# (Towards a) Production Model-Independent Top Mass Measurement Using B-hadron Decay Length

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[with Sagar Airen, Roberto Franceschini, Joseph Incandela, Doojin Kim, Deepak Sathyan: parts of talk is my personal opinion only!]

(Snowmass whitepaper: 2204.02928 and 2212.03929)

# Outline

- Why another method for top quark mass measurement?!
- energy of bottom quark from top quark decay → $m_t$
- Review of bottom quark/$b$-jet "energy-peak" for top quark mass: (quasi-)production model-independent (already done by CMS, but improvement using 13 TeV, NLO…)
- $B$-hadron decay length: "proxy" for bottom quark energy → trade-off: avoid jet energy scale (JES) uncertainty of above, but bring-in hadronization model/fragmentation function (done by CDF/CMS, but assuming SM production)
- Combining above two: new (quasi-)model-independent and JES uncertainty-free ("best of both worlds"!) proposal for measuring top quark mass using $B$-hadron decay length… …but still subject to hadronization model/fragmentation (theory improvement possible?)

# *Motivation for new methods for top quark mass measurement*

*(skip review of why top quark mass is crucial parameter of SM and Beyond)*

# Systematics (statistics not an issue at LHC?!)

## Theoretical

- uncertainties about top quark (pair) production: Beyond SM (BSM) contribution (e.g. light stop decaying into top: see 1407.1043; 1909.09670); PDF's, higher-order effects, even in SM [e.g., top quark $p_T$ (mis-)modeling?!]; hadronization of bottom quark (cf. lepton from $W$ decay)

## Experimental

- JES uncertainty for $b$-jet vs. using ("cleaner"?) leptonic measurements

- each method insensitive to some systematics, but affected by others

# Bottomline

- In my opinion, no "slam dunk" top quark measurement method!

→ motivates new ideas, especially

- independent of details/modeling of production [based on kinematics of (only) decay, thus avoid (some) theory systematics] and/or

- insensitive to some experimental systematics (complementarity)

[Full reconstruction of top quark decay on event-by-event basis not possible due to missing $\nu$ and/or combinatorics (cf. Higgs → di-photon or Z → di-lepton)]

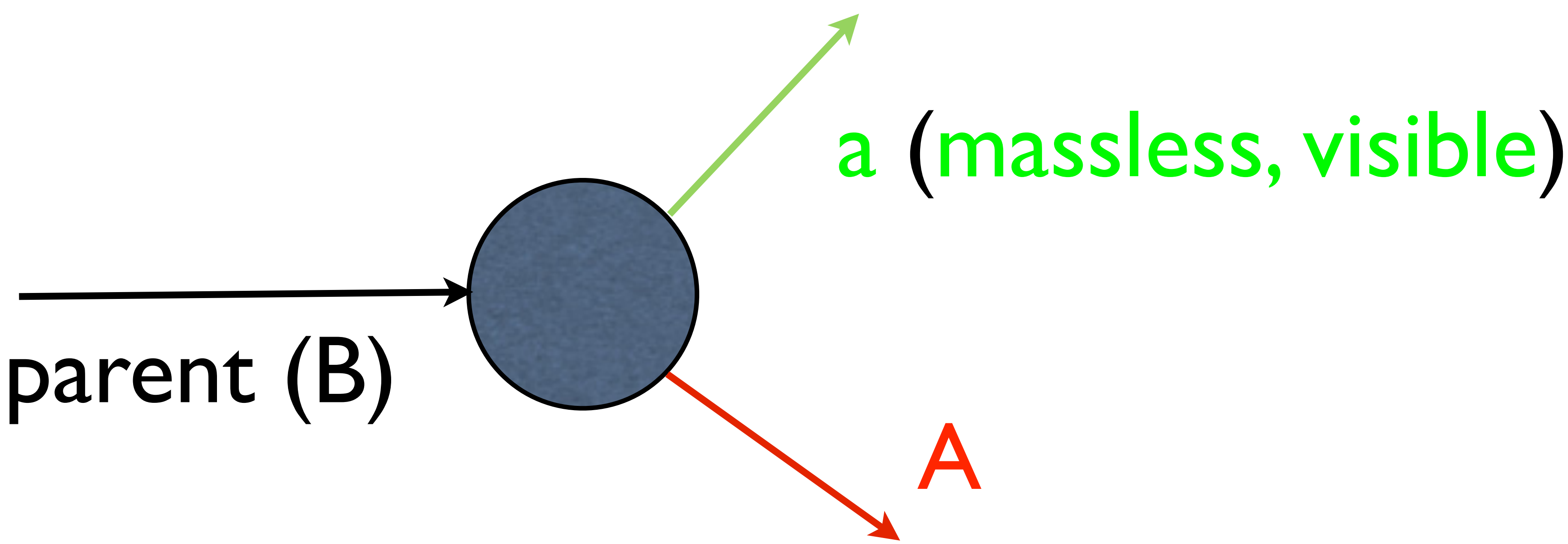
# Review of energy-peak: *general*

*[(quasi-)decay kinematics-based]*

# NEW OBSERVATION/``INVARIANCE"

# Basic set-up/assumptions

- 2-body decay: one child particle visible, massless:




- ...other (A) don't "care" (except for its mass): no need to reconstruct it!
- unpolarized parent (all spin orientations equal) ➡ "quasi" (production) model-independent

# Energy of child particle

- mono-chromatic and simple function of masses in rest frame of parent:

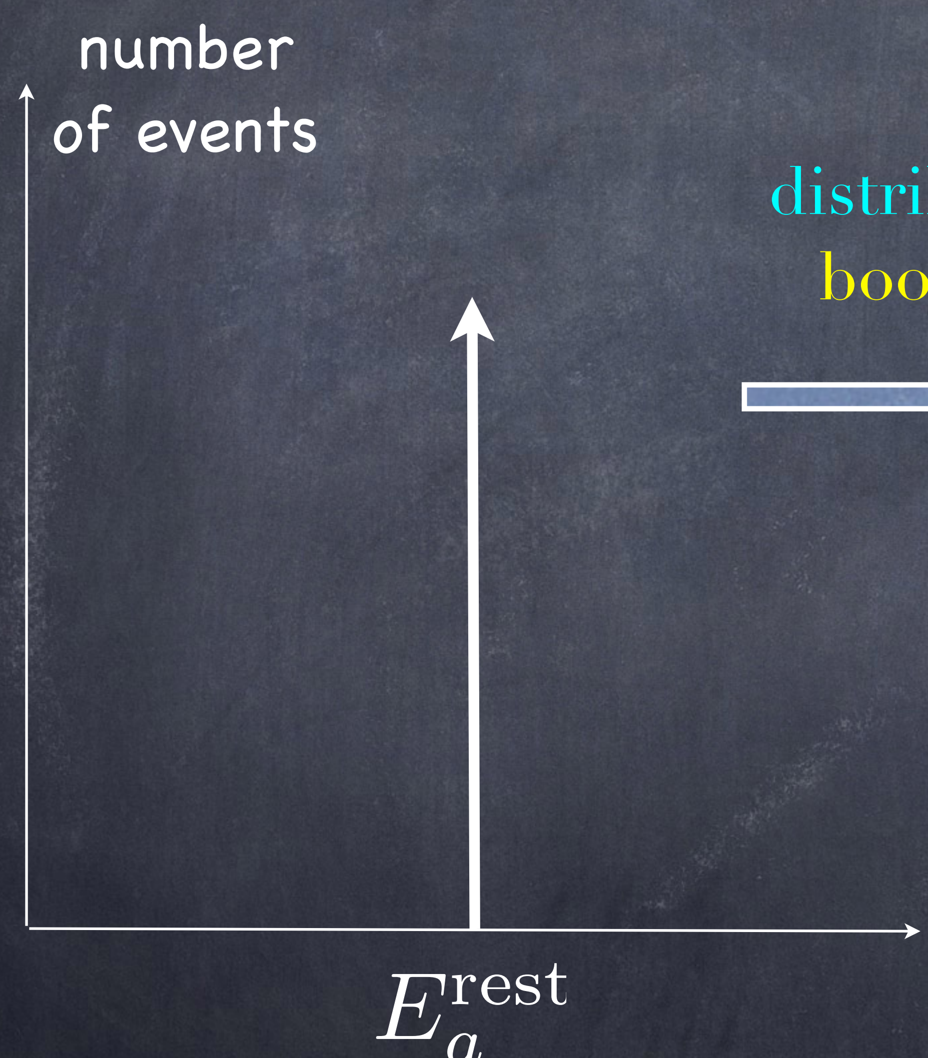
$$E_a^{\mathrm{rest}} = \frac{M_B^2 - M_A^2}{2M_B}$$

- determine $M_B$ if $M_A$ known and $E_a^{\mathrm{rest}}$ measured

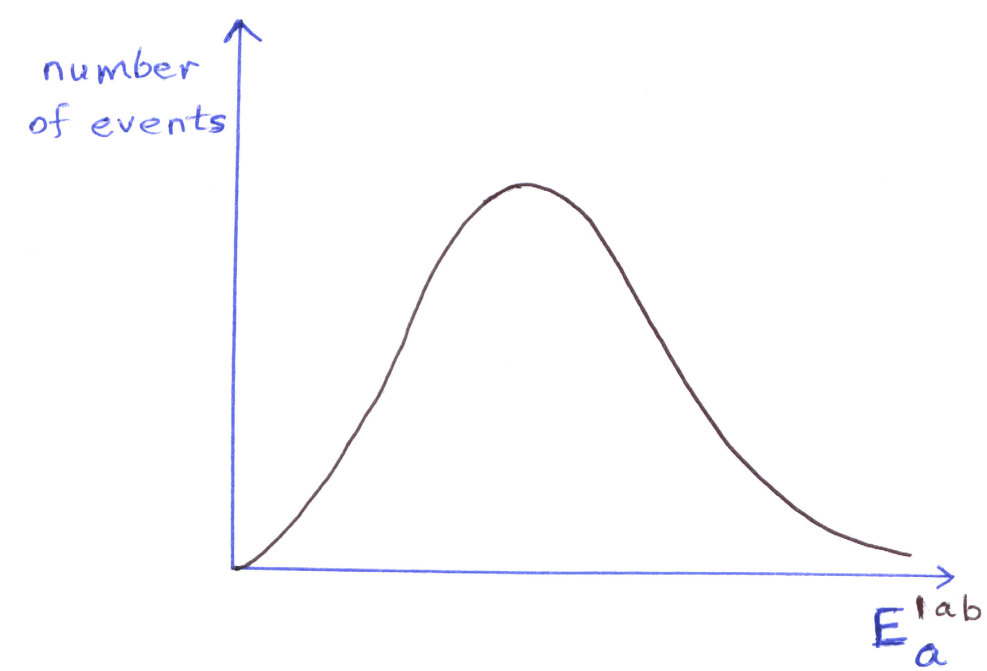
...but not Lorentz (parent boost)-invariant

# ...too simple to be practical/useful?!

- hadron collider: parent has unknown boost; varies event to event ⟹ distribution in $E_a^{\text{lab}}$




distribution of boosts of B ⟹




- "lose" rest-frame information??!!

# "Conservation" of invariance!

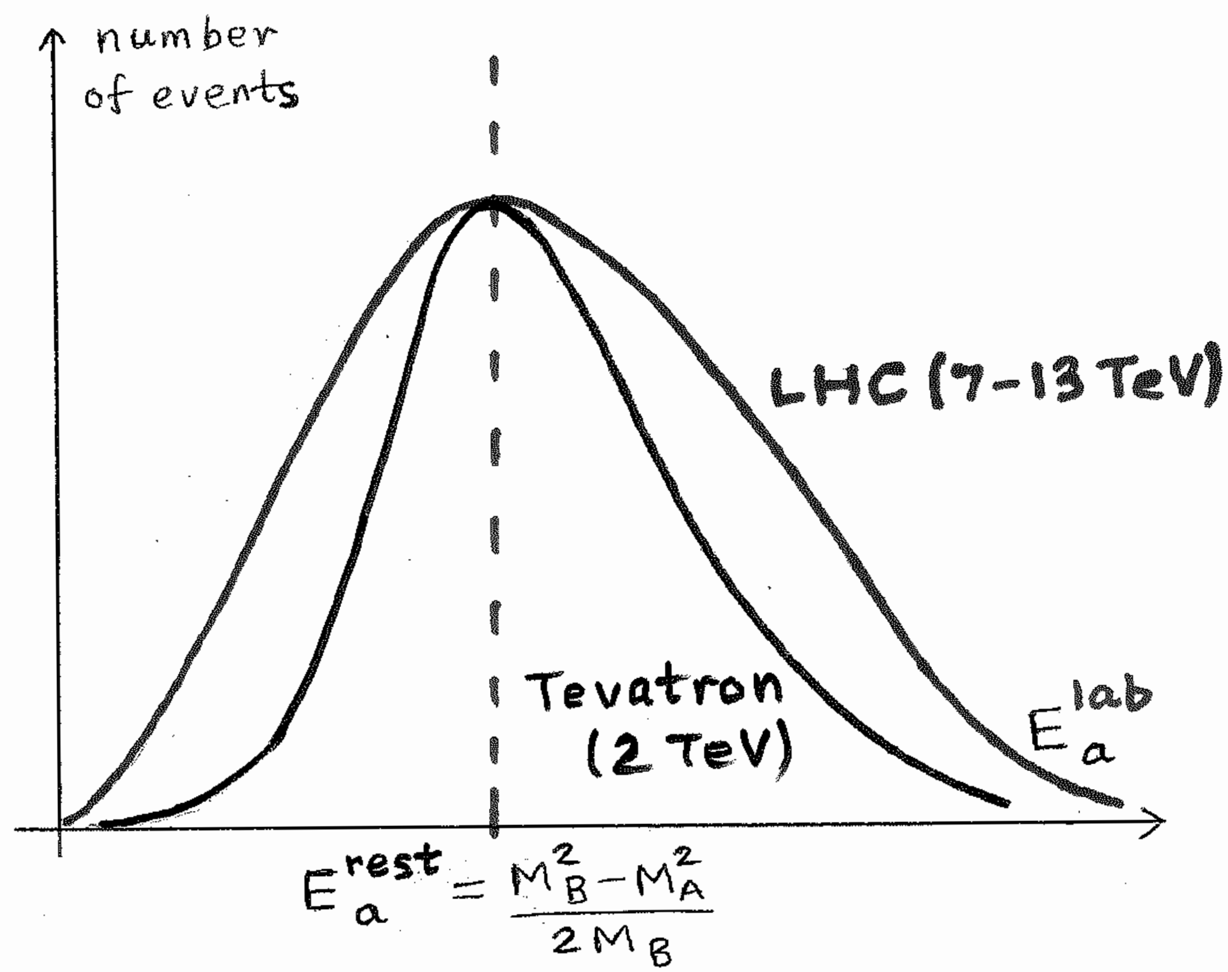
- Show analytically (in 3 back-up slides!): peak (of lab. distribution) still retains this information... simply, precisely, robustly!

independent of boosts of parent, hence production details

- Distribution of log of energy is symmetric about peak

(KA, Franceschini, Kim: 1209.0772)

(see also Stecker: "Cosmic gamma rays")



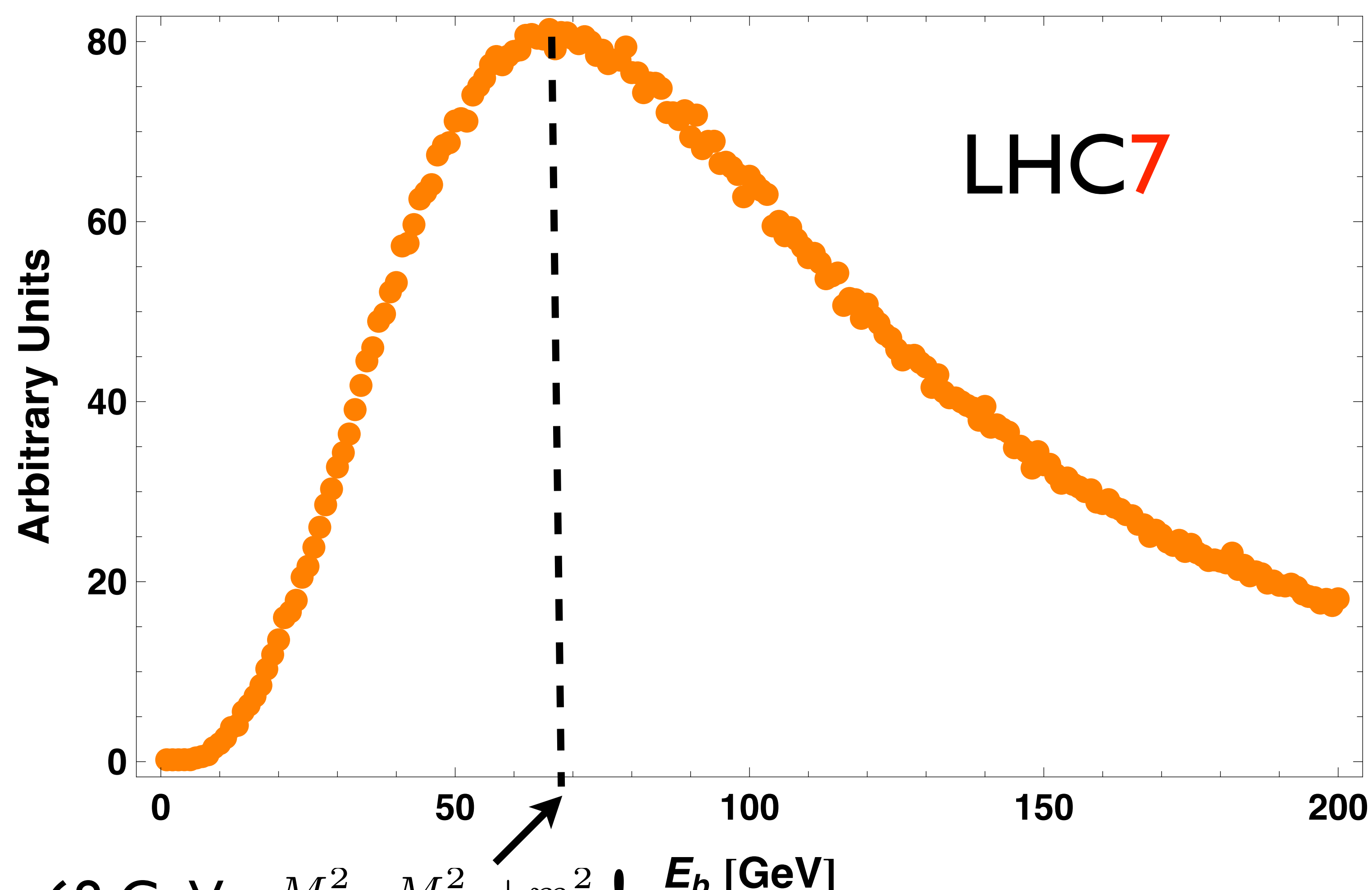
Analytical result (in 3 back-up slides!) → no need really to check via full calculation/simulation, but anyway…

- (“massless”) bottom (parton-level) from 2-body top quark decay (production unpolarized) as example of general result:

bottom mass non-zero, but negligible → peak of energy distribution in lab frame is not expected to shift from single value in top quark rest frame:

$$E_b^{\text{rest}} = \frac{M_t^2 - M_W^2 + m_b^2}{2M_t}$$

modified vs. massless

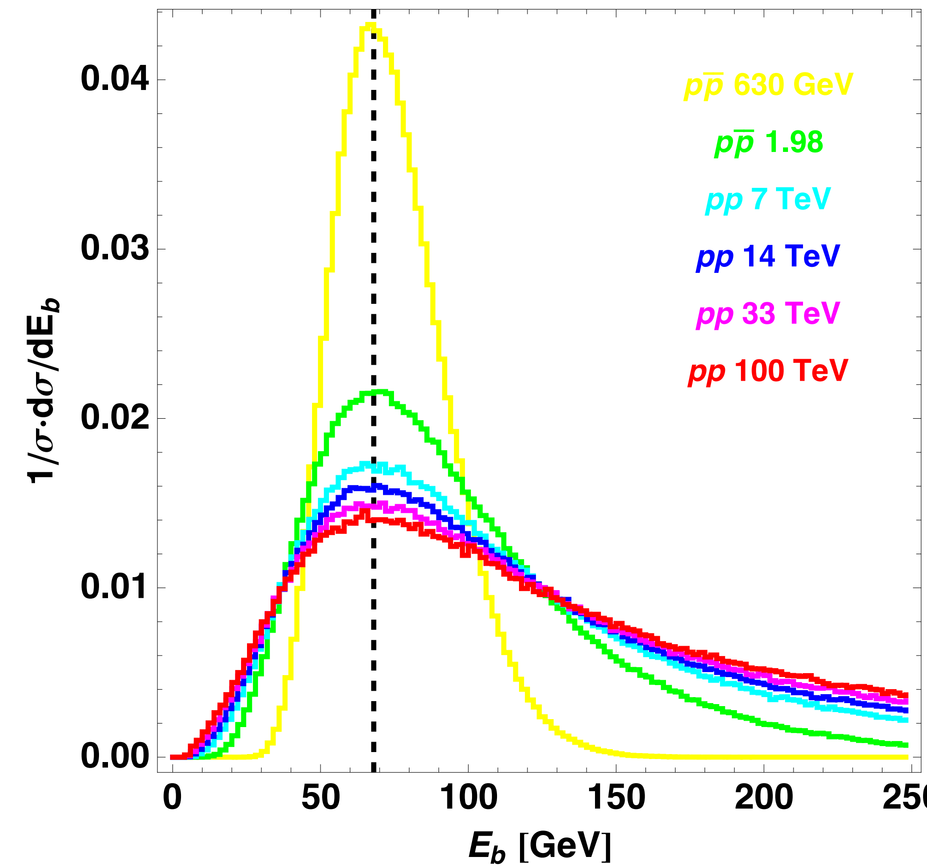



68 GeV $= \frac{M_t^2 - M_W^2 + m_b^2}{2M_t}$ !

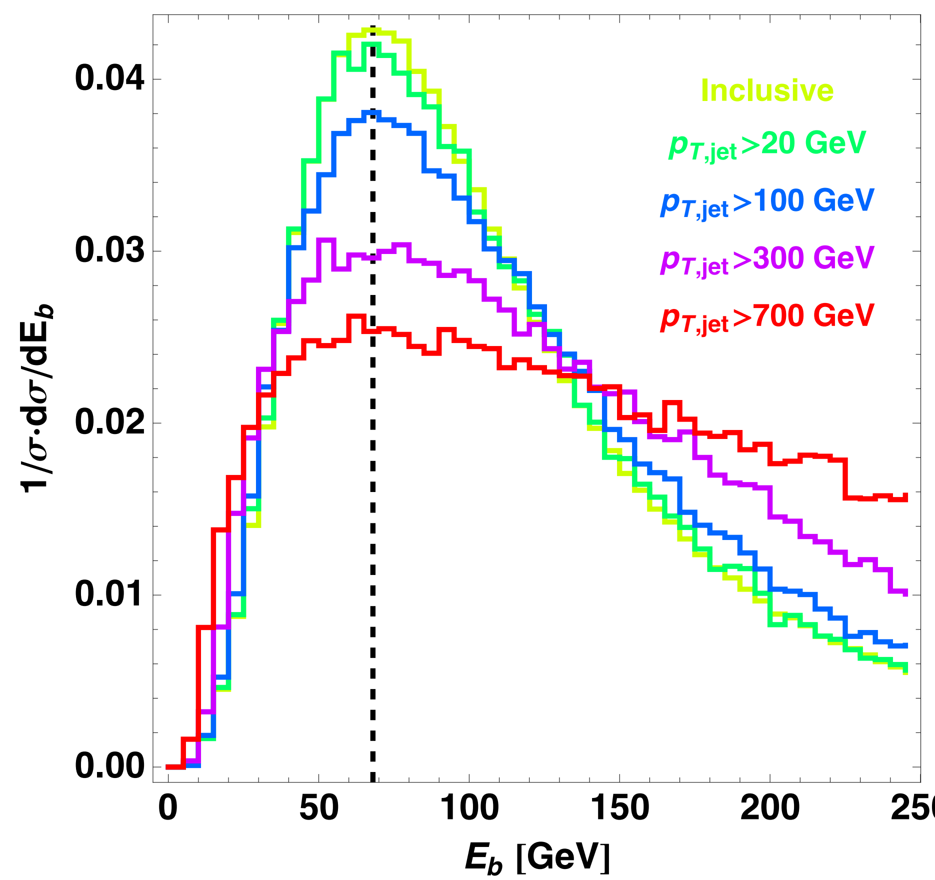
- …maybe an “accident” of specific boost distribution (production model) of top quark, e.g., @ LHC7?!

# ``Invariant'' (under boost distributions) feature in non-invariant (energy) distribution: subtle!

- vary collider energy

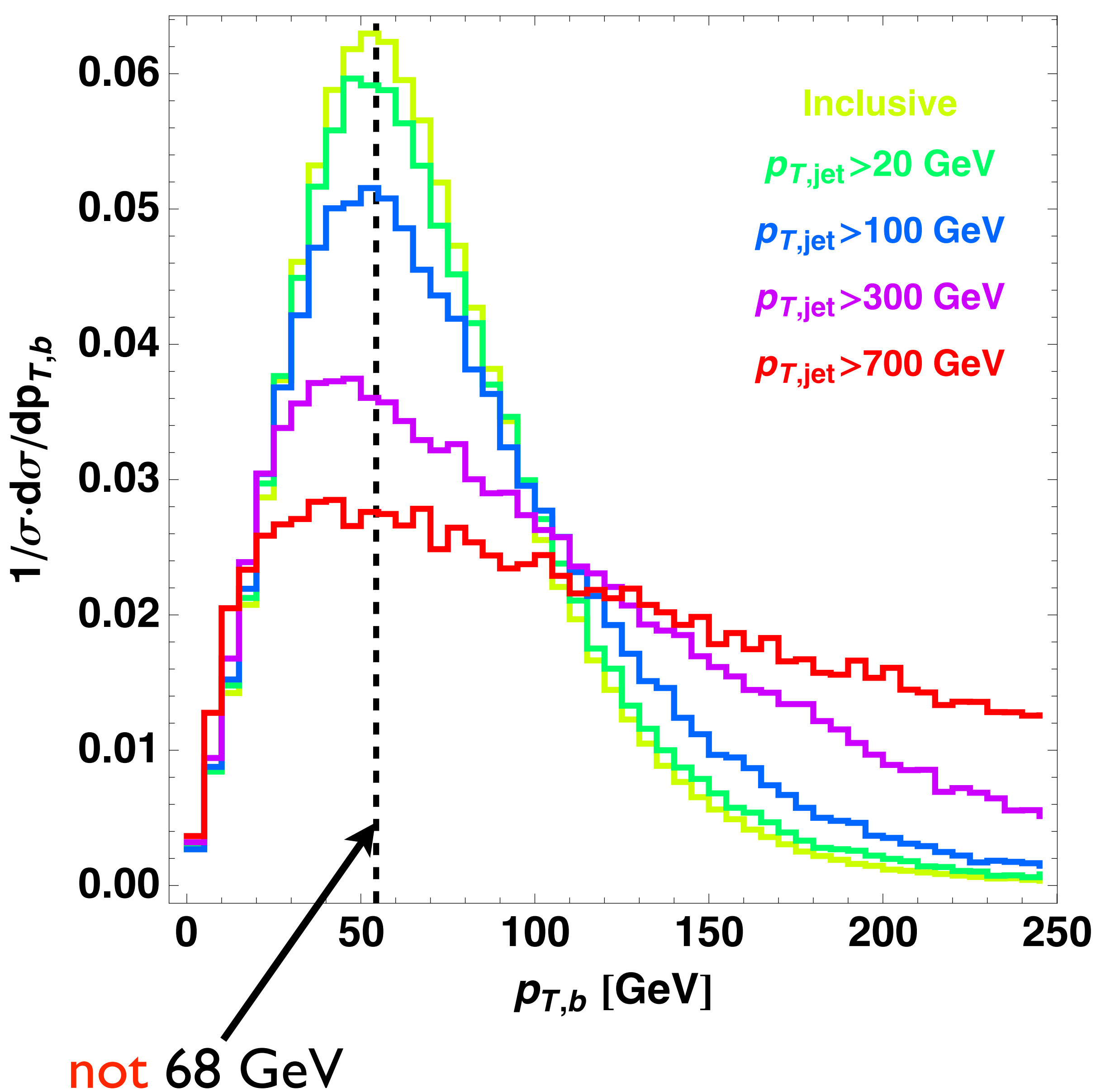


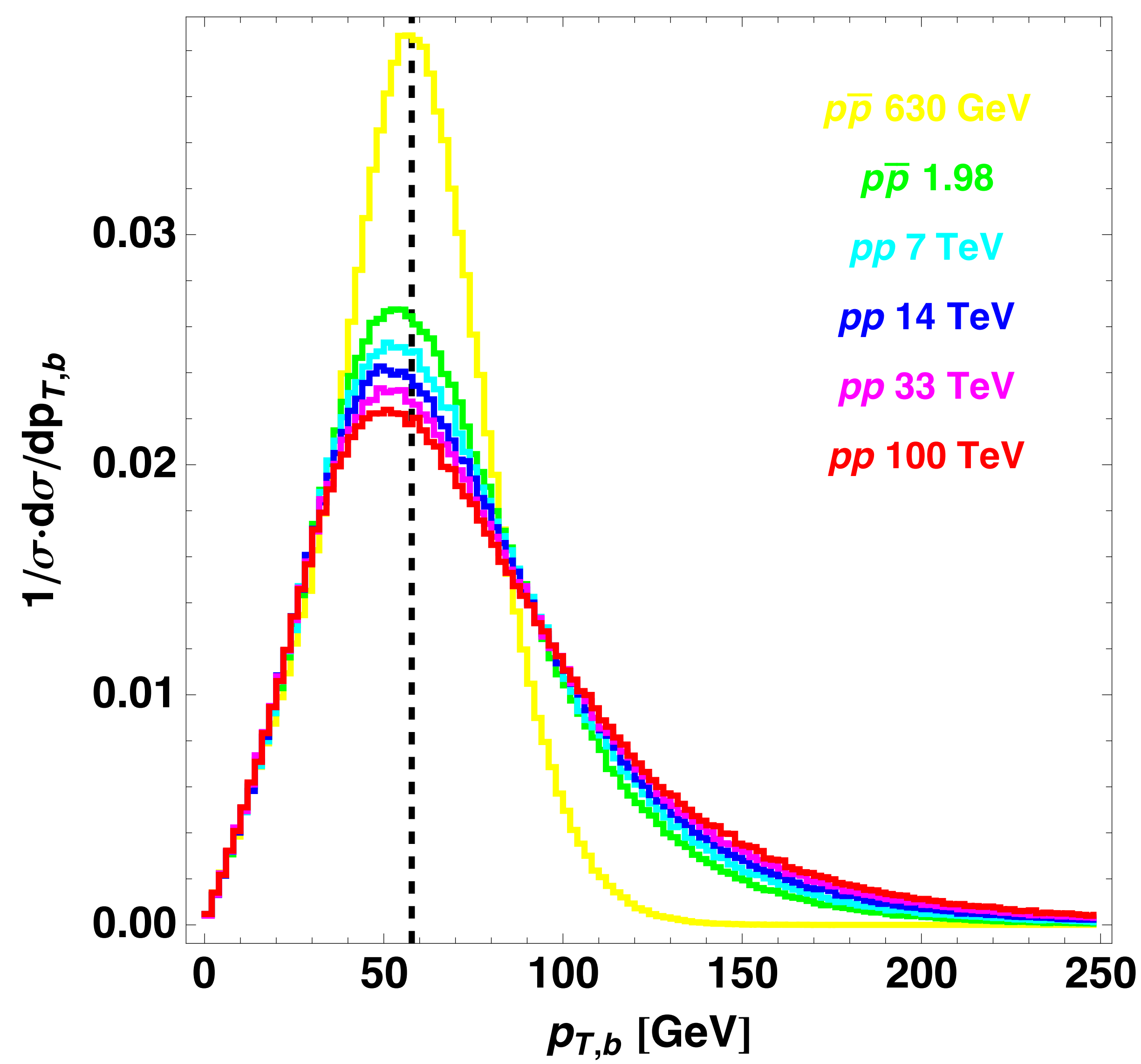

- vary ISR




- ...but, peak stays put, even though shape changes (broadens for more boosted top)

# ...accidents don't happen: no such invariance for $p_T$!




- peak (and shape) change...

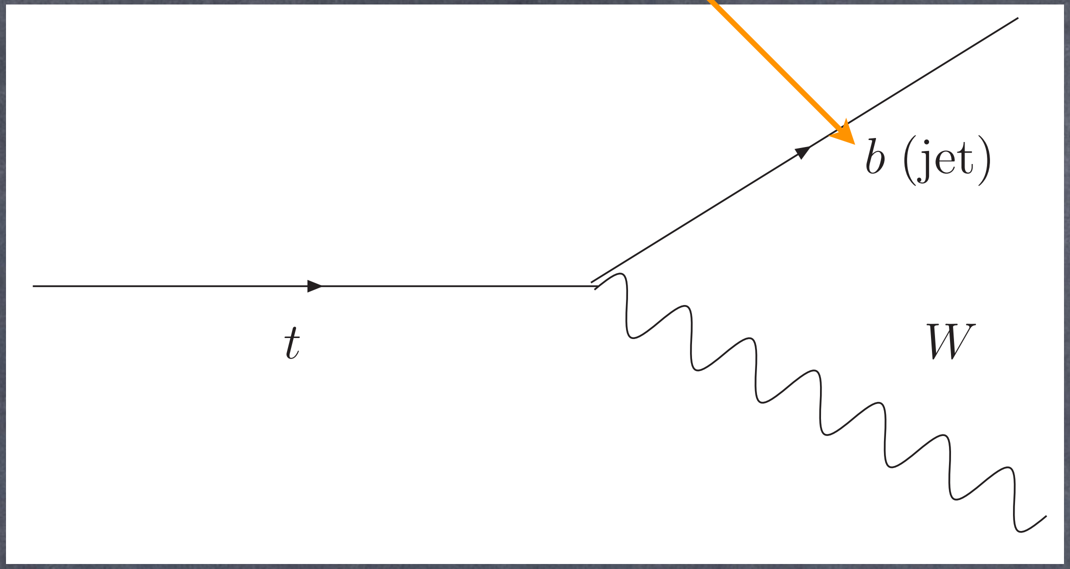
# Technique/Application

# *Review of energy-peak:for top quark decay*

*[mass measurement using energy of (entire) $b$-jet; main motivation: (quasi-)independent of production details]*

# Top quark mass

(almost) massless




- bottom quark energy ( $E_b$ ) $\approx$ energy of $b$-*jet* (inclusive)

- Equate location of peak in measured $b$-jet energy distribution to bottom quark energy in top rest frame, $E_b^{\text{rest}} \left( = \frac{M_t^2 - M_W^2 + m_b^2}{2\ M_t} \right)$:

$$E_{b\text{-jet}}^{\text{lab,mode}} = \frac{m_t^2 - M_W^2 + m_b^2}{2\ m_t}$$

- assuming only top quark unpolarized, independent of boost distribution of top quarks, hence (other) production details (BSM contribution; higher-order effect in SM...)

- Assuming $M_W$ (but no need to reconstruct it!), get $M_t$

....we studied using simulated data, with effects of cuts, detector etc. but...

# ...cut to CMS (real data!)

- implementation on run 1 data in CMS PAS TOP-15-002:

$$m_t = 172.29 \pm 1.17\,(\text{stat.}) \pm 2.66\,(\text{syst.})\ \text{GeV}$$

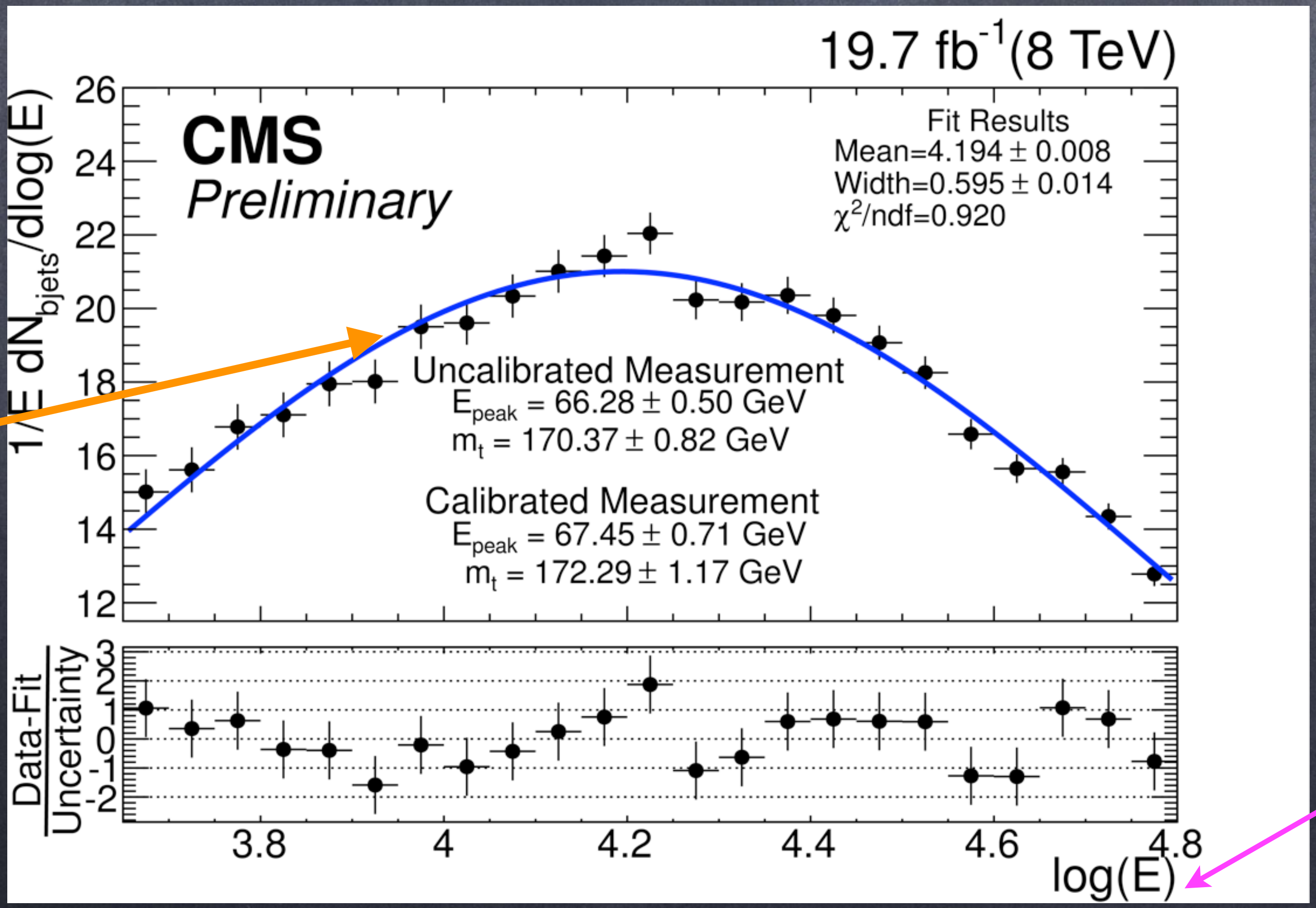
- Complementary (next slide) to other methods (error$\sim$1 GeV)

- Sources of error: JES uncertainty; modeling of top $p_T$

use $B$-hadron decay length (next)

higher-order (theory) calculation (KA, Franceschini, Kim, Schulze: 1603.03445; see also Ravasio, Jezo, Nason, Oleari: 1801.03944 and 1906.09166)

fitting function (see later)




note!

- Can 13 TeV (with NLO)...and/or ATLAS be far behind?!

# Summary of $b$-jet energy-peak method for top quark mass measurement

## advantage

- (quasi-)independent of top quark boost distribution/production details (only assumption: top quark unpolarized), cf. most other methods assume SM matrix element, e.g., compute distribution of decay product as function of $m_t$, find best fit to data:

$$\text{prediction } (m_t; \text{theory}) = \text{data, with theory} = \text{SM}$$

→ (others) valid only if BSM in top production is negligible

- even with SM (only) production, our method might have reduced sensitivity to PDF's, higher-order (QCD) effects (in production)
- similar story with $W$ mass (CDF anomaly due to SM uncertainties)?!

## disadvantage

- ($b$-)JES uncertainty

# Generalizations of energy-peak (a program!)

- Massive child particle from 2-body decay: peak shifts from rest-frame value (in general), but modified ansatz/fitting function still good (KA, Franceschini, Hong, Kim: 1512.02265)
- Direct three-body decay with 2 visible (e.g., off-shell sbottom in gluino decay): for fixed invariant mass of 2 visible, apply 2-body result for massive child particle (KA, Franceschini, Kim, Wardlow: 1503.03836)
- Cascade of 2-body decays: determine masses of 3 new particles [A (invisible), B and C] via (only) visible (a and b) measurements in decay chain $C \to B\, b \to A\, a\, b$ (e.g., gluino decay to on-shell bottom, then neutralino) (KA, Franceschini, Kim: 1309.4776)

# Using energy-peak for searches

- if background is flat or peaks elsewhere from signal
- Stops (Low: 1304.0491):

for $\tilde{t} \to b\tilde{\chi}_1^+$, peak in $E_b^{\text{lab}}$ at $\left(M_{\tilde{t}}^2 - M_{\tilde{\chi}_1^+}^2\right) / \left(2M_{\tilde{t}}\right)$...
can be $\gg \left(M_t^2 - M_W^2\right) / \left(2M_t\right)$ from $t\bar{t}$ background (from SM or from $\tilde{t} \to t\tilde{\chi}_1^0$)

*B-hadron decay length as “proxy” for bottom quark energy (instead of b-jet energy)*

*(motivation: avoid JES uncertainty)*

# (Very) Basic Idea (I): proxy

(more details in 2 slides)

- going from (measured) $B$-hadron decay length ($L_B$) to bottom quark energy

$$L_B \xrightarrow{\text{decay exponential}} \tau_B^{\text{lab}}$$

$$\tau_B^{\text{lab}} \text{ vs. } \tau_B^{\text{rest}} \longrightarrow \gamma_B^{\text{lab}} \text{ or } E_B (\text{energy of } B\text{-hadron})$$

$$E_B \xrightarrow{\text{hadronization model}} E_b$$

# (Very) Basic Idea (II): onto $m_t$

Going from bottom quark energy to top quark mass:

- earlier implementation (Hill, Incandela, Lamb: hep-ex/0501043; CDF: hep-ex/0612061; CMS:PAS TOP-12-030): relate $E_b$ (distribution) to $m_t$ by assuming top quarks boosts from SM production:

$$E_b \xrightarrow{\text{SM production}} m_t$$

- new idea (in this talk): use above energy-peak result instead [(quasi-) model-independent]:

$$E_b \xrightarrow{\text{energy}-\text{peak}} m_t$$

# Working it all out (in "reverse": still schematic/ theory version!): from $E_b$ to $E_B$ distribution...

- Hadronization ( $b \to b\text{-jet} = B\text{-hadron} + X$ ): fixed $E_b$ still gives distribution of $E_B$

- fragmentation function [ $D\left(x; E_b\right)$ ]: probability for $\frac{E_B}{E_b}$ to be $\approx x$

$$\int dx \; D\left(x; E_b\right) = 1 \text{ for any (fixed) } E_b$$

- probability distribution function (pdf's) of two energies related by

$$\underbrace{F\left(E_B\right)}_{\text{pdf}} = \int dE_b \; \underbrace{f\left(E_b\right)}_{\text{pdf}} D\left(\frac{E_B}{E_b}; E_b\right)$$

- (recall) energy-peak result [information about $f\left(E_b\right)$]:

$$\text{location of maximum of } f\left(E_b\right) = E_b^{\text{rest}} \left(= \frac{m_t^2 - m_W^2 + m_b^2}{2m_t}\right)$$

# ...from $E_B$ to mean decay lifetime/length

- Even for fixed $E_B$, (exponential) distribution of decay times with mean (going from $B$-hadron's rest to lab frame):

$$\begin{aligned} \tau_B^{\text{lab}} &= \gamma_B \tau_B^{\text{rest}} \\ &= \frac{E_B}{m_B} \tau_B^{\text{rest}} \end{aligned}$$

not of top quark!

velocity

- convert to mean decay length: $$\begin{aligned} \lambda_B &= c \gamma_B \beta_B \tau_B^{\text{rest}} \\ &= c \frac{E_B}{m_B} \sqrt{1 - \left( \frac{m_B}{E_B} \right)^2} \tau_B^{\text{rest}} . \end{aligned}$$

- $B$-hadron relativistic:

$$\lambda_B \approx c \frac{E_B}{m_B} \tau_B^{\text{rest}} \left[ 1 + \mathcal{O} \left( \left( \frac{m_B}{E_B} \right)^2 \right) \right] .$$

- pdf of mean decay length:

$$\begin{aligned} g(\lambda_B) &= \frac{F(E_B)}{\frac{d\lambda_B}{dE_B}} \\ &\approx F(E_B) \frac{m_B}{c \tau_B^{\text{rest}}} \end{aligned}$$

pdf

# …finally (!) distributions of (measured) decay length ($L_B$) and $E_b$ related

- use **decay** exponential to go from $\lambda_B$ to $L_B$, then previous relations

pdf

$B$-hadron

relativistic

$$
\begin{aligned}
G\left(L_B\right) &= \int d\lambda_B \left[\frac{g\left(\lambda_B^{\text{lab}}\right)}{\lambda_B}\right] \exp\left(-\frac{L_B}{\lambda_B^{\text{lab}}}\right) \\
&\approx \int dE_B \frac{F\left(E_B\right)}{E_B} \frac{m_B}{c\tau_B^{\text{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right) \\
&= \int dE_B \int dE_b \; f\left(E_b\right) D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)
\end{aligned}
$$

$m_t$ and its boost

(double convolution)

$$
\begin{aligned}
G\left(L_B\right) &\rightarrow \text{pdf of decay length of } B\text{-hadron, } L_B \\
f\left(E_b\right) &\rightarrow \text{pdf of energy of bottom quark, } E_b \\
D\left(\frac{E_B}{E_b}; E_b\right) &\rightarrow \text{bottom quark fragmentation function} \\
\tau_B^{\text{rest}} &\rightarrow \text{mean decay lifetime of } B\text{-hadron in its rest frame}
\end{aligned}
$$

# Earlier (CDF/CMS) SM-based implementation (explicitly)

- top quark boosts, hence pdf of $E_b$ ($f$), computed using SM matrix element, with top quark mass as a parameter
- SM "fitting" function for decay length:

fitting function

observable (transverse)

top quark boosts from SM: model-dependence

$$G^{\text{fit,SM}}\left(L_B; m_t\right) = \int dE_B \int dE_b \, f^{\text{SM}}\left(E_b; m_t\right) D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

(unknown) parameter

# *Our Proposal for B-hadron decay length (in detail)*

[same starting point, but use energy-peak result instead of assuming SM production; main motivation: (quasi-)model-independent]

[uses full/3d decay length (cf. transverse in earlier CMS)]

# “Disclaimer”

- Outline/schematic of idea only
- For more details (further plots, cuts used etc.), refer to papers and/or contact my collaborator, Sagar Airen (sairen@umd.edu)

# General (new) idea

- Recall relation between $L_B$ and $E_b$ distributions:

$$G(L_B) = \int dE_B \int dE_b\, f(E_b)\, \overbrace{D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)}^{\text{unchanged}}$$

new proposal: relate to $m_t$ using energy-peak (instead of SM production )

- (twice) "de-convolve" (impossible?!) decay length distribution [$G(L_B)$] to obtain that of bottom quark [$f(E_b)$]

$$\text{location of peak of } f(E_b) \quad \to \quad \frac{m_t^2 - M_W^2 + m_b^2}{2\, m_t}$$

- Or, energy-peak result [+ log symmetry of $f(E_b)$] materializes as "some" robust feature in $G(L_B)$ ?!

# More practically/realistically:

- Model-independent ansatz/fitting function for bottom ($b$-jet) quark energy (peak at $E_b^{\text{rest}}$ + log-symmetric etc.)

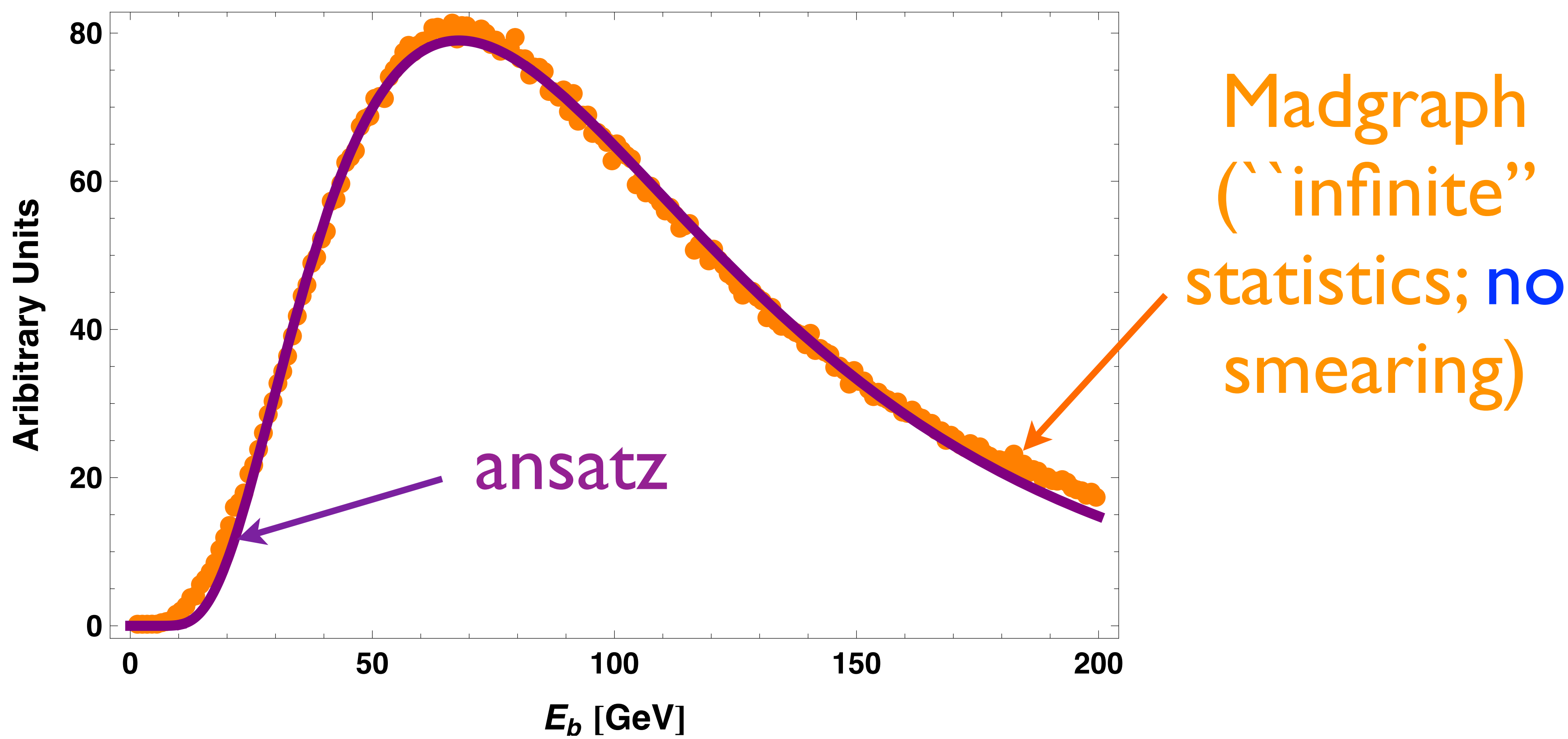
$$f^{\text{fit,us}}\left(E_b; \underbrace{E_b^{\text{rest}}, w}_{\text{parameters}}\right) = \frac{1}{N} \exp\left[-w\left(\frac{E_b}{E_b^{\text{rest}}} + \frac{E_b^{\text{rest}}}{E_b}\right)\right]$$

observable

$$f^{\text{fit,CMS}}\left(E_b; E_b^{\text{rest}}, w\right) = \frac{1}{N} \exp\left[-w \log^2\left(\frac{E_b}{E_b^{\text{rest}}}\right)\right]$$

- fit above function to measured $b$-jet energy distribution
- best fit value of $E_b^{\text{rest}}$ matched to $\frac{M_t^2 - M_W^2 + m_b^2}{2\ M_t}$ ...
- ...as in earlier CMS plot (tested already on actual data!)

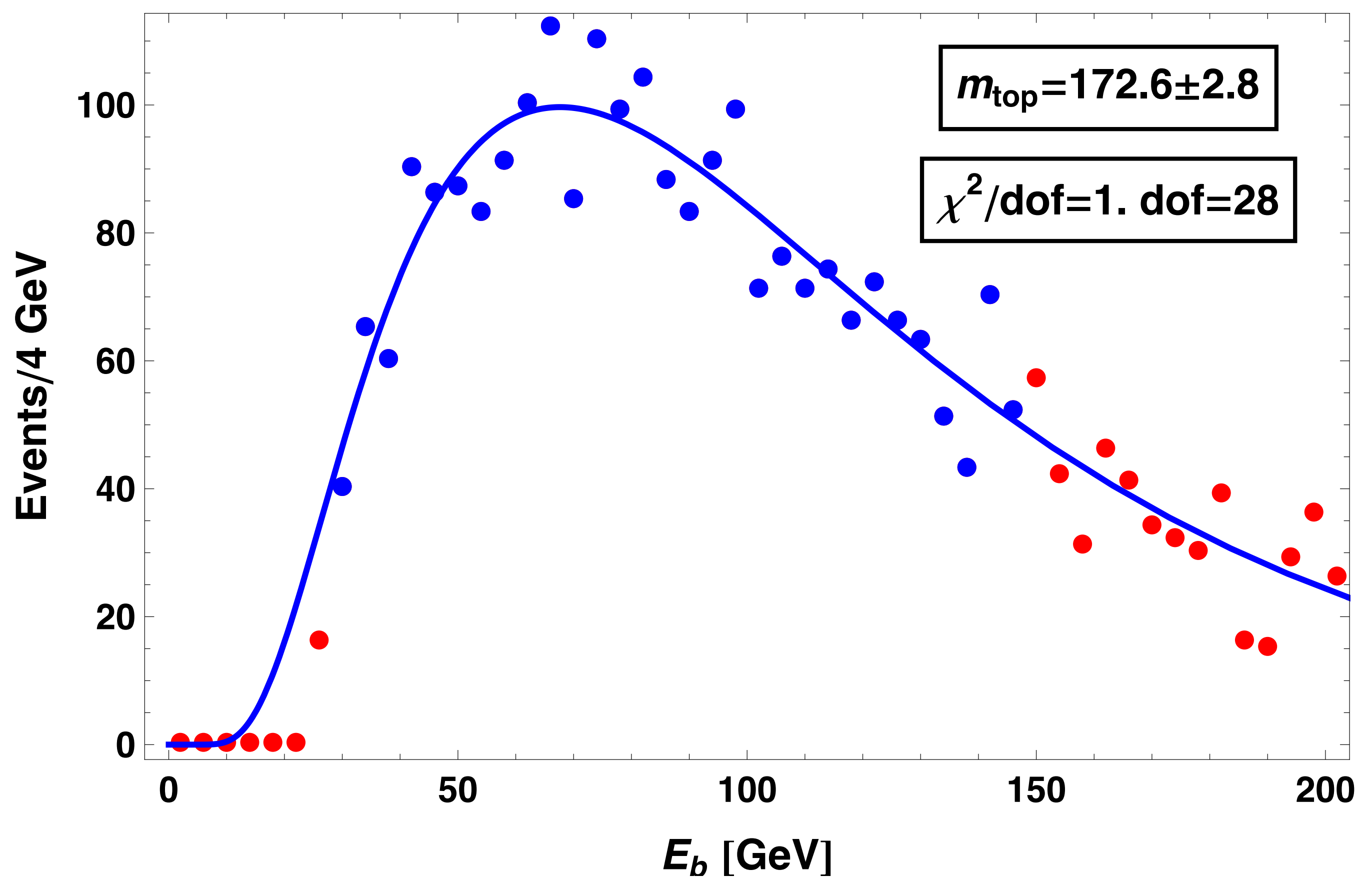
# …``test” of our fitting function on bottom quark energy from top quark decay (@ parton-level)




- bottom (almost) “massless”: peak does not shift, shape property negligibly violated
- good fit for heavier ``top” quark as well: different PDF’s, boost distribution (width parameter encompasses this variation)

# …``test” of our fitting function on b-jet energy from top quark decay on simulated data (5/fb @ LHC7)

(one pseudo-experiment shown)




(use only blue dots)

- measured $m_t$ consistent with input value
- fitting not spoiled by cuts or detector effects

# Bottomline of our proposal (other ways welcome!)

- Plug $f^{\text{fit, us}}\left(E_b; E_b^{\text{rest}}, w\right)$ for $f\left(E_b\right)$ → new fitting function (for decay length distribution now):

$$\underbrace{G^{\text{fit, us}}\left(L_B; E_b^{\text{rest}}, w\right)}_{L_B\text{: observable};\ E_b^{\text{rest}},\, w\text{: parameters}} \approx \int dE_B \int dE_b \overbrace{\frac{1}{N(w)} \exp\left[-w\left(\frac{E_b}{E_b^{\text{rest}}} + \frac{E_b^{\text{rest}}}{E_b}\right)\right]}^{\text{fitting function for } E_b} \times$$
$$D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

[similar procedure to $b$-jet energy-peak method: different observable and (double) convolution in fitting function]

$G^{\text{fit}}\left(L_B; E_b^{\text{rest}}, w\right)$ → fitting function for observed decay length $(L_B)$ distribution

best-fit value of parameter $E_b^{\text{rest}}$ → $\dfrac{m_t^2 - M_W^2 + m_b^2}{2\, m_t}$

$\tau_B^{\text{rest}}$ → mean decay lifetime of $B$-hadron in its rest frame

$D\left(\dfrac{E_B}{E_b}; E_b\right)$ → bottom quark fragmentation function

parameter $w$ → width of fitting function (its extracted value is *not* relevant here)

$N(w)$ → normalization factor

Sensitivity of *measured* $m_t$ (using B-hadron decay length) to top quark $p_T$ (boost) distribution: test of production model-independence (preliminary)

[Neglect other uncertainties, e.g., fragmentation function: focus on $p_T$ of top (largest systematic in CMS analysis)]

# Simulating data (schematic)

- SM production

$$G^{\text{(simulated) data}}_{\infty \text{ statistics}}\left(L_B; m_t^{\text{input}}\right) \approx \int dE_B \int dE_b\, f^{\text{SM}}\left(E_b; m_t^{\text{input}}\right) \times D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

for finite statistics, sample pdf's: $f^{\text{SM}}\left(E_b; m_t^{\text{input}}\right)$, $D\left(\frac{E_B}{E_b}; E_b\right)$ and decay exponential (using Madgraph, Pythia...)

- Modified top quark $p_T$ distribution:

new

$$\frac{\tilde{w}}{w} = 1 + \alpha \cdot \left(p_T^{\text{top}} - 200 \text{ GeV}\right) \text{ for } p_T^{\text{top}} < 400 \text{ GeV}$$

original

re-weighting parameter

- $f^{\text{SM}}\left(E_b; m_t^{\text{input}}\right) \to f^{\widetilde{\text{SM}}}\left(E_b; m_t^{\text{input}}\right)$ due to re-weighting top quark $p_T$

modified vs. SM

new

$$\widetilde{G}^{\text{(simulated) data}}_{\infty \text{ statistics}}\left(L_B; m_t^{\text{input}}\right) \approx \int dE_B \int dE_b\, f^{\widetilde{\text{SM}}}\left(E_b; m_t^{\text{input}}\right) \times D\left(\frac{E_B}{E_b}; E_b\right) \frac{m_B}{c\tau_B^{\text{rest}} E_B} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

# Energy-peak [“flexible/model-independent” $G^{\text{fit,us}}(L_B; E_b^{\text{rest}}, w)$ ] better vs. SM [“hard-wired” $G^{\text{fit,SM}}(L_B; m_t)$]…as expected!

top quark $p_T$ re-weighting $\frac{\tilde{w}}{w} = 1 + \alpha\,\theta(p_t < 400)(p_t - 200)$

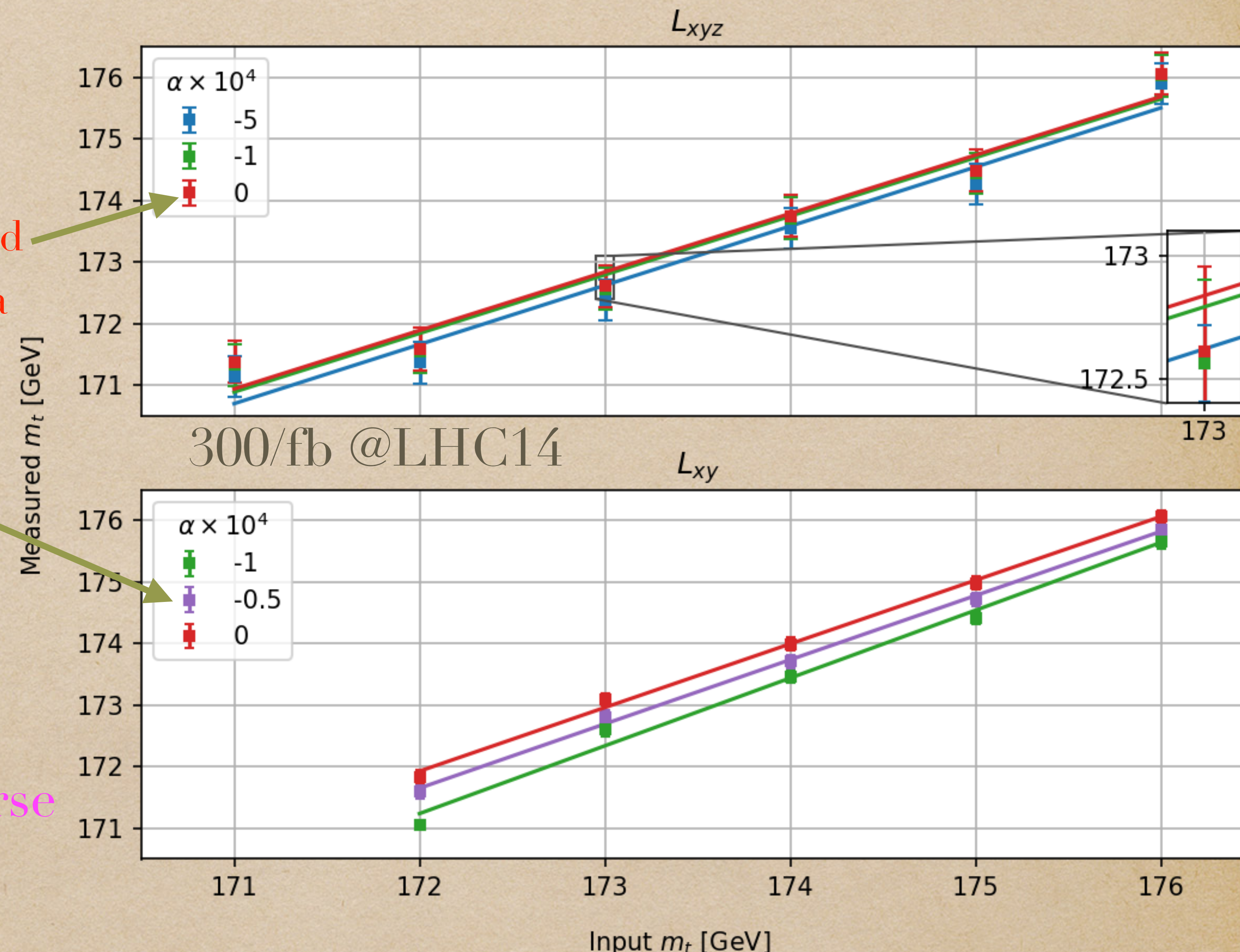
energy-peak

SM production used for simulating data

$\sim 0.5\%$ (*negative*) change (vs. SM) in *average* top quark $p_T$

CMS/SM $L_B^{xy}$

transverse (only)




- 600 MeV “error” for CMS/SM $L_B^{xy}$ (“default” re-weighting)
- negligible error for energy-peak inspired (unless re-weighting x 5!)

# Hadronization model/fragmentation function ($D$): going from bottom quark to

$$b\text{-jet} = B\text{-hadron} + X$$

- important for $B$-hadron decay length (exclusive) vs. not so much for $b$-jet energy (inclusive)
- effects studied by CDF/CMS: error in $m_t$~1 GeV (?)
- similar for our method
- more detailed (theory) work: Corcella, Franceschini, Kim: 1712.05801 (further theory improvements possible?)

(For our analysis, $D$ used in fitting function obtained from Pythia same as in simulated data )

# Summary of new $B$-hadron decay length proposal

- advantage: no JES uncertainty (same as earlier CDF/CMS decay length analysis); (quasi-) model-independence (also for CMS $b$-jet energy peak, cf. SM production assumed for earlier CDF/CMS decay length analysis)

- new systematics (also for earlier CDF/CMS decay length analysis): hadronization modeling (theory); tracker resolution [experimental, but (much) better than JES?!]

# Conclusions

- review of (relatively new, but not really for CMS!) method for top quark mass measurement using bottom quark/$b$-jet energy peak: (quasi-)production model-independent (cf. others assume SM), but afflicted by JES uncertainty (improvement using 13 TeV, NLO…)
- how to "extend" it to $B$-hadron decay length (correlated with bottom quark energy): circumvent JES uncertainty, "replaced" by hadronization model/fragmentation function (theory improvement possible?)
- …(very preliminary) on to $W$ mass from energy-peak of lepton from its decay…but $W$ produced polarized?!

# Outlook: onto W mass

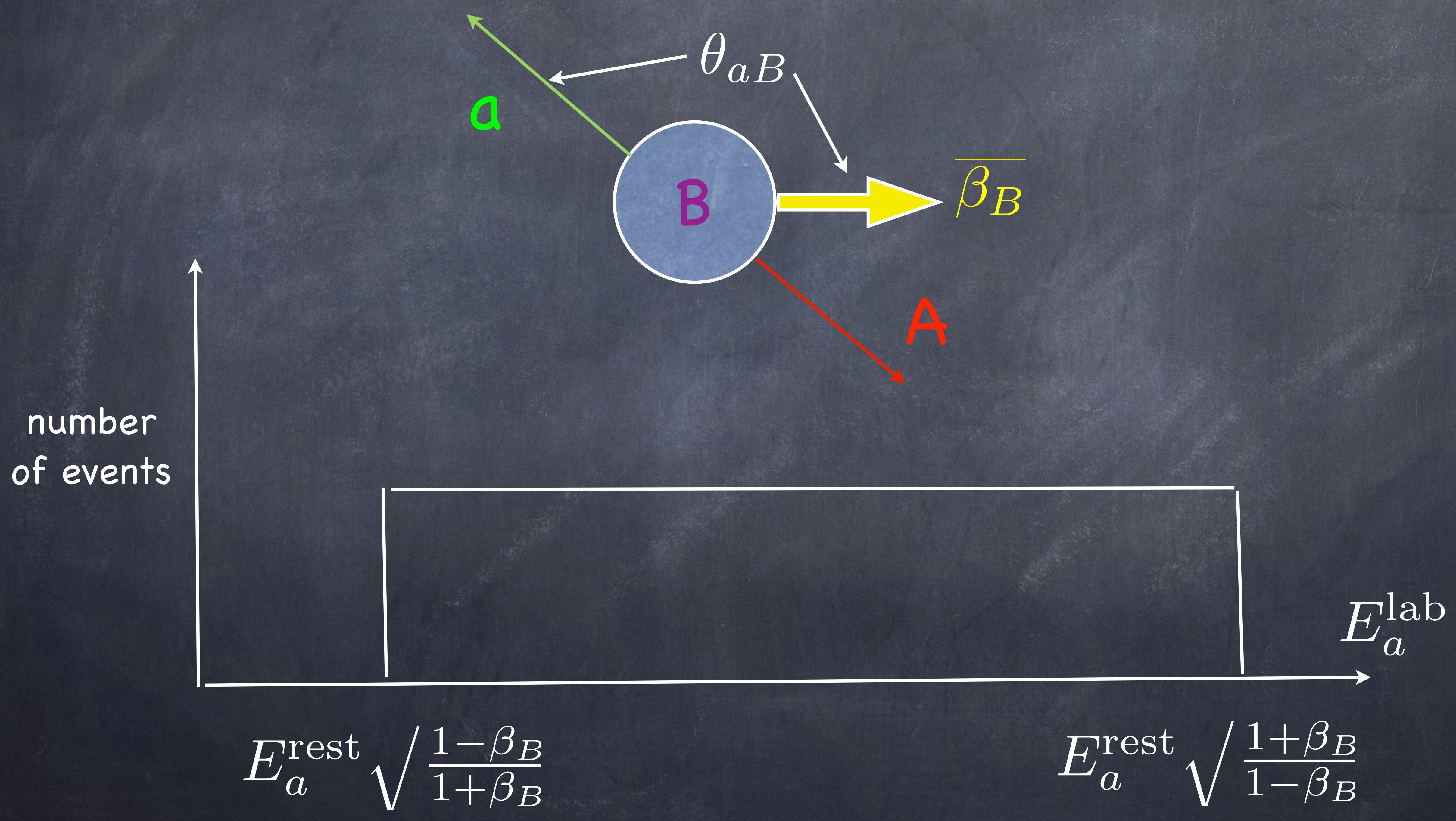
- W produced polarized! Cancellation between W+ and W- not good enough...
- target precision for W mass much higher than for top quark mass → need to control (above and other) corrections to energy-peak result

# BACK-UP

# ``Invariance" of Two-Body Decay Kinematics

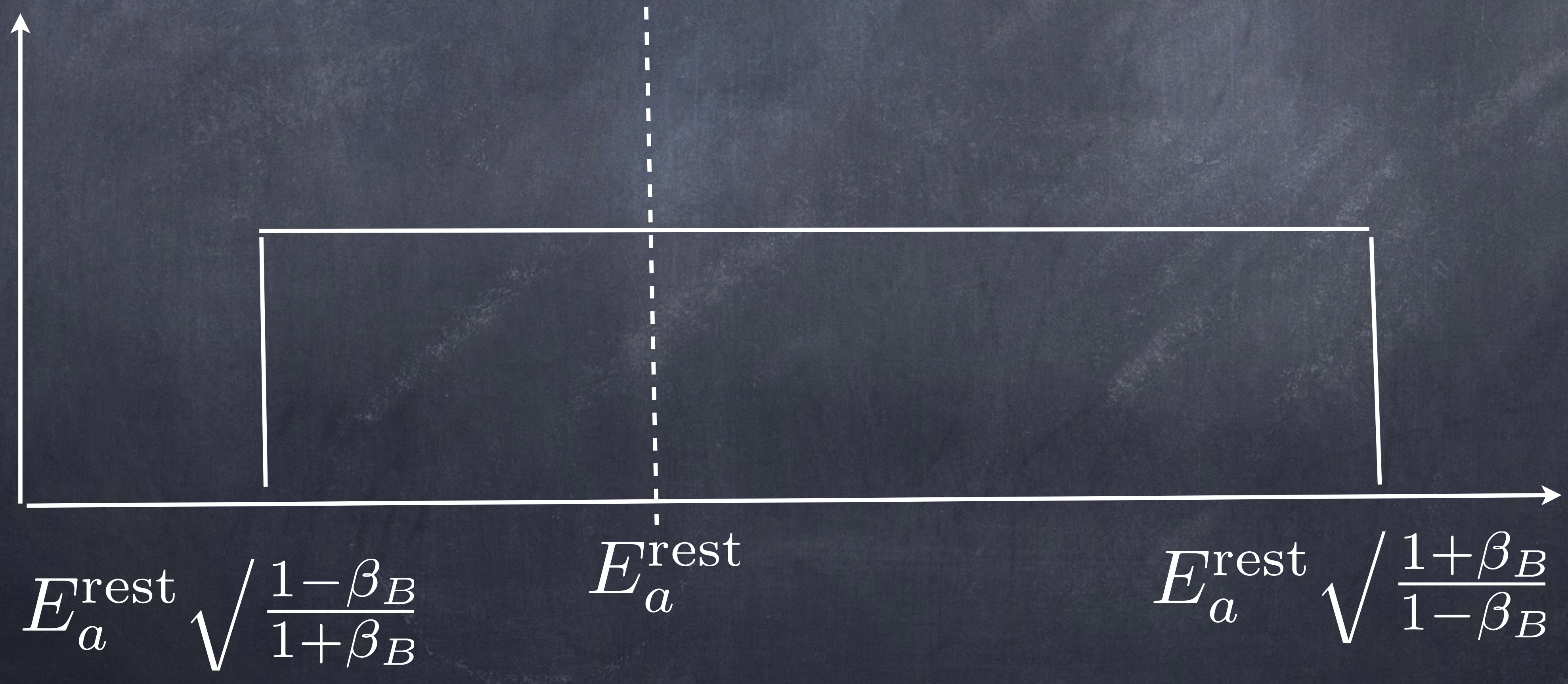
# Rectangle for fixed, but arbitrary boost

- In general: $E_a^{\rm lab} = E_a^{\rm rest}\gamma_B\left(1+\beta_B\cos\theta_{aB}\right)$
- Assume unpolarized parent: $\cos\theta_{aB}$ is flat



# Rectangle vs. rest energy

- contains $E_a^{\rm rest}$ (for any boost)
- no other $E_a^{\rm lab}$ gets larger contribution from given boost than does $E_a^{\rm rest}$
- no other $E_a^{\rm lab}$ is contained in every rectangle (e.g., $\beta_B \to 0$ )
- asymmetric on linear (symmetric on log...)



# (Generic) Boost distribution: "stacking" up rectangles

(KA, Franceschini, Kim: 1209.0772)

(see also Stecker: "Cosmic gamma rays")

- distribution of $E_a^{\rm lab}$ has peak at $E_a^{\rm rest}$
- ....no matter what is the boost distribution!
- boost distribution depends on production mechanism, parent mass, PDF's...