Quantum entanglement in $H \to ZZ, WW$

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Because the Higgs boson has spin zero, in $H \to VV$ [$V = W,Z$] the VV pair is produced in a state of zero total angular momentum.

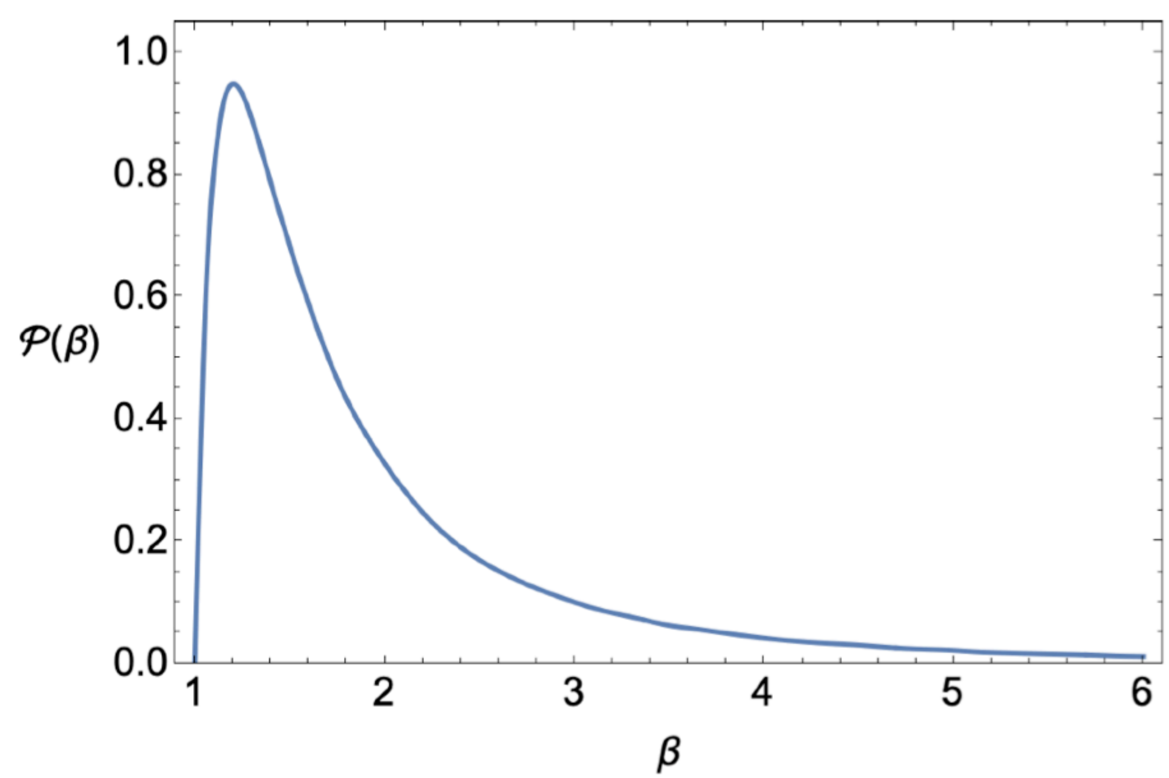
V at rest in *H* c.m. frame → L = 0 ; S = 0

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|+-\rangle - |00\rangle + |-+\rangle\right)$$

V not at rest but yet angular momentum conservation →

$$|\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$

p.d.f. for $H \to ZZ$



mixed state

Basics

For a weak boson, the 3×3 density matrix can be written as a linear combination of the identity [L = 0] plus irreducible tensors T^L_M [L = 1,2]

JAAS, Bernabéu, 1508.04592

$$T^1_1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_0 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T^2_2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^2_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^1_{-1} = -(T^1_1)^\dagger$$
$$T^2_{-2} = -(T^2_2)^\dagger$$
$$T^2_{-1} = -(T^2_1)^\dagger$$

For a systems of two particles, this is done for each one. For a *VV* pair, the density matrix is

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 coefficients corresponding to W^+ polarisation

8 coefficients corresponding to W^- polarisation

64 spin correlation coefficients

The parameterisation of the *VV* spin state, involving 80 independent parameters, is remarkably simple

Entanglement in $H \to VV$

This is a decay $0 \to 1 + 1$. Angular momentum conservation implies that many A and C coefficients are zero. The non-zero ones are

$$
\begin{aligned}
& A_{10}^{1} = -A_{10}^{2}\,, \quad A_{20}^{1} = A_{20}^{2} \\
& C_{1010}\,, \quad C_{2020}\,, \quad C_{1020}\,, \quad C_{2010} \\
& C_{111-1} = C_{1-111}^{*}\,, \quad C_{222-2} = C_{2-222}^{*}\,, \quad C_{212-1} = C_{2-121}^{*}\,, \\
& C_{112-1} = C_{1-121}^{*}\,, \quad C_{211-1} = C_{2-111}^{*}
\end{aligned}
$$

and the 9×9 ρ matrix is sparse [relations among coefficients used below]

$$
\rho = \frac{1}{3}\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 - C_{2020} & 0 & C_{212-1} & 0 & C_{222-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{212-1}^{*} & 0 & -1 + 2C_{2020} & 0 & C_{212-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{222-2}^{*} & 0 & C_{212-1}^{*} & 0 & 2 - C_{2020} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Entanglement in $H \to VV$

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

(n∗m) × (n∗m) matrix

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \quad \Rightarrow \quad \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

Quite difficult in general, but in our case it reduces to

Separability $\xrightarrow{\text{Peres-Horodecki}}$ $C_{212-1} = 0\,, \quad C_{222-2} = 0$

Separability $\xleftarrow{H \to VV \text{ special case}}$ $C_{212-1} = 0\,, \quad C_{222-2} = 0$

Entanglement in $H \to ZZ \to 4\ell$

The decay can be fully reconstructed, and the *A*s and *C*s measured.

The 4-d angular distribution has a very compact form

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad \eta_\ell = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}$$

$$B_2 = \sqrt{\frac{2\pi}{5}}$$

Because spherical harmonics are orthogonal functions, to pick selected terms in the distribution one just has to take averages

constants you can calculate

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)^* Y^{M_2}_{L_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

data follow this distribution

calculate the average of this quantity on your data

the quantity you want

Entanglement in $H \to ZZ \to 4\ell$

Prospects

- Parton level, no detector simulation, approximate eff [0.25] injected
- Background not included [1/4 size of signal]
- Only statistical uncertainties, estimated with pseudo-experiments

	$C_{212\text{-}1}$	$C_{222\text{-}2}$	Significance
Run 2 + 3 : 300 fb^{-1}	-0.98 ± 0.31	0.60 ± 0.37	3σ
HL-LHC : 3 ab^{-1}	-0.95 ± 0.10	0.60 ± 0.12	many σ

Entanglement in $H \to WW \to 2\ell 2\nu$

The decay cannot be reliably reconstructed because of the two neutrinos: the system is underconstrained.

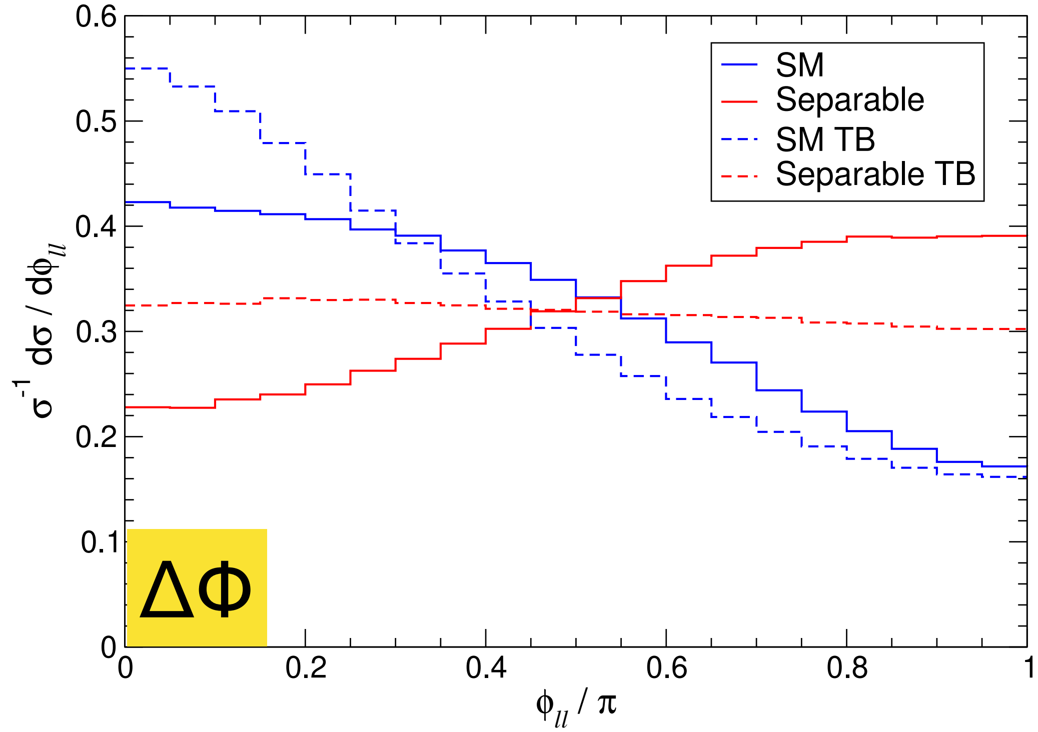
Instead, for entanglement a binary test can be made in lab frame

SM vs separability hypothesis

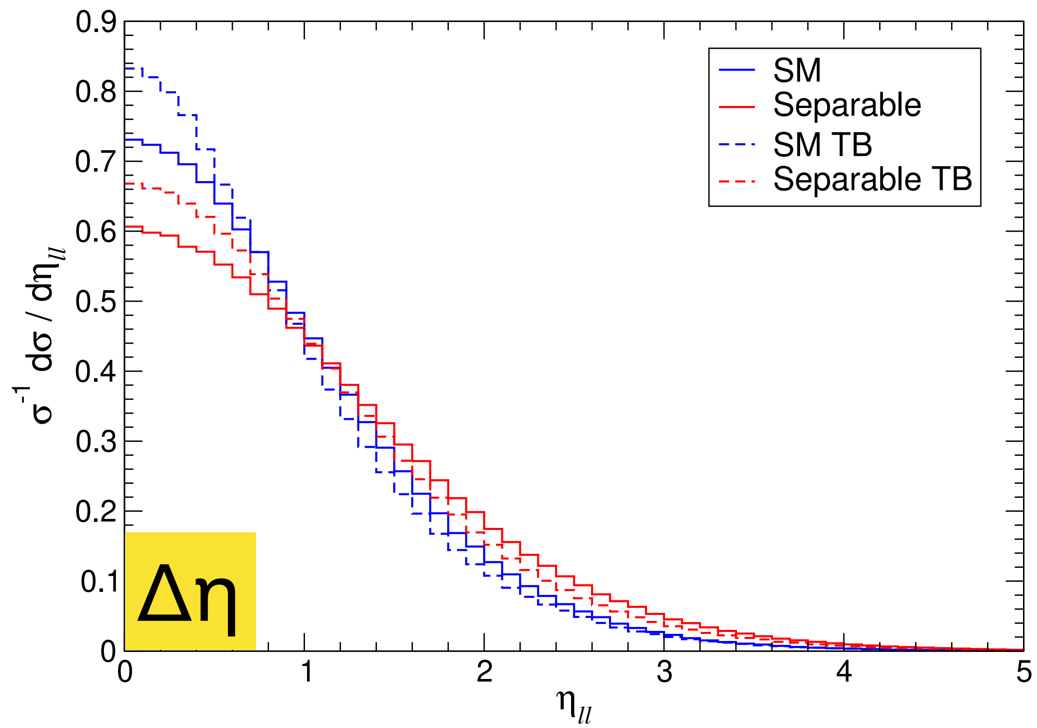
using dilepton kinematical distributions.

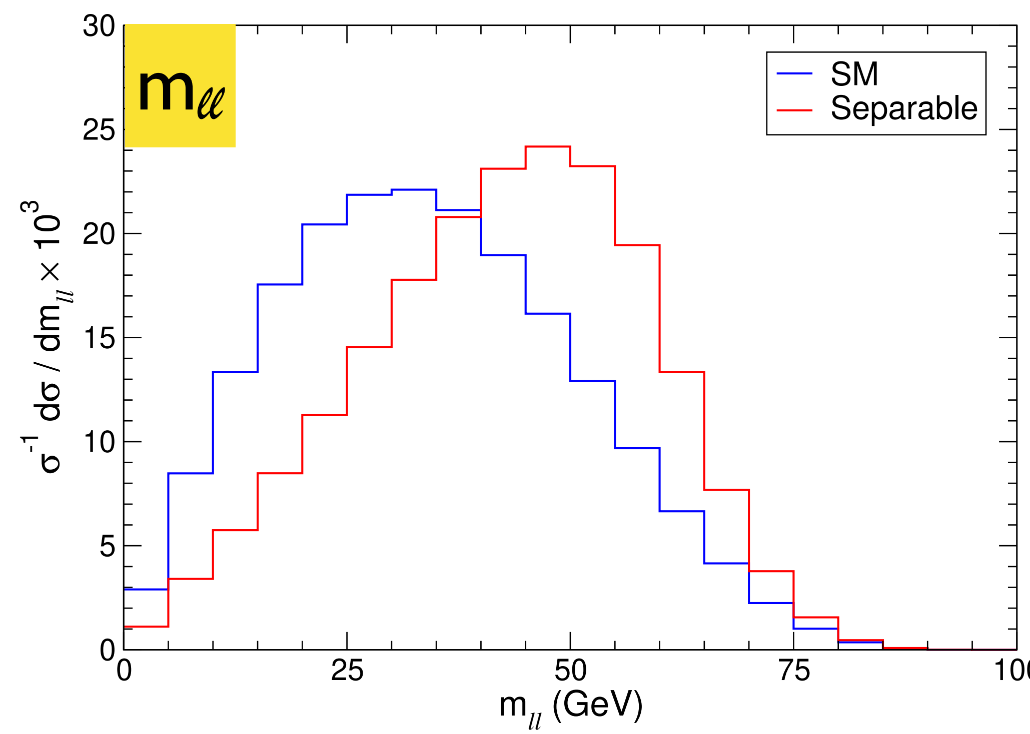
Note: such a trick is not possible to test Bell inequalities 😕

Parton-level plots



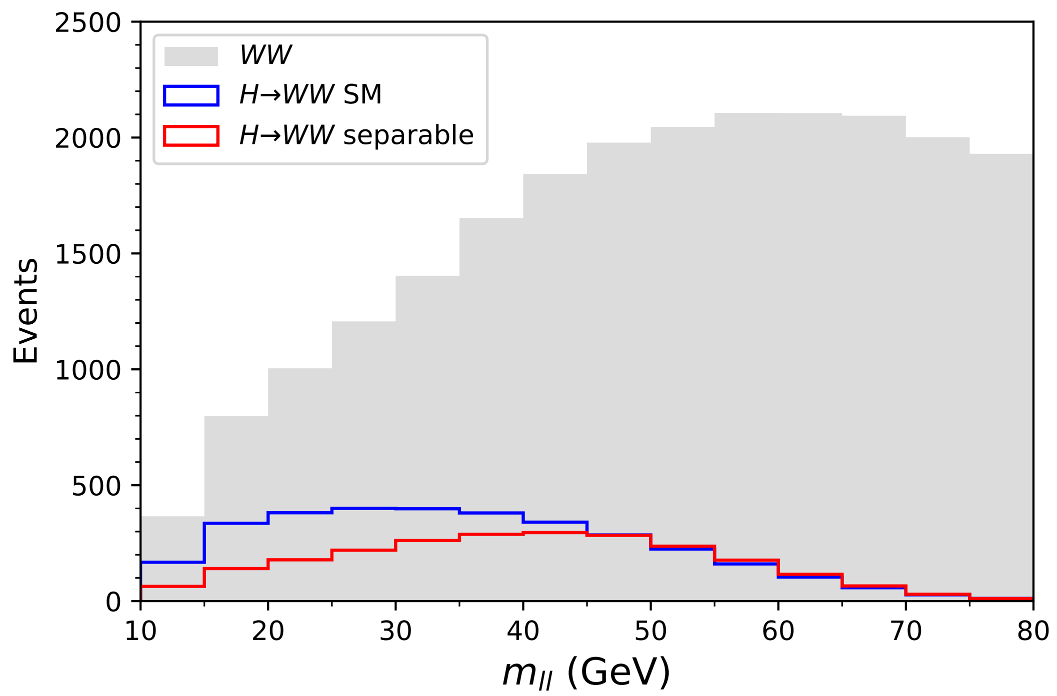
Though the discrimination from ΔΦ is excellent, transverse boosts from ISR [dashed lines] have a significant impact in the distribution.

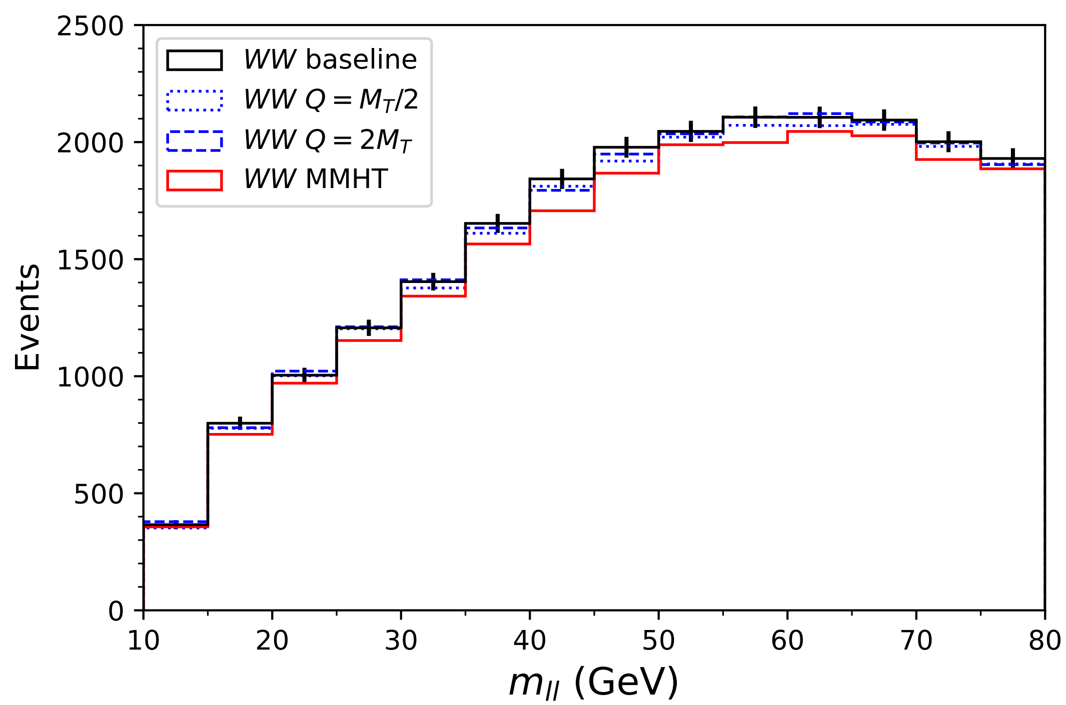




Entanglement in $H \to WW \to 2\ell 2\nu$

Results after Delphes simulation, eμ channel, L = 138 fb^{-1}





The differences between the SM and separable hypotheses arise in the region with smaller bkg

The bkg systematics are small provided we normalise it with a sideband

	Significance
stat only	7.1σ
stat + modeling syst	6.1σ

likely, observation possible already for Run 2

Bell inequalities in $H \to ZZ$

There is an inequality for a pair of spin-1 systems. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B]

CGLMP PRL '02

$$
\begin{aligned}
I_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\
& - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2
\end{aligned}
$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

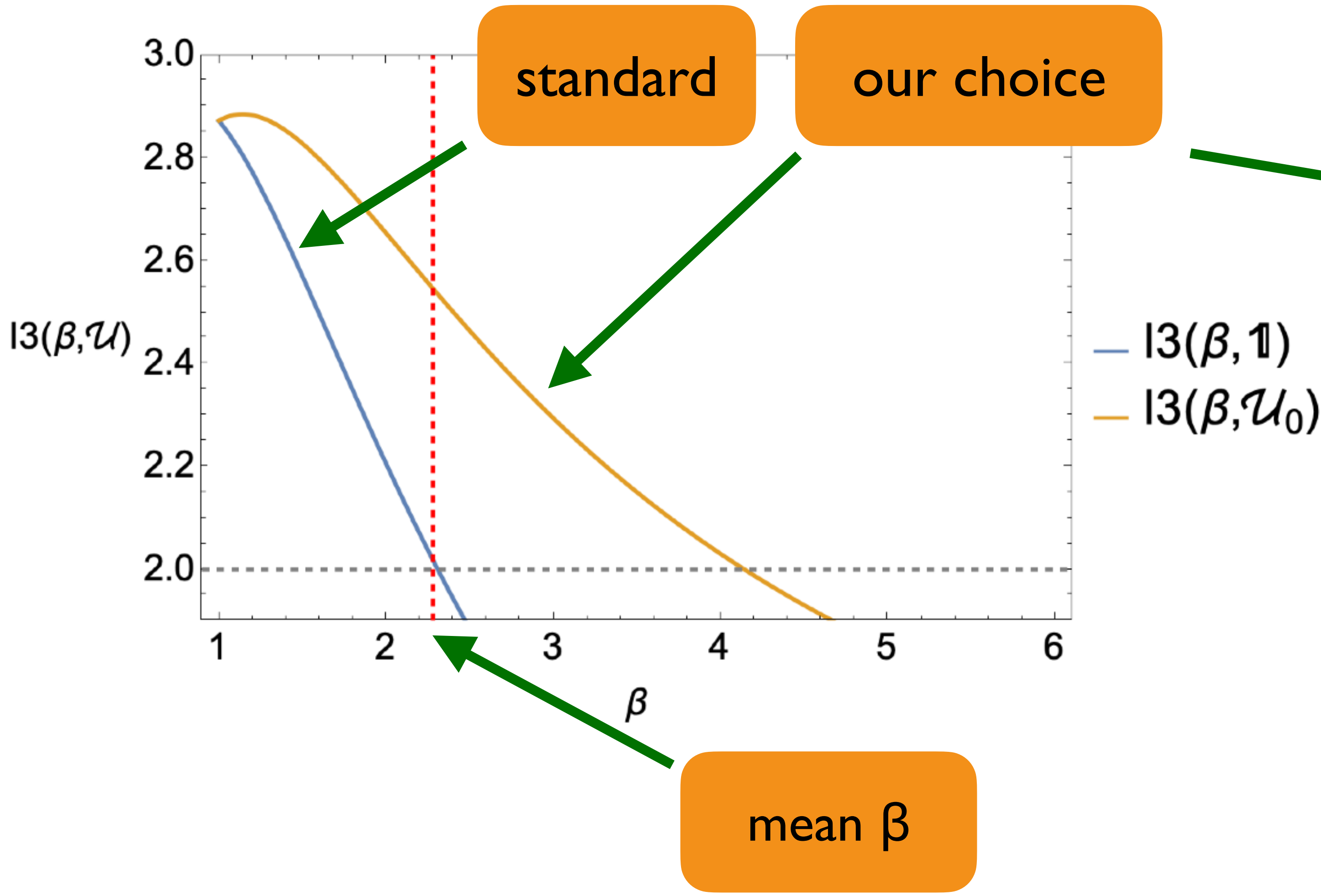
$$|\psi\rangle = \frac{1}{\sqrt{3}} \left(|+-\rangle - |00\rangle + |-+\rangle \right)$$

However, it is not optimal for the mixed spin state of the *ZZ* pair resulting from *H* decay

$$\rho = \int d\beta \; \mathcal{P}(\beta) |\psi_\beta\rangle\langle\psi_\beta| \qquad\qquad |\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}} \left(|+-\rangle - \beta|00\rangle + |-+\rangle \right)$$

Bell inequalities in $H \to ZZ$ 2/2

An improved Bell operator for this case gives a larger I_3 for $\beta > 1$



$$I_3 = \frac{1}{36}\Big[\left(18+16\sqrt{3}\right) - \sqrt{2}\left(9-8\sqrt{3}\right) A^1_{2,0} - 8\left(3+2\sqrt{3}\right) C_{2,1,2,-1} + 6\, C_{2,2,2,-2}\Big]$$

	I_3	Significance
Run 2 + 3 : 300 fb^{-1}	2.66 ± 0.46	1.4σ
HL-LHC : 3 ab^{-1}	2.63 ± 0.15	4.2σ