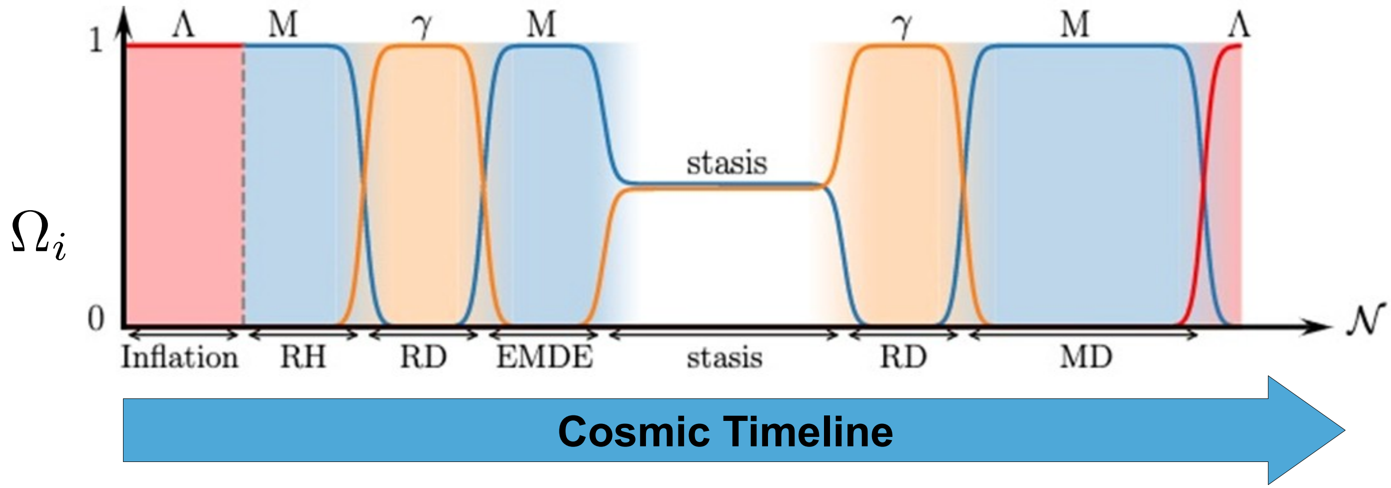
Stasis in an Expanding Universe

Overview, Concrete Realizations, and Observational Consequences



Brooks Thomas

LAFAYETTE COLLEGE

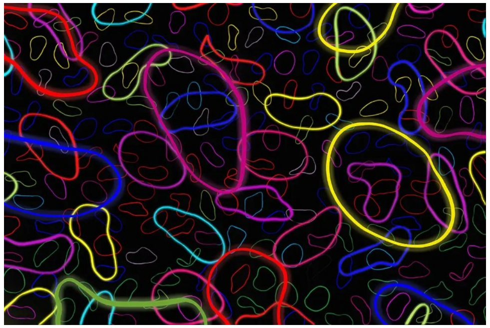
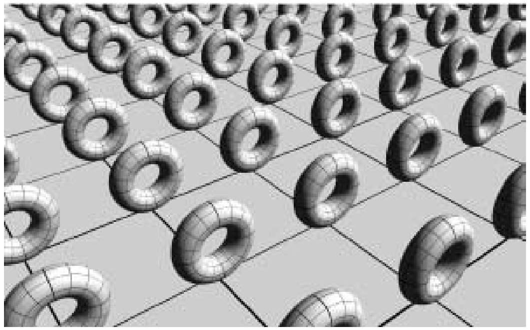
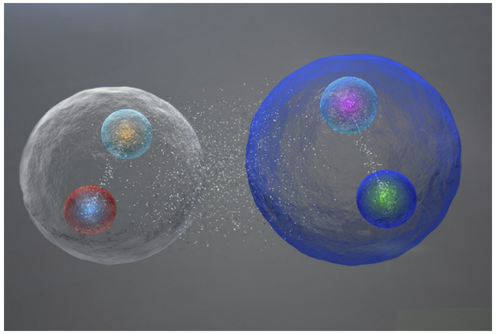
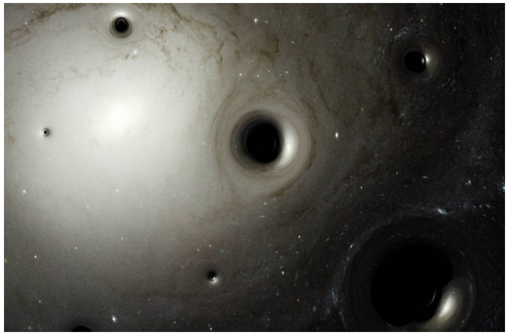
Based on work done in collaboration with:

Keith R. Dienes, Fei Huang, Lucien Heurtier, Doojin Kim, and Tim M. P. Tait [arXiv:2108.02204, 2212.01369]

Mitchell Conference, Texas A&M University, May 19th, 2023

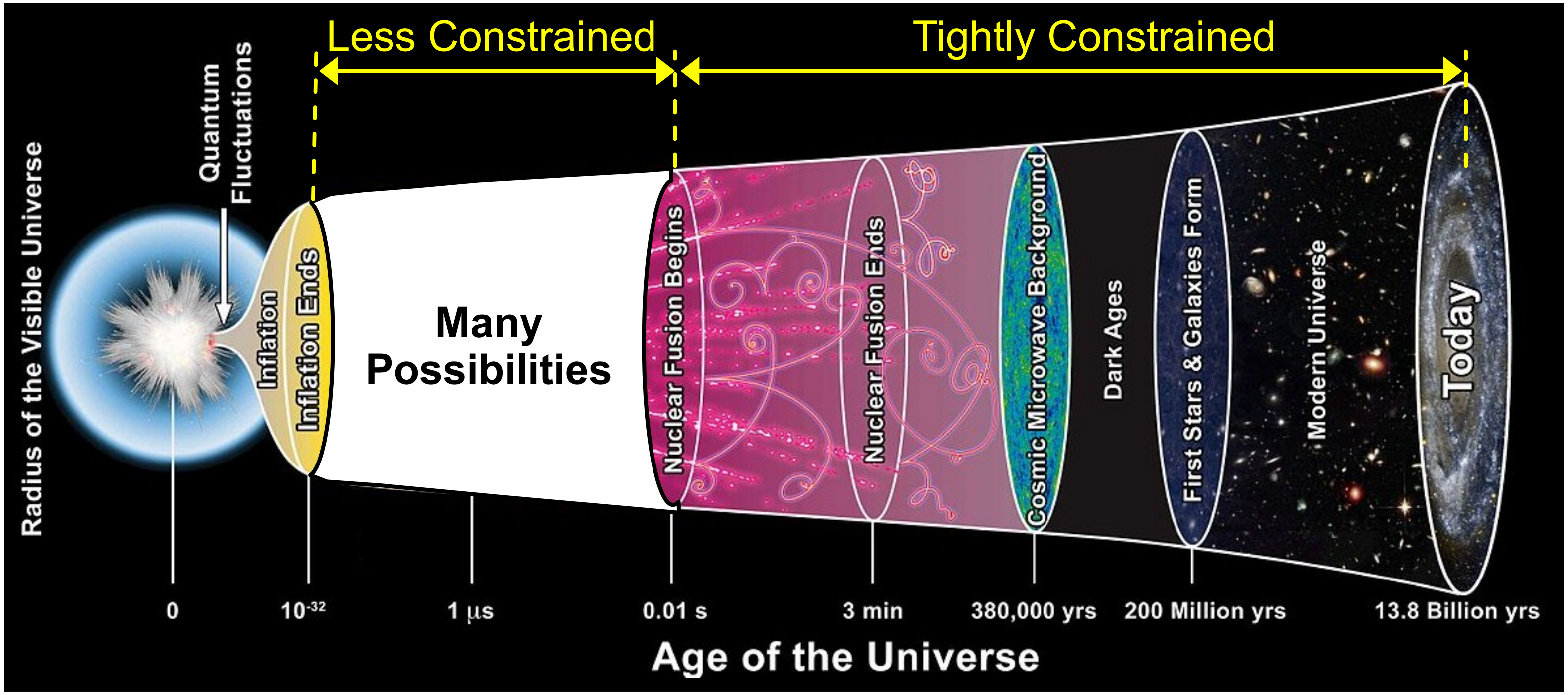
Towers of Unstable States

- A wide variety of scenarios for new-physics predict **towers of massive, unstable states** with a broad spectrum of masses, cosmological abundances, and lifetimes.
- Such towers are a generic feature of, for example,...
  - String theory (string moduli, axions, etc.)
  - Theories with extra spacetime dimensions (KK towers)
  - Scenarios with confining dark/hidden-sector gauge groups (bound-state resonances)
  - Scenarios which lead to the production of primordial black holes with an extended mass spectrum (the black holes themselves)
- In some cases, such states can give rise to astrophysical signals, signals at colliders, etc.; in others, they are too heavy/short-lived.

Cosmological Consequences

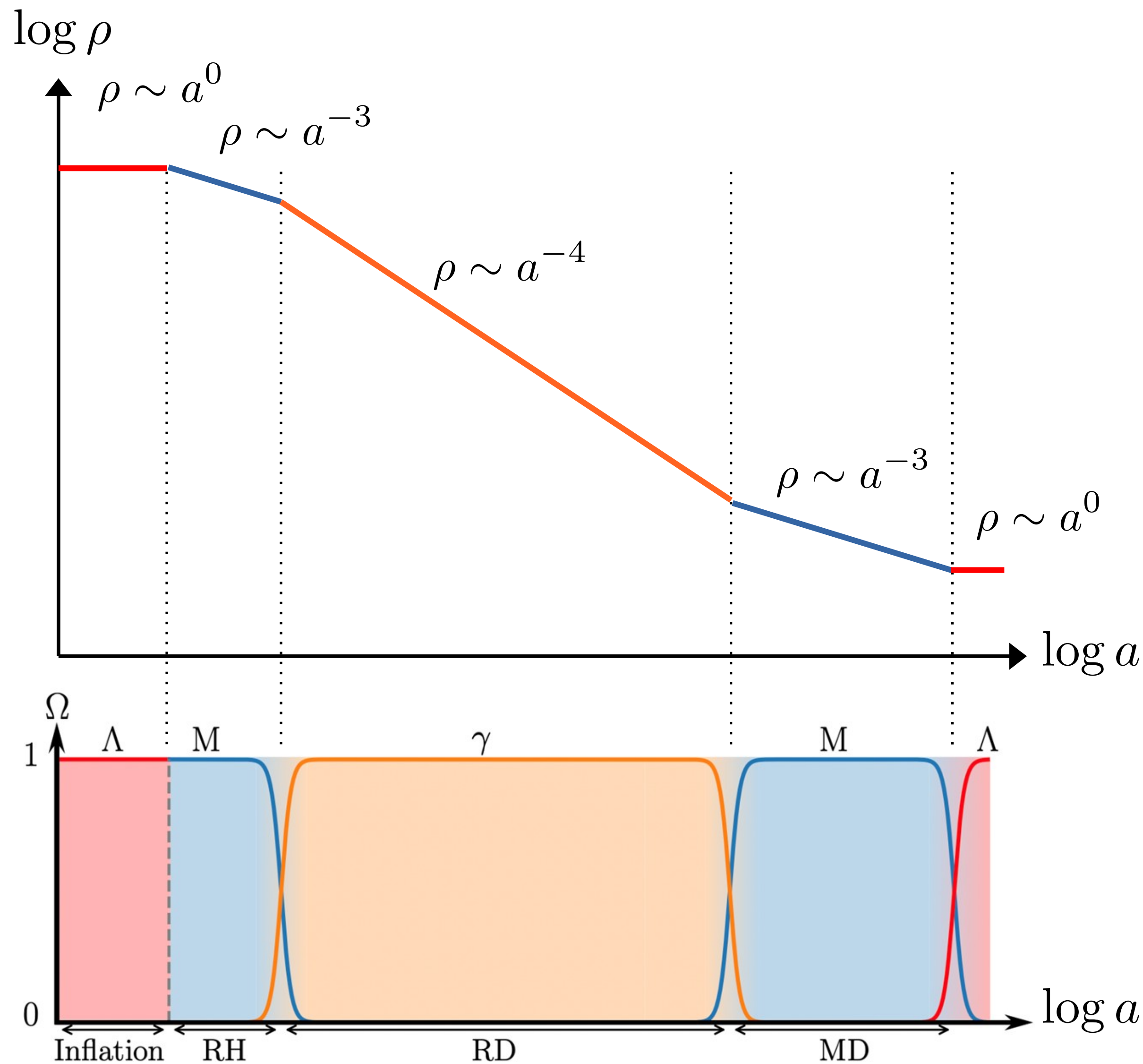
- The presence of such towers can have a significant impact on early-universe cosmology – even if the tower states are too heavy/short-lived to be accessible.



- Indeed, as I'll show in this, talk, such towers can give rise to **stable, mixed-component eras**: eras in which the abundances of multiple cosmological energy components (in this case, matter and radiation) remain effectively constant over an extended period.
- Moreover, these eras are **global attractors**: if the basic conditions under which they arise satisfied, the universe will evolve toward them.

The Traditional Picture

- The energy densities associated with different ***cosmological components*** (matter, radiation, vacuum energy, etc.) with different equations of state scale behave differently under cosmic expansion.
- As a result, except during brief transition periods, the energy density of the universe is ***dominated by one such component***.
- This is certainly the case in the standard cosmology.
- Moreover, it's typically the case event in ***modified cosmologies*** (e.g., with epochs of early matter- or vacuum-energy-domination) as well.

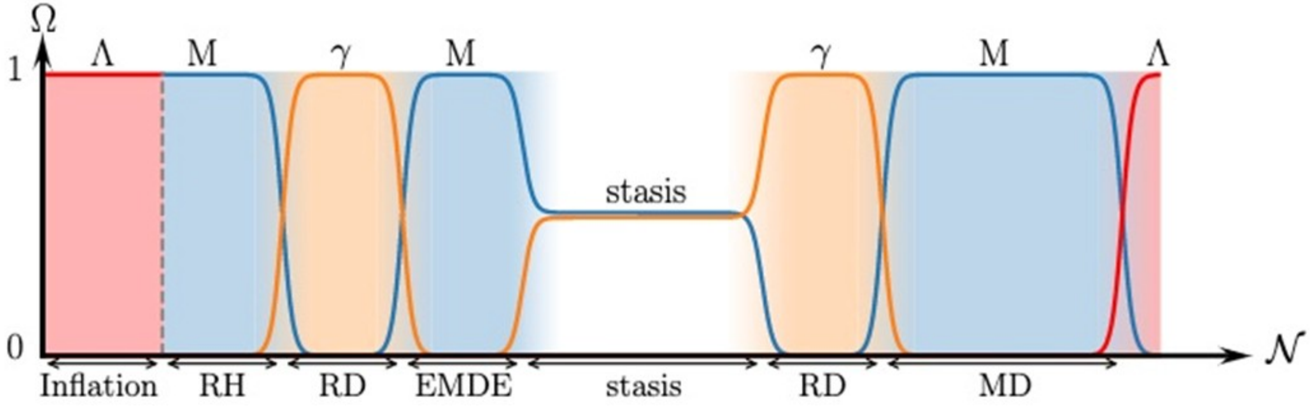


A Stable Mixed-Component Era?

Q So… is it indeed possible to achieve a stable, mixed-component cosmological era in which multiple Ω_i maintain non-neglible, effectively constant values over an extended period?

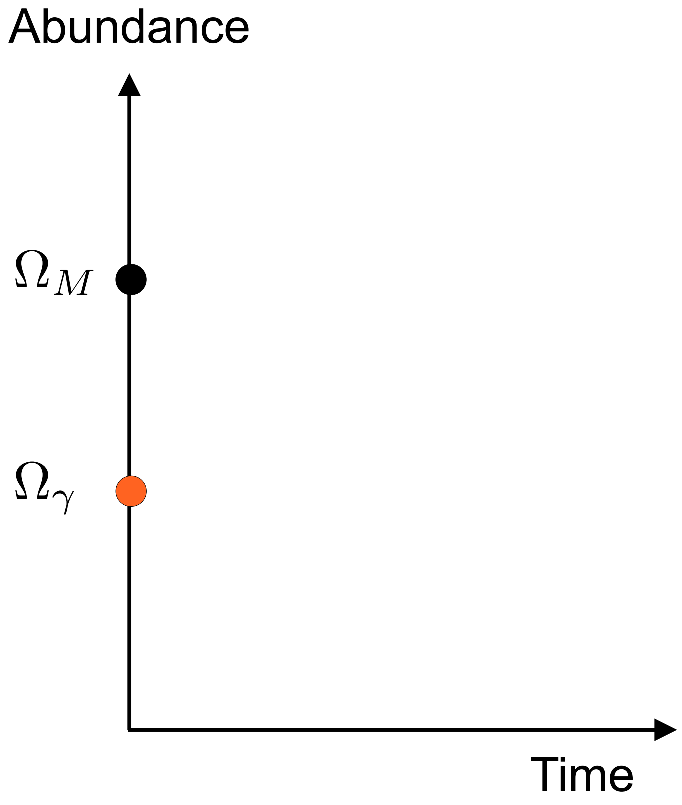
- In other words, can we arrange for slices of the "cosmic pie" corresponding to components with different equations of state to ***remain effectively fixed*** over an extended period?
- At first glance, arranging this may seem impossible – or at least attainable only with a ridiculous amount of fine-tuning.

A However, it turns out that such eras, which we call periods of ***cosmic stasis***, can be realized in a straightforward manner in the presence of towers of unstable states.



Underpinnings of Stasis

- To see how a stasis era can arise, let us consider a universe effectively consisting of matter and radiation alone, with all other Ω_i negligible.



Boltzmann Equations

$$\frac{d\rho_M}{dt} = -3H\rho_M$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma$$

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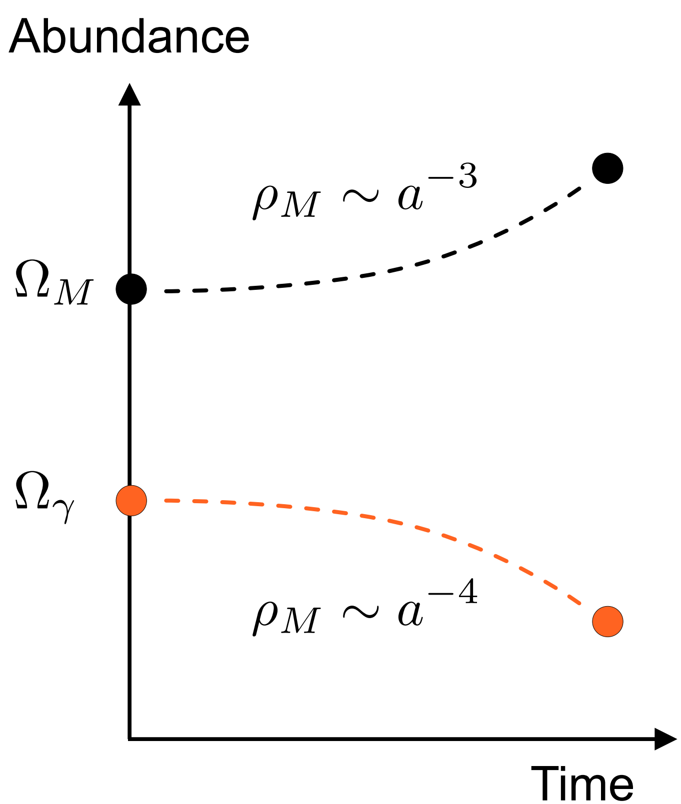

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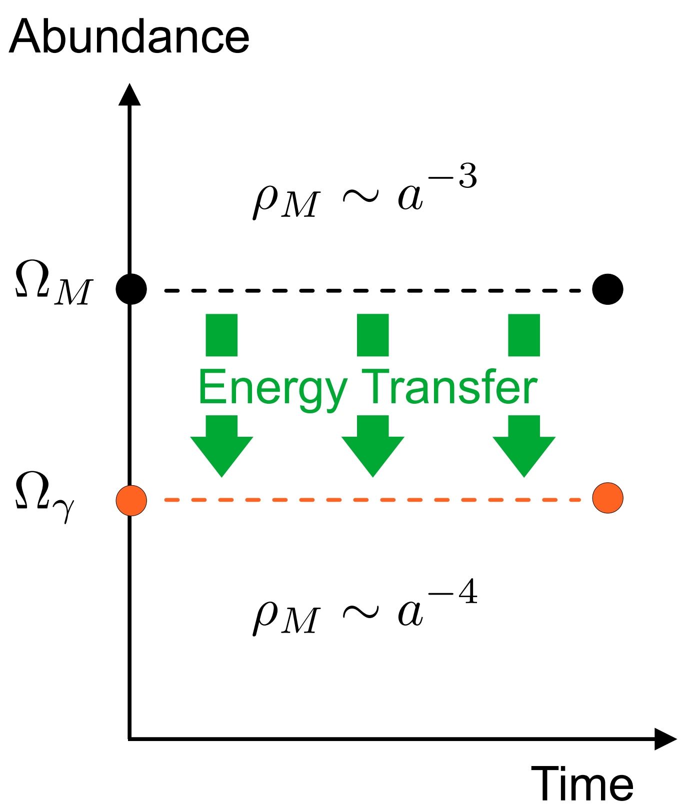
- Since ρ_M and ρ_γ scale differently under cosmic expansion, Ω_M typically increases, while Ω_γ decreases.
- In order to compensate for this effect, what's needed is a ***continuous transfer of energy density*** from matter to radiation.



Boltzmann Equations

$$\frac{d\rho_M}{dt} = -3H\rho_M - S(t)$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + S(t)$$

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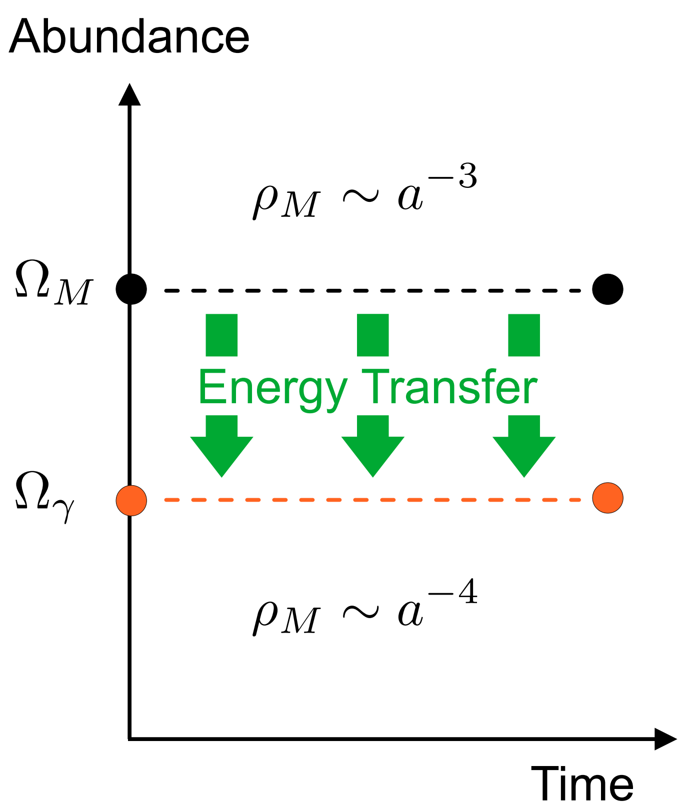

Boltzmann Equations

$$\frac{d\rho_M}{dt} = -3H\rho_M - S(t)$$

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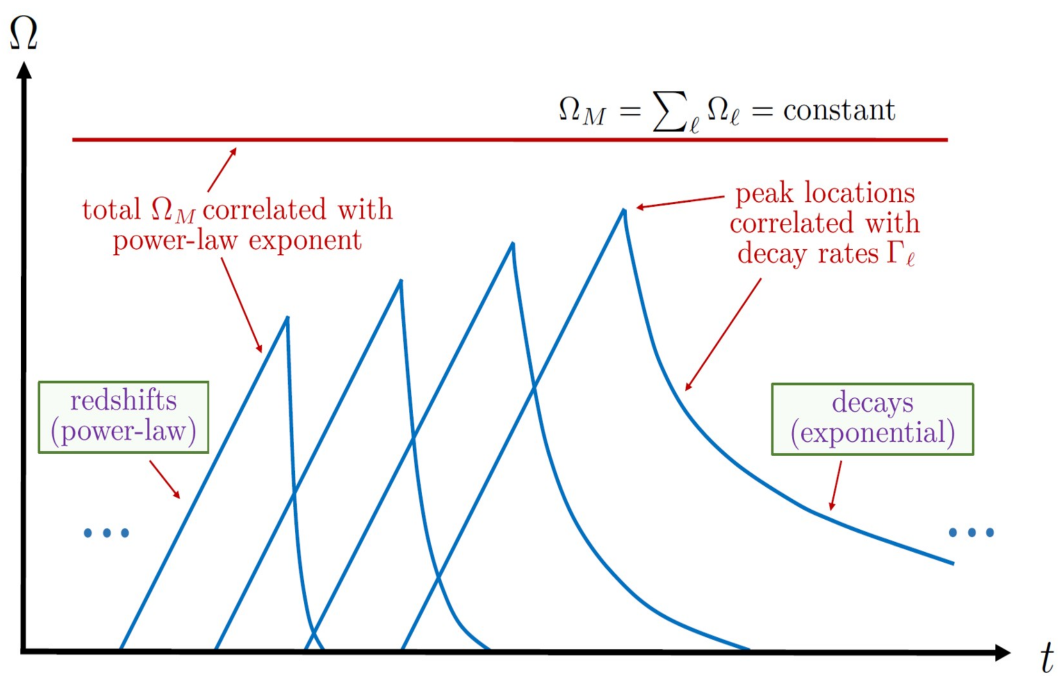
Particle decays provide a natural mechanism for obtaining these source/sink terms.

Underpinnings of Stasis

- The exponential decay of a single matter species, which occurs over a relatively short time period, is insufficient for achieving stasis.
- However in the presence of a ***tower of matter states*** ϕ_ℓ, where $\ell = 0, 1, 2, \ldots, N - 1$, whose decay widths Γ_ℓ and initial abundances $\Omega_\ell^{(0)}$ scale across the tower in such a way that the effect of decays on Ω_M and Ω_γ compensates for the effect of cosmic expansion over a extended period.
- Since the effective equation-of-state parameter for the universe satisfies $0 < w_{\text{eff}} < 1/3$ during stasis, each individual Ω_ℓ rises until $t \sim 1/\Gamma_\ell$, and then plummets as the particle decays.



- Other particles with lower $\Omega_\ell^{(0)}$ but longer lifetimes continue to rise and collectively compensate for the loss of abundance as each state drops out.
- As a result, the total abundance Ω_M of the matter states remains **effectively constant**, despite the rise and fall of the Ω_ℓ!

[Dienes, Huang, Heurtier, Kim, Tait, BT '21]

Conditions for Stasis

- The Boltzmann equations for the individual ρ_ℓ, in conjunction with the relevant Friedmann equation, yield an equation of motion for Ω_M.

Boltzmann Equations
$$\begin{cases} \dfrac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell \rho_\ell \\[2ex] \dfrac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell \end{cases}$$

Friedmann Equation
$$\Big\{\ H^2 = \frac{8\pi G}{3}(\rho_M + \rho_\gamma)$$

$$\Rightarrow \quad \frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H(\Omega_M - \Omega_M^2)$$

- To achieve stasis, we impose $\dfrac{d\Omega_M}{dt} = 0 \Rightarrow$

Stasis Condition (Instantaneous)

$$\boxed{\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)}$$

- In order to achieve an ***extended period*** of stasis, we need this instantaneous stasis condition to be satisfied over a significant range of t.

$\Rightarrow$ The left and right sides of this stasis-condition equation must have the same functional dependence on t.

Conditions for Stasis

- By construction, during a stasis era, $\dfrac{d\Omega_M}{dt} = 0$ $\Rightarrow$ $\Omega_M = \overline{\Omega}_M = [\text{const.}]$

- The Friedmann acceleration equation therefore implies:

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4-\overline{\Omega}_M) \quad \Rightarrow \quad H(t) = \left(\frac{2}{4-\overline{\Omega}_M}\right)\frac{1}{t}$$

- Substituting these results into our instantaneous stasis condition, we find that the condition for realizing an ***extended period of stasis*** can be expressed in two ways:

$$\boxed{\sum_\ell \Omega_\ell = \overline{\Omega}_M}$$

or

$$\boxed{\sum_\ell \Gamma_\ell \Omega_\ell = \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}}$$

- These expresssions can be combined to yield $\boxed{\dfrac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} = \dfrac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\dfrac{1}{t}}$

- During stasis, then, this ratio of sums must be inversely proportional to t.

Stasis from Particle Decays

- Let's consider a tower of N such states states with...

Masses	Decay Widths	Initial Abundances
$m_\ell = m_0 + (\Delta m)\ell^\delta$	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

- Towers of states with mass spectra of this form arise naturally in many extensions of the Standard Model.
  - KK excitations of a 5D scalar: $\begin{cases} mR \ll 1 \longrightarrow \delta \sim 1 \\ mR \gg 1 \longrightarrow \delta \sim 2 \end{cases}$
  - Bound states of a strongly-coupled gauge theory: $\delta \sim \frac{1}{2}$
- Decay through ***contact operators*** of dimension d implies a scaling: $\mathcal{O}_\ell \sim \frac{c_\ell}{\Lambda^{d-4}}\phi_\ell \mathcal{F} \longrightarrow \gamma = 2d - 7$
- Scaling of initial abundances depends on how they're generated:
  - Misalignment production $\longrightarrow$ $\alpha < 0$
  - Thermal freeze-out $\longrightarrow$ $\alpha < 0 \text{ or } \alpha > 0$
  - Universal inflaton decay $\longrightarrow$ $\alpha \sim 1$
  - ...

A Model of Stasis

- The abundance $\Omega_\ell(t)$ of each state at time t is a product of three factors.

$$\boxed{\Omega_\ell(t) \;=\; \underbrace{\Omega_\ell^{(0)}}_{\substack{\text{Initial abundance}\\ \text{(established prior to stasis)}}} \;\times\; \underbrace{h(t^{(0)}, t)}_{\text{Redshift factor}} \;\times\; \underbrace{e^{-\Gamma_\ell (t - t^{(0)})}}_{\text{Exponential-decay factor}}}$$

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- For sufficiently large N and small Δm, we can approximate the sum over $\Gamma_\ell\Omega_\ell$ with an integral:

$$\sum_\ell \Gamma_\ell\Omega_\ell(t) = \Gamma_0\Omega_0^{(0)} h(t^{(0)},t) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0\left(\frac{m_\ell}{m_0}\right)^\gamma (t-t^{(0)})}$$

$$\approx \frac{\Gamma_0\Omega_0^{(0)} h(t^{(0)},t)}{\delta} \int_{m_0}^{m_{N-1}} \frac{dm}{m-m_0} \left(\frac{m-m_0}{\Delta m}\right)^{1/\delta} \left(\frac{m}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0\left(\frac{m}{m_0}\right)^\gamma (t-t^{(0)})}$$

- For $t_{N-1} \ll t \ll t_0$, this is approximately

$$\boxed{\sum_\ell \Gamma_\ell\Omega_\ell(t) \approx \frac{\Gamma_0\Omega_0^{(0)}}{\gamma\delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} h(t^{(0)},t)\underbrace{\Gamma}_{\text{Euler gamma function}}\left(\frac{\alpha+\gamma+1/\delta}{\gamma}\right) \left[\Gamma_0(t-t^{(0)})\right]^{-(\alpha+\gamma+1/\delta)/\gamma}}$$

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Stasis from Particle Decays

- Likewise, the sum over Ω_ℓ is well approximated by

$$\boxed{\sum_\ell \Omega_\ell(t) \approx \frac{\Omega_0^{(0)}}{\gamma\delta}\left(\frac{m_0}{\Delta m}\right)^{1/\delta} h(t^{(0)},t)\Gamma\left(\frac{\alpha+1/\delta}{\gamma}\right)\left[\Gamma_0(t-t^{(0)})\right]^{-(\alpha+1/\delta)/\gamma}}$$

- The ratio of the two sums is

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} \approx \left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t-t^{(0)}} \xrightarrow{t \gg t^{(0)}} \boxed{\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} \approx \left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t}}$$

- Thus, our condition for extended stasis is satisfied! Indeed, we have

$$\boxed{\left(\frac{\alpha+1/\delta}{\gamma}\right)\frac{1}{t} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}}$$

Both sides inversely proportional to t, as desired!

- Solving for $\overline{\Omega}_M$, we find that the ***matter and radiation abundances*** in such a stasis era are

$$\boxed{\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)} \qquad \overline{\Omega}_\gamma = 1 - \overline{\Omega}_M}$$

Stasis from Particle Decays

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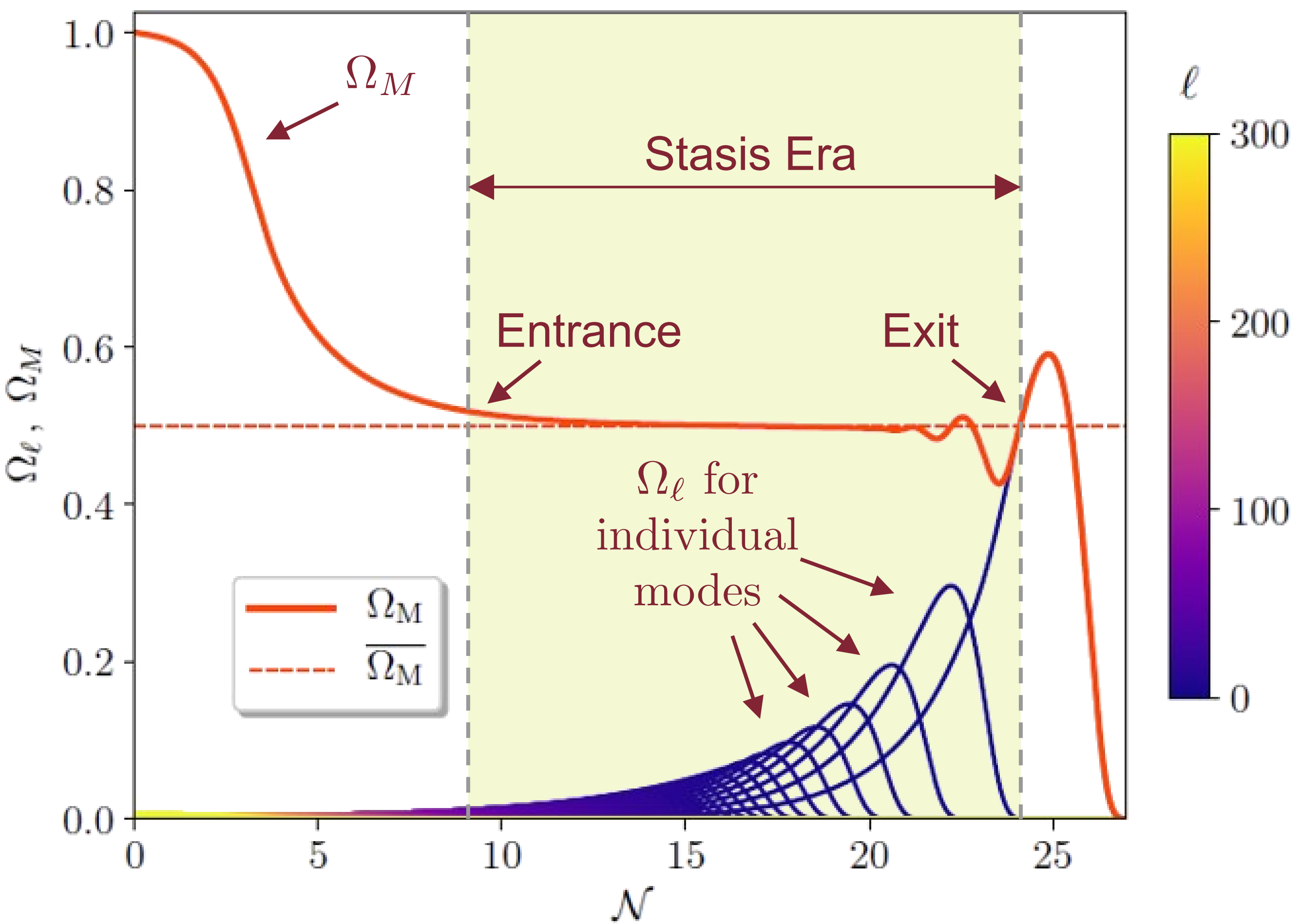
Stasis from Particle Decays

- These analytic results can be cross-checked by solving the Boltzmann equations numerically.
- The results of this analysis confirm our findings and provide additional information about how the stasis epoch ***begins*** and ***ends***.

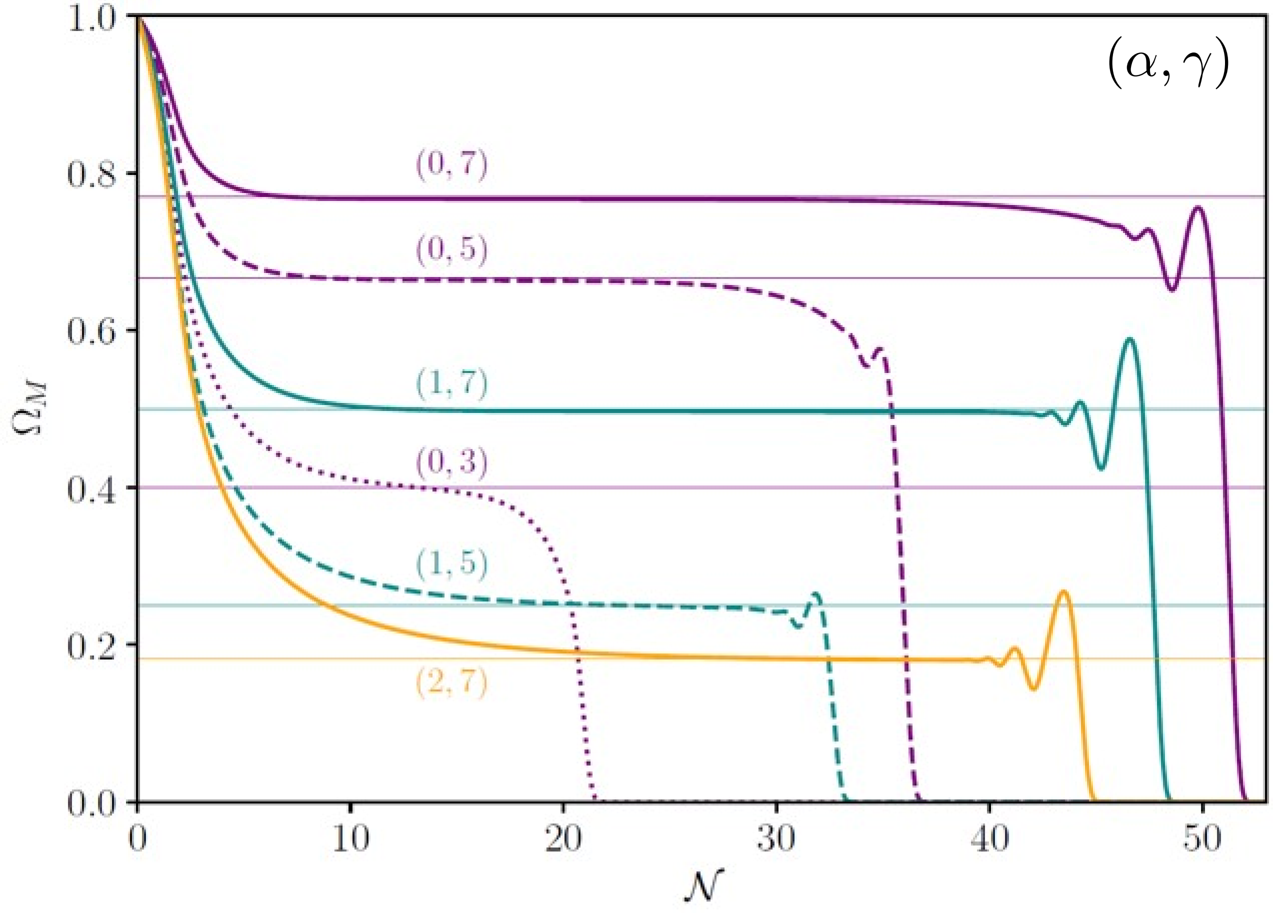


Parameter Choices

$$\alpha = 1$$
$$\gamma = 7$$
$$\delta = 1$$
$$N = 300$$
$$\frac{m_0}{\Delta m} = 1$$
$$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.01$$

Numerical Results

- We obtain similar results for different combinations of α and γ, which yield stasis eras with different values for $\overline{\Omega}_M$.



Parameter Choices

$$\delta = 1$$
$$N = 10^5$$
$$\frac{m_0}{\Delta m} = 1$$
$$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.1$$

- Indeed, our extended stasis condition implies that stasis can arise whenever the scaling parameters satisfy the following criterion:

$$-\frac{1}{\delta} < \alpha < \frac{\gamma}{2} - \frac{1}{\delta}$$

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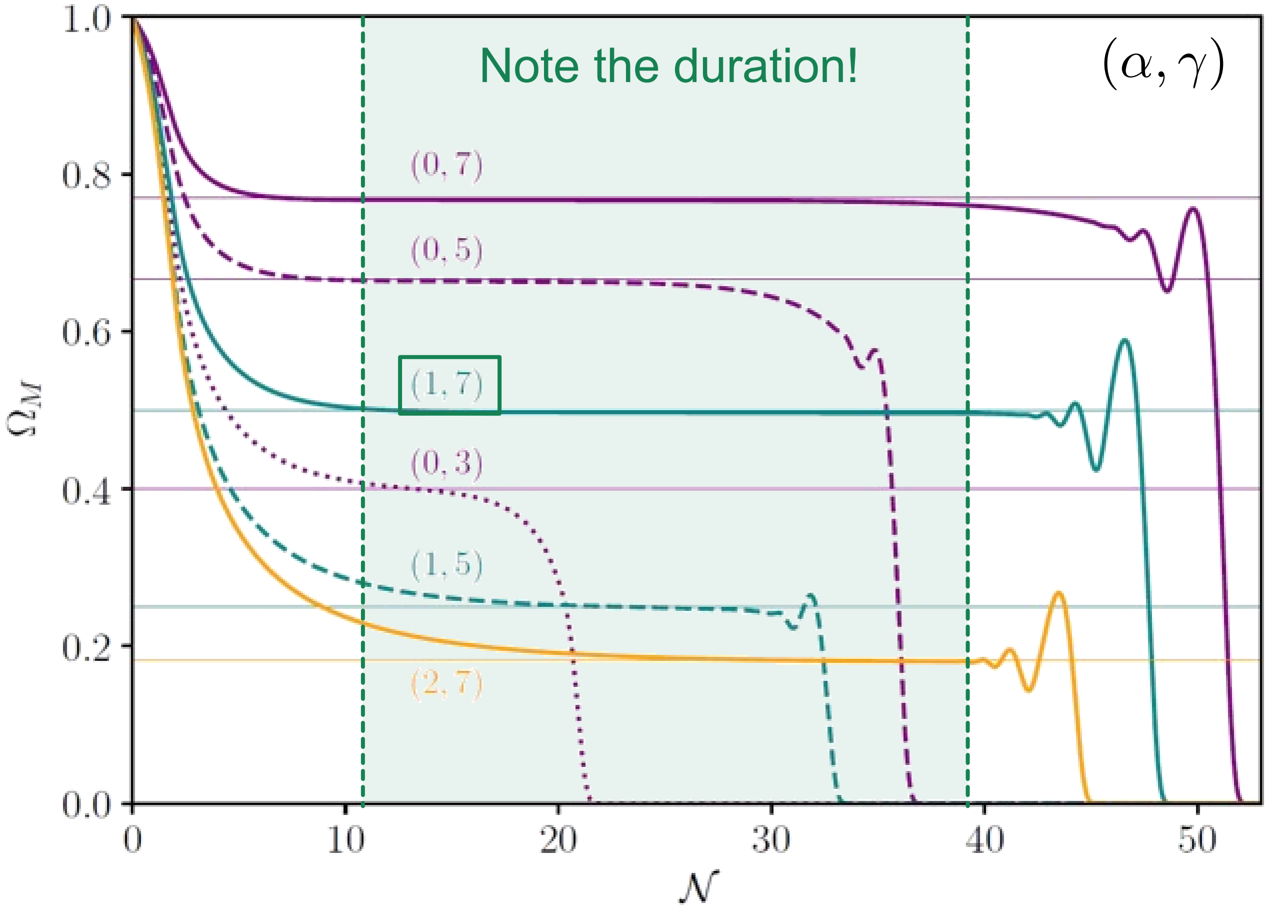

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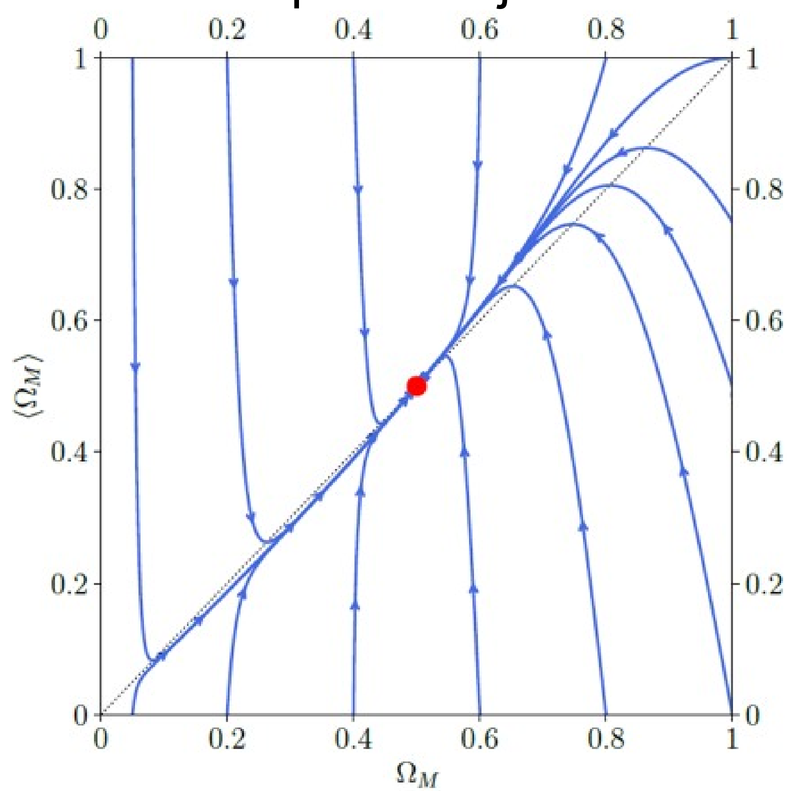
Stasis as a Global Attractor

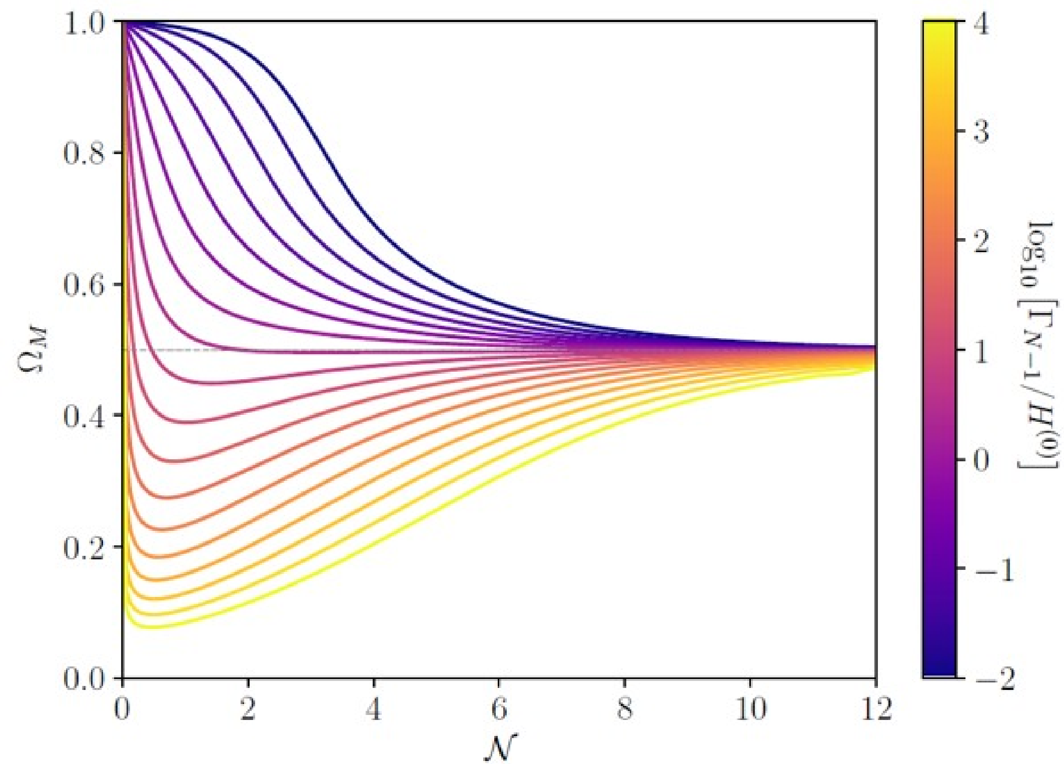
Q Does achieving cosmological stasis require a fine-tuning of the initial conditions for Ω_M and Ω_γ, or for the ratio $\Gamma_{N\text{-}1}/H^{(0)}$?

A No it doesn't. In fact, stasis is a ***global attractor*** in the sense that regardless of what $\Omega_M(t)$ and its time-average $\langle\Omega_M\rangle(t)$ from $t^{(0)}$ to t are at a given $t \geq t^{(0)}$, Ω_M and Ω_γ will ***evolve toward their stasis values***. Stasis doesn't require any special $\Gamma_{N\text{-}1}/H^{(0)}$ value either.

State-Space Trajectories



Ω_M vs. t For Different $\Gamma_{N\text{-}1}/H^{(0)}$

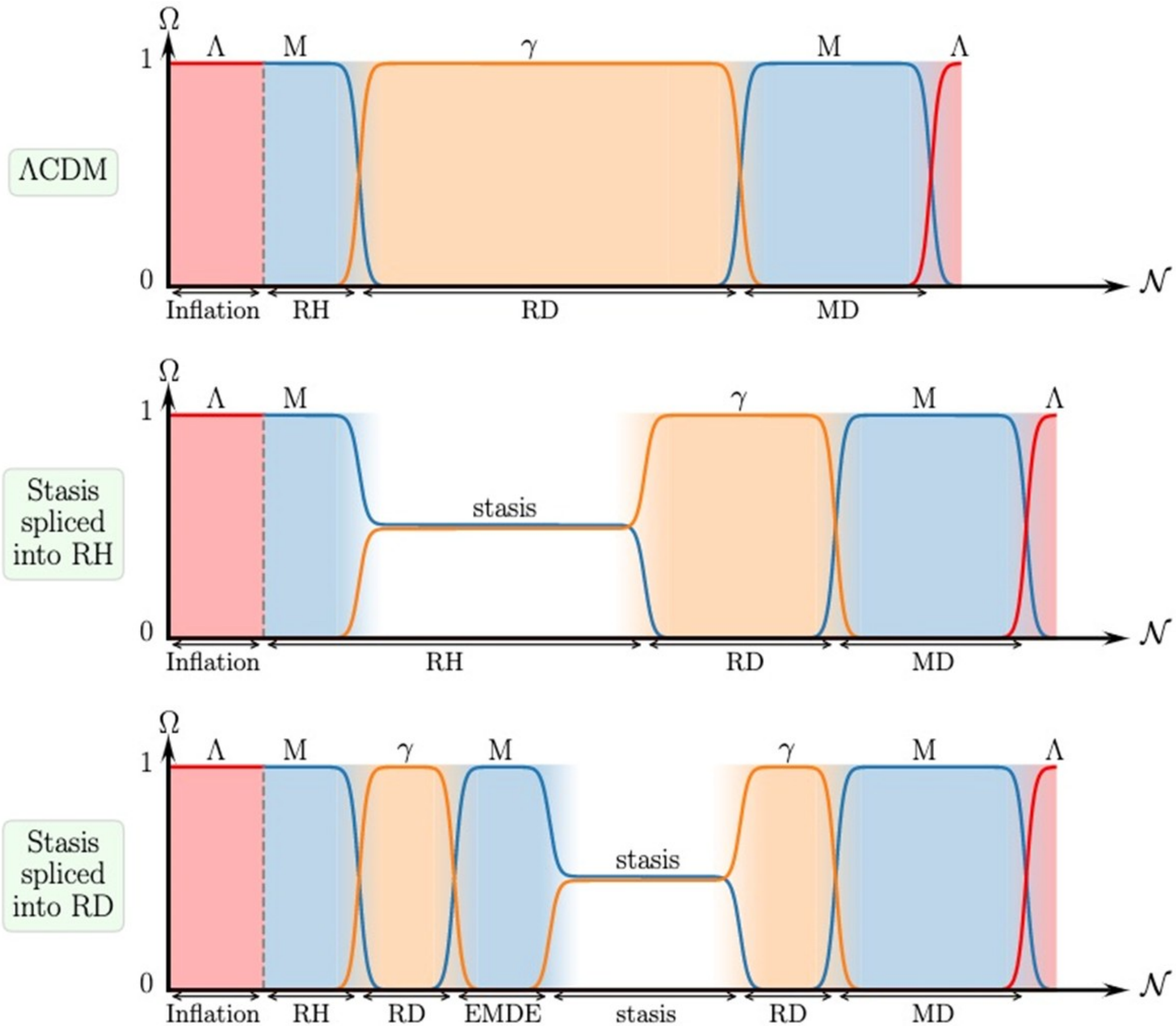


Splicing Stasis Into the Cosmological Timeline

- There are two primary ways in which a stasis epoch which arises from particle decays can be incorporated into the standard cosmological timeline:

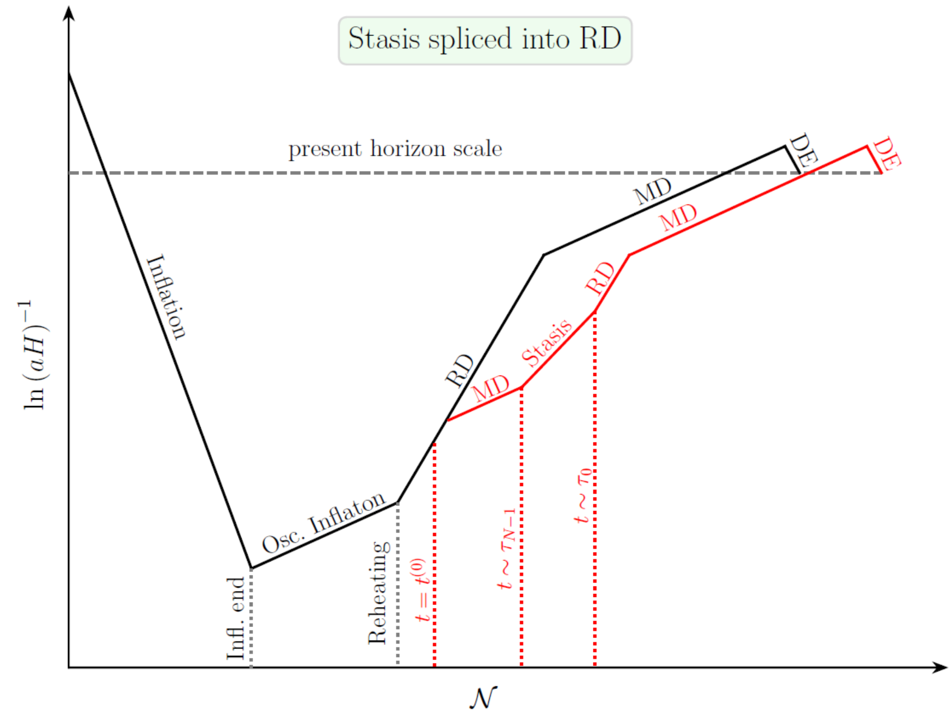
- The stasis epoch could ***follow inflation***. The inflaton produces the ϕ_ℓ directly in this case, and their decays reheat the universe.

- Stasis could occur at some point ***after reheating***, following an EMDE wherein the ϕ_ℓ collectively dominate the energy density of the universe.



Implications of Stasis

- The comoving Hubble radius grows more slowly in cosmologies with a stasis era, so perturbation modes re-enter the horizon at a later time. This has implications for ***inflationary observables***.



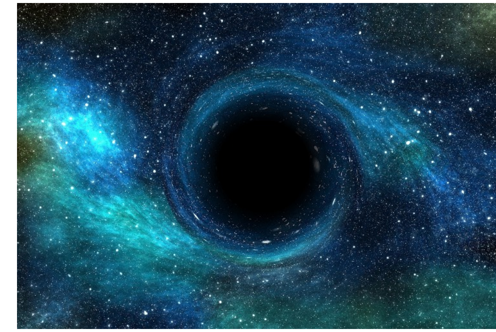
- ***Density perturbations*** grow more quickly during stasis than in an RD era. As a result, compact objects such as PBH or compact minihalos can potentially form during stasis, as they do in an EMDE.

- ***The dark-matter (DM) relic abundance*** would be affected if DM is produced prior to or during stasis, due to the modified expansion history and to the injection of entropy by ϕ_ℓ. The DM could potentially also be produced by the decays of the ϕ_ℓ directly.

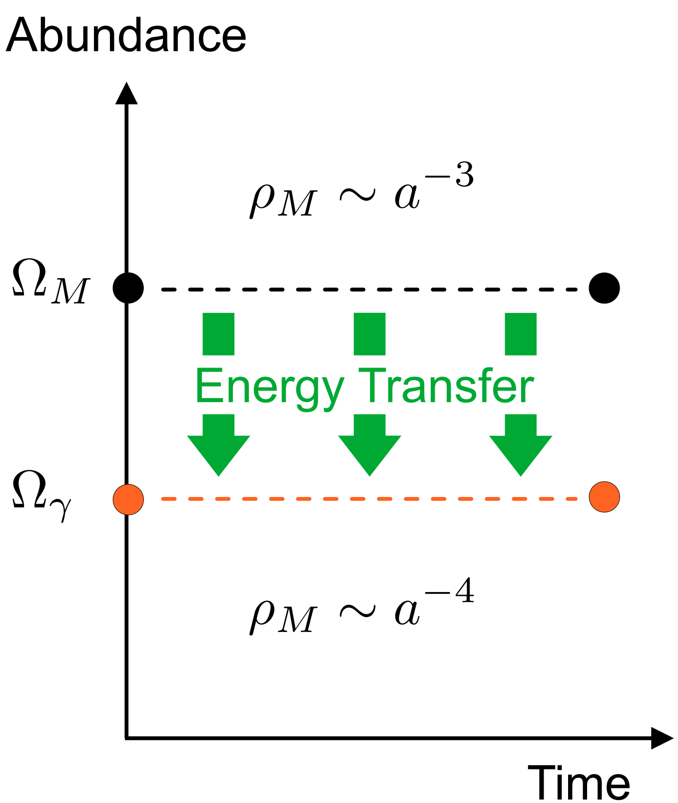
Stasis from Primordial-Black-Hole Evaporation

- A population of ***primordial black holes*** (PBH), whose evaporation via Hawking radiation transfers energy density from matter to radiation, can likewise give rise to a period of stasis.

[Dienes, Huang, Heurtier, Kim, Tait, BT '22]



- In this case, **Hawking radiation** provides the mechanism via which energy density is transferred from matter to radiation.



Boltzmann Equations

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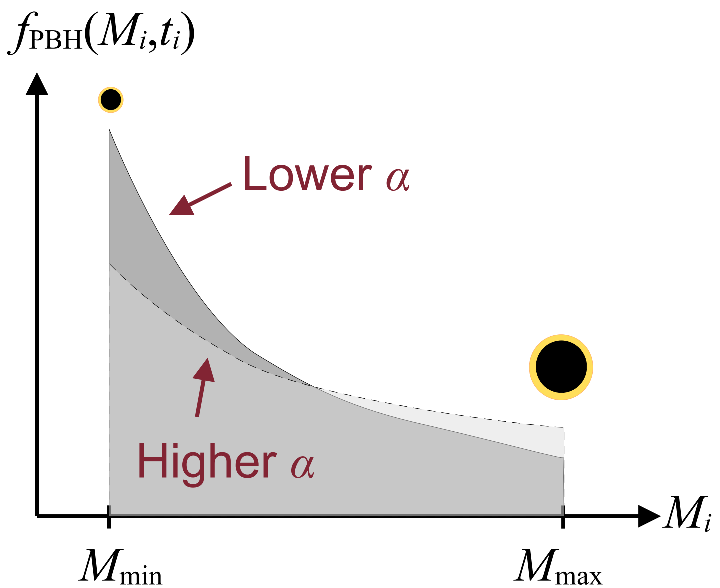
Initial PBH Mass Spectrum



- Let's consider a population of primordial black holes (PBHs) with the mass spectrum

$$f_{\rm BH}(M_i,t_i) = \begin{cases} C M_i^{\alpha-1} & \text{for } M_{\rm min} \le M_i \le M_{\rm max} \\ 0 & \text{otherwise} \end{cases}$$

- Such an **extended mass spectrum** arises naturally in scenarios in which the PBHs form after inflation via the collapse of perturbations with a scale-invariant power spectrum.

[Carr '75; Green, Liddle '97; Kim, Lee, MacGibbon '99; Bringmann, Keifer, Polarski '02; Carr et. al. '17]

- The value of α is determined by the equation-of-state parameter $w_{\rm c}$ for the universe during the epoch wherein the PBHs form.

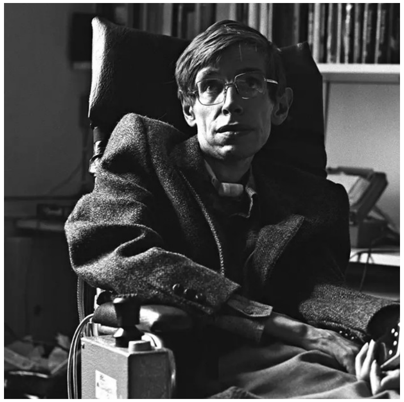
$$\alpha = -\frac{3w_c+1}{w_c+1} \xrightarrow{-1/3 \le w_c \le 1} -2 \le \alpha \le 0$$

- Observational considerations likewise place constraints on the values of $M_{\rm min}$ and $M_{\rm max}$:

$$\underbrace{0.1\text{g} \lesssim M_{\rm min}}_{\text{Planck upper bound on } H_\star} < \underbrace{M_{\rm max} \lesssim 10^9\text{g}}_{\text{Heaviest PBH evaporate completely before BBN}}$$

[Carr, Kohri, Sendouda, Yokoyama '09; Keith, Hooper, Blinov, McDermott '20; Carr, Kohri, Sendouda, Yokoyama '21; Akrami et al. (Planck) '20]

Evaporation



- **Hawking radiation** provides a mechanism via which energy density can be transferred from the PBHs (which behave like massive matter) to radiation. **[Hawking, '74; Hawking '75]**

$$T_{\mathrm{BH}} = \frac{1}{8\pi G M} \sim 1.06\ \mathrm{GeV}\left(\frac{10^{13}\,\mathrm{g}}{M}\right)$$

- The rate of change of the mass M of a single PBH due to this effect is

[MacGibbon, Webber, '90; MAcGibbon '91]

$$\frac{dM}{dt} \equiv -\varepsilon(M)\frac{M_P^4}{M^2}$$

Graybody factor: for this range of M, $\varepsilon(M) \approx \varepsilon$ is approximately constant.

- The time at which a PBH evaporates completely (i.e., at which $M = 0$) as a result of this effect is

$$\tau(M_i) \equiv \frac{{M_i}^3}{3\varepsilon M_P^4}$$

- As a result, the PBH mass spectrum subsequently evolves according to a Boltzmann equation of the form

$$\frac{d\rho_{\mathrm{BH}}}{dt} + 3H\rho_{\mathrm{BH}} = \int_0^\infty dM \ {}_{\mathrm{BH}}(M,t)\frac{dM}{dt}$$

Boltzmann Evolution

- The evolution of the Hubble parameter $H(t)$ is given by the Friedmann acceleration equation, which in thes case takes the form

$$\frac{dH}{dt} = -H^2 - \frac{4\pi G}{3}\left[\rho_{\rm BH}(1 + 3\cancel{w_{\rm BH}}) + \rho_\gamma(1 + w_\gamma)\right] \qquad w_{\rm BH} = 0$$

- Expressed in terms of the cosmological abundance $\Omega_{\rm BH} \equiv \rho_{\rm BH}/\rho_{\rm crit}$, the system of equations governing the expansion of the universe is

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4 - \Omega_{\rm BH})$$
$$\frac{d\Omega_{\rm BH}}{dt} = -\Gamma_{\rm BH}(t)\,\Omega_{\rm BH} + H\left(\Omega_{\rm BH} - \Omega_{\rm BH}^2\right)$$

...where we have defined

$$\Gamma_{\rm BH}(t) \equiv -\frac{\int_0^\infty f_{\rm BH}(M,t)\frac{dM}{dt}\,dM}{\int_0^\infty f_{\rm BH}(M,t)M\,dM}$$

- Alternatively, one can change variables and express this system of equations in terms of $\Omega_{\rm BH}$ and its time-averaged value $\langle\Omega_{\rm BH}\rangle$ since the time t_i at which the PBH spectrum was initially established:

$$\langle\Omega_{\rm BH}\rangle \equiv \frac{1}{t - t_i}\int_{t_i}^t dt'\,\Omega_{\rm BH}(t')$$

PBH-Induced Stasis is a Global Attractor

- One can show that not only do these equations admit a **stasis solution**, but that this stasis solution is a **global attractor**.

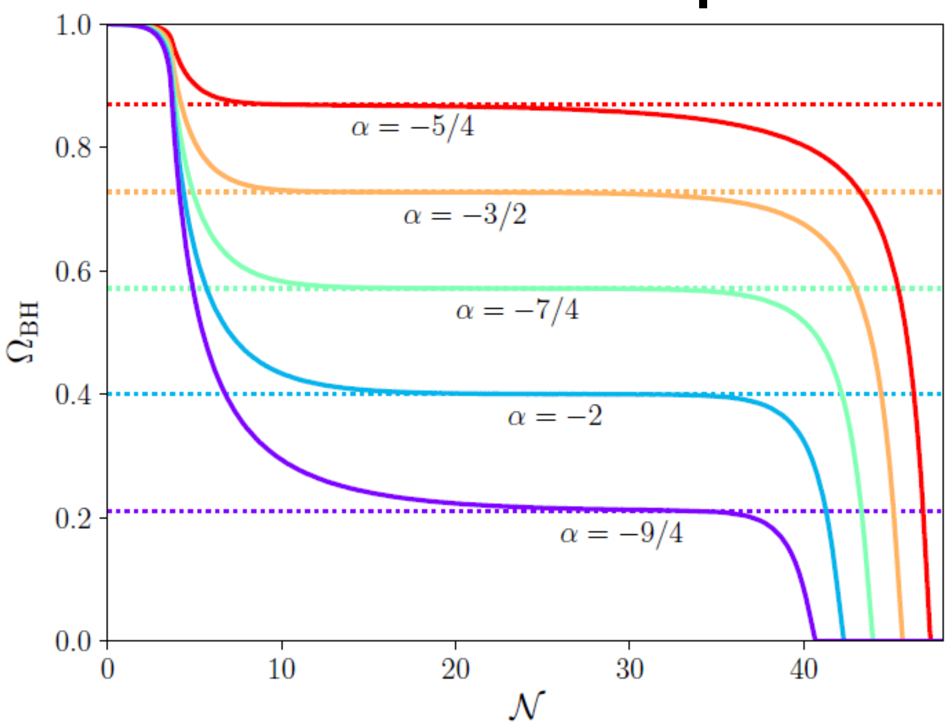
[Barrow, Copeland, Liddle '91; Dienes, Huang, Heurtier, Kim, Tait, BT '22]

- The effective equation-of-state parameter $\overline{w}$ for the universe as a whole during the stasis epoch and the PBH abundance $\Omega_{\rm BH}$ are determined by the value of α:

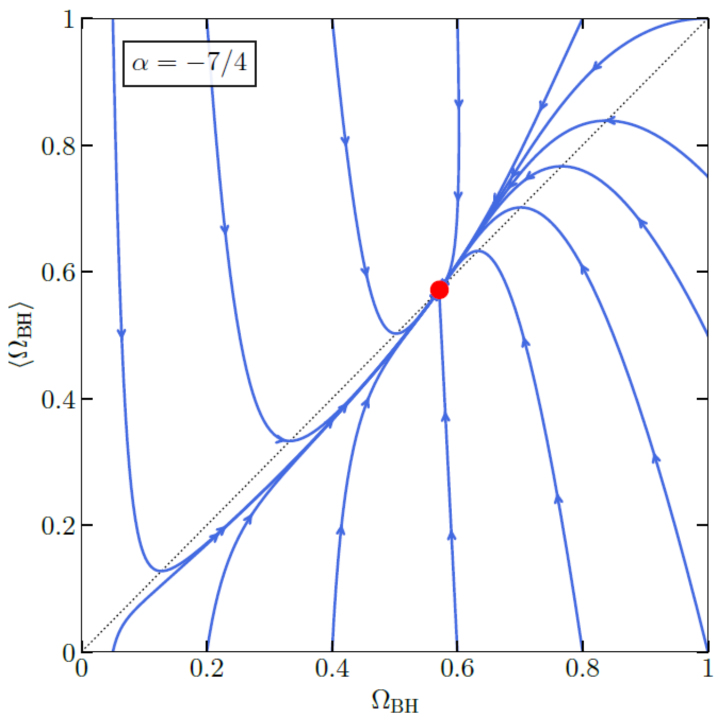
$$\overline{w} = -\frac{\alpha+1}{\alpha+7}$$

$$\overline{\Omega}_{\rm BH} = \frac{4\alpha+10}{\alpha+7}$$

Stasis from PBH Evaporation



Attractor Behavior



Duration of the Stasis Epoch

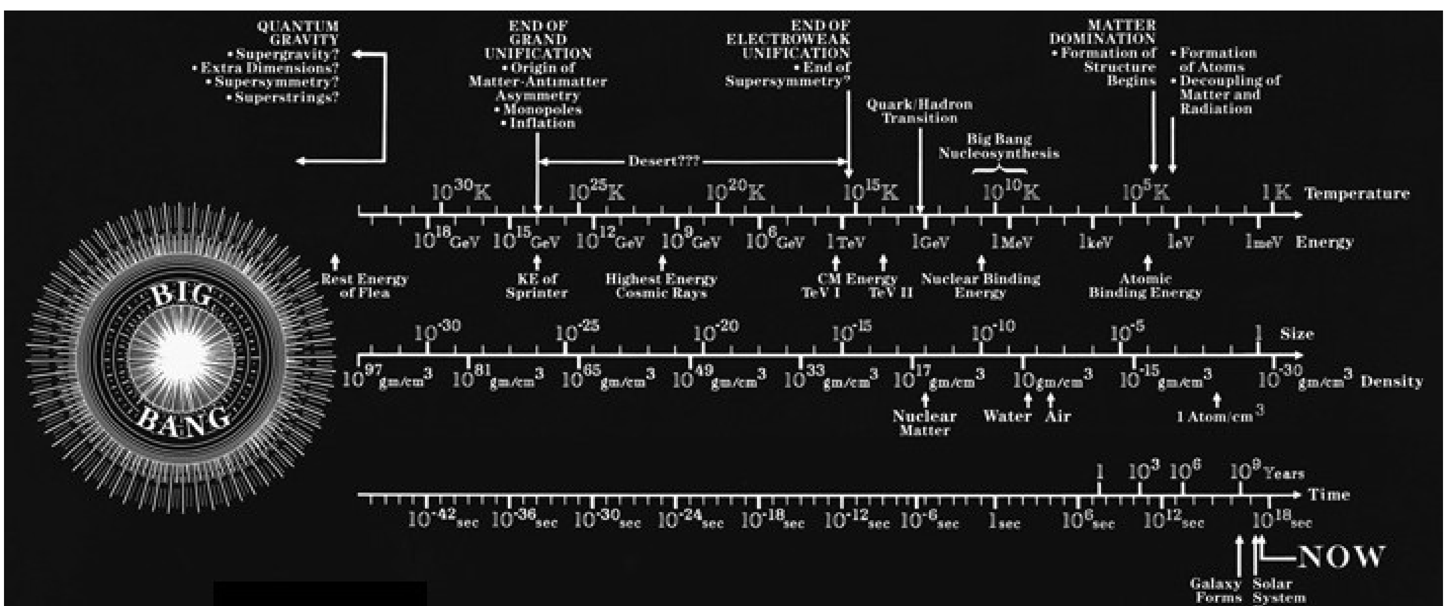
- The duration of this PBH-induced stasis epoch, expressed in terms of the number of e-folds of cosmic expansion that it spans, is given by

$$\boxed{\mathcal{N}_s \approx \log\left[\frac{a(\tau(M_{\max}))}{a(\tau(M_{\min}))}\right] \approx \frac{\alpha+7}{3}\log\left(\frac{M_{\max}}{M_{\min}}\right)}$$

- For $M_{\min} = 0.1$ g at its minimum and $M_{\max} = 10^9$ g at its maximum, this yields a stasis epoch of duration

$$\boxed{\mathcal{N}_s \lesssim 23\left(\frac{\alpha+7}{3}\right)}$$

- This is a significant duration indeed – potentially spanning a range of temperatures $\mathcal{O}(\mathrm{MeV}) \lesssim T \lesssim \mathcal{O}(10^{11}\ \mathrm{GeV})$!



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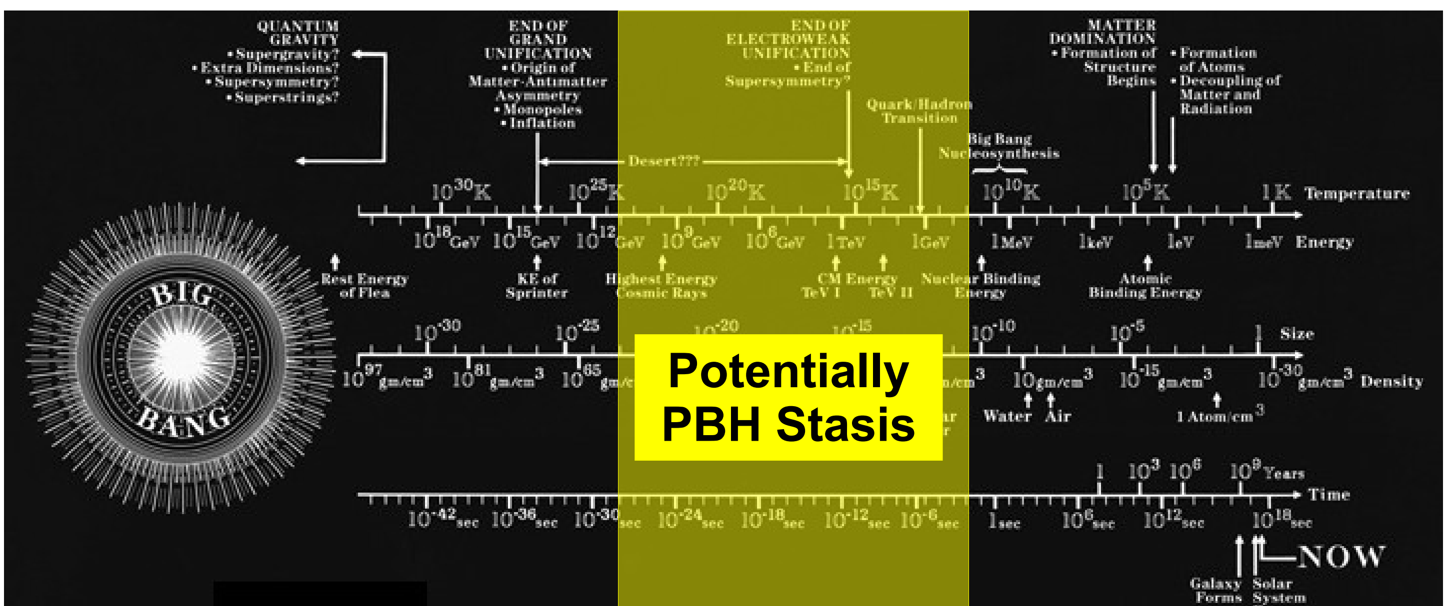
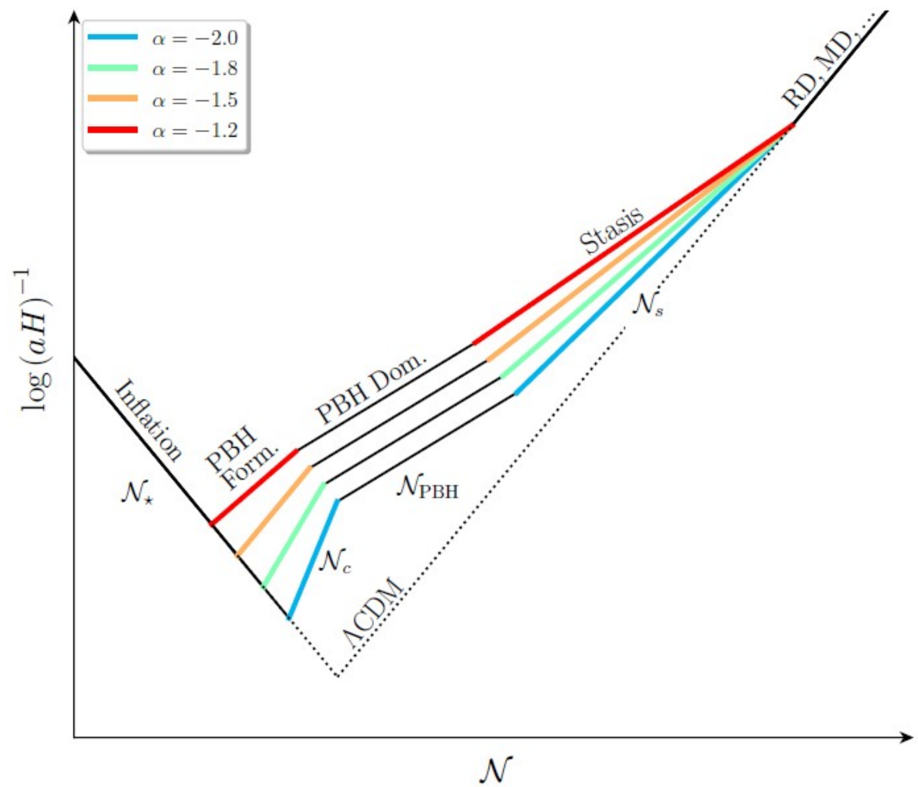


Thus, events such as the electroweak phase transition could have occurred during such a stasis epoch!

Cosmic Expansion History

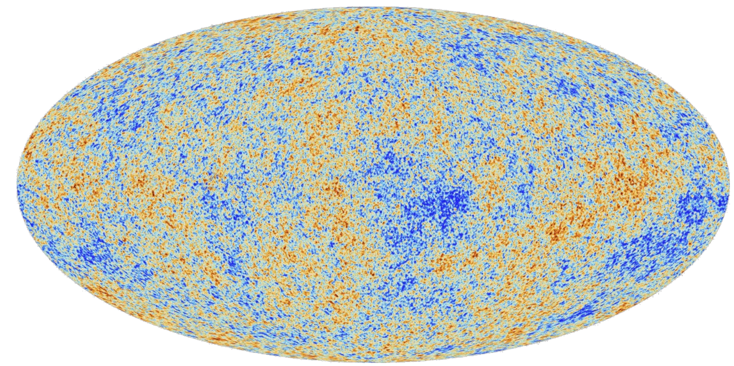
- In cosmologies involving an epoch of PBH-induced stasis, the cosmological timeline includes a series of several different epochs after cosmic inflation ends. Sequentially, these are:
  - The epoch during which the **PBHs are generated**, wherein the equation-of-state parameter w_c determines α.
  - An epoch during which the PBHs come to dominate the energy density of the universe. This epoch is **matter-dominated** ($w = 0$).
  - The **stasis epoch**, which begins once the lightest PBHs begin to evaporate, and wherein $w = \overline{w}$.

Comoving Hubble Horizon



  - The usual **RD epoch** with $w = 1/3$, which begins after the heaviest PBHs evaporate and stasis ends. Once this epoch begins, the expansion history coincides with that of the standard cosmology.

Inflationary Observables



- In the simplest inflationary scenarios, primordial perturbations of the inflaton field give rise to the pattern of inhomogeneities observed in the cosmic microwave background (CMB).
- However, **modifications of the cosmological timeline** beween the end of inflation and last scattering can alter predictions for CMB observables.
- The primary such observables are the **tensor-to-scalar ratio** r and **spectral index** n_s that characterize the primordial perturbation spectrum.
- For example, in single-field, slow-roll models of inflation, these obserables are directly related to the slow-roll parameters ε and η:

$$\boxed{\begin{aligned} n_s &= 1 - 6\epsilon + 2\eta \\ r &= 16\epsilon \end{aligned}}$$

where

$$\boxed{\epsilon \equiv \frac{M_P^2}{16\pi}\left[\frac{V'(\phi_\star)}{V(\phi_\star)}\right]^2 \qquad \eta \equiv \frac{M_P^2}{8\pi}\left|\frac{V''(\phi_\star)}{V(\phi_\star)}\right|}$$

- The quantity $\phi_\star$ denotes the value of the inflaton field at the time at which a perturbation with wavenumber equal to the pivot scale $k_\star$ exits the horizon. Following Planck, we take $k_\star = 0.002\ \mathrm{Mpc}^{-1}$. [Akrami et al. (Planck) '20]

Inflationary Observables

- I order to determine $\phi_\star$ we note that in the slow-roll approximation, the Hubble parameter $H_\star$ and scale factor $a_\star$ at the time at which this same mode exist the horizon are related to $\phi_\star$ by

$$H_\star^2 \approx \frac{8\pi V(\phi_\star)}{3M_P^2} \quad \text{and} \quad \log\left(\frac{a_{\rm end}}{a_\star}\right) = \frac{8\pi}{M_P^2}\int_{\phi_{\rm end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi$$

- Combining these relations yields the integro-differential equation

$$\frac{8\pi}{M_P^2}\int_{\phi_{\rm end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi = \frac{1}{2}\log\left(\frac{8\pi a_{\rm now}^2 V(\phi_\star)}{3M_P^2 k_\star^2}\right) - \log\left(\frac{a_{\rm now}}{a_{\rm end}}\right)$$

...which can be solved for a given form of $V(\phi)$.

- In order to illustrate how r and n_s are modified in cosmologies involving an epoch of PBH-induced stasis, it is useful to work in the context of a concrete model for the inflaton potential… or two. We'll choose

1 **Polynomial potentials**:

$$V(\phi) \sim |\phi|^p$$

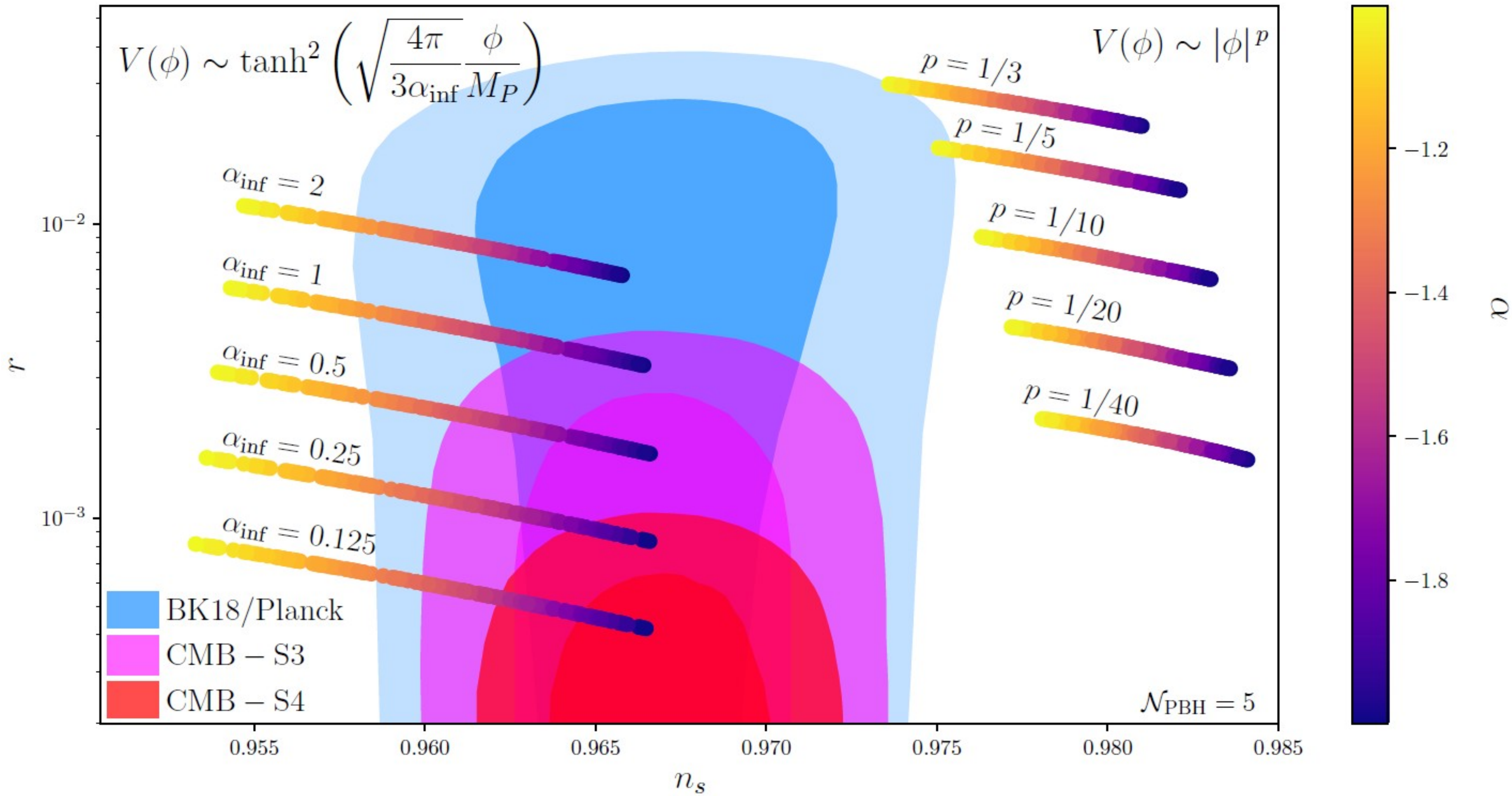
2 **T-Model α-attractors**:

[Kallosh, Linde '13]

$$V(\phi) \sim \tanh^{2n}\left(\sqrt{\frac{4\pi}{3\alpha_{\rm inf}}}\frac{\phi}{M_P}\right)$$

Inflationary Observables: Results

- In general, the modifications of the cosmological timeline associated with PBH-induced stasis serve to increase r and decrease n_s.



- As a result, depending on the inflationary model in question, tensions between the predictions for these observables and CMB data may be **<u>either eased or exacerbated</u>**.

Gravitational-Wave Background

- The cosmological modifications associated with a PBH-induced stasis epoch affect the **gravitational-wave (GW) background** in several ways.
- Perhaps most importantly, the modified expansion history alters the contribution to the GW background generated by other sources.
- For concreteness, we'll consider the simple case of a **stochastic GW background** which is homogeneous, isotropic, Gaussian, and unpolarized.
- The differential GW energy density per logarithmic comoving wavenumber k for this case is:

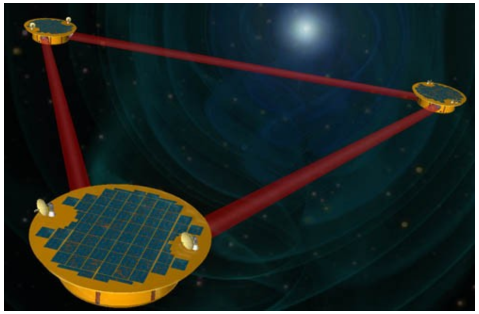
[Caprini, Figueroa '18]

$$\frac{d\rho_{\mathrm{GW}}(a)}{d\log k} = \frac{k^2 h_k^2(a)}{16\pi G a^2}$$

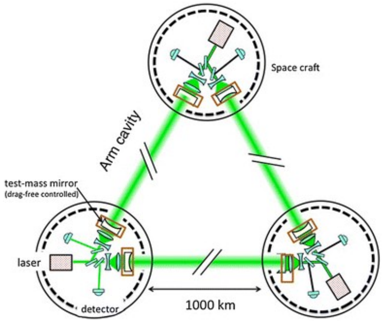
- The differential amplitude $h_k(a)$ depends on when the pertubation mode re-enters the horizon:

$$h_k(a) = \frac{a_k}{a} h_k(a_k)$$

Advanced LIGO



LISA



DECIGO

Gravitational-Wave Background

- During an epoch wherein w is constant, the wavenumber k which enters the horizon at scale factor a_k scales with a_k according to the relation

$$\boxed{k = a_k H_k \propto a_k^{-(1+3w)/2}}$$

- This implies that: $\boxed{\frac{d\rho_{\rm GW}(a)}{d\log k} \propto a^{-4} h_k^2(a_k) k^{\xi(w)}}$ where $\boxed{\xi(w) \equiv \frac{2(3w-1)}{(3w+1)}}$

- In the standard cosmology, wherein the universe remains radiation-dominated ($w = 1/3$) from the end of reheating until matter-radiation equality, $\xi(w) = 0$ throughout the entire duration.

- Thus, the resulting present-day GW spectrum – or, more precisely, the differential present-day GW abundance per unit physical frequency f – is flat (i.e., f-independent) and given by **[Caprini, Figueroa '18]**

$$\boxed{\frac{d\Omega_{\rm GW}^{\rm sc}}{d\log f} = \Omega_\gamma(a_{\rm now}) \left(\frac{g_{\star S}(T_{\rm eq})}{g_{\star S}(T_k)}\right)^{4/3} \frac{g_\star(T_k)}{24\pi^2} \frac{H_\star^2}{M_P^2}}$$

Gravitational-Wave Background

- By contrast, cosmology involving a PBH-induced stasis epoch with $w = w$ – as well as a PBH-production epoch with $w = w_c$ and a PBH-dominated epoch with $w = 1$ – can **differ significantly** from this result.
- In particular, in such a modified cosmology, the corresponding present-day GW spectrum is given by

$$\frac{d\Omega_{\rm GW}}{d\log f} = \frac{d\Omega_{\rm GW}^{\rm sc}}{d\log f} \times \begin{cases} 1 & f \leq f_s \\ \left(\frac{f}{f_s}\right)^{\xi(\overline{w})} & f_s < f \leq f_{\rm PBH} \\ \left(\frac{f_{\rm PBH}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\rm PBH}}\right)^{-2} & f_{\rm PBH} < f \leq f_f \\ \left(\frac{f_{\rm PBH}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\rm PBH}}\right)^{-2} \left(\frac{f}{f_f}\right)^{\xi(w_c)} & f_f < f \leq f_{\rm end} \\ 0 & f_{\rm end} < f \ , \end{cases}$$

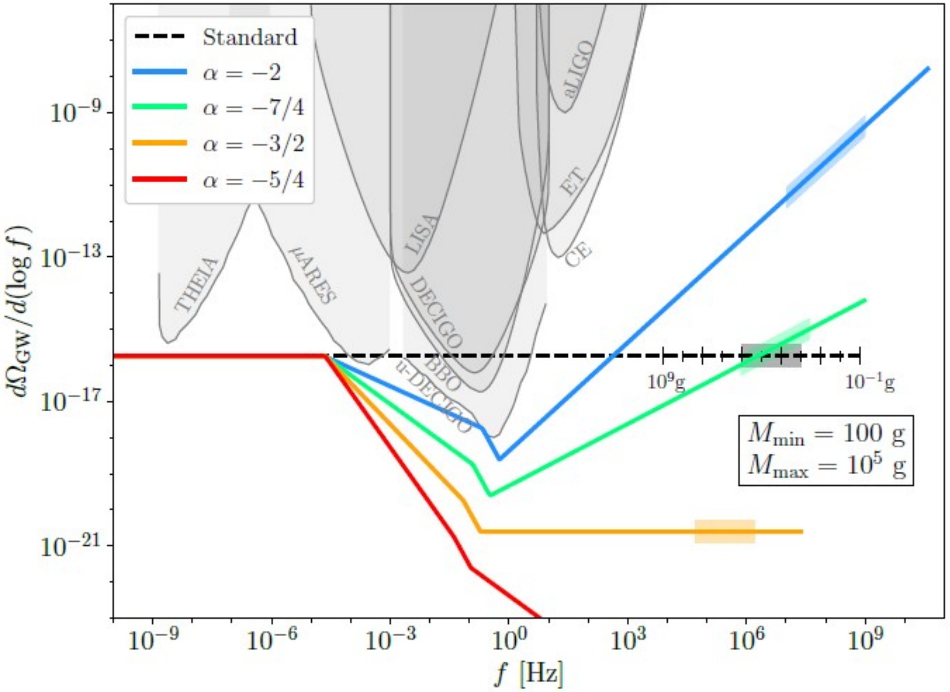
Spectrum obtained in the standard cosmology for the same $H_\star$

Piecewise function with different power-law exponents within different frequency intervals corresponding to different

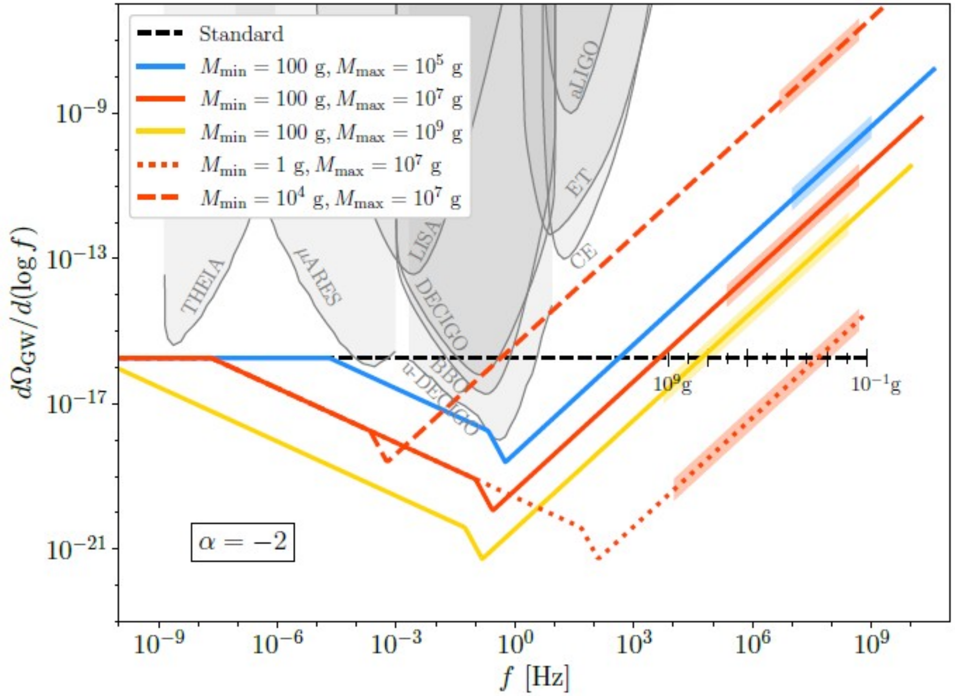
Gravitational-Wave Background: Results

- Given the sensitivities of planned, proposed, and existing **gravitational-wave observatories**, these modifications can have significant implications for the detection of the stochastic GW background.

Effect of Varying α



Effect of Varying $M_{\rm min}$ and $M_{\rm max}$



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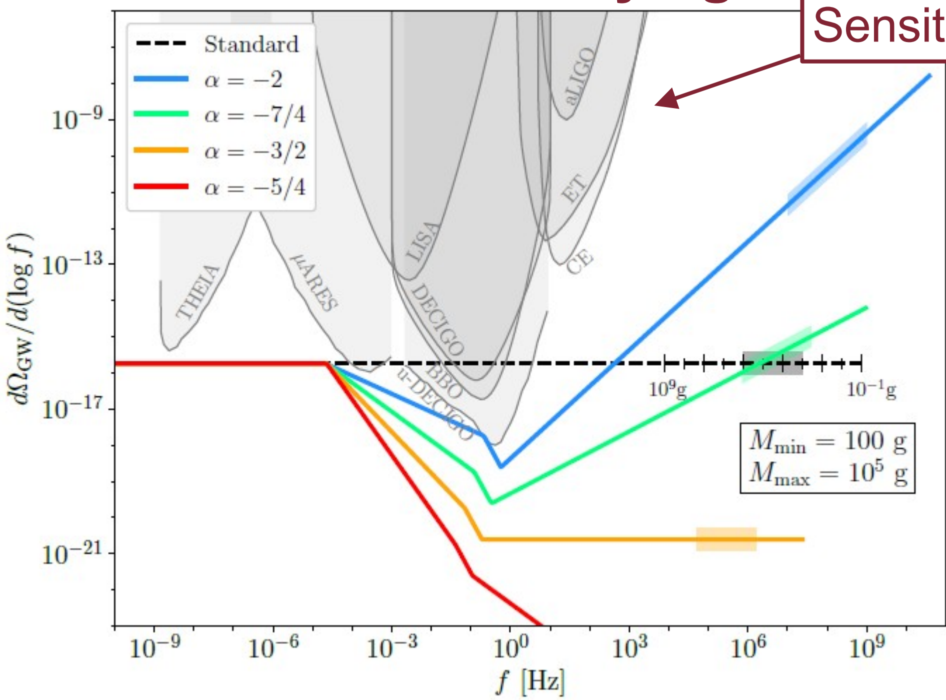
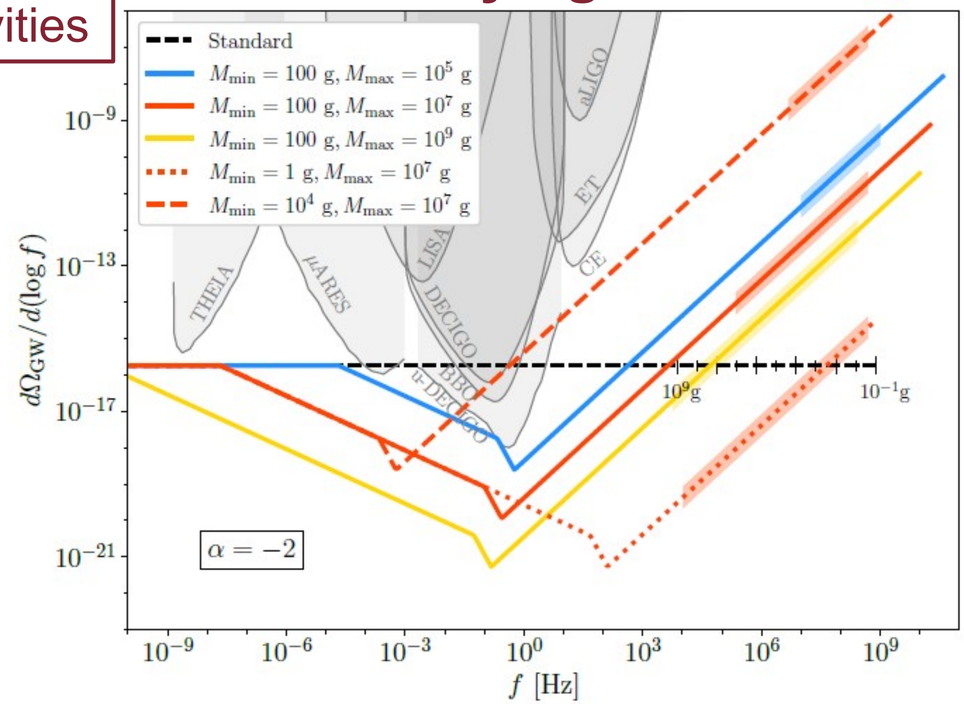
Effect of Varying α

Detector Sensitivities



Effect of Varying M_{min} and M_{max}



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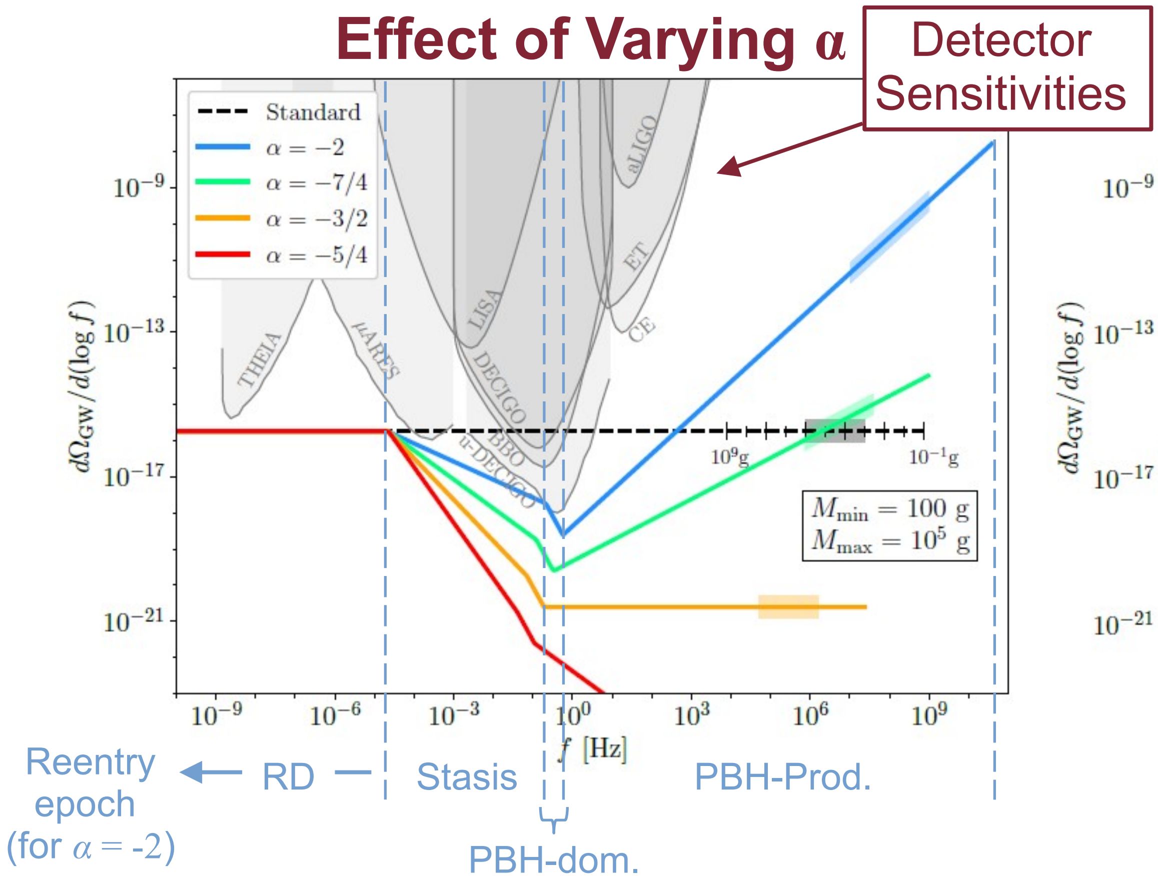

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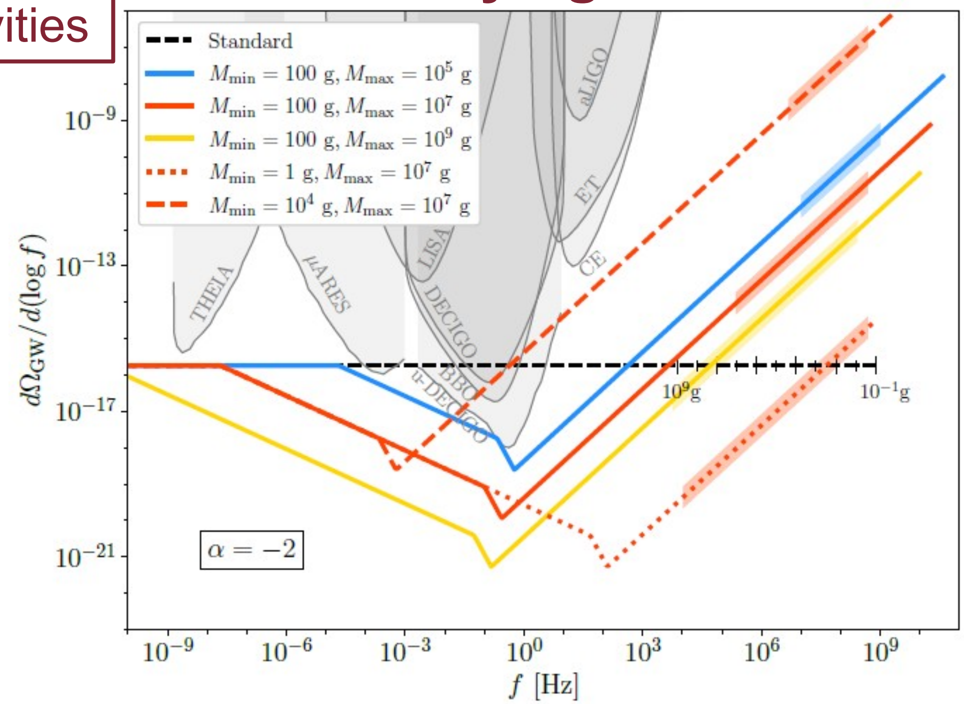

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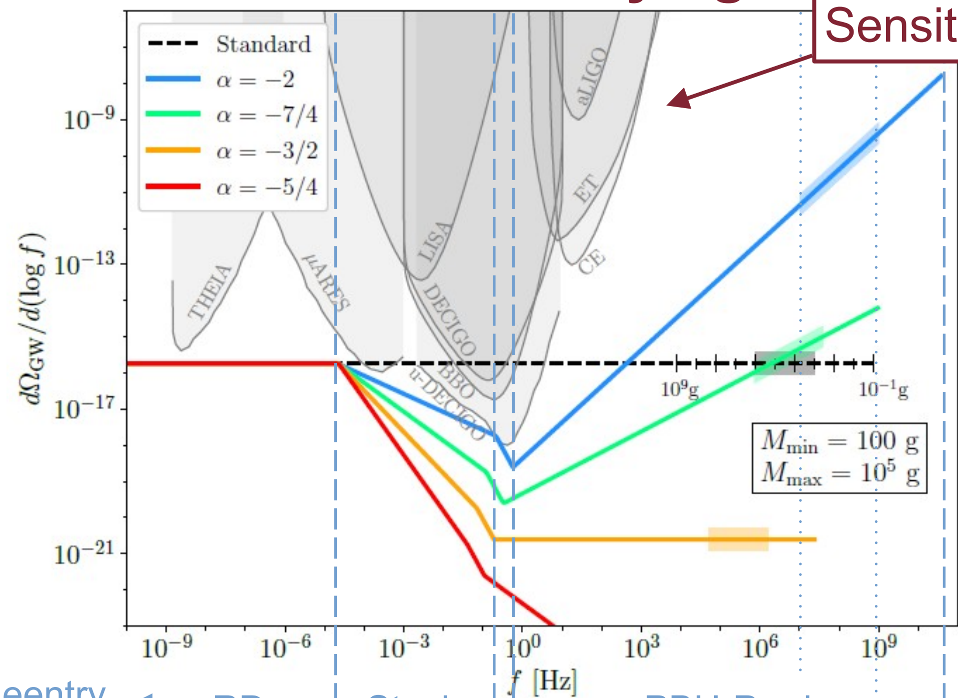
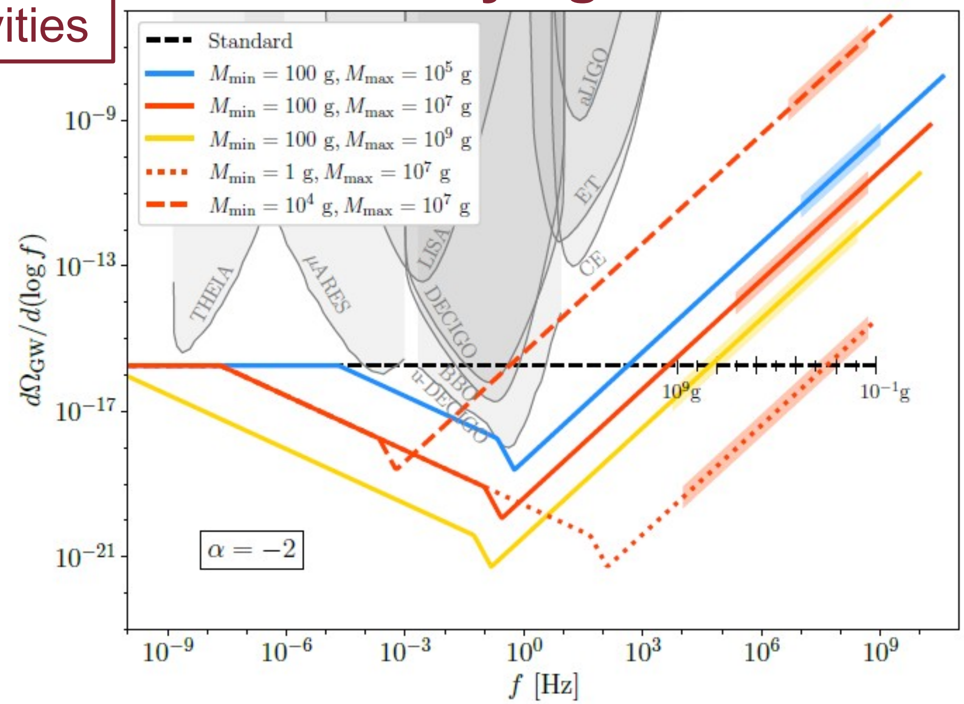


Reentry epoch (for α = -2) ← RD — Stasis PBH-Prod.

PBH-dom.

Likely enhanced (assoc. w/ PBH production)

Effect of Varying $M_{\rm min}$ and $M_{\rm max}$



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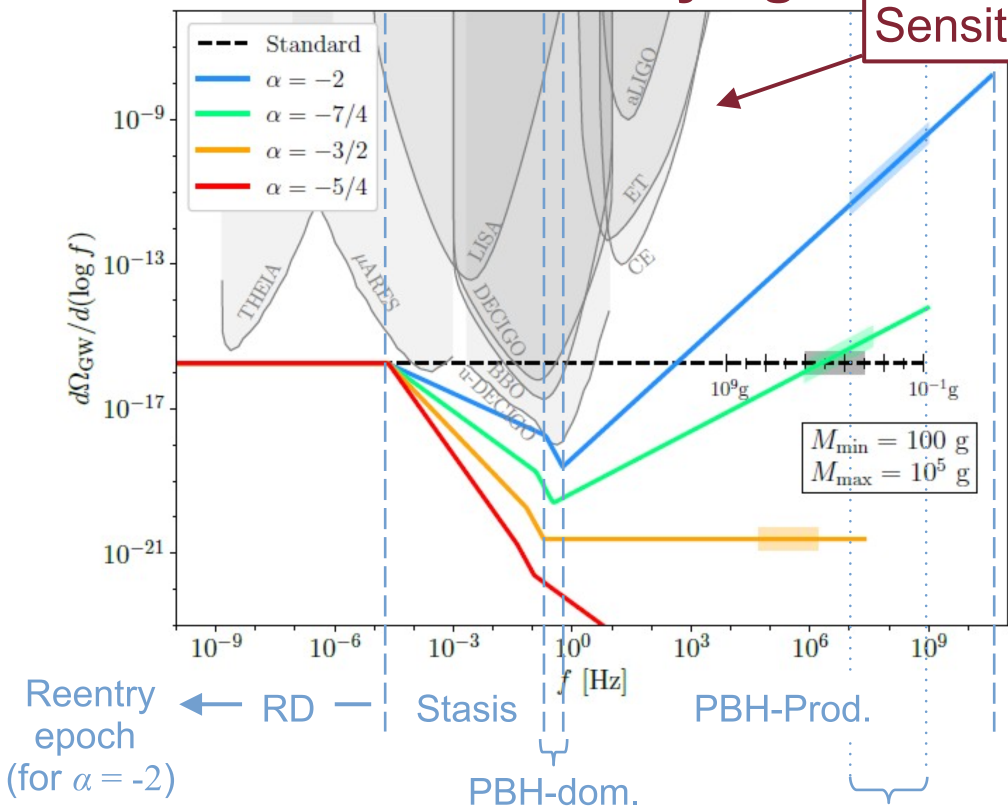
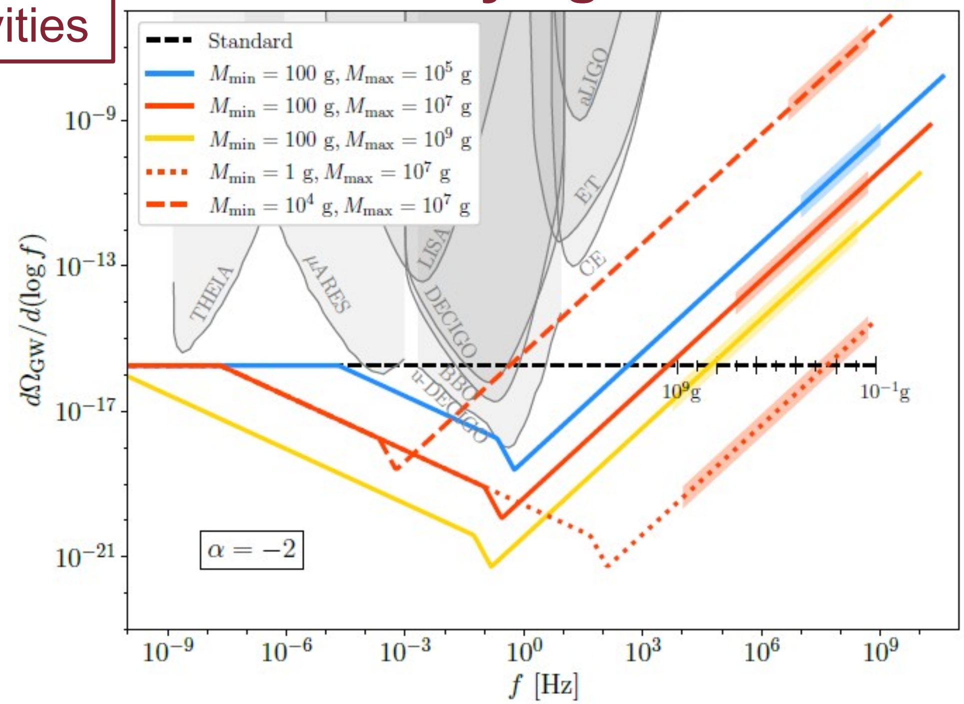


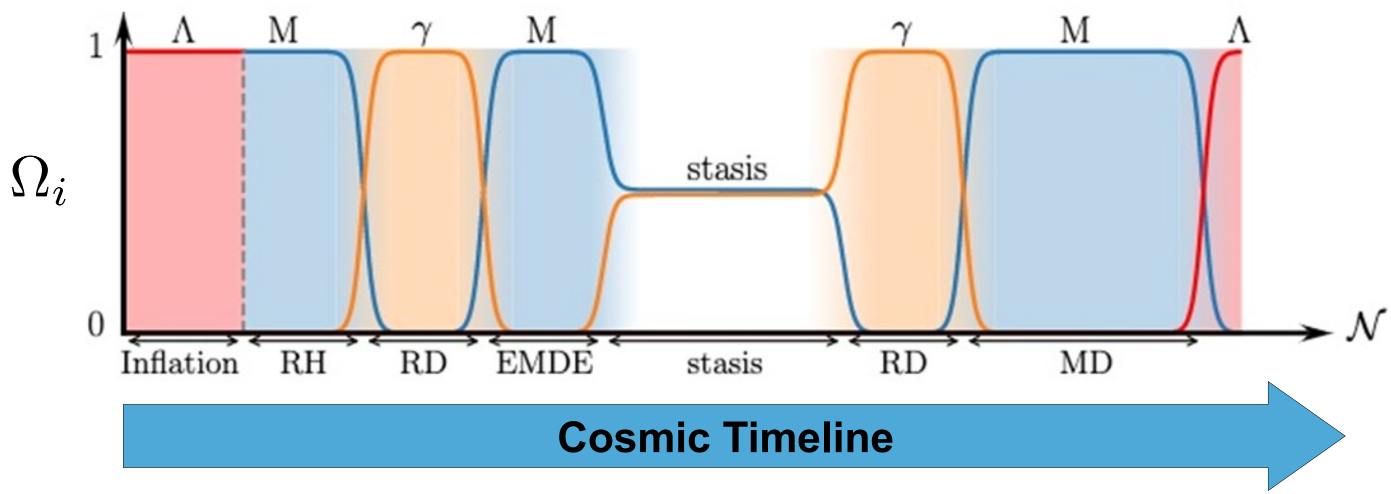


Likely enhanced (assoc. w/ PBH production)

The Upshot: A GW signal can be amplified – or hidden – as a result of PBH-induced stasis. Correlations between slopes in different regions provide an observational handle on α, M_{min}, and M_{max}.

Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* ***stasis eras*** – are indeed a viable cosmological possibility – and one that can arise naturally in many extensions of the Standard Model.
- For example, we have seen that a population of **unstable particles** with a range of lifetimes and a population of **primordial black holes** with an extended mass spectrum can both give rise to a stasis era.
- In both of these realizations, stasis is a ***global attractor***, and achieving it does not require any fine-tuning of initial conditions.
- A period of stasis can have a variety of cosmological implications – including an impact on **inflationary observables**, on the evolution of **density perturbations**, and on the **gravitational-wave spectrum**.



Above All

Since a variety of new-physics scenarios give rise to cosmic stasis, stasis and its consequences are something one must account for in such scenarios!