Neutrino oscillation as a probe of new physics

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Based on arXiv:2210.09436
https://github.com/shengfong/nuprobe

Neutrino oscillation is new physics

For neutrinos to oscillate, we need

$$\begin{aligned}\mathcal{L}_\nu &= \frac{1}{2}\left(i\overline{\nu_{L\alpha}}\partial\!\!\!/\nu_{L\beta} - \overline{\nu_{L\alpha}}m_{\alpha\beta}\nu'_\beta\right) + m_\alpha\overline{\ell_{L\alpha}}\ell_{R\alpha} \\ &\quad - \left(\frac{g}{2}W^-_\mu\overline{\ell_{L\alpha}}\gamma^\mu\nu_{L\alpha} + \frac{g}{\sqrt{2}\cos\theta_W}Z_\mu\overline{\nu_{L\alpha}}\gamma^\mu\nu_{L\alpha}\right) + \text{H.c.} \qquad \alpha = e, \mu, \tau\end{aligned}$$

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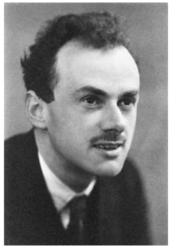
Majorana

$$\nu' = \nu_L^c$$

Dirac



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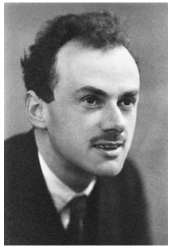

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$$\nu_L \to U\nu_L$$
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$$m = U\hat{m}Z$$

Leptonic (PMNS) mixing matrix

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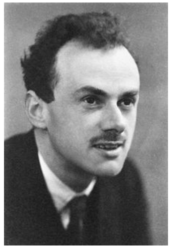

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Leptonic (PMNS) mixing matrix

★ UV completion might result in **new** measurable effects

3+N neutrino oscillations

A brief review

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \qquad \alpha = e, \mu, \tau, s_1, s_2, ..., s_{3+N} \qquad i = 1, 2, ..., 3+N$$

$$i\frac{d}{dt}|\nu_\alpha(t)\rangle = (\mathcal{H}_0 + \mathcal{H}_I)|\nu_\alpha(t)\rangle$$

$$\mathcal{H}_0 |\nu_i\rangle = E_i |\nu_i\rangle, \quad E_i = \sqrt{|\vec{p}_i|^2 + m_i^2}, \quad \langle\nu_\beta|\mathcal{H}_I|\nu_\alpha\rangle = V_{\beta\alpha}$$

Hermitian

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Relativistic $\quad t = x, \quad E_i \simeq E + \frac{m_i^2}{2E}$

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$$i\frac{d}{dx}S_{\beta\alpha}(x) = \sum_\gamma\left[\sum_i U_{\beta i}\Delta_i U^*_{\gamma i} + V_{\beta\gamma}\right]S_{\gamma\alpha}(x)$$

$$\Delta \equiv \frac{1}{2E}\mathrm{diag}\left(m_1^2, m_2^2, ..., m_{3+N}^2\right)$$

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Probability

$$P_{\beta\alpha}(x) = |S_{\beta\alpha}(x)|^2$$

3+N neutrino oscillations

Vacuum mass basis $\widetilde{S}(x) = U^\dagger S(x) U$ $\quad \widetilde{H} = U^\dagger H U = \Delta + U^\dagger V U$

Diagonalization with a unitary X $\quad \widetilde{H} = X \hat{H} X^\dagger$ $\quad \hat{H} = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_{3+N})$

$$\widetilde{S}(x) = X e^{-i\hat{H}x} X^\dagger \qquad S(x) = U X e^{-i\hat{H}x} (UX)^\dagger$$

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Spatially varying matter density $\quad V(x) = V_a, \;\; x_{a-1} < x < x_a$

$$S = T \prod_{a=1} S^{(a)} \qquad S^{(a)} \equiv \left(UX^{(a)}\right) e^{-i\hat{H}^{(a)} x^{(a)}} \left(UX^{(a)}\right)^\dagger$$

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$$S^{(a)}_{\beta\alpha} = \sum_{i,j,k} U_{\beta i} U^*_{\alpha j} X^{(a)}_{ik} X^{(a)*}_{jk} e^{-i\lambda^{(a)}_k x^{(a)}}$$

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We need to solve $S^{(a)}_{\beta\alpha} = \sum_{i,j,k} U_{\beta i} U^*_{\alpha j} X^{(a)}_{ik} X^{(a)*}_{jk} e^{-i\lambda^{(a)}_k x^{(a)}}$

Analytic solutions

Unitarity + diagonalization

$$\begin{aligned}
\sum_k X_{ik} X_{jk}^* &= \delta_{ij}\,, \\
\sum_k \lambda_k X_{ik} X_{jk}^* &= (\widetilde{H})_{ij}\,, \\
\sum_k \lambda_k^2 X_{ik} X_{jk}^* &= (\widetilde{H}^2)_{ij}\,, \\
&\vdots \\
\sum_k \lambda_k^{2+N} X_{ik} X_{jk}^* &= (\widetilde{H}^{2+N})_{ij}\,,
\end{aligned}$$

[Yasuda, arXiv:0704.1531]

3-flavor [Kimura, Takamura, Yokomakura, arXiv:hep-ph/0203099]

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Inversion of Vandermonde matrix

$$X_{ik} X_{jk}^* = \frac{\sum_{p=0}^{2+N} (-1)^p \, (\widetilde{H}^p)_{ij} c_{2+N-p,k}}{Z_k}$$

$$Z_k \equiv \prod_{p \neq k} (\lambda_p - \lambda_k)$$

$$c_{p,k} \equiv \sum_{\{q \neq r \neq \ldots\} \neq k} \underbrace{\lambda_q \lambda_r \ldots}_{p}$$

3-flavor [Kimura, Takamura, Yokomakura, arXiv:hep-ph/0203099]

(Not meant to be read)

Solution of 4-flavor on 1 page

$$\lambda_{1,2} \quad = \quad \frac{\mathcal{T}}{4} - \mathcal{S} \pm \frac{1}{2}\sqrt{2\mathcal{P} - 4\mathcal{S}^2 + \frac{\mathcal{Q}}{\mathcal{S}}}, \quad \lambda_{3,4} = \frac{\mathcal{T}}{4} + \mathcal{S} \pm \frac{1}{2}\sqrt{2\mathcal{P} - 4\mathcal{S}^2 - \frac{\mathcal{Q}}{\mathcal{S}}},$$

$$\mathcal{T} \equiv \mathrm{Tr}\widetilde{H}, \quad \mathcal{D} \equiv \det \widetilde{H}, \quad \mathcal{A} \equiv \frac{1}{2}\left(\mathcal{T}^2 - \mathcal{T}_2\right), \quad \mathcal{T}_p \equiv \mathrm{Tr}(\widetilde{H}^p),$$

$$\mathcal{P} \quad \equiv \quad \frac{3}{8}\mathcal{T}^2 - \mathcal{A}, \quad \mathcal{Q} \equiv -\frac{\mathcal{T}^3}{8} + \frac{\mathcal{T}\mathcal{A}}{2} - \mathcal{A}_2, \quad \mathcal{S} \equiv \frac{1}{2}\sqrt{\frac{2}{3}\mathcal{P} + \frac{2}{3}\mathcal{F}_1 \cos \mathcal{G}_1},$$

$$\mathcal{F}_1 \equiv \sqrt{\mathcal{A}^2 - 3\mathcal{T}\mathcal{A}_2 + 12\mathcal{D}}, \quad \mathcal{G}_1 \equiv \frac{1}{3}\arccos\left(\frac{\Delta_1}{2\mathcal{F}_1^3}\right), \quad \Delta_1 \equiv 2\mathcal{A}^3 - 9\mathcal{T}\mathcal{A}\mathcal{A}_2 + 27\mathcal{T}^2\mathcal{D} + 27\mathcal{A}_2^2 - 72\mathcal{A}\mathcal{D}.$$

$$X_{i1}X_{j1}^* = \frac{\delta_{ij}\lambda_2\lambda_3\lambda_4 - (\widetilde{H})_{ij}\left(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4\right) + (\widetilde{H}^2)_{ij}\left(\lambda_2 + \lambda_3 + \lambda_4\right) - (\widetilde{H}^3)_{ij}}{\left(\lambda_2 - \lambda_1\right)\left(\lambda_3 - \lambda_1\right)\left(\lambda_4 - \lambda_1\right)},$$

$$X_{i2}X_{j2}^* = \frac{\delta_{ij}\lambda_1\lambda_3\lambda_4 - (\widetilde{H})_{ij}\left(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4\right) + (\widetilde{H}^2)_{ij}\left(\lambda_1 + \lambda_3 + \lambda_4\right) - (\widetilde{H}^3)_{ij}}{\left(\lambda_1 - \lambda_2\right)\left(\lambda_3 - \lambda_2\right)\left(\lambda_4 - \lambda_2\right)},$$

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For (n>4)-flavor, eigenvalues have to be numerical

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3-flavor scenario

One interesting quantity for CP violation (Jarlskog combination)

$$\widetilde{J}^{jk}_{\beta\alpha} \equiv \mathrm{Im}\left(\widetilde{U}_{\beta j}\widetilde{U}^{*}_{\alpha j}\widetilde{U}^{*}_{\beta k}\widetilde{U}_{\alpha k}\right), \quad \beta \neq \alpha, j \neq k$$

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Unitary scenario (arbitrary matter potential)

$$\widetilde{J}^{jk}_{\beta\alpha} = \frac{\mathrm{Im}\left[\left(H^2\right)_{\alpha\beta}\left(H\right)_{\beta\alpha}\right]}{\lambda_{21}\lambda_{31}\lambda_{32}} \sum_l \epsilon_{jkl}$$

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★ Unitary relations $\widetilde{J}^{12}_{\beta\alpha} + \widetilde{J}^{13}_{\beta\alpha} = \widetilde{J}^{21}_{\beta\alpha} + \widetilde{J}^{23}_{\beta\alpha} = \widetilde{J}^{31}_{\beta\alpha} + \widetilde{J}^{32}_{\beta\alpha} = 0$

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n-flavor

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★ <u>Diagonal</u> matter potential

$$\lambda_{21}\lambda_{31}\lambda_{32}\widetilde{J}^{jk}_{\beta\alpha} = \mathrm{Im}\left\{\left(H_0^2\right)_{\alpha\beta}\left(H_0\right)_{\beta\alpha}\right\}\sum_l \epsilon_{jkl} = \Delta_{21}\Delta_{31}\Delta_{32}J^{jk}_{\beta\alpha}$$

[Naumov, IJMPD 1, 379 (1992)] [Harrison & Scott, 21, arXiv:hep-ph/0203021]

Low scale “nonunitarity”

Definition: light steriles states (3+N) kinematically allowed to participate in oscillation but heavy enough that their fast oscillations are averaged out

Leading order in small “active-heavy” mixing $\widetilde{H} = \Delta + U^\dagger V U$

[CSF, Minakata & Nunokawa, arXiv:1609.08623, arXiv:1712.02798]

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[CSF, Minakata & Nunokawa, arXiv:1609.08623, arXiv:1712.02798]

Nonunitary!

Low scale "nonunitarity"

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In vacuum

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J^{12}_{\beta\alpha} + J^{13}_{\beta\alpha} &= -\mathrm{Im}\left\{\left(UU^\dagger\right)_{\beta\alpha} U_{\alpha 1}U^*_{\beta 1}\right\}, \\
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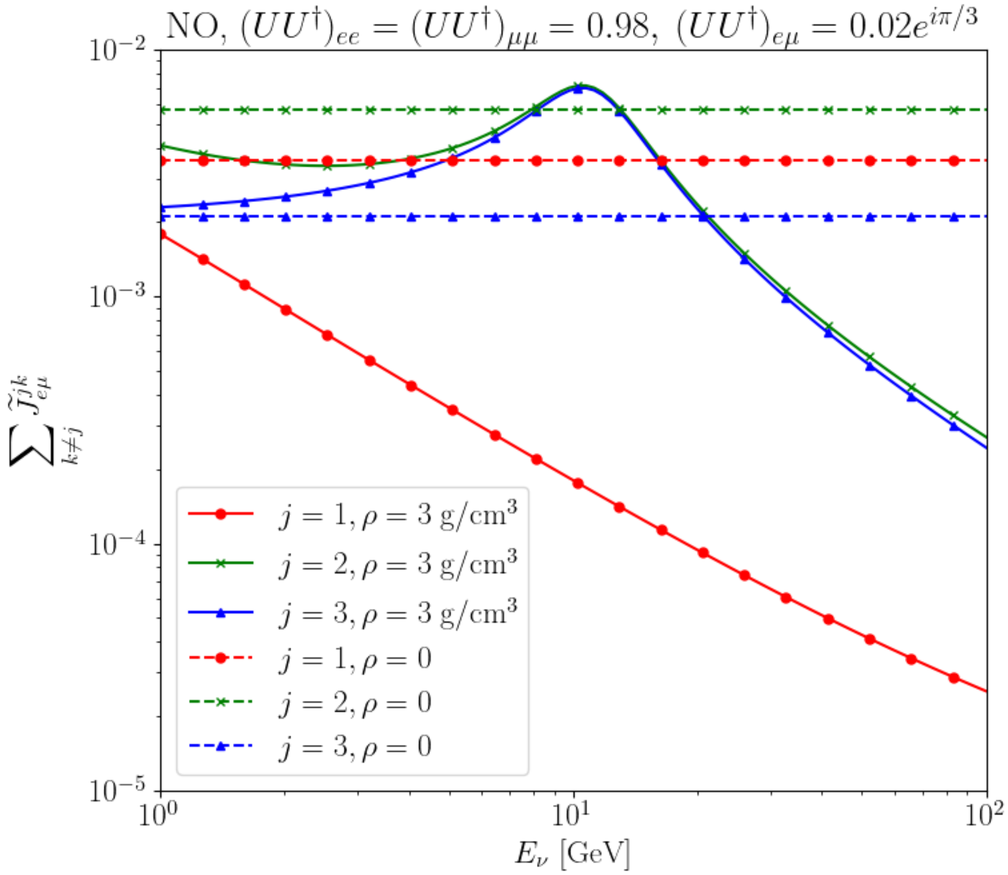
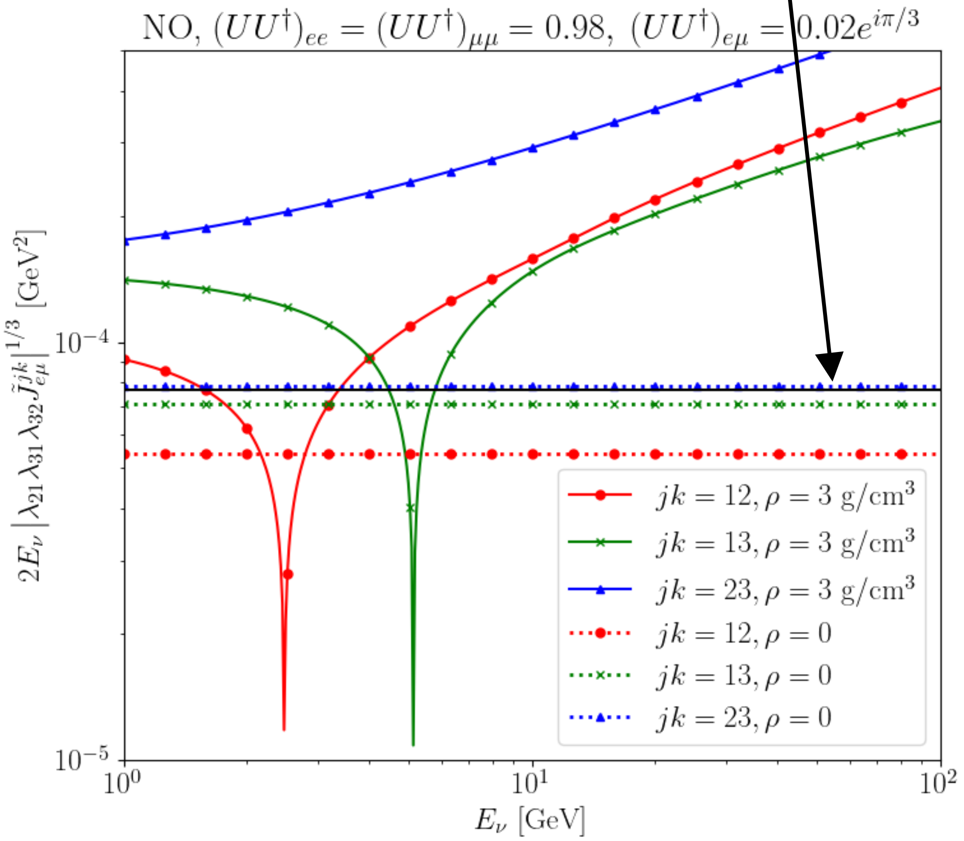
★ NHS identity violated as well

Low scale "nonunitarity"

"Unitary NHS"





Unitary relations violated

NHS identity violated

Nonstandard neutrino interactions (NSI)

We can parametrize the matter potential with NSI as [Dev et al., arXiv:1907.00991]

$$\mathrm{V} = \sqrt{2} G_F n_e \begin{pmatrix} 1 + \epsilon_{ee} - \frac{1}{2} n_n / n_e & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} - \frac{1}{2} n_n / n_e & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} - \frac{1}{2} n_n / n_e \end{pmatrix}$$

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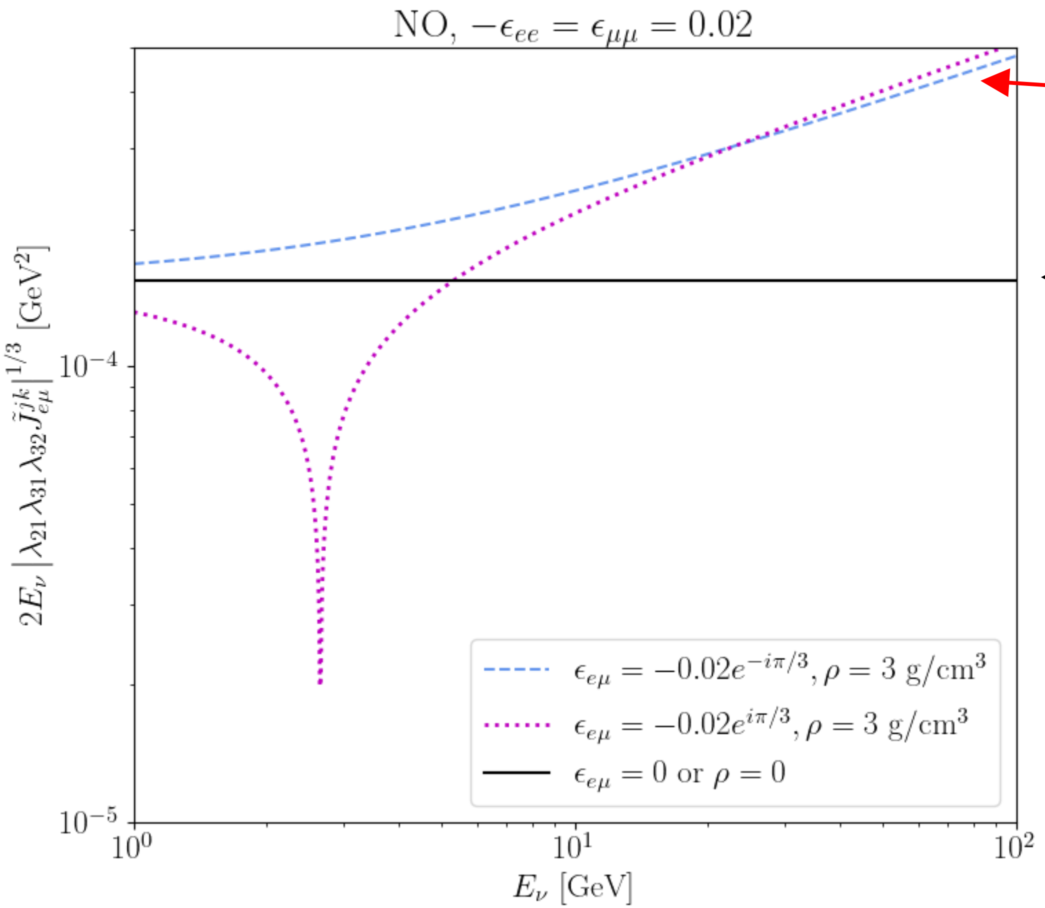
★ Modified NHS identity

$$\lambda_{21}\lambda_{31}\lambda_{32}\widetilde{J}^{jk}_{\beta\alpha} = \Delta_{21}\Delta_{31}\Delta_{32}J^{jk}_{\beta\alpha} + \boxed{\mathrm{Im}\left\{\sum_\gamma \left[(H_0)_{\alpha\gamma} V_{\gamma\beta} + V_{\alpha\gamma}(H_0)_{\gamma\beta}\right](H_0)_{\beta\alpha}\right\}}$$

Nonstandard neutrino interactions



Modified NHS

NHS for diagonal potential

Strategy

Unitary relations

$$\sum_k \widetilde{J}^{jk}_{\beta\alpha} = 0$$

Yes

No

NHS identity

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Low scale nonunitarity

NSI? A challenging task
but doable in principle

Yes

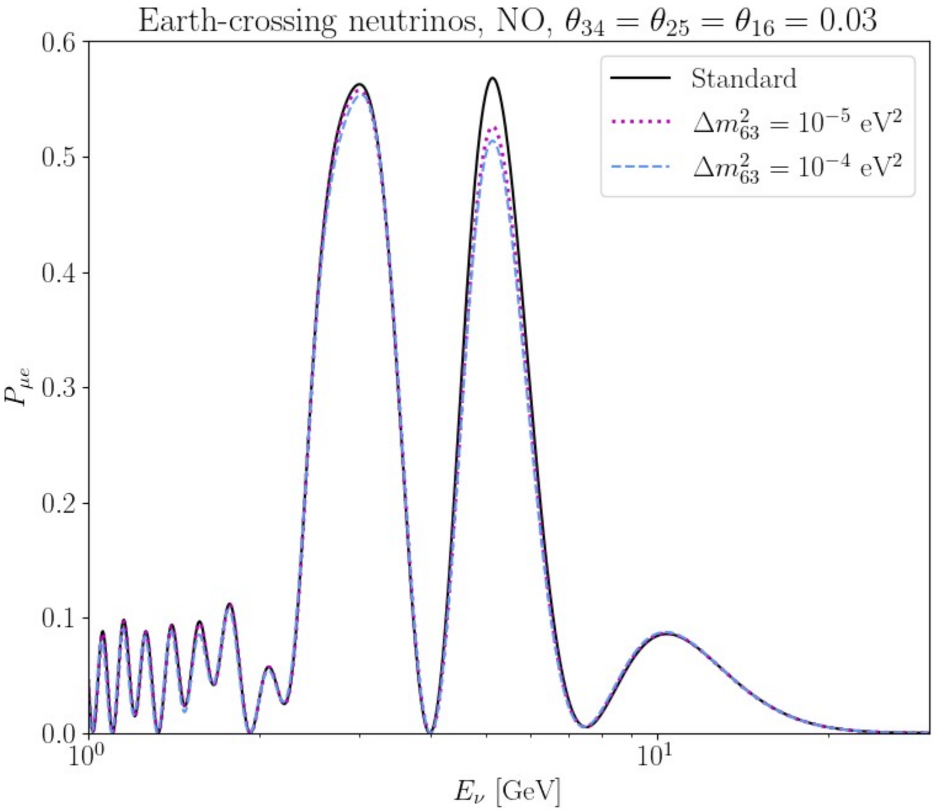
No

Diagonal matter potential
NSI?

NSI

Other new physics

Quasi (pseudo)-Dirac scenario for Earth-crossing neutrinos



[Anamiati, Fonseca & Hirsch, arXiv:1710.06249]

[Anamiati, De Romeri, Hirsch, Ternes & Tórtola, arXiv:1907.00980]

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

$$\mathrm{m}_L, m_R \ll m_D$$

$$\mathrm{m}_{i\pm} \simeq \hat{m}_{Di} \left[1 + \mathcal{O}(\tfrac{m_L, m_R}{m_D})\right]$$

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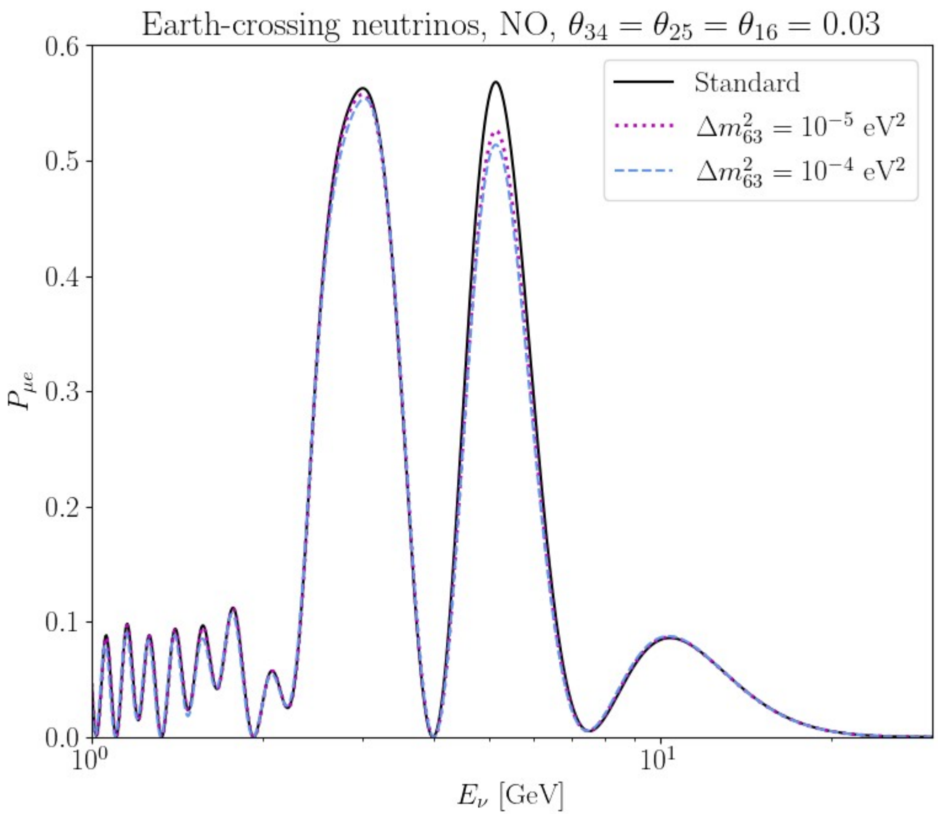

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Connection to leptogenesis

$$|\epsilon^{\max}| \simeq \frac{\delta m}{2 m_\nu}$$

[CSF, Gregoire & Tonero, arXiv:2007.09158]

NuProbe

The analytical solutions (up to 7-flavor) is implemented in NuProbe (https://github.com/shengfong/nuprobe).

The last plot is produced using simplified density profile for the Earth

```
####### Matter density profile #######
# Earth radius [km]
R0 = 6371

# Layers in [km]
LL = np.array([0, 0.1, 0.45, 0.81, 1.19, 1.55, 1.9, 2])*R0

# Matter densities in [g/cm^3]
rho_LL = [3.6, 5, 10, 13, 10, 5, 3.6]
```

Takeaway

- Neutrino oscillations can be a probe to new physics (beyond the standard 3-flavor paradigm)
- Analytic solutions render new physics more transparent
- Low scale nonunitarity is qualitatively (quantitatively) different from NSI
  - For the former, unitary relations and NHS identity are violated
  - For the latter, unitary relations are satisfied while the NHS identity is violated only for nondiagonal matter profile
- Analytic solutions implemented in a NuProbe up 7-flavor and accept arbitrary mixing matrix and matter potential

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Thanks! Comments?