Milli-Magnetic Monopole Dark Matter and the Survival of Galactic Magnetic Fields

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based on MG, Ian Shoemaker, Natalia Tapia Arellano, arXiv:2105:05769, JHEP 2022

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Dark Sectors and Dark Magnetic Monopoles

- Broad theory space for models of dark matter
  - Focus on broad paradigms, with novel physical properties, that have perhaps been overlooked
- Monopoles fascinating objects since the time of Dirac

Dark Sectors and Dark Monopoles

Dark sectors kinetically mixed with Standard Model particles have a rich phenomenology and deserved focused attention

Dark magnetic monopoles lead to novel features and signatures not ``covered'' by the dark photon/dark electric charges paradigm [Hook, Huang, 2018]

Natural to investigate whether they can comprise all or some fraction of the dark matter [Terning, Verhaaren, 2018, 2019]

Dark Sectors and Dark Monopoles

Toy Model

- Light Dirac monopoles in dark sector
- Massive dark photon
  - Confinement of magnetic charge
  - Physical flux tube (Nielsen-Olesen string), thickness set by inverse dark photon mass
  - String tension

- Electric-electric kinetic mixing $\boxed{\mathcal{L} \supset \varepsilon F'_{\mu\nu} F^{\mu\nu}}$ Holdom 1985

Monopole Interactions in Kinetically Mixed Theories [Hook, Huang, 2018], [Terning, Verhaaren, 2018]

- SM electrically charged particles acquire a tiny electric charge under the dark photon

$$\varepsilon q_e$$

- Dark sector magnetically charged particles acquire a tiny magnetic charge under ordinary electromagnetism

$$\varepsilon g_D \qquad \partial_\mu \tilde{F}^{\mu\nu}_{em} = \varepsilon g_D K^\nu_D$$

Monopole Interactions [Hook, Huang, 2018]

(magnetic charges only)

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F^2 + \frac{1}{2}\tilde{F}G + K \cdot B \ , \ \ G = dB \\
&= -\frac{1}{4}F^2 + B \cdot (-\partial\tilde{F} + K) \\
&= -\frac{1}{4}F^2 \ , \quad [\text{eliminate } B, \ \partial\tilde{F} = K] \\
&= \frac{1}{4}\tilde{F}^2 + \frac{1}{2}\tilde{F}G + K \cdot B \\
&= \frac{1}{4}(\tilde{F} + G)^2 - \frac{1}{4}G^2 + K \cdot B \\
&= -\frac{1}{4}G^2 + K \cdot B \ , \quad [\text{eliminate } \tilde{F}]
\end{aligned}
$$

Monopole Interactions [Hook, Huang, 2018]

(magnetic charges only)

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{2}\tilde{F}G - \frac{1}{4}F_D^2 + \frac{1}{2}\tilde{F}_D G_D + m_D^2 A_D^2$$
$$+\varepsilon F F_D + K \cdot B \; + K_D \cdot B_D$$

$$A \to A + \varepsilon A_D$$

Introduces $\varepsilon \frac{1}{2}\tilde{F}_D G$ cancelled with $B_D \to B_D - \varepsilon B$

Monopole Interactions

$$\mathcal{L} \to \mathcal{L} = \boxed{(gK - \varepsilon g_D K_D)} \cdot B + g_D K_D \cdot B_D$$
$$= gK \cdot B + g_D K_D \cdot \boxed{(B_D - \varepsilon B)}$$

Source for ordinary magnetic potential

Effective magnetic potential experienced by a dark magnetic monopole

Question of how monopoles interact with electric charges is subtle, because of the impossibility of constructing a local, Lorentz invariant action for simultaneously both electric and magnetic charges

Terning and Verhaaren (2018) reproduced and generalized these conclusions to include both electric and magnetic charges, using two-potential formalism of Zwanziger

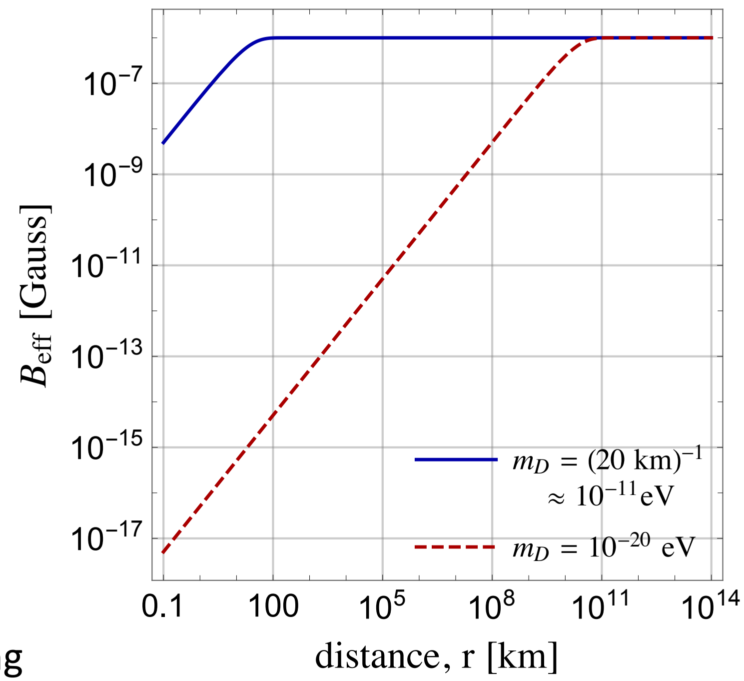
Effective magnetic field felt by dark magnetic monopole

$$B_{\text{eff}} = B_D - \varepsilon B \rightarrow \varepsilon B\left(1 - e^{-m_D r}\right)$$

[Hook, Huang, 2018]

- ``unsuppressed'' far from ordinary source
- Suppressed close to ordinary source: decoupling

- Turnover scale set by dark photon mass,
  the thickness of the magnetic flux tube/Nielsen-Olesen string



Dark magnetic monopole properties

Non-relativistic Dark Magnetic Monopoles

$$H = \frac{p^2}{2\mu} - \frac{\alpha_D}{r}e^{-m_D r} + C\pi v_D^2 r + gQ_m Bz$$

$$\boxed{Q_m = \varepsilon g_D/g}$$

- Properties of ground state:
  - Hydrogen-like if $\boxed{m_D \ll \alpha_D M}$ otherwise "Airy-like"
  - Magnetically neutral
  - Absolutely stable (if dark monopoles have opposite flavors)

SIDM bounds on dark magnetic monopoles

SIDM bounds on dark magnetic monopoles

- SIDM bounds on long-range Coulomb interactions explored by a number of authors

[Feng, Kaplinghat, Tu, Yu, '09],
[Cyr-Racine, Sigurdson '13],
[Cline, Liu, Moore, Xue, '13],
[Agrawal, Cyr-Racine, Randall, Scholtz, '16]

- Bound state dark magnetic monopoles have long- and short- range interactions with other bound states
  - long-range van der Waals interactions, short distance Coulomb interactions

- ``Free '' mmCPs have long range Coulomb interactions, regulated by interparticle spacing or dark Debye length

Strength of interactions constrained by Bullet cluster and, independently, by the existence of elliptical galaxies ("halo ellipticity")

Cross-section

- Numerical: Cline et al (2013) considered the elastic scattering between atomic DM states via an effective inter-atomic potential {that at long distances describes the van der Waals interaction)

$$\sigma_T \quad = \quad \pi L_0^2 \left(b_1 + b_2(E/E_0) + b_3(E/E_0)^2\right)^{-1}$$

- Hard scattering: change in kinetic energy comparable to change in potential energy. This has an impact parameter less than the Bohr radius, provided we're in the ``fast limit" where

$$\boxed{v \gg \alpha_D}$$

  - dominated by Rutherford scattering
  - forward singularity cut off by appropriate length scale (Bohr radius/interparticle length/Dark Debye length)

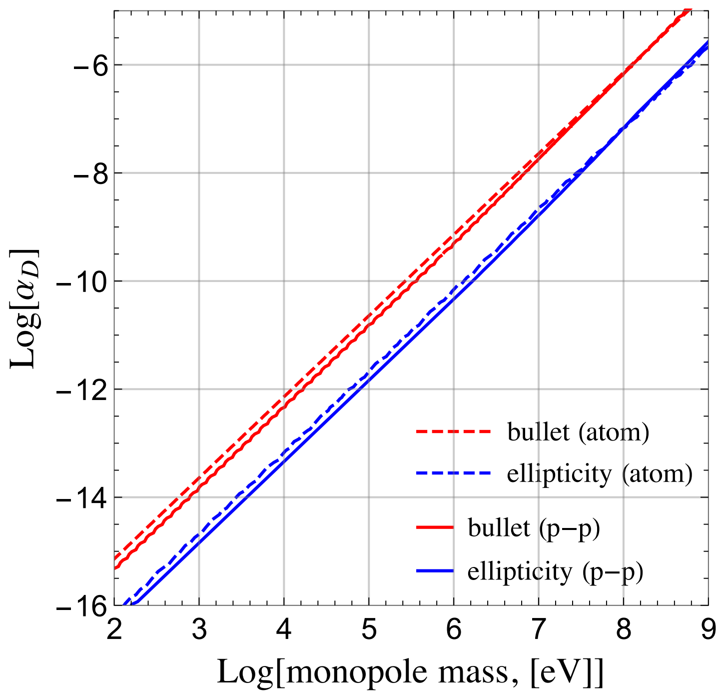
- Soft scattering

  - approximately solve classical equation of motion for vdW and integrate over impact parameters
    - [adapted from Ackerman, Buckley, Carroll, Kamionowski, 2006 to vdW instead of Coulomb]
  - Parametrically the same as the hard scattering cross-section

Halo Ellipticity and Bullet Cluster bounds

$$\sigma_T \lesssim 0.7\ \mathrm{cm}^2/\mathrm{g} \qquad \text{(bullet)}$$
$$\Gamma \lesssim 10 H_0 \qquad \text{(halo ellipticity)}$$

Use bounds as input to energy loss from Parker effect



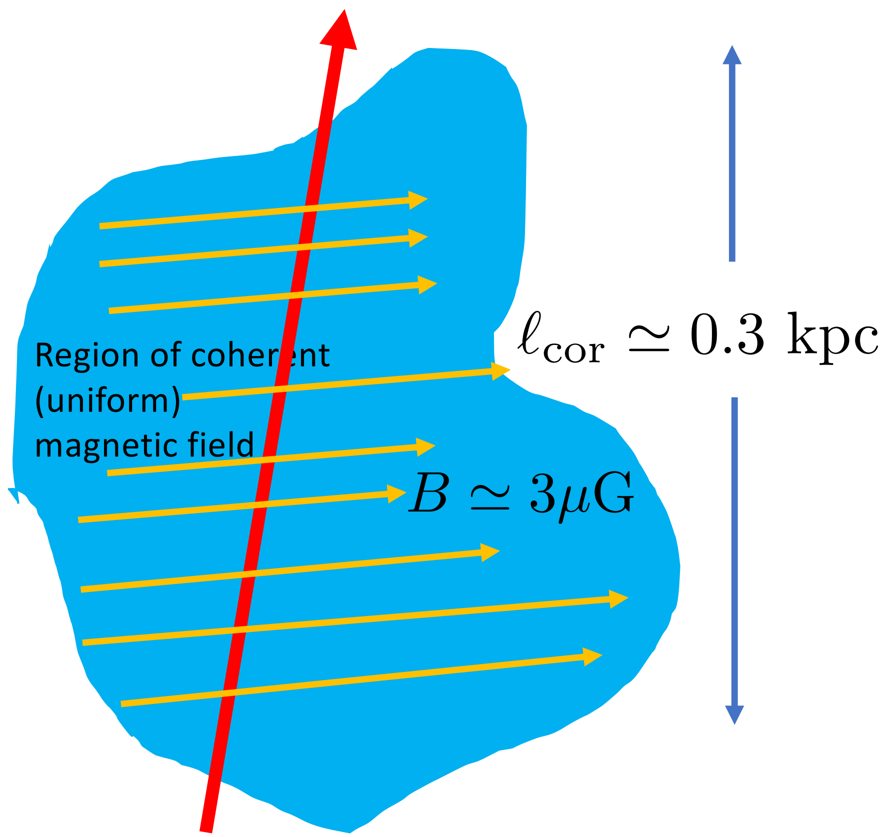
Parker effect in the Milky Way [Parker ’70; Turner, Parker, Bogdan, ’82; Parker ’83; Parker ’87; Adams et al ‘93]

- Magnetic monopoles accelerated by magnetic fields, dissipating magnetic field energy

- Require dissipation of magnetic field energy occurs on timescales greater than the galactic dynamo timescale

$$\tau_{\rm dyn} \simeq 10^8 \text{ yr}$$

$$\boxed{\tau_{\rm diss} \simeq \frac{B^2}{j \cdot B}}$$



Magnetic monopole (accelerated, with little deflection since for mmCPs $\Delta v \ll v_0$)

Parker Effect (Turner, Parker, Bogdan, 1982)

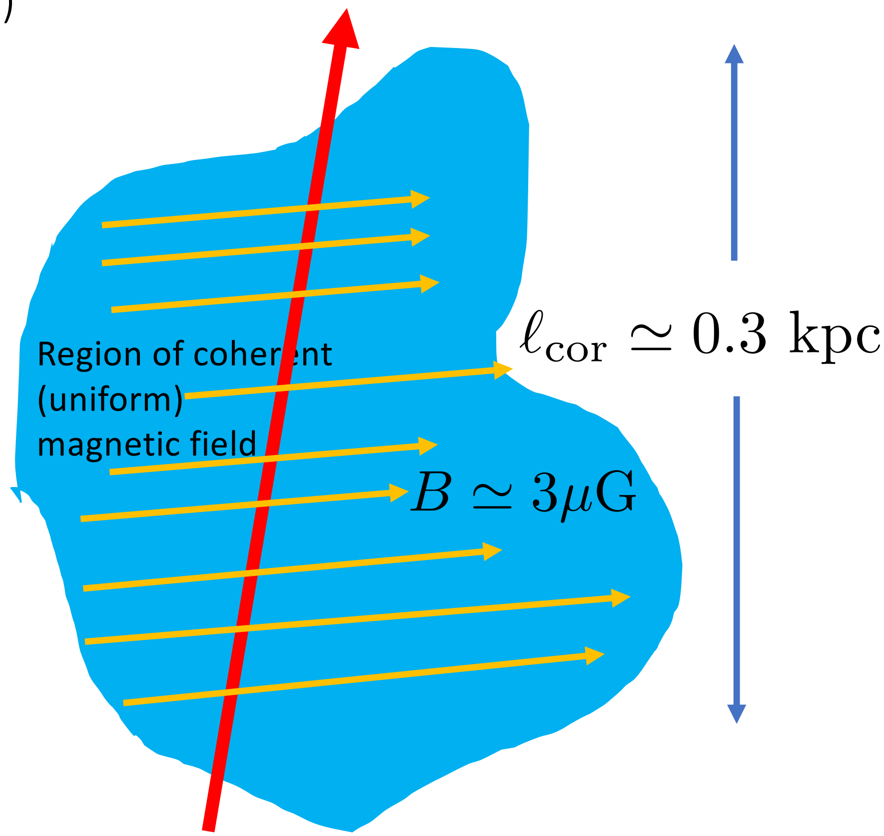
$$\langle \Delta E \rangle \times 2\mathcal{F} \times 4\pi \ell^2 < \frac{B^2}{8\pi}\frac{4\pi}{3}\ell^3 \frac{1}{\tau_{\rm dyn}},$$

$$\langle \Delta E \rangle \equiv T_{\rm f} - T_{\rm i} \simeq \frac{1}{2}(\Delta t)^2 \frac{d^2T}{dt^2}$$

Estimate $\frac{d^2T}{dt^2}$ from the Lorentz force law,

assuming static magnetic field.

Bound flux or, given a density, the magnetic coupling (mmCP)



Magnetic monopole (accelerated, with little deflection since for mmCPs $\Delta v \ll v_0$)

Revisited Parker Bound

- Repeated energy-loss analysis of Turner, Parker, and Bodgan (1982), (trivially) updated for modified coupling

- Stringent bound driven by large size of coherent magnetic domain ~ 0.3 kpc and galactic dynamo timescale ~ 30 Myrs, and the large number density of dark monopole dark matter

$$
\begin{aligned}
Q_m &< \sqrt{\frac{1}{12\pi}\frac{1}{\tau_{\rm dyn}}\frac{v\,M^2}{g^2\rho_{DM}\,\ell}\left(\frac{\rho_{DM}}{\rho_M}\right)} \\
&\simeq 4.5\times 10^{-26}\left(\frac{M}{10^4\ {\rm eV}}\right)\left(\frac{\rho_{DM}}{\rho_M}\right)^{1/2}
\end{aligned}
$$

$$\boxed{Q_m = \varepsilon g_D/g}$$

Parker Effect for Dark Magnetic Monopoles

Considered two naïve simplified and extreme scenarios for astrophysical populations

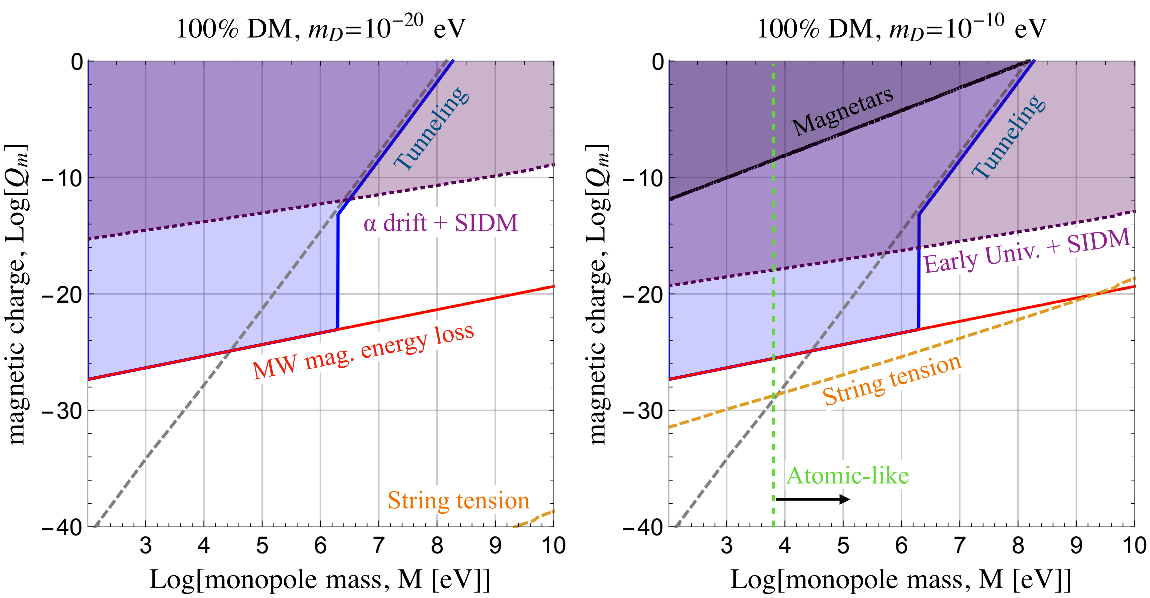
1. Dark magnetic monopoles essentially "free"
   - magnetic charges accelerated by background field and Parker's energy-loss argument applies
2. All Magnetic monopoles in atomic-like ground state (conservative)
   - In background magnetic field of Milky Way, ground state is unstable to decay

$$\Gamma = K S_0 e^{-2S_0} = \left(\frac{4\alpha_D^5 \mu^3}{g Q_m B(d)}\right) e^{-\frac{2\alpha_D^3 \mu^2}{3 g Q_m B(d)}} \qquad \text{(WKB)}$$

(This is a problem in Landau and Liftshitz, that happily enough comes with a solution)

- After tunneling, essentially ``free'' magnetic charges accelerated by background field and Parker's energy-loss argument applies

Parker bound on dark magnetic monopoles



$$\boxed{Q_m = \varepsilon g_D / g}$$

Magnetar bounds from Hook and Huang (2018)

Light purple exclusion: cone of Milky Way magnetic energy loss (red) and requirement that tunneling occurs on timescales less than the galactic dynamo timescale

Dark purple exclusion (dotted): SIDM bounds on g_D combined with other independent bounds on ε

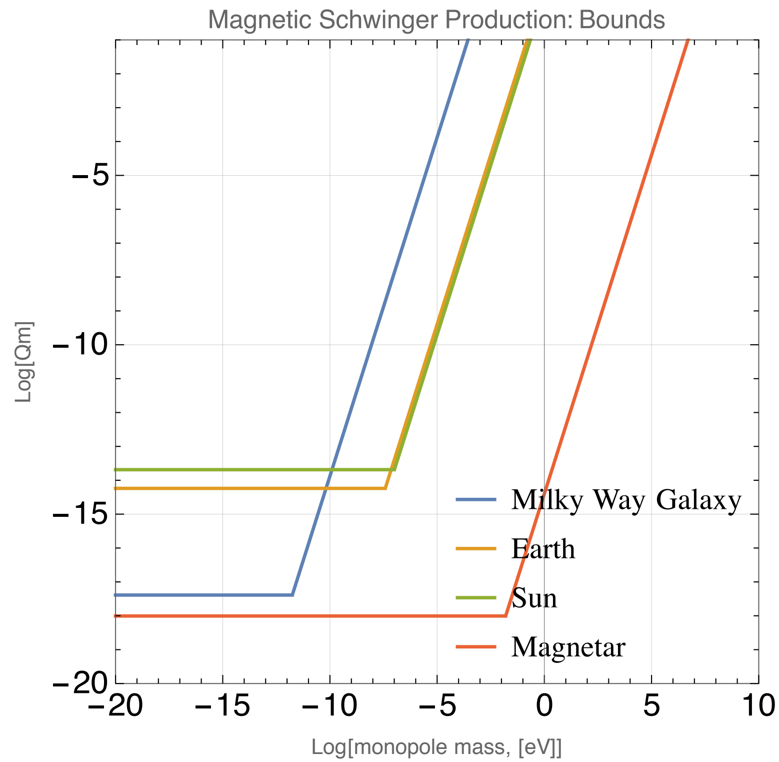
Magnetar bounds weaker than Parker bounds, especially at small dark photon masses, but our Parker bound have more caveats

Conclusions

Assuming magnetic monopoles of a dark sector comprise a significant fraction of the dark matter:

- Stringent limits on magnetic fine structure constant of milli-magnetic monopoles from SIDM bounds
- Survival of Galactic Magnetic field places stringent limits on Q_m
- Heavier dark magnetic monopole masses more weakly constrained

Magnetar bounds evaded for smaller dark photon masses, constrained by survival of magnetic fields of other astrophysical systems



Summary

- Dark sectors + kinetic mixing can be a rich area of phenomenology
- Milli-magnetic monopole interactions with ordinary photon strongly constrained by magnetars as well as by the Galactic Parker effect
- Currently investigating:
  - magnetic Schwinger effect in other astrophysical systems
  - cosmological ``freeze-in'' targets
  - signatures and bounds from laboratory experiments

Backups

Magnetic Schwinger Mechanism and Magnetars [Hook and Huang, '18] $B_{\text{magnetar}} = 10^{15}\ \text{G}$

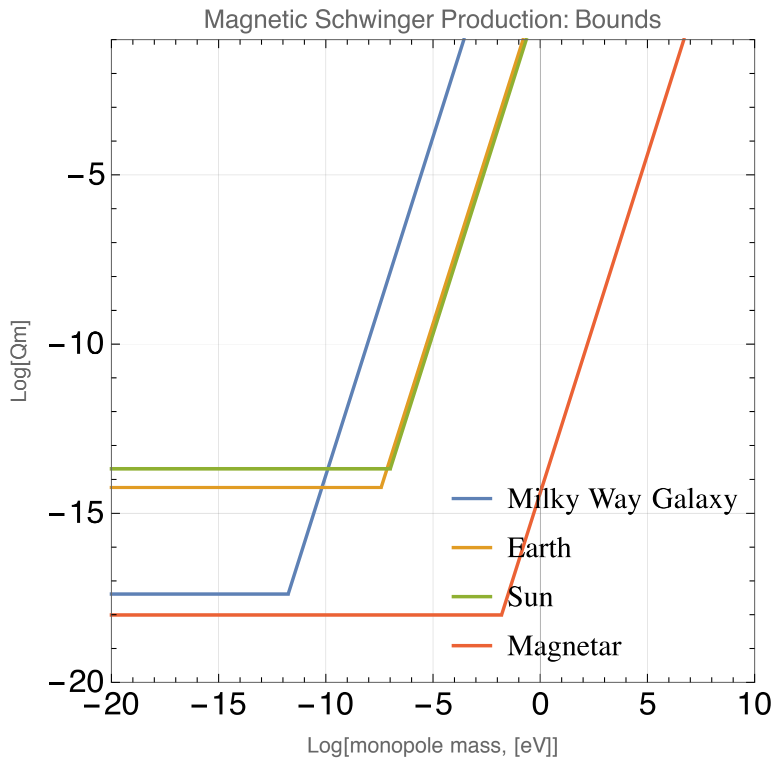
- Magnetic fields can pair produce milli-magnetic monopoles, even if they are not dark matter
- Provided magnetic field is more than m_D^{-1} away from source
- Constraint obtained if magnetic field is drained on a timescale shorter than known persistence (e.g., magnetic dynamo) timescale

$$\underbrace{E_{\text{loss}}}_{\text{Energy loss}} \underbrace{\frac{g^2 Q_m^2 B^2(d)}{4\pi^3} \exp\left[\frac{-\pi M^2}{g Q_m B(d)}\right]}_{\text{mmCP production rate}} < \underbrace{\frac{B^2}{2\, t_{\text{life}}}}_{\text{Rate of energy loss from magnetic field}}$$

$$E_{\text{loss}} = g Q_m B(d) d$$

Magnetic Schwinger Mechanism and other astrophysical objects

- If dark photon mass is lighter than d=(20 km)$^{-1}$, magnetar bounds don't apply
- Other astrophysical objects in the Universe have more extensive magnetic fields – so bounds apply to even smaller dark photon masses
- And last longer, but have weaker magnetic fields.
- Still, they can lead to interesting bounds.
- What is the relevant (dynamo) timescale?
- Magnetars can be source of dark and ordinary photons



Magnetic monopole features

- Dirac charge quantization condition
- Naturally occur in non-Abelian theories + spontaneous symmetry breaking (t'Hooft-Polyakov monopoles)
- Electron-magnetic monopole scattering strongly coupled, not Lorentz invariant (Weinberg)
    - Promising solution provided in Terning, Verhaaren, 2019 "Resolving Weinberg Paradox with Topology" for using kinetically mixed theories as a toy model
- In certain N=2 supersymmetric gauge theories (Seiberg-Witten theories) magnetic monopoles become weakly coupled massless at certain points on moduli space
    - At liberty to consider effective field theories of light magnetic monopoles [Csaki, Shirmin, Terning, 2010; Terning, Verhaaren, 2018, 2019 (2)]

Throughout will be considering Dirac magnetic monopoles only

Dark magnetic monopoles + kinetic mixing

At long distances:

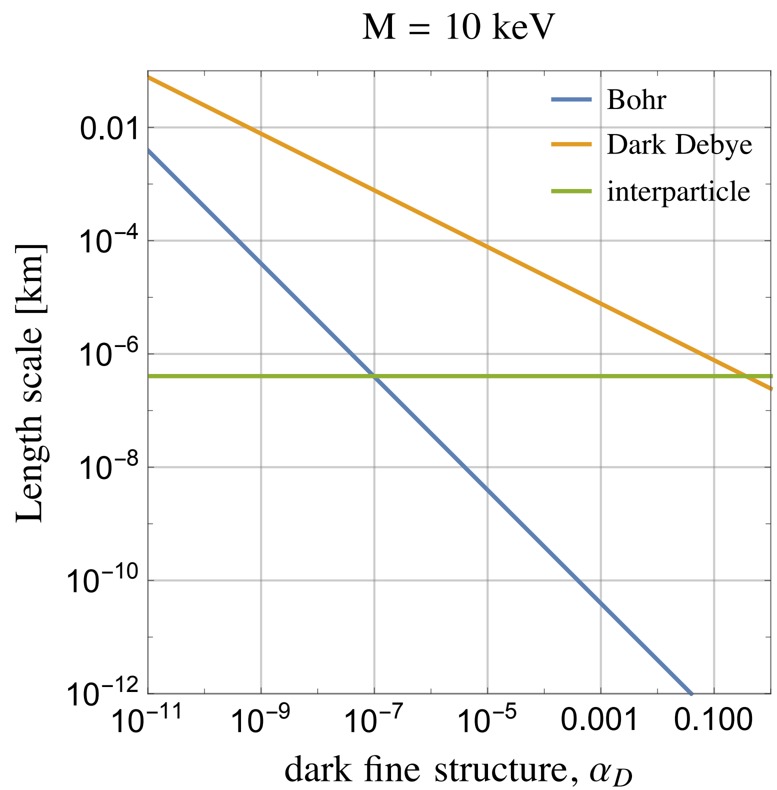
$$\boxed{\begin{array}{rcl} \text{ordinary photon} & : & \left\{ \begin{array}{c} J, \\ K - \varepsilon K_D \end{array} \right. \\ \text{dark photon} & : & \left\{ \begin{array}{c} J_D + \varepsilon J, \\ K_D \end{array} \right. \end{array}}$$

- "Diagonal" Dirac charge quantization condition is preserved
- Dirac charge quantization in each sector is violated, because the Dirac string is physical
- Basis for detecting millimagnetic monopoles using Aharonov-Bohm phase detectors (Terning and Verhaaren '18)

Length scales

- Bohr radius less than interparticle spacing :
- Ground state has no magnetic properties and is stable, but in a background magnetic field it becomes **unstable**, provided effective magnetic field larger than internal tension
- Need to wait for bound monopoles to tunnel before Parker effect can initiate

- Bohr radius bigger than interparticle spacing :
- Galactic population is a dark plasma, more complicated story, but don't have to wait for tunneling to occur, and each dark monopole is accelerated by the effective magnetic field



Transition depends on dark monopole mass and magnetic fine structure constant

Cross-section

- Usually quoted in terms of a “momentum transfer” or “transport” cross-section which cuts out forward scattering, effectively capturing only hard scattering

$$\boxed{\sigma_T = \int \frac{d\sigma}{d\cos\theta}(1-\cos\theta)d\cos\theta}$$

- Numerous soft scattering vs. few hard scatterings: both could be important
  - soft scattering needs a different treatment than computing the transfer cross-section
- Bound-state-bound-state monopole scattering is a 4-body problem
  - Resort to several approximations

Soft Scattering [following Ackerman, Buckley, Carroll, Kamionowski, 2006]

- Each soft scatter contributes a small momentum-transfer q, but over many scatterings these can add up to a sizable change in kinetic energy.
- To estimate this effect we approximately solved the classical equations of motion of a single monopole bound state moving in the van der Waals potential of another bound state.
- This will give an estimate of the number of soft scatterings, and therefore timescale, needed to cause a change in kinetic energy comparable to the initial kinetic energy

Soft Scattering [adapted from Ackerman, Buckley, Carroll, Kamionowski, 2006]

Approximate inter-bound state potential as $\boxed{V_{\mathrm{vdW}} \sim -\frac{\alpha_D}{L_0}\frac{L_0^6}{r^6}}$

$$\boxed{\delta q \simeq \pm V'_{\mathrm{vdW}}(b)\left(\frac{b}{v}\right)}$$

to first order in transit time $T \simeq b/v$

Bound state undergoes a random walk as it orbits the halo, with $\boxed{\langle \delta v^2 \rangle = (\delta v)^2 \delta n}$ non-vanishing

Estimate the timescale for $\boxed{\langle \delta v^2 \rangle \simeq v^2}$ after a certain number of orbits, and require that timescale τ_{soft} to be on the age of the galaxy or longer

Interpreting τ_{soft} in terms of a scattering rate, $\boxed{\tau_{\mathrm{soft}} \simeq \left(\langle n \sigma_{\mathrm{soft}} v \rangle\right)^{-1}}$

$$\boxed{\sigma_{\mathrm{soft}} \sim \frac{\alpha_D^2}{M^2 v^4}}$$

Cross Section

- In summary, in the fast limit

$$\boxed{v \gg \alpha_D}$$

we see the cross-section for both hard and soft scattering is parametrically the same as Coulomb scattering

- In the opposite limit $\boxed{v \ll \alpha_D}$
  the cross-section is too large as it is set by the Bohr radius; then milli-magnetic monopoles can't be all of the dark matter

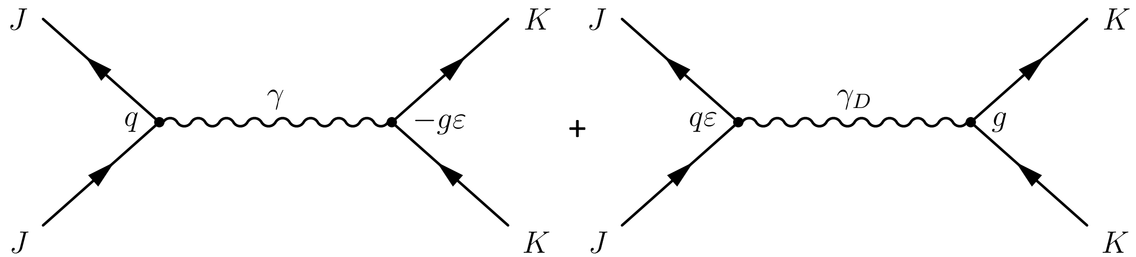
[Terning, Verhaaren, 2019]

Tunneling of bound monopole ground state in background magnetic field

- For a mixed monopole mass, SIDM constraints bound α_D from above, so using this as input, we obtain the largest upper bound on Q_m by requiring the decay happens on time scales longer than the dynamo timescale.

- If α_D is smaller than the SIDM bound, then the upper bound on Q_m decreases

mmCP production



Terning, Verhaaren, 2020

- If CP is a good symmetry, then s-channel pair production of dark magnetic monopoles from (Standard Model) fermions and single (dark/ordinary) photon exchange vanishes at leading order in ε

$$\mathcal{A} = 0$$

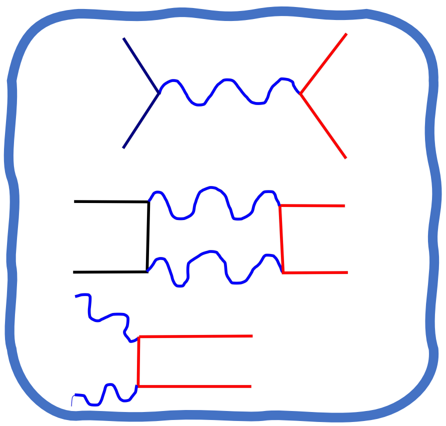
- Originates from different CP properties of electric field (J^{PC} =1^{--}) and magnetic field (J^{PC} = 1^{-+})

mmCP production in early Universe: freeze-in

Dark magnetic monopoles are weakly coupled to the SM, and have unusual production mechanisms

- Dark photon-dark photon fusion induced production non-zero

  (Terning, Verhaaren, 2020)
- Expect dark photon-photon fusion induced production to be non-vanishing
- Expect box diagrams to be non-vanishing



Investigating whether this leads to qualitatively different preferred regions: stay tuned!

Monopole Interactions: Zwanziger's two-potential formalism [Csaki, Shirman, Terning]

- With both electric and magnetic sources present, Maxwell equations can't be described by a single gauge potential A
- Introduce two gauge potentials A, B, and constant Lorentz-violating vector n; naïve extra-degrees of freedom are projected out by n
- Action is Lorentz-violating, but EOM are Lorentz-covariant

$$
\begin{aligned}
\partial_\nu (F^{\mu\nu} + i^* F^{\mu\nu}) &= J^\mu + iK^\mu \\
F_{\mu\nu} &= \frac{n^\alpha}{n^2}\left(n_\mu F^A_{\alpha\nu} - n_\nu F^A_{\alpha\mu} - \varepsilon^\beta_{\mu\nu\alpha} n^\gamma F^B_{\gamma\beta}\right) \\
{}^*F_{\mu\nu} &= \frac{n^\alpha}{n^2}\left(n_\mu F^B_{\alpha\nu} - n_\nu F^B_{\alpha\mu} - \varepsilon^\beta_{\mu\nu\alpha} n^\gamma F^A_{\gamma\beta}\right)
\end{aligned}
$$

For dark + ordinary sector: (A,B) and (A_D ,B_D) [Terning, Verhaaren, 2018]

Caveats about astrophysical populations

- We assumed that all of bound monopole systems are in their ground state, at least when it is self-consistent to do so.
- This leads to a conservative upper bound on the dark magnetic monopole coupling, arising from bounds on self-interacting dark matter.
- The above assumption is not self-consistent when the interparticle spacing becomes smaller than the Bohr radius, and here we resorted to approximating the galactic population as a non-degenerate, collisionless plasma.
- Characterizing the different occupation numbers requires following the coupled Boltzmann equations, including dissipative processes, over the history of the Milky Way galaxy
- Inelastic collisions between bound monopoles can produce long-lived excited states, dissipating initial kinetic energy, since we're in the limit

$$\boxed{\alpha_D \ll v}$$

Magnetic plasma oscillations

- Could the decay of galactic magnetic field in the presence of magnetic monopoles be the first half of an oscillation in the galactic magnetic field?
- In other words, could magnetic monopoles support the galactic magnetic field?
- Turner, Bogdan, Parker (1982) and Parker (1987) reach a negative conclusion: Monopoles could induce plasma oscillations in the Galactic magnetic field, given by the plasma frequency w.
- But need the oscillation timescale of B to occur on timescales longer than the Galactic dynamo timescale
- Moreover, to avoid Landau damping on kpc scales, the phase velocity w/k of oscillations needs to be greater than the monopole virial velocity
- For mmCP: we repeated same analysis and no region of effective magnetic coupling is possible, so for mmCPs magnetic plasma oscillations are irrelevant

Magnetic plasma oscillations: Half-velocity effect [Parker '87]

The magnetohydrodynamic equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

implying that in the limit of infinite conducting fluid, the magnetic fields travel at the same speed as the fluid

If magnetic monopoles support plasma oscillations on timescales less than the Galactic dynamo timescale, Parker found magnetic fields follow the conducting fluid at half the velocity:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\frac{\mathbf{v}}{2} \times \mathbf{B}) - \nabla \times (\frac{1}{2} \eta \nabla \times \mathbf{B})$$

Maxwell Equations with kinetic mixing

$$\boxed{\begin{aligned} \partial_\mu F^{\mu\nu} - \varepsilon \partial_\mu F_D^{\mu\nu} &= e J^\nu \ , \\ \partial_\mu F_D^{\mu\nu} - \varepsilon \partial_\mu F^{\mu\nu} &= e_D J_D^\nu + m_D^2 A_D^\nu \ , \\ \partial_\mu \tilde{F}^{\mu\nu} &= 0 \ , \\ \partial_\mu \tilde{F}_D^{\mu\nu} &= g_D K_D^\nu , \end{aligned}}$$

Undo the kinetic mixing with $A \to A + \varepsilon A_D$ giving

Maxwell Equations with kinetic mixing

$$\partial_\mu F^{\mu\nu} = eJ^\mu \ ,$$

$$\partial_\mu F_D^{\mu\nu} = e_D J_D^\nu + m_D^2 A_D^\nu + \varepsilon e J^\nu \ ,$$

$$\partial_\mu \tilde{F}^{\mu\nu} = \boxed{-\varepsilon g_D K_D^\nu} \ ,$$

$$\partial_\mu \tilde{F}_D^{\mu\nu} = g_D K_D^\nu \ .$$

- At large distances compared to the inverse dark photon mass, these reduce to ordinary Maxwell's equations sourced by a dark magnetic monopole having a ``milli-magnetic'' charge
- For static EM currents, both an ordinary magnetic field and a dark magnetic field are generated, the latter exponentially suppressed by the dark photon mass