

New Physics hints from τ scalar interactions and $(g - 2)_{e,\mu}$

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Outline

- 1 Motivation and 2HDMs I-g ℓ FC and II-g ℓ FC
- 2 ($g - 2$) $_{e,\mu}$ new contributions and solution regions
- 3 Constraints and results

Based on work done in collaboration with:

Francisco J. Botella, Fernando Cornet-Gómez & Carlos Miró

 arXiv:2302.05471

 arXiv:2205.01115, EPJC82 (2022)

 arXiv:2006.01934, PRD102 (2020)

Motivation

“Anomalies” in the anomalous magnetic moments of μ and e

$$\delta a_\mu^{\text{Exp}} \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = +(2.5 \pm 0.6) \times 10^{-9}$$

Muon $g - 2$ collaboration, *Phys. Rev. Lett.* 126 (2021) 14

$$\delta a_e^{\text{Exp,Cs}} \equiv a_e^{\text{Exp,Cs}} - a_e^{\text{SM}} = -(8.7 \pm 3.6) \times 10^{-13}$$

Parker et al. *Science* (2018) 360:191, α from ^{133}Cs recoil

Also

$$\delta a_e^{\text{Exp,Rb}} \equiv a_e^{\text{Exp,Rb}} - a_e^{\text{SM}} = +(4.8 \pm 3.0) \times 10^{-13}$$

Morel et al. *Nature* (2020) 588:61, α from ^{87}Rb recoil

N.B. $a_\ell = (g_\ell - 2)/2$

Vertex $\bar{\ell} \ell A^\mu \not\rightarrow \gamma^\mu \rightarrow \Gamma^\mu = \gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2)$, $a_\ell = F_2(0)$

Motivation

$$\frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq - \left(\frac{m_e}{m_\mu} \right)^{1.494}$$

- $\delta a_\mu^{\text{Exp}}$ and $\delta a_e^{\text{Exp,Cs}}$ have *opposite signs!*
- Not only the sign,
 - if NP model gives $\delta a_\ell \propto m_\ell$

$$\frac{\delta a_e}{\delta a_\mu} \sim \frac{m_e}{m_\mu} = \left(\frac{m_e}{m_\mu} \right)^{-0.494} \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq -13.9 \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}}$$

- if NP model gives $\delta a_\ell \propto m_\ell^2$

$$\frac{\delta a_e}{\delta a_\mu} \sim \left(\frac{m_e}{m_\mu} \right)^2 = \left(\frac{m_e}{m_\mu} \right)^{0.506} \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq -\frac{1}{14.8} \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}}$$

⌚ Serious obstacle for many New Physics solutions

Motivation

- If the origin of both anomalies is beyond SM, some sort of *effective decoupling* between e and μ should be in place
- 2 Higgs Doublets Models (2HDMs) incorporate *new flavour structures* that can implement that property but
 - not in symmetry-shaped 2HDMs of types I, II, X, Y
(new couplings proportional to masses)
 - not in “aligned 2HDMs” (proportionality to masses again)
 - maybe in general flavour conserving 2HDMs (gFC-2HDMs)! ↗

2HDMs

- In 2HDMs the Yukawa sector is

$$\begin{aligned}\mathcal{L}_Y = -\bar{Q}_L^0 & \left(\Phi_1 Y_{d1} + \Phi_2 Y_{d2} \right) d_R^0 - \bar{Q}_L^0 \left(\tilde{\Phi}_1 Y_{u1} + \tilde{\Phi}_2 Y_{u2} \right) u_R^0 \\ & - \bar{L}_L^0 \left(\Phi_1 Y_{\ell 1} + \Phi_2 Y_{\ell 2} \right) \ell_R^0 + \text{H.c.}\end{aligned}$$

N.B. $\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$, neutrinos are massless

- Expansion around vacuum appropriate for electroweak symmetry breaking

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{v_j + \rho_j + i\eta_j}{\sqrt{2}} \end{pmatrix}$$

- Higgs basis, $c_\beta \equiv \cos \beta = \frac{v_1}{v}$, $s_\beta \equiv \sin \beta = \frac{v_2}{v}$, $t_\beta \equiv \tan \beta$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1} \Phi_1 \\ e^{-i\theta_2} \Phi_2 \end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

2HDMs

- Higgs basis

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad v^2 = v_1^2 + v_2^2 = \frac{1}{\sqrt{2}G_F}$$

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix}$$

- would-be Goldstone bosons G^0, G^\pm
- physical charged scalar H^\pm
- neutral scalars $\{H^0, R^0, I^0\}$, not the mass eigenstates
- Yukawa couplings again

$$\begin{aligned} \mathcal{L}_Y = & -\frac{\sqrt{2}}{v} \bar{Q}_L^0 (H_1 M_d^0 + H_2 N_d^0) d_R^0 - \frac{\sqrt{2}}{v} \bar{Q}_L^0 (\tilde{H}_1 M_u^0 + \tilde{H}_2 N_u^0) u_R^0 \\ & - \frac{\sqrt{2}}{v} \bar{L}_L^0 (H_1 M_\ell^0 + H_2 N_\ell^0) \ell_R^0 + \text{H.c.} \end{aligned}$$

2HDMs

- Going to the fermion mass bases

$$\begin{aligned}\mathcal{L}_Y = & -\frac{\sqrt{2}}{v} \bar{Q}_L (H_1 \textcolor{blue}{M}_{\textcolor{blue}{d}} + H_2 \textcolor{blue}{N}_{\textcolor{blue}{d}}) d_R - \frac{\sqrt{2}}{v} \bar{Q}_L (\tilde{H}_1 \textcolor{blue}{M}_{\textcolor{blue}{u}} + \tilde{H}_2 \textcolor{blue}{N}_{\textcolor{blue}{u}}) u_R \\ & - \frac{\sqrt{2}}{v} \bar{L}_L (H_1 \textcolor{blue}{M}_{\textcolor{blue}{e}} + H_2 \textcolor{blue}{N}_{\textcolor{blue}{e}}) \ell_R + \text{H.c.}\end{aligned}$$

where

- $\textcolor{blue}{M}_f$ are the diagonal fermion mass matrices
- $\textcolor{blue}{N}_f$ are the new flavour structures
(the ones that may explain the anomalies!)

The I-g ℓ FC and II-g ℓ FC models

Finally

- Model I-g ℓ FC is defined by

$$\mathbf{N}_u = t_\beta^{-1} \mathbf{M}_u, \quad \mathbf{N}_d = t_\beta^{-1} \mathbf{M}_d, \quad \mathbf{N}_\ell = \text{diag}(\mathbf{n}_e, \mathbf{n}_\mu, \mathbf{n}_\tau)$$

The couplings $\mathbf{N}_u, \mathbf{N}_d$ are the same as in 2HDMs of types I or X

- Model II-g ℓ FC is defined by

$$\mathbf{N}_u = t_\beta^{-1} \mathbf{M}_u, \quad \mathbf{N}_d = -t_\beta \mathbf{M}_d, \quad \mathbf{N}_\ell = \text{diag}(\mathbf{n}_e, \mathbf{n}_\mu, \mathbf{n}_\tau)$$

The couplings $\mathbf{N}_u, \mathbf{N}_d$ are the same as in 2HDMs of types II or Y

- \mathbf{N}_ℓ is diagonal, arbitrary and one loop RGE stable
- Effective decoupling among new e and μ couplings required for the $g - 2$ anomalies \leftrightarrow independence of n_e and n_μ

The I-g ℓ FC and II-g ℓ FC models

Completing the model

- since the quark sector is a type I or type II 2HDM, adopt a \mathbb{Z}_2 symmetric scalar potential

$$\begin{aligned}\mathcal{V}(\Phi_1, \Phi_2) = & \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \left(\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + 2\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left(\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right)\end{aligned}$$

$\mu_{12}^2 \neq 0 \Rightarrow$ softly broken \mathbb{Z}_2 symmetry

- Mass matrix of the neutral scalars \mathcal{M}_0^2 , diagonalised by a 3×3 real orthogonal matrix \mathcal{R}

$$\mathcal{R}^T \mathcal{M}_0^2 \mathcal{R} = \text{diag}(\textcolor{blue}{m_h^2}, \textcolor{blue}{m_H^2}, \textcolor{blue}{m_A^2}), \quad \mathcal{R}^{-1} = \mathcal{R}^T$$

- Physical neutral scalars {h, H, A}:

$$\begin{pmatrix} \textcolor{blue}{h} \\ \textcolor{blue}{H} \\ \textcolor{blue}{A} \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} H^0 \\ R^0 \\ I^0 \end{pmatrix}$$

The I-g ℓ FC and II-g ℓ FC models

- Flavour conserving Yukawa couplings of the neutral scalars

$$\mathcal{L}_N = - \sum_{S=\text{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^3 \frac{m_{f_j}}{v} S \bar{f}_j (a_{f_j}^S + i b_{f_j}^S \gamma_5) f_j$$

- Further simplifications

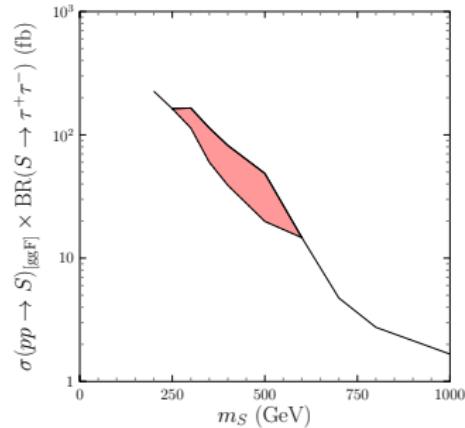
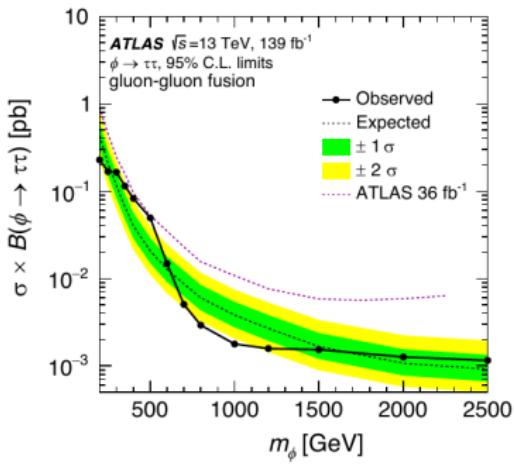
- 1 the new Yukawa couplings are real, $\text{Im}(n_\ell) = 0$
- 2 there is no CP violation in the scalar sector,

$$\mathcal{R} = \begin{pmatrix} s_{\alpha\beta} & -c_{\alpha\beta} & 0 \\ c_{\alpha\beta} & s_{\alpha\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{cases} s_{\alpha\beta} \equiv \sin(\alpha - \beta) \\ c_{\alpha\beta} \equiv \cos(\alpha - \beta) \end{cases}$$

$\alpha - \frac{\pi}{2}$: mixing angle in $\{\rho_j, \eta_j\} \rightarrow \{G^0, \text{h, H, A}\}$

The τ motivation

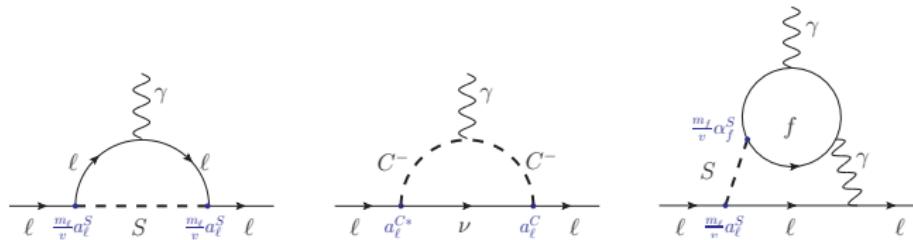
- We have 3 new couplings n_e , n_μ , n_τ ,
and 2 “New Physics observations”, δa_e , δa_μ
- ... is there some other “New Physics hint” for the τ ? YES



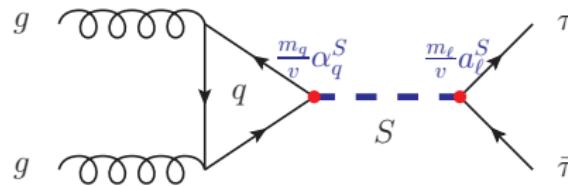
ATLAS collaboration, *Phys. Rev. Lett.* 125 (2020) 051801, 2002.12223

Ingredients

- New contributions to δa_ℓ at one loop and at two loops (Barr-Zee)



- $[pp]_{\text{ggF}} \rightarrow S \rightarrow \tau\bar{\tau}$



The new contributions to δa_ℓ

Two types of solutions

- ✎ “Solution [A]/heavy”: scalars with masses in the 1–2 TeV range, $t_\beta \sim 1$, and $(g - 2)_{e,\mu}$ anomalies produced by two loop Barr-Zee contributions.
 $\text{Re}(n_e)$ in the few GeV range, $\text{Re}(n_\mu) \sim -15\text{Re}(n_e)$
Solution a priori present in both I-g ℓ FC and II-g ℓ FC
- ✎ “Solution [B]/light”: $t_\beta \gg 1$, lighter H, $m_H \in [200; 400]$ GeV, and a heavier A. δa_e is obtained with two loop contributions while δa_μ is one loop controlled. Contrary to solution [A], there is no linear relation among $\text{Re}(n_\mu)$ and $\text{Re}(n_e)$, and in fact both signs of $\text{Re}(n_\mu)$ can work.
- + “Intermediate regions”
- ? Impact of the $[pp]_{\text{ggF}} \rightarrow S \rightarrow \tau\bar{\tau}$ excess

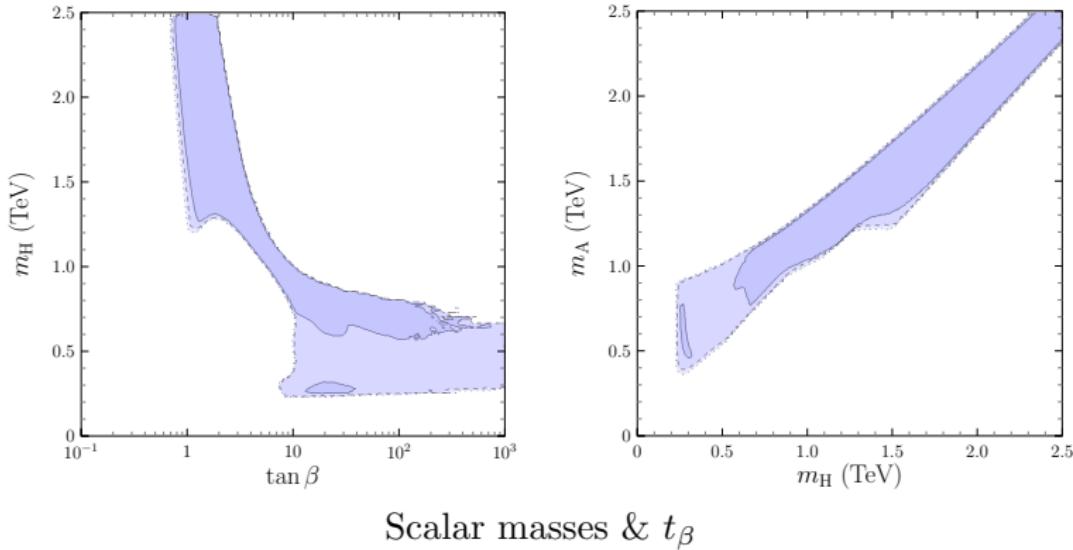
N.B. Solution [B] available in the I-g ℓ FC model, but not in II-g ℓ FC.

Constraints for full numerical analysis

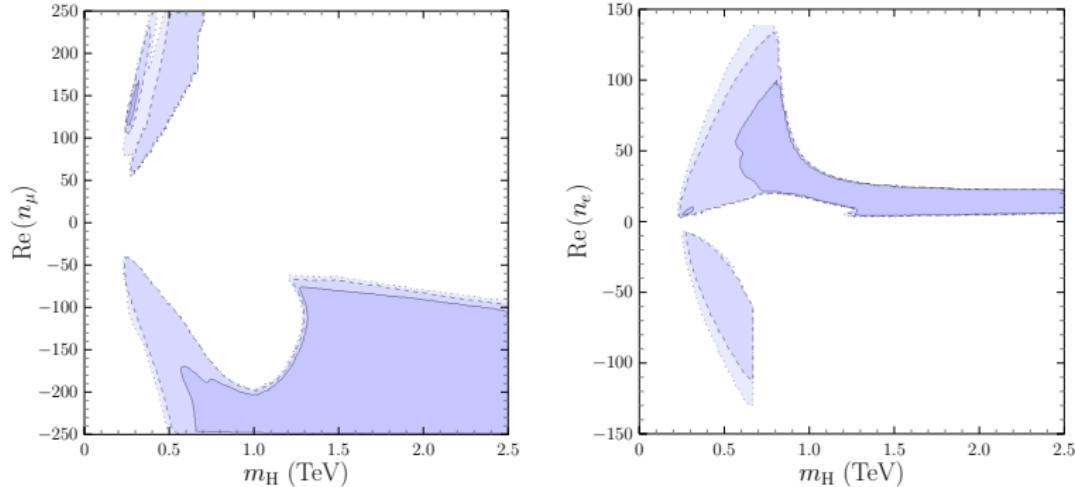
Shopping list

- $\delta a_\ell^{\text{Exp}}$ constraints
- $[pp]_{\text{ggF}} \rightarrow S \rightarrow \tau\bar{\tau}$ excess
- Scalar sector
- Fermion sector
- Higgs signal strengths
- H^\pm mediated contributions
 - Lepton flavour universality
 - $b \rightarrow s\gamma$, $B_q^0 - \bar{B}_q^0$ mixing
- $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$ at LEP
- LHC searches
 - searches of dilepton resonances: $\sigma(pp \rightarrow S)_{[\text{ggF}]} \times \text{Br}(S \rightarrow \ell^+\ell^-)$,
 $S = H, A$ and $\ell = \mu, \tau$
 - searches of charged scalars: $\sigma(pp \rightarrow H^\pm tb) \times \text{Br}(H^\pm \rightarrow f)$,
 $f = \tau\nu, tb$
- CDF W mass shift (optional)

Results, model I-g ℓ FC – without $\tau\bar{\tau}$ excess

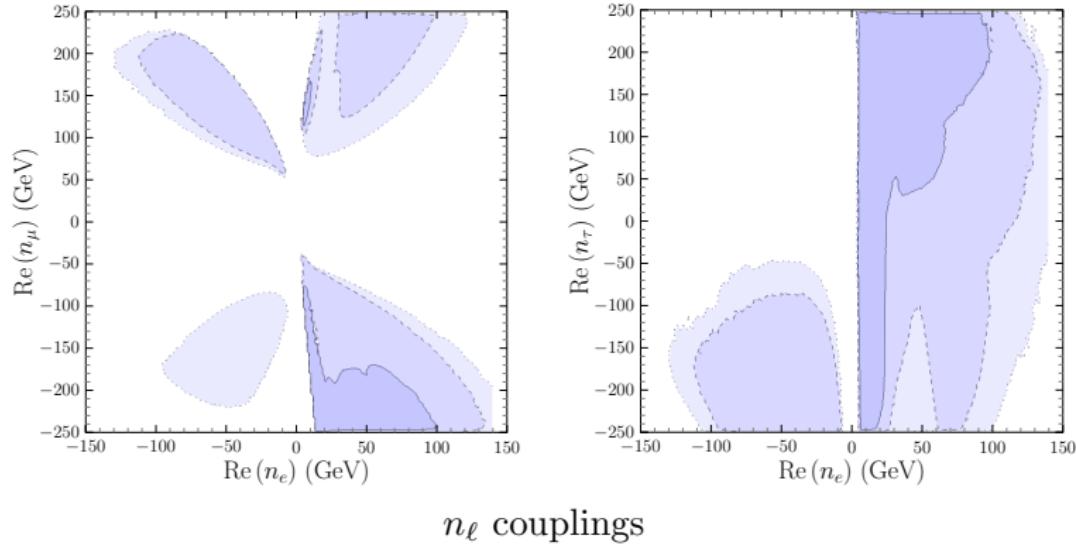


Results, model I-g ℓ FC – without $\tau\bar{\tau}$ excess

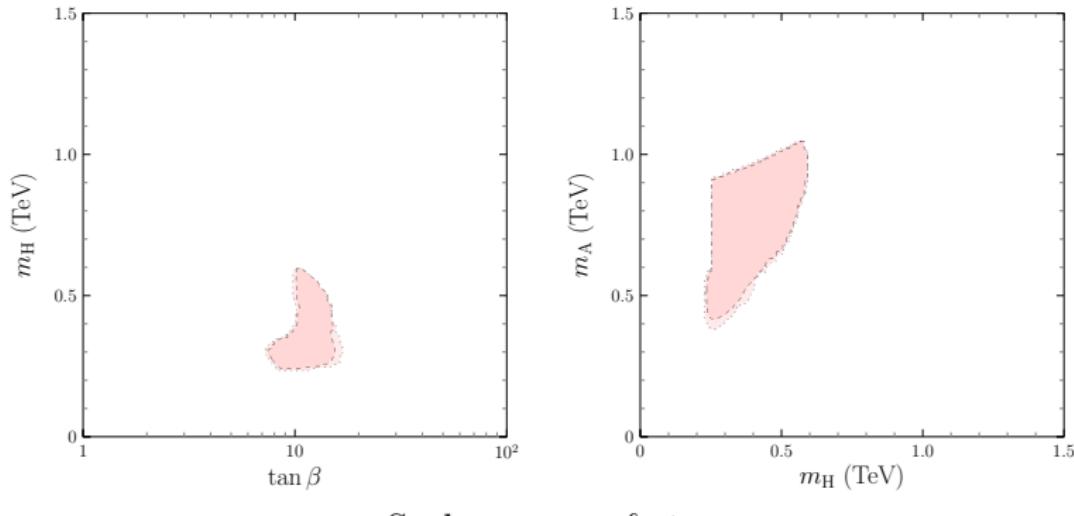


$n_{e,\mu}$ (GeV) & m_H

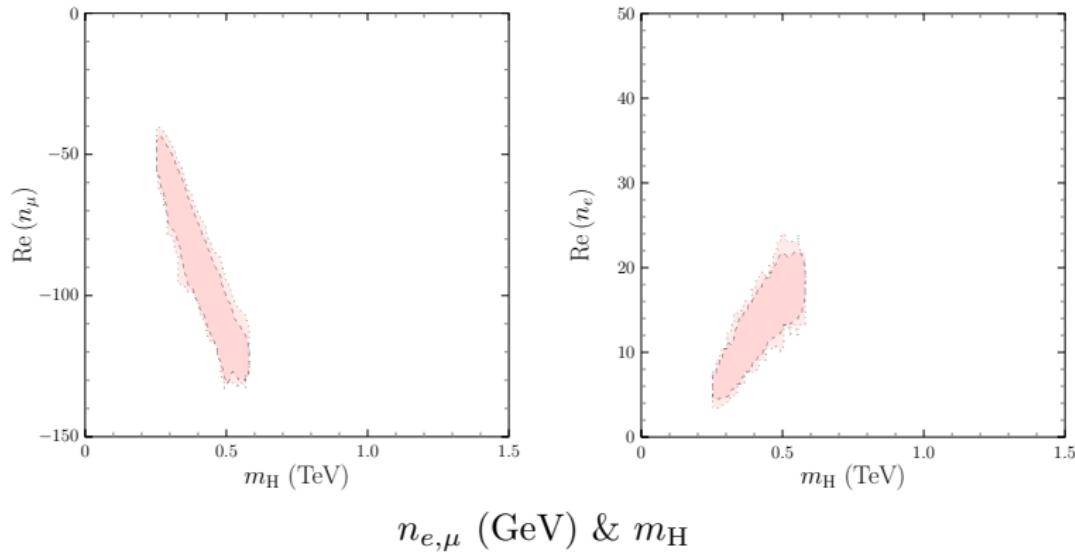
Results, model I-g ℓ FC – without $\tau\bar{\tau}$ excess



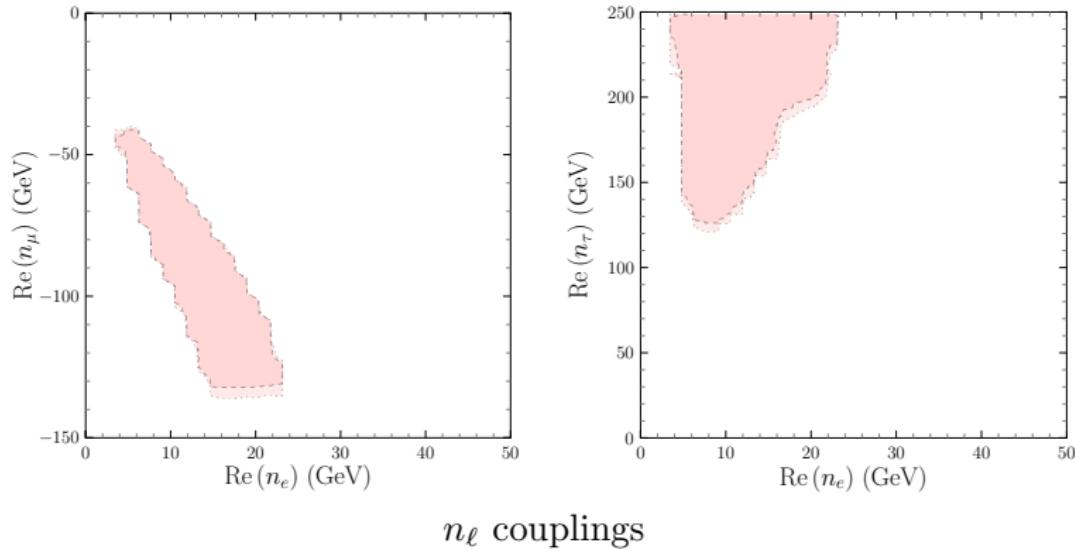
Results, model I-g ℓ FC – with $\tau\bar{\tau}$ excess



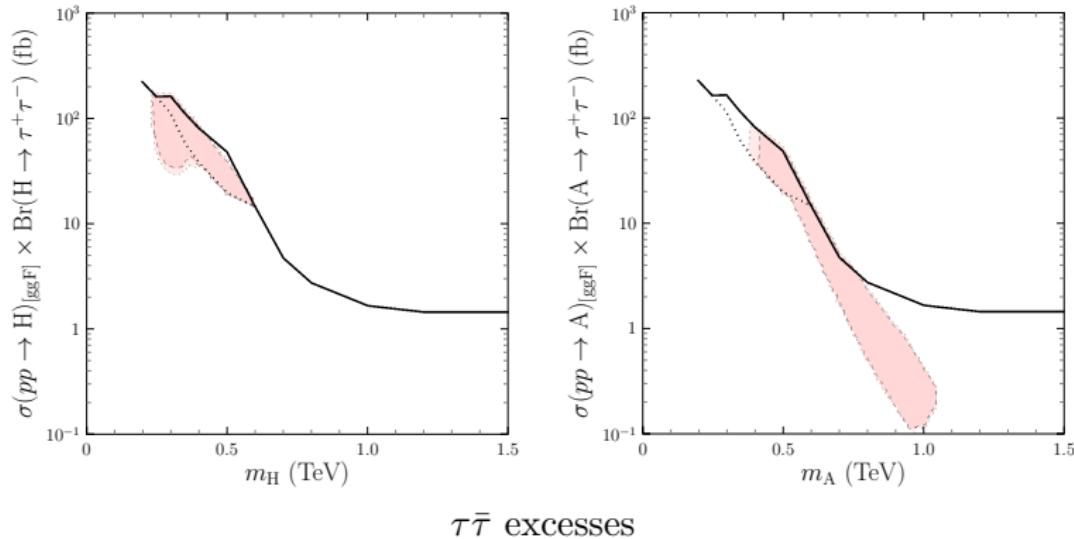
Results, model I-g ℓ FC – with $\tau\bar{\tau}$ excess



Results, model I-g ℓ FC – with $\tau\bar{\tau}$ excess



Results, model I-g ℓ FC – with $\tau\bar{\tau}$ excess



Conclusions

- General 2HDMs without SFCNC in the lepton sector are a robust framework (stable under RGE)
- Lepton flavour universality is broken beyond \propto mass
- Two models, I-g ℓ FC & II-g ℓ FC, to address the δa_ℓ anomalies
- Quark & scalar sector as type I, II 2HDMs, softly broken \mathbb{Z}_2
- Different types of solutions in agreement with constraints
 - 1 “Heavy”, present in both models
 - new scalars have masses in the 1–2.5 TeV range,
 - $v_1 \sim v_2$,
 - both δa_ℓ from two loop Barr-Zee contributions
 - 2 “Light”, present in I-g ℓ FC, not in II-g ℓ FC
 - new scalars have masses below 1 TeV,
 - $v_1 \ll v_2$,
 - δa_e from two loop Barr-Zee contributions, δa_μ from one loop
 - 3 “Intermediate” regions

Conclusions

- Including $[pp]_{\text{ggF}} \rightarrow S \rightarrow \tau\bar{\tau}$ excess
 - obtained with H, or A, or both (two \neq excesses within the region)
 - selects the “Intermediate”-“Light” region
 - only model I-g ℓ FC can do the work
- Involved numerical analysis (subtleties & unexpected regions)
 - Gluon-gluon fusion production of scalar vs pseudoscalar
 - Role of $A \rightarrow HZ$
 - τ loop in Barr-Zee contributions
- Extra balls
 - Role of different δa_e^{Exp} values
 - CDF M_W through ΔS and ΔT
 - $(g - 2)_\tau$
 - Example points

Thank you!

Backup

- 2HDMs
- δa_ℓ in detail
- Constraints
- Scalar vs pseudoscalar gluon-gluon fusion
- M_W from CDF
- δa_τ
- Example points
- Different δa_e^{Exp}
- Yukawa couplings

2HDMs

$$\begin{aligned}\mathcal{L}_Y = & -\frac{\sqrt{2}}{v} \bar{Q}_L (H_1 \textcolor{blue}{M}_d + H_2 \textcolor{blue}{N}_d) d_R - \frac{\sqrt{2}}{v} \bar{Q}_L (\tilde{H}_1 \textcolor{blue}{M}_u + \tilde{H}_2 \textcolor{blue}{N}_u) u_R \\ & - \frac{\sqrt{2}}{v} \bar{L}_L (H_1 \textcolor{blue}{M}_\ell + H_2 \textcolor{blue}{N}_\ell) \ell_R + \text{H.c.}\end{aligned}$$

- Natural Flavour Conservation:

only one Yukawa matrix $\neq 0$ in each sector

\mathbb{Z}_2 symmetry, types I, II, X, Y, with $\textcolor{blue}{N}_f = \pm t_\beta^{\mp 1} M_f$

Glashow & Weinberg, *PRD15* (1977)

- “Aligned” 2HDM: $\textcolor{blue}{N}_f = \zeta_f M_f$

Pich & Tuzón, *PRD80* (2009)

RGE: unstable quark sector, stable lepton sector

Botella, Branco, Coutinho, Rebelo & Silva-Marcos, *EPJC75* (2015)

- General flavour conserving: diagonal $\textcolor{blue}{N}_f$

Peñuelas & Pich, *JHEP 12* (2017)

RGE: unstable quark sector, stable lepton sector

Botella, Cornet-Gómez & N, *PRD98* (2018)

The new contributions to δa_ℓ

- Full prediction

$$a_\ell^{\text{Th}} = a_\ell^{\text{SM}} + \delta a_\ell$$

a_ℓ^{SM} : SM contribution; δa_ℓ : corrections due to the model

- To solve the discrepancies, the aim is

$$\delta a_e \simeq \delta a_e^{\text{Exp,Cs}}, \quad \delta a_\mu \simeq \delta a_\mu^{\text{Exp}}$$

within models I-g ℓ FC and II-g ℓ FC

- Introduce Δ_ℓ

$$\delta a_\ell = K_\ell \Delta_\ell, \quad K_\ell = \frac{1}{8\pi^2} \left(\frac{m_\ell}{v} \right)^2 = \frac{1}{8\pi^2} \left(\frac{gm_\ell}{2M_W} \right)^2$$

K_ℓ are typical factors arising in one loop contributions

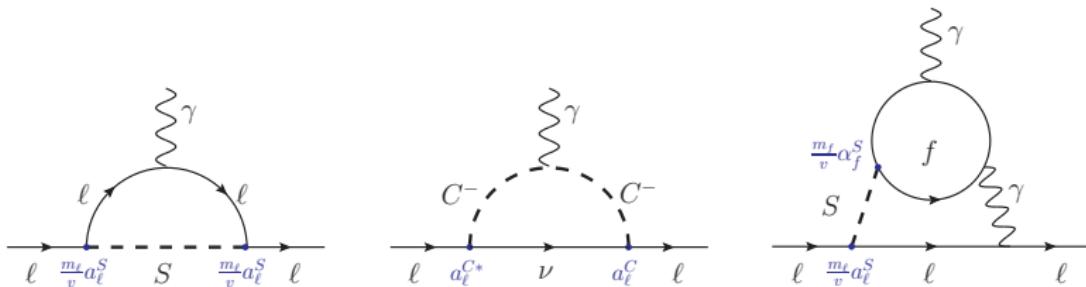
$$K_e \simeq 5.5 \times 10^{-14}, \quad K_\mu \simeq 2.3 \times 10^{-9}$$

The new contributions to δa_ℓ

- With these values, $K_e \simeq 5.5 \times 10^{-14}$, $K_\mu \simeq 2.3 \times 10^{-9}$, we need

$$\Delta_e \simeq -16, \quad \Delta_\mu \simeq 1$$

- Contributions at one loop and at two loops (Barr-Zee type) can be relevant



The new contributions to δa_ℓ

- To gain some insight consider the leading terms in $(m_\ell/m_S)^2$ of the one loop contributions in the alignment limit $s_{\alpha\beta} \rightarrow 1$

$$\Delta_\ell^{(1)} \simeq [\text{Re}(n_\ell)]^2 \left(\frac{\mathcal{I}_{\ell H}}{m_H^2} - \frac{\mathcal{I}_{\ell A} - 2/3}{m_A^2} - \frac{1}{6m_{H^\pm}^2} \right)$$

where

$$\mathcal{I}_{\ell S} = -\frac{7}{6} - 2 \ln \left(\frac{m_\ell}{m_S} \right)$$

[N.B. Same in both models I-g ℓ FC & II-g ℓ FC]

- We do not consider light scalars/pseudoscalars,
we assume $m_h < m_H, m_A$
- Typical values of the loop function for $m_S \in [0.2; 2.0] \text{ TeV}$

$$\mathcal{I}_{eS} \in [24.6; 29.2], \quad \mathcal{I}_{\mu S} \in [13.9; 18.5]$$

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq [\text{Re}(n_\ell)]^2 \left(\frac{I_{\ell H}}{m_H^2} - \frac{I_{\ell A} - 2/3}{m_A^2} - \frac{1}{6m_{H^\pm}^2} \right)$$

$$I_{eS} \in [24.6; 29.2], \quad I_{\mu S} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced),
 $\Delta_e^{(1)} \simeq -16$ can only come from A:

$$\Delta_e^{(1)} \simeq -[\text{Re}(n_e)]^2 I_{eA}/m_A^2 \text{ requires } [\text{Re}(n_e)]^2 \sim m_A^2$$

\Rightarrow violate perturbativity requirements for Yukawa couplings or constraints from resonant dilepton searches

- ☞ We *do not* expect an explanation of $\delta a_e^{\text{Exp}, \text{Cs}}$ in terms of one loop contributions

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq \text{Re}(n_\ell)^2 \left(\frac{I_{\ell H}}{m_H^2} - \frac{I_{\ell A} - 2/3}{m_A^2} - \frac{1}{6m_{H^\pm}^2} \right)$$

$$I_{eS} \in [24.6; 29.2], \quad I_{\mu S} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced),
 $\Delta_\mu^{(1)} \simeq 1$ can only come from H:

$$\Delta_\mu^{(1)} \simeq [\text{Re}(n_\mu)]^2 I_{\mu H} / m_H^2 \text{ requires } [\text{Re}(n_\mu)]^2 \sim [m_H/4]^2$$

\Rightarrow a not too heavy H (reasonably perturbative n_μ)

$\Rightarrow m_A > m_H$ in order to avoid cancellations

- ☞ An explanation of $\delta a_\mu^{\text{Exp}}$ in terms of one loop contributions
might be possible

The new contributions to δa_ℓ

- Dominant two loop contributions: Barr-Zee diagrams
- In the same approximation (leading m_ℓ/m_S terms, $s_{\alpha\beta} \rightarrow 1$)

$$\Delta_\ell^{(2)} = - \left(\frac{2\alpha}{\pi} \right) \left(\frac{\text{Re}(n_\ell)}{m_\ell} \right) \textcolor{blue}{F}$$

$\textcolor{blue}{F}$ depends on

- masses of the fermions in the closed loop,
- couplings of those fermions to H and A,
- m_H and m_A

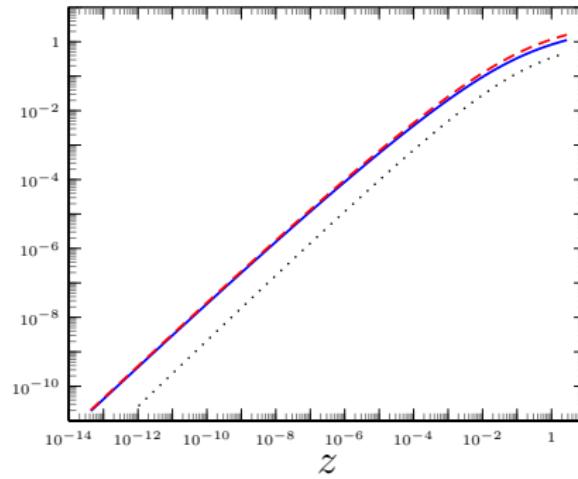
$$F_I = \frac{\cot\beta}{3} [4(f_{tH} + g_{tA}) + (f_{bH} - g_{bA})] + \frac{\text{Re}(n_\tau)}{m_\tau} (f_{\tau H} - g_{\tau A}),$$

$$F_{II} = \frac{\cot\beta}{3} [4(f_{tH} + g_{tA}) - \tan^2\beta (f_{bH} - g_{bA})] + \frac{\text{Re}(n_\tau)}{m_\tau} (f_{\tau H} - g_{\tau A})$$

The new contributions to δa_ℓ

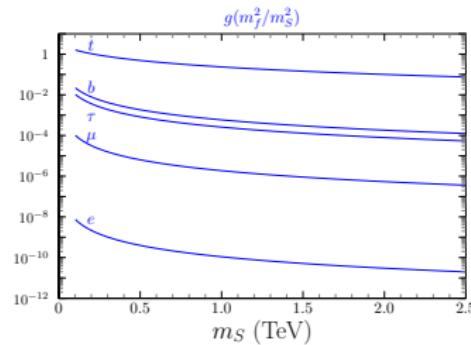
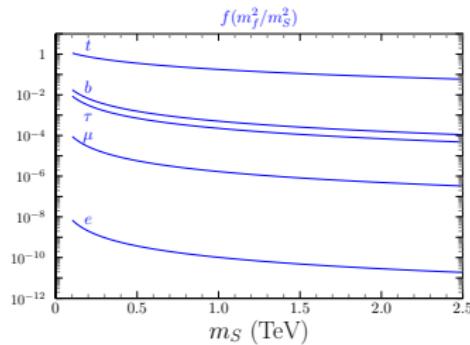
$$f_{\text{FS}} \equiv f \left(\frac{m_f^2}{m_S^2} \right), \quad g_{\text{FS}} \equiv g \left(\frac{m_f^2}{m_S^2} \right)$$

$f(z)$ ——— $g(z)$ - - - $g(z) - f(z)$



The new contributions to δa_ℓ

$$f_{\text{fS}} \equiv f \left(\frac{m_f^2}{m_S^2} \right), \quad g_{\text{fS}} \equiv g \left(\frac{m_f^2}{m_S^2} \right)$$



The new contributions to δa_ℓ

- Relevant aspects
 - $f(z) \simeq g(z)$ in the range of interest
 - the largest values correspond to the heavier fermion
 - the values of f and g for the top quark contributions vary between 0.1 and 1
- Considering the dominant top quark terms, for $t_\beta \simeq 1$ and $m_H \simeq m_A$, one can realize that for $m_H \sim 1 - 2$ TeV,
 $\delta a_e^{\text{Exp,Cs}}$ can be explained with $\text{Re}(n_e) \sim 3 - 7$ GeV
- To obtain $\delta a_\mu^{\text{Exp}}$ from the same type of contribution

$$\text{Re}(n_\mu) = \frac{\delta a_\mu}{\delta a_e} \frac{m_e}{m_\mu} \text{Re}(n_e) \simeq -15 \text{Re}(n_e)$$

Different signs of δa_e and $\delta a_\mu \rightarrow$ freedom to have

opposite $\text{Re}(n_e)$ and $\text{Re}(n_\mu)$

Same assumptions $t_\beta \sim 1$, $m_A \sim m_H \sim 1 - 2$ TeV

$\rightarrow \text{Re}(n_\mu) \in -[45; 105]$ GeV

Argument applies to both models I-g ℓ FC and II-g ℓ FC

The new contributions to δa_ℓ

Beyond $t_\beta \sim 1$

- $t_\beta \ll 1$ excluded in 2HDMs of types I and II by flavour constraints \Rightarrow excluded in I-g ℓ FC and II-g ℓ FC as well
- What about $t_\beta \gg 1$ and δa_ℓ ?
- The factor F

$$\Delta_\ell^{(2)} = - \left(\frac{2\alpha}{\pi} \right) \left(\frac{\text{Re}(n_\ell)}{m_\ell} \right) F$$

is quite model dependent

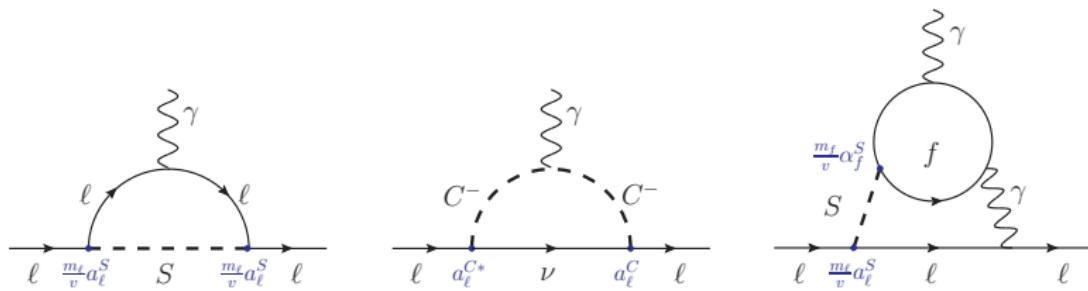
- We consider for reference $t_\beta \sim 1$ and $m_A \sim m_H \sim 1 - 2$ TeV, which can reproduce the anomalies, and analyse how to maintain that prediction if, for definiteness, $t_\beta \mapsto t_\beta = 50$

The new contributions to δa_ℓ

$$F_I = \frac{\cot \beta}{3} [4(f_{tH} + g_{tA}) + (f_{bH} - g_{bA})] + \frac{\text{Re}(n_\tau)}{m_\tau} (f_{\tau H} - g_{\tau A})$$

- In I-g ℓ FC, the $\cot \beta$ suppression can be compensated with *smaller* m_H , m_A and *larger* $\text{Re}(n_e)$: e.g. $m_A \sim m_H \sim 200$ GeV gives a factor of 10 with respect to $m_A \sim m_H \sim 1 - 2$ TeV, $\text{Re}(n_e) \mapsto 5\text{Re}(n_e)$ required to fully compensate the factor of 50
- That is, $\delta a_e^{\text{Exp,Cs}}$ can be reproduced by the two loop contributions in the $t_\beta \gg 1$ regime with light H, A and $\text{Re}(n_e) \sim 15 - 35$ GeV
- What about δa_μ ?
 $\text{Re}(n_\mu) \mapsto 5\text{Re}(n_\mu)$ gives $\text{Re}(n_\mu) \in -[225; 505]$ GeV,
in conflict with perturbativity requirements!
Fortunately, for light m_H , e.g. $m_H \in [200; 400]$ GeV and
 $|\text{Re}(n_\mu)| \sim m_H/4 \in [50; 100]$ GeV, the one loop contributions
can reproduce $\delta a_\mu^{\text{Exp}}$!

Loop contributions to δa_ℓ



One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathcal{L}_{S\ell\ell} = -\frac{m_\ell}{v} S \bar{\ell} (a_\ell^S + i b_\ell^S \gamma_5) \ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2} \frac{m_\ell^2}{v^2} \sum_S \left\{ [a_\ell^S]^2 (2I_2(x_{\ell S}) - I_3(x_{\ell S})) - [b_\ell^S]^2 I_3(x_{\ell S}) \right\},$$

with $x_{\ell S} \equiv m_\ell^2/m_S^2$ and

$$I_2(x) = 1 + \frac{1 - 2x}{2x\sqrt{1 - 4x}} \ln \left(\frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}} \right) + \frac{1}{2x} \ln x$$

$$I_3(x) = \frac{1}{2} + \frac{1}{x} + \frac{1 - 3x}{2x^2\sqrt{1 - 4x}} \ln \left(\frac{1 + \sqrt{1 - 4x}}{1 - \sqrt{1 - 4x}} \right) + \frac{1 - x}{2x^2} \ln x$$

One loop contributions to δa_ℓ

For $x \ll 1$,

$$I_2(x) \simeq x \left(-\frac{3}{2} - \ln x \right) + x^2 \left(-\frac{16}{3} - 4 \ln x \right) + \mathcal{O}(x^3)$$

$$I_3(x) \simeq x \left(-\frac{11}{6} - \ln x \right) + x^2 \left(-\frac{89}{12} - 5 \ln x \right) + \mathcal{O}(x^3)$$

For $m_\ell \ll m_S$,

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2} \frac{m_\ell^2}{m_S^2} \frac{m_\ell^2}{v^2} \left\{ -[a_\ell^S]^2 \left(\frac{7}{6} + \ln \left(\frac{m_\ell^2}{m_S^2} \right) \right) + [b_\ell^S]^2 \left(\frac{11}{6} + \ln \left(\frac{m_\ell^2}{m_S^2} \right) \right) \right\}$$

One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathcal{L}_{C\ell\nu} = -C^- \bar{\ell}(a_\ell^C + i b_\ell^C \gamma_5) \nu - C^+ \bar{\nu}(a_\ell^{C*} + i b_\ell^{C*} \gamma_5) \ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = -\frac{1}{8\pi^2} \sum_C \left\{ |a_\ell^C|^2 + |b_\ell^C|^2 \right\} H(x_{\ell C})$$

where $x_{\ell C} = m_\ell^2/m_{C^\pm}^2$, and

$$H(x) = -\frac{1}{2} + \frac{1}{x} + \frac{1-x}{x^2} \ln(1-x), \quad H(x) \simeq \frac{x}{6} + \frac{x^2}{12} + \mathcal{O}(x^3) \text{ for } x \ll 1$$

Two loop contributions to δa_ℓ

For quarks

$$\mathcal{L}_{S\bar{f}f} = -\frac{m_f}{v} S \bar{f} (\alpha_f^S + i\beta_f^S \gamma_5) f$$

The two loop Barr-Zee contributions to the anomalous magnetic moment of lepton ℓ

$$\Delta a_\ell^{(2)} = -\frac{\alpha^2}{4\pi^2 s_W^2} \frac{m_\ell^2}{M_W^2} \sum_f \sum_S N_c^f Q_f^2 \{ a_\ell^S \alpha_f^S f(z_{fS}) - b_\ell^S \beta_f^S g(z_{fS}) \}$$

f : fermions in the closed fermion loop

(N_c^f colour, Q_f electric charge, $z_{fS} = m_f^2/m_S^2$)

S : neutral scalar connecting the closed fermion loop with the external lepton line

Two loop contributions to δa_ℓ loop contributions to δa_ℓ

Loop functions

$$f(z) = \frac{z}{2} \int_0^1 dx \frac{1 - 2x(1-x)}{x(1-x) - z} \ln \left(\frac{x(1-x)}{z} \right)$$
$$g(z) = \frac{z}{2} \int_0^1 dx \frac{1}{x(1-x) - z} \ln \left(\frac{x(1-x)}{z} \right)$$

Constraints: δa_ℓ

- The anomalies

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13}, \quad \delta a_\mu^{\text{Exp}} = (2.5 \pm 0.6) \times 10^{-9}.$$

- The “natural” δa_ℓ constraint

$$\chi_0^2(\delta a_e, \delta a_\mu) = \left(\frac{\delta a_e - c_e}{\sigma_e} \right)^2 + \left(\frac{\delta a_\mu - c_\mu}{\sigma_\mu} \right)^2,$$

$$\text{with } \delta a_\ell^{\text{Exp}} = c_\ell \pm \sigma_\ell$$

- We impose a stronger requirement

$$\chi^2(\delta a_e, \delta a_\mu) = 16\chi_0^2(\delta a_e, \delta a_\mu)$$

that is $\sigma_\ell \mapsto \sigma_\ell/4$

Constraints

- Scalar sector
 - potential bounded from below
 - perturbativity and perturbative unitarity of $2 \rightarrow 2$ high energy scattering
 - electroweak precision (oblique parameters S, T)
 - ☞ one can play with M_W and the CDF value!
- Fermion sector: perturbative Yukawa couplings

$$|n_\ell| \leq n_0$$

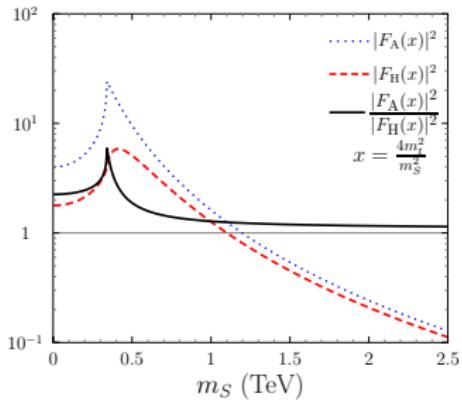
with two different choices $n_0 = 100$ GeV or $n_0 = 250$ GeV

- Higgs signal strengths:
 - production \times decay signal strengths of the usual channels
 - large lepton couplings: also include $h \rightarrow \mu^+ \mu^-, e^+ e^-$ information
 - \Rightarrow Higgs alignment

Constraints

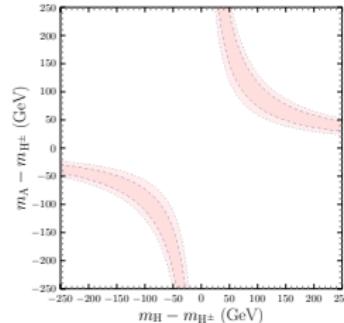
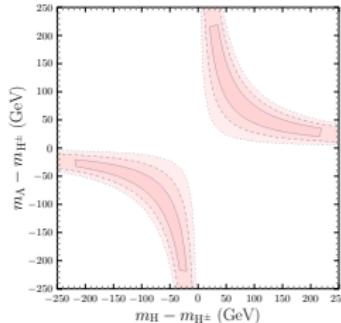
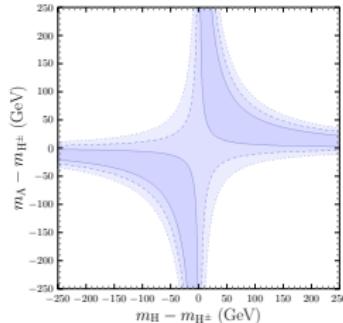
- H^\pm mediated contributions
 - Lepton flavour universality
 - purely leptonic decays $\ell_j \rightarrow \ell_k \nu \bar{\nu}$
 - decays with light pseudoscalar mesons $K, \pi \rightarrow e\nu, \mu\nu$ and $\tau \rightarrow K\nu, \pi\nu$
 - $b \rightarrow s\gamma$, $B_q^0 - \bar{B}_q^0$ mixing
- $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$ at LEP
 - (cross sections up to $\sqrt{s} = 208$ GeV)
- LHC searches
 - searches of dilepton resonance: $\sigma(pp \rightarrow S)_{[\text{ggF}]} \times \text{Br}(S \rightarrow \ell^+\ell^-)$,
 $S = H, A$ and $\ell = \mu, \tau$
 - searches of charged scalars: $\sigma(pp \rightarrow H^\pm tb) \times \text{Br}(H^\pm \rightarrow f)$,
 $f = \tau\nu, tb$

Gluon-gluon-scalar vs gluon-gluon-pseudoscalar



$F_H(x)$ from $S\bar{t}t$, $F_A(x)$ from $S\bar{t}\gamma_5 t$

Oblique parameters and M_W from CDF



$m_A - m_{H^\pm}$ vs $m_H - m_{H^\pm}$ for $m_{H^\pm} = 1$ TeV

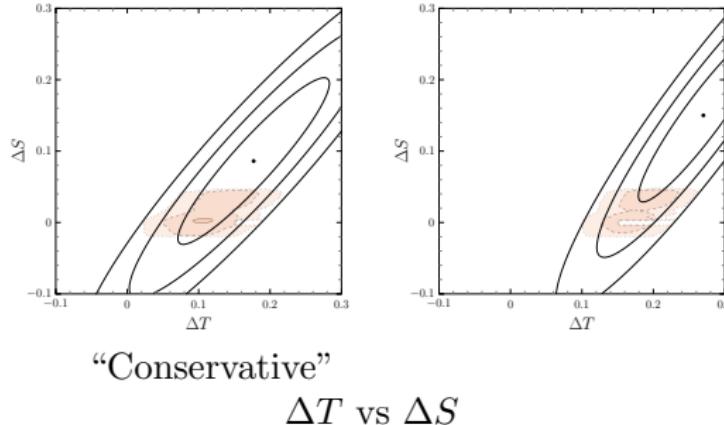
$$\begin{aligned}\Delta S &= 0.00 \pm 0.07 \\ \Delta T &= 0.05 \pm 0.06 \\ \rho &= 0.92\end{aligned}$$

$$\begin{aligned}\Delta S &= 0.086 \pm 0.077 \\ \Delta T &= 0.177 \pm 0.070 \\ \rho &= 0.89 \\ \text{“Conservative”} & \\ \textcolor{blue}{2204.04204} &\end{aligned}$$

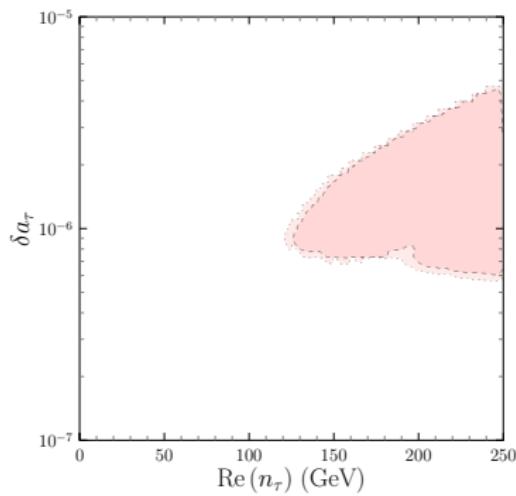
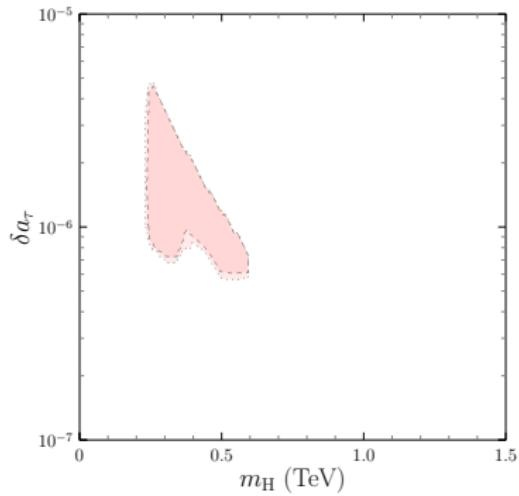
$$\begin{aligned}\Delta S &= 0.15 \pm 0.08 \\ \Delta T &= 0.27 \pm 0.06 \\ \rho &= 0.93 \\ \textcolor{blue}{2204.03796} &\end{aligned}$$

Oblique parameters and M_W from CDF

Bottom line: since m_{H^\pm} is rather irrelevant for δa_ℓ ,
a simple shift in m_{H^\pm} may work



δa_τ



$\delta a_\tau \& m_H, n_\tau$

Example points

Pt	m_H	m_A	m_{H^\pm}	t_β	$\text{Re}(\mu_{12}^2)$	$\text{Re}(n_e)$	$\text{Re}(n_\mu)$	$\text{Re}(n_\tau)$
1	362	775	778	10.7	-1.19×10^4	8.16	-72.8	222.5
2	277	494	502	10.8	-6.61×10^3	9.12	-58.0	191.2
3	278	539	554	7.93	-8.90×10^5	6.71	-53.1	189.7

Parameters in GeV ($\text{Re}(\mu_{12}^2)$ in GeV^2)

Pt	$\delta a_e^{(*)}$	1 loop			2 loop					
		H	A	H^\pm	tH	tA	τH	τA	μH	μA
1	-7.24	0	0	0	0.307	0.199	0.779	-0.259	-0.036	0.010
2	-9.84	0	0	0	0.310	0.258	0.845	-0.388	-0.038	0.015
3	-8.87	0	0	0	0.347	0.264	0.683	-0.271	-0.028	0.010

Relative contributions to δa_e .

(*) $: 10^{13} \delta a_e$

Example points

Pt	$\delta a_\mu^{(*)}$	1 loop			2 loop					
		H	A	H^\pm	tH	tA	τH	τA	μH	μA
1	2.42	0.588	-0.136	-0.003	0.169	0.110	0.429	-0.143	-0.020	0.006
2	2.29	0.650	-0.218	-0.005	0.175	0.146	0.477	-0.219	-0.021	0.009
3	2.35	0.531	-0.147	-0.003	0.214	0.163	0.422	-0.168	-0.017	0.006

Relative contributions to δa_μ . (*) : $10^9 \delta a_\mu$

Pt	$S \rightarrow$	$e\bar{e}$	$\mu\bar{\mu}$	$\tau\bar{\tau}$	$t\bar{t}$	Hz
1	H	1.2×10^{-3}	0.096	0.902	7×10^{-4}	—
	A	0.0004	0.028	0.265	0.004	0.703
2	H	0.002	0.084	0.914	—	—
	A	0.001	0.049	0.528	0.008	0.415
3	H	0.001	0.073	0.926	—	—
	A	0.0005	0.033	0.415	0.012	0.540

Decay branching ratios of H and A

Example points

$\sigma(pp \rightarrow S)_{[\text{ggF}]} \times \text{Br}(S \rightarrow \tau^+ \tau^-)$ (fb)					
Point		Value	Expected bound	Observed bound	Excess
1	H	83.6	54.9	106.0	✓
	A	1.50	3.2	3.2	—
2	H	84.0	136.0	164.0	—
	A	27.3	20.8	50.4	✓
3	H	160.1	135.4	164.0	✓
	A	27.6	17.6	31.1	✓

Different δa_e^{Exp} assumptions

Analyses with different “measurements”

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13},$$

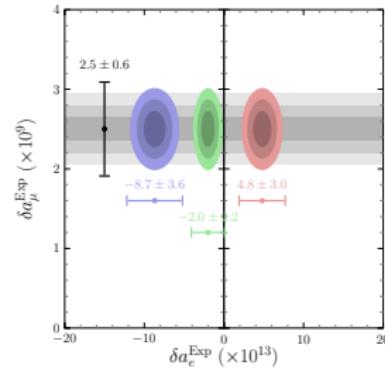
$$\delta a_e^{\text{Exp,Rb}} = (4.8 \pm 3.0) \times 10^{-13}$$

$$\delta a_e^{\text{Exp,Avg}} = -(2.0 \pm 2.2) \times 10^{-13},$$

$$\delta a_e^{\text{Exp,Bound}} = 20 \times 10^{-13}$$

N.B. Last case: $|\delta a_e| < \delta a_e^{\text{Exp,Bound}}$

In all cases $n_0 = 250$ GeV



Different δa_e^{Exp} assumptions

Simple analysis

- For a point in parameter space explaining $\delta a_e^{\text{Exp,Cs}}$,
 - 1 consider $\text{Re}(n_e) \mapsto \text{Re}(n_e) \frac{\delta a_e^{\text{Exp}}}{\delta a_e^{\text{Exp,Cs}}}$, which gives $\delta a_e \simeq \delta a_e^{\text{Exp}}$
 - 2 analyse if this new value conflicts with other observables sensitive to $\text{Re}(n_e)$: muon decay and pseudoscalar mesons decays
- Answer
 - Short version: $\delta a_e^{\text{Exp,Cs}}$ is “worst case”
because of absolute value and sign
 - Long version in the next slides
 - ☒ all cases can be reproduced at least with the regions arising from the previous $\text{Re}(n_e)$ transformation

Different δa_e^{Exp} assumptions

- For muon decay

$$\left| \frac{\text{Re}(n_e) \text{Re}(n_\mu)}{m_{H^\pm}^2} \right| < 0.035$$

$\Rightarrow |\text{Re}(n_e)|$ for $\delta a_e^{\text{Exp,Cs}}$ is “worse” than other cases

- For pseudoscalar meson decays, consider

$$R_{\mu e}^P = \frac{\Gamma(P^+ \rightarrow \mu^+ \nu)}{\Gamma(P^+ \rightarrow \mu^+ \nu)_{\text{SM}}} \frac{\Gamma(P^+ \rightarrow e^+ \nu)_{\text{SM}}}{\Gamma(P^+ \rightarrow e^+ \nu)}$$

Experimental values:

$$R_{\mu e}^\pi = 1 + (4.1 \pm 3.3) \times 10^{-3}, \quad R_{\mu e}^K = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

$$R_{\mu e}^P = \frac{|1 - \Delta_\mu^P|^2}{|1 - \Delta_e^P|^2}, \quad |1 - \Delta_\ell^P|^2 = \left| 1 - \frac{M_P^2}{t_\beta m_{H^\pm}^2} \frac{\text{Re}(n_\ell)}{m_\ell} \right|^2$$

Different δa_e^{Exp} assumptions

Experimental values:

$$R_{\mu e}^\pi = 1 + (4.1 \pm 3.3) \times 10^{-3}, \quad R_{\mu e}^K = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

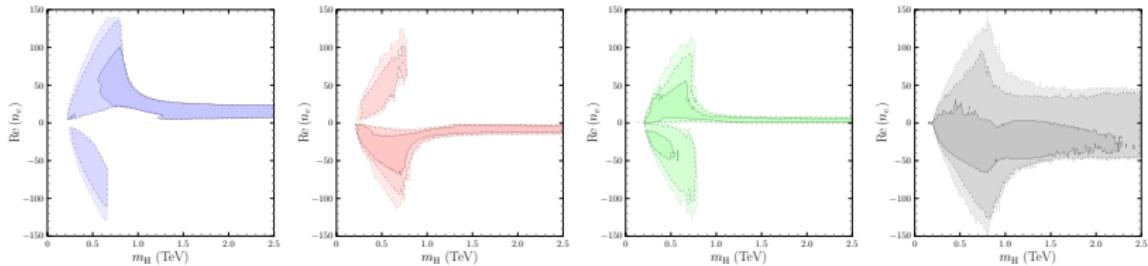
$$R_{\mu e}^P = \frac{|1 - \Delta_\mu^P|^2}{|1 - \Delta_e^P|^2}, \quad |1 - \Delta_\ell^P|^2 = \left| 1 - \frac{M_P^2}{t_\beta m_{H^\pm}^2} \frac{\text{Re}(n_\ell)}{m_\ell} \right|^2$$

- For $\Delta_\ell^P \ll 1$,

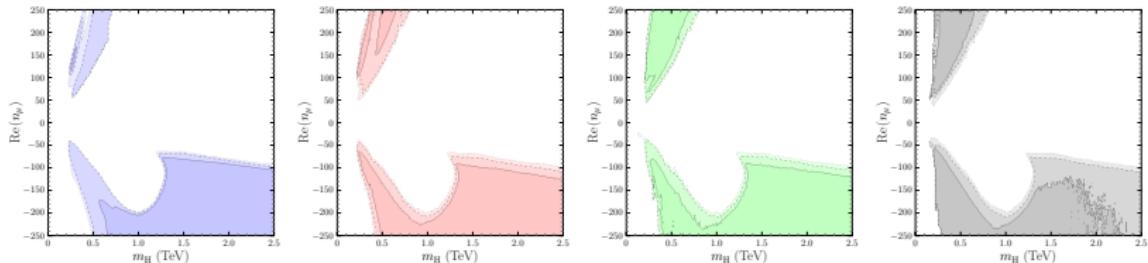
$$R_{\mu e}^P \simeq 1 + 2 \frac{M_P^2}{t_\beta m_{H^\pm}^2} \left(\frac{\text{Re}(n_e)}{m_e} - \frac{\text{Re}(n_\mu)}{m_\mu} \right)$$

- Concentrate on $R_{\mu e}^K$, neglect the $\text{Re}(n_\mu)$ contribution:
 $\text{Re}(n_e) > 0$ for $\delta a_e^{\text{Exp,Cs}}$ is “worse” than other cases

Different δa_e^{Exp} assumptions

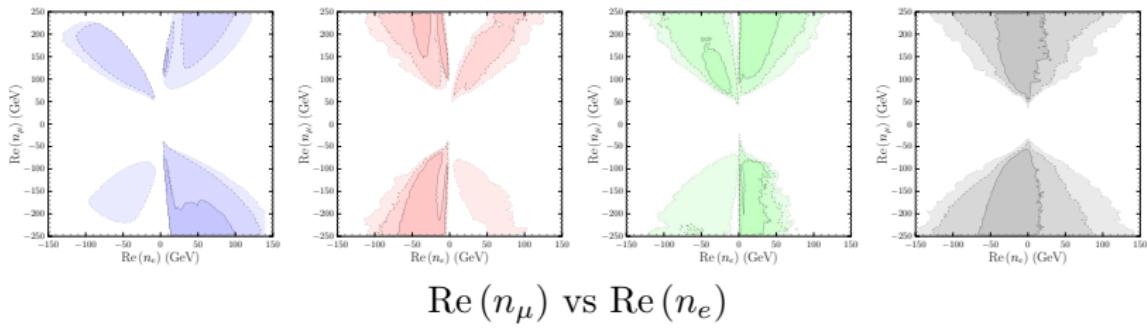


$\text{Re}(n_e)$ vs m_H



$\text{Re}(n_\mu)$ vs m_H

Different δa_e^{Exp} assumptions



Yukawa couplings

Neutral scalars

$$\begin{aligned}\mathcal{L}_{S\bar{f}f} = & -\frac{S}{v} \bar{f} \left[\mathcal{R}_{1s} M_f + \mathcal{R}_{2s} \frac{N_f + N_f^\dagger}{2} + i\epsilon_{(f)} \mathcal{R}_{3s} \frac{N_f - N_f^\dagger}{2} \right] f \\ & - \frac{S}{v} \bar{f} \gamma_5 \left(\mathcal{R}_{2s} \frac{N_f - N_f^\dagger}{2} + i\epsilon_{(f)} \mathcal{R}_{3s} \frac{N_f + N_f^\dagger}{2} \right) f\end{aligned}$$

where $s = 1, 2, 3$ in correspondence with $S = h, H, A$; $f = u, d, \ell$; in terms proportional to \mathcal{R}_{3s} , $\epsilon_{(d)} = \epsilon_{(\ell)} = -\epsilon_{(u)} = 1$

Yukawa couplings

Charged scalars

$$\begin{aligned}\mathcal{L}_{H^\pm ud} = & \frac{H^-}{\sqrt{2}v} \bar{d} \left[V^\dagger N_u - N_d^\dagger V^\dagger + \gamma_5 (V^\dagger N_u + N_d^\dagger V^\dagger) \right] u \\ & + \frac{H^+}{\sqrt{2}v} \bar{u} \left[N_u^\dagger V - V N_d + \gamma_5 (N_u^\dagger V + V N_d) \right] d\end{aligned}$$

and

$$\mathcal{L}_{H^\pm \ell \nu} = -\frac{\sqrt{2}}{v} H^+ \bar{\nu}_L U^\dagger N_\ell \ell_R - \frac{\sqrt{2}}{v} H^- \bar{\ell}_R N_\ell^\dagger U \nu_L$$

V and U are, respectively, the CKM and PMNS mixing matrices
(massless neutrinos assumed, one can set $U \rightarrow \mathbf{1}$)

Couplings in the I-g ℓ FC and II-g ℓ FC models

$$\mathcal{L}_N = - \sum_{S=h,H,A} \sum_{f=u,d,\ell} \sum_{j=1}^3 \frac{m_{f_j}}{v} S \bar{f}_j (a_{f_j}^S + i b_{f_j}^S \gamma_5) f_j$$

Quark couplings

		a_u^S	b_u^S	a_d^S	b_d^S
I-g ℓ FC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0
	A	0	$-t_\beta^{-1}$	0	$+t_\beta^{-1}$
II-g ℓ FC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} - c_{\alpha\beta} t_\beta$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} - s_{\alpha\beta} t_\beta$	0
	A	0	$-t_\beta^{-1}$	0	$-t_\beta$

Couplings in the I-g ℓ FC and II-g ℓ FC models

$$\mathcal{L}_N = - \sum_{S=\text{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^3 \frac{m_{f_j}}{v} S \bar{f}_j (a_{f_j}^S + i b_{f_j}^S \gamma_5) f_j$$

Lepton couplings

		a_ℓ^S	b_ℓ^S
I-g ℓ FC	h	$s_{\alpha\beta} + c_{\alpha\beta} \frac{\text{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} \frac{\text{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\text{Re}(n_\ell)}{m_\ell}$
II-g ℓ FC	h	$s_{\alpha\beta} + c_{\alpha\beta} \frac{\text{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} \frac{\text{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\text{Re}(n_\ell)}{m_\ell}$

Couplings in the I-g ℓ FC and II-g ℓ FC models back

Yukawa couplings of the charged scalar

$$\mathcal{L}_{Ch} = -\frac{1}{\sqrt{2}v} \sum_{f=q,\ell} \sum_{j,k=1}^3 \left\{ H^- \bar{f}_{-\frac{1}{2},j} (\alpha_{jk}^f + i\beta_{jk}^f \gamma_5) f_{\frac{1}{2},k} + H^+ \bar{f}_{\frac{1}{2},k} (\alpha_{jk}^{f*} + i\beta_{jk}^{f*} \gamma_5) f_{-\frac{1}{2},j} \right\}$$

with $q_{+\frac{1}{2},j} = u_j$; $q_{-\frac{1}{2},j} = d_j$; $\ell_{+\frac{1}{2},j} = \nu_j$; $\ell_{-\frac{1}{2},j} = \ell_j$

	α_{ij}^q	β_{ij}^q
I-g ℓ FC	$V_{ji}^* t_\beta^{-1} (m_{u_j} - m_{d_i})$	$V_{ji}^* t_\beta^{-1} (m_{u_j} + m_{d_i})$
II-g ℓ FC	$V_{ji}^* (t_\beta^{-1} m_{u_j} + t_\beta m_{d_i})$	$V_{ji}^* (t_\beta^{-1} m_{u_j} - t_\beta m_{d_i})$

	α_{ij}^ℓ	β_{ij}^ℓ
I-g ℓ FC	$-\text{Re}(n_{\ell_i}) \delta_{ij}$	$\text{Re}(n_{\ell_i}) \delta_{ij}$
II-g ℓ FC	$-\text{Re}(n_{\ell_i}) \delta_{ij}$	$\text{Re}(n_{\ell_i}) \delta_{ij}$