

# **Neutrinos**

**a few selected myths about them**

Rupert Leitner

## Myths:

M1. The neutrino mass eigenstates have the same energy or momentum, the oscillation curve will be the same. Which quantity differs more, energy or momentum?

M2. On Earth we see only one third of the electron neutrino flux because there are three flavours of neutrinos? But the solution is the effect of the dense mass of the solar core.

M3. Neutrino oscillations don't conserve energy? Of course they do.

M4. Relic neutrinos move at practically the speed of light? At least two states of matter out of three are moving at (much) less than the speed of light.

## (Almost) facts:

F1. The fourth family of quarks and leptons does not exist.

No 4<sup>th</sup> ("light") neutrino is evidenced by the measurement of the Z0 meson line shape.

The 4<sup>th</sup> family of heavy quarks is ruled out by the measurement of the Higgs boson.

**M1. Neutrino mass eigenstates have the same energy or the same momentum, oscillation formula will be the same. What is more different, energies or momenta?**

Neutrino flavor eigenstates  $|\nu_f\rangle$   $f = e, \mu, \tau$

produced in weak interactions are different from mass eigenstates  $|\nu_i\rangle$   $i = 1, 2, 3$

They are related by the unitary mixing matrix  $U_{fi} \equiv \langle \nu_f | \nu_i \rangle$  PMNS Matrix

$$|\nu_f\rangle = ( |\nu_1\rangle\langle\nu_1| + |\nu_2\rangle\langle\nu_2| + |\nu_3\rangle\langle\nu_3| ) |\nu_f\rangle$$

$$|\nu_f\rangle = \mathbf{U}_{f1}^* |\nu_1\rangle + \mathbf{U}_{f2}^* |\nu_2\rangle + \mathbf{U}_{f3}^* |\nu_3\rangle$$

Anti neutrinos

$$|\bar{\nu}_f\rangle = \mathbf{U}_{f1} |\bar{\nu}_1\rangle + \mathbf{U}_{f2} |\bar{\nu}_2\rangle + \mathbf{U}_{f3} |\bar{\nu}_3\rangle$$

$$|\nu_f(L)\rangle = \left( \sum_i e^{-i\frac{m_1^2 L}{2\hbar c E}} |\nu_i\rangle \langle \nu_i| \right) |\nu_f\rangle = U_{f1}^* e^{-i\frac{m_1^2 L}{2\hbar c E}} |\nu_1\rangle + U_{f2}^* e^{-i\frac{m_2^2 L}{2\hbar c E}} |\nu_2\rangle + U_{f3}^* e^{-i\frac{m_3^2 L}{2\hbar c E}} |\nu_3\rangle$$

$$A_{\nu_f \rightarrow \nu_f}(L) = \langle \nu_f | \nu_f(L) \rangle = |U_{f1}|^2 e^{-i\frac{m_1^2 L}{2\hbar c E}} + |U_{f2}|^2 e^{-i\frac{m_2^2 L}{2\hbar c E}} + |U_{f3}|^2 e^{-i\frac{m_3^2 L}{2\hbar c E}}$$

$$P_{\nu_f \rightarrow \nu_f}(L) = |U_{f1}|^4 + |U_{f2}|^4 + |U_{f3}|^4$$

$$+ 2|U_{f1}|^2 |U_{f2}|^2 \cos\left(\frac{m_2^2 - m_1^2}{2\hbar c} \frac{L}{E}\right) + 2|U_{f1}|^2 |U_{f3}|^2 \cos\left(\frac{m_3^2 - m_1^2}{2\hbar c} \frac{L}{E}\right) + 2|U_{f2}|^2 |U_{f3}|^2 \cos\left(\frac{m_3^2 - m_2^2}{2\hbar c} \frac{L}{E}\right)$$

$$= 1 - 4|U_{f1}|^2 |U_{f2}|^2 \sin^2\left(\frac{m_2^2 - m_1^2}{4\hbar c} \frac{L}{E}\right) - 4|U_{f1}|^2 |U_{f3}|^2 \sin^2\left(\frac{m_3^2 - m_1^2}{4\hbar c} \frac{L}{E}\right) - 4|U_{f2}|^2 |U_{f3}|^2 \sin^2\left(\frac{m_3^2 - m_2^2}{4\hbar c} \frac{L}{E}\right)$$

Neutrino oscillations require „non-diagonal“ U matrix elements and **different masses of neutrino mass eigenstates**. They are therefore clear evidence of non-zero neutrino mass.

$$e^{-i\frac{m_i^2 L}{2\hbar c E}}$$

$$\nu_i(L, ct = 0) = e^{+\frac{i}{\hbar c} P_i L} \nu_i$$

$$\nu_i(L, ct) = e^{-\frac{i}{\hbar c} (E_i ct - P_i L)} \nu_i \cong e^{-\frac{i}{\hbar c} (\textcolor{blue}{E}_i - \textcolor{blue}{P}_i)L} = e^{-\frac{i}{\hbar c} \frac{E_i^2 - P_i^2}{E_i + P_i} L} \cong e^{-\frac{i}{\hbar c} \frac{m_i^2}{2E} L}$$

$$E_1 = E_2 = E_3 = E_0 \\ P_i = \sqrt{E_0^2 - m_i^2} \cong E_0 - \frac{m_i^2}{2E_0}$$

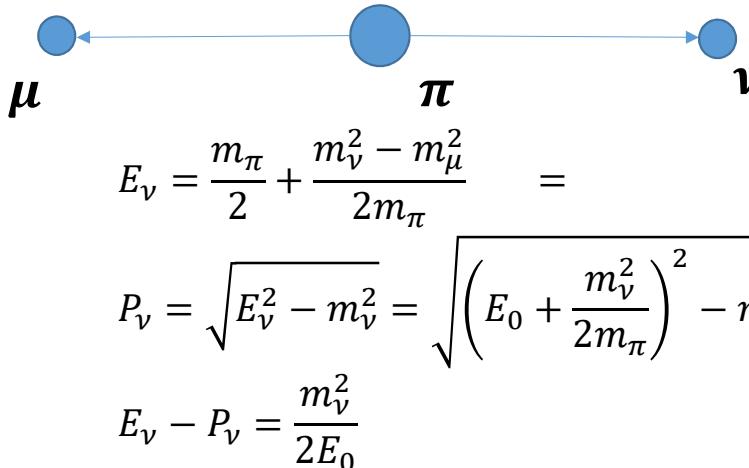
$$\textcolor{blue}{E}_i - \textcolor{blue}{P}_i = \frac{m_i^2}{2E_0}$$

$$P_1 = P_2 = P = P_0 \\ E_i = \sqrt{P_0^2 + m_i^2} \cong P_0 + \frac{m_i^2}{2E_0} \\ \textcolor{blue}{E}_i - \textcolor{blue}{P}_i = \frac{m_i^2}{2P_0} \cong \frac{m_i^2}{2E_0}$$

All 3 ways give the same answer. How is it in reality?

Do neutrinos mass eigenstates have the same momenta but different energies or same energies and different momenta, ...?

**Neutrinos have both different energies and momenta. Momenta usually differ (much) more.**



**Neutrinos from pion decay at rest**

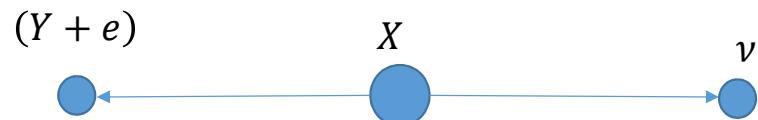
$$\pi \rightarrow \mu + \nu$$

Reactor neutrinos of  $\sim 4$  MeV from decays of  $\sim 100$  GeV heavy nuclei

$$E_\nu = \frac{m_X}{2} + \frac{m_\nu^2 - m_{Ye}^2}{2m_X} = E_0 + \frac{m_\nu^2}{2m_X} \cong 4\text{MeV} + \frac{m_\nu^2}{100\text{GeV}}$$

$$P_\nu = \sqrt{E_\nu^2 - m_\nu^2} \cong E_0 + \frac{m_\nu^2}{2m_X} - \frac{m_\nu^2}{2E_0} \cong 4\text{MeV} + \frac{m_\nu^2}{100\text{GeV}} - \frac{m_\nu^2}{4\text{MeV}}$$

$$X \rightarrow (Y + e) + \nu$$

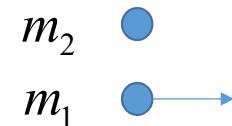


**In real experiment, energies of different mass eigenstates are almost the same, momenta differs much more**

$$E_i = E_0 + \delta_i \left( = \frac{m_i^2}{2m_X} \right)$$

$$P_i \cong E_0 + \delta_i \left( = \frac{m_i^2}{2m_X} \right) - \Delta_i \left( = \frac{m_i^2}{2E_0} \right)$$

**Neutrinos have mass. Let us take two neutrinos and boost them to the rest frame of the heavier one.**



Let us go to the rest frame of heavier neutrino ( $m_2$ )

$$\beta = \frac{P_2}{E_2} = \frac{E_0 + \delta_2 - \Delta_2}{E_0 + \delta_2} = 1 - \frac{\Delta_2}{E_0 + \delta_2} \cong 1 - \frac{\Delta_2}{E_0} = 1 - \frac{m_2^2}{2E_0^2} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{\frac{m_2^2}{2E_0^2} \left( 2 - \frac{m_2^2}{2E_0^2} \right)}} \cong \frac{E_0}{m_2}$$

$$P^* = \gamma(P - \beta E) \quad E^* = \gamma(E - \beta P)$$

$$P_2^* = \frac{E_0}{m_2} \left( E_0 + \delta_2 - \Delta_2 - \left( 1 - \frac{m_2^2}{2E_0^2} \right) (E_0 + \delta_2) \right) = \frac{E_0}{m_2} \left( -\Delta_2 + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{E_0}{m_2} \left( -\frac{m_2^2}{2E_0} + \frac{m_2^2}{2E_0} \right) = \mathbf{0}$$

$$E_2^* = \frac{E_0}{m_2} \left( E_0 + \delta_2 - \left( 1 - \frac{m_2^2}{2E_0^2} \right) (E_0 + \delta_2 - \Delta_2) \right) = \frac{E_0}{m_2} \left( \Delta_2 + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{E_0}{m_2} \left( -\frac{m_2^2}{2E_0} + \frac{m_2^2}{2E_0} \right) = \mathbf{m}_2$$

$$P_1^* = \frac{E_0}{m_2} \left( E_0 + \delta_1 - \Delta_1 - \left( 1 - \frac{m_2^2}{2E_0^2} \right) (E_0 + \delta_1) \right) = \frac{E_0}{m_2} \left( -\Delta_1 + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{E_0}{m_2} \left( -\frac{m_1^2}{2E_0} + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{m_2^2 - m_1^2}{2m_2}$$

$$E_1^* = \frac{E_0}{m_2} \left( E_0 + \delta_1 - \left( 1 - \frac{m_2^2}{2E_0^2} \right) (E_0 + \delta_1 - \Delta_1) \right) = \frac{E_0}{m_2} \left( \Delta_1 + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{E_0}{m_2} \left( -\frac{m_1^2}{2E_0} + \frac{m_2^2}{2E_0^2} E_0 \right) = \frac{m_2^2 + m_1^2}{2m_2}$$

## The rest frame of heavier neutrino 2:

$$P_2^* = 0 \quad E_2^* = m_2$$

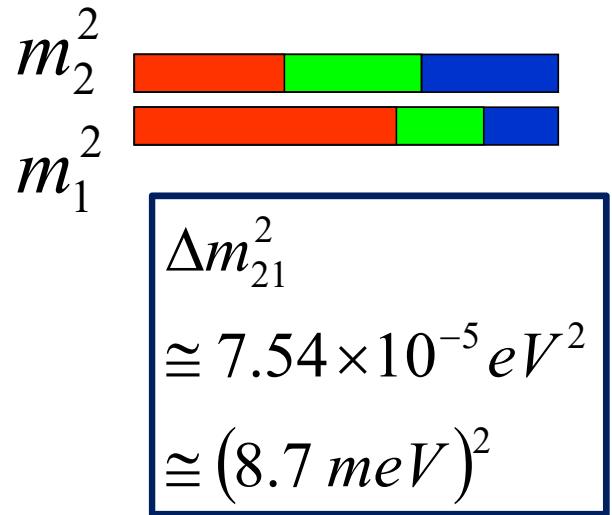
$$P_1^* = \frac{m_2^2 - m_1^2}{2m_2} > 0 \quad E_1^* = \frac{m_2^2 + m_1^2}{2m_2} < E_2^*$$

$$E_2^* - E_1^* = \frac{m_2^2 - m_1^2}{2m_2}$$

$$\Rightarrow \beta_1^* = \frac{m_2^2 - m_1^2}{m_2^2 + m_1^2}$$

$$m_1 = 0 \Rightarrow \beta_1^* = 1 \dots P_1^* = E_1^* = \frac{(9 \text{ meV})^2 / 2}{(9 \text{ meV})} \cong 4.5 \text{ meV}$$

$$m_1 = 0.1 \text{ eV} \Rightarrow \beta_1^* = \frac{(9 \text{ meV})^2}{2 (100 \text{ meV})^2} = 0.004$$



In the rest frame of heavier neutrino mass eigenstates, the difference in momenta and energies are the same.

$$P_1^* - P_2^* = E_2^* - E_1^* = \frac{m_2^2 - m_1^2}{2m_2}$$

## M1. Neutrino mass eigenstates have the same energy or the same momentum, oscillation formula will be the same. What is more different, energies or momenta?

In real experiment, energies of different mass eigenstates are almost the same, momenta differs much more (E0 is neutrino energy, MX is the mass of decaying particle (ranging from pi mezon 140 MeV to nuclei ~100 GeV)

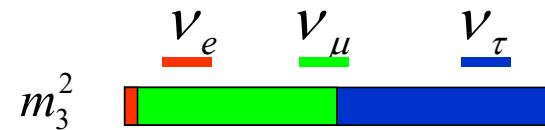
$$P_1 - P_2 = \frac{m_2^2 - m_1^2}{2E_0} \ll E_2 - E_1 = \frac{m_2^2 - m_1^2}{2M_X}$$

**M2. On Earth we see only one third of predicted electron neutrino flux because there are three flavours of neutrinos?**

**But the solution is the effect of the dense mass of the solar core.**

# OSCILLATION PARAMETERS

## NORMAL MASS HIERARCHY (NH)



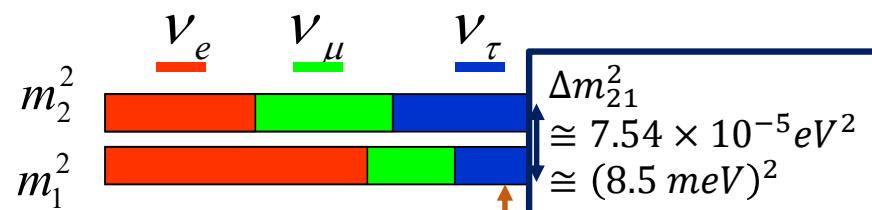
$$|U_{e3}|^2 |U_{\mu 3}|^2 |U_{\tau 3}|^2$$

$$|U_{e2}|^2 |U_{\mu 2}|^2 |U_{\tau 2}|^2$$



$$|U_{e1}|^2 |U_{\mu 1}|^2 |U_{\tau 1}|^2$$

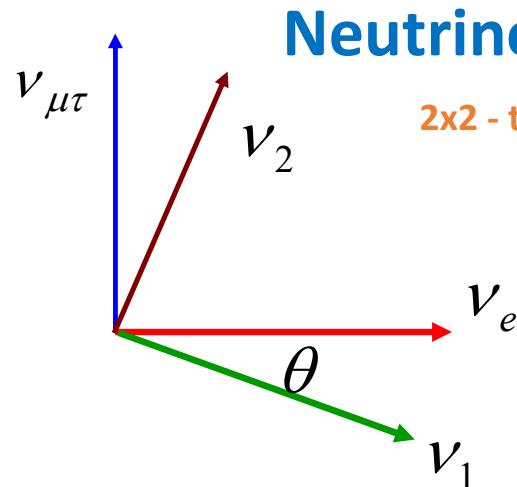
## INVERSE MASS HIERARCHY (IH)



$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix}$$



$$|\Delta m_{21}^2| \cong 7.54 \times 10^{-5} \text{ eV}^2 \cong (8.5 \text{ meV})^2$$



## Neutrino oscillation – 2x2

2x2 - two types (flavors) and two mass states of neutrinos:

$$|\nu_e\rangle = \cos(\theta)|\nu_1\rangle + \sin(\theta)|\nu_2\rangle$$

$$|\nu_{\mu\tau}\rangle = -\sin(\theta)|\nu_1\rangle + \cos(\theta)|\nu_2\rangle$$

$$\begin{pmatrix} \nu_e \\ \nu_{\mu\tau} \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{e2}^* \\ U_{\mu\tau 1}^* & U_{\mu\tau 2}^* \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$|\nu_e\rangle = \left( \sum_i |\nu_i\rangle \langle \nu_i| \right) |\nu_e\rangle = U_{e1}^* |\nu_1\rangle + U_{e2}^* |\nu_2\rangle$$

$$|\nu_{\mu\tau}\rangle = \left( \sum_i |\nu_i\rangle \langle \nu_i| \right) |\nu_{\mu\tau}\rangle = U_{\mu\tau 1}^* |\nu_1\rangle + U_{\mu\tau 2}^* |\nu_2\rangle$$

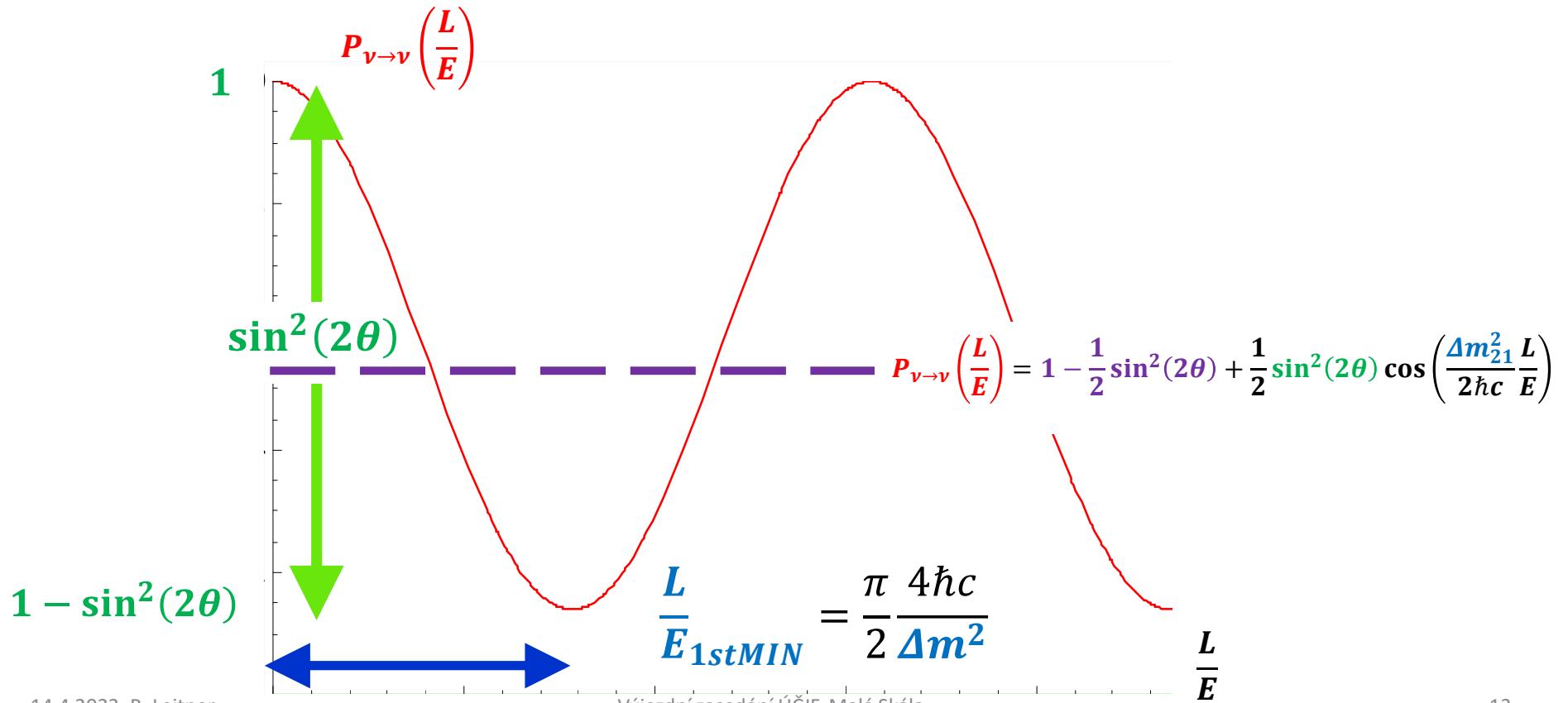
$$U_{fi}^* = \begin{pmatrix} U_{e1}^* & U_{e2}^* \\ U_{\mu\tau 1}^* & U_{\mu\tau 2}^* \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$L = 0 \dots |\nu_e\rangle \Rightarrow |\nu_e(L)\rangle = +\cos(\theta) \cdot e^{-i\frac{m_1^2 L}{2\hbar c E}} |\nu_1\rangle + \sin(\theta) \cdot e^{-i\frac{m_2^2 L}{2\hbar c E}} |\nu_2\rangle$$

$$L = 0 \dots |\nu_{\mu\tau}\rangle \Rightarrow |\nu_{\mu\tau}(L)\rangle = -\sin(\theta) \cdot e^{-i\frac{m_1^2 L}{2\hbar c E}} |\nu_1\rangle + \cos(\theta) \cdot e^{-i\frac{m_2^2 L}{2\hbar c E}} |\nu_2\rangle$$

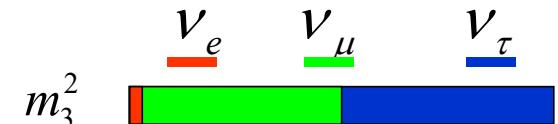
**2x2 Mixing Amplitude of oscillations =  $\sin^2(2\theta)$ ,  
oscillation length is inversely proportional to  $\Delta m^2$**

$$P_{\nu_f \rightarrow \nu_f}(L) = 1 - 4|U_{f1}|^2 |U_{f2}|^2 \sin^2 \left( \frac{m_2^2 - m_1^2}{4\hbar c} \frac{L}{E} \right) = 1 - \sin^2(2\theta) \sin^2 \left( \frac{\Delta m_{21}^2}{4\hbar c} \frac{L}{E} \right)$$



$\theta_{23} \cong 45^\circ$  Half of both muon and tauon neutrinos in m3

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23}) \end{pmatrix}.$$



$\theta_{13} \cong 8.5^\circ$  Very small fraction of electron neutrinos in m3  $|U_{e3}|^2 |U_{\mu 3}|^2 |U_{\tau 3}|^2$

$$\begin{pmatrix} \cos(\theta_{13}) & 0 & \sin(\theta_{13}) \cdot e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin(\theta_{13}) \cdot e^{i\delta} & 0 & \cos(\theta_{13}) \end{pmatrix}.$$

$|U_{e2}|^2 |U_{\mu 2}|^2 |U_{\tau 2}|^2$

$$\begin{pmatrix} \cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\ -\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$m_2^2$

$m_1^2$

$|U_{e1}|^2 |U_{\mu 1}|^2 |U_{\tau 1}|^2$

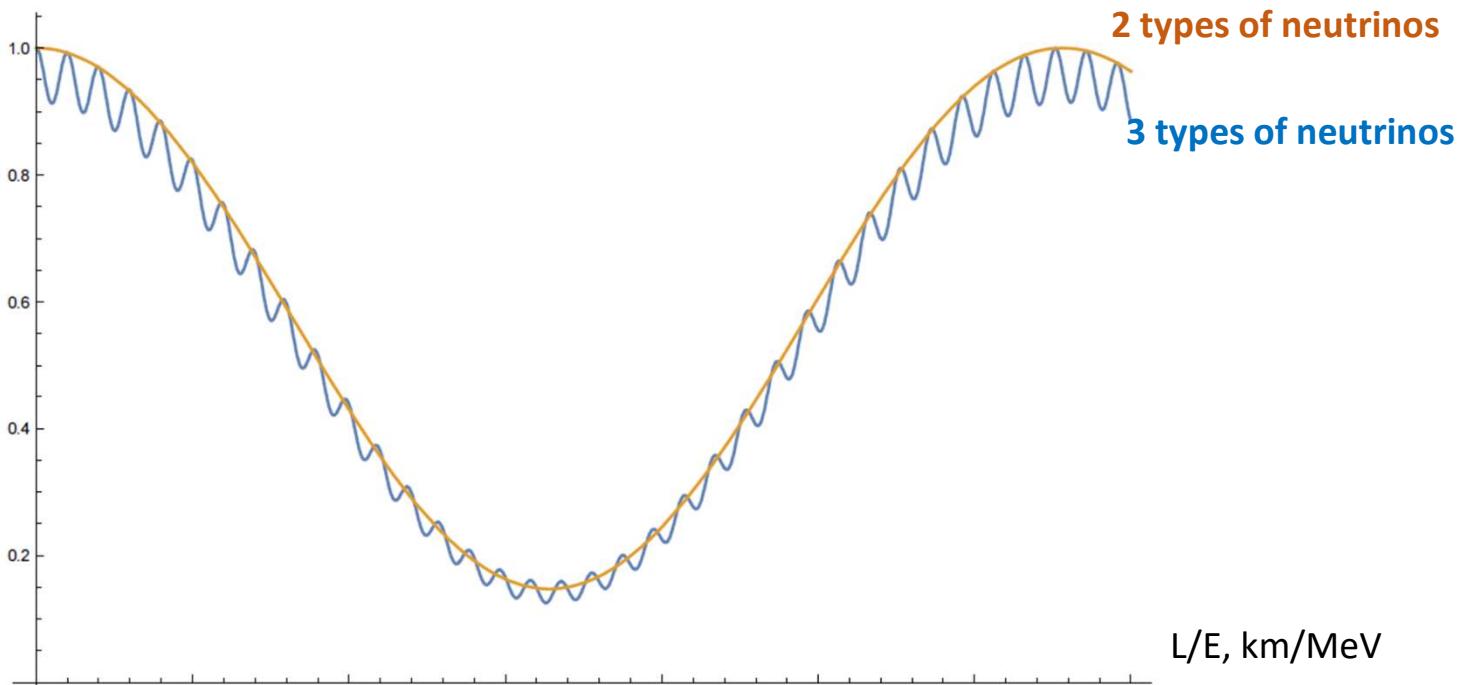
$\theta_{12} \cong 34^\circ$  2/3 of electron neutrinos is in m1 and 1/3 in m2

$$P_{\nu e \rightarrow \nu e}^{2x2} = 1 -$$

$$\sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m_{21}^2}{4\hbar c} \frac{L}{E}\right)$$

$$P_{\nu e \rightarrow \nu e}^{3x3} = 1 - \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m_{21}^2}{4\hbar c} \frac{L}{E}\right) - \sin^2(2\theta_{13}) \left( \cos^2(\theta_{12}) \sin^2\left(\frac{\Delta m_{31}^2}{4\hbar c} \frac{L}{E}\right) + \sin^2(\theta_{12}) \sin^2\left(\frac{\Delta m_{32}^2}{4\hbar c} \frac{L}{E}\right) \right)$$

**Probability of oscillations for electron  
(anti)neutrinos for 2 types and 3 types of neutrinos**



# Oscillations in matter

**Variable matter density**  $V(x) = (\hbar c)^3 \sqrt{2} G_F N_e(x)$  electron density

$$\cos 2\theta(x) = \frac{\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2}\right)^2}}$$

Mixing angle in matter is sensitive to the sign of  $\Delta m_{21}^2$

In the core of the Sun, for most energetic neutrinos

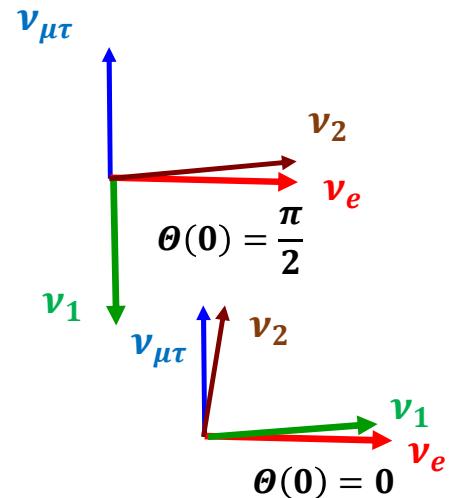
the term  $\frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(0)}{2}$  dominates:

$$\cos 2\theta(0) = -1 \quad \theta(0) = \frac{\pi}{2} \quad \text{for } m_2 > m_1$$

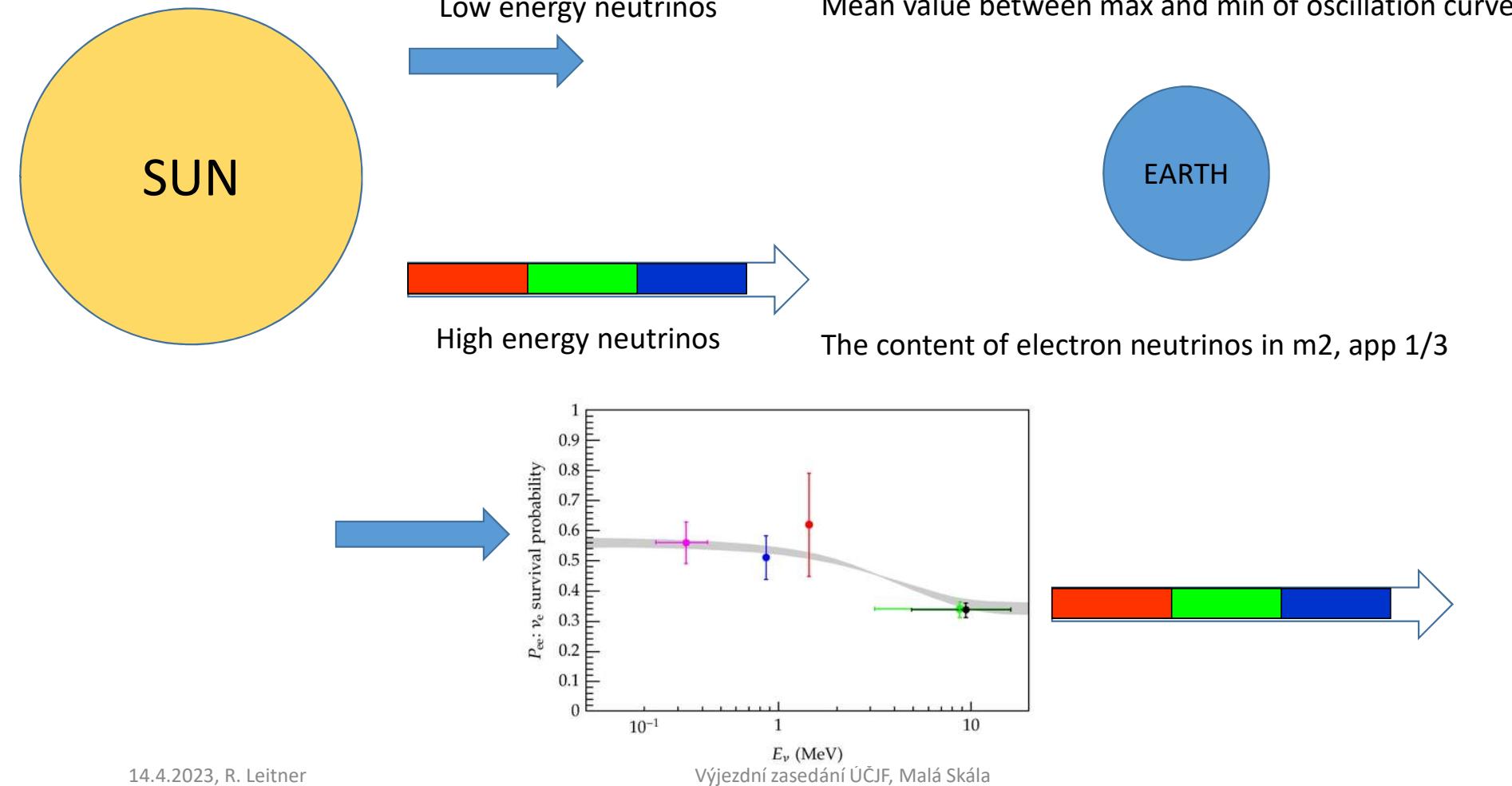
Electron neutrino is equal to  $m_2$

$$\cos 2\theta(0) = +1 \quad \theta(0) = 0 \quad \text{for } m_1 > m_2$$

Electron neutrino is equal to  $m_1$

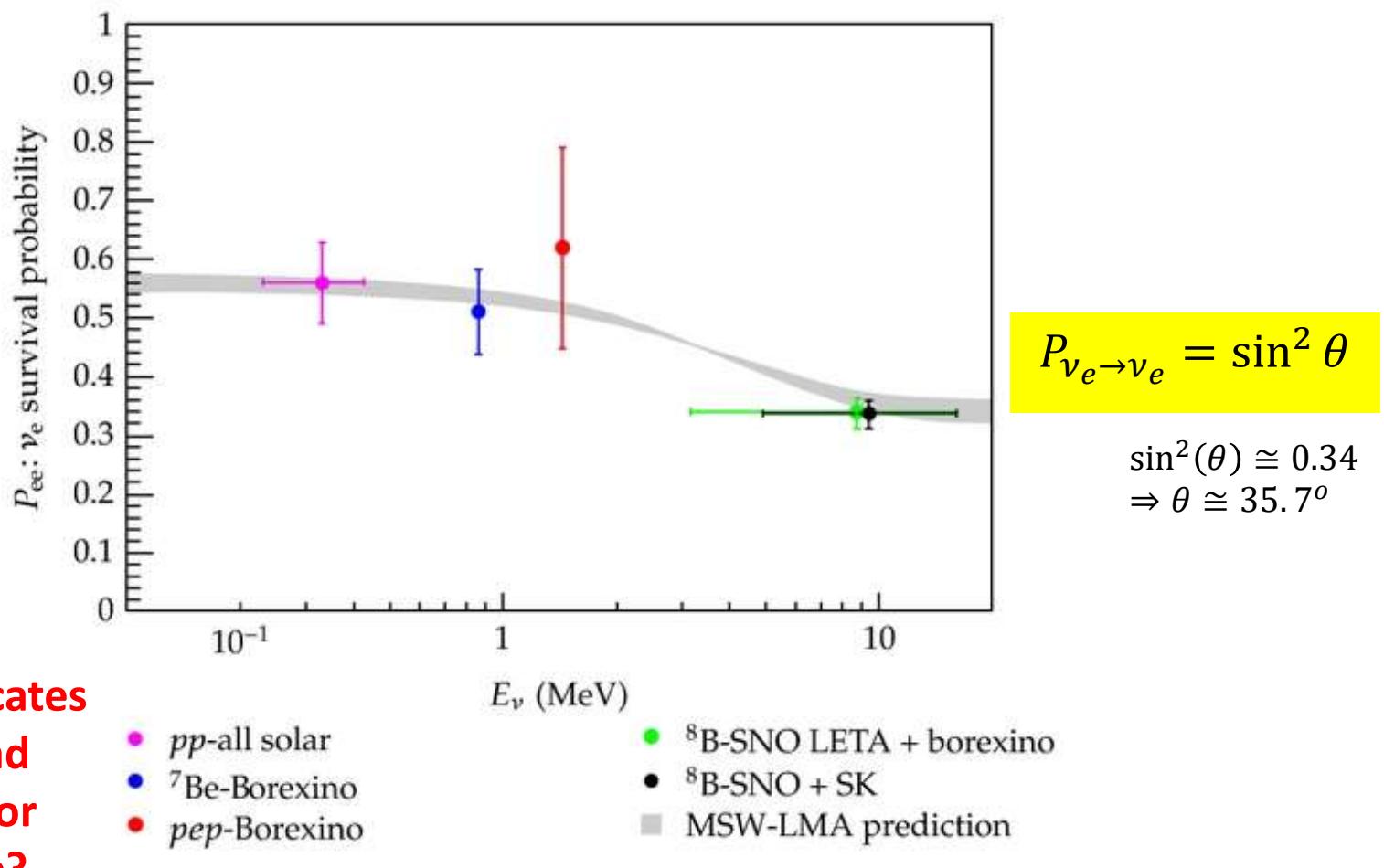


Electron neutrino is equal to heavier mass eigenstate of  $m_1$   $m_2$



## Probability of solar electron neutrinos at the Earth

$$1 - \frac{1}{2} \sin^2(2\theta) \cong 0.57$$
$$\Rightarrow \theta \cong 34.9^\circ$$



This measurement indicates that  $m_2 > m_1$ . How to find that  $m_3$  is the heaviest or lightest mass eigenstate?



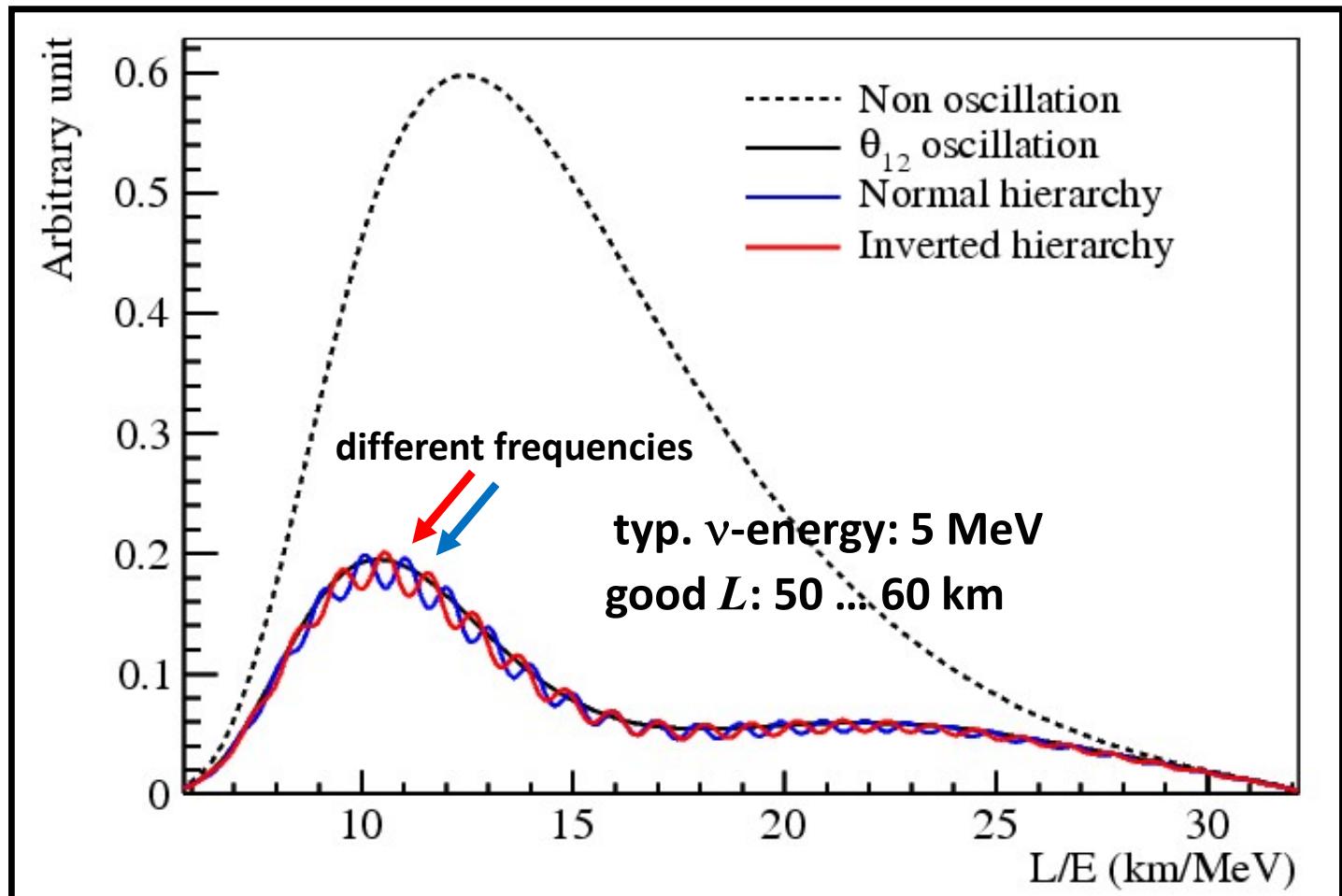
$$\begin{aligned}
 A_{\nu_e \rightarrow \nu_e}(x) &= e^{-i\frac{m_1^2 x}{2\hbar c E}} |U_{e1}|^2 + e^{-i\frac{m_2^2 x}{2\hbar c E}} |U_{e2}|^2 + e^{-i\frac{m_3^2 x}{2\hbar c E}} |U_{e3}|^2 \\
 &= e^{-i\frac{m_1^2 x}{2\hbar c E}} \left( |\textcolor{red}{U}_{e1}|^2 + e^{-i\frac{m_2^2 - m_1^2 x}{2\hbar c E}} |\textcolor{blue}{U}_{e2}|^2 + e^{-i\frac{m_3^2 - m_1^2 x}{2\hbar c E}} |\textcolor{red}{U}_{e3}|^2 \right)
 \end{aligned}$$

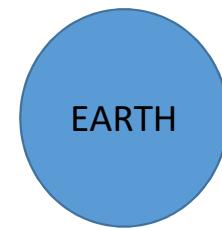
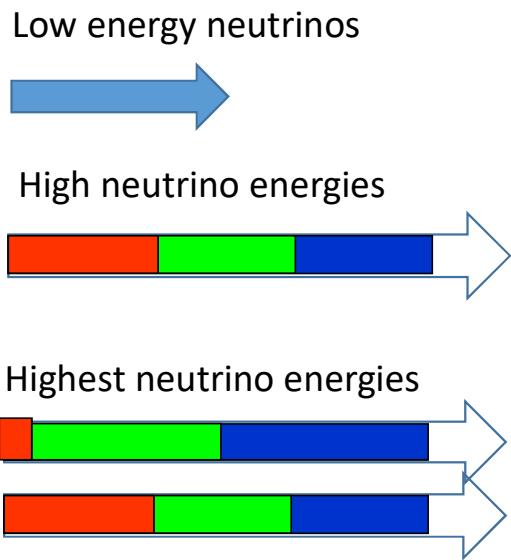
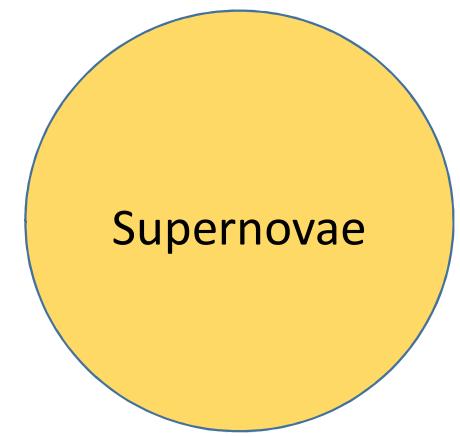


At certain distances, Total amplitudes will be different for and INVERTED hierarchy

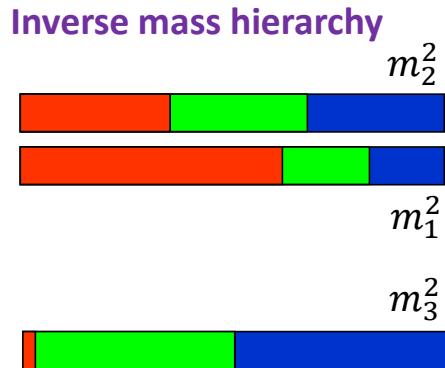
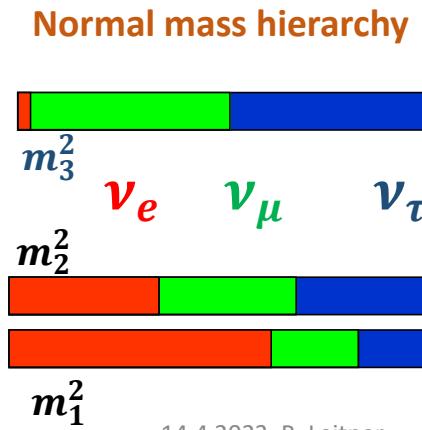
$$\begin{aligned}
 A_{\nu_e \rightarrow \nu_e}(x) &= e^{-i\frac{m_1^2 x}{2\hbar c E}} |U_{e1}|^2 + e^{-i\frac{m_2^2 x}{2\hbar c E}} |U_{e2}|^2 + e^{-i\frac{m_3^2 x}{2\hbar c E}} |U_{e3}|^2 \\
 &= e^{-i\frac{m_1^2 x}{2\hbar c E}} (|U_{e1}|^2 + e^{-i\frac{m_2^2 - m_1^2 x}{2\hbar c E}} |U_{e2}|^2 + e^{-i\frac{m_3^2 - m_1^2 x}{2\hbar c E}} |U_{e3}|^2)
 \end{aligned}$$

## Expected spectrum in future JUNO experiment



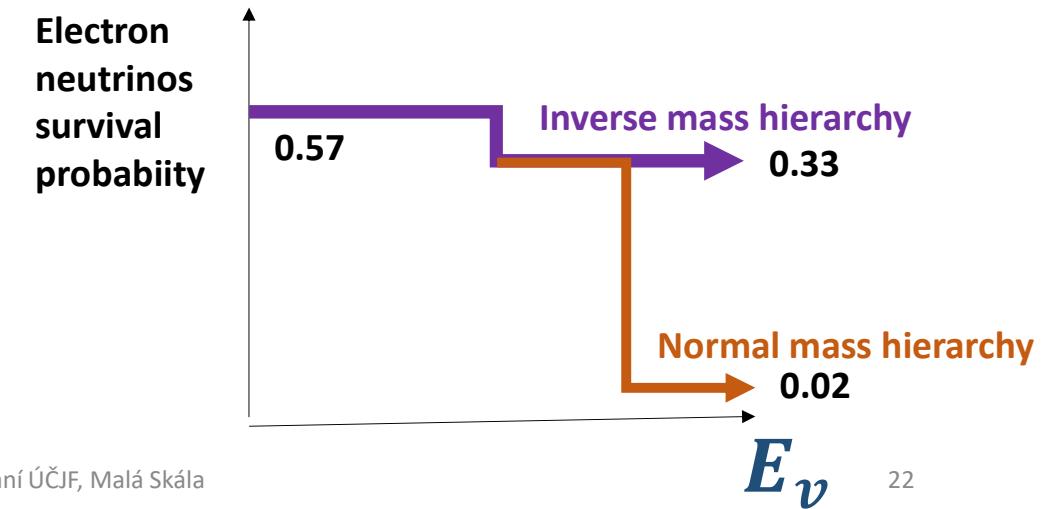


The content of electron neutrinos in m<sub>3</sub> (Normal hierarchy app. only 2%) or m<sub>2</sub> (Inverse hierarchy app 1/3) or



14.4.2023, R. Leitner

Výjezdní zasedání ÚČJF, Malá Skála



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### M3. Neutrino oscillations do not conserve energy? Of course they do.

$$\begin{aligned} E_e(L=0) &= \langle \nu_e | H | \nu_e \rangle = \\ &\langle \nu_1 \cos \vartheta + \nu_2 \sin \vartheta | H | \nu_1 \cos \vartheta + \nu_2 \sin \vartheta \rangle \\ &= \langle \nu_1 | H | \nu_1 \rangle \cos^2 \vartheta + \langle \nu_2 | H | \nu_2 \rangle \sin^2 \vartheta = E_1 \cos^2 \vartheta + E_2 \sin^2 \vartheta \end{aligned}$$

$$\begin{aligned} E_e(L) &= \langle \nu_e(L) | H | \nu_e(L) \rangle \\ &= \left\langle \nu_1 \left( e^{-i \frac{m_1^2 L}{2\hbar c E}} \right)^* \cos \vartheta + \nu_2 \left( e^{-i \frac{m_2^2 L}{2\hbar c E}} \right)^* \sin \vartheta \middle| H \middle| \nu_1 e^{-i \frac{M_1^2 L}{2E}} \cos \vartheta + \nu_2 e^{-i \frac{M_2^2 L}{2E}} \sin \vartheta \right\rangle \\ &= E_1 \cos^2 \vartheta + E_2 \sin^2 \vartheta \end{aligned}$$

**Is it true even if (electron) neutrino completely changes to muon neutrino?**

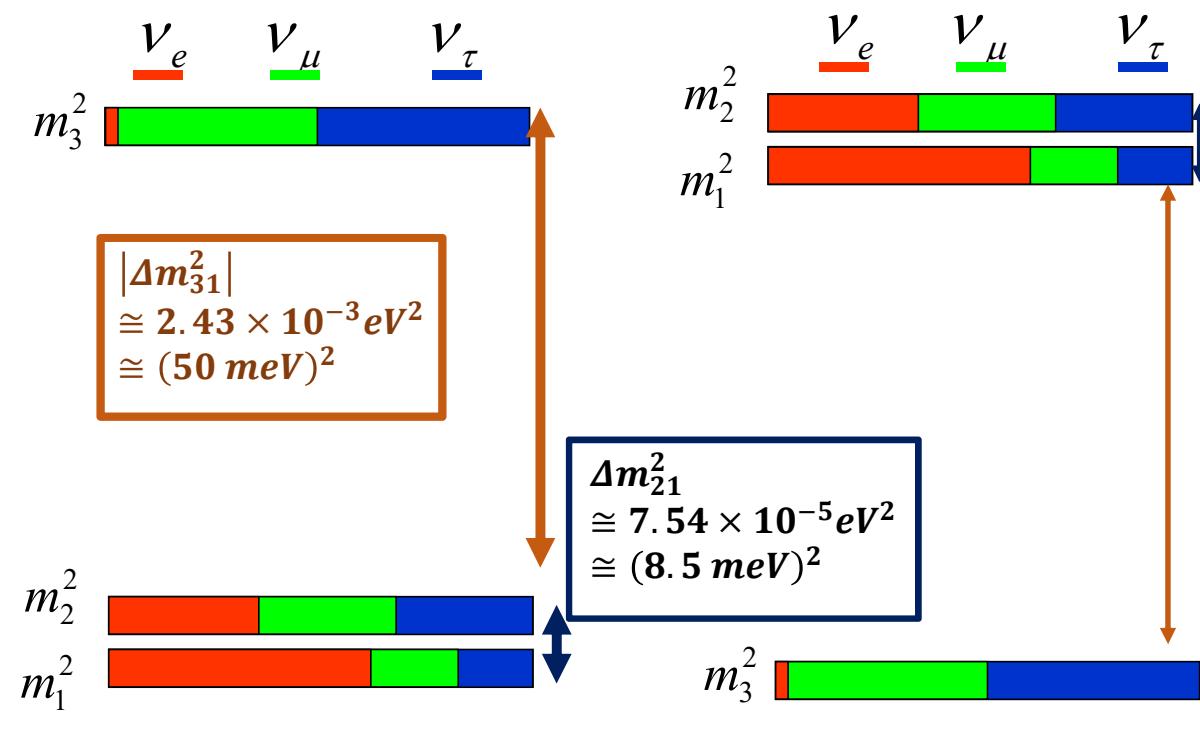
$$\begin{aligned}E_{\mu} &= \langle \nu_{\mu} | H | \nu_{\mu} \rangle = \langle -\nu_1 \sin \vartheta + \nu_2 \cos \vartheta | H | -\nu_1 \sin \vartheta + \nu_2 \cos \vartheta \rangle \\&= E_1 \sin^2 \vartheta + E_2 \cos^2 \vartheta\end{aligned}$$

$$\begin{aligned}E_e &= \langle \nu_e | H | \nu_e \rangle = \langle \nu_1 \cos \vartheta + \nu_2 \sin \vartheta | H | \nu_1 \cos \vartheta + \nu_2 \sin \vartheta \rangle \\&= \langle \nu_1 | H | \nu_1 \rangle \cos^2 \vartheta + \langle \nu_2 | H | \nu_2 \rangle \sin^2 \vartheta = E_1 \cos^2 \vartheta + E_2 \sin^2 \vartheta\end{aligned}$$

**The energies look different**

But a full change to another neutrino flavor is possible only for maximal mixing angle  
 $\vartheta = 45^\circ \Rightarrow \sin^2 \vartheta = \cos^2 \vartheta \Rightarrow E_1 \sin^2 \vartheta + E_2 \cos^2 \vartheta = E_1 \cos^2 \vartheta + E_2 \sin^2 \vartheta = (E_1 + E_2)/2$   
and energy is also conserved

## M4. Relic neutrinos move at practically the speed of light? At least two states of matter out of three are moving at (much) less than the speed of light.



Normal mass hierarchy  
(ordering)  $m_1 \geq 0, m_2 \geq 8.5$  meV,  $m_3 \geq 50$  meV

Inverted mass hierarchy  
(ordering)  $m_1 \geq 0, m_2 \geq 8.5$  meV,  $m_3 \geq 50$  meV

Are relic neutrinos ( $T=2$  K) relativistic?

$$T=2 \text{ K} \rightarrow kT = k \cdot 300 \text{ K} \quad (2 \text{ K} / 300 \text{ K}) = 25 \text{ meV} / 150 = 0.17 \text{ meV}$$

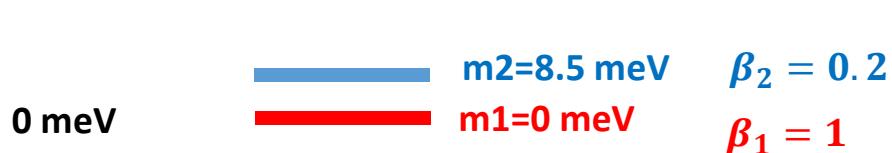
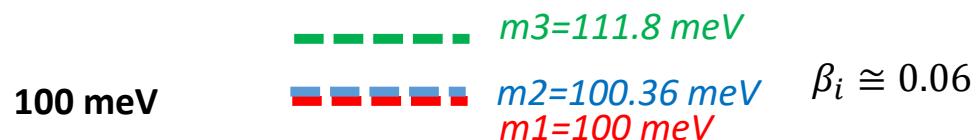
**Normal hierarchy  $m_1 \geq 0, m_2 \geq 8.5 \text{ meV}, m_3 \geq 50 \text{ meV}$**

For normal mass ordering, at least 2 relic neutrino mass eigenstates ( $m_2, m_3$ ) are non-relativistic

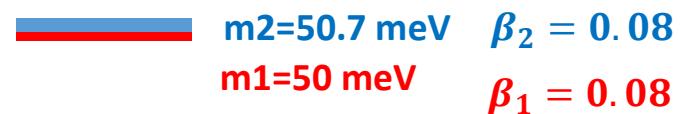
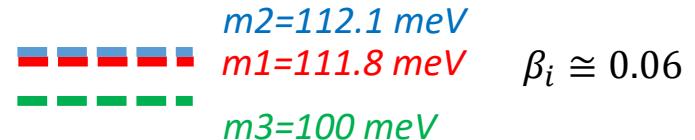
$$\beta_2 \leq \frac{P}{E} = \frac{\sqrt{(kT + m_2)^2 - m_2^2}}{kT + m_2} = \frac{\sqrt{2kT m_2 + kT^2}}{m_2 + kT} \cong \sqrt{\frac{2kT}{m_2}} = 0.2$$

$$\beta_3 \leq \frac{P}{E} = \frac{\sqrt{(kT + m_3)^2 - m_3^2}}{kT + m_3} \cong \sqrt{\frac{2kT}{m_3}} = 0.08$$

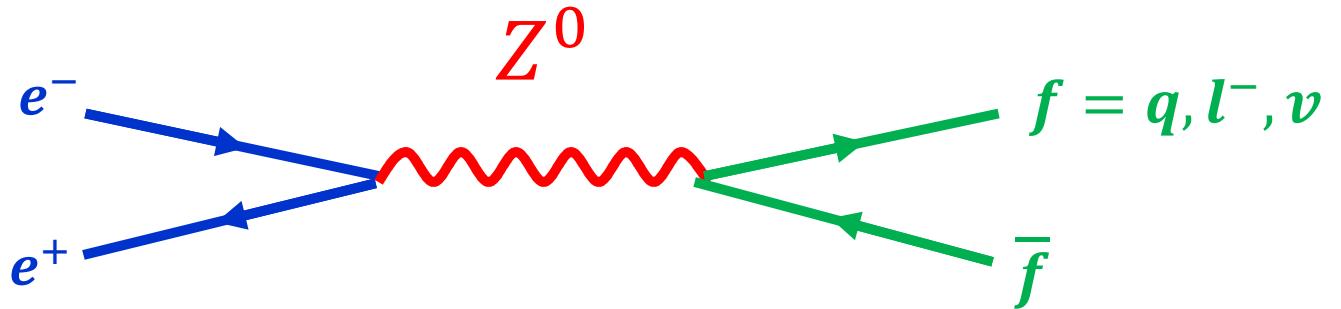
## NORMAL MASS ORDERING (Hierarchy)



## INVERTED MASS ORDERING (Hierarchy)



**F1. The fourth family of quarks and leptons does not exist.  
No 4<sup>th</sup> (“light”) neutrino is evidenced by the measurement of the Z0 meson line shape.  
The 4<sup>th</sup> family of heavy quarks is ruled out by the measurement of the Higgs boson.**



$$\sigma(s) = \frac{12\pi(\hbar c)^2}{M_Z^2} \frac{\Gamma_{ee}}{\Gamma_Z} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \frac{\Gamma_{ff}}{\Gamma_Z}$$

$$\sigma(s = M_Z^2) = \frac{12\pi(\hbar c)^2}{M_Z^2} \left( \frac{\Gamma_{ee}}{\Gamma_Z} \frac{\Gamma_{qq}}{\Gamma_Z} \right)$$

Using this relation we can measure the decay width to neutrinos and thus determine the number of neutrino families.

$$\sigma(s = M_Z^2) = \frac{12\pi(\hbar c)^2}{M_Z^2} \left( \frac{\Gamma_{ee}}{\Gamma_Z} \frac{\Gamma_{qq}}{\Gamma_Z} \right)$$

$$\frac{12\pi(\hbar c)^2}{M_Z^2} = \frac{12\pi(0.197 \text{ GeV fm})^2}{(91.19 \text{ GeV})^2} = 1760 \text{ nb}$$

$$\Gamma_{ee} = \Gamma_{\mu\mu} = \Gamma_{\tau\tau} = \Gamma_{ll} \quad \Gamma_{vv} = 2 \Gamma_{ll} \quad \Gamma_Z = \Gamma_{tot} = 3 \Gamma_{ll} + N_v 2\Gamma_{ll} + \Gamma_{had}$$

$N_\nu = 3$  neutrinos:

$$\frac{\Gamma_{ll}}{\Gamma_Z} = 3 \times 3.33\% = 10\% \quad \frac{\Gamma_{vv}}{\Gamma_Z} = 3 \times 2 \times 3.33\% = 20\% \quad \frac{\Gamma_{had}}{\Gamma_Z} = 70\%$$

$N_\nu = 2$  neutrinos:

$$\Gamma_Z - 2 \times 3.33\% \Gamma_Z \quad \frac{\Gamma_{ll}}{\Gamma_Z - 2 \times 3.33\% \Gamma_Z} = 10.7\% \quad \frac{\Gamma_{vv} - 2 \times 3.33\% \Gamma_Z}{\Gamma_Z - 2 \times 3.33\% \Gamma_Z} = 14.3\%$$

$N_\nu = 4$  neutrinos:

$$\Gamma_Z + 2 \times 3.33\% \Gamma_Z \quad \frac{\Gamma_{ll}}{\Gamma_Z + 2 \times 3.33\% \Gamma_Z} = 9.4\% \quad \frac{\Gamma_{vv} + 2 \times 3.33\% \Gamma_Z}{\Gamma_Z + 2 \times 3.33\% \Gamma_Z} = 25\%$$

$$\frac{\Gamma_{had}}{\Gamma_Z + 2 \times 3.33\% \Gamma_Z} = 75\%$$

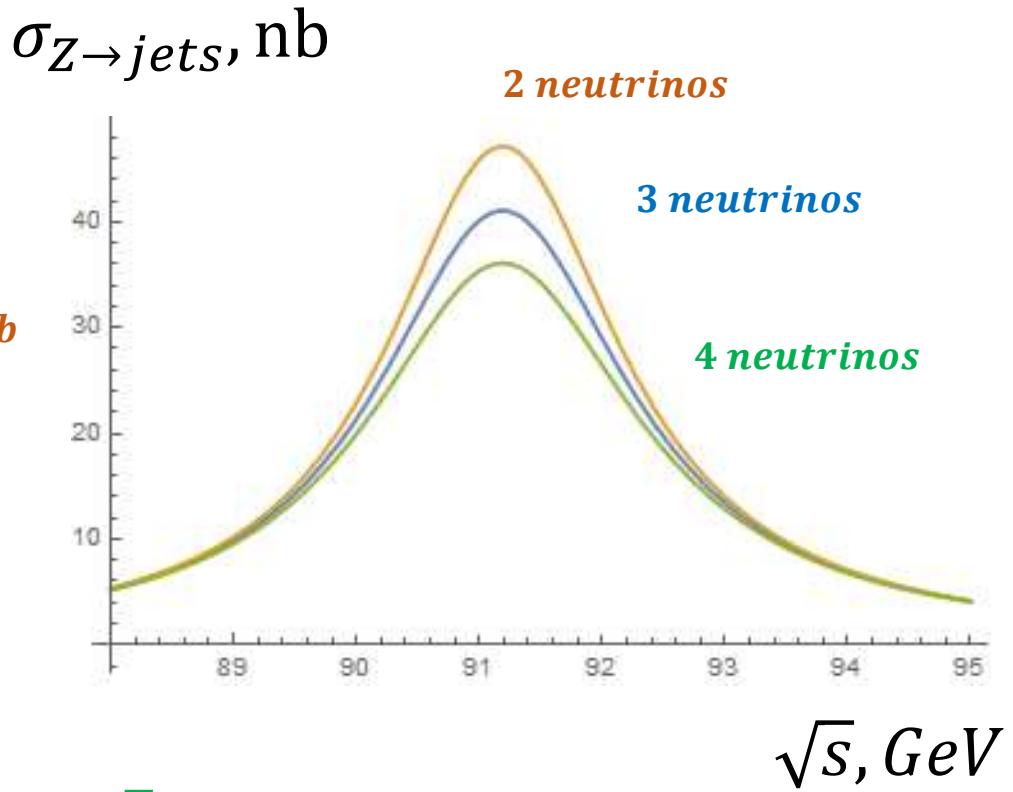
$$\frac{\Gamma_{had}}{\Gamma_Z + 2 \times 3.33\% \Gamma_Z} = 65.6\%$$

$$\sigma(M_Z) = 1760 \text{ nb} \left( \frac{\Gamma_{ee}}{\Gamma_Z} \frac{\Gamma_{qq}}{\Gamma_Z} \right)$$

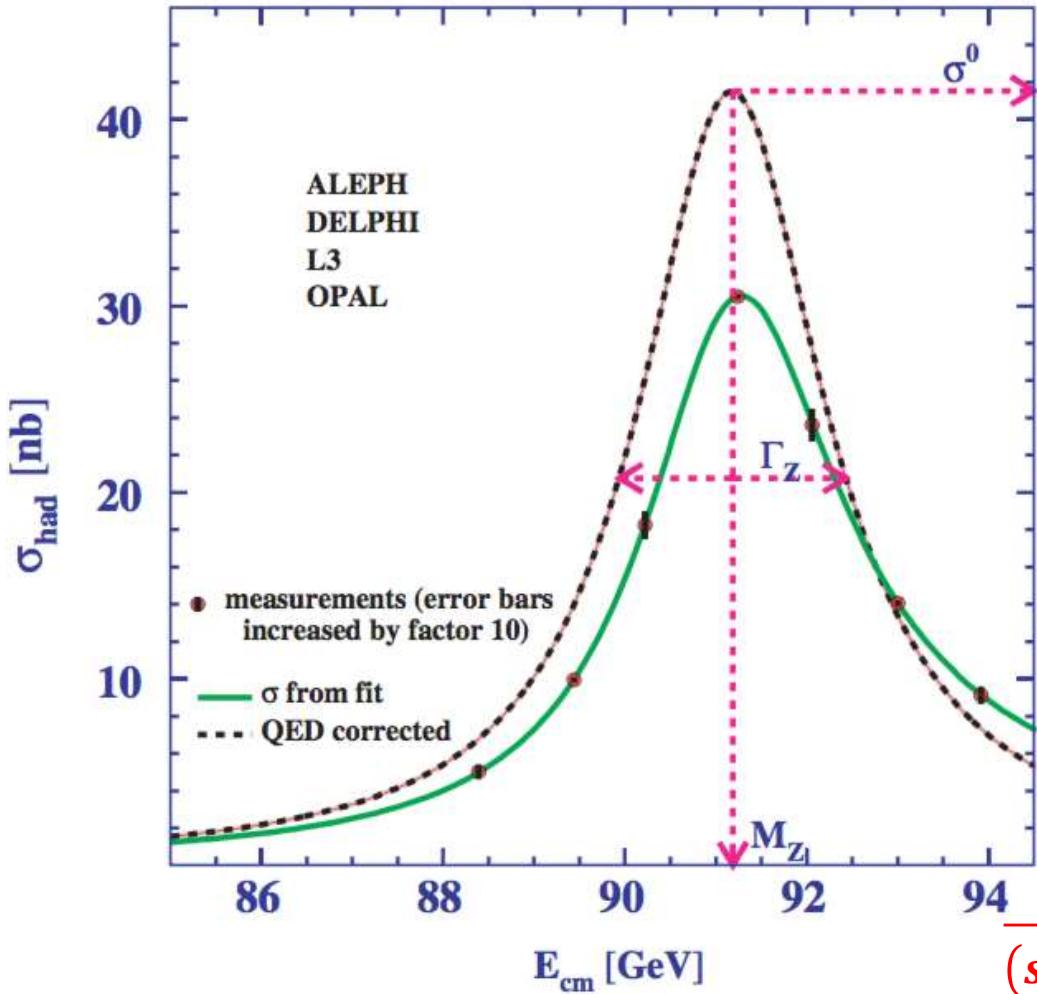
**2 neutrinos:**  $\sigma(M_Z) = 1760 \mu b \left( \frac{10.7\%}{3} 75\% \right) = 47.1 \text{ nb}$

**3 neutrinos:**  $\sigma(M_Z) = 1760 \text{ nb} \left( \frac{10\%}{3} 70\% \right) = 40.7 \text{ nb}$

**4 neutrinos:**  $\sigma(M_Z) = 1760 \text{ nb} \left( \frac{9.4\%}{3} 65.6\% \right) = 36.2 \text{ nb}$



$$\sigma(s) = \frac{12\pi(\hbar c)^2}{M_Z^2} \frac{\Gamma_{ee}}{\Gamma_Z} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \frac{\Gamma_{ff}}{\Gamma_Z}$$



The shape changes due to the emission of a braking photon by an electron or positron before the interaction. This changes the  $\sqrt{s}$  (Ecm) value

$$\frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \quad s = (E + E)^2 - (E - E)^2 = (2E)^2$$

If either electron or positron emits gamma with an energy  $E_\gamma$  before e+e- collide (Initial State Radiation), electron and positron collide at lower value of  $s^{cor}$

$$s^{cor} = (E - E_\gamma + E)^2 - (E - E_\gamma - E)^2 = (2E)^2 - 2EE_\gamma \\ = s - \sqrt{s} E_\gamma$$

$$\frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \rightarrow \frac{(s - \sqrt{s} E_\gamma)\Gamma_Z^2}{(s - \sqrt{s} E_\gamma - M_Z^2)^2 + (s - \sqrt{s} E_\gamma)\Gamma_Z^2}$$

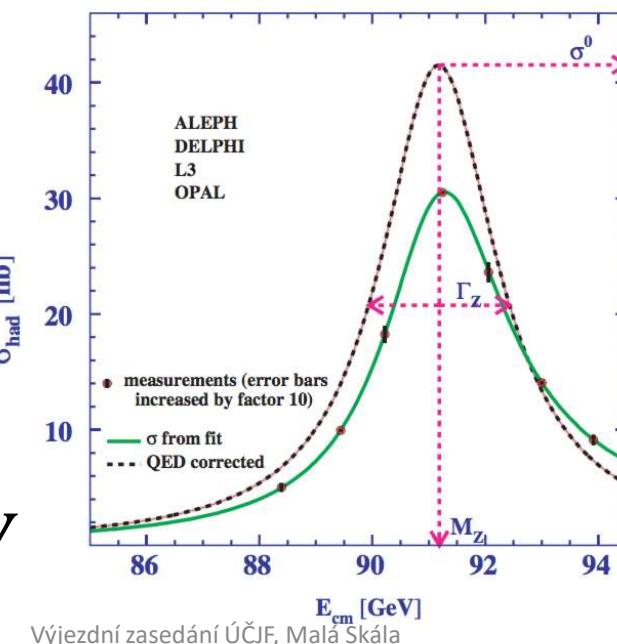
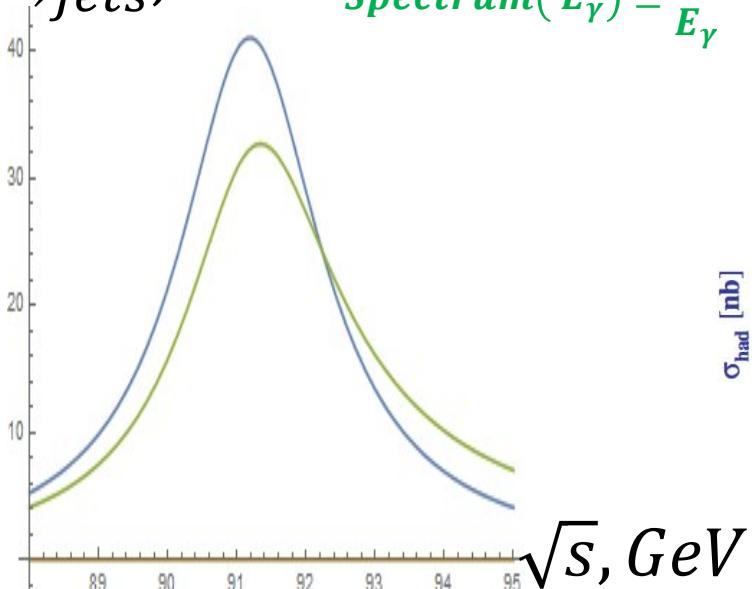
$$\frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \rightarrow \frac{(s - \sqrt{s} E_\gamma)\Gamma_Z^2}{(s - \sqrt{s} E_\gamma - M_Z^2)^2 + (s - \sqrt{s} E_\gamma)\Gamma_Z^2}$$

And we shall integrate over the spectrum Eg

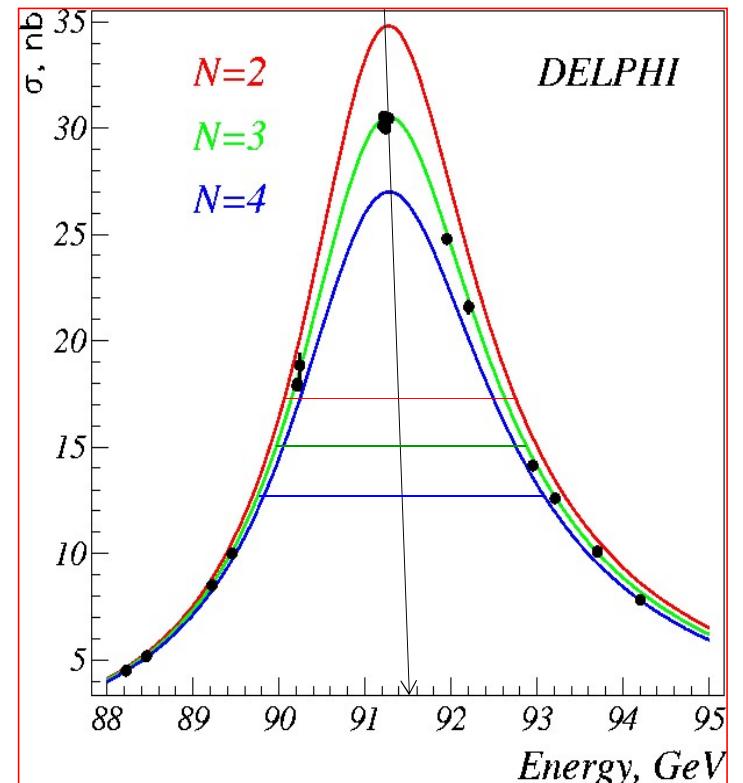
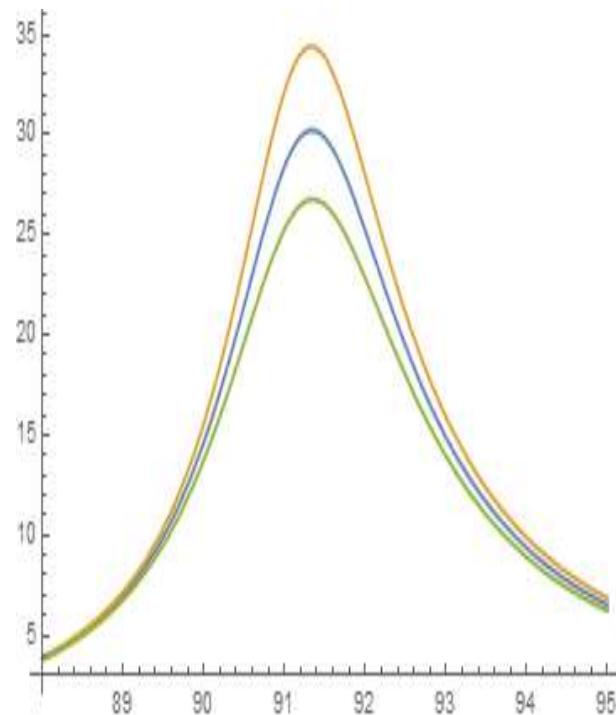
$$\sigma(s) = 1760 \text{ nb} \frac{\Gamma_{ee}}{\Gamma_Z} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s\Gamma_Z^2} \frac{\Gamma_{ff}}{\Gamma_Z} \rightarrow 1760 \text{ nb} \frac{\Gamma_{ee}}{\Gamma_Z} \left( \frac{\int_{E_{\gamma min} \approx 1 \text{ MeV}}^{E_{\gamma max} \approx \frac{\sqrt{s}}{2}} \frac{(s - \sqrt{s} E_\gamma)\Gamma_Z^2}{(s - \sqrt{s} E_\gamma - M_Z^2)^2 + (s - \sqrt{s} E_\gamma)\Gamma_Z^2} \text{Spectrum}(E_\gamma) dE_\gamma}{\int_{E_{\gamma min} \approx 1 \text{ MeV}}^{E_{\gamma max} \approx \frac{\sqrt{s}}{2}} \text{Spectrum}(E_\gamma) dE_\gamma} \right) \frac{\Gamma_{ff}}{\Gamma_Z}$$

Green curve is the result for the

$$\sigma_{Z \rightarrow jets, \text{ nb}} \quad \text{Spectrum}(E_\gamma) \approx \frac{1}{E_\gamma}$$



The measurement of Z0 shows that there are 3 kinds of neutrinos.  
There is no neutrino with mass  $>\sim 40$  GeV.

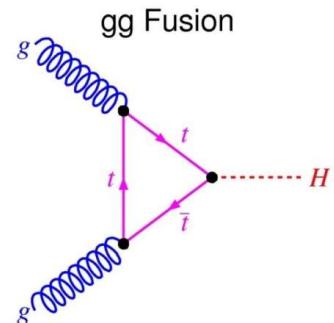


$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \quad \begin{pmatrix} \nu_4 \\ l4^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{pmatrix} t4 \\ b4 \end{pmatrix}$$

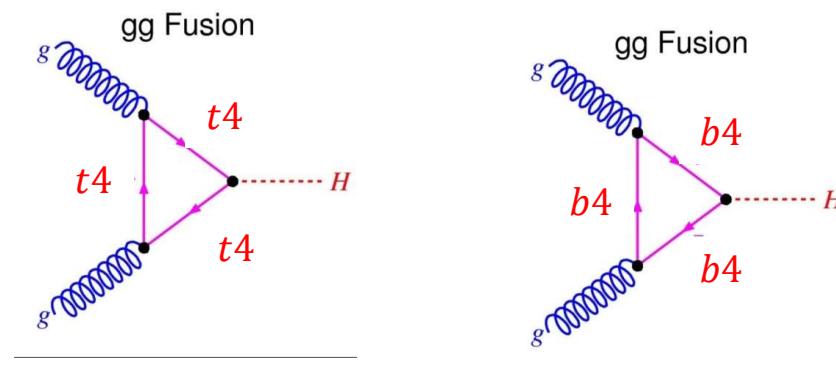
**Z0 line shape:  $M_{\nu 4} > 45 \text{ GeV}$**

Dominant Higgs production is via gluon gluon fusion. The biggest contribution is from the loop with the heaviest quark - top quark.



14.4.2023, R. Leitner

If there were two other very heavy quarks  $t4$  and  $b4$ , they would contribute by even larger amplitudes. The production of the Higgs boson would be an order of magnitude higher.



Výjezdní zasedání ÚČJF, Malá Skála

This would mean that Higgs would have been observed at the Tevatron accelerator in the US.

Measured production of the Higgs boson at the LHC is a clear exclusion of the existence of a 4th family of quarks.

Thank you for your attention and enjoy your stay in Malá Skála

