

Collider, Dark Matter, and Neutrino

Physics 2023

19 May, 2023

Mitchell Institute, Texas A&M University

Effective field theory approach for
radiative corrections in neutron beta
decay



Los Alamos
NATIONAL LABORATORY

Oleksandr (Sasha) Tomalak

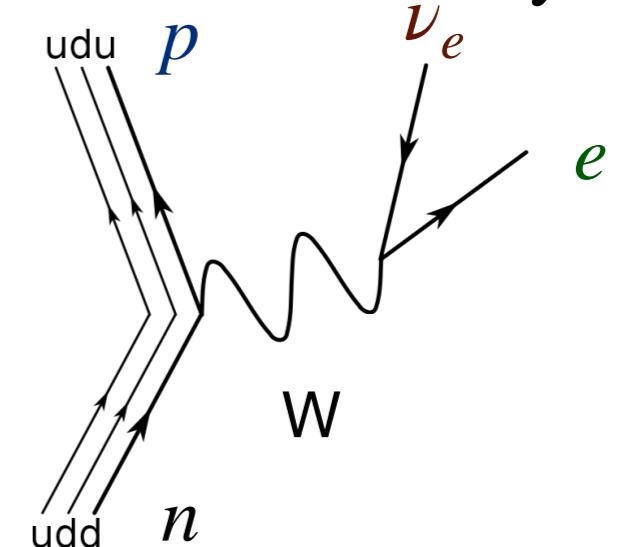
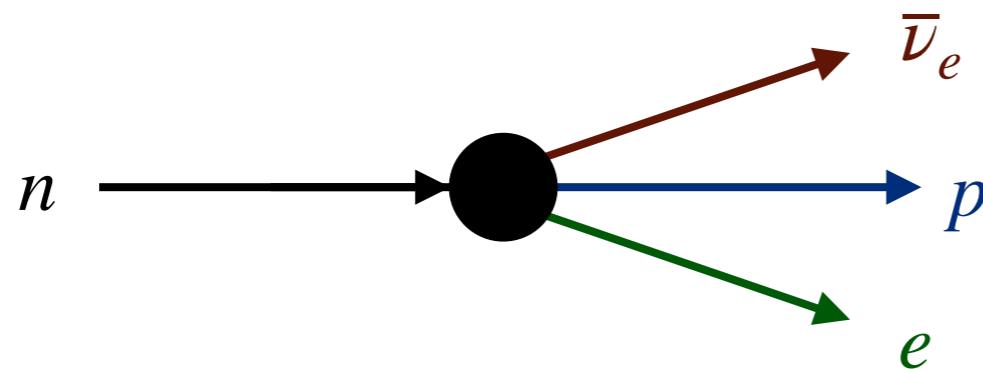
LA-UR-23-25227

Outline

- 1) neutron lifetime: experiment and theory updates
- 2) effective field theory approach to beta decay and pions
S. Ando et al., Phys. Lett. B 595, 250 (2004)
Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen,
Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)
- 3) Standard Model to LEFT matching
O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)
W. Dekens, P. Stoffer, JHEP 07, 107 (2019)
- 4) LEFT to HB χ PT matching
O. T., Few-Body Syst. 64, 23 (2023)
Vincenzo Cirigliano, Wouter Dekens, Emanuele Mereghetti, and O. T. (arXiv: very soon)
- 5) Vector coupling constant and V_{ud}

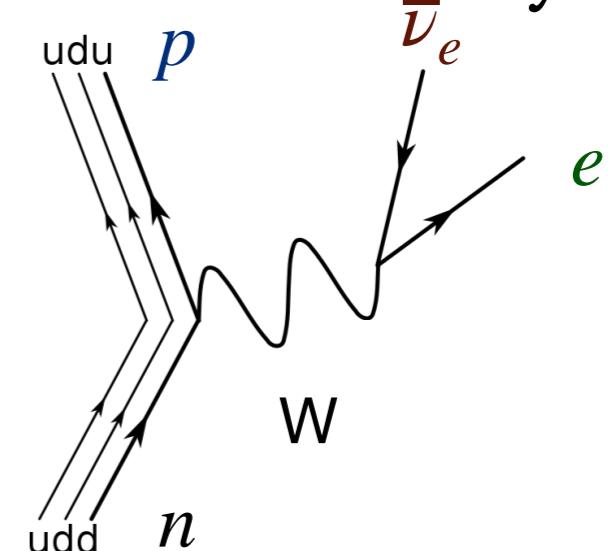
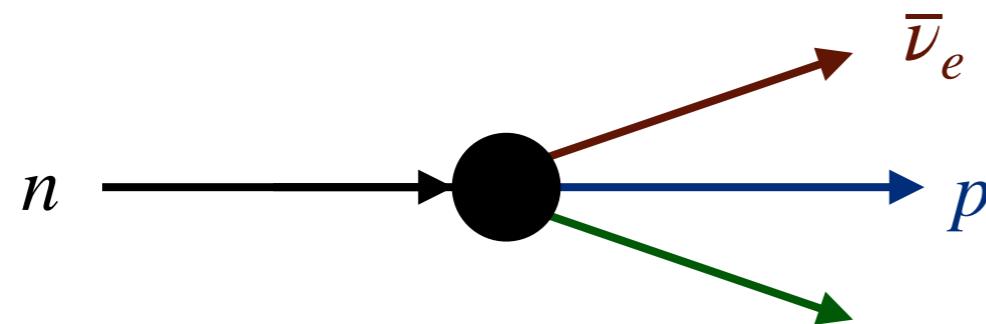
Neutron decay

- neutron is heavier than proton by 1.3 MeV and can decay
- neutron lifetime is around 15 mins



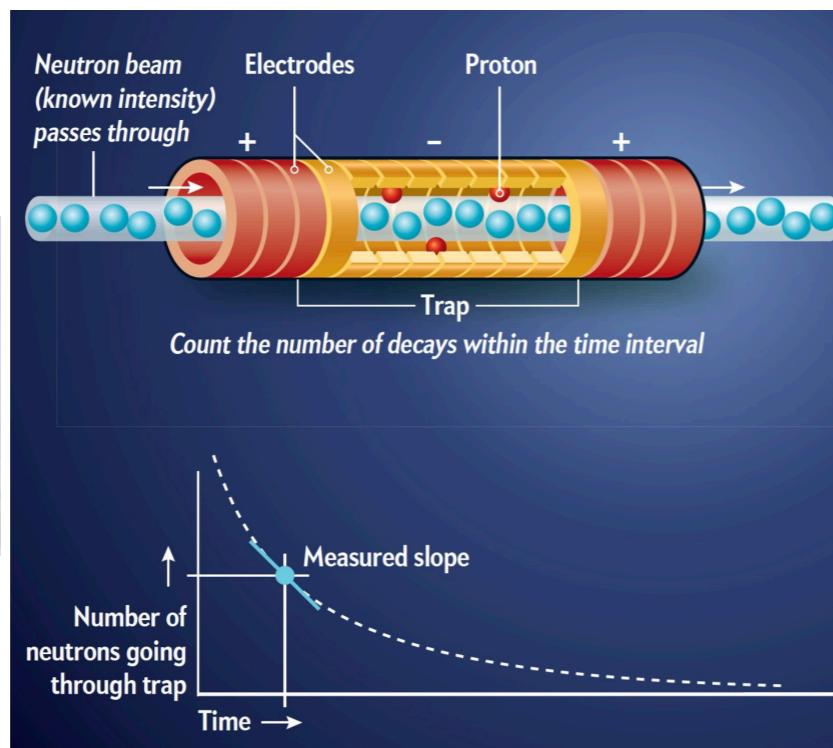
Neutron decay

- neutron is heavier than proton by 1.3 MeV and can decay
- neutron lifetime is around 15 mins

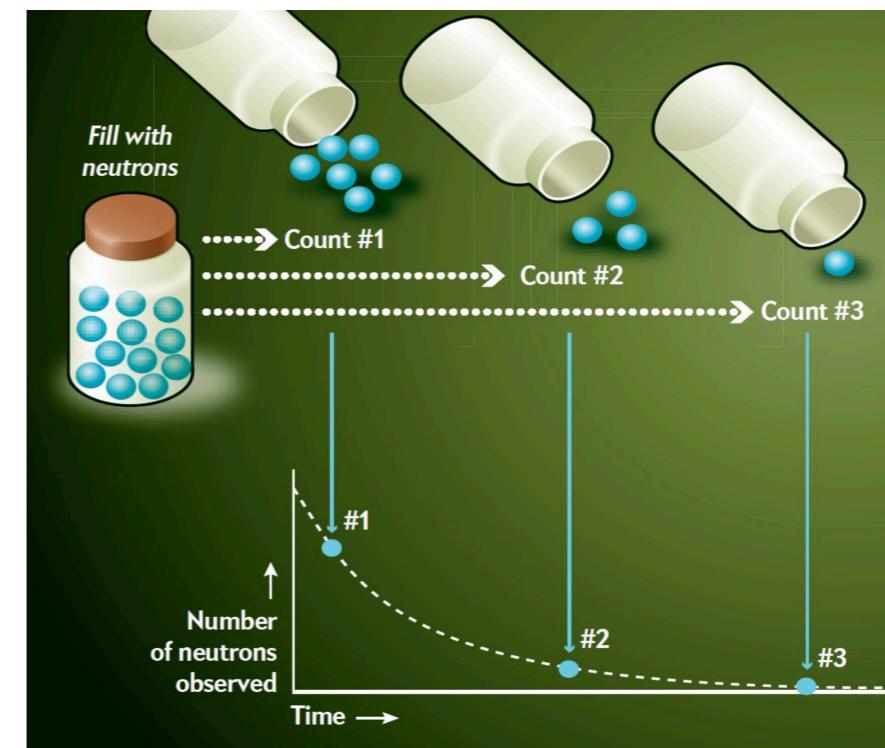


Neutron lifetime measurements

beam method

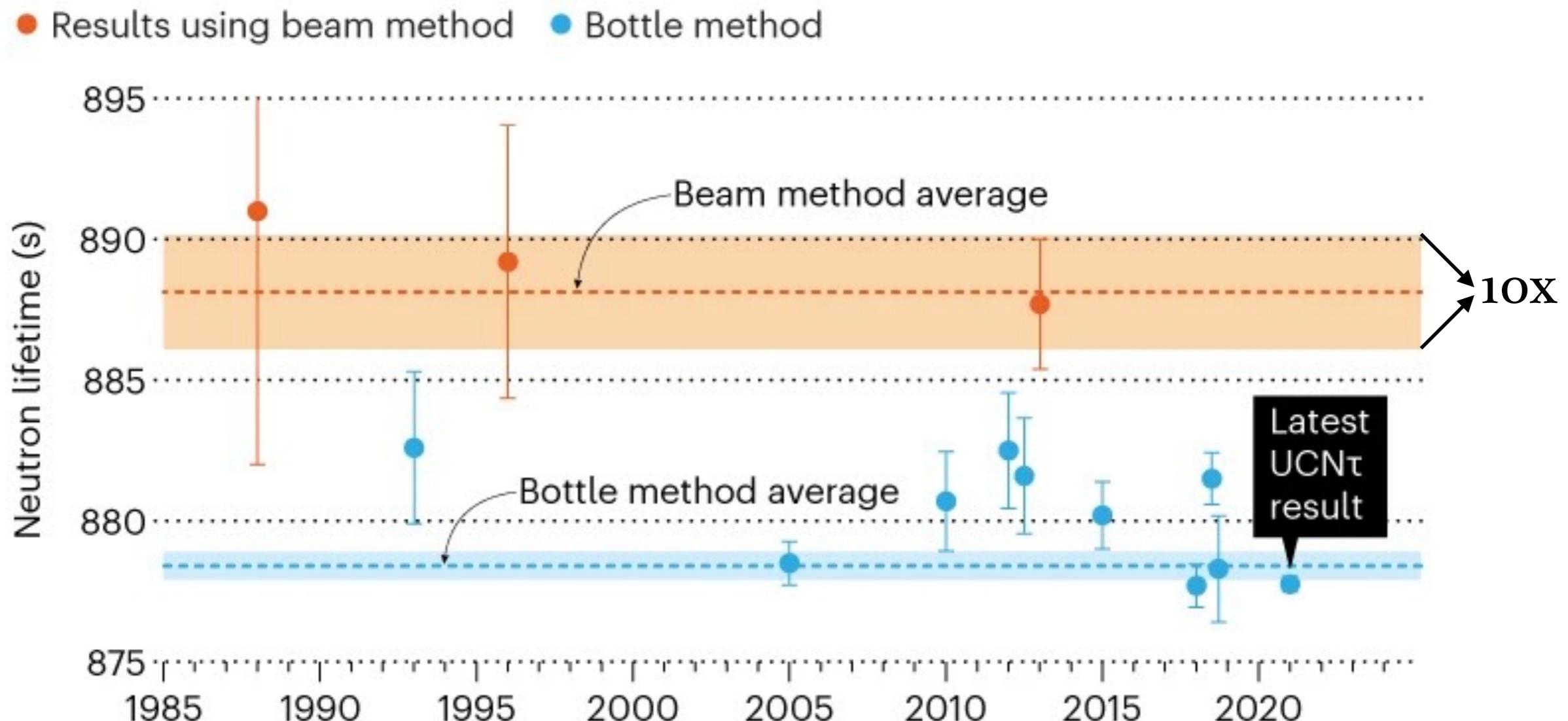


bottle method



how many neutrons pass? how many neutrons survive?

Neutron lifetime



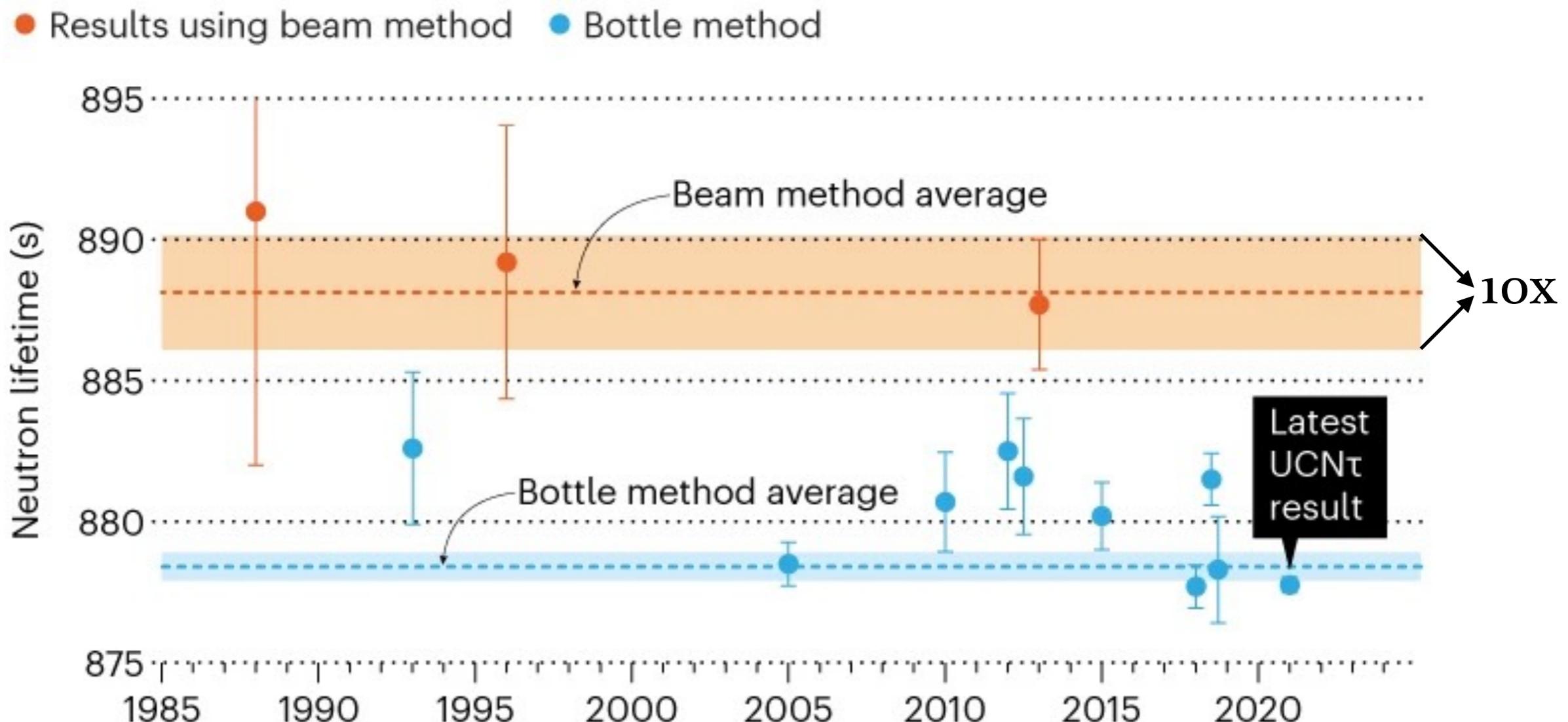
©nature

UCN τ : F. Gonzales et al., Phys. Rev. Lett. 127, 162501 (2021)

D. Castelvecchi, Nature 598, 549 (2021)

- 8-9 seconds discrepancy beam vs bottle method : $3-5\sigma$
- 0.3 seconds uncertainty of UCN τ @LANL : $(3 - 4) \times 10^{-4}$

Neutron lifetime



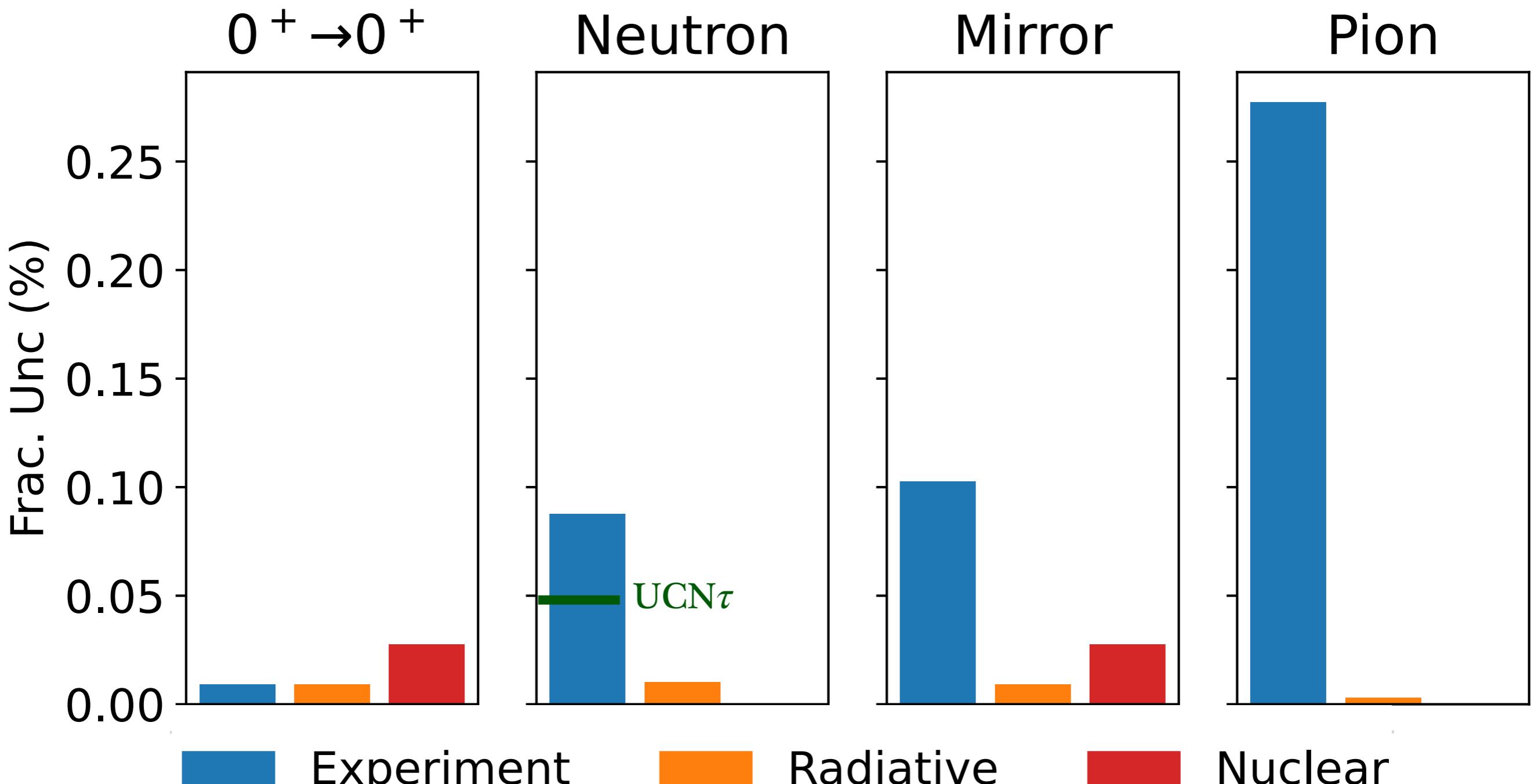
©nature

UCN τ : F. Gonzales et al., Phys. Rev. Lett. 127, 162501 (2021)

D. Castelvecchi, Nature 598, 549 (2021)

- complementary way to determine V_{ud}
- test of CKM unitarity and search for BSM at low energies

V_{ud} determinations



2022 Fundamental Symmetries, Neutrons, and Neutrinos (FSNN) white paper

- neutron decay becomes competitive with $0^+ \rightarrow 0^+$

Low-energy description

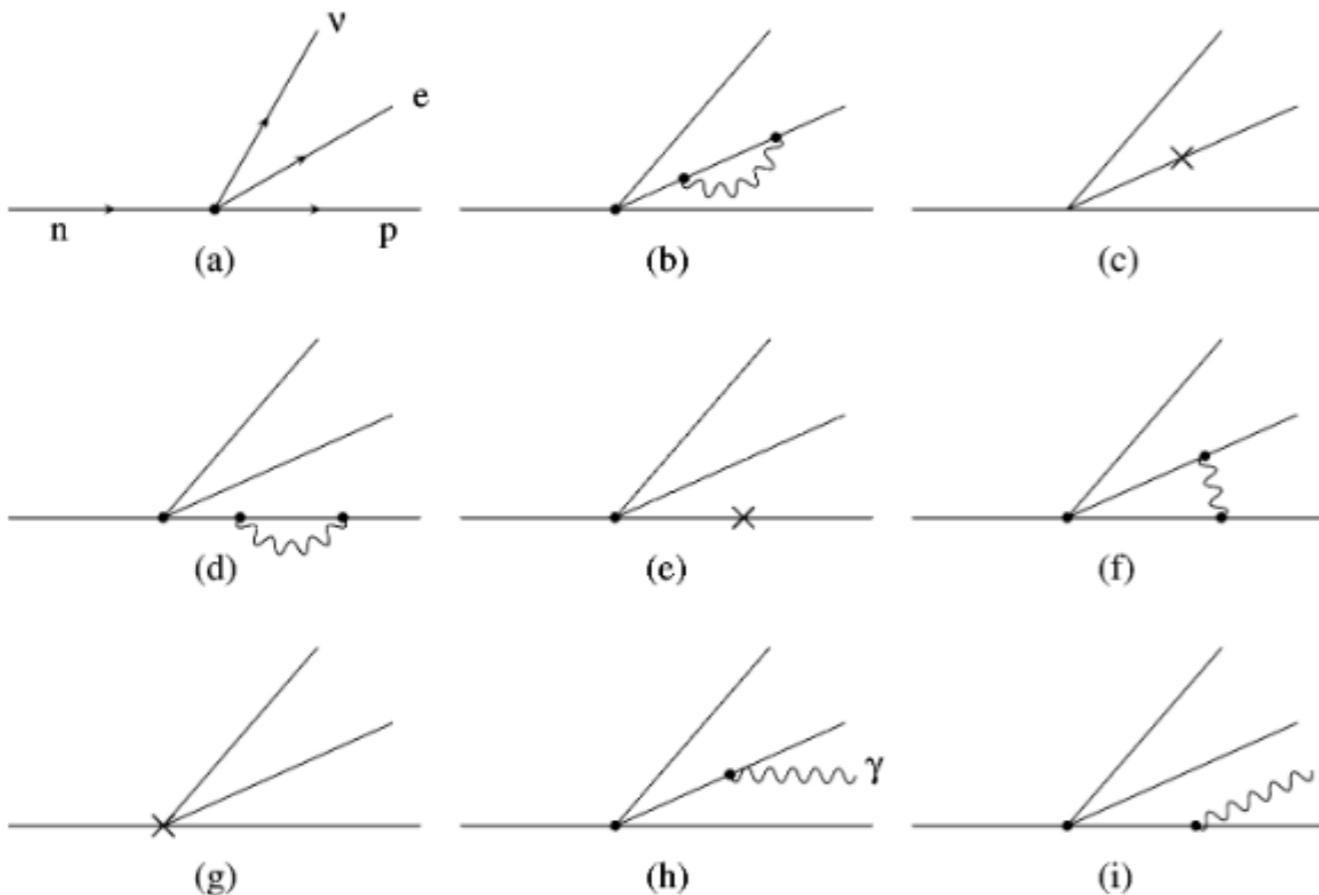
- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\text{eff}} = -\sqrt{2}G_F V_{ud} \bar{e} \gamma_\mu P_L \nu_e \cdot \overline{N} (g_V v^\mu - 2g_A S^\mu) \tau^+ N + O\left(\frac{m_e}{M_p}, \alpha, \alpha \frac{m_\pi}{M_p}, \alpha \frac{m_e}{m_\pi}\right)$$

for uncertainty $m_e \sim M_p - M_n$

A. Sirlin, Phys. Rev. 164, 50 (1967)

- radiative corrections formulated in modern EFT language



vector and axial-vector counterterms (diagrams c, e, g)

The diagram consists of two separate blue arrows, each pointing diagonally upwards and to the right. The arrow on the far left is associated with the word "data" written in a large, bold, black serif font directly beneath it. The arrow on the far right is associated with the words "Standard Model" written in a large, bold, black serif font directly beneath it.

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- two coupling constants predict all observables

Radiative corrections to neutron decay

- current-algebra formulation of radiative corrections
A. Sirlin, Rev. Mod. Phys. 50, 573 (1978)
- β decay ($0^+ \rightarrow 0^+$ Fermi transition, g_V) corrects by overall factor

short-distance short-distance long-distance

Sirlin's function EW pQCD γW

$$RC_{EW} = \frac{\alpha}{2\pi} \left(g(E_m) + 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_A} + A_g + 2C \right)$$

W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 56, 22 (1986)

A. Sirlin, Phys. Rev. 50, 164 (1967)

- EW logarithms resummed by renormalization group analysis
- assuming same relative change of g_V and g_A , theory error:

$$8 \times 10^{-4}$$

A. Czarnecki, W. J. Marciano and A. Sirlin, Phys. Rev. D 70, 093006 (2004) and before

- perturbative logarithms separated; subpermille uncertainty

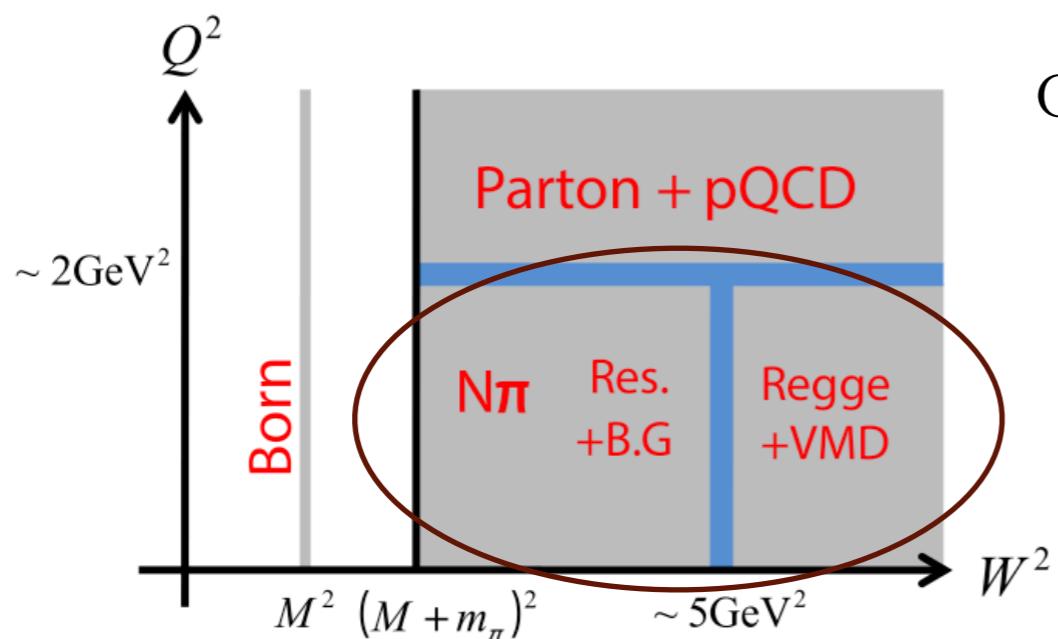
Radiative corrections to neutron decay

- updated calculations of γW contributions

$$\frac{\alpha}{2\pi} C^V = 3.83(11) \times 10^{-3} \quad \text{vs} \quad \frac{\alpha}{2\pi} C^V = 3.26(19) \times 10^{-3}$$

Ch.-Y. Seng, M. Gorchtein et al., Phys. Rev. Lett. 121, 24 (2018)

- γW box with Born, $N\pi$, resonance and Regge physics



$$C^V = \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \int_0^1 dx f\left(\frac{x^2}{Q^2}\right) F_3^{(0)}(x, Q^2)$$

A. Czarnecki, W. J. Marciano and A. Sirlin,
Phys. Rev. D 100, 073008 (2019)

K. Shiells, P. G. Blunden and W. Melnitchouk,
Phys. Rev. D 104, 033003 (2021)

L. Hayen, Phys. Rev. D 103, 113001 (2021)

- dispersive validation of the same relative change of g_V and g_A :

$$\frac{\alpha}{2\pi} (C^A - C^V) = 0.13(11)_V (6)_A \times 10^{-3}$$

M. Gorchtein and Ch.-Y. Seng, JHEP 10, 053 (2021)

$$\frac{\alpha}{2\pi} (C^A - C^V) = 0.6(5) \times 10^{-3}$$

L. Hayen, Phys. Rev. D 103, 113001 (2021)

- hadron physics -> precise evaluations of long-distance γW

Effective field theory for β decay

M_Z

full content of Standard Model (SM)

integrate out top, Z, W, h

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

W. Dekens, P. Stoffer, JHEP 07, 107 (2019)

m_b

integrate out GeV particles

m_c

α_s becomes too strong going to lower energies

this talk

m_π

dynamical pions

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen,
Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

m_e

photons, neutrinos, electrons, external nucleons

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- coupling constants from SM + small QED corrections

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
mass $\approx 2.2 \text{ MeV}/c^2$	charge $2/3$	spin $1/2$	U up	g gluon
$\approx 1.28 \text{ GeV}/c^2$	$2/3$	$1/2$	C charm	H higgs
$\approx 173.1 \text{ GeV}/c^2$	$2/3$	$1/2$	t top	
	0 0 1			
$\approx 4.7 \text{ MeV}/c^2$	$-1/3$	$1/2$	d down	γ photon
$\approx 96 \text{ MeV}/c^2$	$-1/3$	$1/2$	S strange	Z Z boson
$\approx 4.18 \text{ GeV}/c^2$	$-1/3$	$1/2$	b bottom	W W boson
	0 0 1			
$\approx 0.511 \text{ MeV}/c^2$	-1	$1/2$	e electron	
$\approx 105.66 \text{ MeV}/c^2$	-1	$1/2$	μ muon	
$\approx 1.7768 \text{ GeV}/c^2$	-1	$1/2$	τ tau	
	0 1			
$<1.0 \text{ eV}/c^2$	0	$1/2$	ν_e electron neutrino	
$<0.17 \text{ MeV}/c^2$	0	$1/2$	ν_μ muon neutrino	
$<18.2 \text{ MeV}/c^2$	0	$1/2$	ν_τ tau neutrino	
	0 1			

QUARKS

LEPTONS

SCALAR BOSONS
VECTOR BOSONS

GAUGE BOSONS
VECTOR BOSONS



hadron physics

Low-energy description

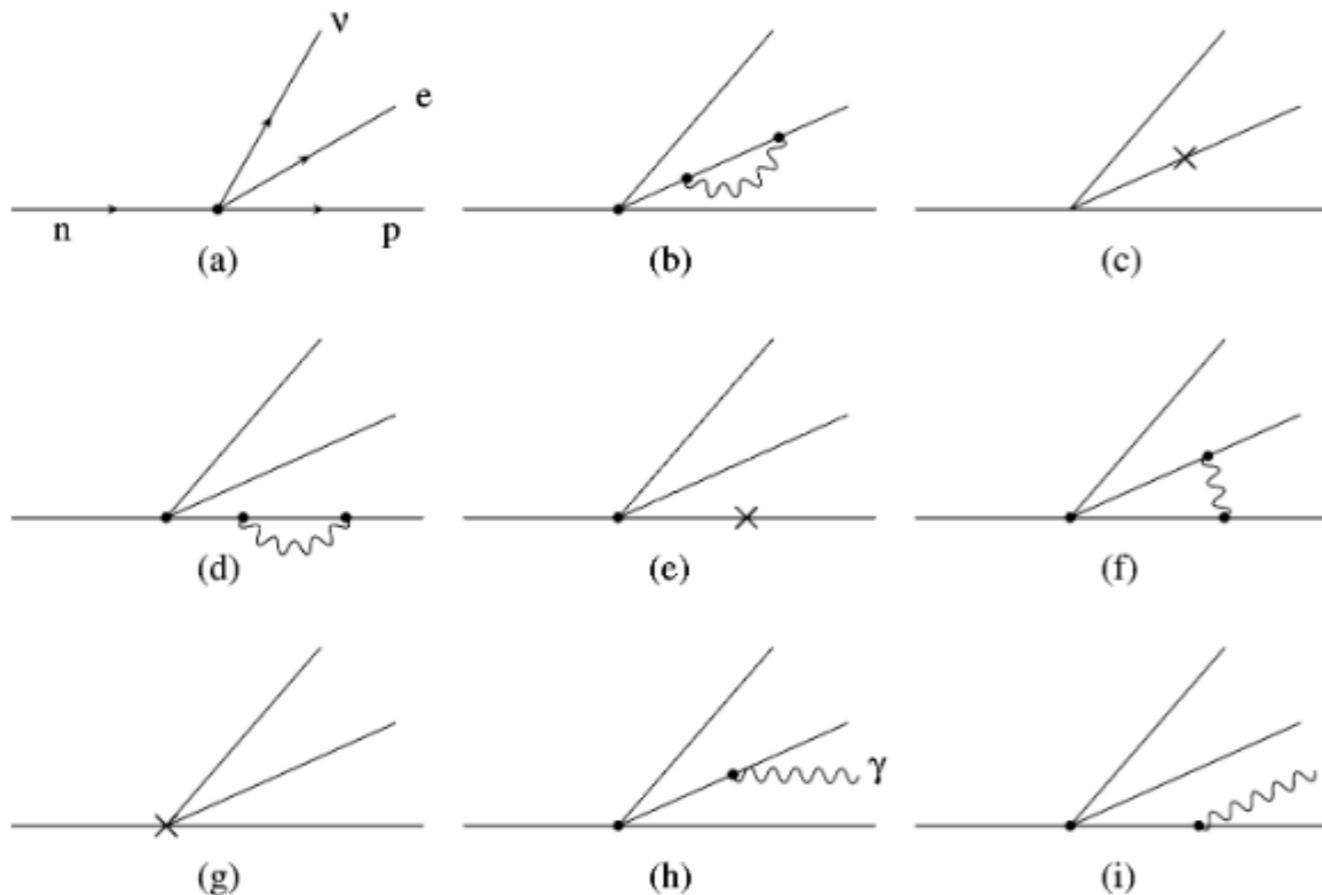
- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\text{eff}} = -\sqrt{2}G_F V_{ud} \bar{e} \gamma_\mu P_L \nu_e \cdot \overline{N} (g_V v^\mu - 2g_A S^\mu) \tau^+ N + O\left(\frac{m_e}{M_p}, \alpha, \alpha \frac{m_\pi}{M_p}, \alpha \frac{m_e}{m_\pi}\right)$$

for uncertainty $m_e \sim M_p - M_n$

A. Sirlin, Phys. Rev. 164, 50 (1967)

- radiative corrections formulated in modern EFT language



vector and axial-vector counterterms (diagrams c, e, g)

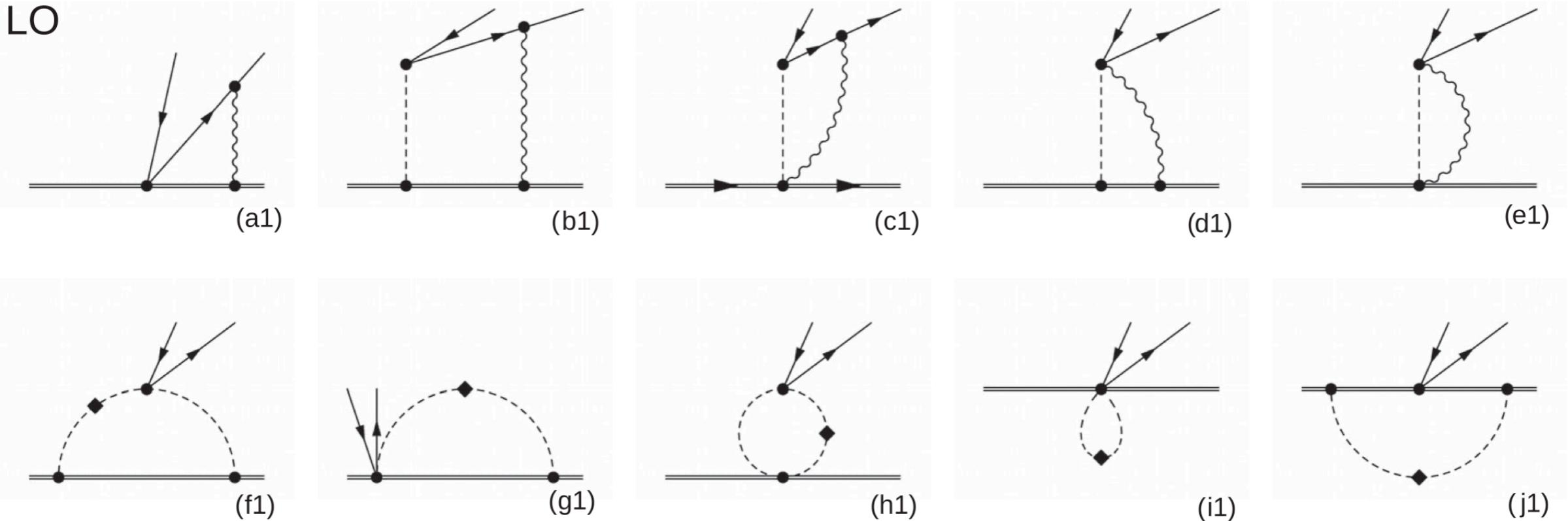
The diagram consists of two separate parts. On the left, the word "data" is written in a large, bold, black serif font. A single blue arrow originates from the bottom-left corner of this text and points diagonally upwards towards the top-right. On the right, the words "Standard Model" are written in a similar large, bold, black serif font. Another blue arrow originates from the bottom-left corner of this text and points diagonally upwards towards the top-right.

S. Ando et al., Phys. Lett. B 595, 250 (2004)

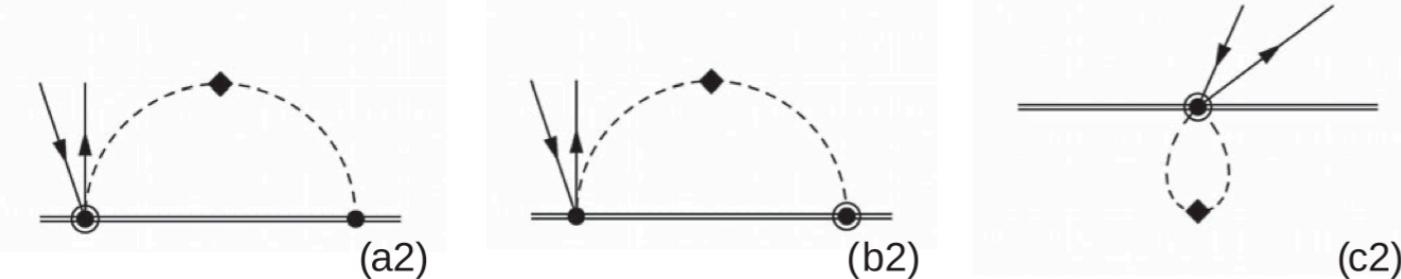
- two coupling constants predict all observables

π EFT and role of pions

LO



NLO



$$\mathcal{L}_{e^2 p^0}^\pi = 2e^2 F_\pi^2 Z_\pi \pi^+ \pi^-$$

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen,
Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

- pion-mediated correction to g_A : for data vs SM comparison
- first steps in matching to χ PT with baryons

Effective field theory for β decay

M_Z

full content of Standard Model (SM)

integrate out top, Z, W, h

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

W. Dekens, P. Stoffer, JHEP 07, 107 (2019)

m_b

integrate out GeV particles

m_c

α_s becomes too strong going to lower energies



m_π

dynamical pions

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen,
Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

m_e

photons, neutrinos, electrons, external nucleons

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- coupling constants from SM + small QED corrections

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
mass charge spin	I U up	II C charm	III t top	g gluon
	$\approx 2.2 \text{ MeV}/c^2$ $2/3$ $1/2$	$\approx 1.28 \text{ GeV}/c^2$ $2/3$ $1/2$	$\approx 173.1 \text{ GeV}/c^2$ $2/3$ $1/2$	$\approx 124.97 \text{ GeV}/c^2$ 0 0 1
QUARKS	d down	s strange	b bottom	H higgs
	$\approx 4.7 \text{ MeV}/c^2$ $-1/3$ $1/2$	$\approx 96 \text{ MeV}/c^2$ $-1/3$ $1/2$	$\approx 4.18 \text{ GeV}/c^2$ $-1/3$ $1/2$	0 0 1
LEPTONS	e electron	μ muon	τ tau	γ photon
	$\approx 0.511 \text{ MeV}/c^2$ -1 $1/2$	$\approx 105.66 \text{ MeV}/c^2$ -1 $1/2$	$\approx 1.7768 \text{ GeV}/c^2$ -1 $1/2$	Z Z boson
	$<1.0 \text{ eV}/c^2$ 0 $1/2$	$<0.17 \text{ MeV}/c^2$ 0 $1/2$	$<18.2 \text{ MeV}/c^2$ 0 $1/2$	W W boson
SCALAR BOSONS	V_e electron neutrino	V_μ muon neutrino	V_τ tau neutrino	
GAUGE BOSONS VECTOR BOSONS				

SCALAR BOSONS

GAUGE BOSONS
VECTOR BOSONS

LEPTONS

QUARKS

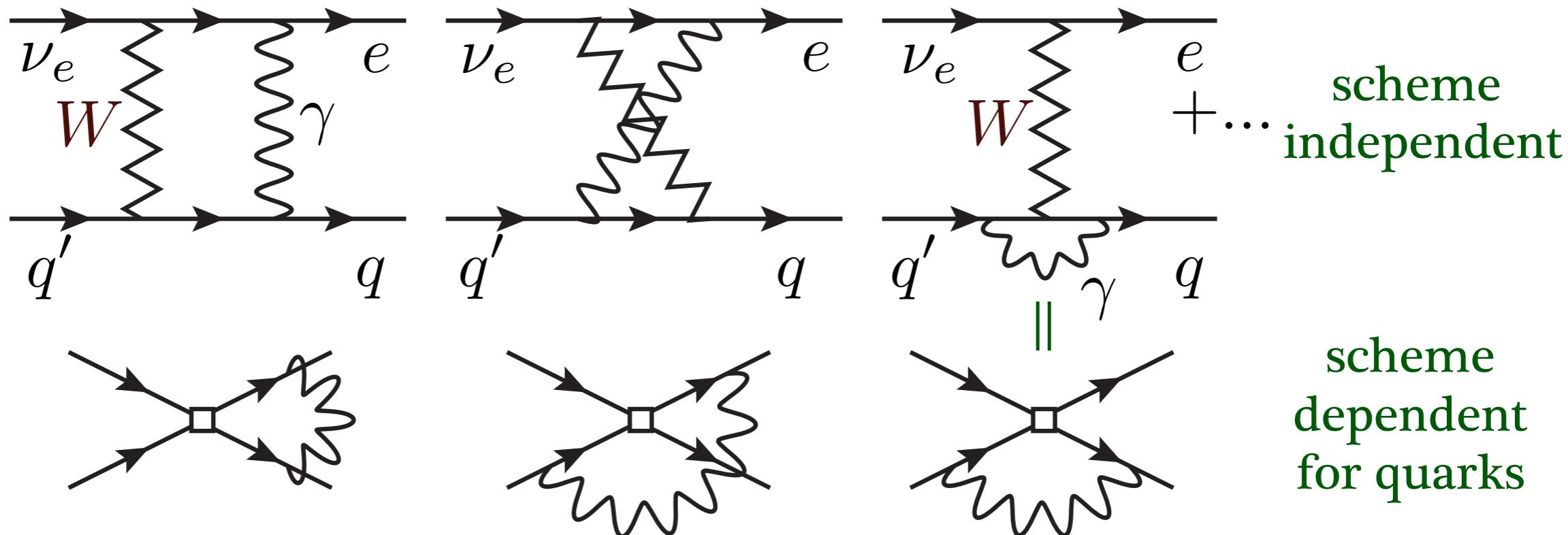
hadron physics



Semileptonic operators and muon decay

$$\mathcal{L}_{\text{LEFT}} \sim G_F \bar{e}_L \gamma_\rho \mu_L \bar{\nu}_{\mu L} \gamma^\rho \nu_{e L} + G_F V_{ud} C_\beta^r(a, \mu) \bar{e}_L \gamma_\rho \nu_{e L} \bar{u}_L \gamma^\rho d_L$$

- M_Z - Fermi coupling is scale independent



- NDR scheme for γ_5 with $a=-1$ for evanescent operators E
- $$\gamma^\alpha \gamma^\beta \gamma^\mu P_L \otimes \gamma_\mu \gamma_\beta \gamma_\alpha P_L = 4(1 + a(4-d)) \gamma^\mu P_L \otimes \gamma_\mu P_L + E(a)$$
- Buras and Weisz (1990)
- Wilson coefficient C_β^r depends on scheme and scale

- one-loop matching with explicit scheme dependence

LEFT running

- M_Z - one renormalization group equation + α, α_s running

$$\mu \frac{dC_{\beta}^r(a, \mu)}{d\mu} = \left(\gamma_0 \frac{\alpha}{\pi} + \gamma_1 \left(\frac{\alpha}{\pi} \right)^2 + \gamma_{se} \frac{\alpha}{\pi} \frac{\alpha_s}{4\pi} \right) C_{\beta}^r(a, \mu)$$

- m_b - 2 anomalous dimensions verified by 3+ groups

- m_{τ}

$$\gamma_0 = -\gamma_{se} = -1$$

- m_c

- γ_{se} and γ_1 are combined at the diagram level

Buras and Weisz (1990)

$$\gamma_1^{NDR}(a) = \frac{\tilde{n}}{18} (2a + 1)$$

- effective fermion number \tilde{n} corrections \sim UCN τ precision

- updated value for NLO anomalous dimension

Effective field theory for β decay

M_Z

full content of Standard Model (SM)

integrate out top, Z, W, h

O.T., R. J Hill, Phys. Lett. B 805, 3, 135466 (2020)

W. Dekens, P. Stoffer, JHEP 07, 107 (2019)

m_b

integrate out GeV particles

m_c

α_s becomes too strong going to lower energies



m_π

dynamical pions

Vincenzo Cirigliano, Jordy de Vries, Leendert Hayen,
Emanuele Mereghetti, and Andre Walker-Loud, Phys. Rev. Lett. 129, 12801 (2022)

m_e

photons, neutrinos, electrons, external nucleons

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- coupling constants from SM + small QED corrections

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
mass charge spin	I U up	II C charm	III t top	g gluon
	$\approx 2.2 \text{ MeV}/c^2$ $2/3$ $1/2$	$\approx 1.28 \text{ GeV}/c^2$ $2/3$ $1/2$	$\approx 173.1 \text{ GeV}/c^2$ $2/3$ $1/2$	$\approx 124.97 \text{ GeV}/c^2$ 0 0 1
QUARKS	d down	s strange	b bottom	H higgs
	$\approx 4.7 \text{ MeV}/c^2$ $-1/3$ $1/2$	$\approx 96 \text{ MeV}/c^2$ $-1/3$ $1/2$	$\approx 4.18 \text{ GeV}/c^2$ $-1/3$ $1/2$	0 0 1
LEPTONS	e electron	μ muon	τ tau	γ photon
	$\approx 0.511 \text{ MeV}/c^2$ -1 $1/2$	$\approx 105.66 \text{ MeV}/c^2$ -1 $1/2$	$\approx 1.7768 \text{ GeV}/c^2$ -1 $1/2$	Z Z boson
	$<1.0 \text{ eV}/c^2$ 0 $1/2$	$<0.17 \text{ MeV}/c^2$ 0 $1/2$	$<18.2 \text{ MeV}/c^2$ 0 $1/2$	W W boson
SCALAR BOSONS	V_e electron neutrino	V_μ muon neutrino	V_τ tau neutrino	
GAUGE BOSONS VECTOR BOSONS				

SCALAR BOSONS

GAUGE BOSONS
VECTOR BOSONS

GAUGE BOSONS
VECTOR BOSONS

hadron physics



ChPT notations

- SU(2) strong interaction low-energy effective theory

$$u = e^{i \frac{\pi^a T^a}{2}}$$

$$\mathbf{q} = \mathbf{q}^a T^a$$

$$u_\mu = i \left(u^\dagger (\partial_\mu - ir_\mu + i\mathbf{q}_R A_\mu) u - u (\partial_\mu - il_\mu + i\mathbf{q}_L A_\mu) u^\dagger \right)$$

$$\tilde{D}_\mu = \partial_\mu + \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u l_\mu u^\dagger$$

$$c_\mu^\pm = -\frac{i}{2} \left(u (i\partial_\mu \mathbf{q}_L + [l_\mu, \mathbf{q}_L]) u^\dagger \pm u^\dagger (i\partial_\mu \mathbf{q}_R + [r_\mu, \mathbf{q}_R]) u \right)$$

$$Q_L = u \mathbf{q}_L u^\dagger, \quad Q_R = u^\dagger \mathbf{q}_R u, \quad Q_\pm = \frac{Q_L \pm Q_R}{2}$$

$$\mathbf{q}_V = \mathbf{q}_L + \mathbf{q}_R \quad v = l + r$$

$$\mathbf{q}_A = \mathbf{q}_L - \mathbf{q}_R \quad a = l - r$$

- notations for next slides

Electromagnetic coupling constants

- SU(2) strong interaction Lagrangian with $\langle Q_L - Q_R \rangle = 0$

$$\mathcal{L}_{\pi N}^{e^2 p} = e^2 \sum_{i=1}^{14} g_i \bar{N} O_i N$$

extension of S. Steininger's Ph.D. thesis

- spurion-dependent operators relevant to β decay

$$O_{13} = \langle Q_+^2 + Q_-^2 \rangle u \cdot S$$

$$O_1 = \langle Q_+^2 - Q_-^2 \rangle u \cdot S$$

$$O_2 = \langle Q_+ \rangle^2 u \cdot S$$

$$O_9 = -\frac{i}{2M} [Q_+, c^+ \cdot \tilde{D}] + \text{h.c.}$$

$$O_{11} = i [Q_+, c^- \cdot S]$$



- 5 LECs enter β decay in 2 combinations

Electromagnetic coupling constants

- relevant Lagrangian in LEFT

$$\mathcal{L}_{\text{4-Fermi}}^{\text{spurions}} = -eA_\mu \bar{q} \mathbf{q}_V \gamma^\mu q + \dots$$

$$g_9 \bar{N} T^c N = \frac{i\varepsilon_{abc} e^2}{F_0^2} v_\mu \frac{\partial}{\partial r_\mu} \left(\int d^d x e^{ir \cdot x} \langle N | \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A)}{\delta \mathbf{q}_{V^b}(x) \delta \mathbf{q}_{V^a}(0)} \Big|_{\mathbf{q}=0} |N\rangle \right) \Big|_{r_\mu=0}$$

$$g_9 \bar{N} T^c N = -\frac{2\varepsilon_{abc} e^2}{F_0^2} \int \frac{d^d k}{(2\pi)^d} \frac{g_{\lambda\rho} k \cdot v}{(k^2 - \lambda^2 + i\varepsilon)^2} \int d^d x e^{ik \cdot x} \langle N | \bar{q} T^b \gamma^\lambda q(x) \bar{q} T^a \gamma^\rho q(0) | N \rangle$$

+ χ PT loops

+ LEFT counterterms

forward Compton tensor

$$T_{V^a V^b}^{\lambda\rho}(v, k)$$

- 2-point functions: product of quark currents in LEFT

- coupling constants in terms of non-perturbative physics

Electroweak coupling constants

- SU(2) weak interaction Lagrangian with $\langle Q_L^W \rangle = 0$

$$\mathcal{L}_{\pi N \ell}^{e^2 p} = e^2 \sum_{i=1}^6 (V_i v^\mu - A_i S^\mu) \bar{e}_L \gamma_\mu \nu_e \bar{N}_v O_i N_v + \text{h.c.}$$

- spurion-dependent operators

$$O_1 = [Q_L, Q_L^W]$$

$$O_3 = \{Q_L, Q_L^W\}$$

$$O_5 = \langle Q_L Q_L^W \rangle$$

$$O_2 = [Q_R, Q_L^W]$$

$$O_4 = \{Q_R, Q_L^W\}$$

$$O_6 = \langle Q_R Q_L^W \rangle$$

- scattering amplitude determined by spurion's physical values

$$T = e^2 ((V_1 + V_2 + V_3 + V_4)v^\mu - (A_1 + A_2 + A_3 + A_4)S^\mu) \bar{e}_L \gamma_\mu \nu_e \bar{N}' T^a N$$

- 8 LECs enter physical observables in 2 combinations

Electroweak coupling constants

- functional derivatives of generating functional w.r.t. isoscalar and isovector spurious

$$T =$$

$$e^2 ((V_1 + V_2 + V_3 + V_4)v^\mu - (A_1 + A_2 + A_3 + A_4)S^\mu) \bar{e}_L \gamma_\mu \nu_e \bar{N}' T^a N =$$
$$\int d^d x \langle e \bar{\nu}_e N' | \left(\frac{\varepsilon_{abc}}{2} \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A, \mathbf{q}_W)}{\delta \mathbf{q}_{V^b}(x) \delta \mathbf{q}_{W^c}(0)} + i \frac{\delta^2 W(\mathbf{q}_V, \mathbf{q}_A, \mathbf{q}_W)}{\delta \mathbf{q}_{V^0}(x) \delta \mathbf{q}_{W^a}(0)} \right) \Big|_{\mathbf{q}=0} |N\rangle$$

- vector-vector and vector-axial products of quark currents

- 2 correlation functions determine all electroweak LECs

HB χ PT to π EFT matching

- purely leptonic counterterm

$$\mathcal{L}_{\text{lept}}^{\text{CT}} = e^2 X_6 \bar{e} (i\partial_\mu + eA_\mu) \gamma^\mu e$$

- π EFT counterterms from χ PT couplings in baryon sector

$$g_V = 1 + e^2 \left[-\frac{X_6}{2} + 2(V_1 + V_2 + V_3 + V_4) - g_9 \right]$$

- vector and axial products of quark currents

- LECs in terms of correlation functions

One-loop result

- clear illustration of leading logarithms

$$g_V(\mu_\chi) = 1 - \frac{\alpha}{2\pi} \left[\frac{5}{8} + \frac{3}{4} \ln \frac{\mu_\chi^2}{\mu_0^2} + \ln \frac{\mu_0^2}{M_Z^2} \right] - e^2 \int \frac{i d^4 q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \left[\frac{T_3(\nu, Q^2)}{2\nu} - \frac{2}{3} \frac{1}{Q^2 + \mu_0^2} \right]$$

- same Regge and resonance inputs as in traditional approach
- updates in elastic; DIS is part of running in LEFT

- agreement with current algebra evaluation

Low-energy running of g_V

M_Z

- one renormalization group equation + α running

$$\mu \frac{dg_V(\mu)}{d\mu} = \left(\tilde{\gamma}_0 \frac{\alpha}{\pi} + \tilde{\gamma}_1 \left(\frac{\alpha}{\pi} \right)^2 \right) g_V(\mu)$$

m_b

- well-known heavy-light anomalous dimension

$$\tilde{\gamma}_0 = -\frac{3}{4}$$

matching scale

- $\tilde{\gamma}_1$ is combined at the diagram level

Gimenez (1991)

$$\tilde{\gamma}_1 = \frac{5\tilde{n}}{24} + \frac{5}{32} - \frac{\pi^2}{6}$$

m_π

- inclusion of $\tilde{\gamma}_1 \sim \text{UCN}\tau$ precision

m_e

- first time evolution with NLO anomalous dimension

Effect of running

M_Z

- renormalization group equations

$$\mu \frac{dg_V(\mu)}{d\mu} = \left(\tilde{\gamma}_0 \frac{\alpha}{\pi} + \tilde{\gamma}_1 \left(\frac{\alpha}{\pi} \right)^2 \right) g_V(\mu)$$

$$m_b \mu \frac{dC_\beta^r(a, \mu)}{d\mu} = \left(\gamma_0 \frac{\alpha}{\pi} + \gamma_1 \left(\frac{\alpha}{\pi} \right)^2 + \gamma_{se} \frac{\alpha}{\pi} \frac{\alpha_s}{4\pi} \right) C_\beta^r(a, \mu)$$

m_c

- resummed result with leading $\alpha\alpha_s$ corrections

matching scale

$$g_V(\mu_\chi = m_e) - 1 = (2.499 \pm 0.012) \%$$

m_π

- one-loop result without $\alpha\alpha_s$ contributions

$$g_V^{1\text{-loop}}(\mu_\chi = m_e) - 1 = (2.430 \pm 0.012) \%$$

m_e

- cancellation of various contributions -> 0.07% effect

Low-energy description

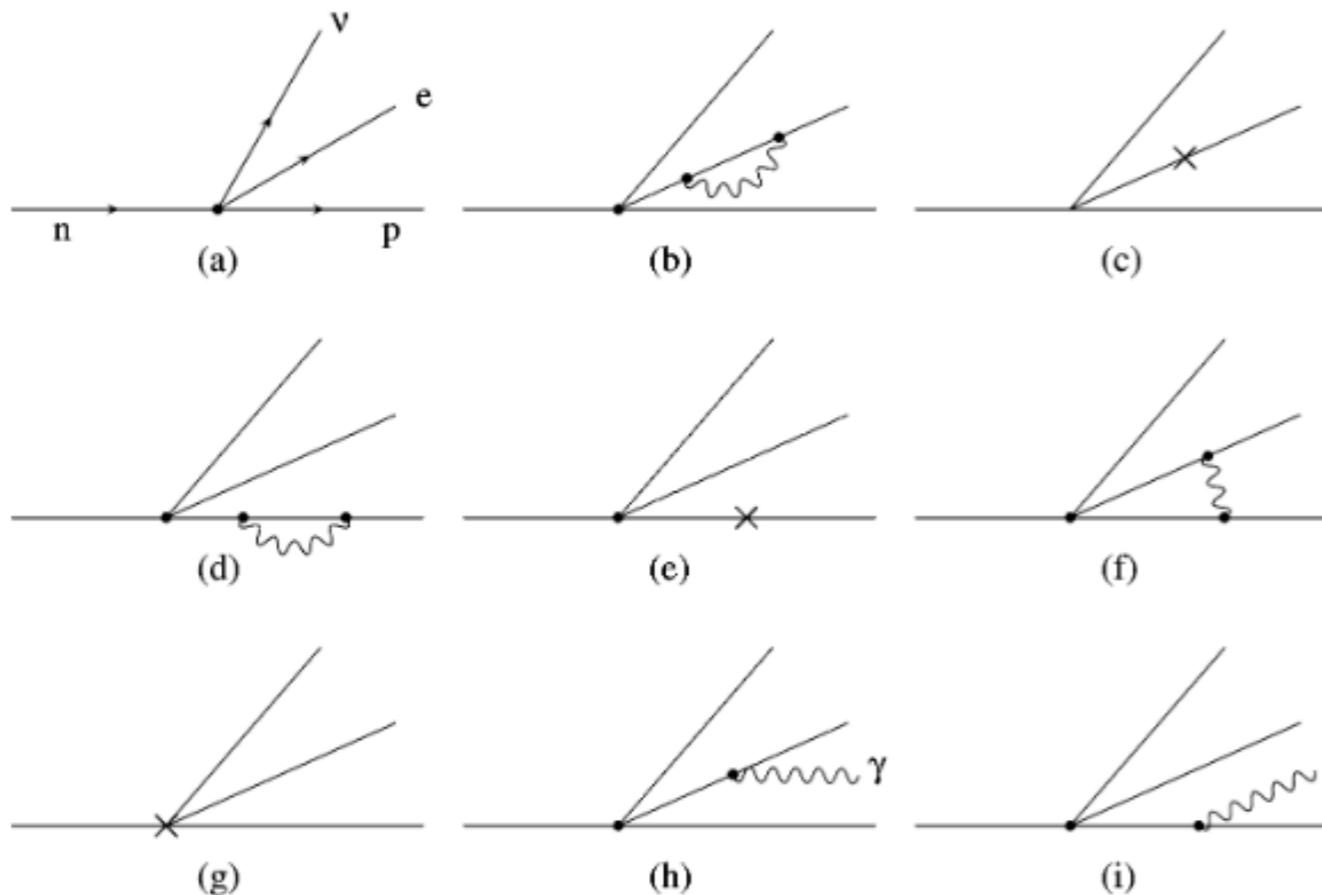
- four-fermion interaction between leptons and heavy nucleons

$$\mathcal{L}_{\text{eff}} = -\sqrt{2}G_F V_{ud} \bar{e} \gamma_\mu P_L \nu_e \cdot \overline{N} (g_V v^\mu - 2g_A S^\mu) \tau^+ N + O\left(\frac{m_e}{M_p}, \alpha, \alpha \frac{m_\pi}{M_p}, \alpha \frac{m_e}{m_\pi}\right)$$

for uncertainty $m_e \sim M_p - M_n$

A. Sirlin, Phys. Rev. 164, 50 (1967)

- radiative corrections formulated in modern EFT language



vector and axial-vector counterterms (diagrams c, e, g)

The diagram consists of two separate blue arrows, each pointing diagonally upwards and to the right. The arrow on the far left is associated with the word "data" written in a large, bold, black serif font directly beneath it. The arrow on the far right is associated with the words "Standard Model" written in a large, bold, black serif font directly beneath it.

S. Ando et al., Phys. Lett. B 595, 250 (2004)

- two coupling constants predict all observables

Decay rate and neutron lifetime

- new expression for differential decay rate resumming $\left(\frac{\pi\alpha}{\beta}\right)^n$

$$\frac{d\Gamma_n}{dE_e} = \frac{G_F^2 |V_{ud}|^2}{(2\pi)^5} (1 + 3\lambda^2) p_e E_e (E_0 - E_e)^2 [g_V(\mu_\chi)]^2 F_{NR}(\beta) \left(1 + \delta_{\text{RC}}(E_e, \mu_\chi)\right) \left(1 + \delta_{\text{recoil}}(E_e)\right)$$

$$\lambda = \frac{g_A}{g_V}$$

- remove dependence on UV regulator (nucleon radius)

$$F(\beta) = 4(2E_e\beta R)^{2(\gamma-1)} e^{\pi y} \frac{|\Gamma(\gamma + iy)|^2}{(\Gamma(1 + 2\gamma))^2}, \quad y = \frac{\alpha}{\beta}, \quad \gamma = \sqrt{1 - \alpha^2}$$

$$F(\beta) = F_{NR}(\beta) [1 - \alpha^2 (\gamma_E - 3 + \ln(2E_e R \beta)) + \mathcal{O}(\alpha^4)]$$

- include α^2/β consistently with matching to pNRQED
idea from Hoang (1997), Czarnecki and Melnikov (1997) and others
- non-relativistic Fermi function and $\alpha^2/\beta \sim \text{UCN}\tau$ precision

- separation of UV-finite Coulomb-enhanced corrections

V_{ud} from neutron decay

- λ from electron spin correlation measurements

$$|V_{ud}|^2 \tau_n (1 + 3\lambda^2) \left(1 + \Delta_{\text{TOT}}\right) = 5283.321(5) \text{ s}$$

$$\Delta_{\text{TOT}}^{\text{EFT}} = 7.756(27)\% \quad \text{vs} \quad \Delta_{\text{TOT}} = 7.735(27)\%$$

preliminary

- extraction with PDG lifetime and λ

$$|V_{ud}^{n,PDG}| = 0.97432(4)_{fs}(11)_{g_V}(82)_{\lambda}(28)_{\tau_n}[87]_{\text{total}}$$

- extraction with UCN τ lifetime and PERKEO-III λ

$$|V_{ud}^{n,best}| = 0.97404(4)_{fs}(11)_{g_V}(35)_{\lambda}(20)_{\tau_n}[42]_{\text{total}}$$

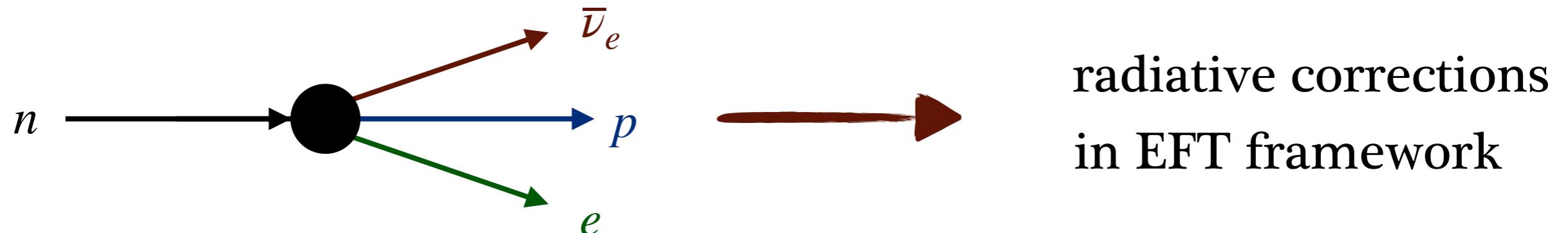
- in agreement with superallowed β decays, similar error

$$|V_{ud}^{0^+ \rightarrow 0^+}| = 0.97373(31)$$

Hardy and Towner (2020)

- clean neutron lifetime extraction vs superallowed β decays

Conclusions



radiative corrections
in EFT framework

- radiative corrections in effective field theory approach

Standard Model \rightarrow LEFT \rightarrow HB χ PT \rightarrow π EFT

- electroweak large logs and Coulomb enhancement
- next-to-leading logarithm resummation
- updated extraction of V_{ud}

Thanks for your attention !!!

Beta Decay Parameters

Jackson, Treiman and Wyld (Phys. Rev. **106** and Nucl. Phys. **4**, 1957)

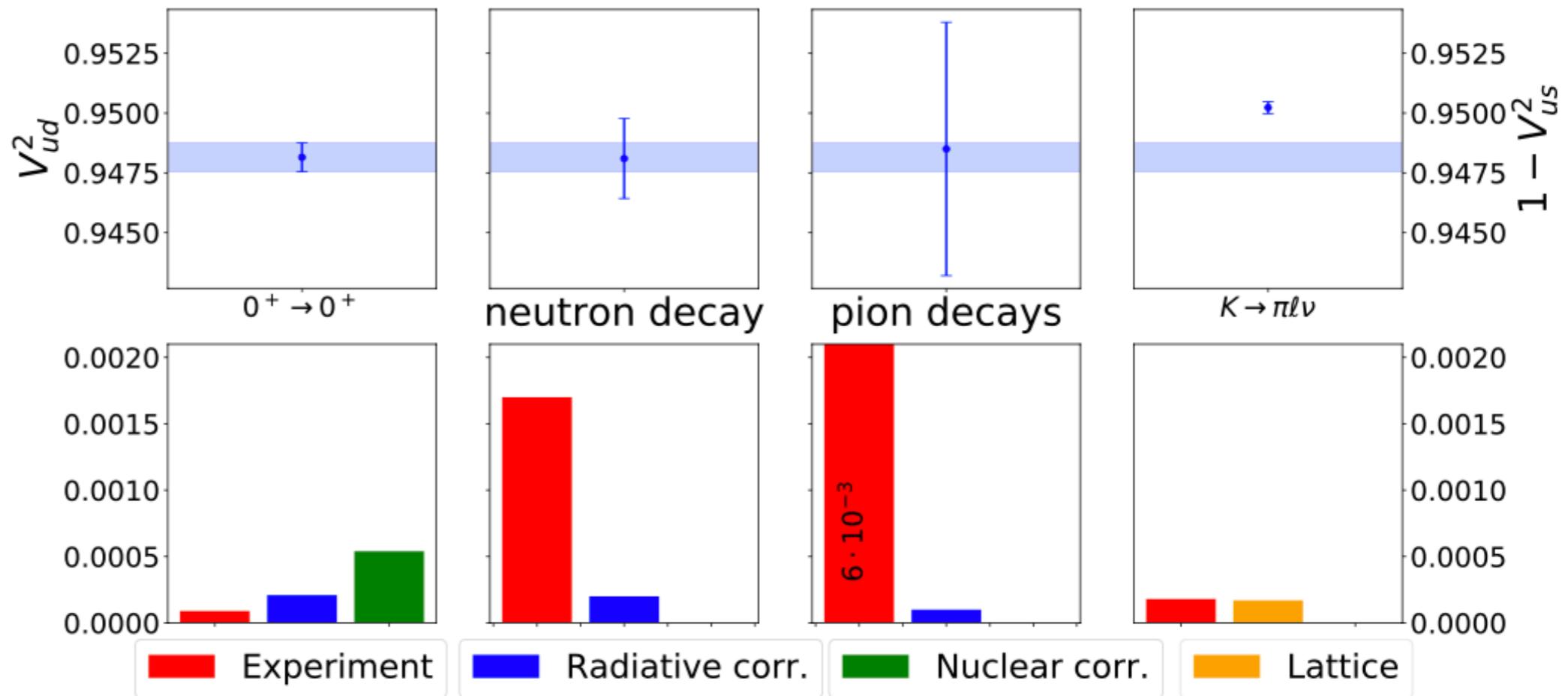
$$\frac{d^5W}{dE_e d\Omega_e d\Omega_{\nu_e}} = \overbrace{\frac{G_F^2 |\mathbf{V}_{ud}|^2}{(2\pi)^5} p_e E_e (A_\circ - E_e)^2 \xi}^{\text{basic decay rate}} \left(1 + \overbrace{a_{\beta\nu} \frac{\vec{p}_e \cdot \vec{p}_{\nu_e}}{E_e E_{\nu_e}}}^{\beta-\nu \text{ correlation}} + \overbrace{b \frac{\Gamma m_e}{E_e}}^{\text{Fierz term}} \right. \\ \left. + \frac{\langle \vec{I} \rangle}{I} \cdot \left[\underbrace{\mathbf{A}_\beta \frac{\vec{p}_e}{E_e}}_{\beta \text{ asym}} + \underbrace{\mathbf{B}_\nu \frac{\vec{p}_\nu}{E_\nu}}_{\nu \text{ asym}} + \underbrace{\mathbf{D} \frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu}}_{T\text{-violating}} \right] \right) + \dots$$

On-going or planned efforts to measure:

Note: to specify both C_v and C_A , 2 meas. needed

- (1) Decay rates and β -spectra ($G_F V_{ud}, \xi, b$) 
 - (2) Unpolarized angular correlations ($a_{\beta\nu}, b$)
 - (3) Polarized angular correlations (A_β, B_ν, b, b_ν)

CKM unitarity tests



V_{us} and V_{ub}

- extracted from $K \rightarrow \pi \ell \nu$ and $K \rightarrow \mu \nu / \pi \rightarrow \mu \nu$
- theory input on $f_+(0)$ and f_K/f_π from Lattice QCD
- $|V_{ub}|^2 \sim 10^{-5}$, beyond present accuracy

slide of Emanuele Mereghetti@INT Workshop “New physics searches at the precision frontier”