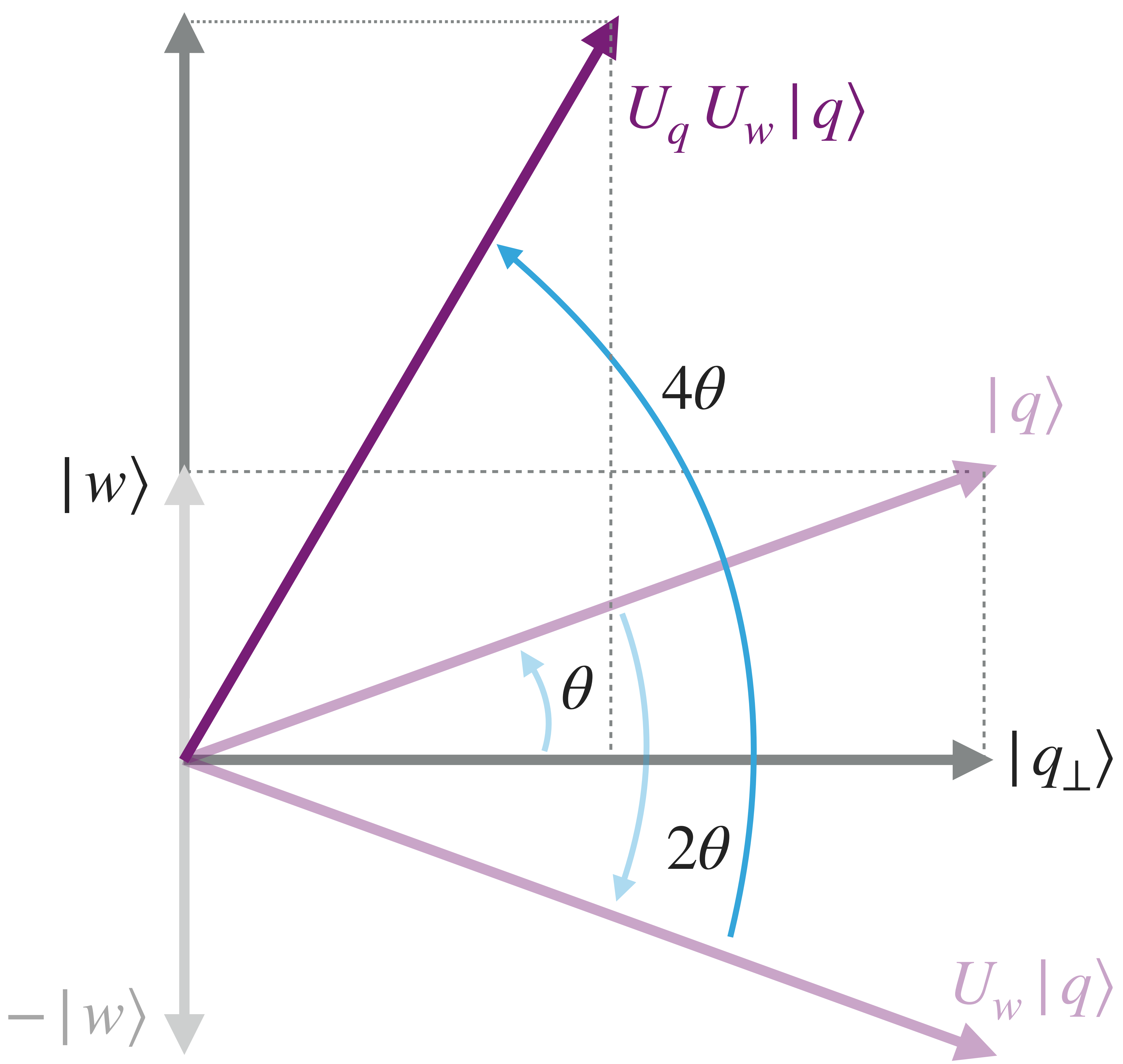

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4 Iterative application t times leads to

$$(U_q U_w)^t |q\rangle = \sin\theta_t |w\rangle + \cos\theta_t |q_\perp\rangle \qquad \theta_t = (2t+1)\,\theta$$

- if $r \ll N$, requires $\mathcal{O}(\sqrt{N/r})$ iterations instead of $\mathcal{O}(N)$ from a **classical** computation

LOOP QUANTUM ALGORITHM

- **The $|q\rangle$ register** encodes the states of the edges/internal propagators: the qubit q_i is in the state $|1\rangle$ if the momentum flow of the corresponding edge is oriented in the direction of the original assignment, and $|0\rangle$ if it is in the opposite direction
- **The $|c\rangle$ register** stores the binary clauses that probe if two qubits representing two adjacent edges are in the same state (oriented in the same direction)

$$c_{ij} \equiv (q_i = q_j) \qquad \bar{c}_{ij} \equiv (q_i \neq q_j)$$

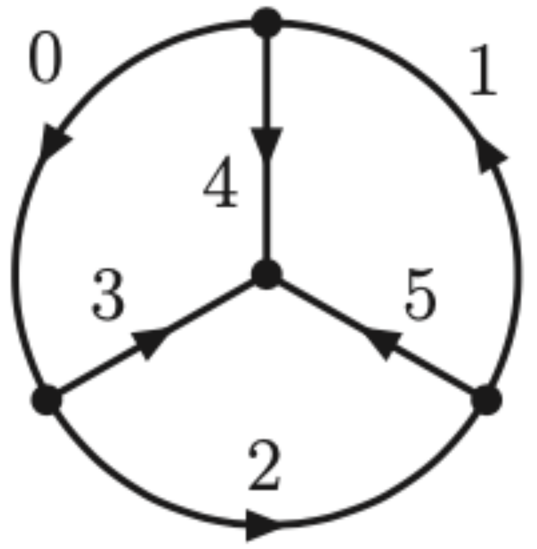
- **The $|a\rangle$ register** stores the loop clauses that probe if all the qubits (edges) in each subloop form a cyclic circuit
- **The Grover's marker** initialized to the Bell state $|out_0\rangle = |-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$
- **The oracle**

$$U_w|q\rangle|c\rangle|a\rangle|out_0\rangle = |q\rangle|c\rangle|a\rangle|out_0 \otimes f(a,q)\rangle$$

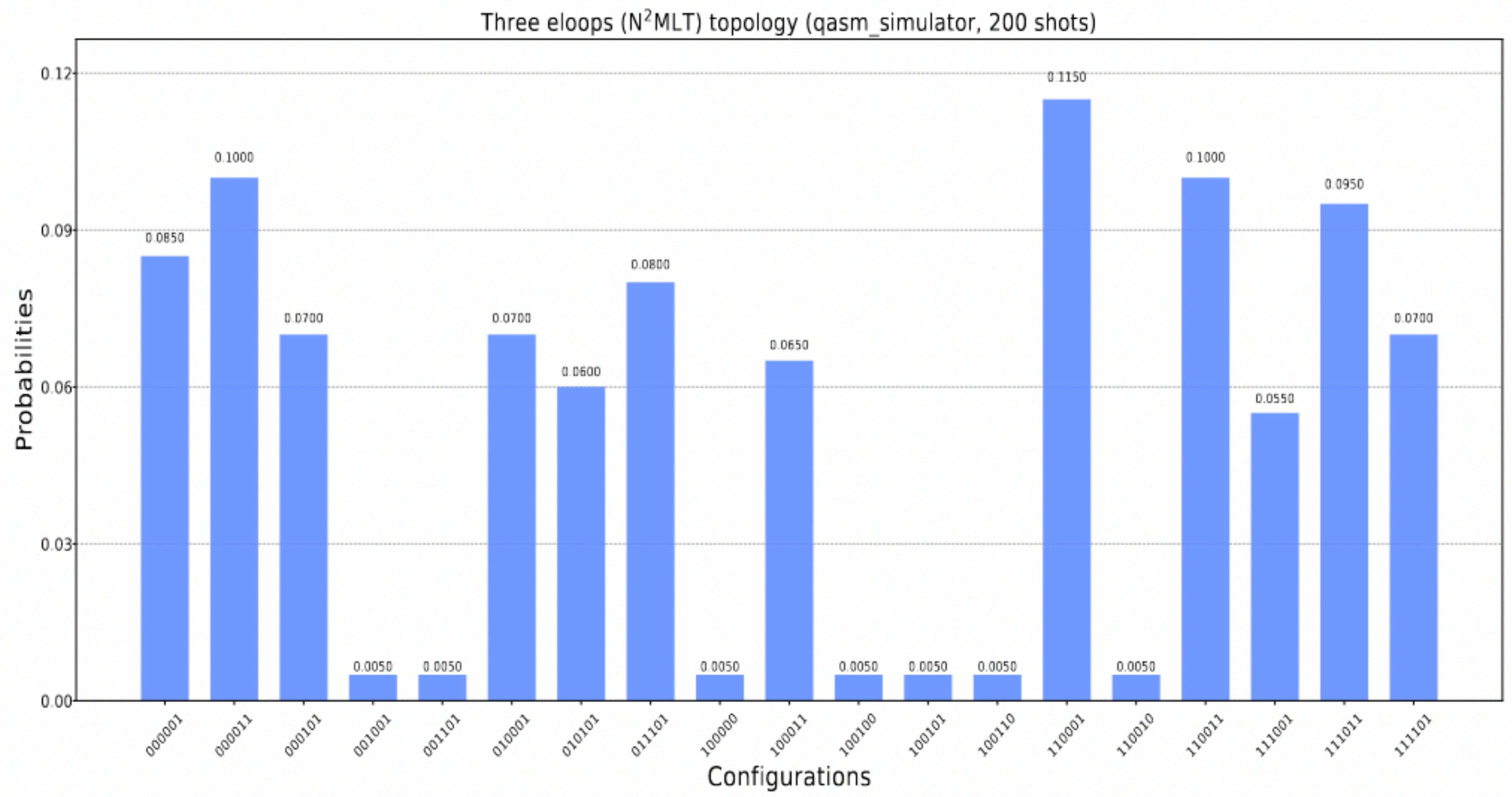
$$|out_0 \otimes 0\rangle = |out_0\rangle$$
$$|out_0 \otimes 1\rangle = -|out_0\rangle$$

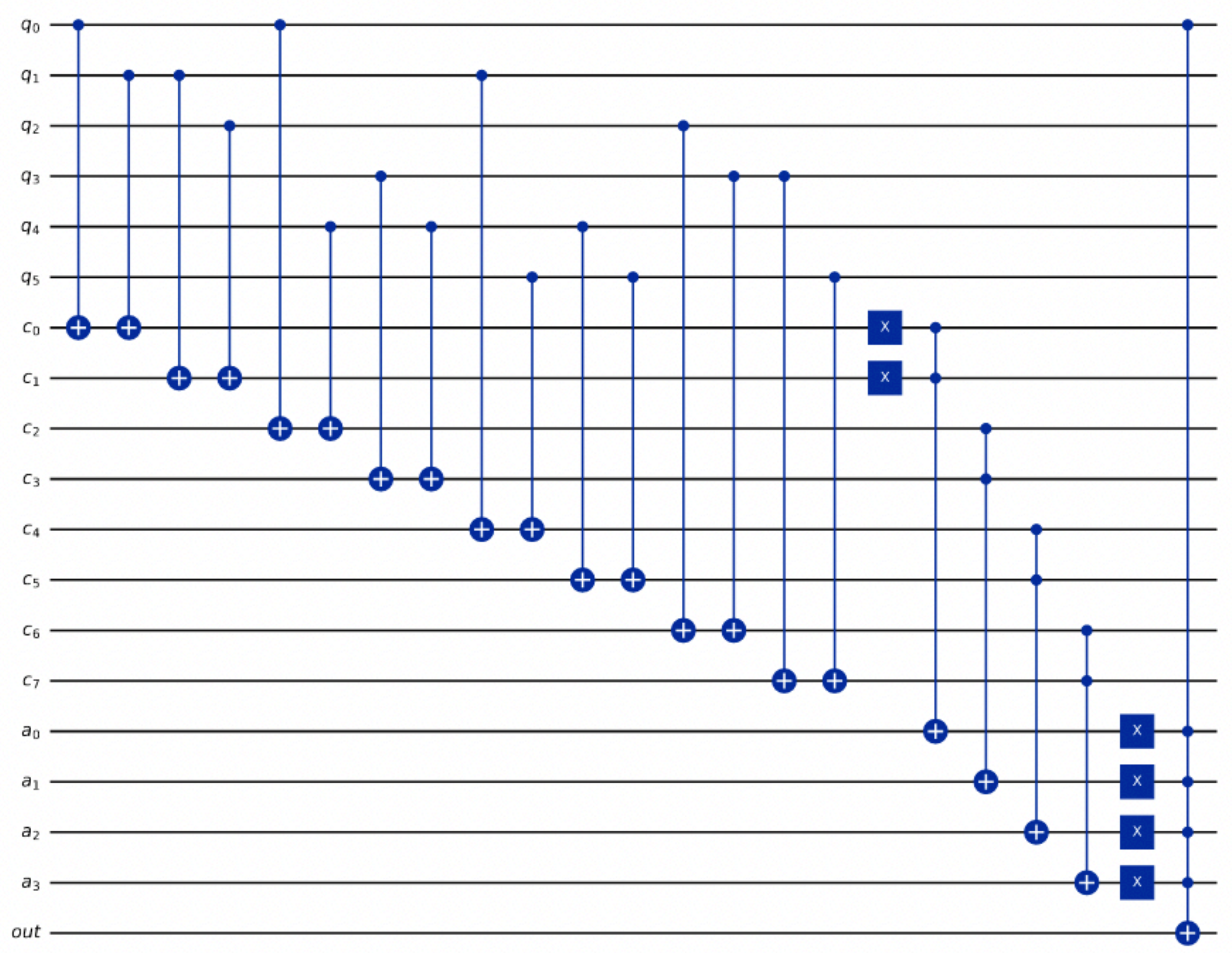
- **The diffuser U_q** from IBM Qiskit

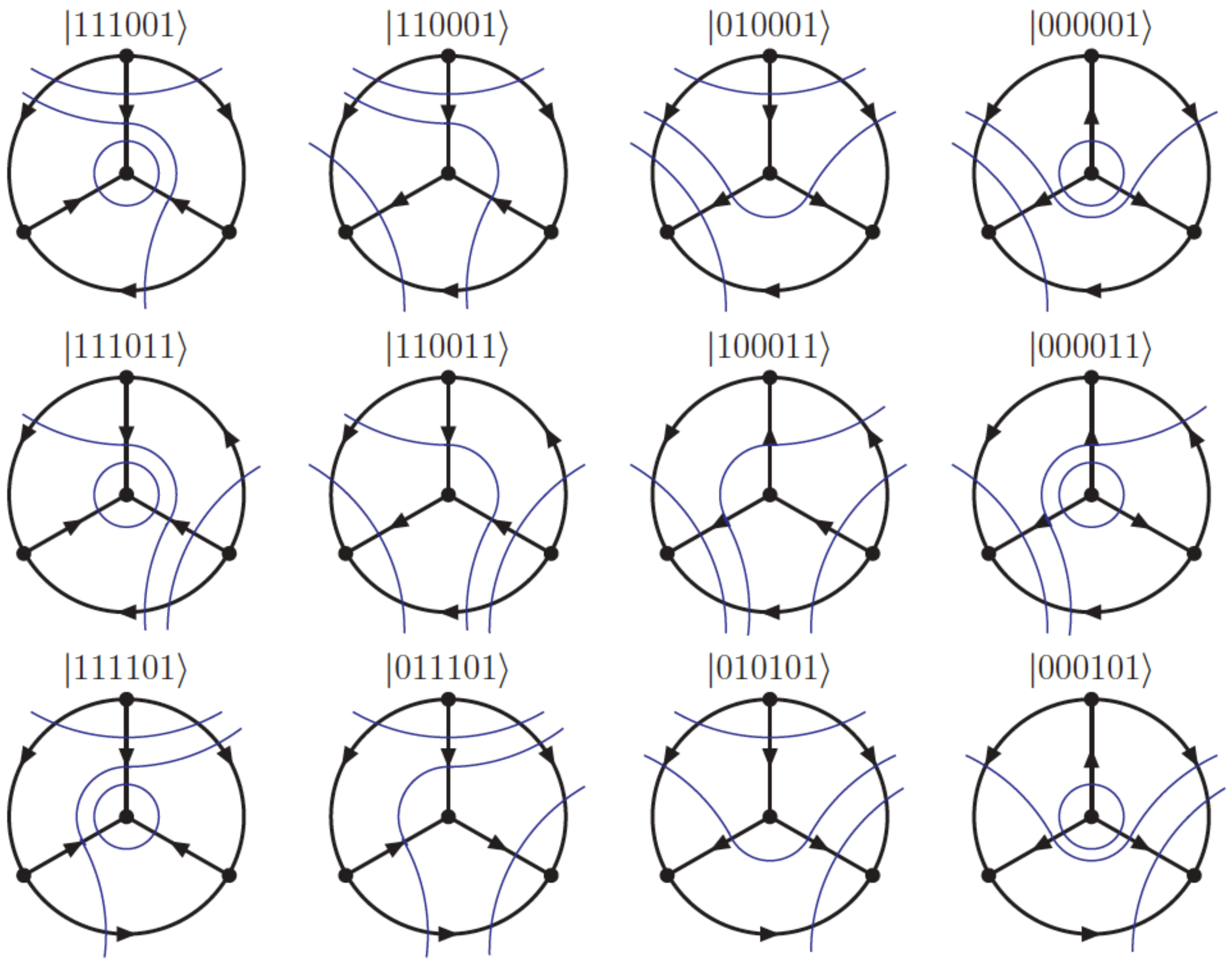
THREE ELOOPS Qiskit



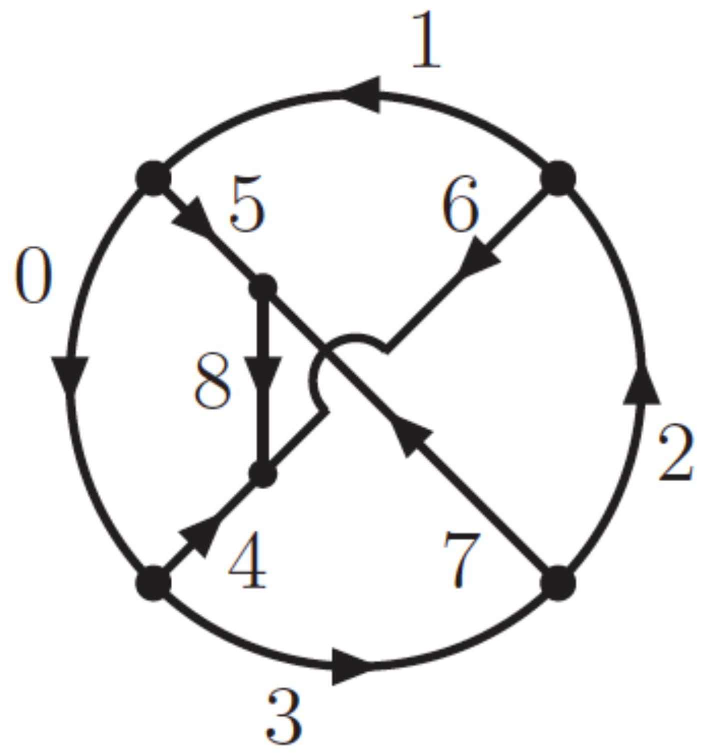
$$a_0 = \neg(c_{01} \wedge c_{12}) \qquad a_1 = \neg(\bar{c}_{04} \wedge \bar{c}_{34})$$
$$a_2 = \neg(\bar{c}_{15} \wedge \bar{c}_{45}) \qquad a_3 = \neg(\bar{c}_{23} \wedge \bar{c}_{35})$$
$$f^{(3)}(a, q) = (a_0 \wedge \ldots \wedge a_3) \wedge q_0$$



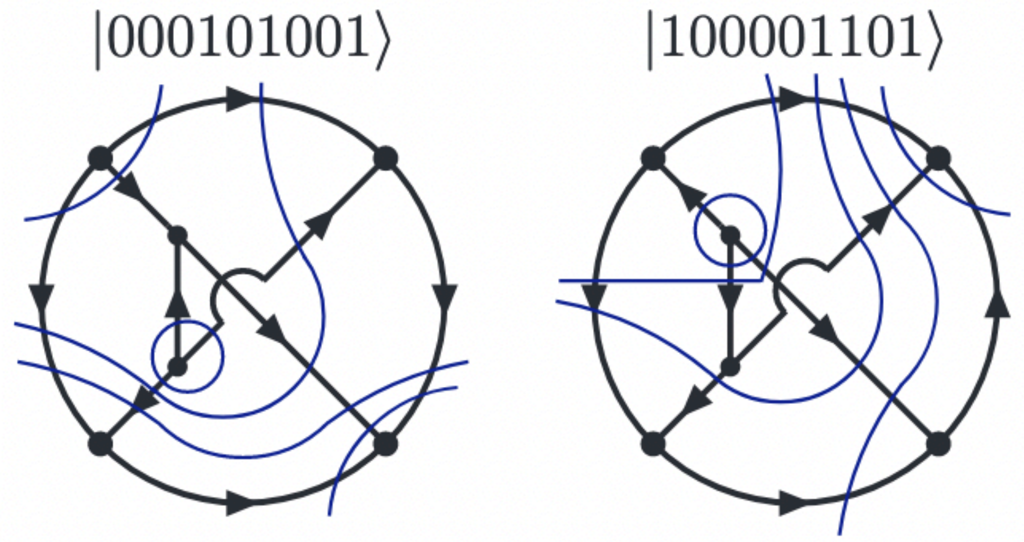


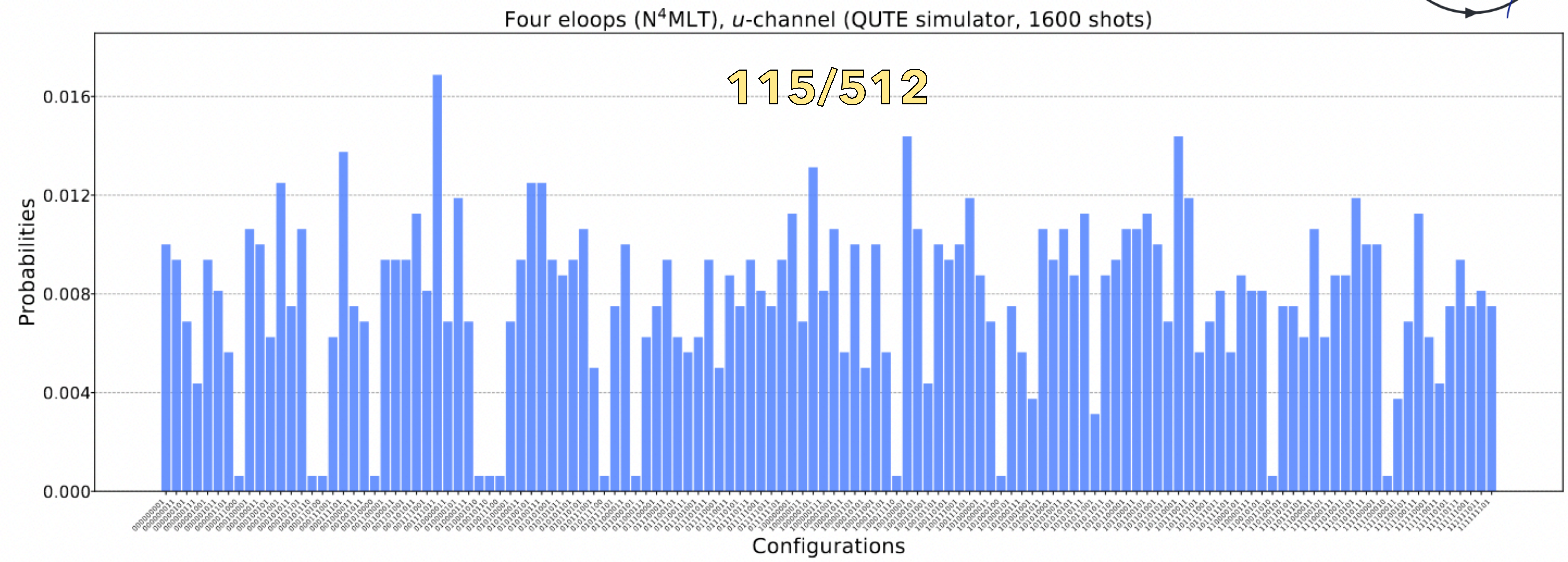


FOUR ELOOPS

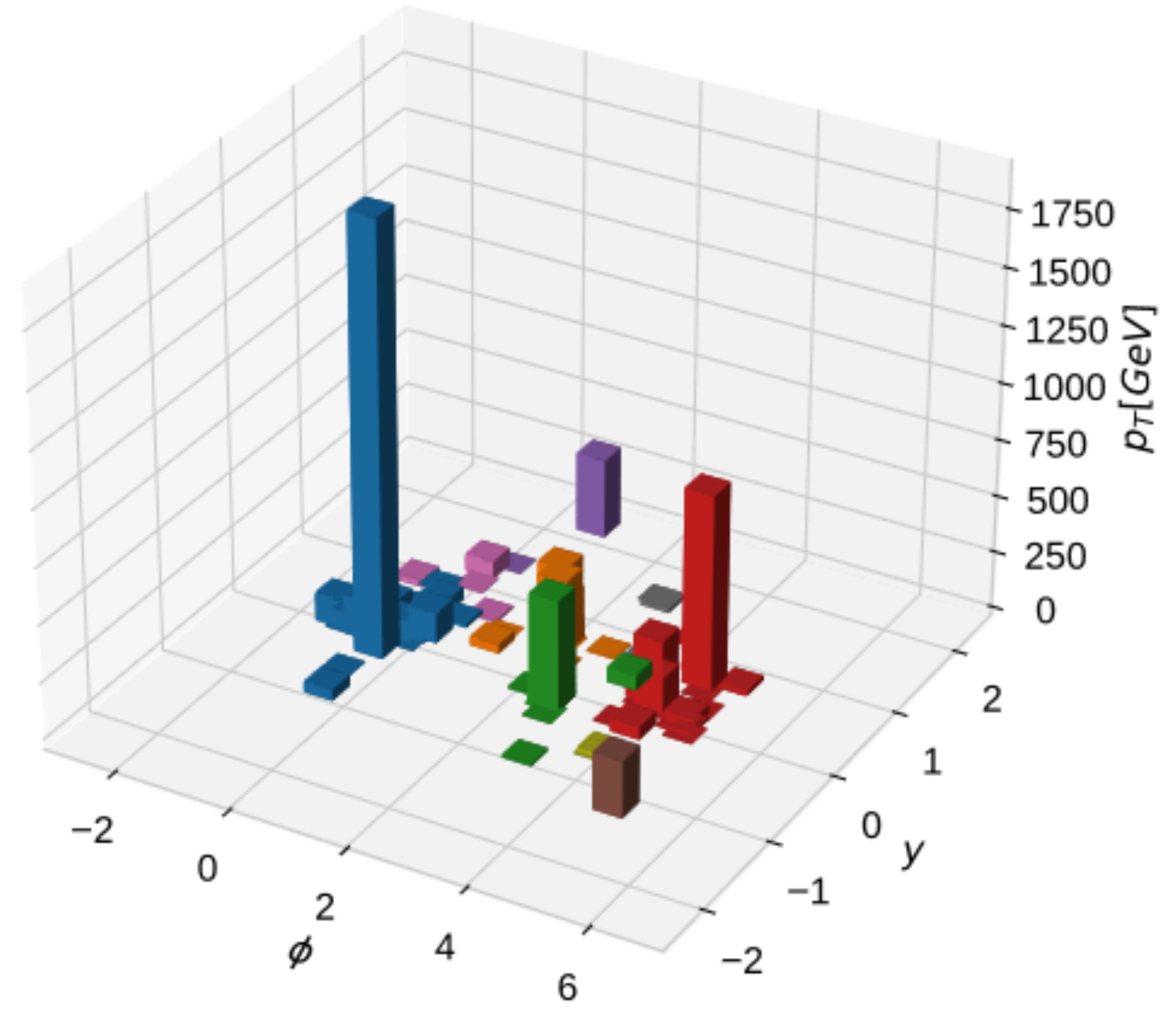


- First **nonplanar** Feynman diagram starting at four loops
- 115/512 causal states
- **QUTE simulator**, up to 38 logical qubits
  Fundación Centro Tecnológico de la Información y la Comunicación (CTIC), Gijón, Spain



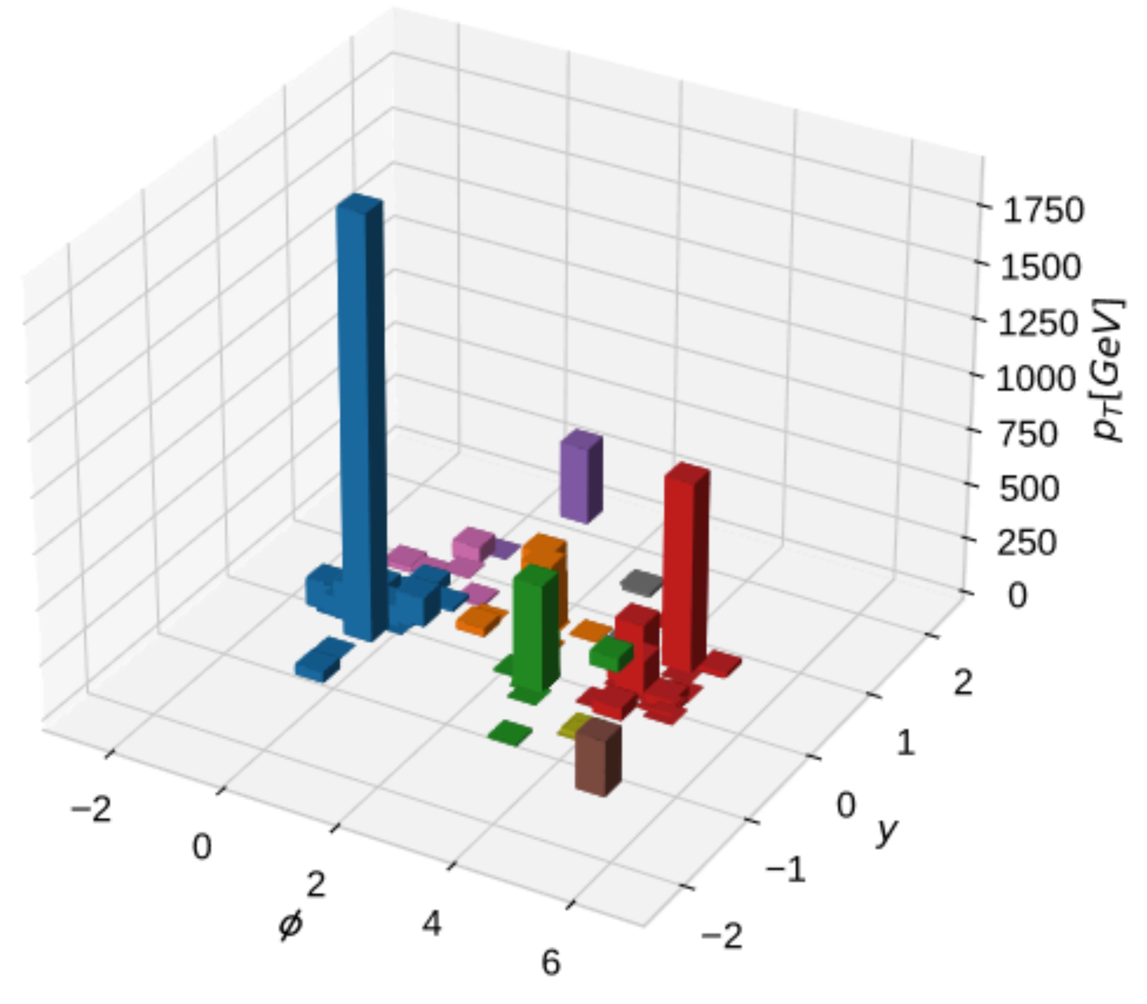


JET CLUSTERING AT THE LHC

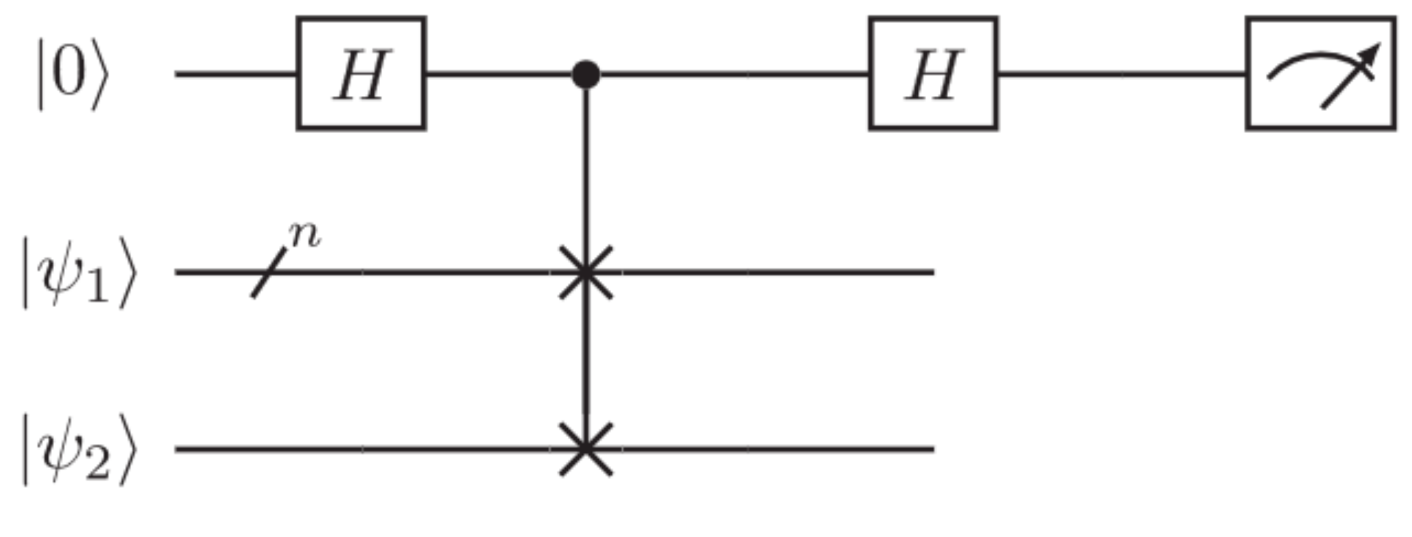


(a) Classical anti-k_T, $p=-1$, $R=1$.

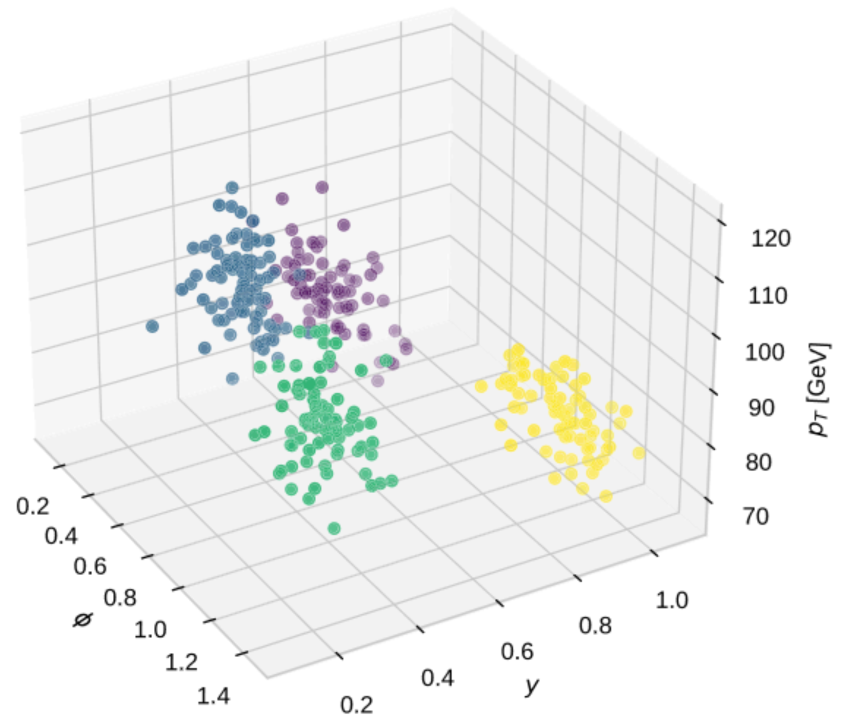
- Three clustering algorithms: K-means, Affinity Propagation, k_T-jet
- Hierarchical classical clustering (e.g. k_T-jet) requires to find the absolute minimum distance at each intermediate steep: very costly
- Mininum-distance quantum algorithm is probabilistic and less expensive



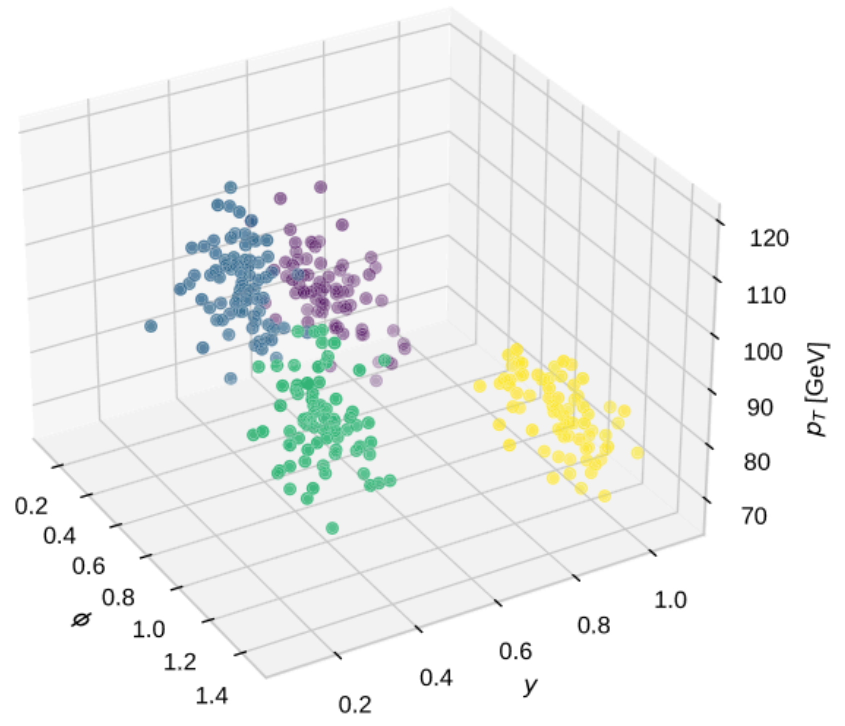
(b) Quantum anti-k_T, $p=-1$, $R=1$, $\epsilon_c=0.99$.



SwapTest to calculate distances



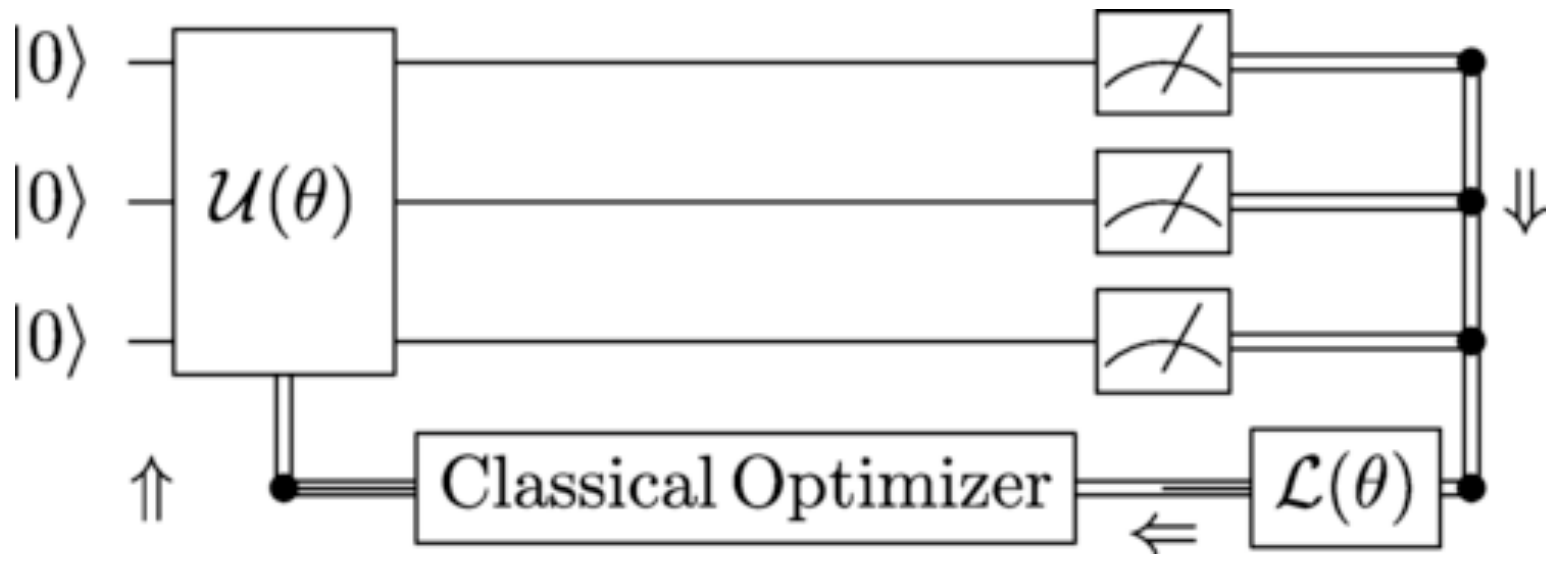
(a) Classical K-means clustering, $\varepsilon_t = 1.00$.

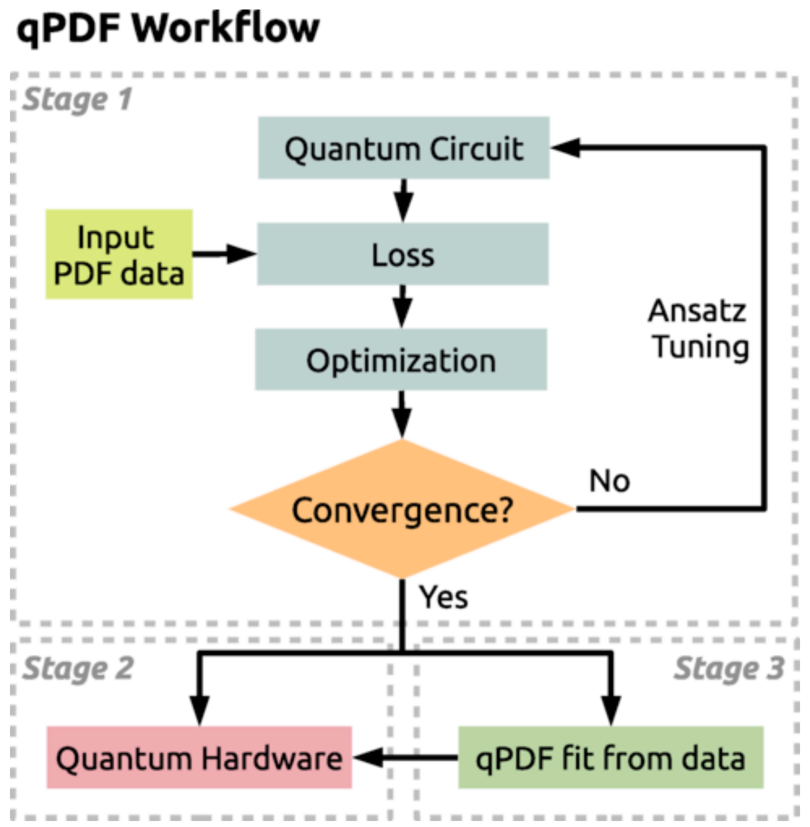


(b) Quantum K-means clustering, $\varepsilon_t = 1.00$.

Pérez-Salinas, Cruz-Martínez, Alhajri, Carrazza, PRD 103, 034027 (2021)

QUANTUM MACHINE LEARNING

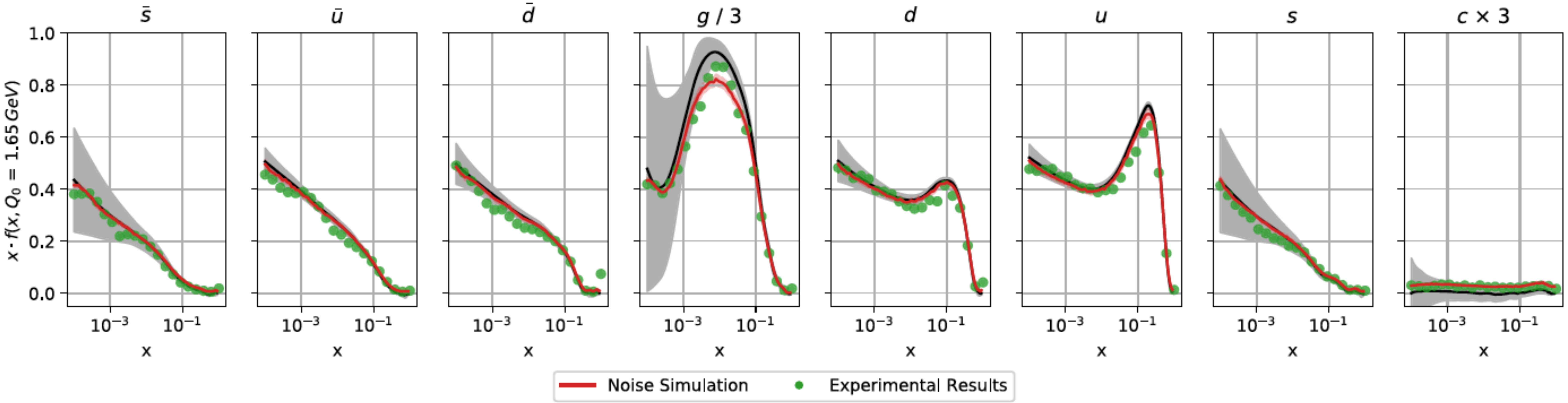




- A classical computer doing most of the work, a quantum computer **solving the bottlenecks**
- A parametrized quantum circuit (PQC) whose inner parameters depend both on PDF data and trainable parameters

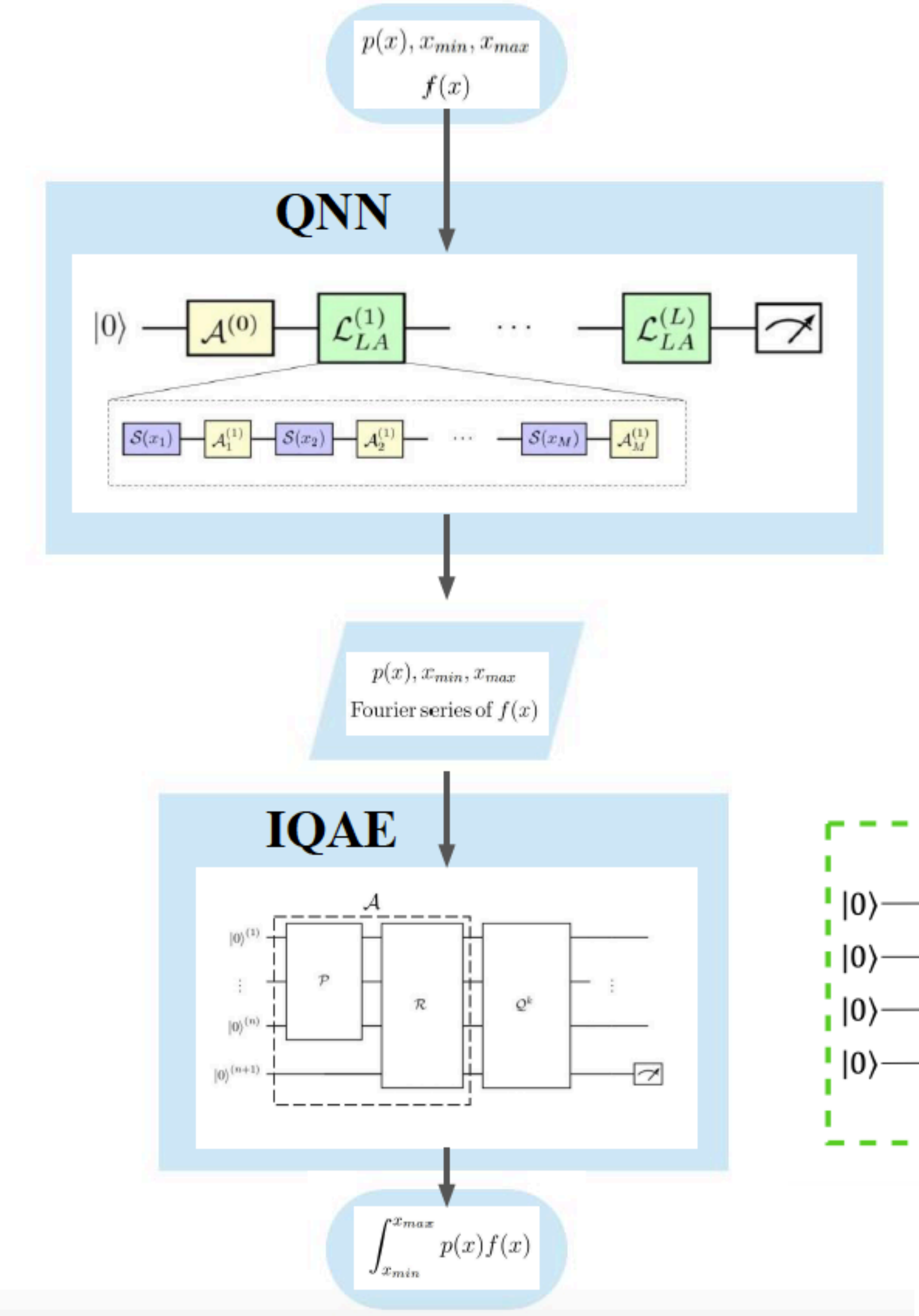
$$U_w(\alpha, x) = R_z\left(\alpha_3 \log(x) + \alpha_4\right) R_y(\alpha_1 x + \alpha_2)$$

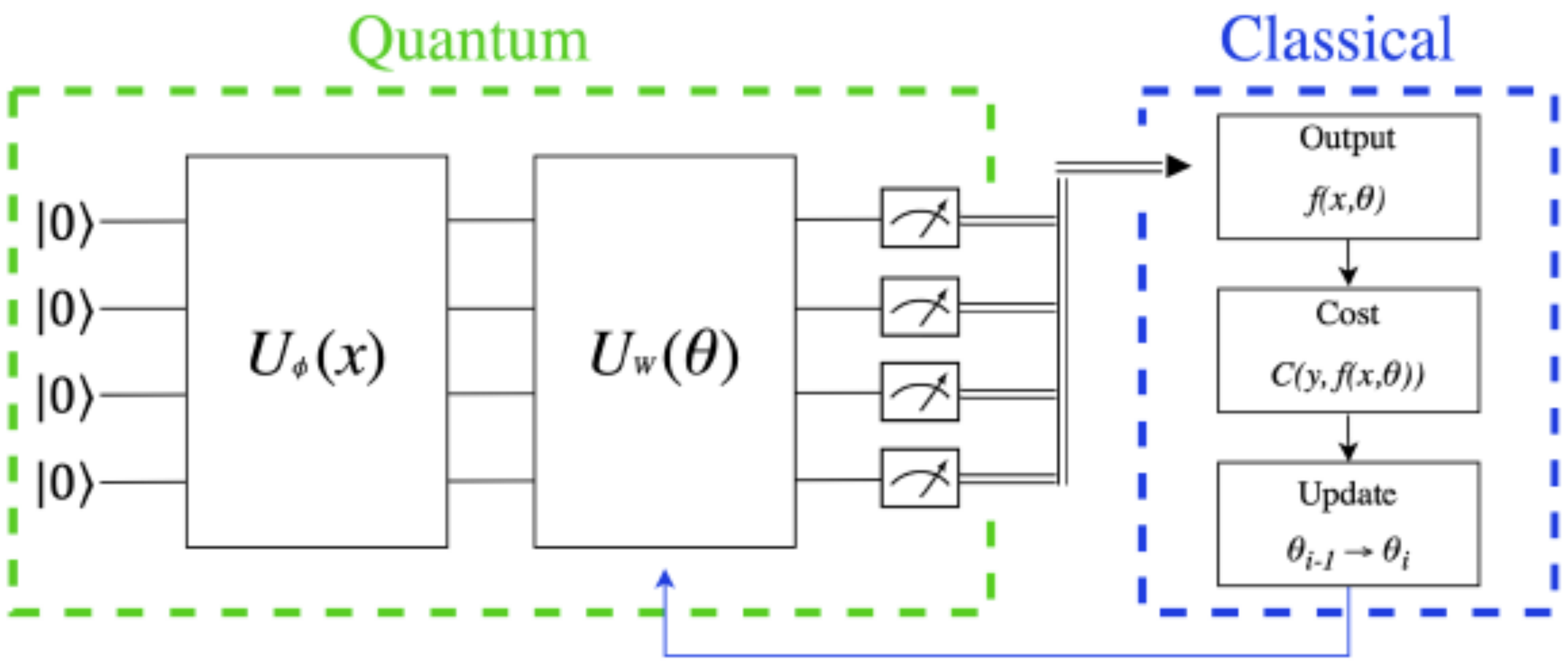
- The circuit applied to an initial quantum state, the output state contains information on PDFs
- Circuit parameters determined with classical optimization and a predefined cost function
- each qubit represents a flavour

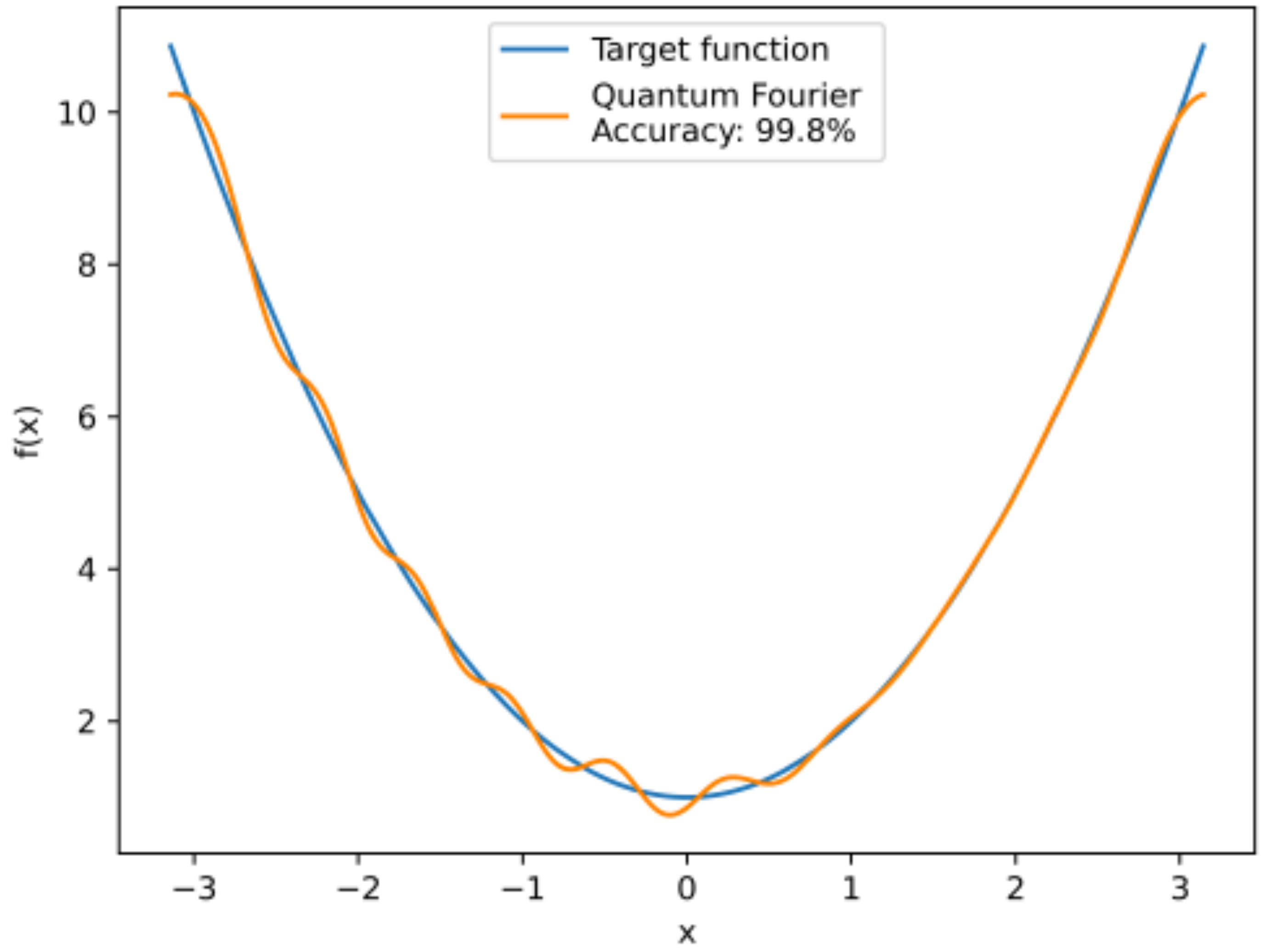


QUANTUM FOURIER ITERATIVE AMPLITUDE ESTIMATION (QFIAE)

- Quantum Machine Learning + Grover's amplification
- Fourier series using a Quantum Neural Network (QNN)
- Integrates each component using Iterative Quantum Amplitude Estimation (IQAE), a variant of Grover's algorithm







CONCLUSIONS

- **Quantum algorithms** are an interesting quantum pathway to QFT
- Apart from potential speedups, a new quantum perspective on theoretical predictions and experimental analysis at high-energy colliders
- Many promising applications in particle physics, despite current hardware limitations