

# Precise determination of W/Z transverse-momentum spectrum including higher-order QCD-EW corrections.



**German F. R. SBORLINI**

Departamento de Física Fundamental  
Universidad de Salamanca (USAL)

10.05.2023

7th RED LHC Workshop  
(IFT - Madrid, Spain)



This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No.101034371

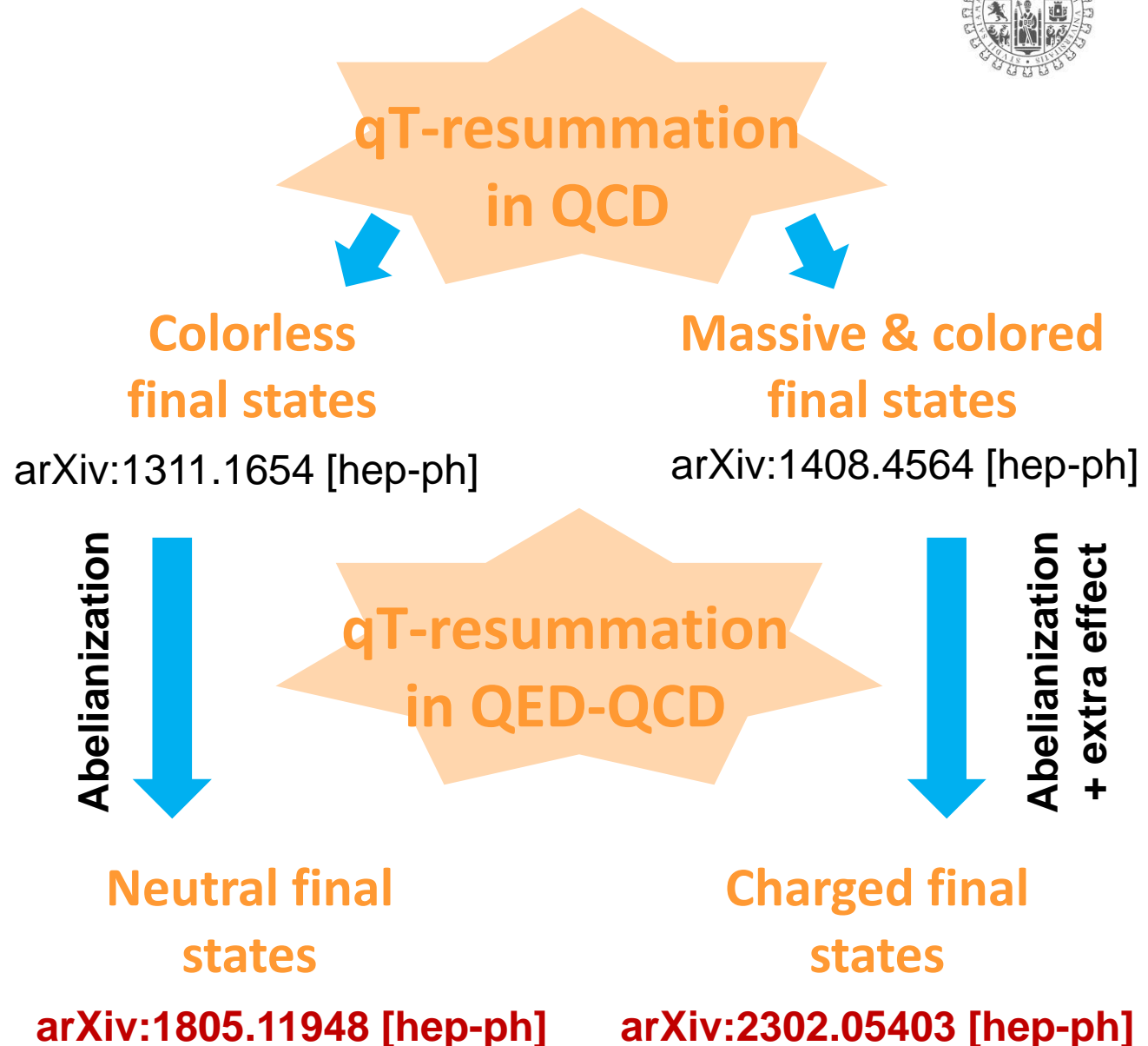


VNIVERSIDAD  
D SALAMANCA





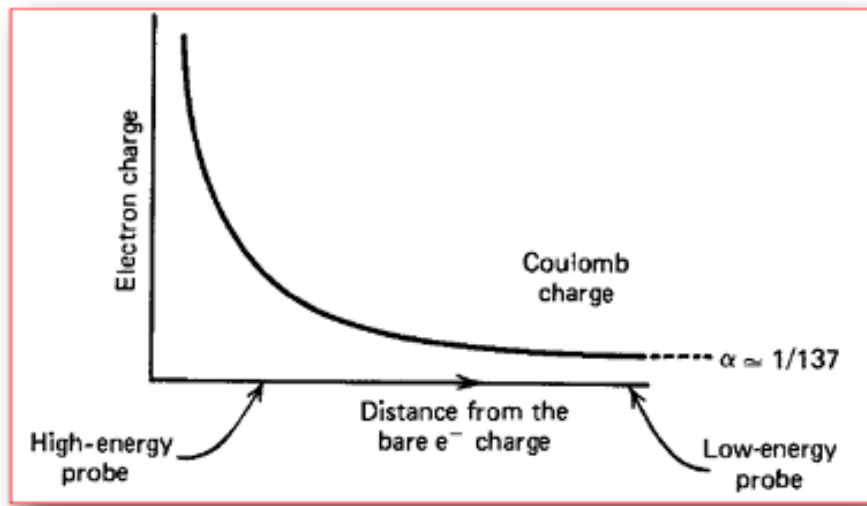
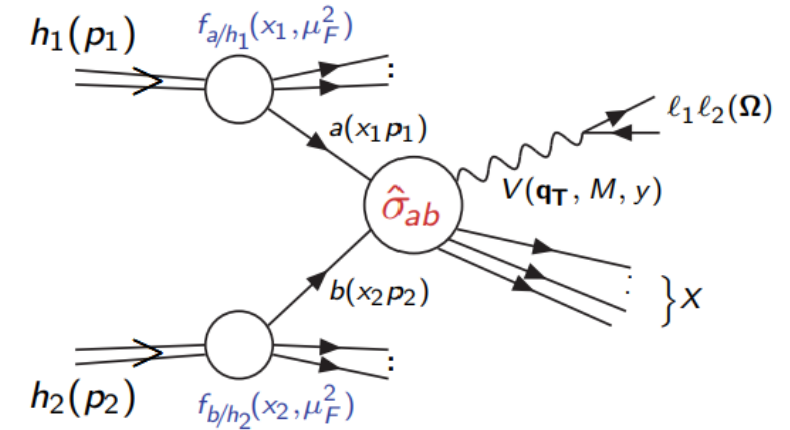
1. Motivation and introduction
  - A. qT-resummation in QCD
  - B. **qT-resummation in QED-QCD (neutral final state)**
  - C. **qT-resummation in QED-QCD (charged final state)**
2. Phenomenological results
  - A. **QED-QCD corrections to Z/W production**
  - B. **Impact on W/Z ratio**
3. Conclusions



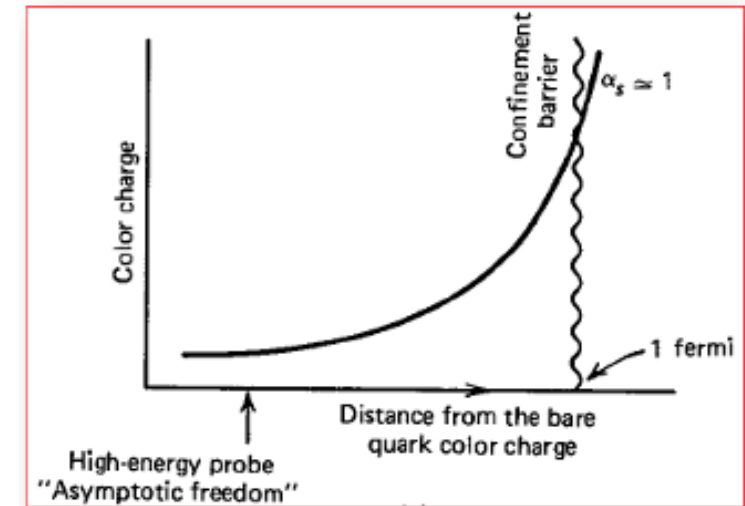
# Why do we need QED-QCD and resummation for DY?



- Drell-Yan (DY) is a paramount at hadron colliders: it is used for SM and BSM physics, calibration, SM parameter extraction, ...
- DY is measured with an astonishing precision (**per-thousand level!**)
- **Inclusion of EW/QED beyond LO could lead to novel effects:**
  - Quark-gluon interacting with leptons and photons
  - **Dependence on the photon content of the proton!**
  - Enhanced contributions at high-energies (due to the running EM coupling)



**QED vs. QCD**

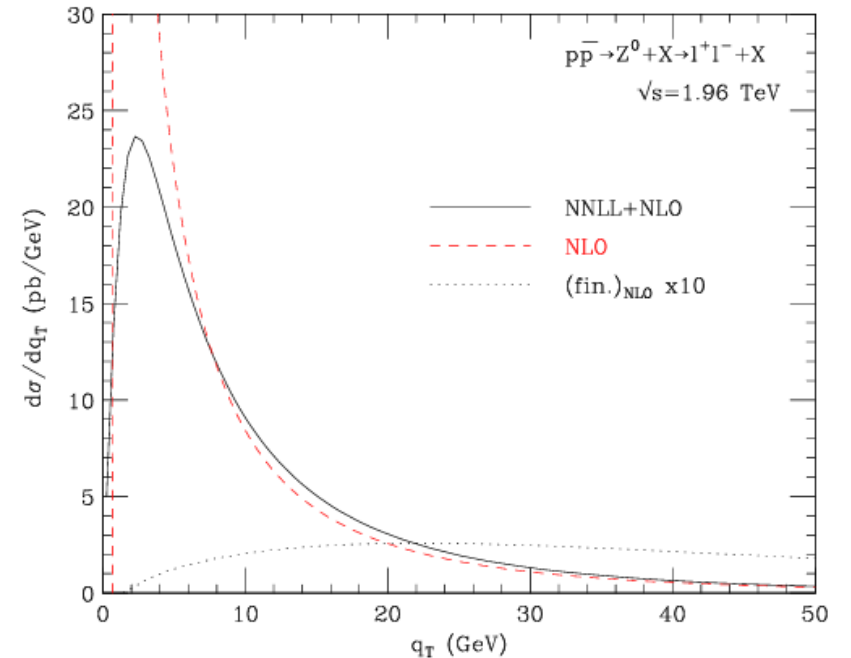
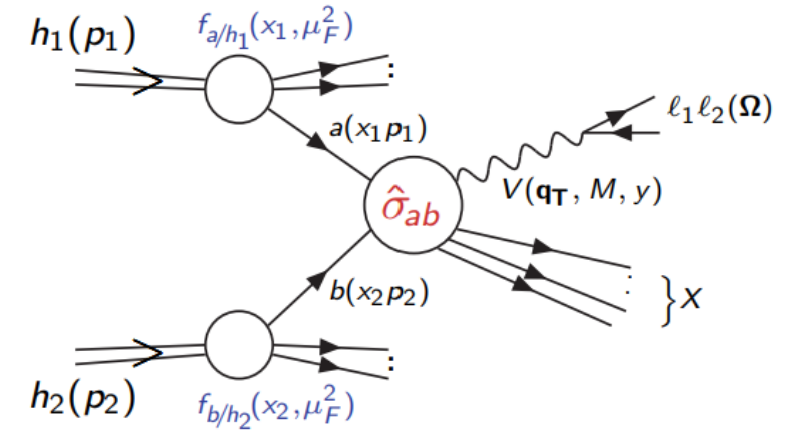


Michiel Botje, "Lectures on Particle Physics II"


# Why do we need QED-QCD and resummation for DY?



- Drell-Yan (DY) is a paramount at hadron colliders: it is used for SM and BSM physics, calibration, SM parameter extraction, ...
- DY is measured with an astonishing precision (**per-thousand level!**)
- **Inclusion of EW/QED beyond LO could lead to novel effects:**
  - Quark-gluon interacting with leptons and photons
  - **Dependence on the photon content of the proton!**
  - Enhanced contributions at high-energies (due to the running EM coupling)
- **Particular interest in the low  $q_T$ -region:**
  - Fixed-order calculations are no longer reliable
  - **Presence of logarithmically enhanced contributions!**
  - $q_T$  spectrum is phenomenologically relevant



arXiv:1007.2351 [hep-ph]

- Soft gluon radiation leads to a modification of the spectrum in the low-qT region
- **Perturbative fixed-order expansion not reliable**  **Re-arrange or resum:**

$$\int_0^{q_T^2} dq_T'^2 \frac{d\hat{\sigma}}{dq_T'^2} \approx 1 + \alpha_S [c_{12}L^2 + c_{11}L + \dots] + \alpha_S^2 [c_{24}L^4 + c_{23}L^3 + \dots] + \dots$$

- **Master formulae for qT-resummation in QCD:**

- The singular (i.e. divergent) part has an universal structure:

$$\frac{d\sigma_F^{(\text{sing})}(p_1, p_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T dM^2 dy d\Omega} = \frac{M^2}{s} \sum_{c=q, \bar{q}, g} \left[ d\sigma_{c\bar{c}, F}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} S_c(M, b) \times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [H^F C_1 C_2]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

- The **Sudakov factor** resums all the soft/collinear-emissions from the incoming legs; it is process independent
- The **“hard-collinear”** coefficients **H** and **C** are related with the hard-virtual and collinear parts, and also contain the process dependence.

- **Components of the resummation master formula:**

- The Sudakov factor contains the logarithmically enhanced contributions.

$$S_c(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[ A_c(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

with

$$\begin{cases} A_c(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n A_c^{(n)} \\ B_c(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n B_c^{(n)} \end{cases}$$

- $A_c$  and  $B_c$  depend on the leg responsible for the emission. **They are related to the splitting functions!**
- Also,  $C$  and  $H$  are calculable within perturbation theory.  $C$  is process independent ( $H$  contains the virtuals):

$$H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S) = 1 + \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n H_q^{F(n)}(x_1 p_1, x_2 p_2; \Omega)$$

$$C_{qa}(z; \alpha_S) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n C_{qa}^{(n)}(z)$$

Loop information  
(finite parts)

Radiation from incoming legs  
(transitions)

- The resummed component can be alternatively expressed as:

$$\frac{d\hat{\sigma}_{a_1 a_2 \rightarrow F}^{(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(b q_T) \mathcal{W}_{a_1 a_2}^F(b, M, \hat{s}; \mu_F)$$

with

$$\mathcal{W}_N^F(b, M; \mu_F) = \hat{\sigma}_F^{(0)}(M) \mathcal{H}_N^F(\alpha_S; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2)$$

Hard collinear part

$$\times \exp \{ \mathcal{G}_N(\alpha_S, L; M^2/\mu_R^2, M^2/Q^2) \}$$

Logarithmically-enhanced  
contributions

- **Photon radiation introduced by “Abelianization” of gluon emission:** transforms color factors into charges by defining suitable replacements

More details in arXiv's: 1512.00612, 1606.02887, 1805.11948, 1805.12214, 2005.10705

- **Our master formulae for simultaneous QED-QCD resummation with neutral final states:**

- We extend the *colorless QCD formalism* starting from:

$$\mathcal{W}_N'^F(b, M; \mu_F) = \hat{\sigma}_F^{(0)}(M) \mathcal{H}_N'^F(\alpha_S, \alpha; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp \left\{ \mathcal{G}_N'(\alpha_S, \alpha, L; M^2/\mu_R^2, M^2/Q^2) \right\}$$

- The hard-collinear part is expanded in a power series:

$$\mathcal{H}_N'^F(\alpha_S, \alpha) = \underbrace{\mathcal{H}_N^F(\alpha_S)}_{\text{Pure QCD}} + \underbrace{\frac{\alpha}{\pi} \mathcal{H}_N'^F(1) + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}_N'^F(n)}_{\text{Pure QED part}} + \underbrace{\sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}_N'^F(n,m)}_{\text{Mixed QCD-QED}}$$

- The Sudakov is also expanded in the same fashion:

$$\mathcal{G}_N'(\alpha_S, \alpha, L) = \underbrace{\mathcal{G}_N(\alpha_S, L)}_{\text{Pure QCD}} + \underbrace{L g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g_N'^{(n)}(\alpha L)}_{\text{Pure QED part}} + \underbrace{g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{\substack{n,m=1 \\ n+m \neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L)}_{\text{Mixed QCD-QED}}$$

with

$$\lambda = \frac{1}{\pi} \beta_0 \alpha_S L \quad \text{Large log!!!}$$

$$\lambda' = \frac{1}{\pi} \beta_0' \alpha L$$

... and G-functions in terms of A and B coefficients (perturbative)



- When final state is electrically charged, we “Abelianize” the QCD result for colorful final states.
- We start from the master formula for QCD resummation in  $t\bar{t}$  production **Catani et al [arXiv:1408.4564]**

$$W_N^V(b, M) = \sum_{ca_1a_2} \sigma_{c\bar{c},V}^{(0)}(\alpha_S(M^2)) f_{a_1/h_1,N}(b_0^2/b^2) f_{a_2/h_2,N}(b_0^2/b^2) \times S_c(M, b) \times [(\mathbf{H}^V \Delta C_1 C_2)]_{c\bar{c},a_1,a_2;N}(M^2, b_0^2/b^2)$$

- The soft factor  $\Delta$  accounts for soft (non-collinear) wide-angle radiation from final state and from initial-final state interferences
- Full EW loop-effects can be included within  $\mathbf{H}$
- **Master formulae for simultaneous QED-QCD resummation with charged final states:**
  - The soft factor part is expanded in a power series, exploiting the fact that EM charge is Abelian:

$$\Delta(\alpha; Q, b) = \exp \left\{ - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\} \quad \text{with} \quad D'(\alpha) = \frac{\alpha}{\pi} D'^{(1)} + \sum_{n=2}^{+\infty} \left( \frac{\alpha}{\pi} \right)^n D'^{(n)}$$

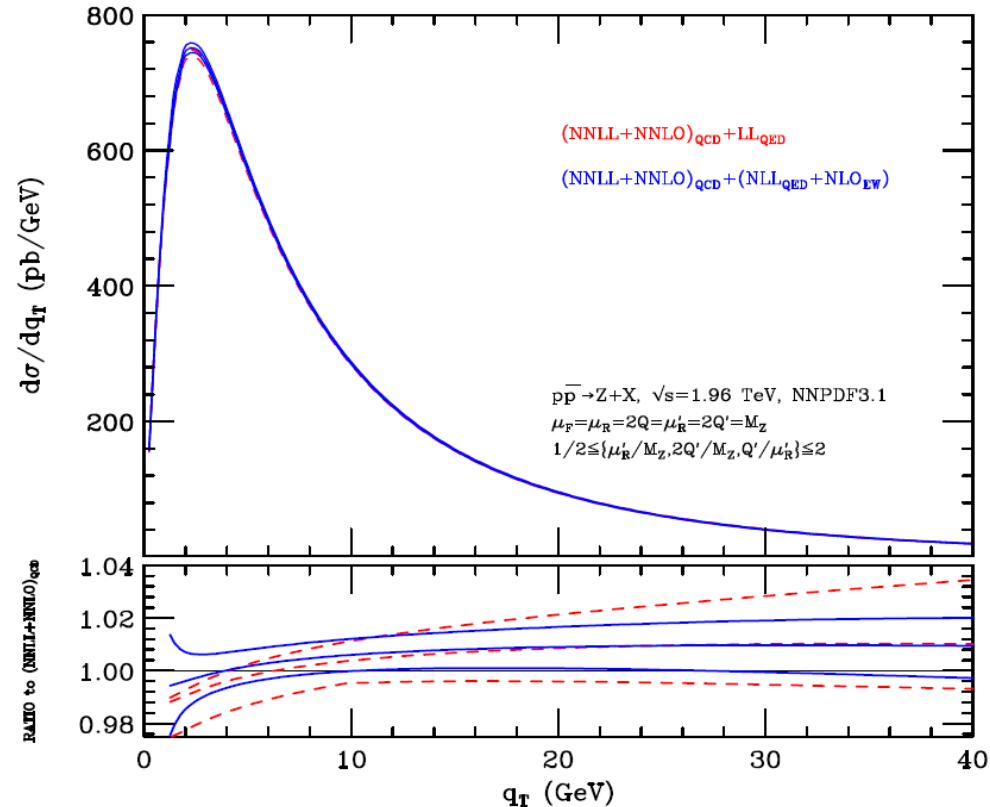
For W-boson production:

$$D'^{(1)} = -\frac{e_V^2}{2}$$

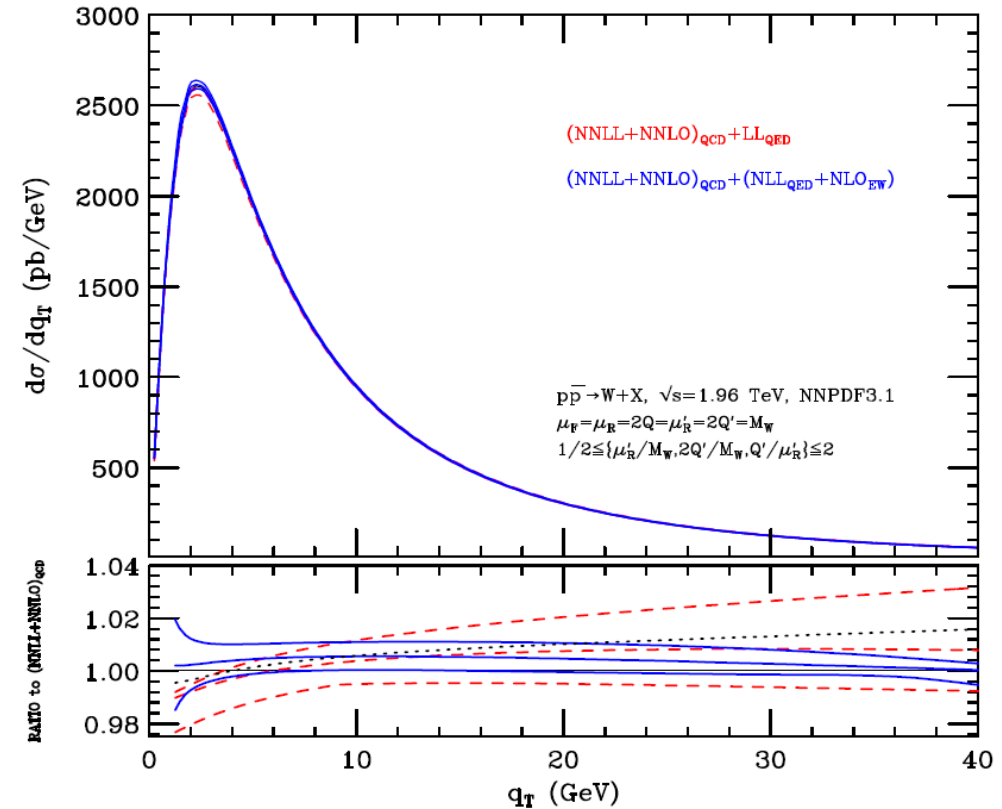
- The Sudakov factor also includes an additional term; in the exponential G-function we have:

$$\mathcal{G}'_N(\alpha, L) = - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left( A'(\alpha(q^2)) \log \left( \frac{M^2}{q^2} \right) + \tilde{B}'_N(\alpha(q^2)) + D'(\alpha(q^2)) \right)$$

- Vector boson production at Tevatron ( $E_{\text{CM}} = 1.96$  TeV)

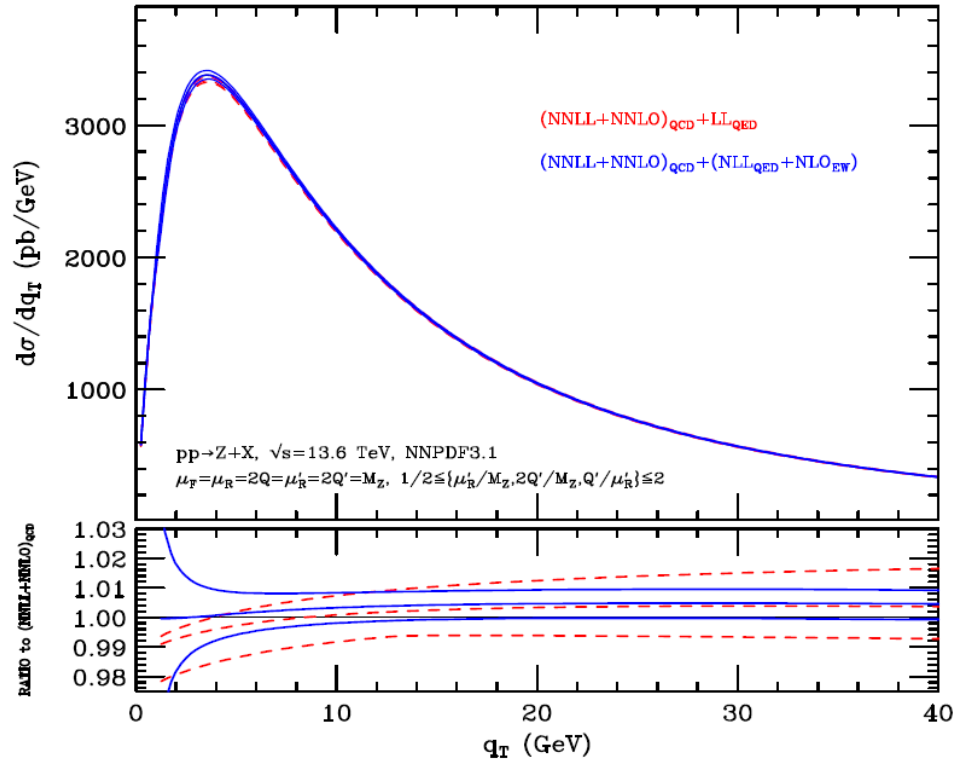


- LL<sub>QED</sub>**: spectrum slightly harder (O(1%) correction), and scale variation O(2-4%)
- NLL<sub>QED</sub>+NLO<sub>EW</sub>**: effects of O(0.5%) but scale variation band reduced by a factor 2

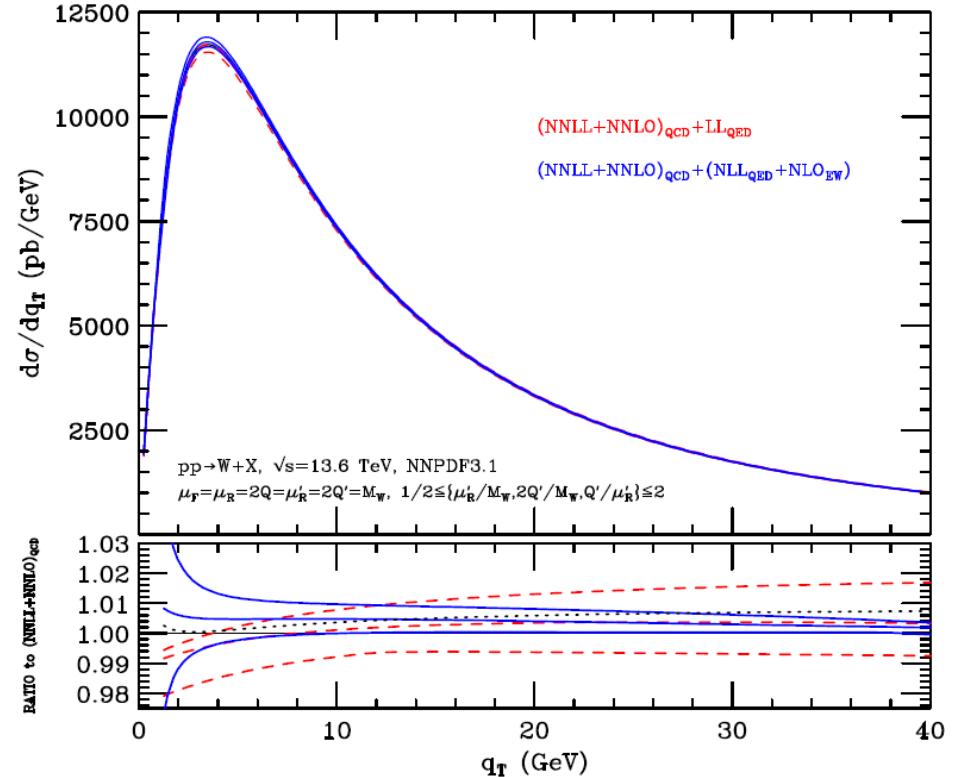


- LL<sub>QED</sub>**: spectrum slightly harder (O(<1%) correction), and scale variation O(2-3%)
- NLL<sub>QED</sub>+NLO<sub>EW</sub>**: effects of O(1%), scale variation band reduced by a factor 1.5-2.  $D_1$  makes the spectrum softer (enhances  $B_1$  term)

- **Vector boson production at LHC ( $E_{\text{CM}} = 13.6$  TeV)**
  - *Impact of QED effects reduced w.r.t. Tevatron (enhanced gluon contribution)*



- **LL<sub>QED</sub>**: spectrum slightly modified (-1% to 0.5% correction), and scale variation O(2%)
- **NLL<sub>QED</sub>+NLO<sub>EW</sub>**: effects of O(+0.5%) but scale variation band reduced by a factor 1.5-2



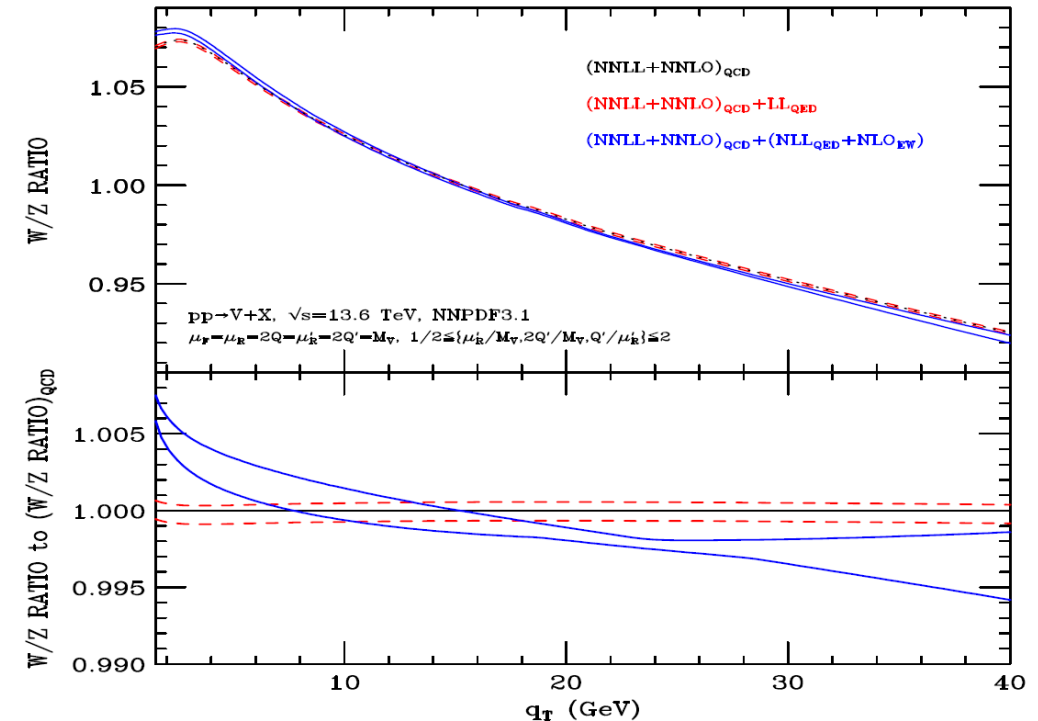
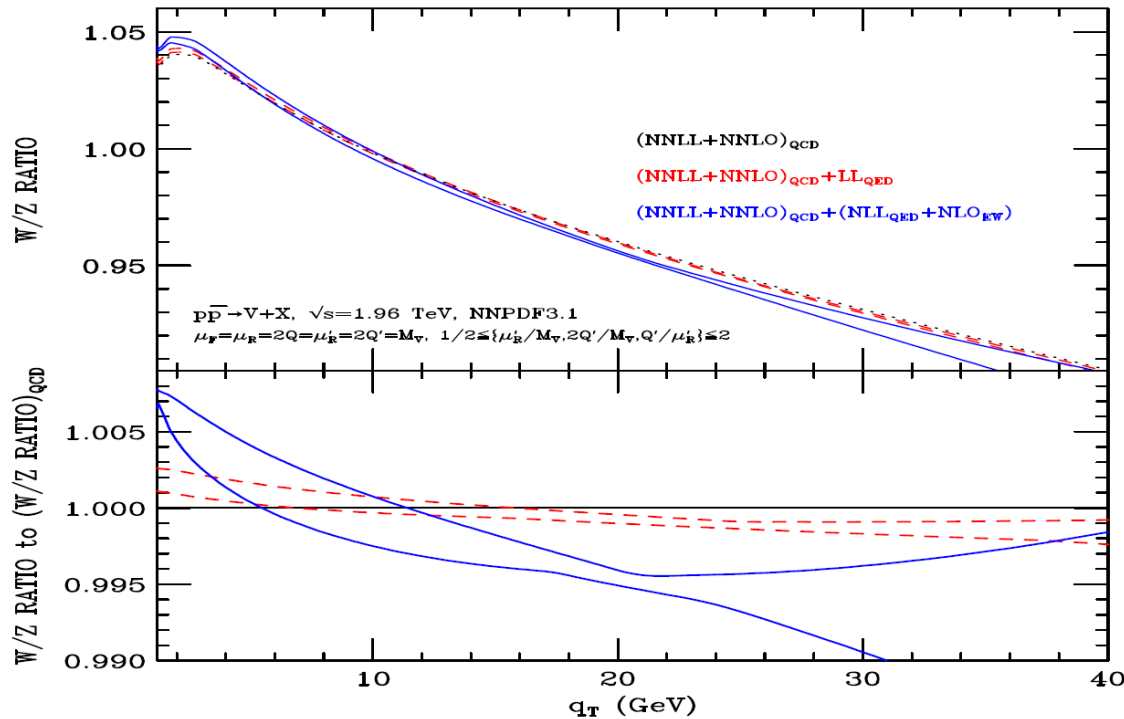
- **LL<sub>QED</sub>**: spectrum slightly harder (O(<1%) correction), and scale variation O(2%)
- **NLL<sub>QED</sub>+NLO<sub>EW</sub>**: effects of O(1%), scale variation band reduced by factor 1.5-2 (up to 4).  $D_1$  makes the spectrum softer.



- **R-ratio for Tevatron (left) and LHC (right)**

- This ratio is useful for  $m_W$  determination
- QED-EW effects at LHC reduced w.r.t. Tevatron due to enhanced gluon luminosity

$$R(q_T) = \frac{\frac{1}{\sigma_W} \frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z} \frac{d\sigma_Z}{dq_T}}$$



- $LL_{QED}$  effects are per-mille level, as well as the scale variation.
- $NLL_{QED}+NLO_{EW}$  effects are larger than at the lowest order (larger error bands w.r.t. LL calculation!!).
- This is due to new QED-EW effects that do not cancel in the ratio. *Further investigation required!*



- We introduced mixed QCD-QED/EW effects in the qT-resummation formalism
- The formalism was applied to Z/W boson production (EW one-loop included!)
- **Implemented in DYqT; ongoing efforts to migrate to DYTurbo**

**arXiv:1910.07049 [hep-ph]**

- Results for Z production are in agreement with the previous findings; EW one-loop effects are very small w.r.t. QED ones.
- Corrections to W boson production range from per-mille to percent level, with a reduction of the error band when considering  $NLL_{QED}+NLO_{EW}$  effects
- $LL_{QED}$  corrections induced to  $R(qT)$  are very small (per-mille level), but they are **slightly enhanced** when considering the  $NLL_{QED}+NLO_{EW}$  effects. The error bands become wider!



**THANKS!**