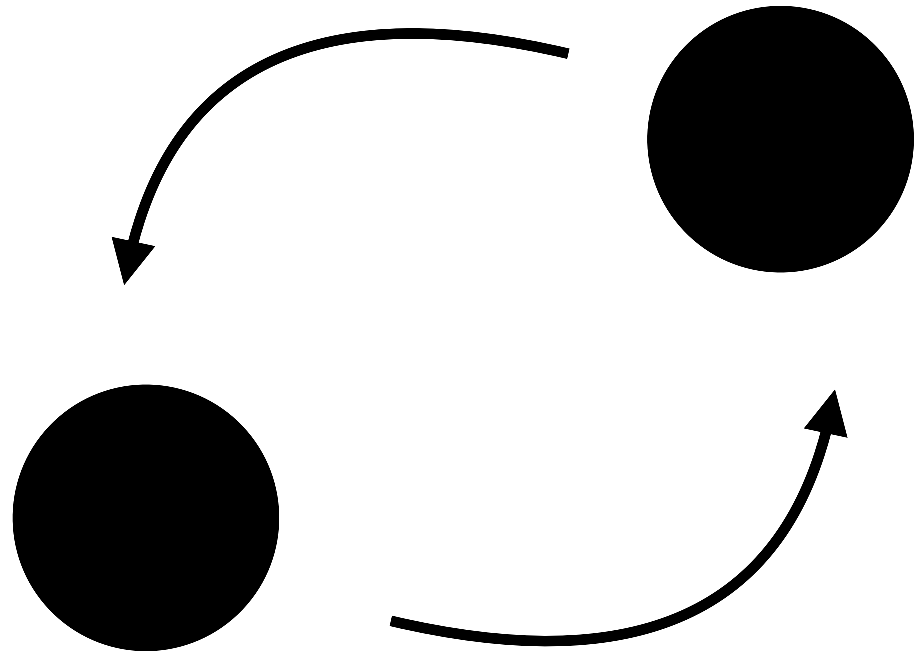
Ringdown beyond Kerr

Aaron Zimmerman (UT Austin),
Asad Hussain (UT Austin)
arXiv:2206:10653

Mitchell Conference
May 18, 202



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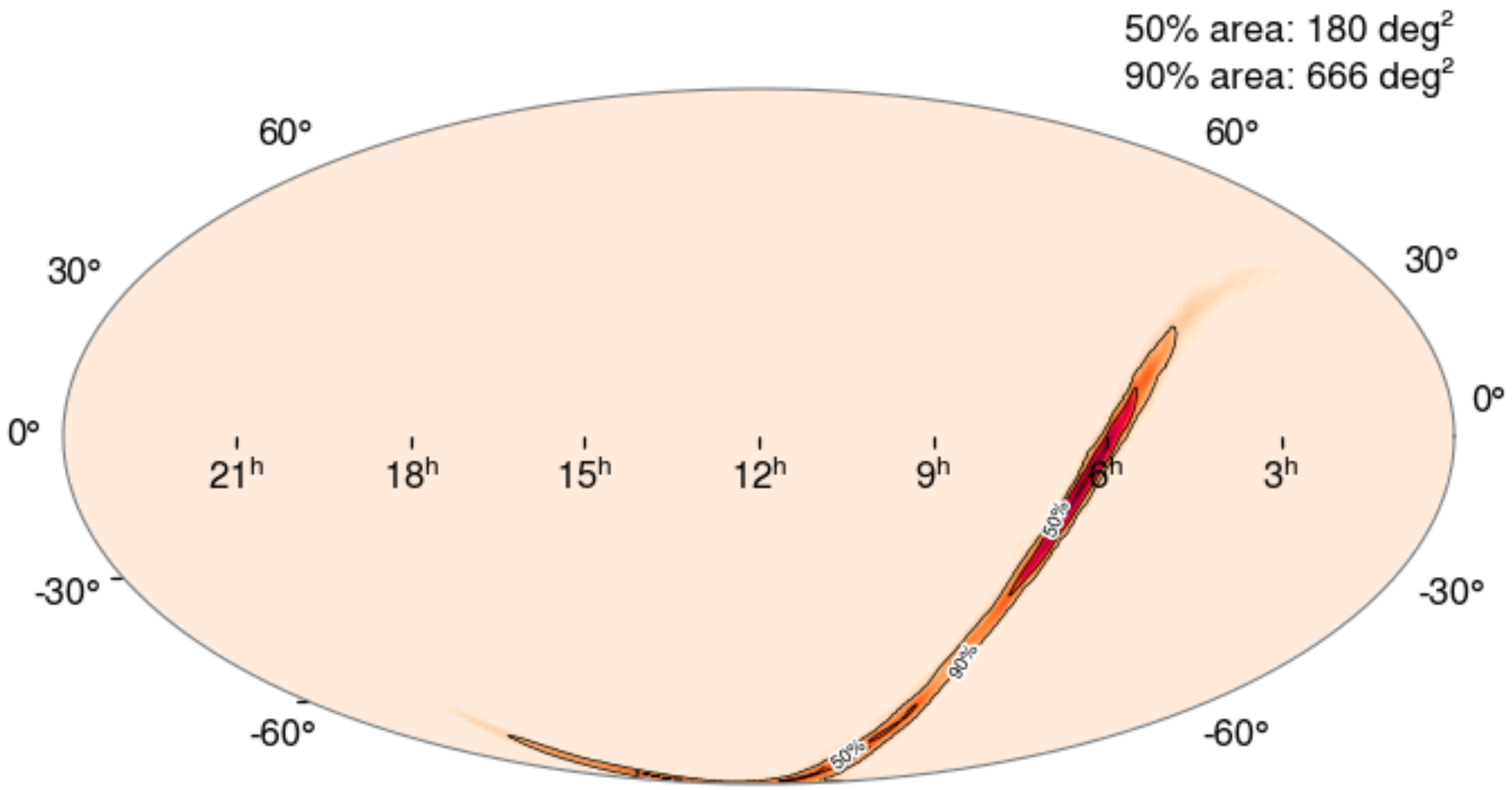
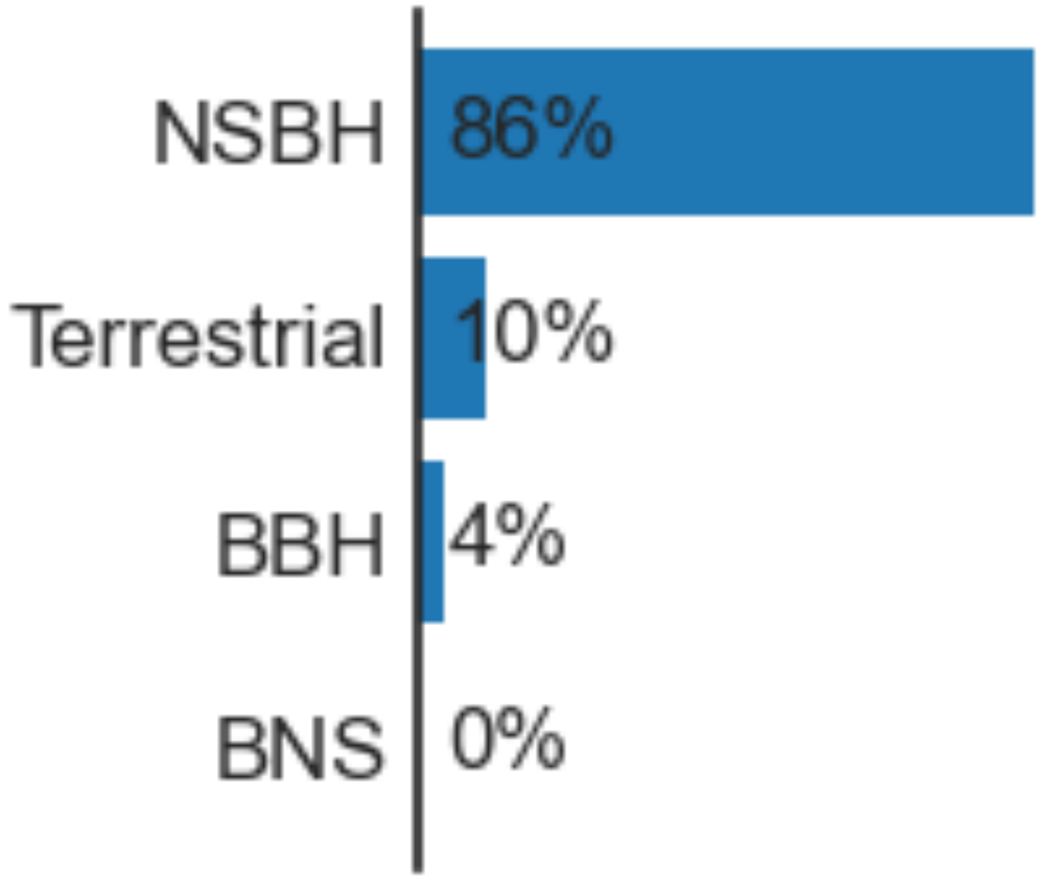
*Not on behalf of the LVK; references not at all exhaustive

First event from O4 (kind of)

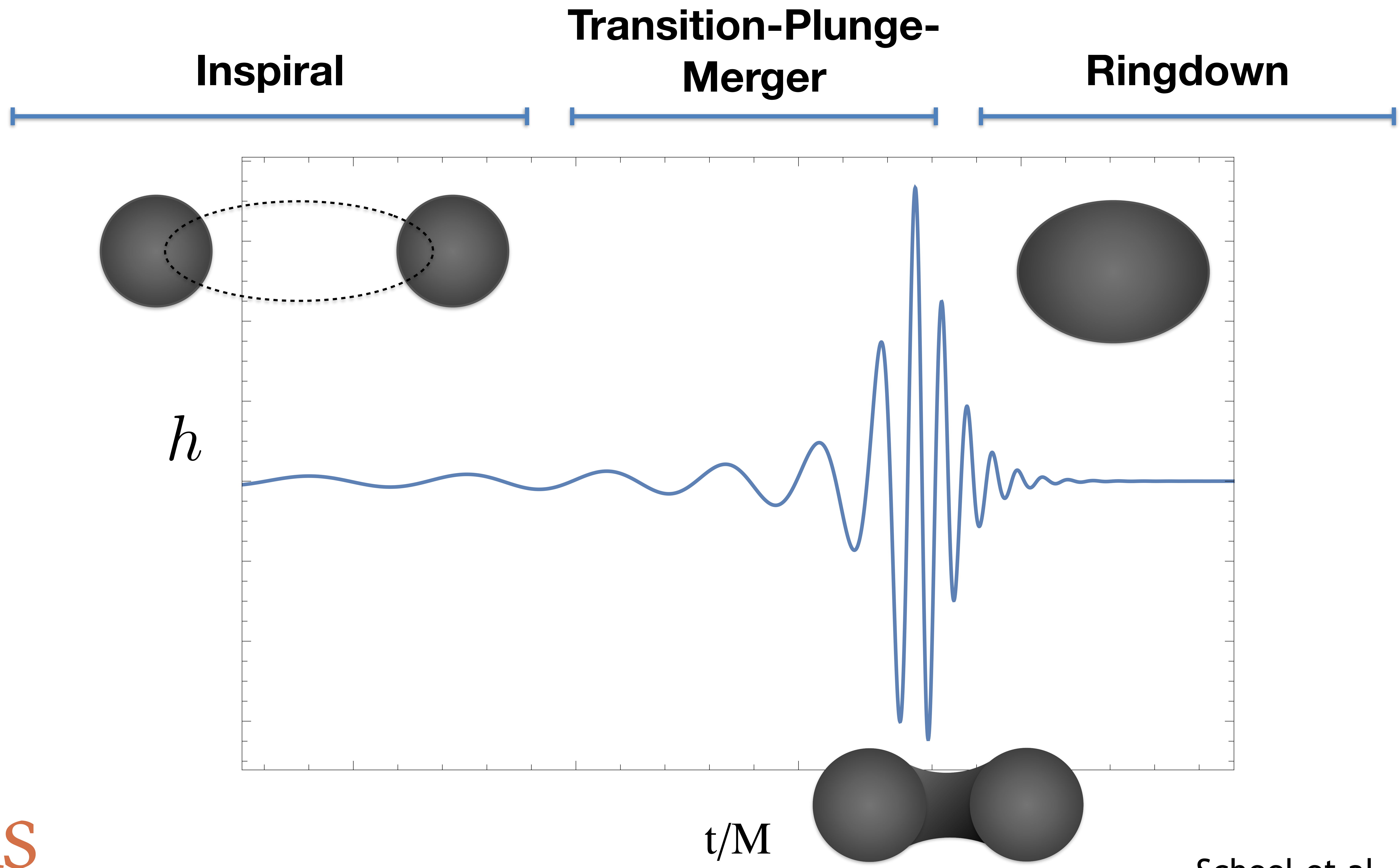
- Still in ER15
- S230518h: Likely NSBH
- FAR: 1 in 98 yr





https://gracedb.ligo.org/superevents/S230518h/view/

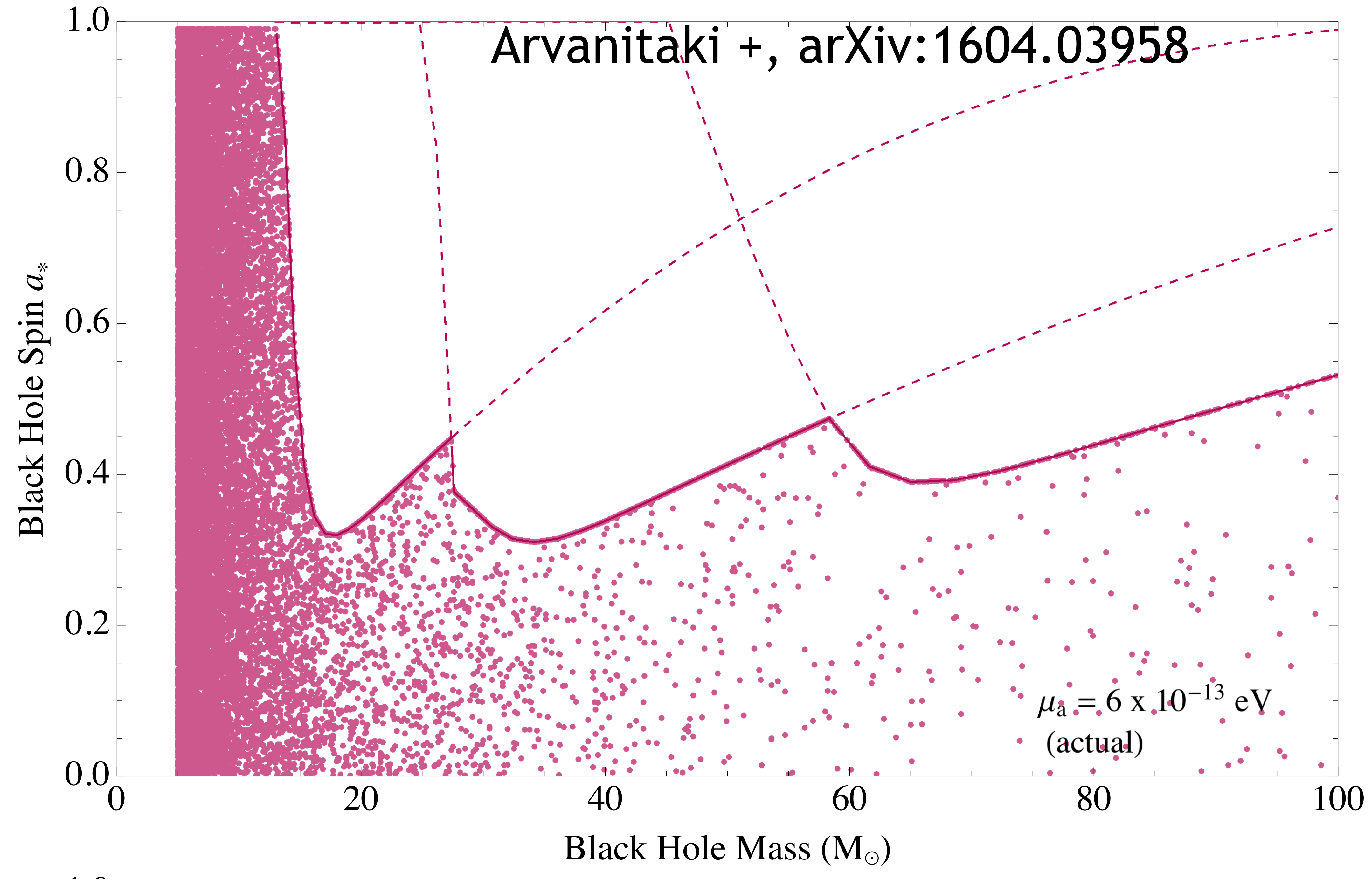
Binary black hole merger



Scheel et al. arXiv:0810.1767 (2008)

Dark matter and compact binaries

- Energy of 200 Hz GWs ~ peV
- Energy scale of 1 km ~ 200 peV
- Measure finite-size effects: response to spin and tides, hence equation of state
- ALPs: can produce “gravitational atoms”
  - Finite-size effects, monochromatic emission, population effects



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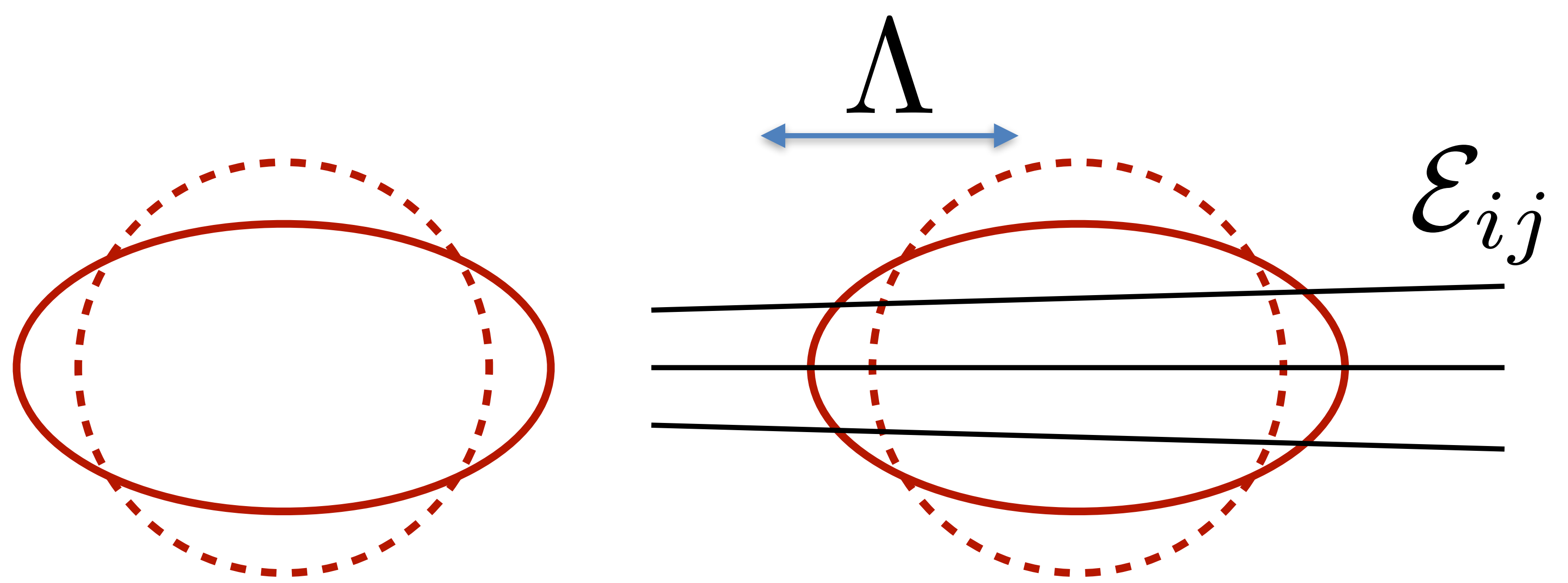

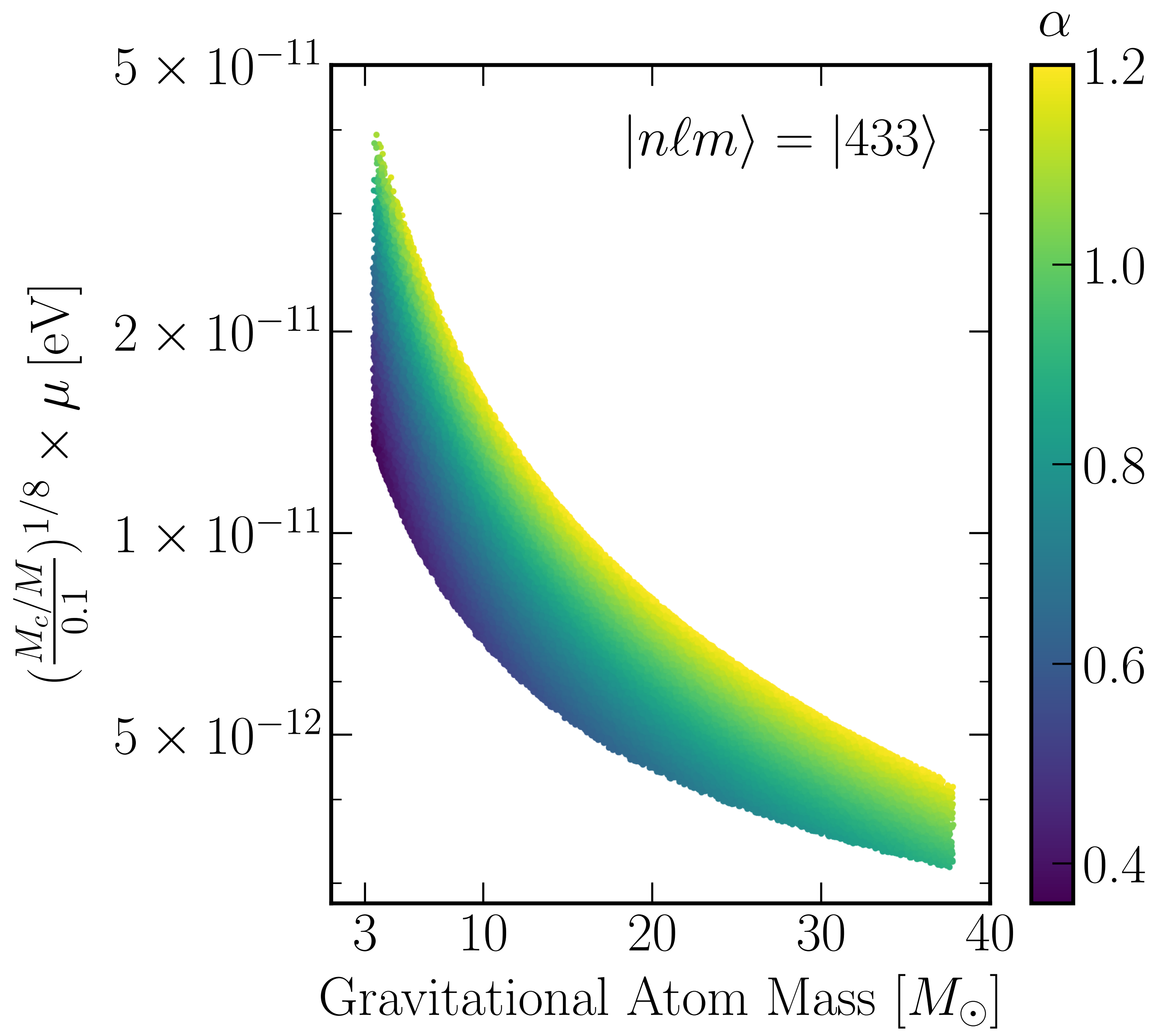

Chia, Edwards, Wadekar, AZ +, arXiv:2306.XXXXX

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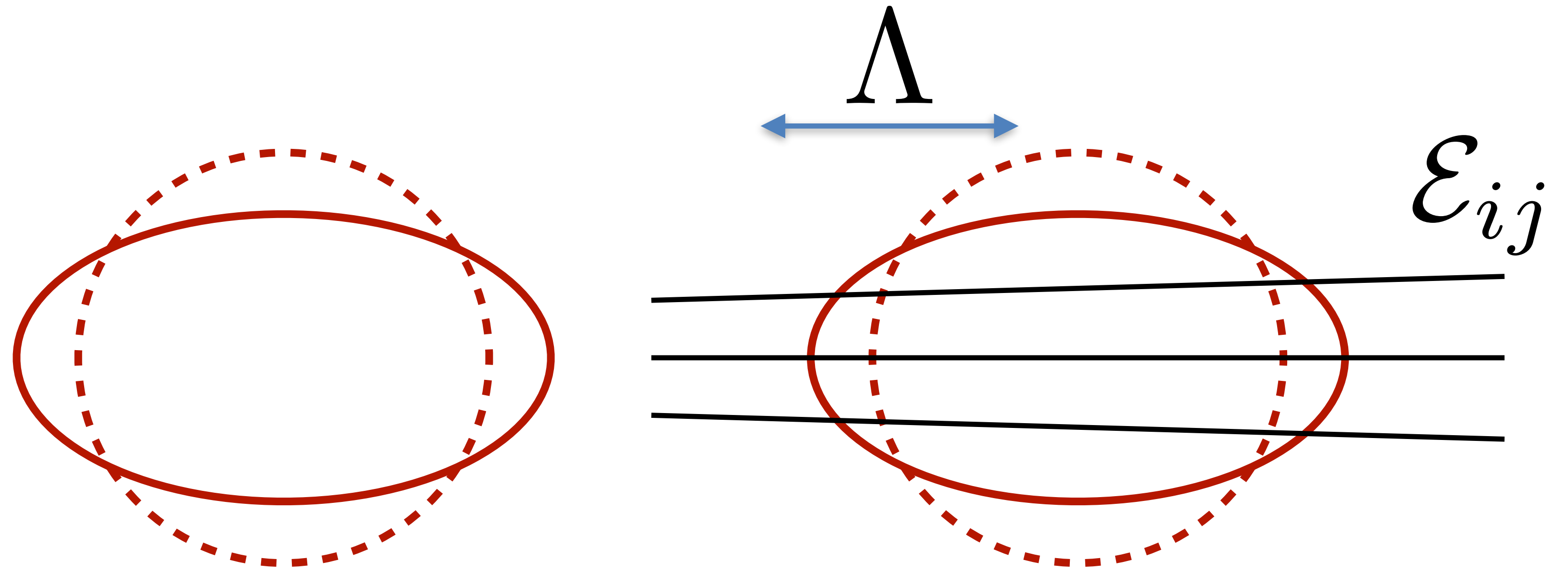
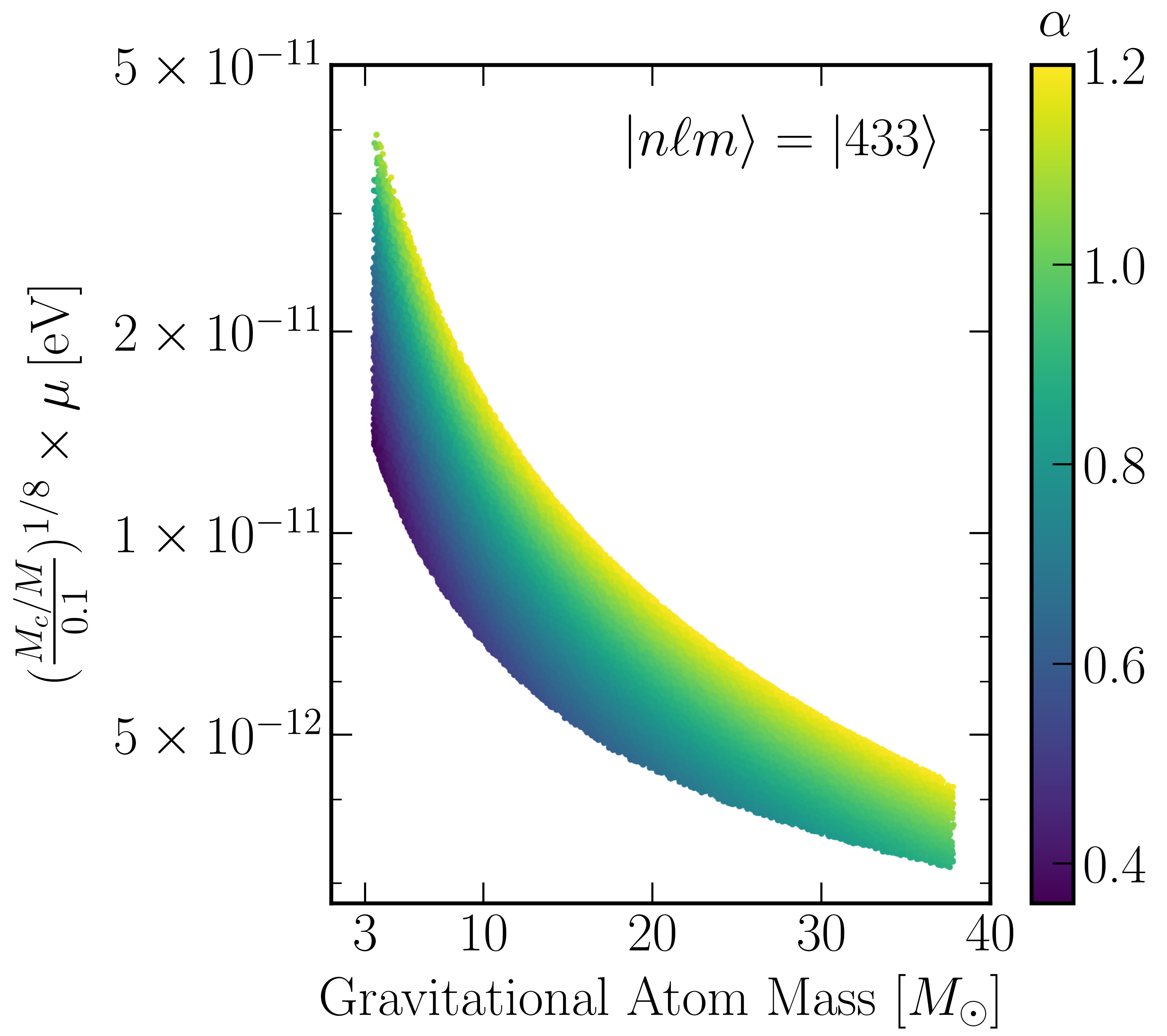
- Can detect entirely new km-scale compact objects
- …or focus on tests of gravity





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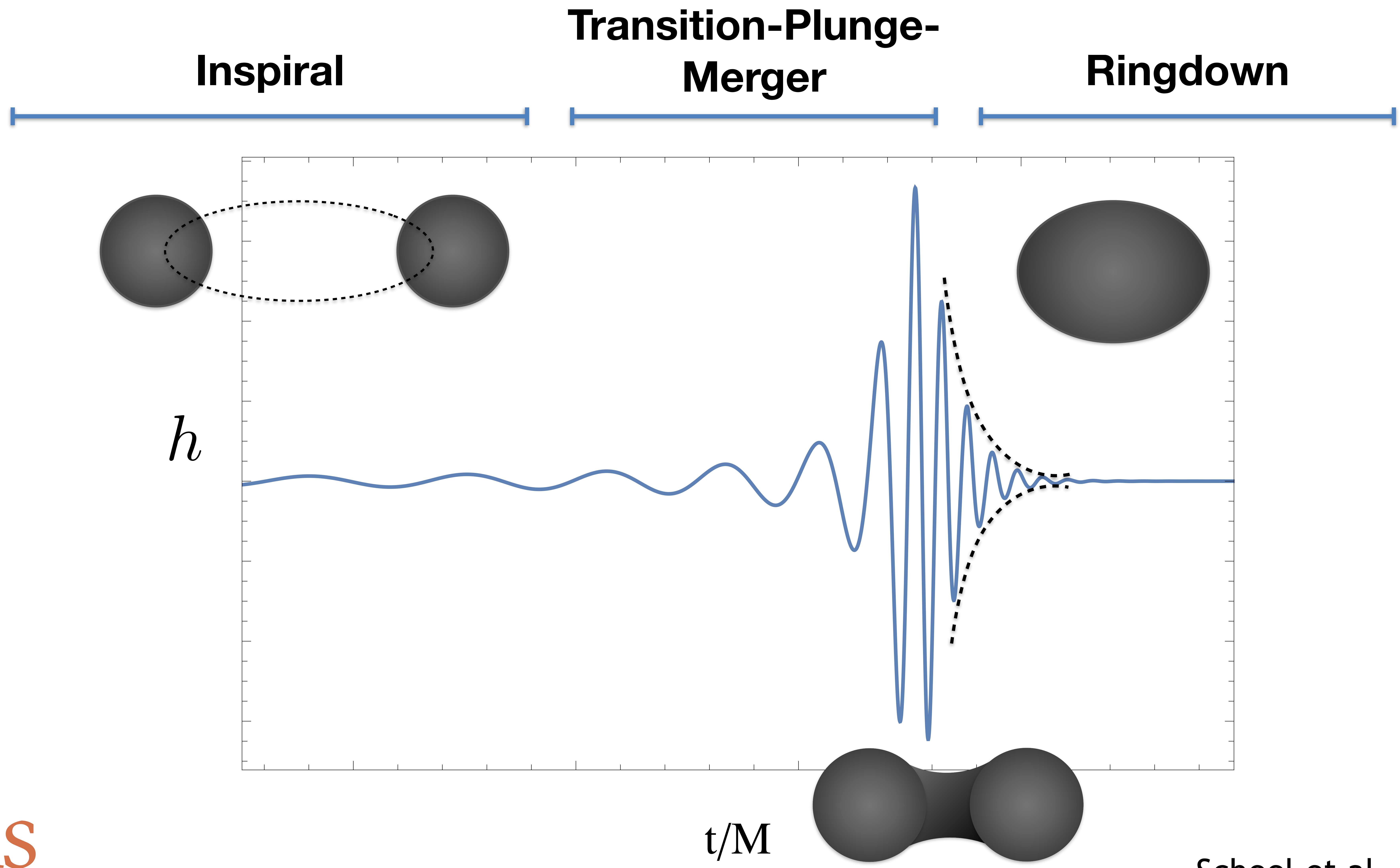
Black hole ringdown

Binary black hole merger



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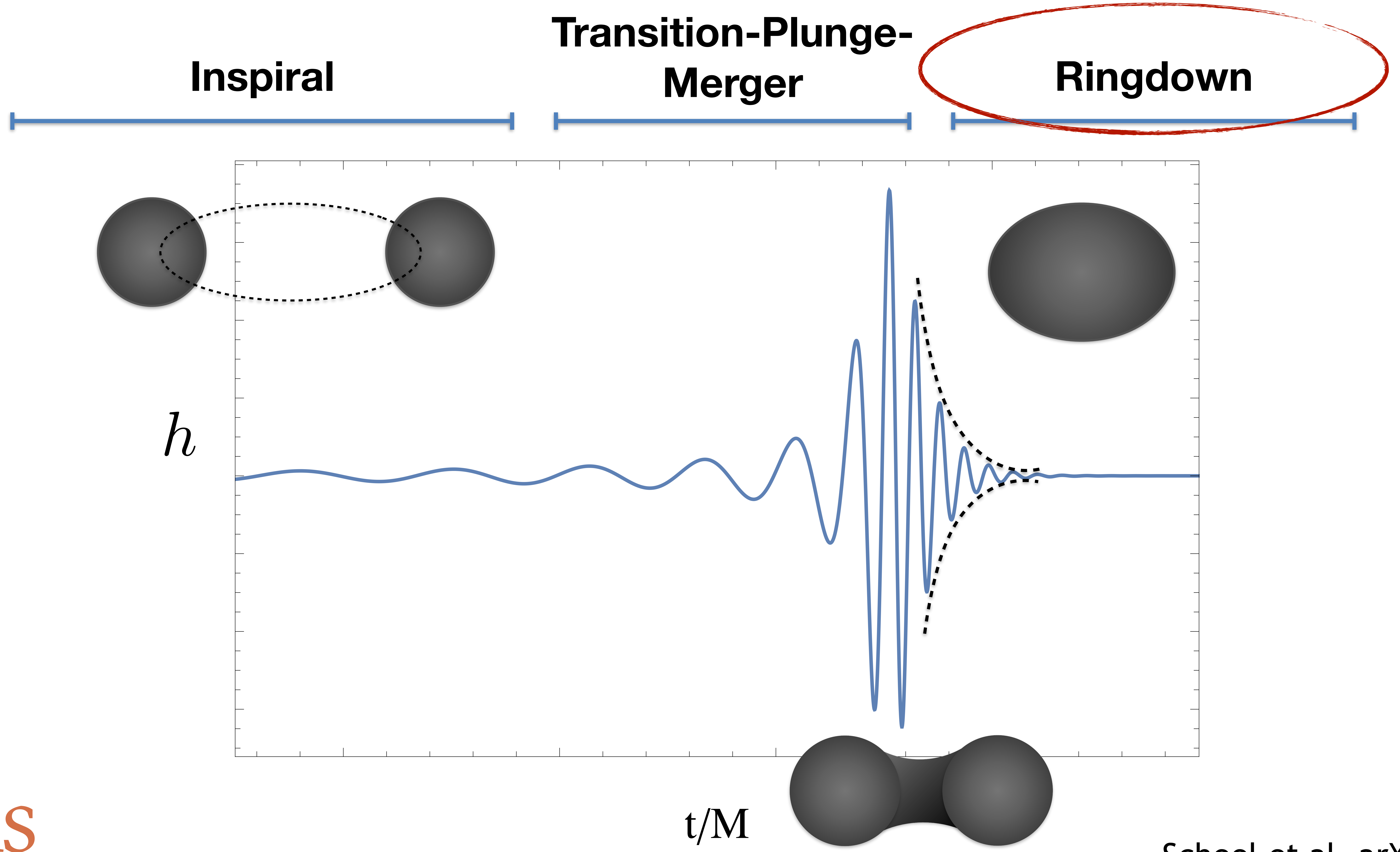

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Waves around black holes

$$\Box_g \Phi = 0$$

- Schw: separation of variables:

$$\Phi_{\omega lm} \sim e^{-i\omega t} \frac{u_{\omega lm}(r)}{r} Y_{lm}(\theta, \phi)$$

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- Radial wave equation

$$\frac{d^2 u_{\omega lm}}{dr_*^2} + \left(\omega^2 - V\right) u_{\omega lm} = 0$$

$$V = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$$

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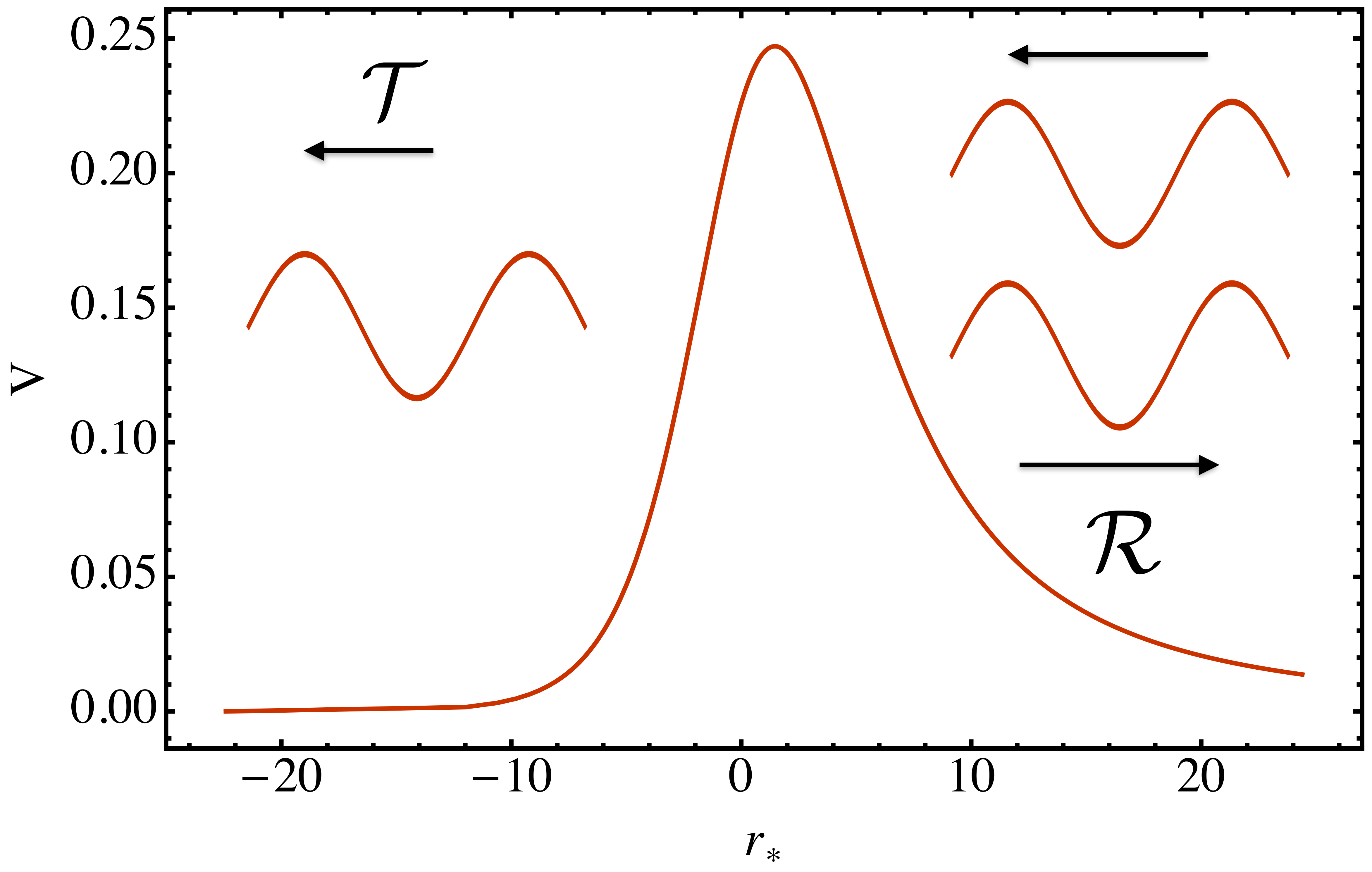

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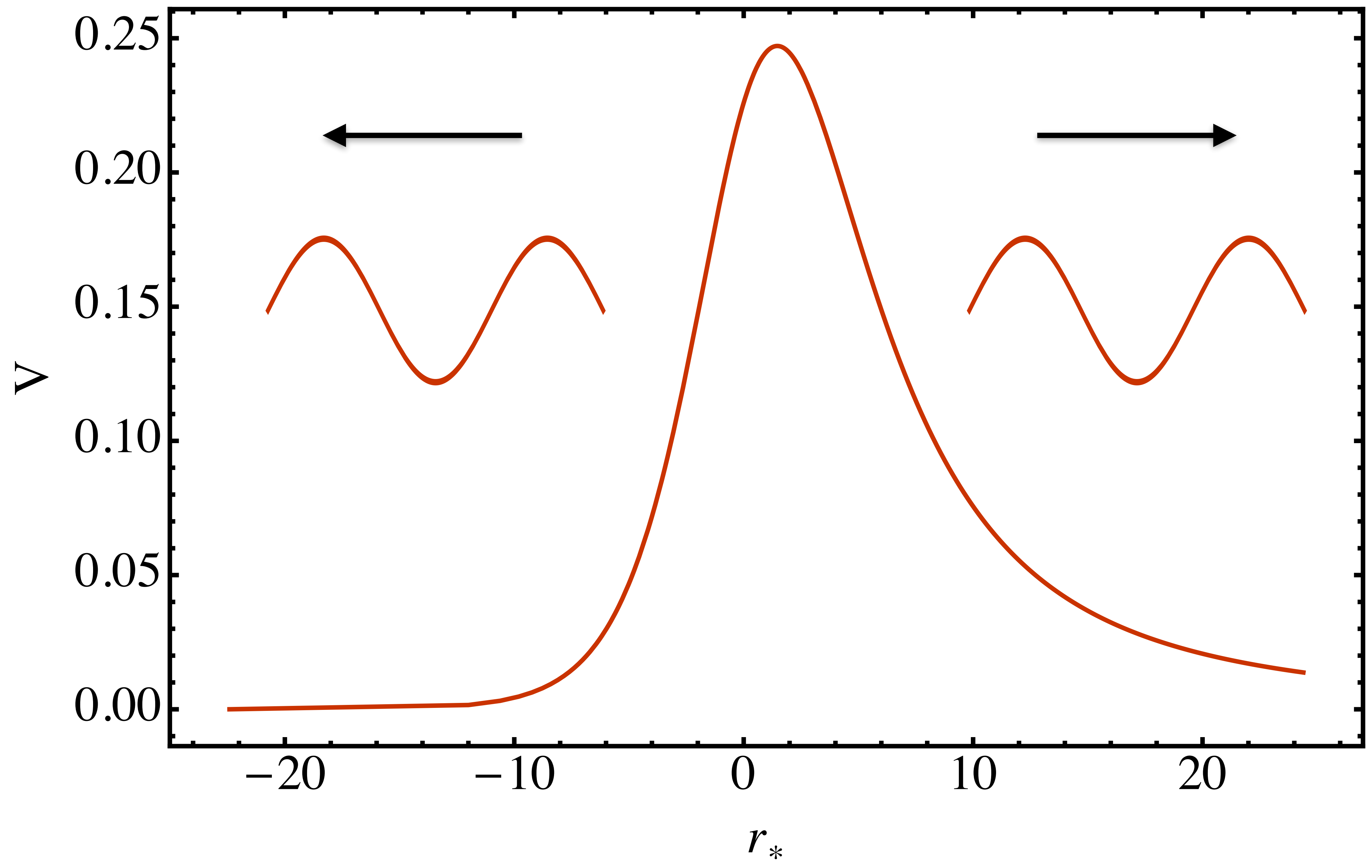
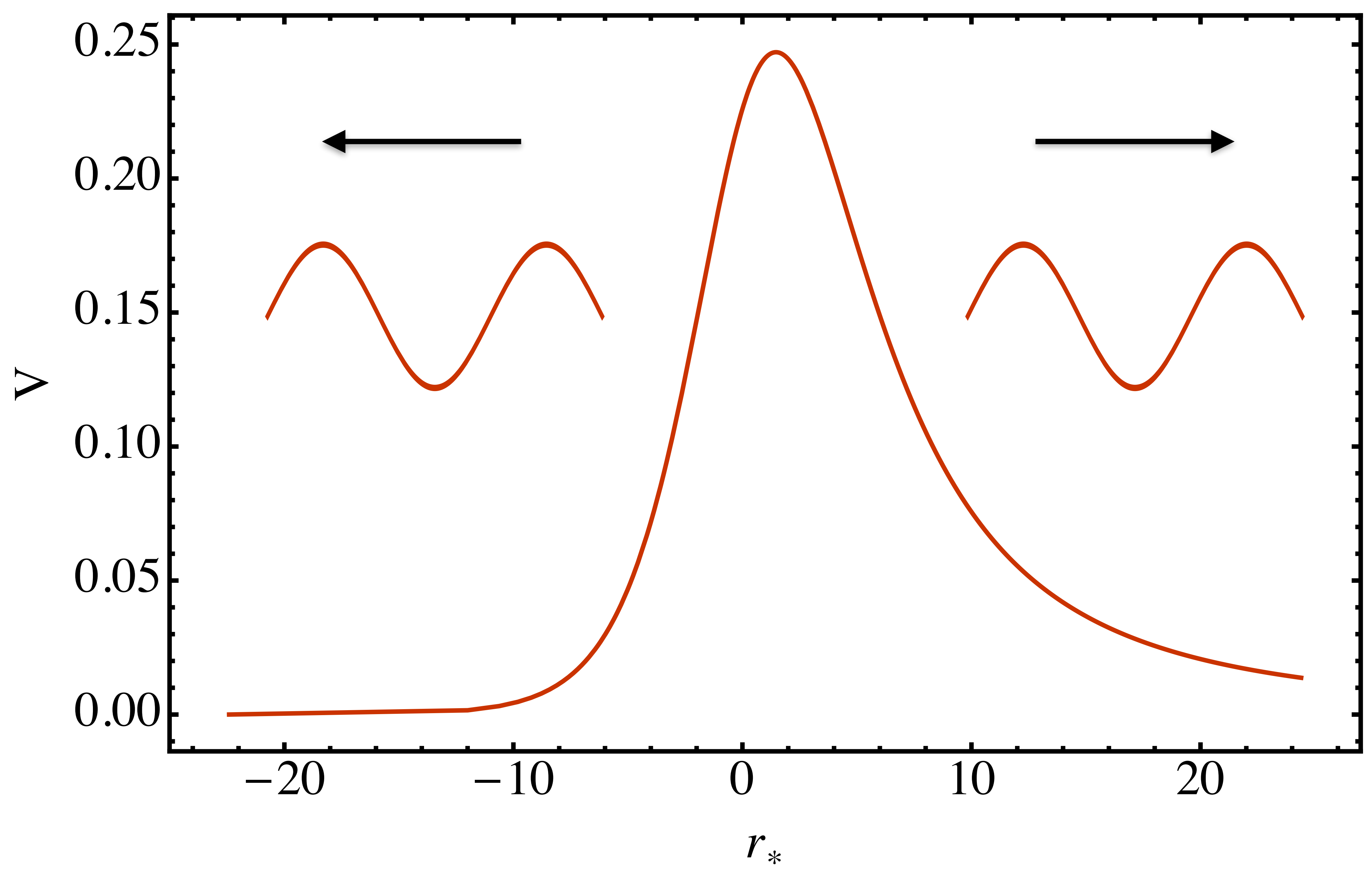
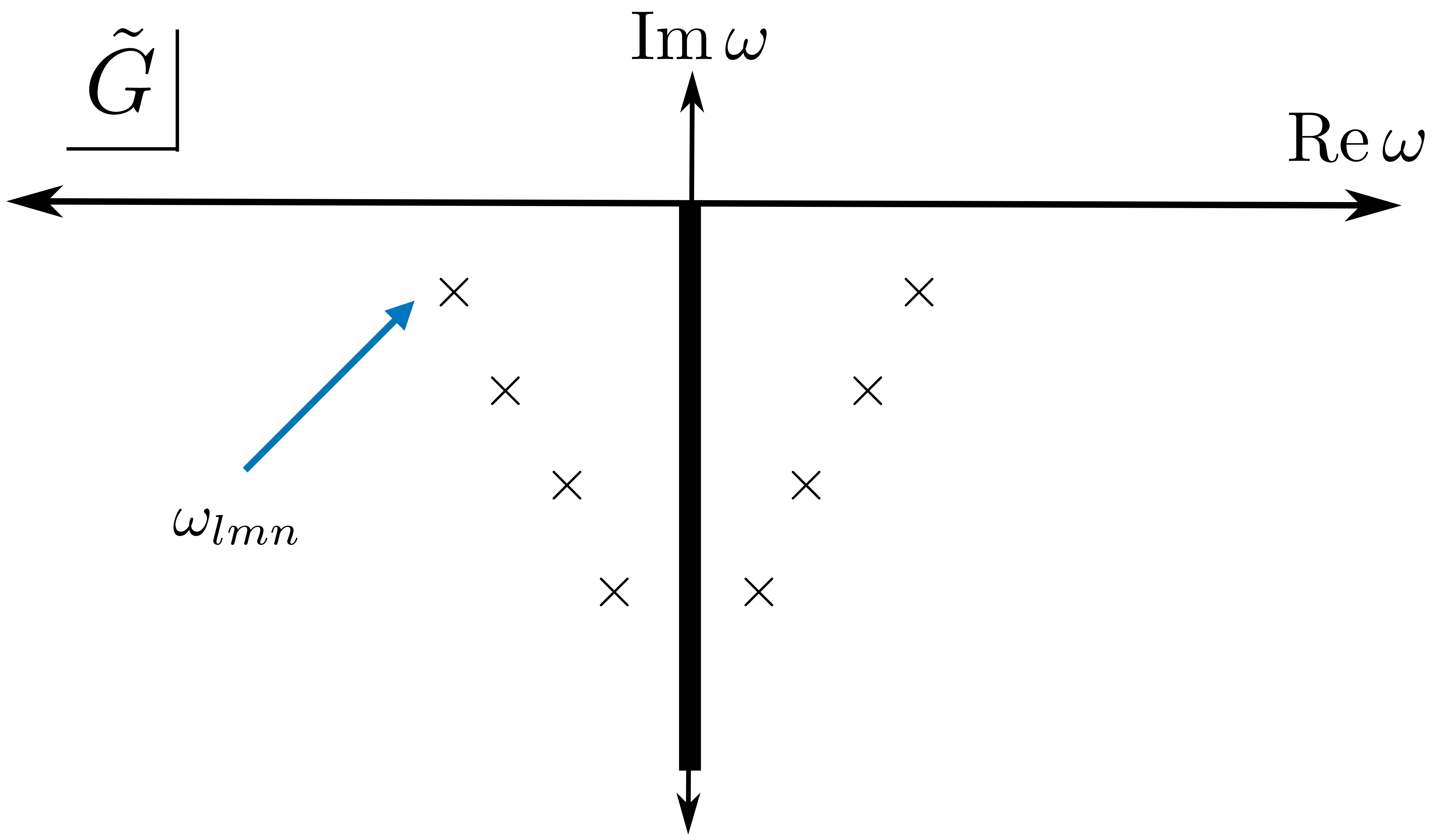
Quasinormal modes



Leaver 1985, 1986

Quasinormal modes





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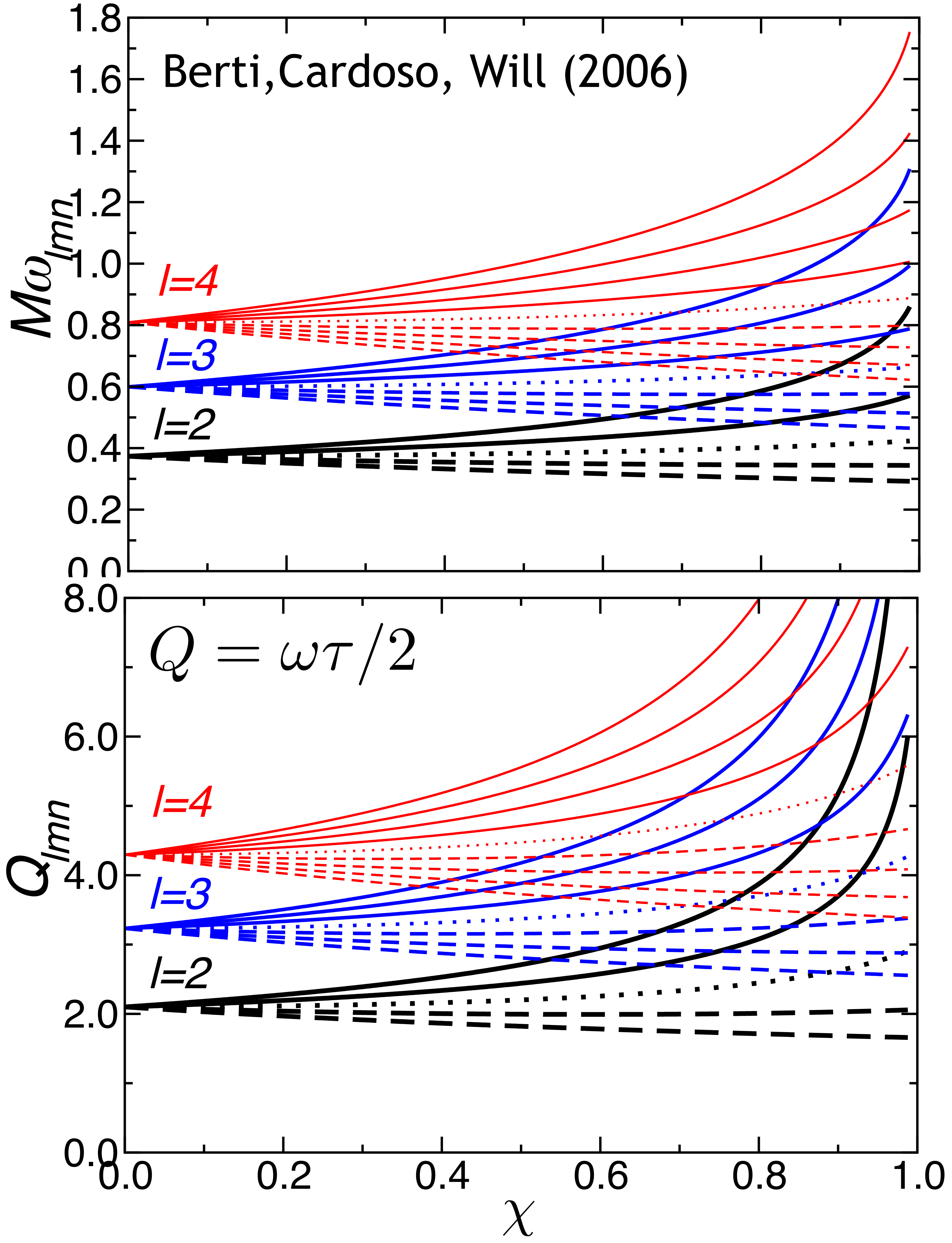
Leaver 1985, 1986

Black hole spectroscopy

- Spectra determined by mass and spin
- Mass sets overall frequency scale

$$f \approx 16 \left(\frac{M_\odot}{M}\right) \mathrm{kHz}$$

- Low quality oscillator: hard to measure ringdown
- One mode: mass and spin
- Two modes: clean test of Kerr spacetime



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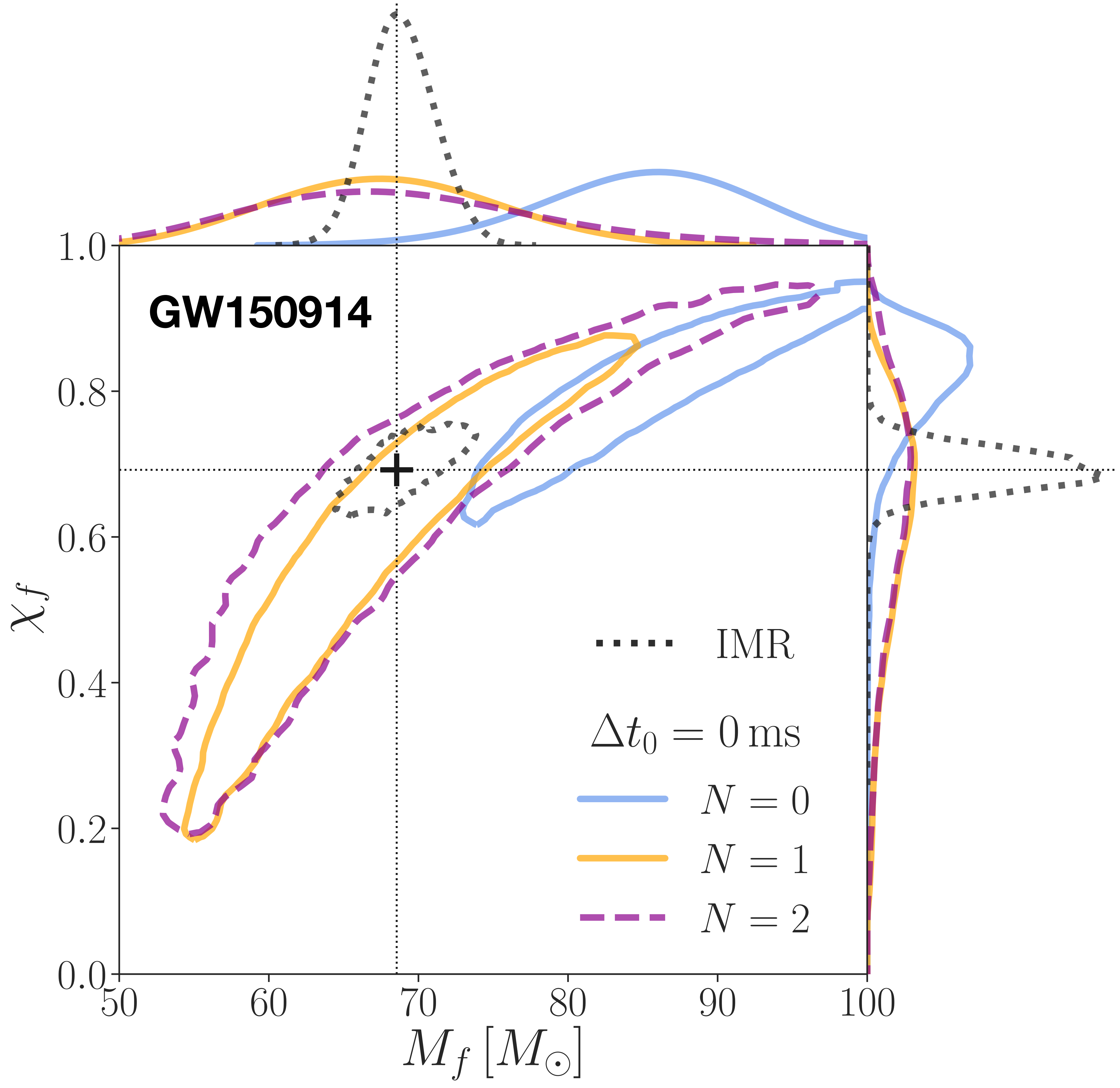
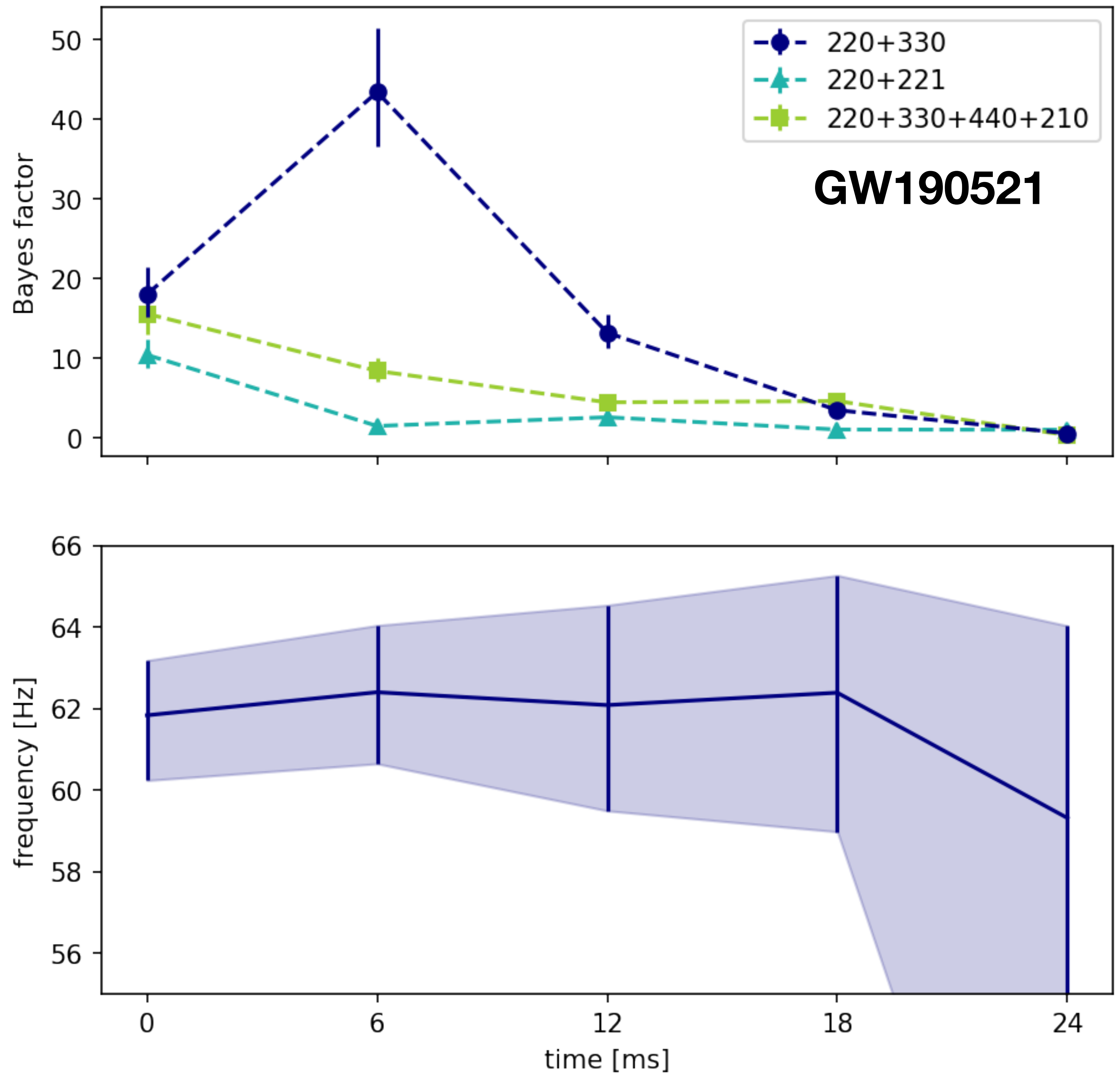
Multiple modes in ringdown





Isi, Gielser +, arXiv:1905.00869
c.f. Cotesta +, arXiv:2201.00822

Capano, Cabero +, arXiv:2105.05238
c.f. LVK arXiv:2010.14529

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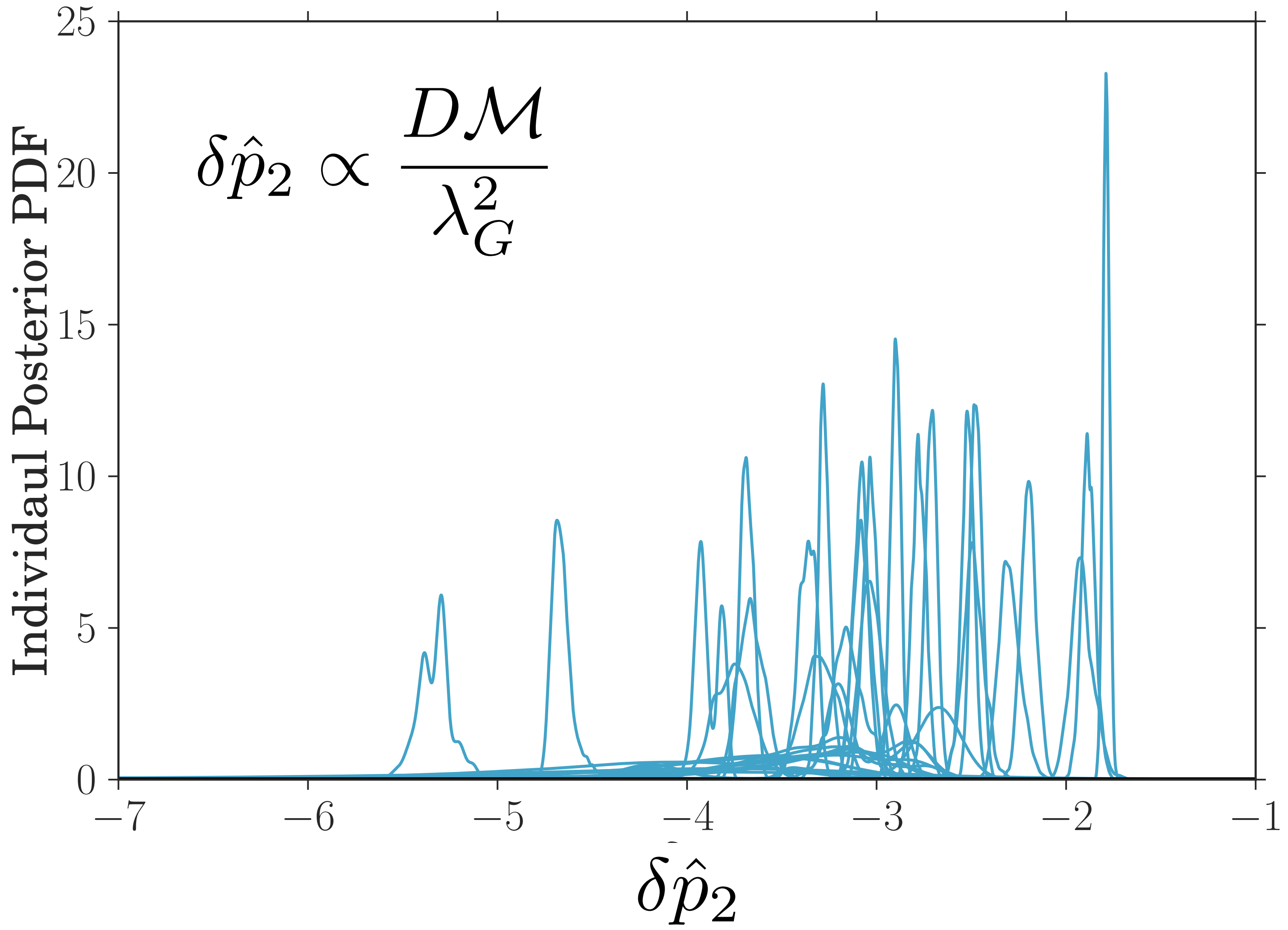
Constraining deviations

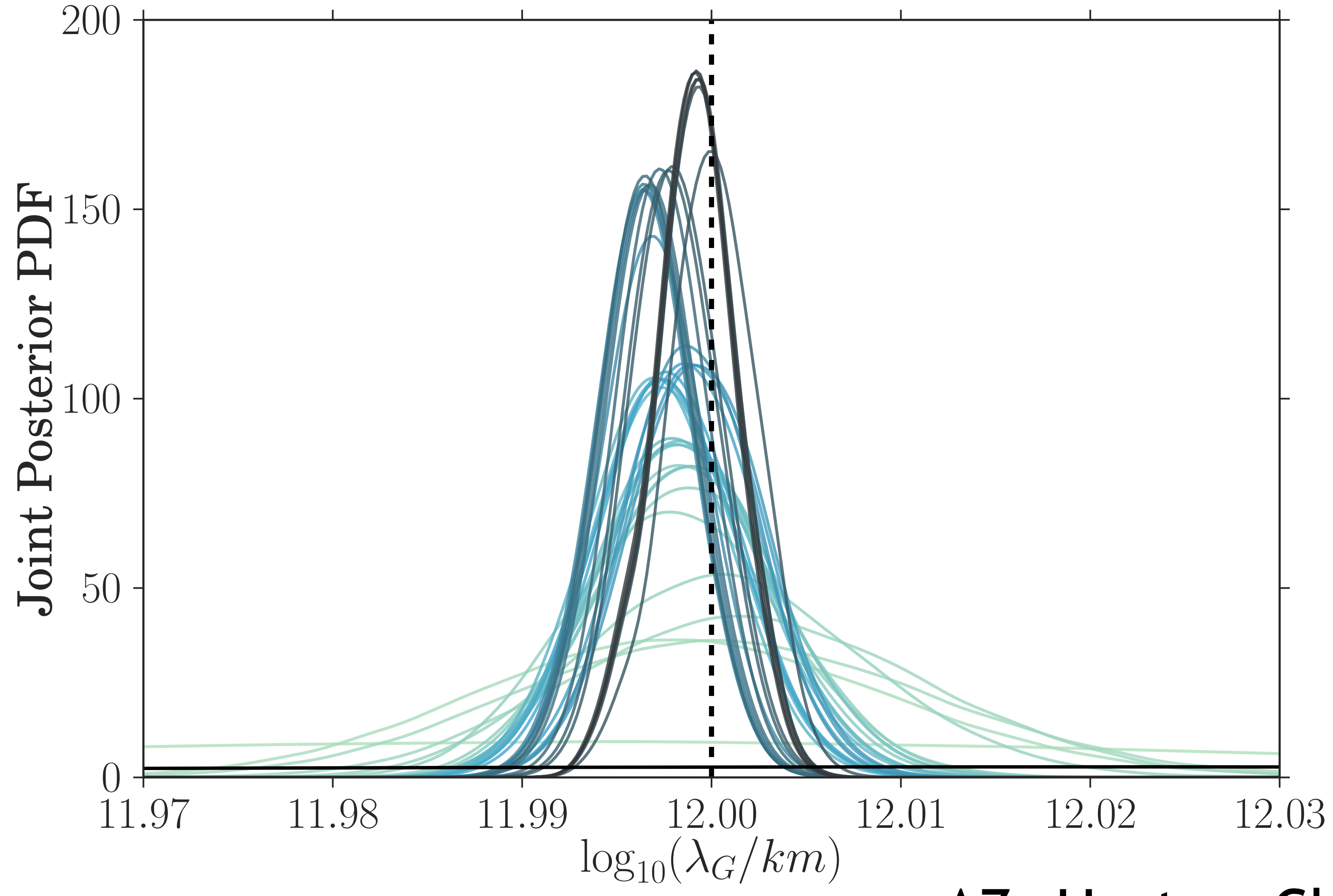
- Primarily null tests

$$h(t) = \sum A_{lmn} e^{-t/\tau_{lmn}} \cos(2\pi f_{lmn} t + \phi_{lmn})$$

$$f \rightarrow f(1 + \delta \hat{f})$$
$$\tau \rightarrow \tau(1 + \delta \hat{\tau})$$

- How to combine multiple constraints?
  - Need specific theory
  - Hierarchical analysis





AZ, Haster, Chatziioannou, PRD 99, 124044 (2019)

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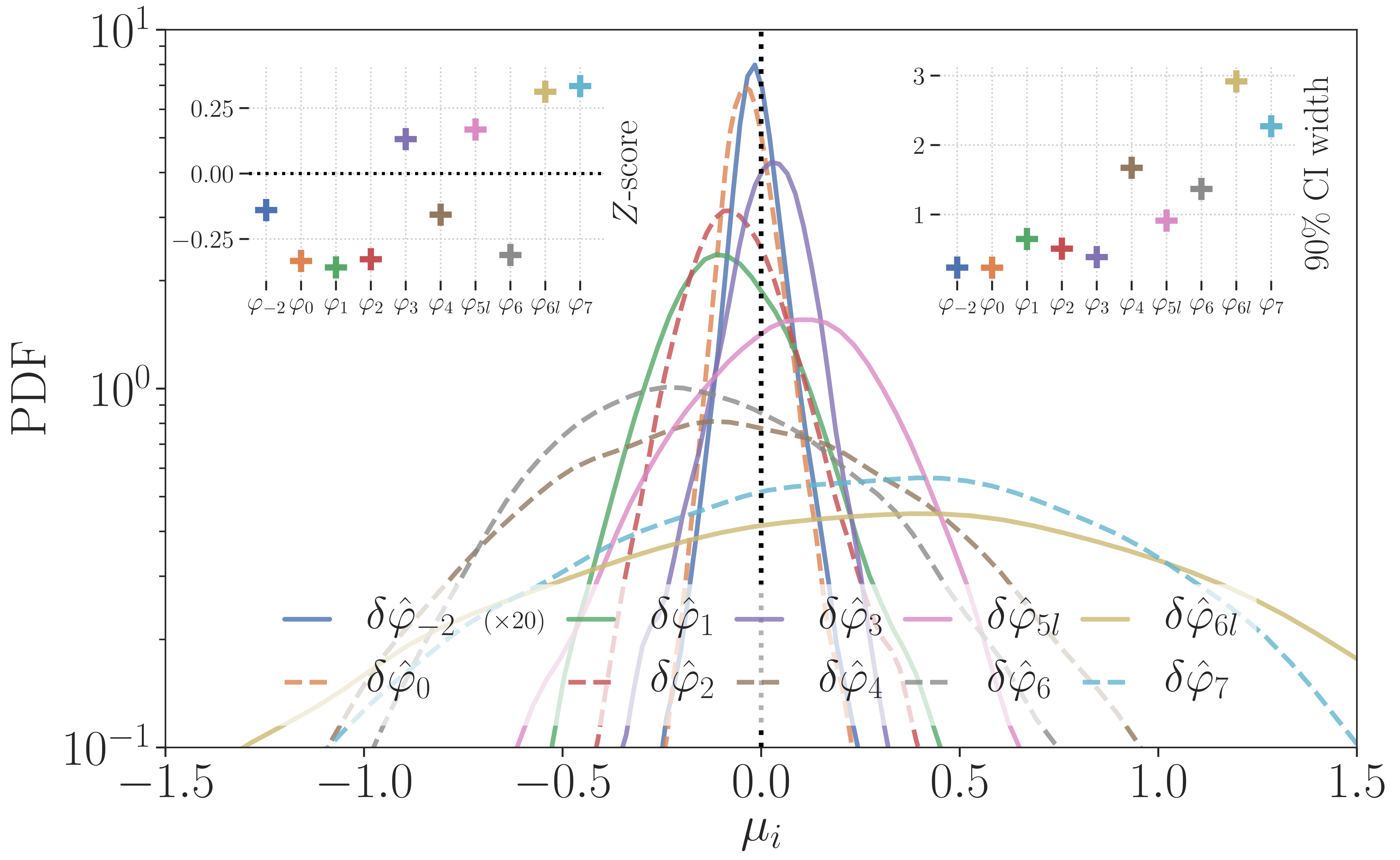
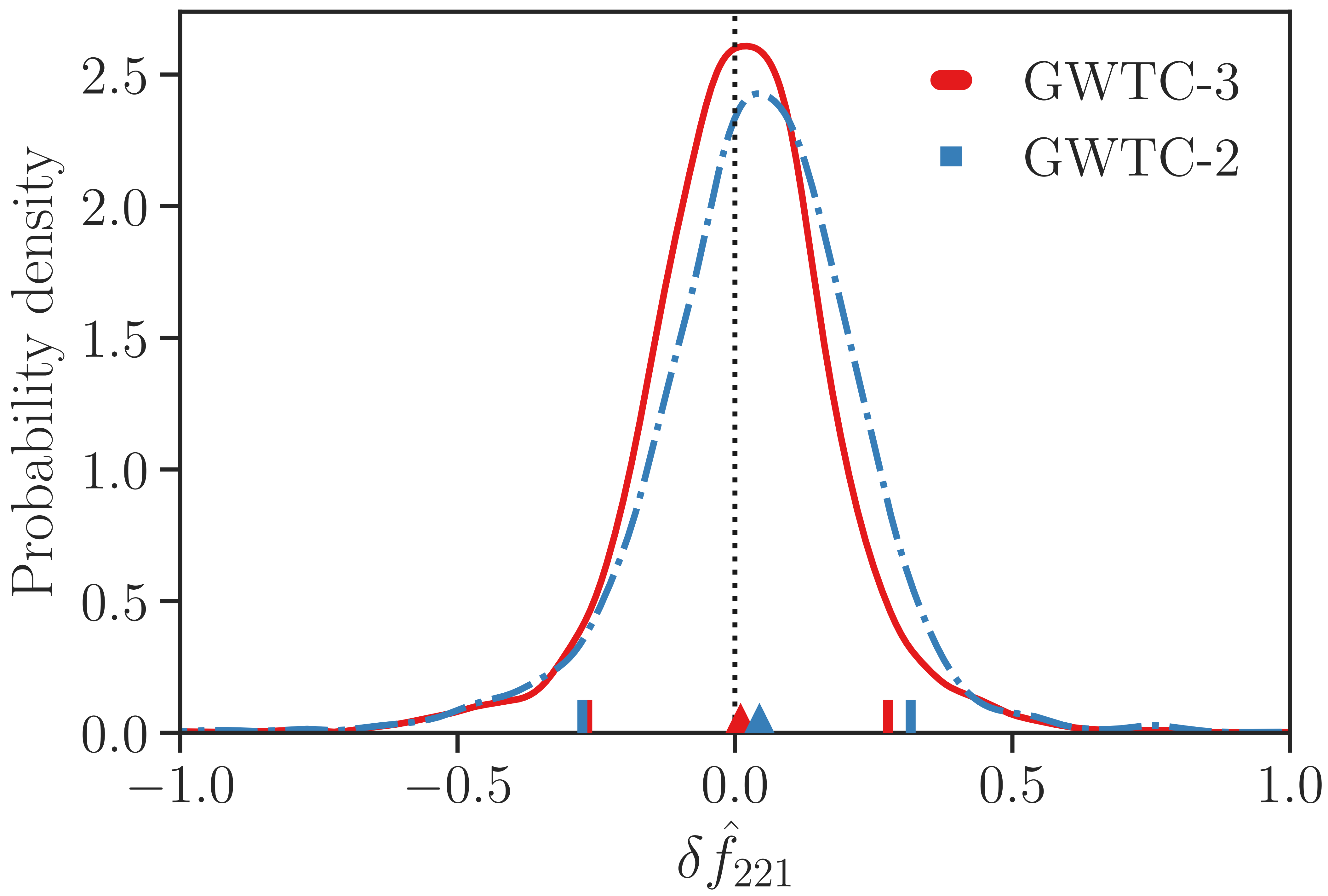


$$\delta\hat{\varphi}_i \sim \mathcal{N}(\mu_i, \sigma_i)$$

Isi, Chatziioannou, Farr,
PRL 123, 121101 (2019)

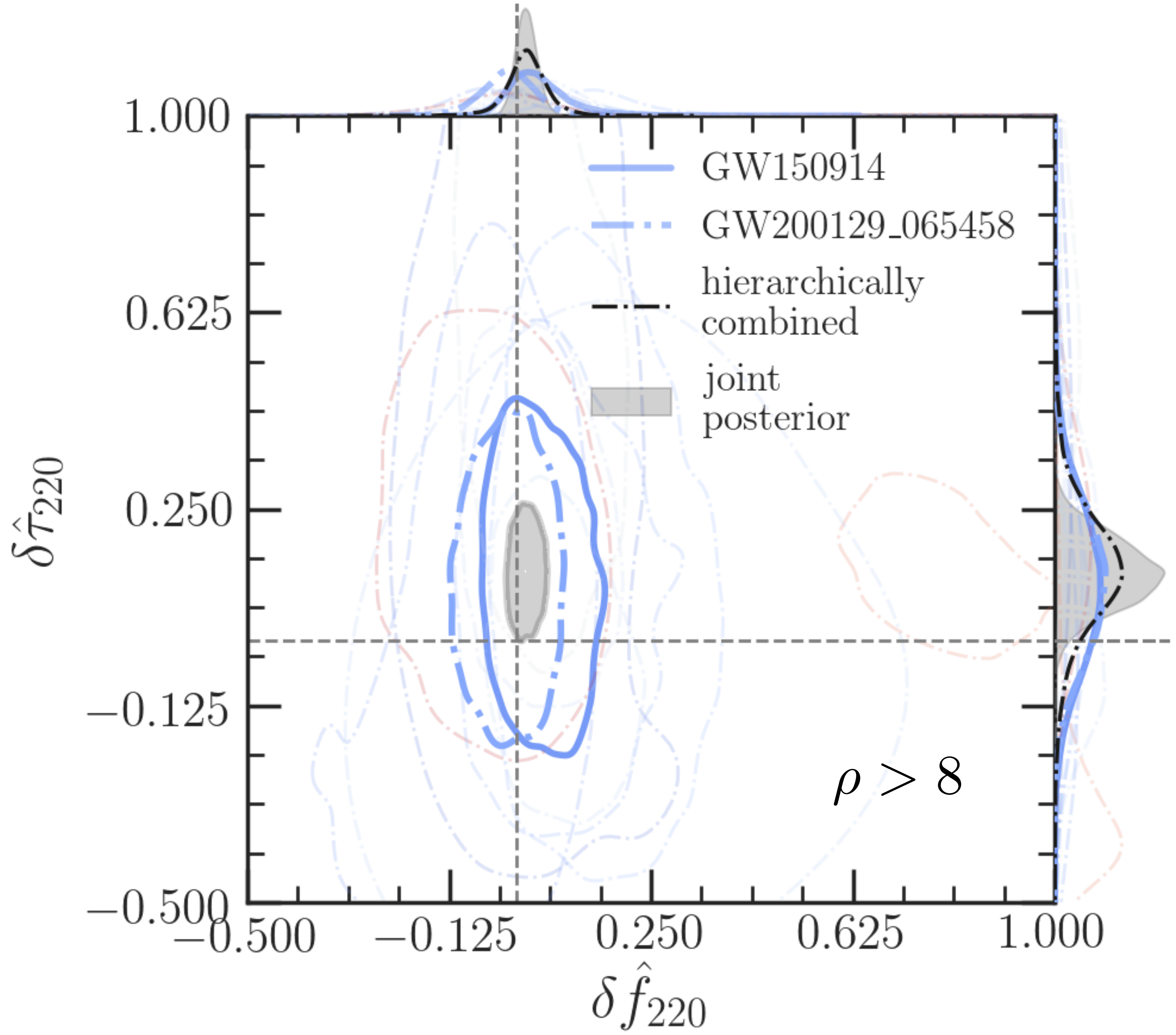
Ringdown tests from O3

Ringdown only



PyRING

Full waveform, no overtones



pSEOBNRv4HM

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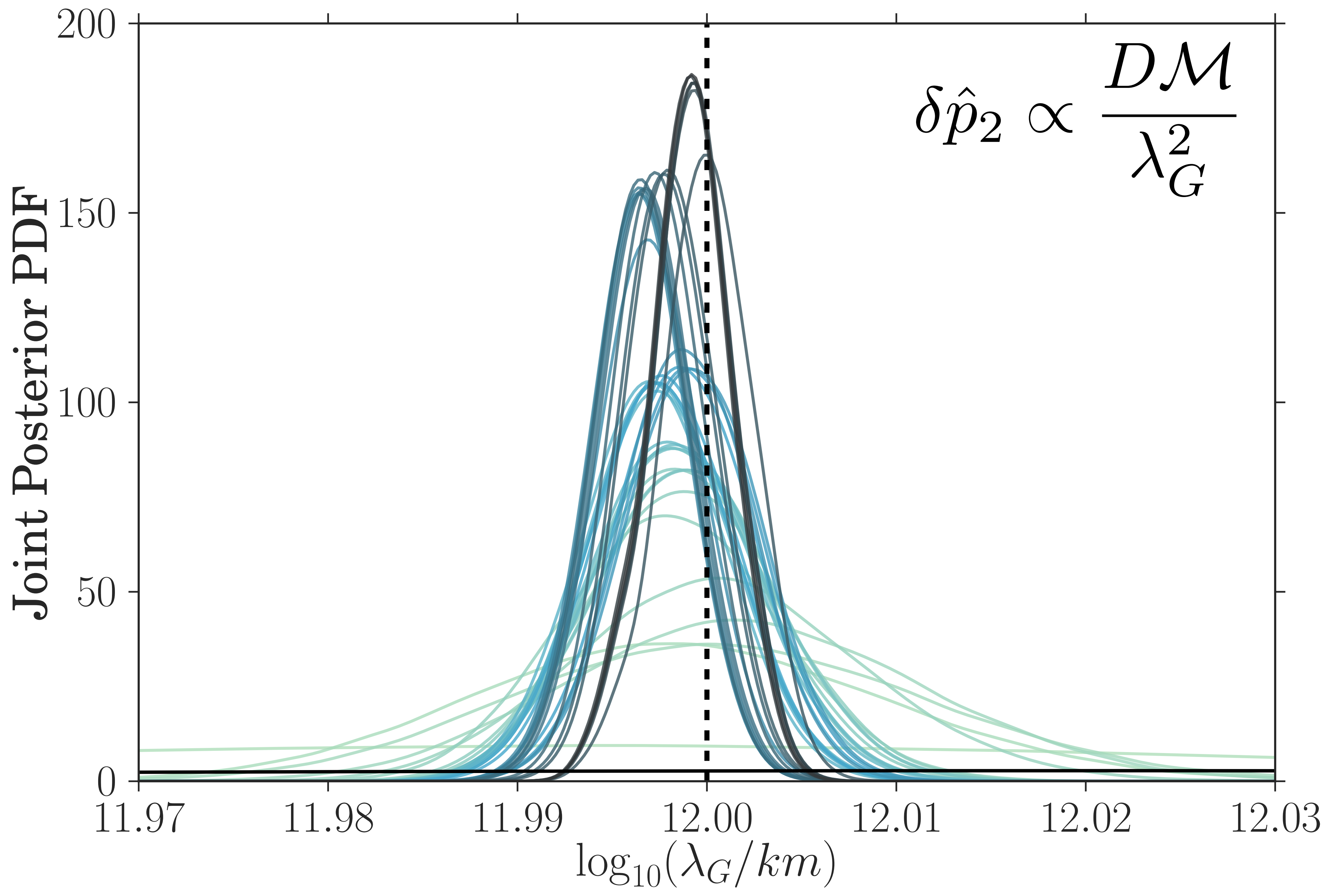
LVK arXiv:2112.06861

Towards precision tests

- Test specific theories
  - Constraints mapped to theory params
  - Incorporate higher harmonics and overtones
- Much work on QNMs beyond-GR, expansions in small spin
  - McManus et al. arXiv:1906.05155
  - Cano, Fransen, Hertog arXiv:2005.03671
- But merged black holes have $\chi \sim 0.7$



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Ringdown beyond Kerr

Gravitational perts for Kerr

- Metric perts don't separate or decouple in Kerr

$$G_{ab}(g) = \kappa_0 \eta T_{ab} \qquad\qquad g_{ab} = g^{(0)}_{ab} + \eta h_{ab}$$

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- Teukolsky (1973): Use Newman-Penrose eqns to decouple scalar quantites

$$\begin{array}{llcll} s=0: & \Phi & & \Phi & \\ s=\pm 1: & F_{\mu\nu} & \longrightarrow & \phi_0,\ \phi_2 & \longrightarrow \quad \mathcal{O}_s[\psi_s] = 4\pi T_s \\ s=\pm 2: & C_{\mu\nu\rho\sigma} & & \Psi_0,\ \Psi_4 & \end{array}$$

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Gravitational perts for Kerr

- Master eqn separates

$$\psi_{slm\omega} = e^{-i\omega t} e^{im\phi} R_{slm\omega}(r) S_{slm\omega}(\theta)$$

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$$\mathcal{S}_s^{ab} \mathcal{E}_{ab}[h] = \mathcal{O}_s[\psi_s]$$

- Metric can be reconstructed (in special gauges)

$$h_{ab}[\psi_s, \bar{\psi}_s]$$

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Black holes beyond GR

- Focus on theories which perturb off GR in decoupling limit

$$S = S_{EH} + \int d^4x \sqrt{-g}[\mathcal{L}_\vartheta + \epsilon \mathcal{L}_{\rm int} + \mathcal{L}_{\rm matter}]$$

Hussain, AZ arXiv: 2206.10653

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$$\longrightarrow \vartheta_A = 0 + \epsilon \vartheta^{(1)}_A \longrightarrow g_{ab} = g^{(0)}_{ab} + \epsilon^2 g^{(2)}_{ab}$$

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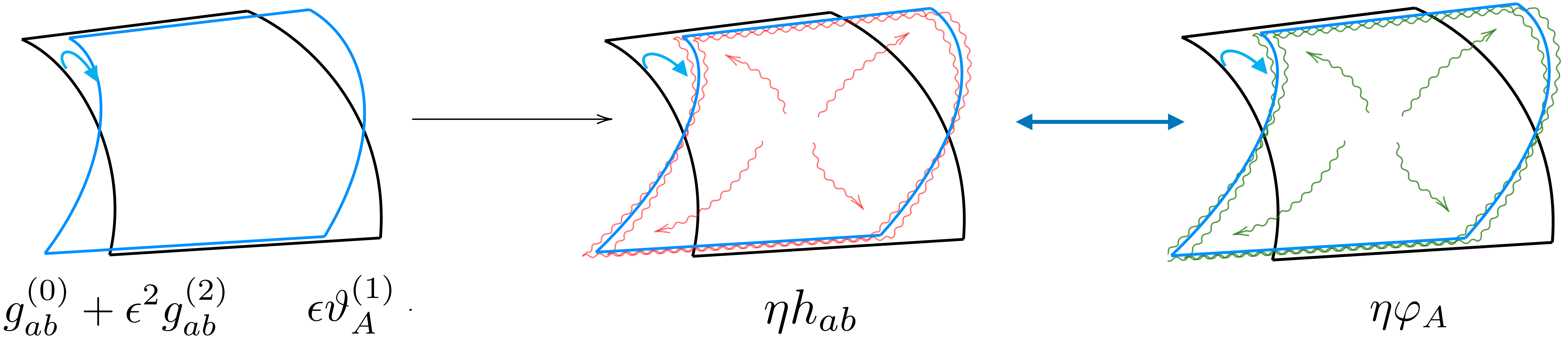
Hussain, AZ arXiv: 2206.10653

Perturbed black holes beyond Kerr

- Now add dynamical perturbations to all fields

$$\vartheta_A = \epsilon\vartheta_A^{(1)} + \eta\varphi_A + \ldots \qquad g_{ab} = g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)} + \eta h_{ab} + \ldots$$



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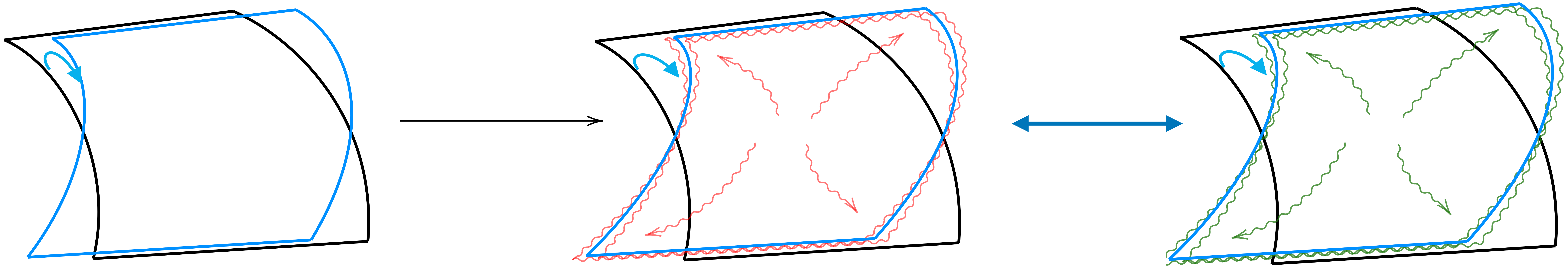

$g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)}$ $\quad$ $\epsilon \vartheta_A^{(1)}$ $\qquad\qquad$ ηh_{ab} $\qquad\qquad$ $\eta \varphi_A$

- Preferred basis partially decouples: $\quad h_{ab} = h_{ab}^{(0)} + \epsilon^2 h_{ab}^{(2)} \qquad \varphi = 0 + \epsilon \varphi_A^{(1)}$

Hussain, AZ arXiv: 2206.10653

Modified Teukolsky equation

- First solve $\varphi_A^{(1)}[h^{(0)}] = \mathcal{W}_A^{-1}[h^{(0)}]$
- Deriving modified Teukolsky equation very involved

$$\mathcal{E}_{ab}[h] + \epsilon^2(\delta\mathcal{E}_{ab}[h] - \delta T^{\vartheta}_{ab}[h] + C_{ab}[h])$$

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See also Li + arXiv: 2206.10652

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- View as perturbed eigenvalue problem

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Eigenvalue perturbations

- For a spacetime deformed from Kerr, can apply perturbative approach

$$H|n\rangle = E_n|n\rangle \to (H + \delta H)|n\rangle = (E_n + \delta E_n|n\rangle)$$

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Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ +, arXiv:1406.4206

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- Scalar wave equation straightforward: $g_{ab} = g_{ab}^{(0)} + \epsilon g_{ab}^{(1)}$

$$\Box_{g^{(0)}+\epsilon g^{(1)}}\Phi = [\Box^{(0)} + \epsilon\,\delta\Box]\Phi$$

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$$H|n\rangle = E_n|n\rangle \to (H+\delta H)|n\rangle = (E_n + \delta E_n|n\rangle)$$

$$\langle n^{(0)}|H|n^{(1)}\rangle = E_n\langle n^{(0)}|n^{(1)}\rangle \longrightarrow \delta E_n = \frac{\langle n^{(0)}|\delta H|n^{(0)}\rangle}{\langle n^{(0)}|n^{(0)}\rangle}$$

- Scalar wave equation straightforward: $g_{ab} = g_{ab}^{(0)} + \epsilon g_{ab}^{(1)}$

$$\Box_{g^{(0)}+\epsilon g^{(1)}}\Phi = [\Box^{(0)} + \epsilon\,\delta\Box]\Phi \qquad \delta\omega = -\frac{\langle\Phi^{(0)}|\delta\Box|\Phi^{(0)}\rangle}{\langle\Phi^{(0)}|\partial_\omega\Box^{(0)}|\Phi^{(0)}\rangle}$$

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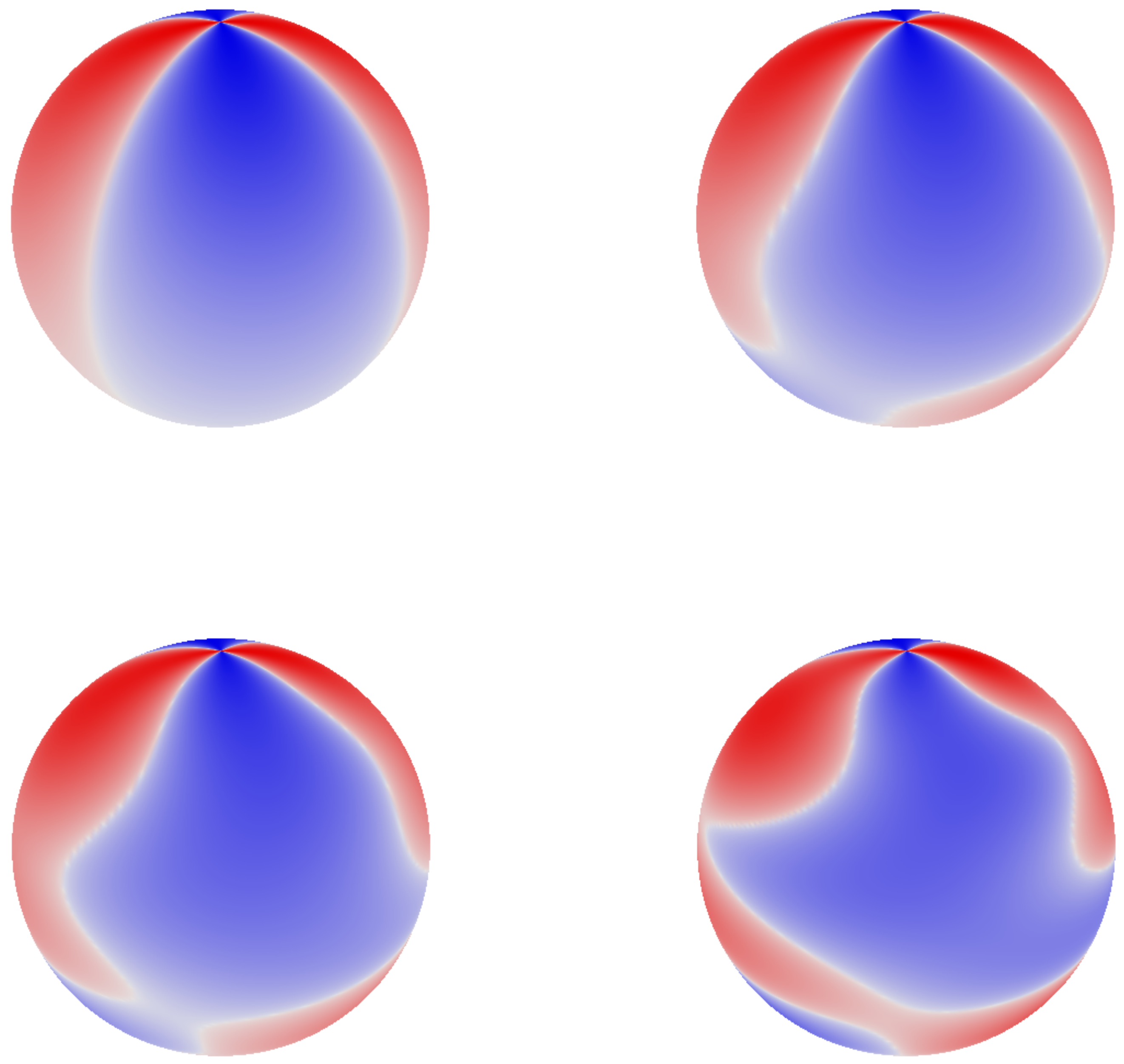
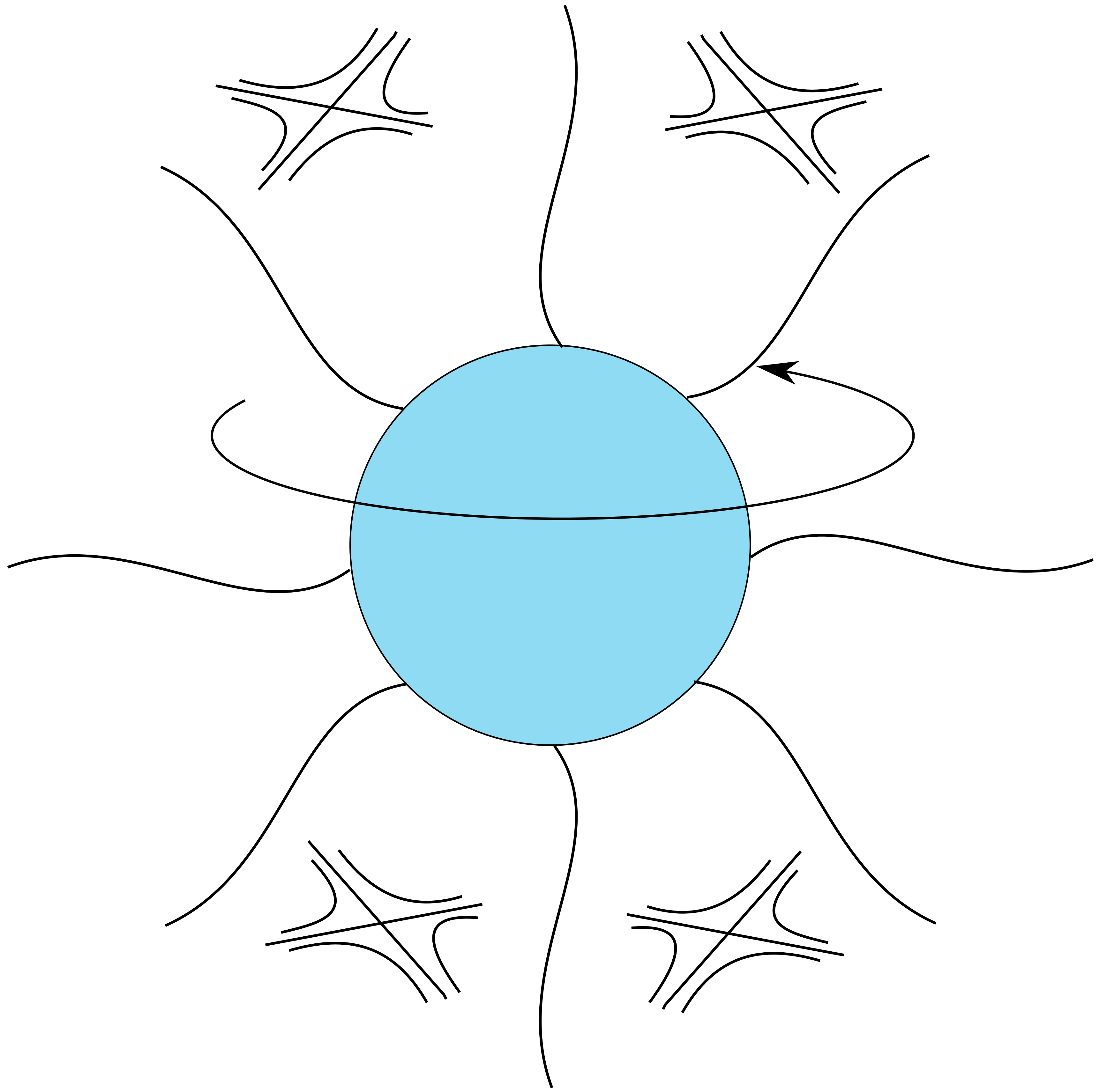
Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ +, arXiv:1406.4206

Eigenvalue perturbations



Transient "turbulence" of scalar perts



Weakly charged Kerr-Newman

Yang, AZ, Lehner, arXiv:1402.4859

Mark, Yang, AZ, Chen, arXiv:1409.5800

Roadmap

$\mathcal{L}_\vartheta + \epsilon\mathcal{L}_{\rm int}$

Choose theory

$\vartheta_A^{(1)} \qquad g_{ab}^{(2)}$

Stationary deformation

$\psi_{slmn}^{(0)}$

Choose mode

$h_{ab}^{(0)}[\psi_s, \bar{\psi}_s]$

Reconstruct metric

$\varphi_A^{(1)}[h]$

Coupled dynamical fields

$\mathcal{O}[\psi_s] + \epsilon^2\mathcal{V}[h] + \epsilon^2\mathcal{C}[h]$

Modified Teukolsky eqn

$\omega_{lmn}^{(2)}$

EVP

Ringdown inference

New physics?

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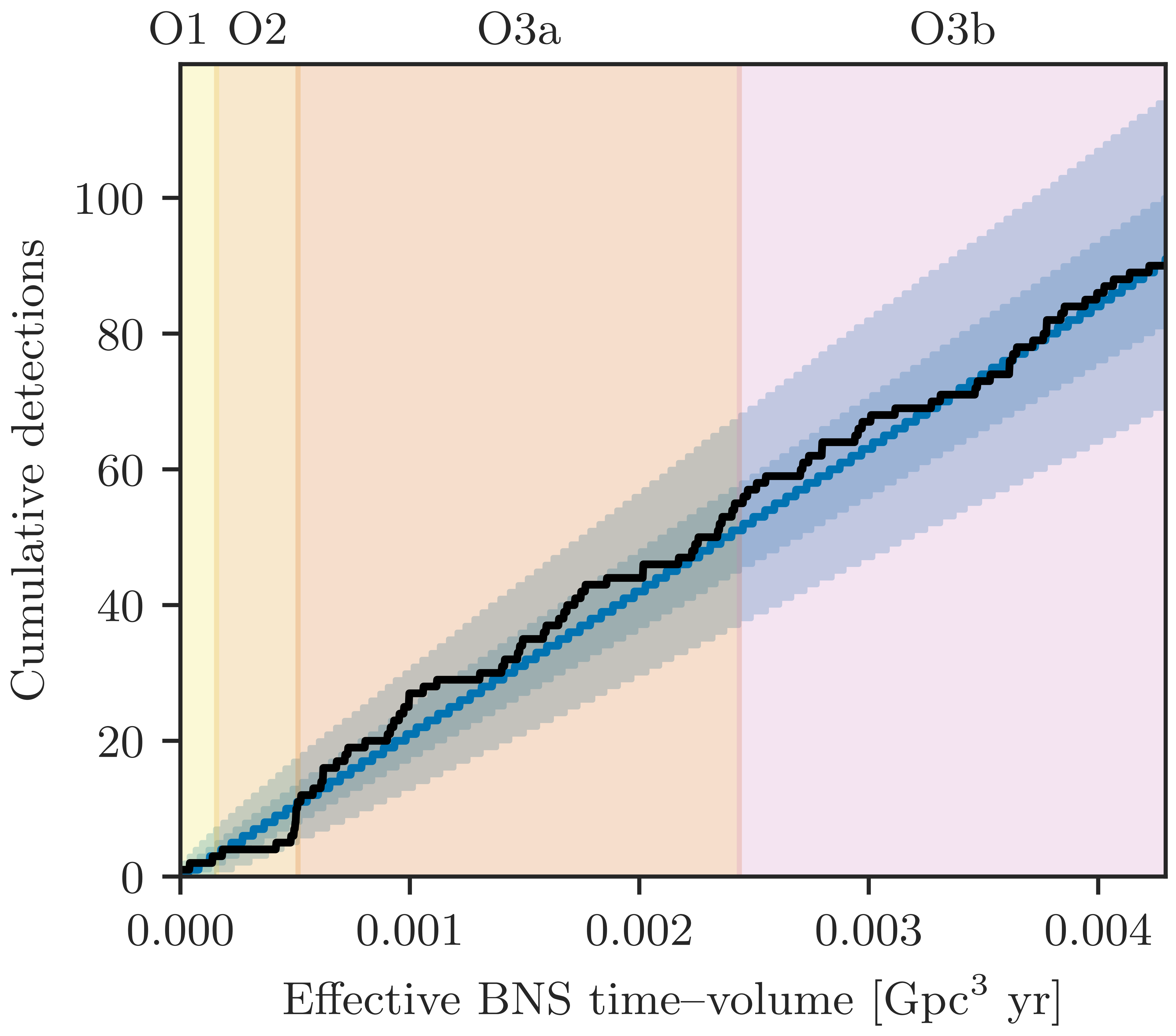
Summary and future

- Predicting QNMs allow for multi-mode ringdown tests of Kerr
  - Modified Teukolsky eqn
  - EVP method: allows for high spins
  - Several challenges ahead in implementation
- Many detections in the coming years
  - Combine constraints
- 3rd gen and LISA: precision predictions needed



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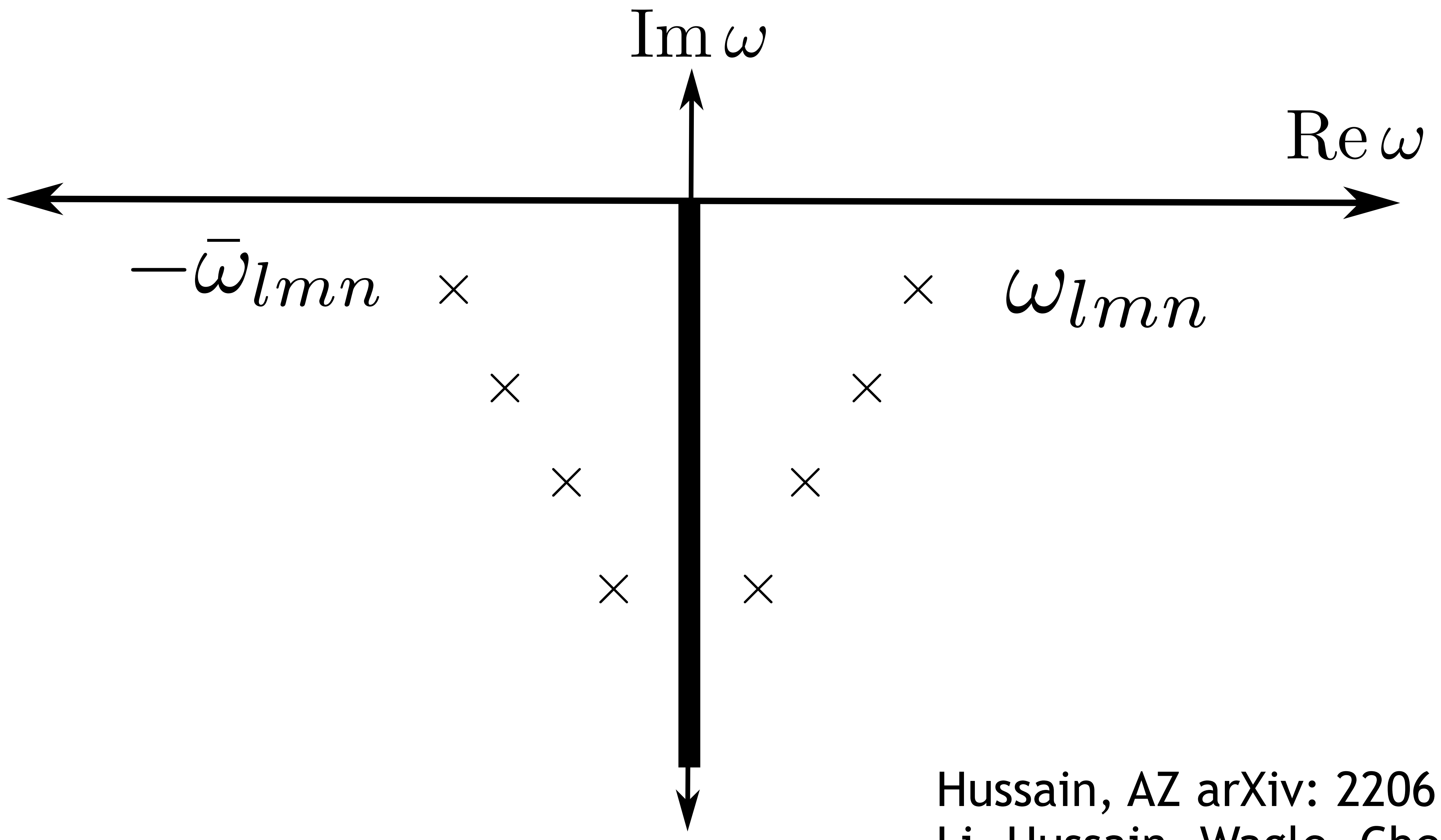
LVK arXiv:2111.03606

Extras

Breaking isospectrality

- One conceptual issue: metric reconstruction couples ψ_s and $\bar{\psi}_s$
- Couples two families of modes: ω_{lmn} and $-\bar{\omega}_{lmn}$
  - Equality of modes: even and odd parity modes have same spectrum (Nichols et al. 2012)



- Really degenerate perturbation theory

$$\omega^{(2)}_{\text{even}} \neq \omega^{(2)}_{\text{odd}}$$

- Ongoing work on parity breaking: Li et al.

Hussain, AZ arXiv: 2206.10653
Li, Hussain, Wagle, Chen, Yunes, AZ arXiv:2306.XXXX

Degenerate EVP

- Formally write metric reconstruction as

$$h_{ab}^{(0)} = \mathcal{K}_{ab}[\psi] + \bar{\mathcal{K}}_{ab}[\bar{\psi}] \qquad \mathcal{V}[h] = \mathcal{V}\mathcal{K}[\psi] + \mathcal{V}\bar{\mathcal{K}}[\bar{\psi}]$$

- Consider superposition of states that don't mix

$$\psi = \psi_+ + \alpha\psi_-$$

- Apply EVP approach

$$\omega_+^{(2)} = -\frac{\langle\psi_+|(\mathcal{V}+\mathcal{C})\mathcal{K}|\psi_+\rangle + \alpha\langle\psi_+|(\mathcal{V}+\mathcal{C})\bar{\mathcal{K}}|\bar{\psi}_-\rangle}{\langle\psi_+|\partial_\omega\mathcal{O}|\psi_+\rangle}$$

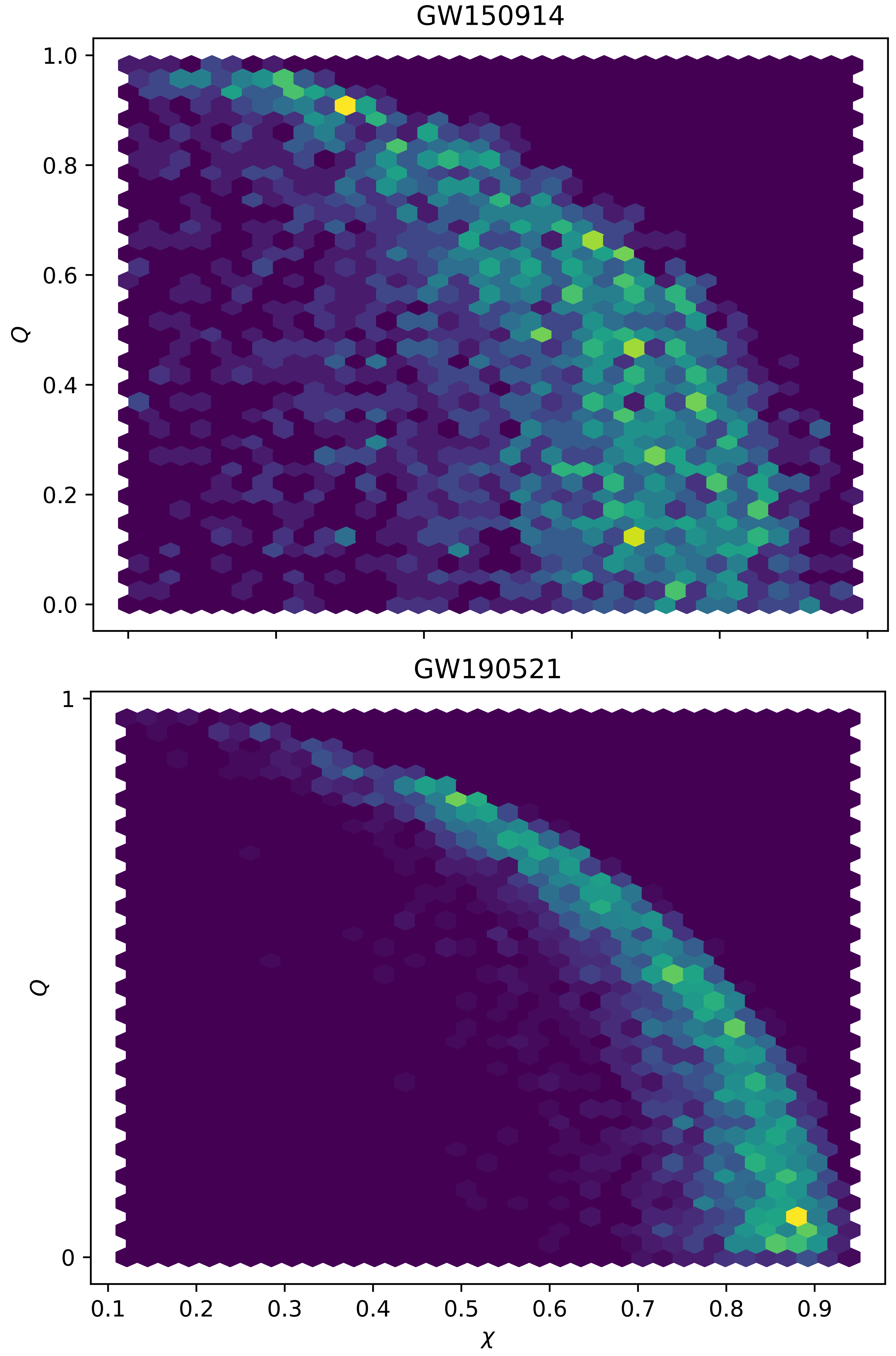
Hussain, AZ arXiv: 2206.10653
Li, Hussain, Wagle, Chen, Yunes, AZ arXiv:2306.XXXX

Combining events

- Beyond-GR parameter common to all events
- Beyond-GR parameter varies
  - Need population modeling (hierarchical modeling) to combine events
  - Modeling needs to account for degeneracies

$$p(\vec{\theta}) \to p(\vec{\theta}|\vec{\Lambda})p(\vec{\Lambda})$$

- Example: charged black holes
  - Use `ringdown` package (Isi, Farr)
  - Use multiple tones, infer $M,\ \chi,\ Q$
  - Start from peak of full IMR waveform



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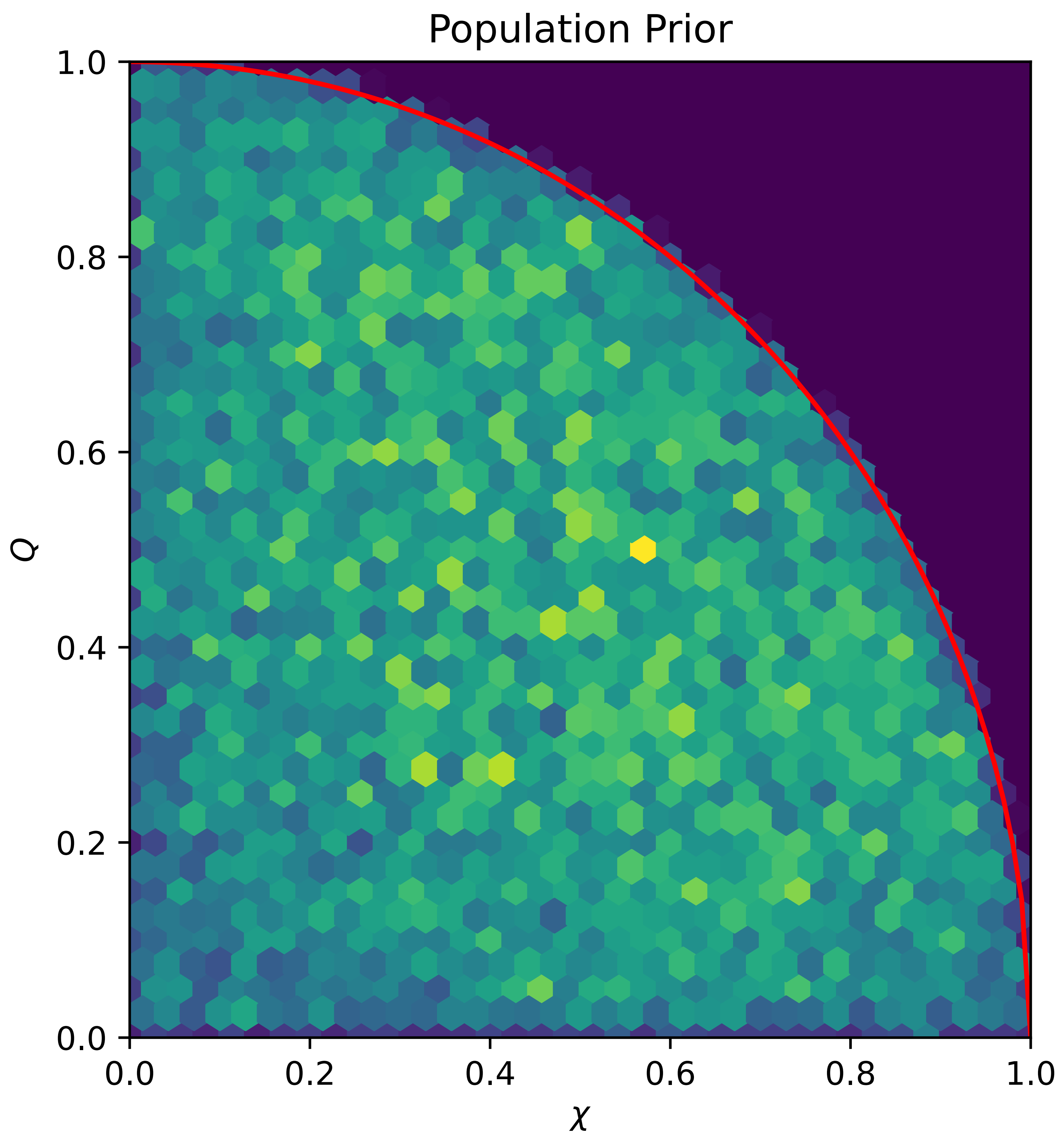
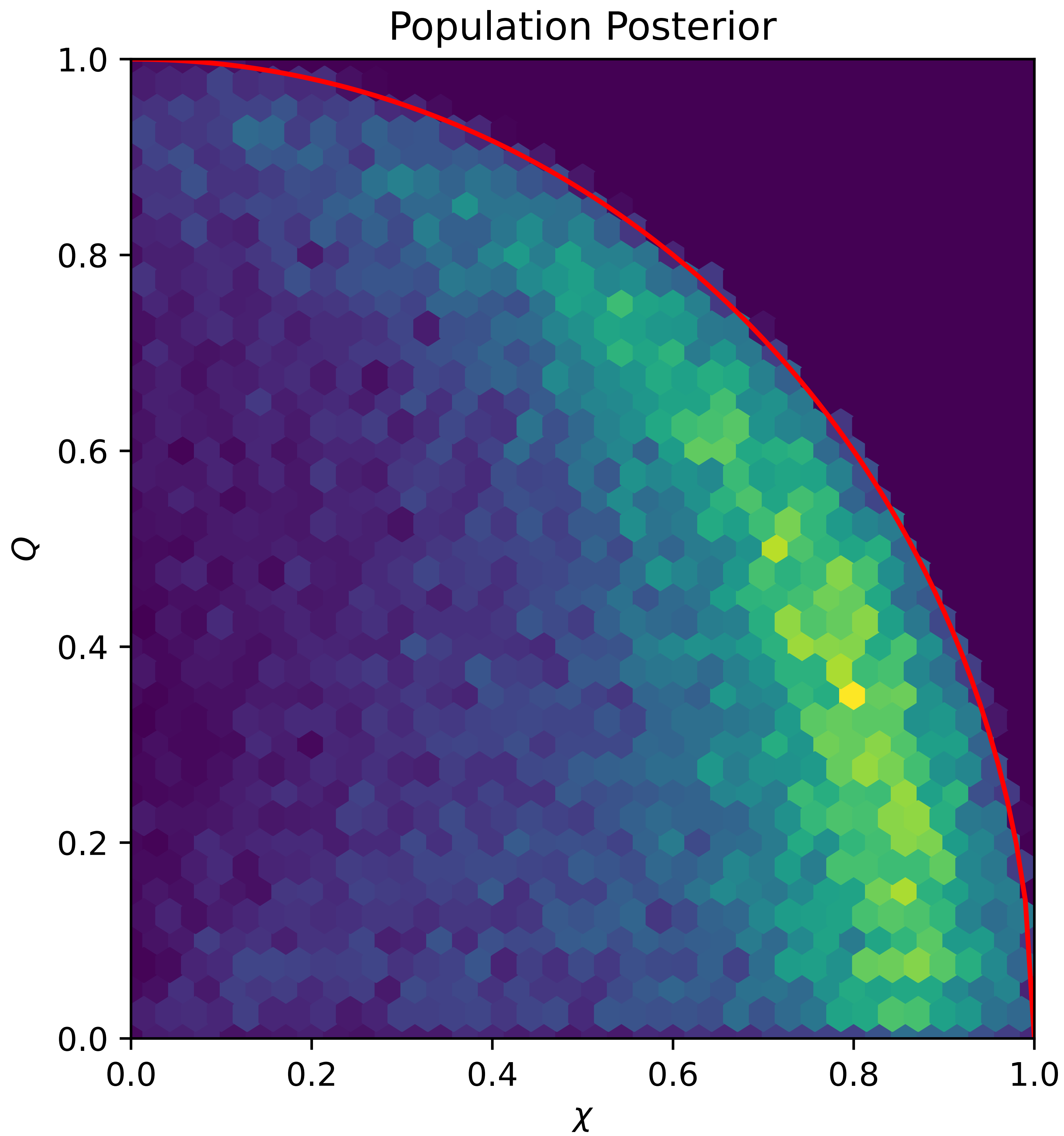
Hussain, Isi, AZ in prep

Example: Charged BHs





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Hussain, Isi, AZ in prep

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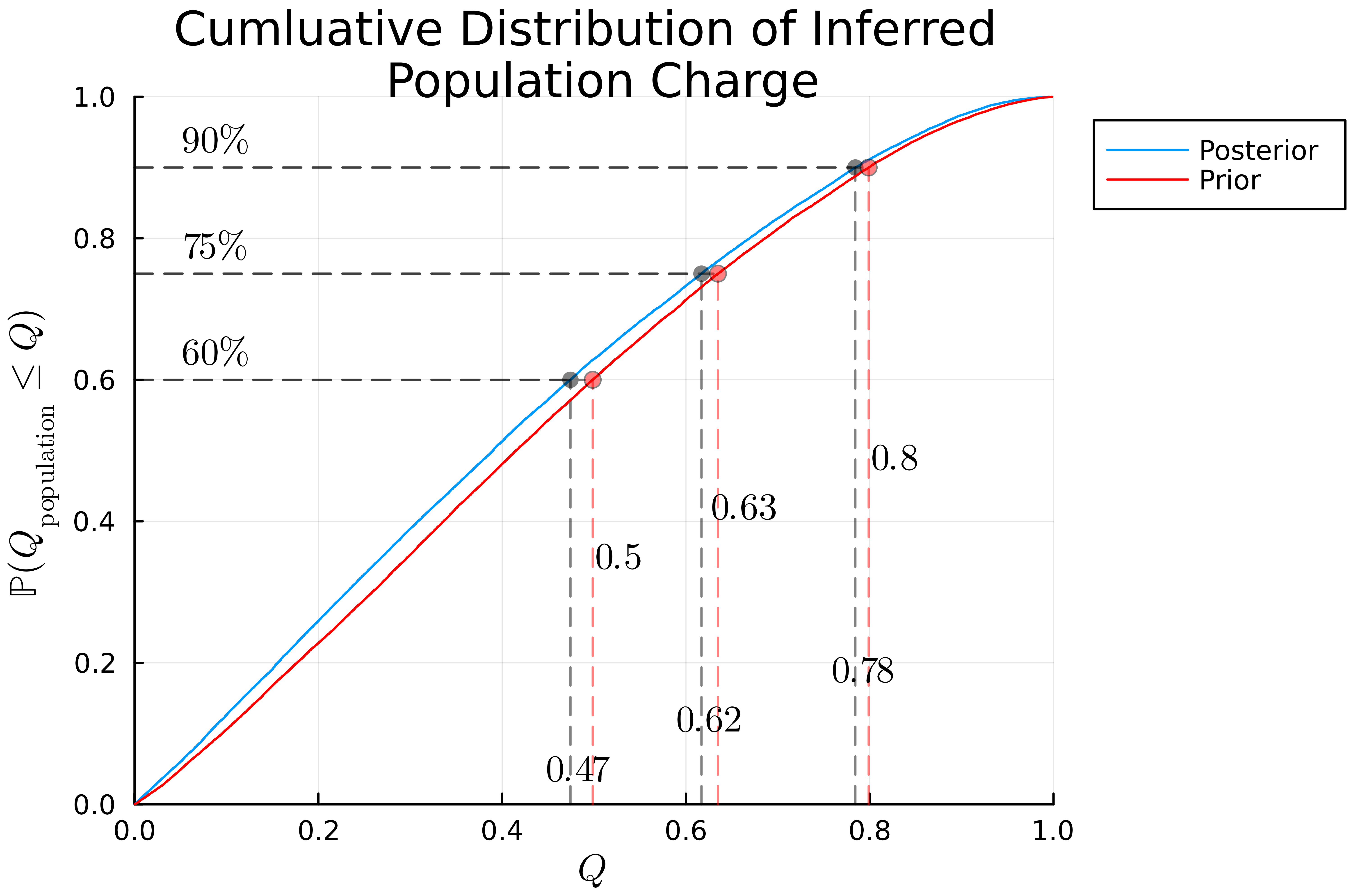

Hussain, Isi, AZ in prep

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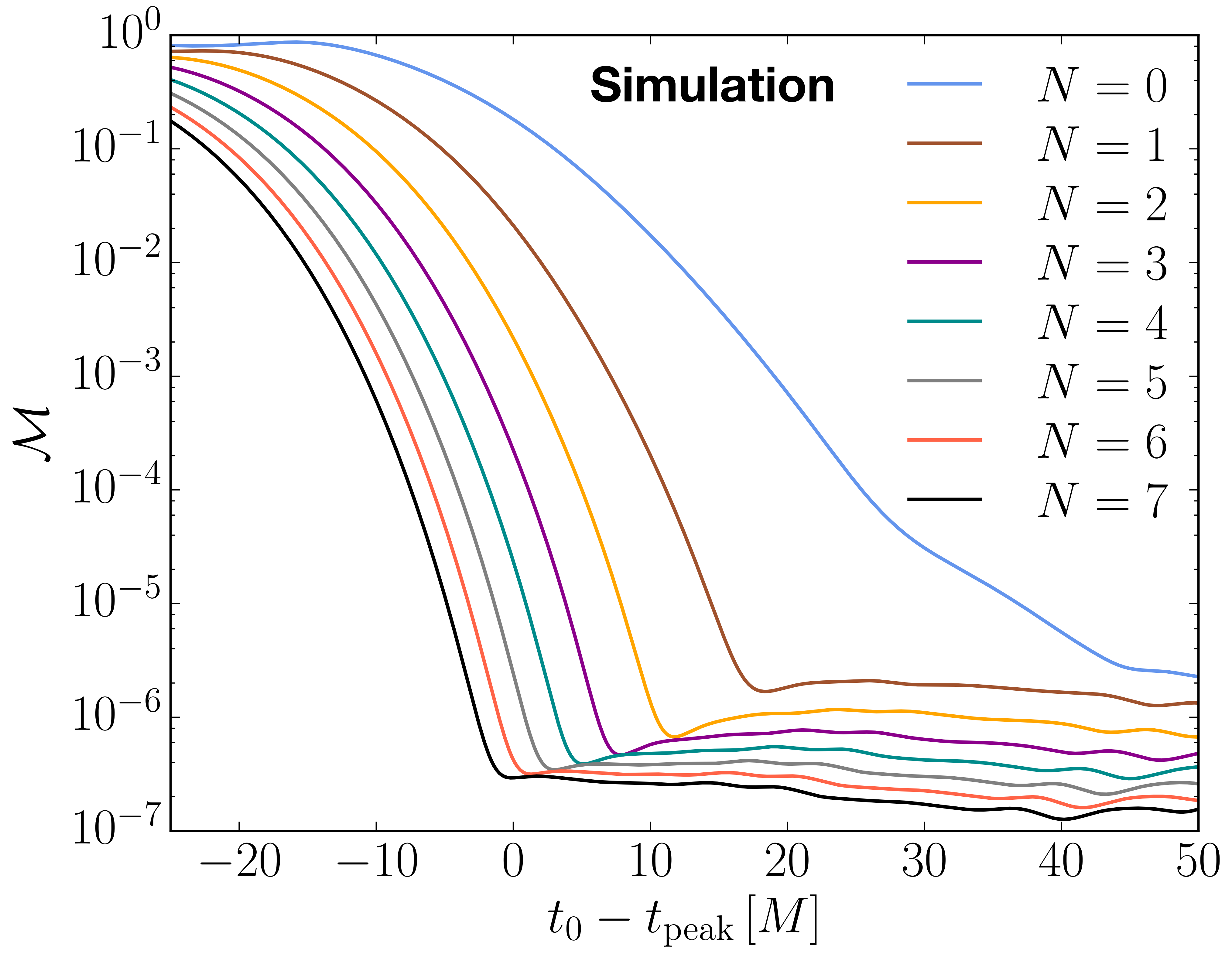
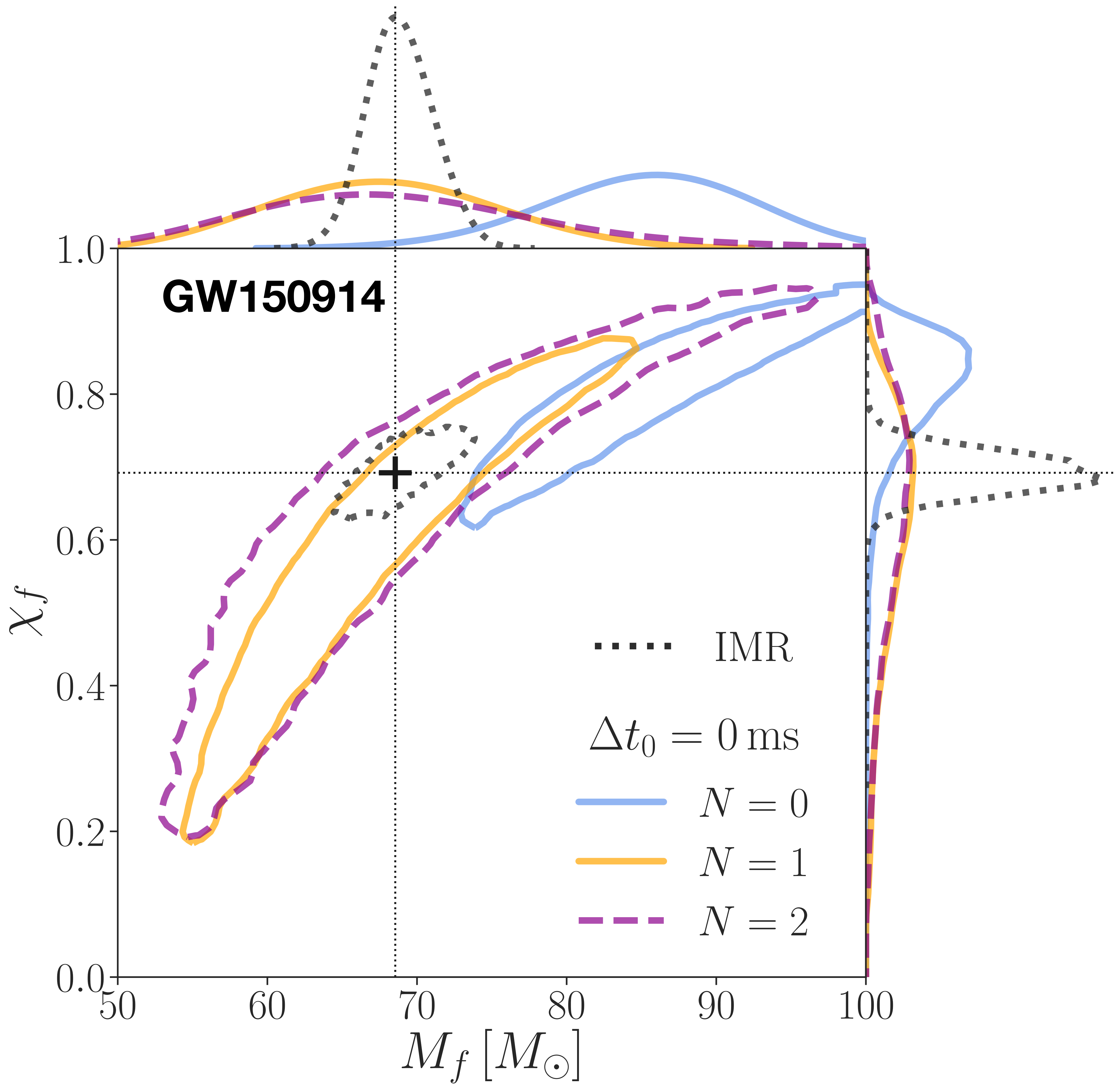
Overtones in ringdown



Giesler, Isi, Scheel, Teukolsky
arXiv:1903.08284



Isi, Gielser et al. arXiv:1905.00869

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Gravitational perts for Kerr

- Angular equation: (spin-weighted) spheroidal harmonics

$$ {}_s\psi_{lm\omega} = e^{-i\omega t} e^{im\phi} \, {}_sR_{lm\omega}(r) \, {}_sS_{lm\omega}(\theta) $$

- Standard Sturm-Liouville eigenvalue problem

$$ \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\,S_{lm\omega}}{d\theta}\right) + V_\theta(\omega, A_{lm})S_{lm\omega} = 0 $$

$$ V_\theta = {}_sE_{lm\omega} - \frac{m^2}{\sin^2\theta} - s^2\cot^2\theta - s^2 + a^2\omega^2\cos^2\theta - 2a\omega s\cos\theta $$

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Gravitational perts for Kerr

- Radial equation: Schroedinger-like with complex potential

$$\frac{d^2 u_{lm\omega}}{dr_*^2} + V_r\, u_{lm\omega} = S_{lm\omega}(r) \qquad R_{lm\omega} = \frac{u_{lm\omega}}{[(r-r_+)^s (r-r_-)^s (r^2+a^2)]^{1/2}}$$

$$V_r = \left(\omega - \frac{am}{r^2+a^2}\right)^2 - 2is\frac{r-M}{r^2+a^2}\left(\omega - \frac{am}{r^2+a^2}\right) + F(r, s, E_{lm\omega}, \omega)$$

$$u_{\rm in} \sim \begin{cases} A_{\rm in} e^{-i\omega r_*} + A_{\rm out} e^{i\omega r_*} & r_* \to \infty \\ e^{-i(\omega - m\Omega_H) r_*} & r_* \to -\infty \end{cases}$$

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$$\left(1 - \frac{m\Omega_H}{\omega}\right)|\mathcal{T}|^2 = 1 - |\mathcal{R}|^2$$

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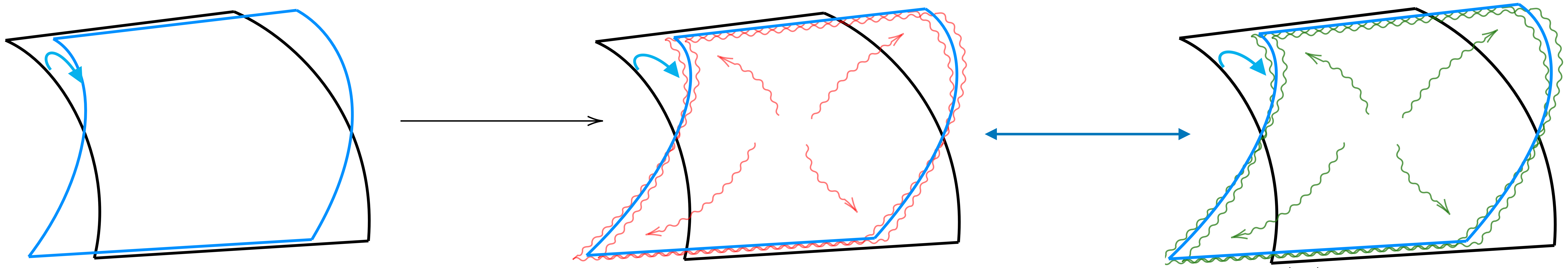
Perturbed black holes beyond Kerr

- Now add dynamical perturbations to all fields

$$\vartheta_A = \epsilon \vartheta_A^{(1)} + \eta \varphi_A + \ldots \qquad g_{ab} = g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)} + \eta h_{ab} + \ldots$$



$g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)}$ $\qquad$ ηh_{ab} $\qquad$ $\epsilon \vartheta_A^{(1)} + \eta \varphi_A + \ldots$

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Hussain, AZ arXiv: 2206.10653

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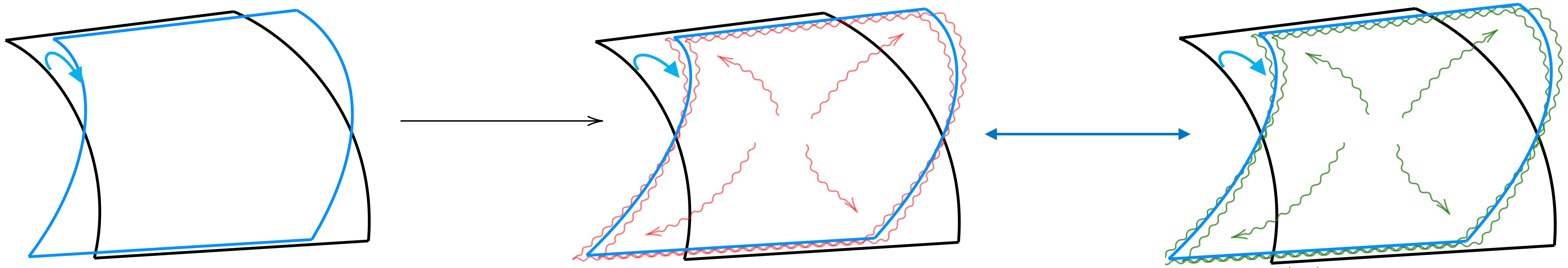

$$g_{ab}^{(0)} + \epsilon^2 g_{ab}^{(2)} \qquad \eta h_{ab} \qquad \epsilon\vartheta_A^{(1)} + \eta\varphi_A + \ldots$$

- Coupled equations for perts

$$\begin{pmatrix} \mathcal{E}_{ab} + \epsilon^2(\delta\mathcal{E}_{ab} - \delta T^{\vartheta}_{ab}) & \epsilon\mathcal{C}_{ab} \\ \epsilon\,\mathcal{F}_A & \mathcal{W}_A + \epsilon(\delta\mathcal{W}_A - \delta\rho_A) \end{pmatrix} \begin{pmatrix} h_{cd} \\ \varphi_B \end{pmatrix} = 0$$

Hussain, AZ arXiv: 2206.10653

Eigenvalue perturbations

- Perturb eigenvalue and eigenstate

$$\Phi = (\Phi^{(0)}_{m\omega} e^{-i\epsilon\,\delta\omega\,t} + \epsilon\Phi^{(1)}_{m\omega})e^{im\phi - i\omega^{(0)}t}$$

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Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ +, arXiv:1406.4206

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- Need finite product where wave operator is self-adjoint

$$\langle \Psi | \Phi \rangle = C \qquad\qquad \langle \Psi | \Box^{(0)} \Phi \rangle = \langle \Box^{(0)} \Psi | \Phi \rangle$$

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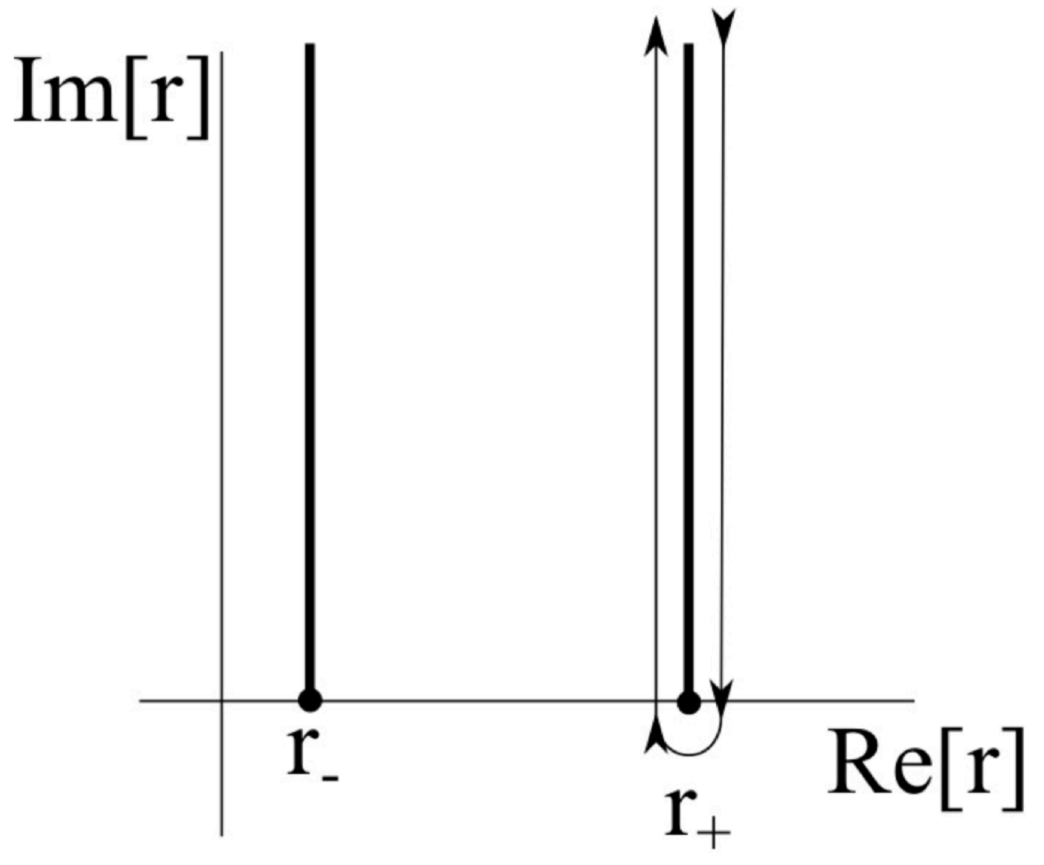

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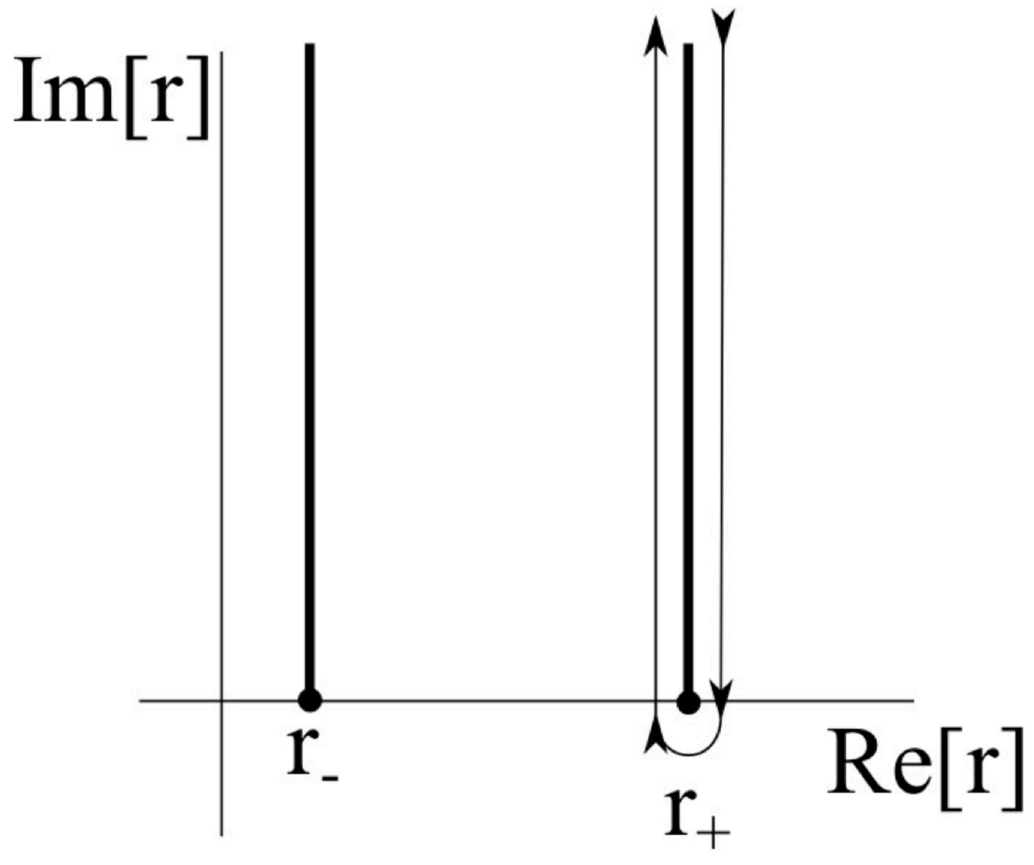

$$\delta\omega = -\frac{\langle\Phi^{(0)}|\delta\Box|\Phi^{(0)}\rangle}{\langle\Phi^{(0)}|\partial_\omega\Box^{(0)}|\Phi^{(0)}\rangle}$$

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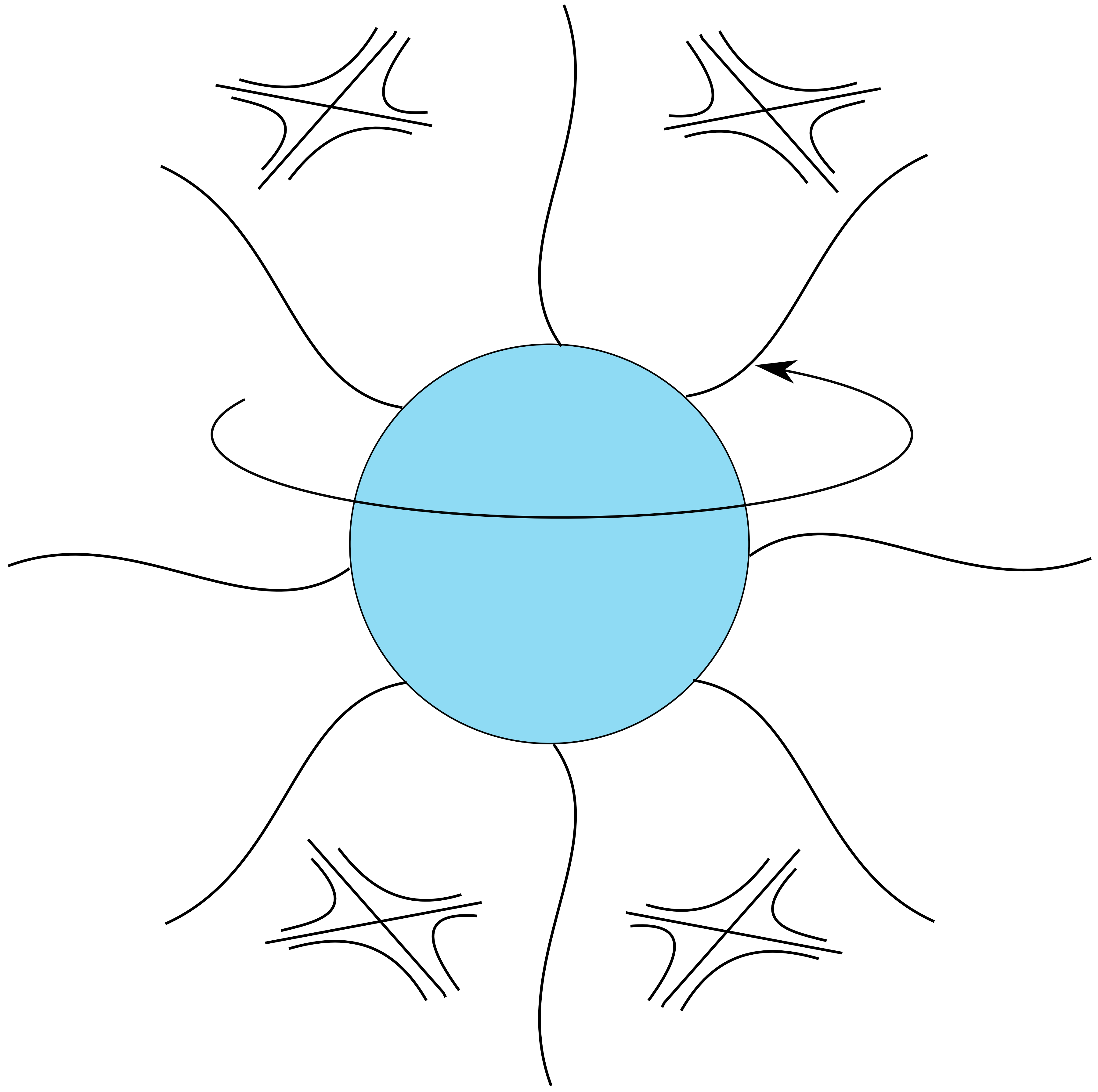
Gravitational example: charged black holes

- Coupled equations

$$G_{ab} = 8\pi T^{\mathrm{EM}}_{ab}$$

$$g^{ab}\nabla_a F_{bc} = 0$$

- Cannot decouple and separate: gravito-electromag perturbations



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$$G_{ab} = 8\pi T_{ab}^{\mathrm{EM}}$$

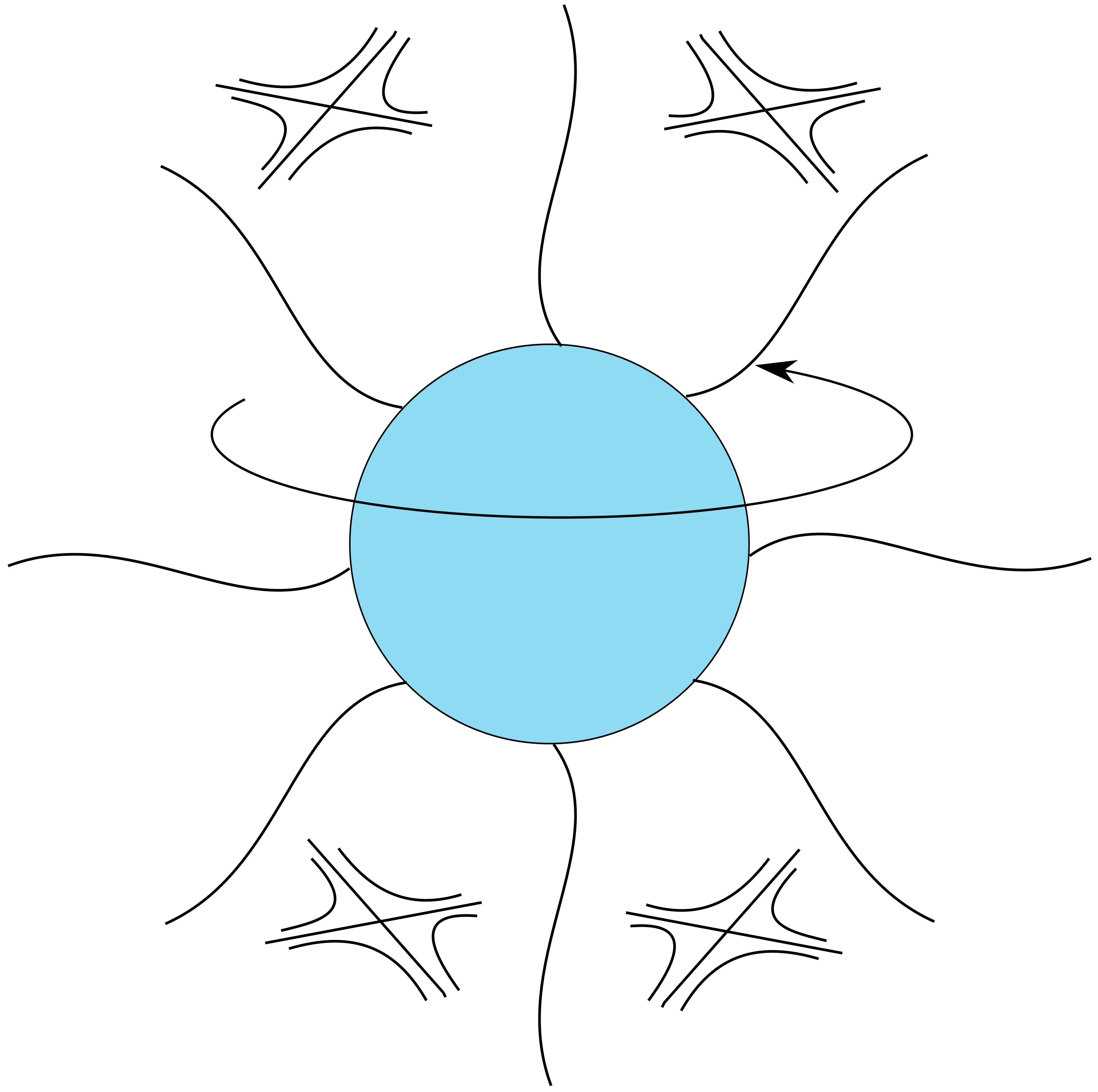
$$g^{ab}\nabla_a F_{bc} = 0$$

- Cannot decouple and separate: gravito-electromag perturbations

- Small charge: can decouple and apply EVP

$$g_{ab} = g_{ab}^{(0)} + Q^2\, g_{ab}^{(2)} + \eta\, h_{ab}$$

$$F_{ab} = Q\, F_{ab}^{(1)} + \eta\, f_{ab}$$



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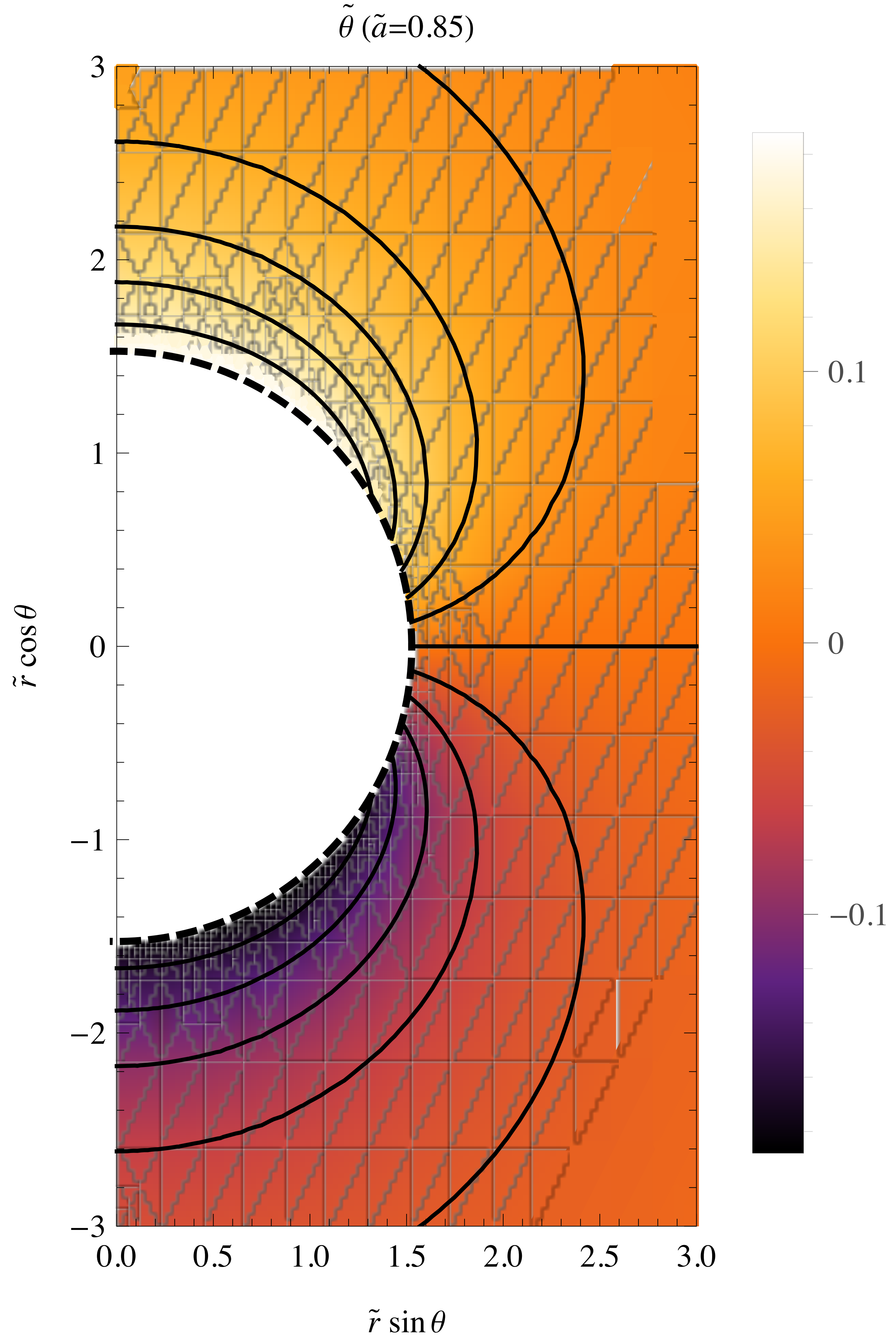
Quadratic gravity example: dCS

- Dynamical Chern-Simons: couple total derivative to scalar field, new length scale $\epsilon \sim \ell^2/M^2$

$$\mathcal{L}_{\rm int} = \vartheta \mathcal{R}_{\rm dCS} \qquad \mathcal{R}_{\rm dCS} = -\frac{1}{8}{}^*RR := -\frac{1}{8}{}^*R^{abcd}R_{abcd}$$



Stein arXiv:1407.2350

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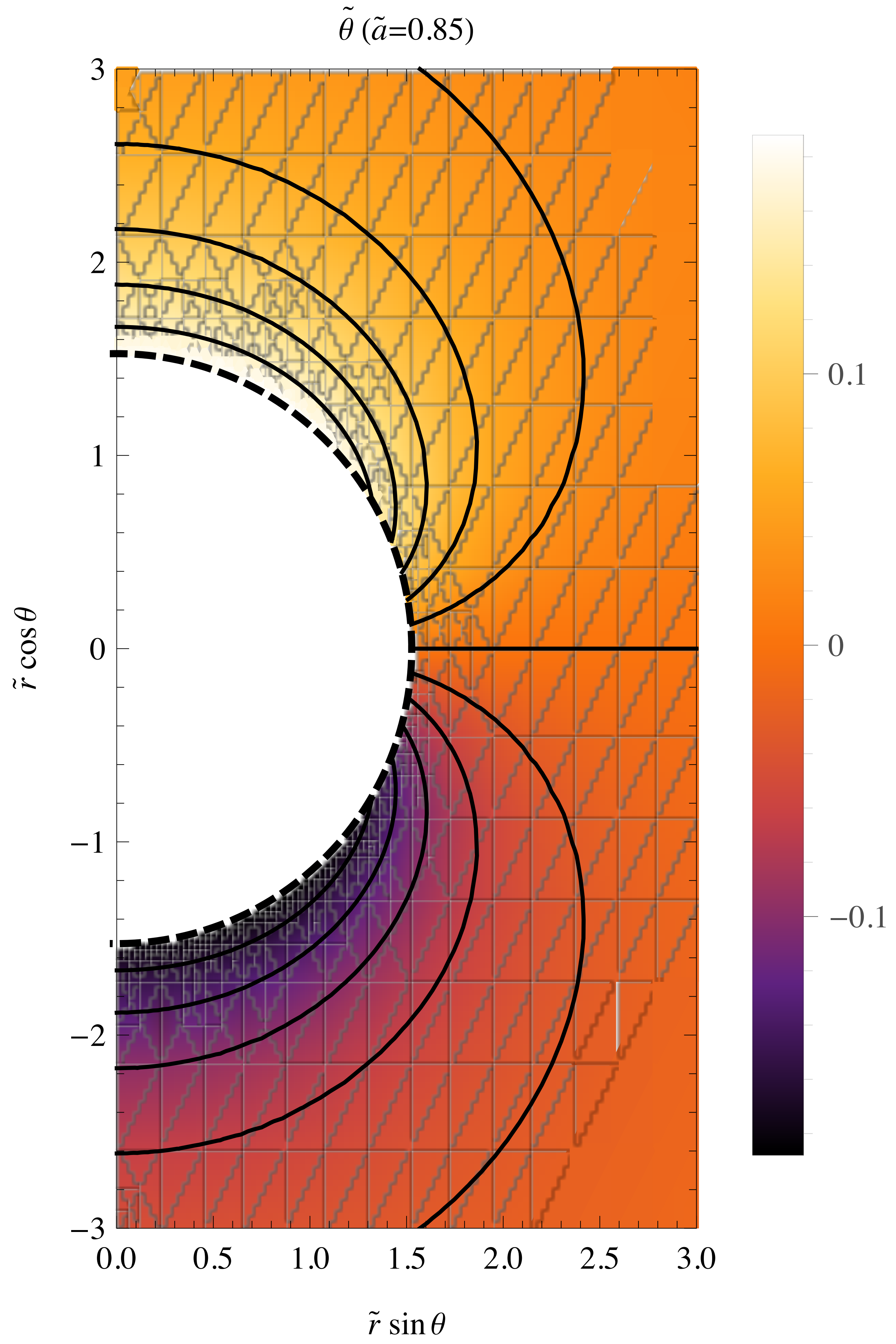

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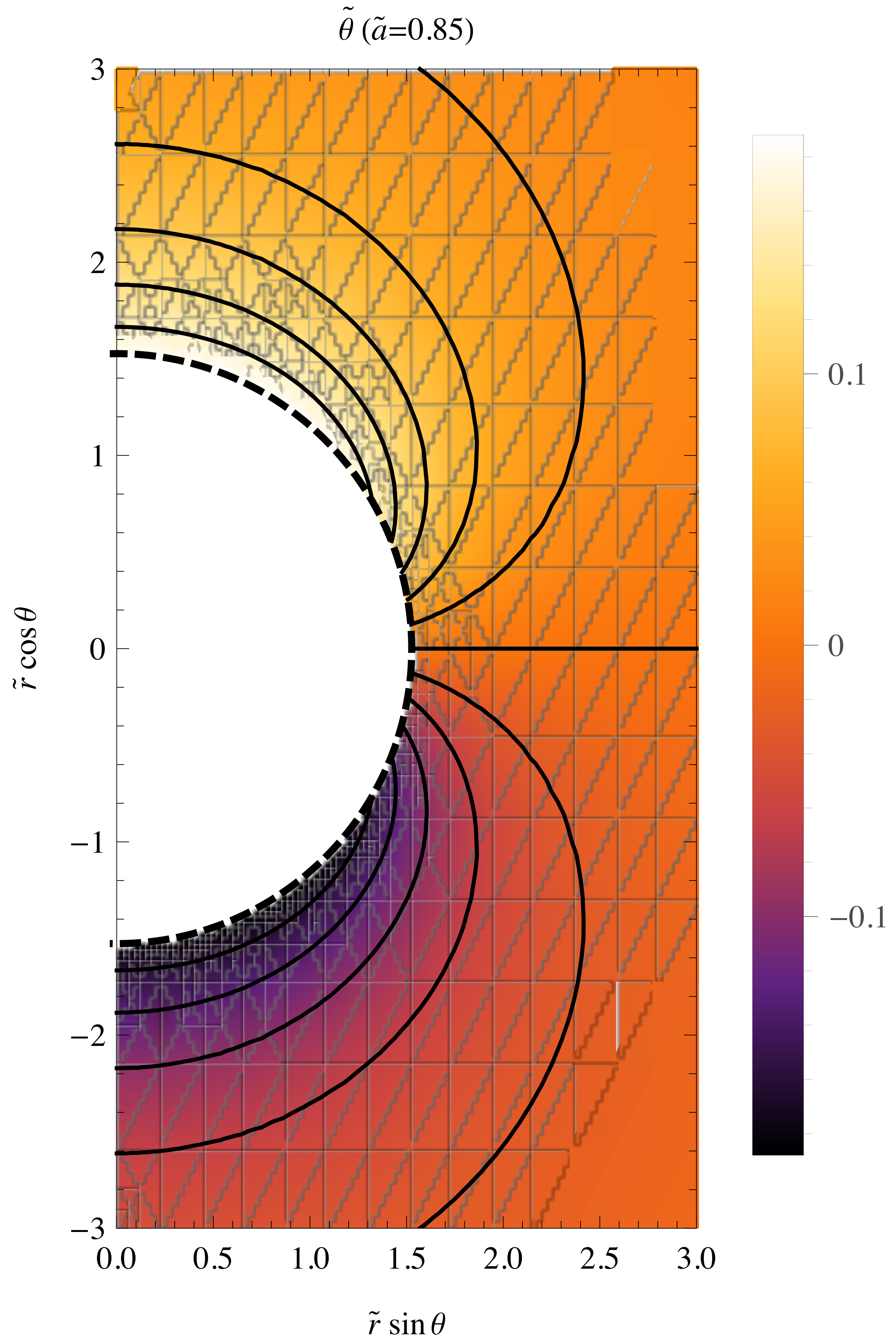
- Post-Newtonian predictions (Yagi et al. 2012)



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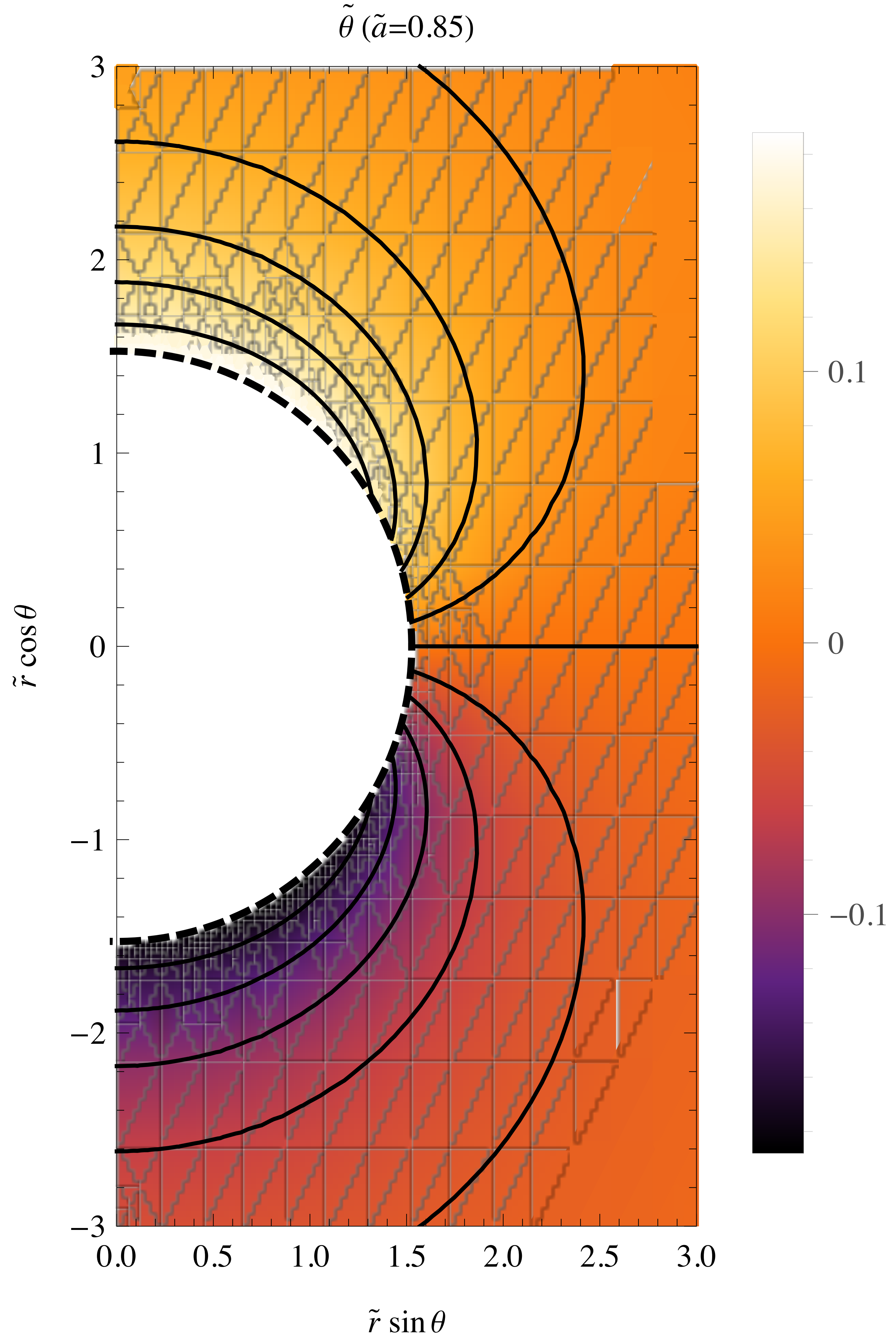

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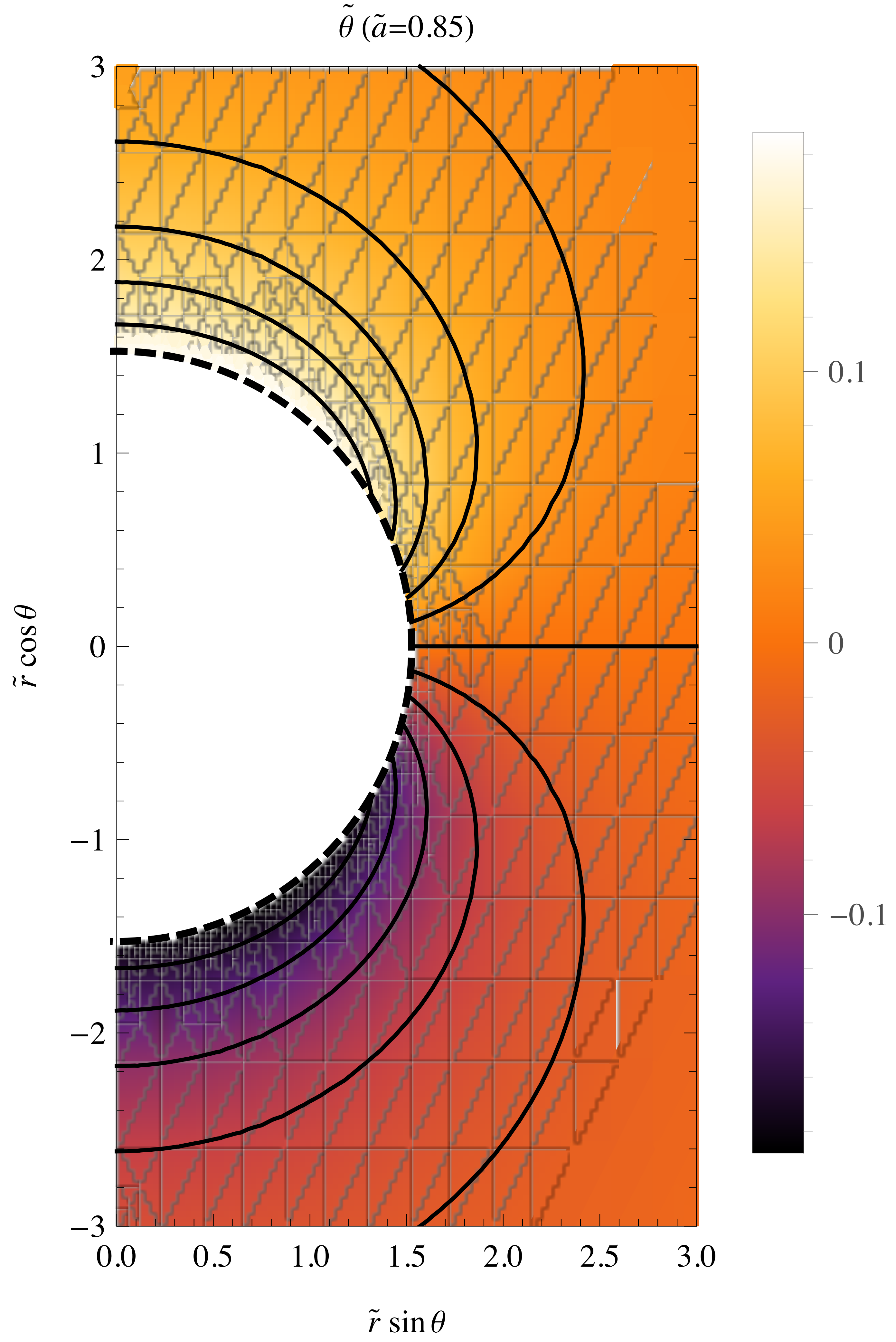

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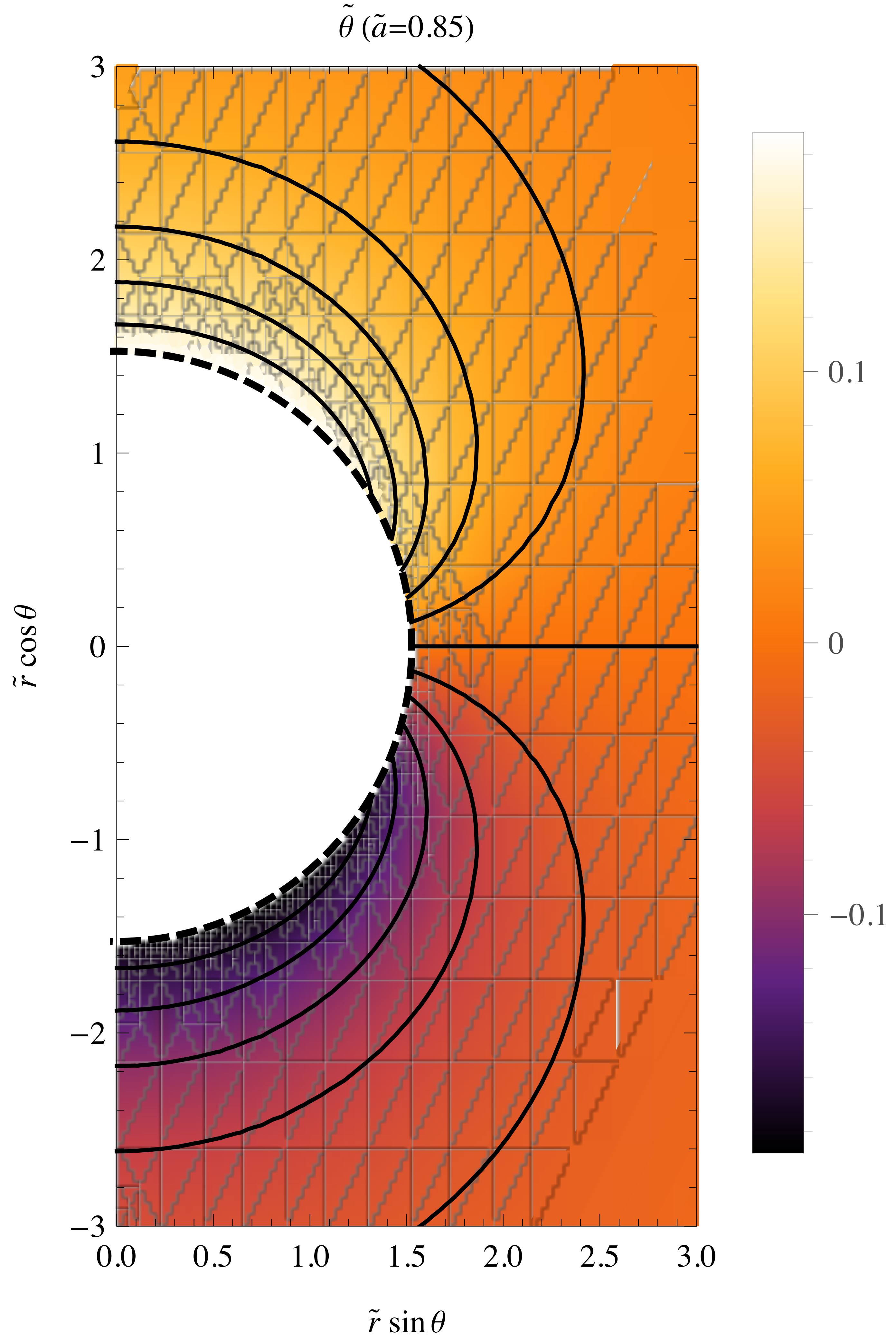

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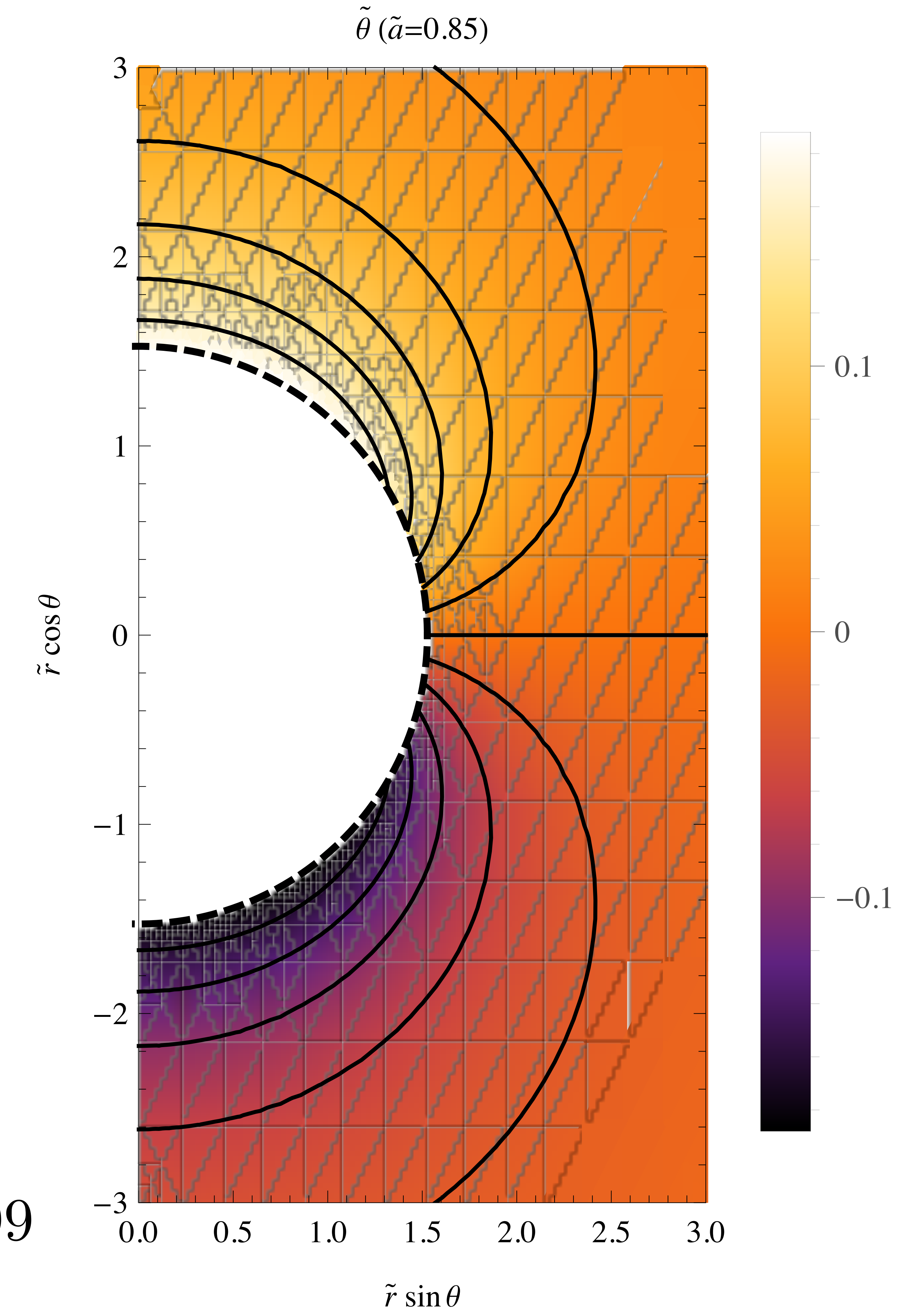
- Slow-spin expansion for deform and ringdown (Cano et al. 2020; Wagle et al. 2021; Srivastava et al. 2021)
- But parameter inference requires results at high spins $0 \leq \chi \leq 0.99$

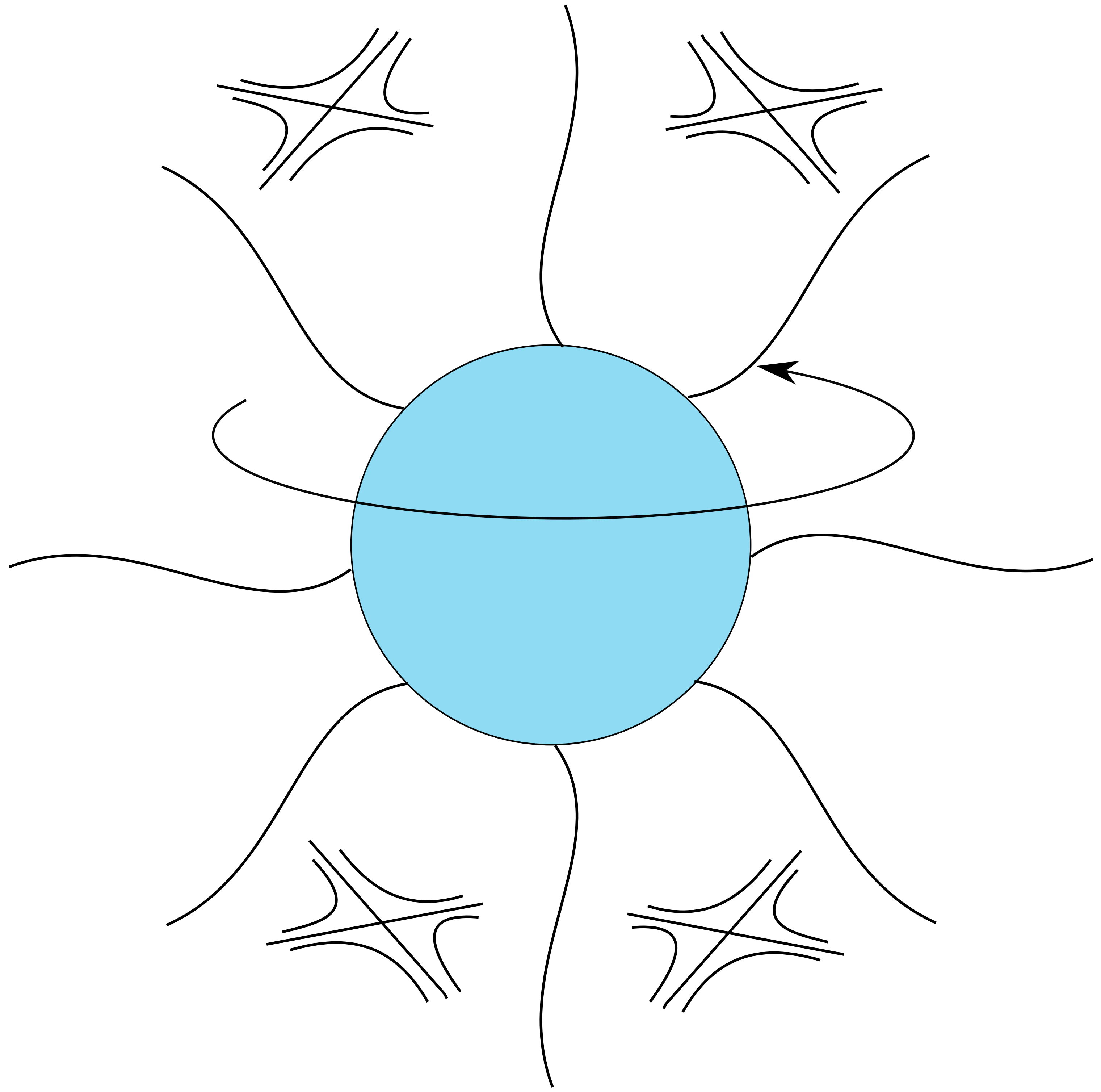


Stein arXiv:1407.2350

Example: charged black holes

- Chandrasekhar: NP derivation

$$\begin{pmatrix} \mathcal{O}_2 + Q^2\delta O_2 & Q^2\,\mathcal{G}_2 \\ Q^2\,\mathcal{G}_1 & \mathcal{O}_1 + Q^2\delta O_1 \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix} = 0$$



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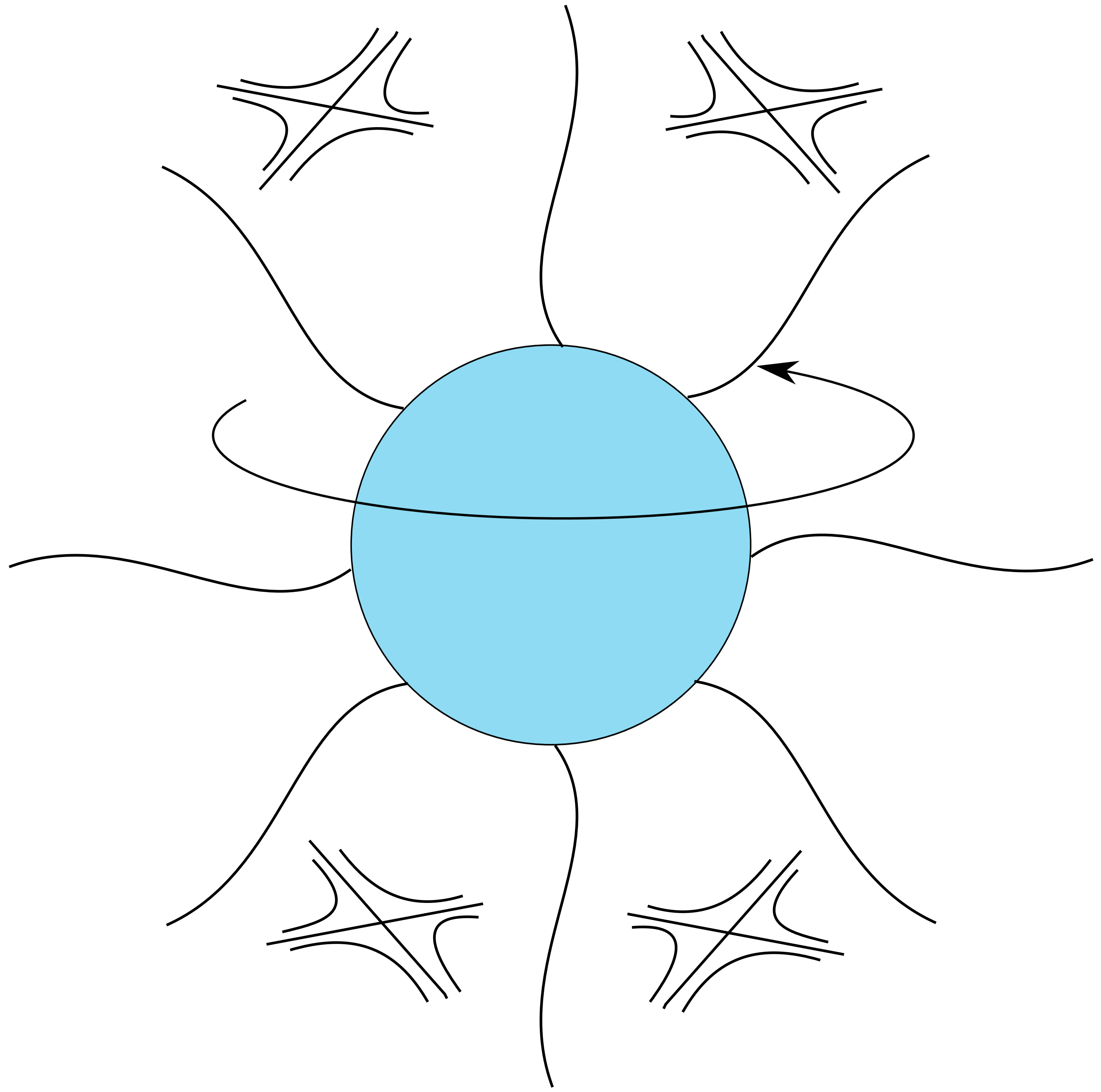

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$$\psi_2 = \psi_2^{(0)} + Q^2 \psi_2^{(2)}$$

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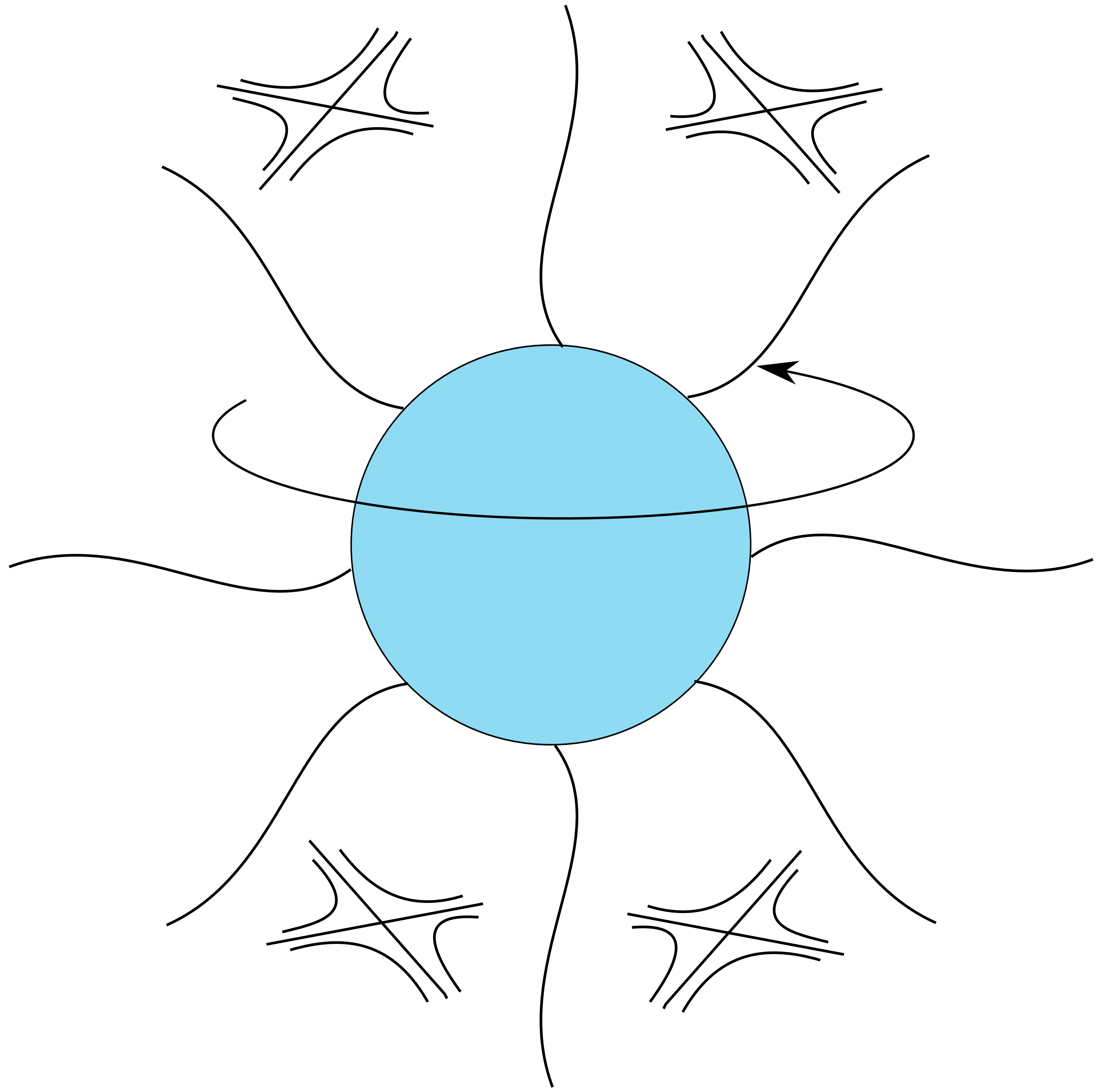
- This decouples everything



$$\omega^{(2)} = -\frac{\langle \psi_2^{(0)} | \delta \mathcal{O}_2 | \psi_2^{(0)} \rangle}{\langle \psi_2^{(0)} | \partial_\omega \mathcal{O}_2 | \psi_2^{(0)} \rangle}$$

Mark, Yang, AZ, Chen, arXiv:1409.5800

Example: charged black holes

- Chandrasekhar: NP derivation

$$\begin{pmatrix} \mathcal{O}_2 + Q^2\delta O_2 & Q^2\,\mathcal{G}_2 \\ Q^2\,\mathcal{G}_1 & \mathcal{O}_1 + Q^2\delta O_1 \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix} = 0$$

- We know the eigenmodes for Q = 0

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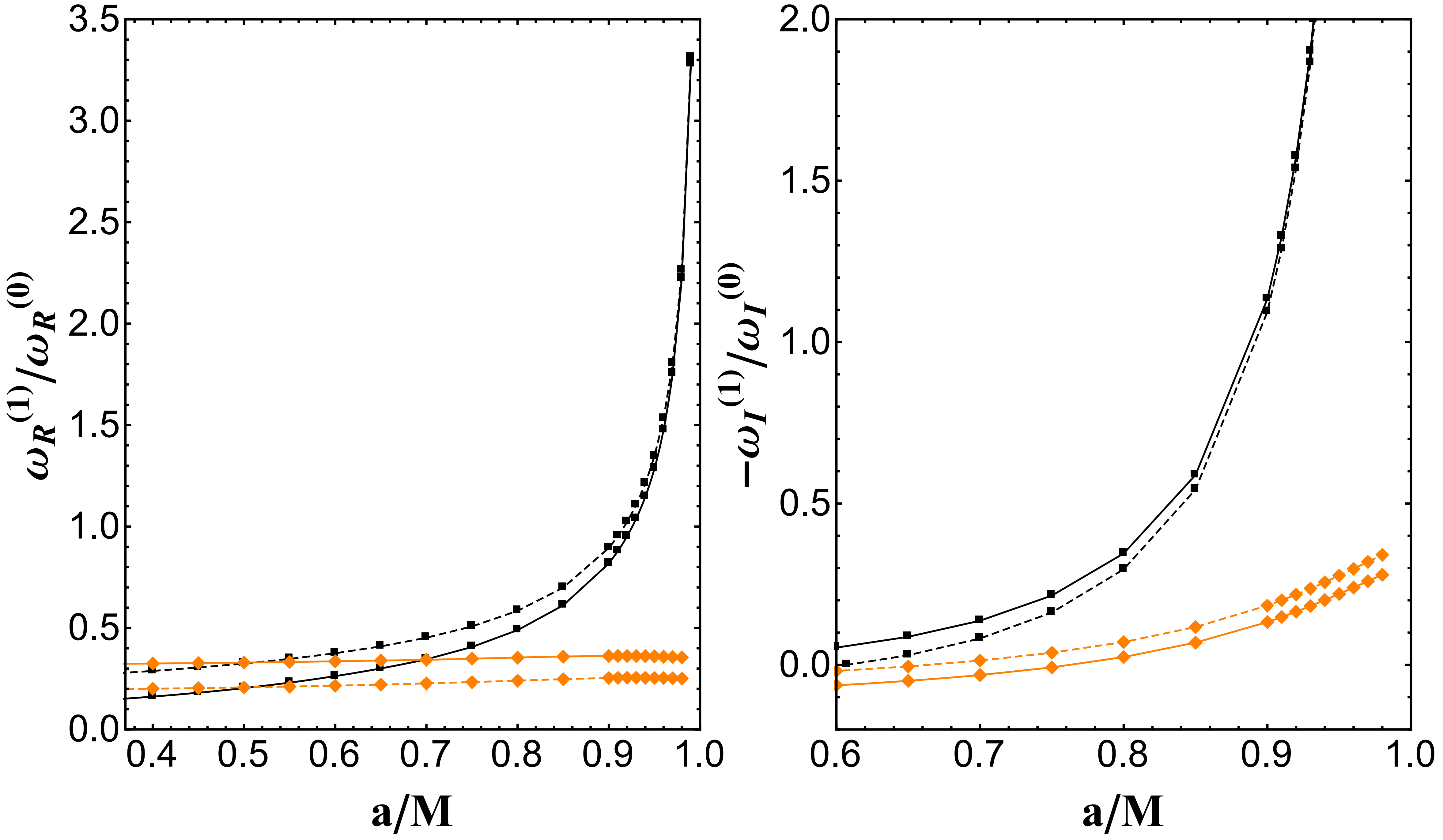

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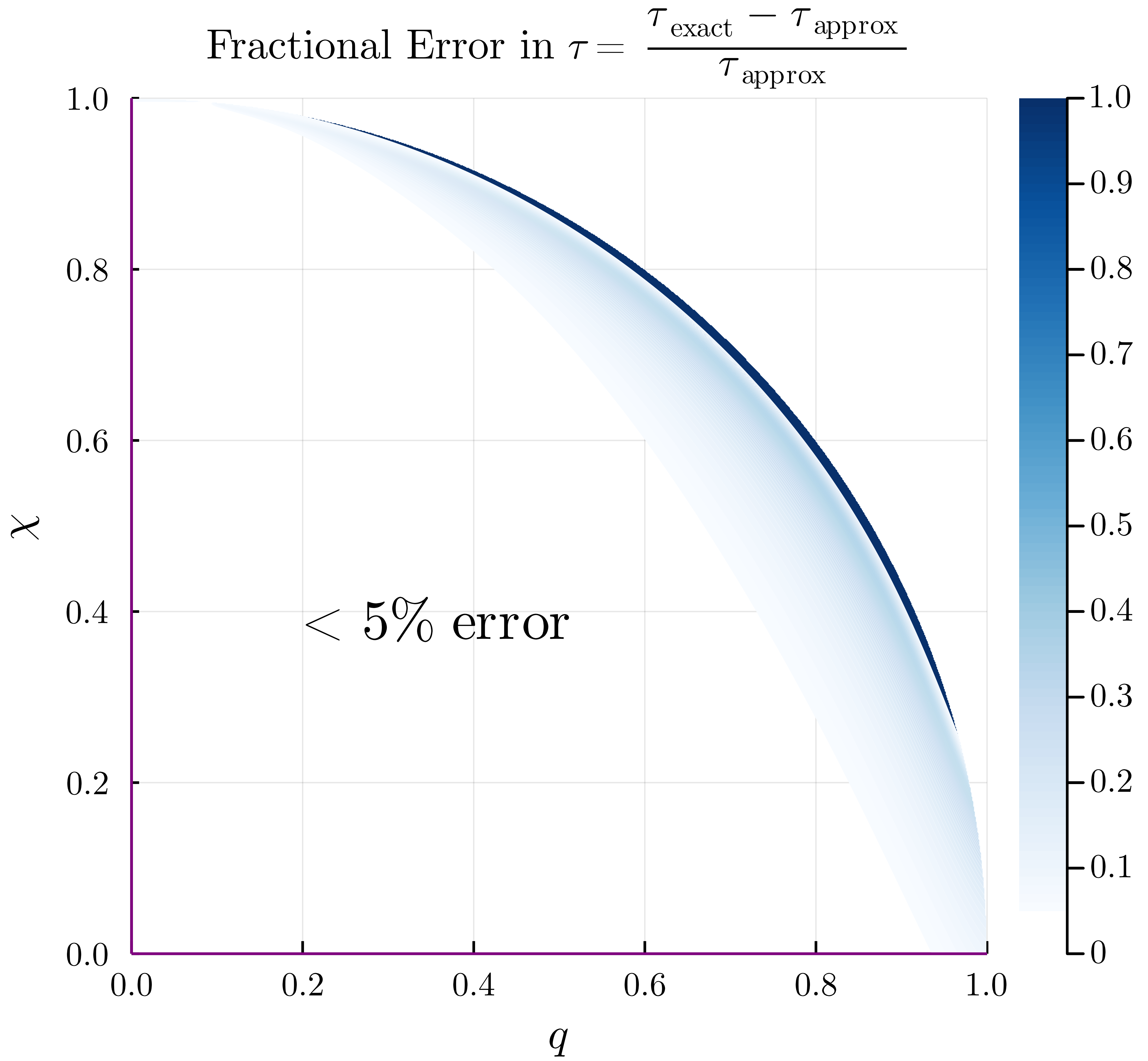

Dias, Godazgar, Santos, arXiv:1501.04625
Carullo et al. arXiv:2109.13961