

Dipole resonances, neutron skins and nuclear Equation of State

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Outline

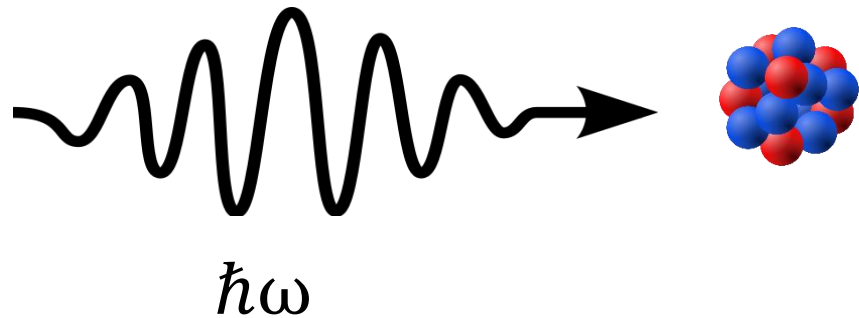
- Collective states in nuclei
dipole excitations, pygmy modes, polarizability
- Neutron skin thickness
- Nuclear Equation of State

Nuclear collective vibrations

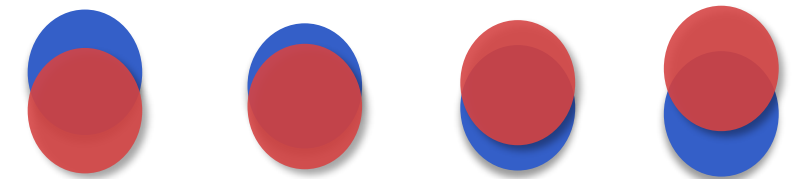
- γ -rays – an ideal external probe because interaction with nucleus is well understood
- photoabsorption cross section can be decomposed to different multipoles and electric (E) and magnetic (M) parts

$$\sigma_{\gamma}^{abs}(\hbar\omega, i \rightarrow f) \sim \sum_{\lambda, X=E, M} (\hbar\omega)^{2\lambda-1} S(X\lambda, \omega) \approx \hbar\omega S(E1, \omega)$$

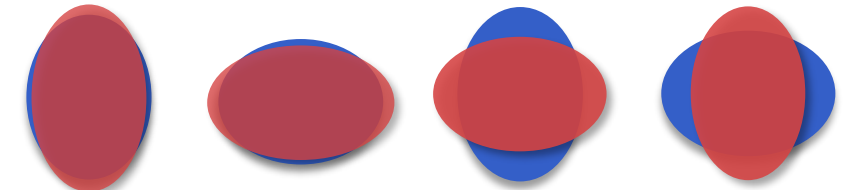
dominant contribution: E1 (electric dipole)



Isovector electric dipole (IV E1)



Isoscalar and isovector electric quadrupole (E2)

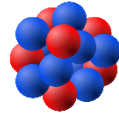
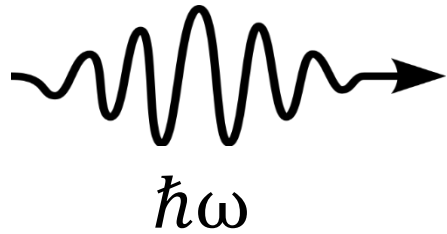


Scissor magnetic dipole (orbital M1)



→ isoscalar or isovector type

Electric dipole excitations



$$\hat{H} = \hat{H}_{nuc} + \hat{H}_{int}$$

periodic perturbation
with frequency ω

Reduced transition probability

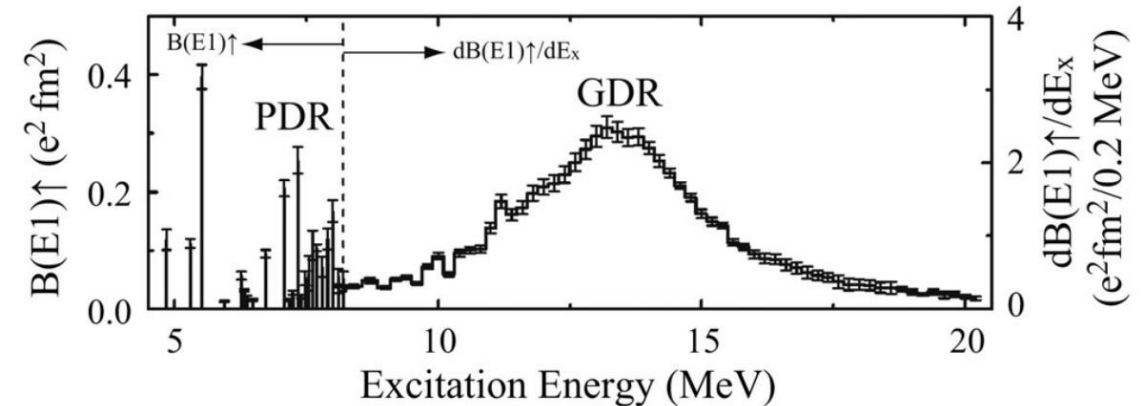
$$B(E1, i \rightarrow f) \sim \left| \left\langle f, J_f^{\pi_f} \right| \widehat{M}_1^E \left| i, J_i^{\pi_i} \right\rangle \right|^2$$

dipole operator

nuclear states calculated in some many-body approximation scheme

(γ -ray) dipole strength function

$$S(E1, \omega) = \sum_f B(E1, i \rightarrow f) \rho_{\Delta}(\omega - \omega_{fi}), \quad \hbar\omega_{fi} = E_f - E_i$$

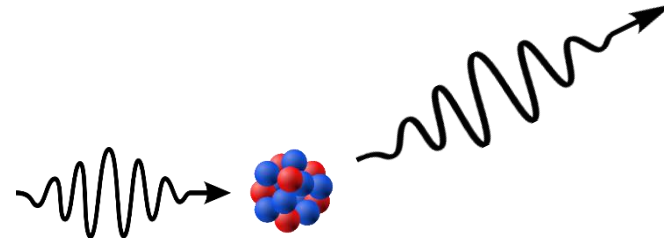


Tools for studying dipole response

- **Electromagnetic probes**

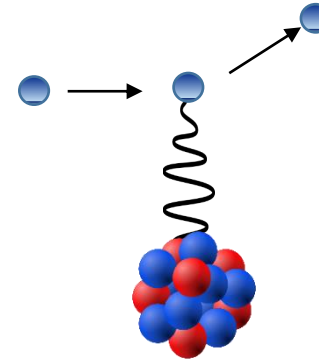
real photons

photonuclear (γ, γ') or photon scattering

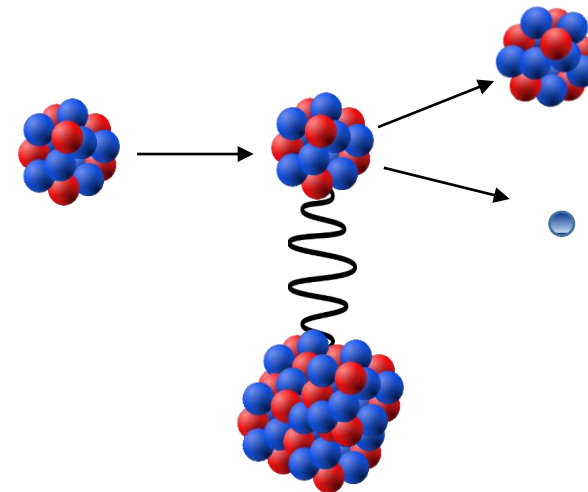


virtual photons

Coulomb excitations: target nucleus e.g. via (p, p')



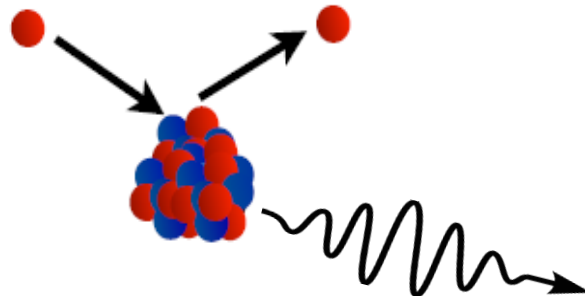
radioactive beam (exotic nuclei)



- **Hadronic probes**

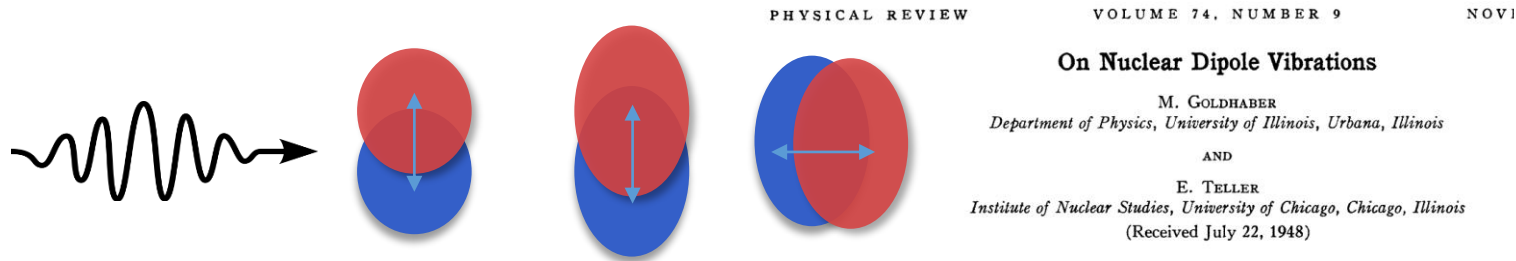
$(\alpha, \alpha' \gamma)$

$(^{17}\text{O}, ^{17}\text{O}' \gamma)$



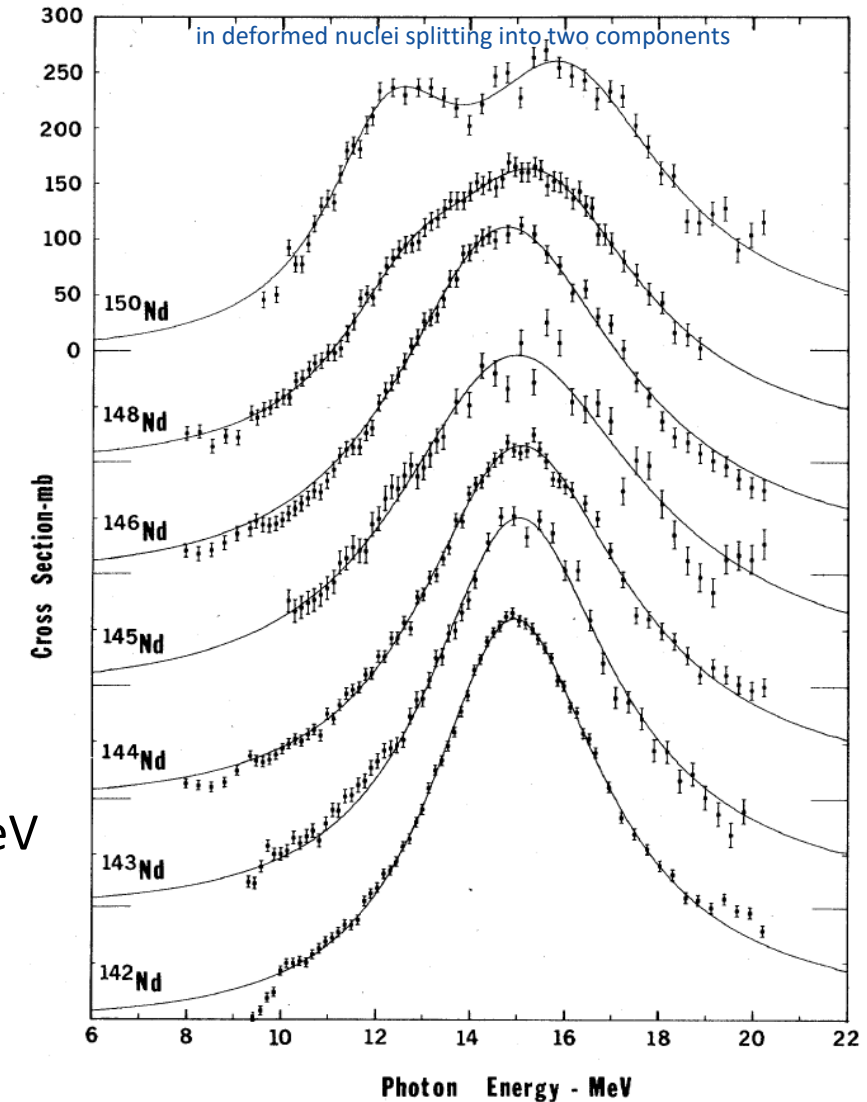
Isovector giant dipole resonance (IVGDR)

- dominant component of the photoabsorption cross section
- first measurement of IVGDR: Bothe and Gentner (1937) in $^{63}\text{Cu}(\gamma, n)$ reaction
- Goldhaber, Teller (1948): **isovector** dipole collective oscillation of protons against neutrons



- a general phenomenon observed in all nuclei
- energy centroid $E_x = 31.2 A^{-1/3} + 20.6 A^{-1/6}$ MeV, width 2.5 – 5 MeV
- Thomas-Reiche-Kunh (TRK) sum rule

$$\int_{S_n \approx 8 \text{ MeV}}^{E_{\text{max}} \approx 25 \text{ MeV}} \sigma_{\gamma}^{\text{abs}} dE \approx \frac{60NZ}{A} \text{ MeV} \cdot \text{mb}$$



B.L. Berman and S.C. Fultz, Rev. Mod. Phys. 47, 713 (1975).

Nuclear dipole response

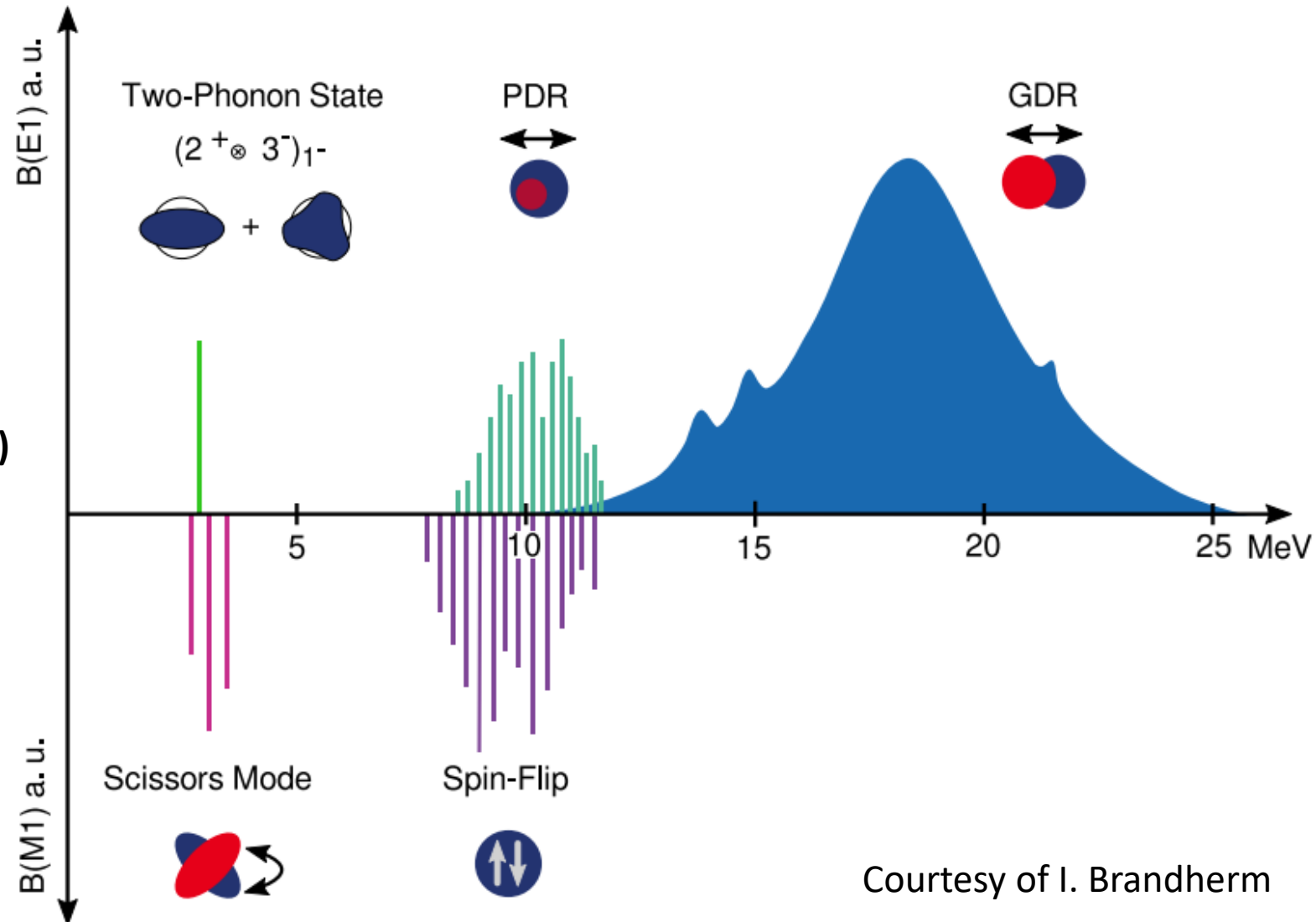
Giant resonances → coherent motion of many nucleons → large transition probabilities ($B(E1)$)
collectivity → TRK sum rule

(quasi) harmonic vibrations

- low-lying 2-phonon vibrations
oscillations of shape (density)
- Pygmy dipole resonance (PDR)
soft mode around S_n
observed in neutron-rich nuclei
exhausts small fraction of TRK (less than %)

oscillation of **neutron skin** against
symmetric *pn* core?

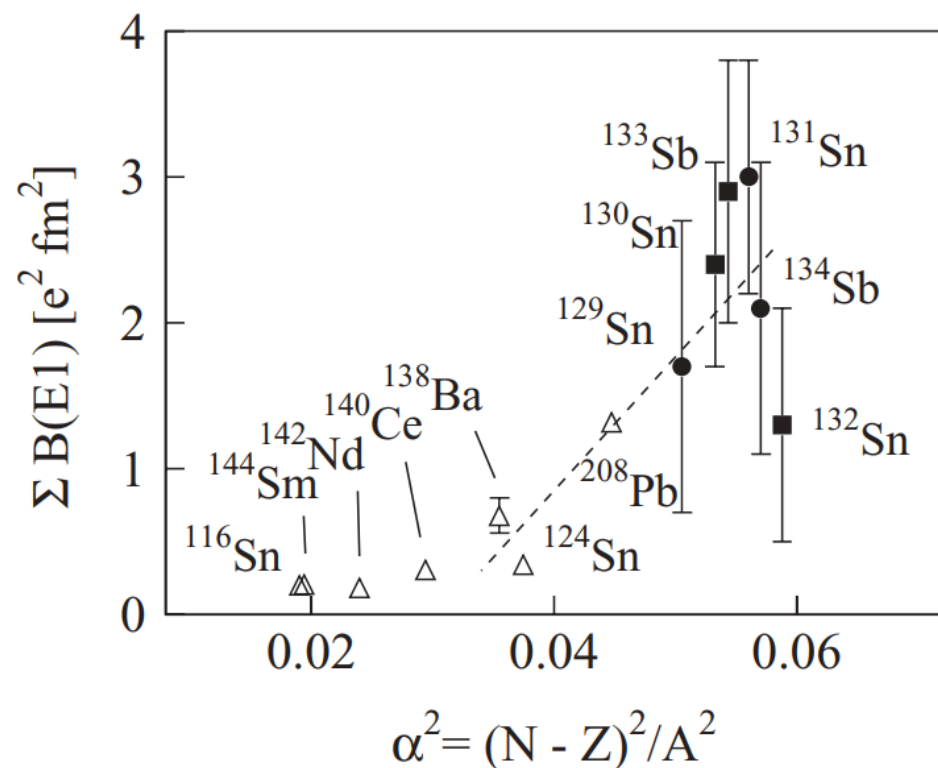
- Giant dipole resonance (GDR)
high frequency collective oscillation of
protons against neutrons
exhausts TRK sum rule



Courtesy of I. Brandherm

Pygmy dipole resonance

- enhancement on the low-energy tail of the GDR observed in neutron-rich nuclei
- concentrated around the neutron separation energy
- **related to nuclear dipole polarizability and neutron skin thickness**
- „traditional“ macroscopic picture → oscillation of pn symmetric core against neutron skin questioned by *Repko et al.* → compressional and toroidal collective motion



A. Klimkiewicz et al., Phys. Rev. C 76, 051603(R), (2007).

PHYSICAL REVIEW C **87**, 024305 (2013)

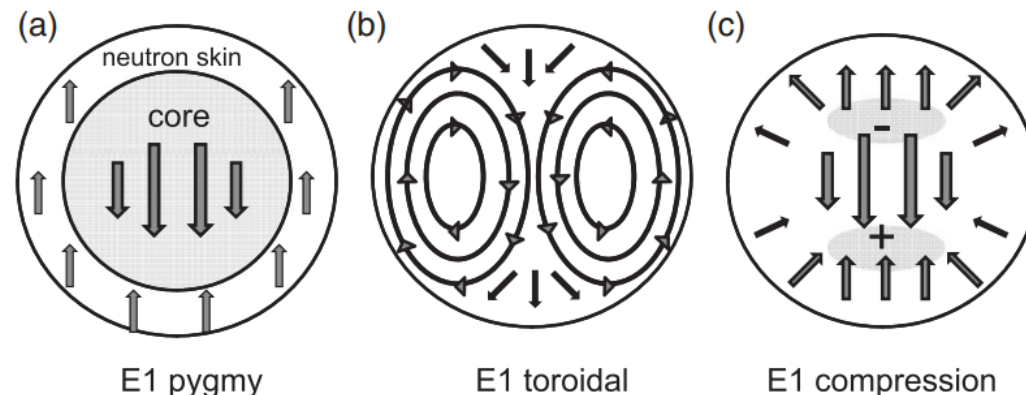
Toroidal nature of the low-energy $E1$ mode

A. Repko,¹ P.-G. Reinhard,² V. O. Nesterenko,^{3,*} and J. Kvasil¹

¹Institute of Particle and Nuclear Physics, Charles University, CZ-18000, Praha 8, Czech Republic

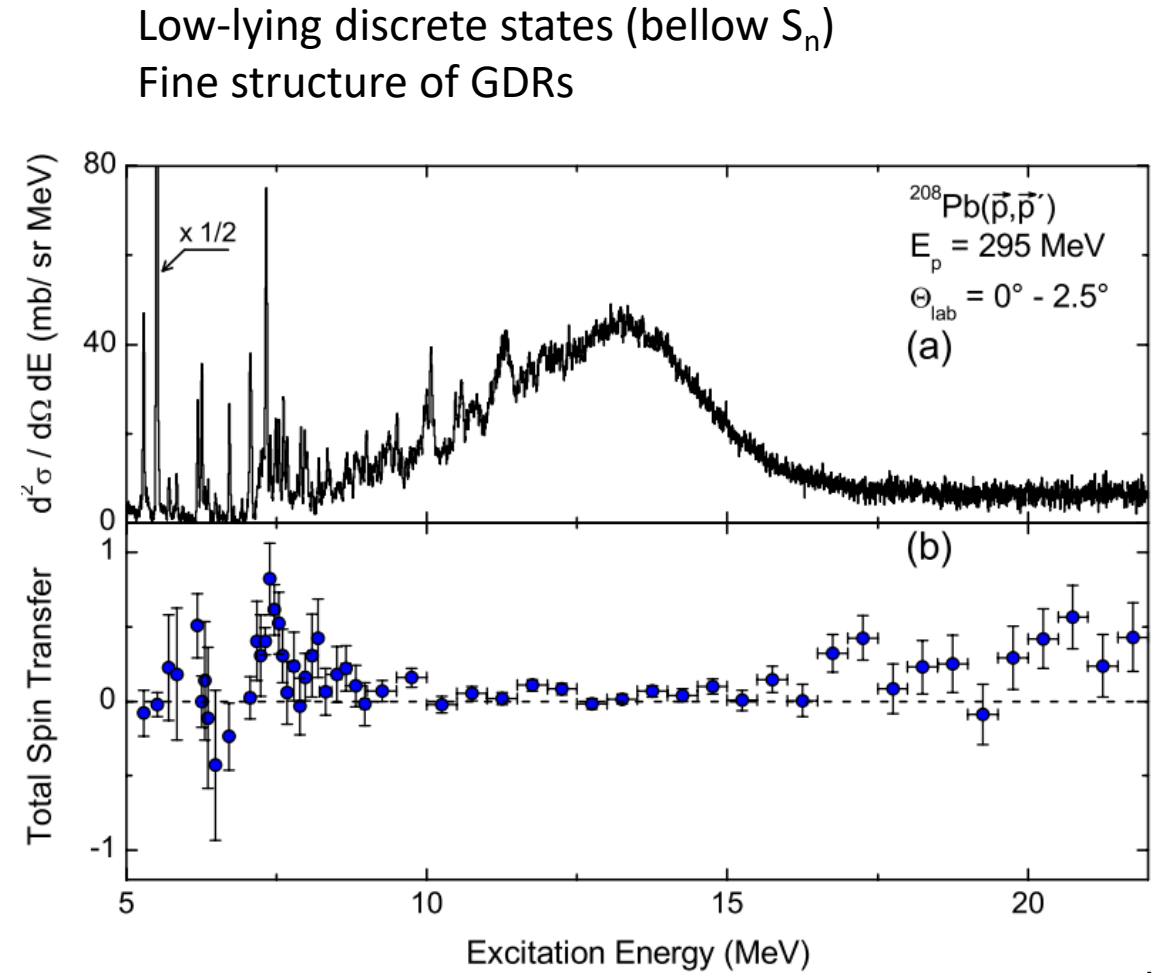
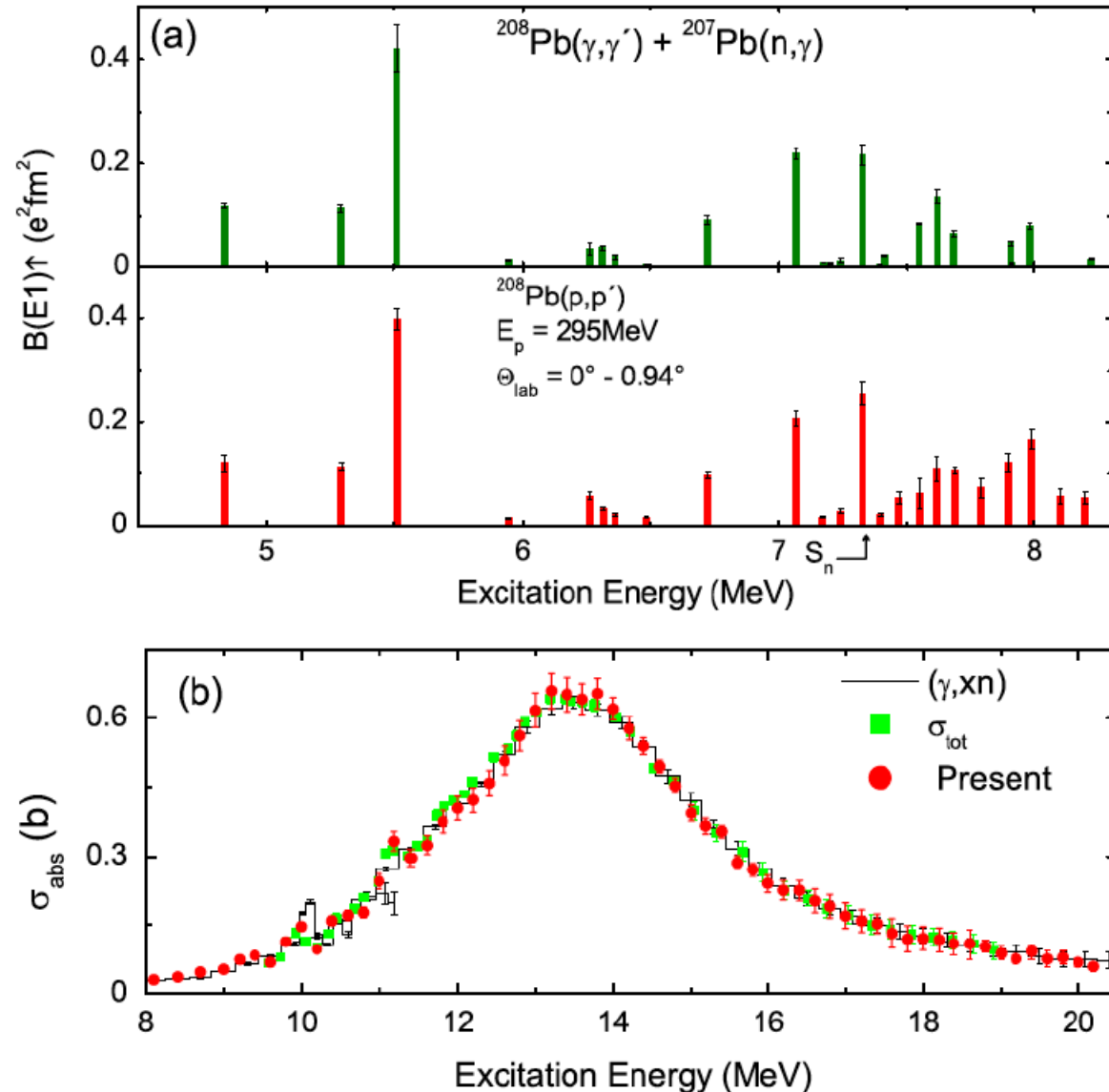
²Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany

³Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia



Nuclear dipole response

For some nuclei complementary measurements exist, good example is ^{208}Pb



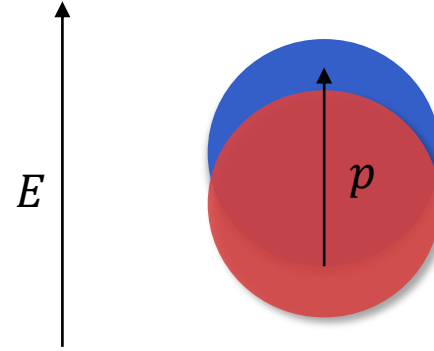
Tamii et al. Phys. Rev. Lett. 107, 062502 (2011)

RCNP Osaka

Dipole polarizability

- response to an external static electric field E
→ electric dipole moment p

electric dipole polarizability α_D

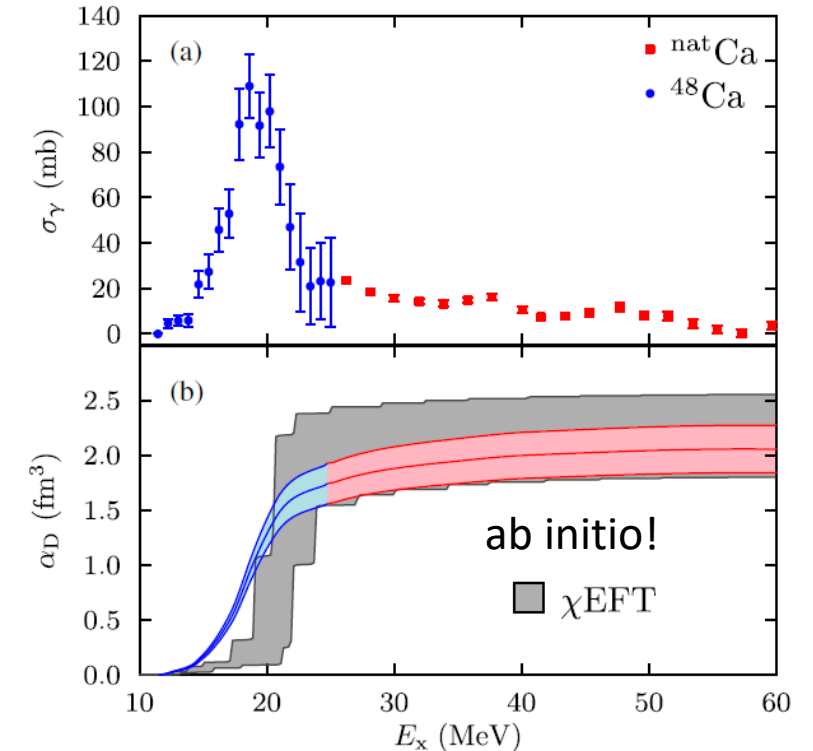
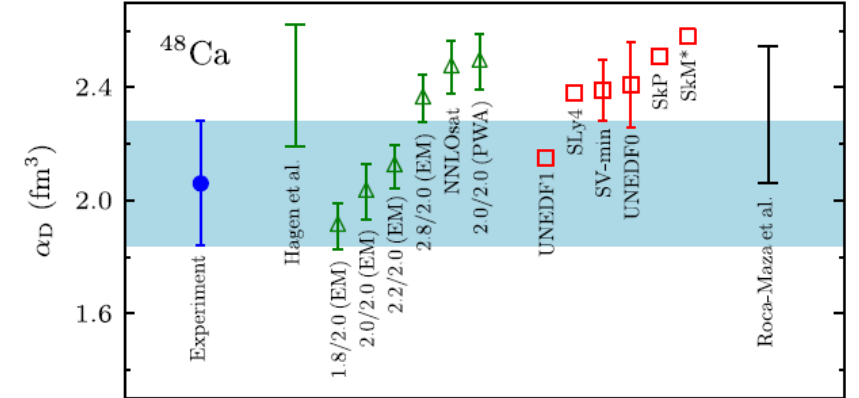


$$\alpha_D = \frac{p}{E} \sim \int \frac{S(E1, \omega)}{\omega} d\omega \sim \int \frac{\sigma_{\gamma}^{abs, E1}(\omega)}{\omega^2} d\omega$$

strength function (or photoabsorption cross section) → dipole polarizability
important contribution from PDR (neutron-rich systems)

Why is α_D useful?

correlation with **neutron skin** thickness → nuclear structure
neutron skin thickness sensitive to symmetry energy
important for understanding of neutron matter → neutron stars



Neutron skin thickness

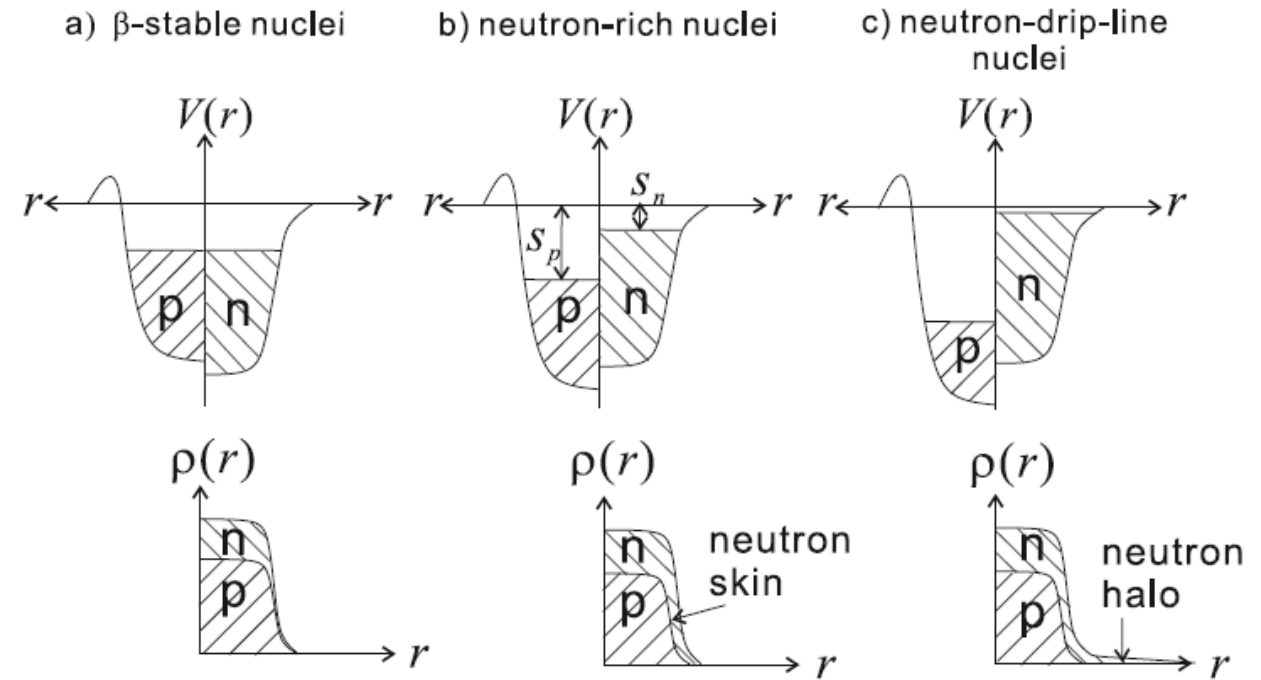
- charged radii and densities well measured

electron scattering → information about size, surface thickness, saturation density, (e.g. rms charged radius of ^{208}Pb is known with error 0.02%).

- (not?)surprisingly neutron densities are not known very precisely

hadronic probes suffer from large uncertainties (reaction mechanisms, „unknown“ interaction)

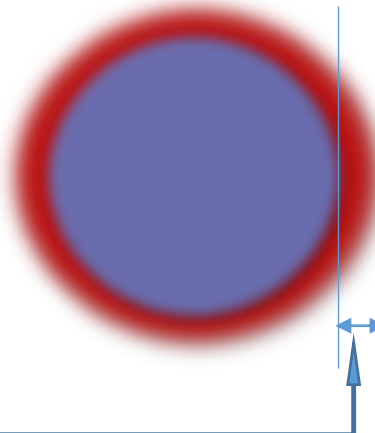
neutron-rich nuclei → development of neutron skin



Auman T. , Nakamura T., Phys. Scr. T152, 014012(2013).

How large is the neutron skin?

$$\Delta r_{np} = \sqrt{\langle r_N^2 \rangle} - \sqrt{\langle r_P^2 \rangle}$$



Neutron skin thickness

Simplified droplet model (DM) → approximately linear increase of neutron skin thickness with pn asymmetry

X. Roca-Maza, N. Paar / Progress in Particle and Nuclear Physics 101 (2018) 96–176

importance of symmetry energy (at the surface and in the core)

In real world, it is more complicated (e.g. shell effects ...)

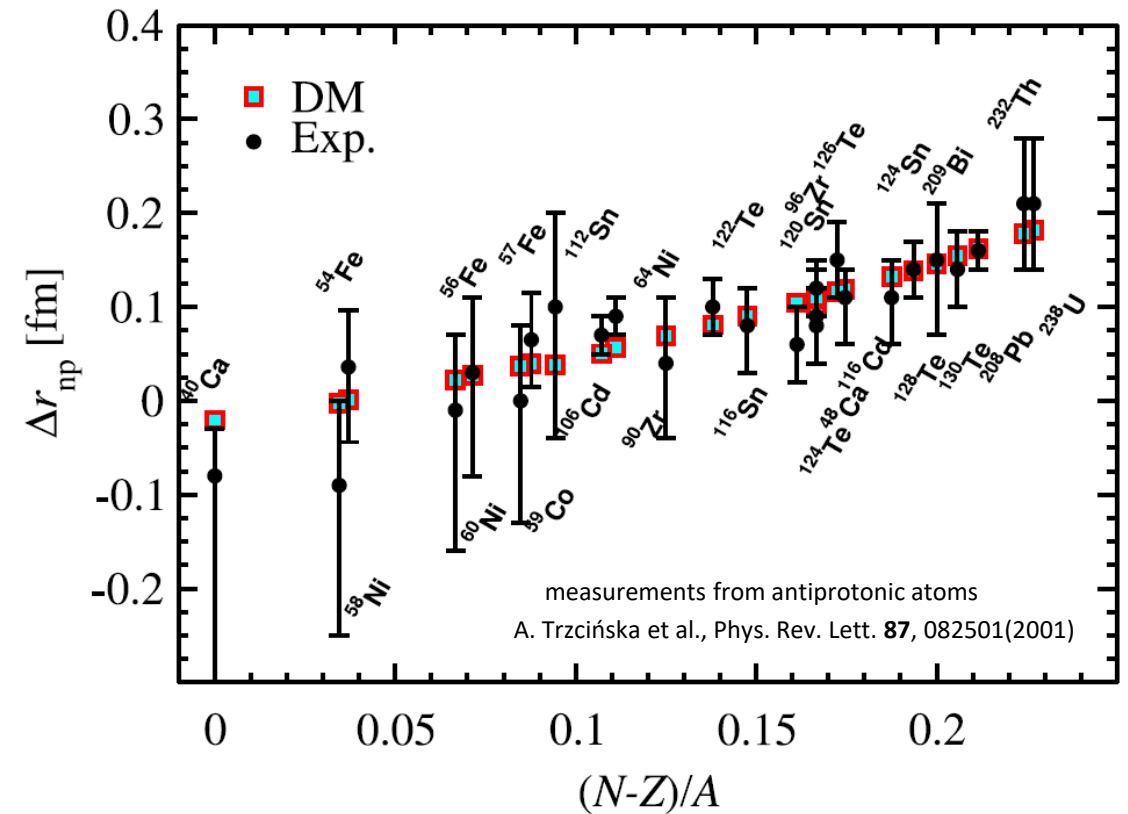
$$\Delta r_{np}^{\text{DM}} = \frac{2\langle r^2 \rangle^{1/2}}{3} \frac{a_{AS}}{a_A} (I - I_C) + \Delta r_{np}^C$$

$$I \equiv (N - Z)/A$$

$$\langle r^2 \rangle^{1/2} \approx 0.9A^{1/3}$$

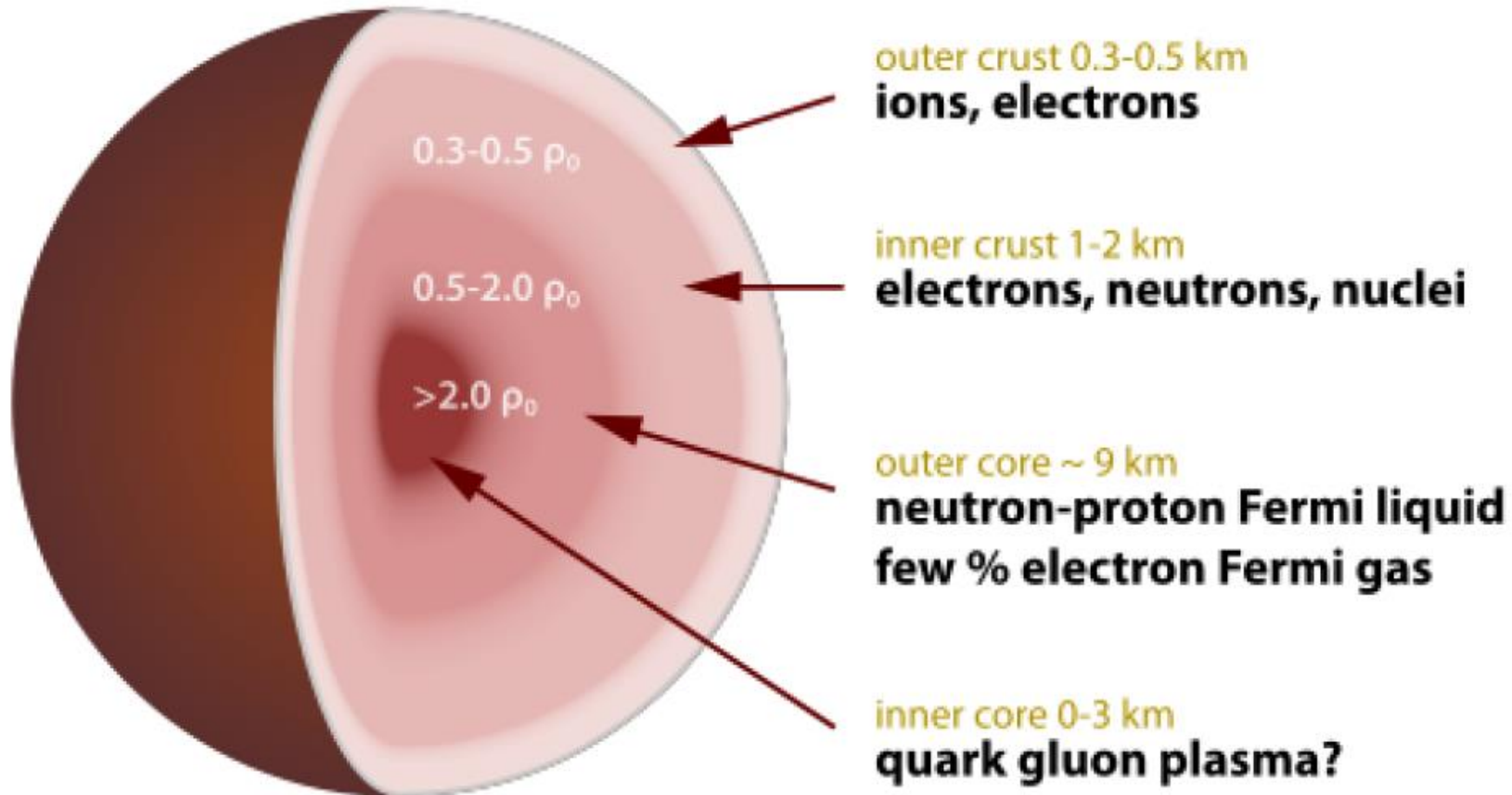
$$B(A, Z) = (a_V - a_SA^{-1/3})A - a_C \frac{Z(Z-1)}{A^{1/3}} - (a_A - a_{AS}A^{-1/3}) \frac{(A-2Z)^2}{A}$$

competition between surface tension and symmetry energy difference at core (high density) and surface (low density)



Nuclear Equation of state (EoS)

- EoS describes relationships between the energy, pressure, temperature, density and proton-neutron asymmetry.



- stellar mass depends on EoS at the highest densities
- radius is sensitive to properties of the EoS in the vicinity of nuclear matter saturation density

Nuclear Equation of state (EoS)

- parametrization of energy per nucleon dependence of uniform infinite system ($A \rightarrow \infty$, $V \rightarrow \infty$, $\frac{A}{V} \rightarrow \text{const.}$)
- unknown parameters \rightarrow data from finite nuclei

polarizability, neutron skin thickness \rightarrow symmetry energy part of EoS

\rightarrow EoS for neutron matter

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$

$$\rho \equiv \rho_n + \rho_p$$

$$\delta \equiv \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

parametrization of **symmetry energy**

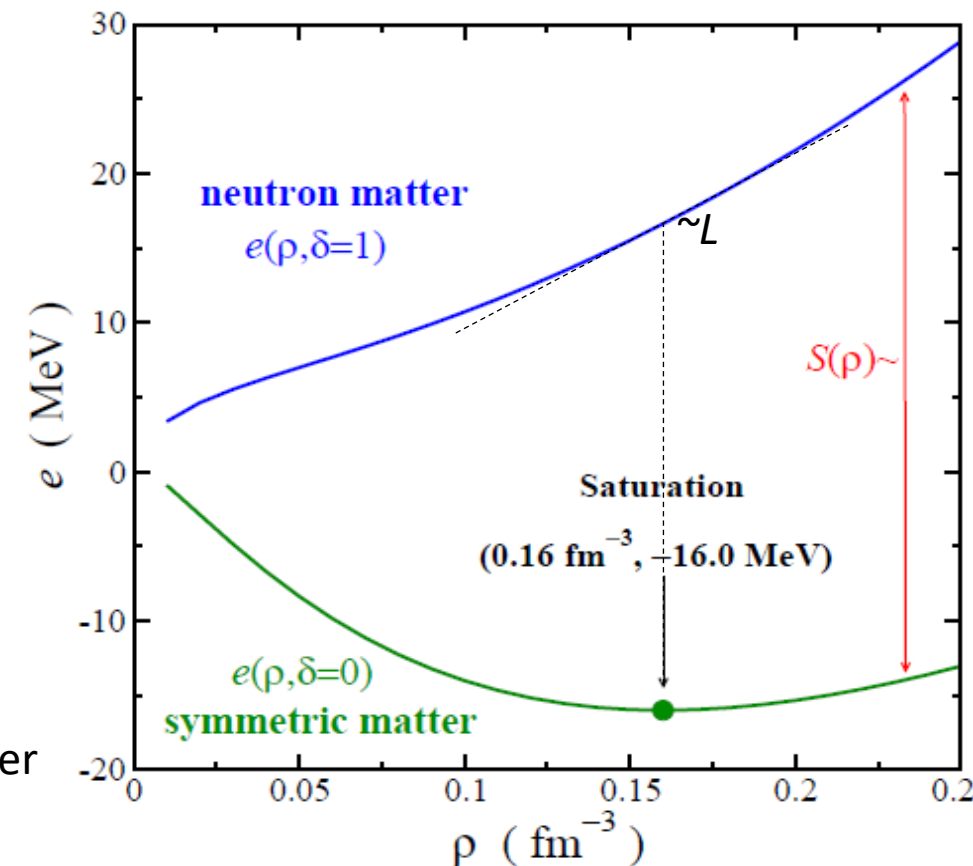
$$S(\rho) = J + L \frac{\rho - \rho_0}{3\rho_0} +$$

Saturation density

$$\rho_0 = \frac{3A}{4\pi R^3}$$

Pressure of neutron matter

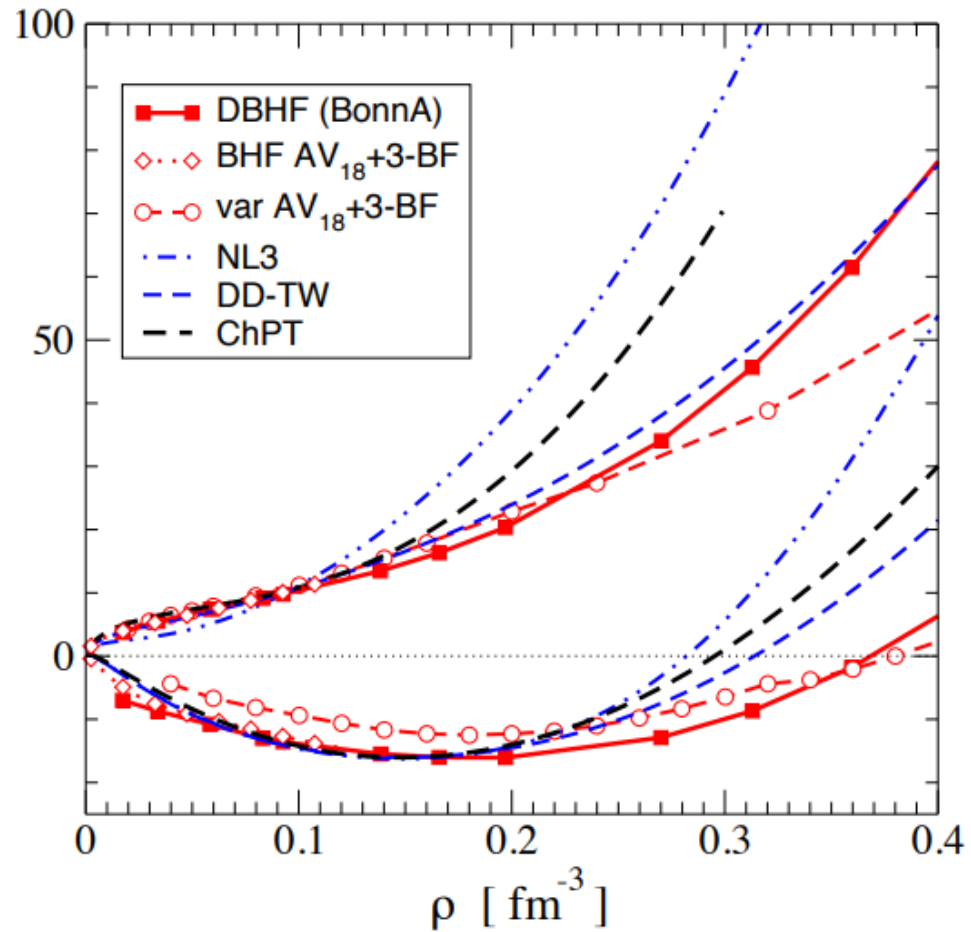
$$P_0 \approx \frac{1}{3}\rho_0 L$$



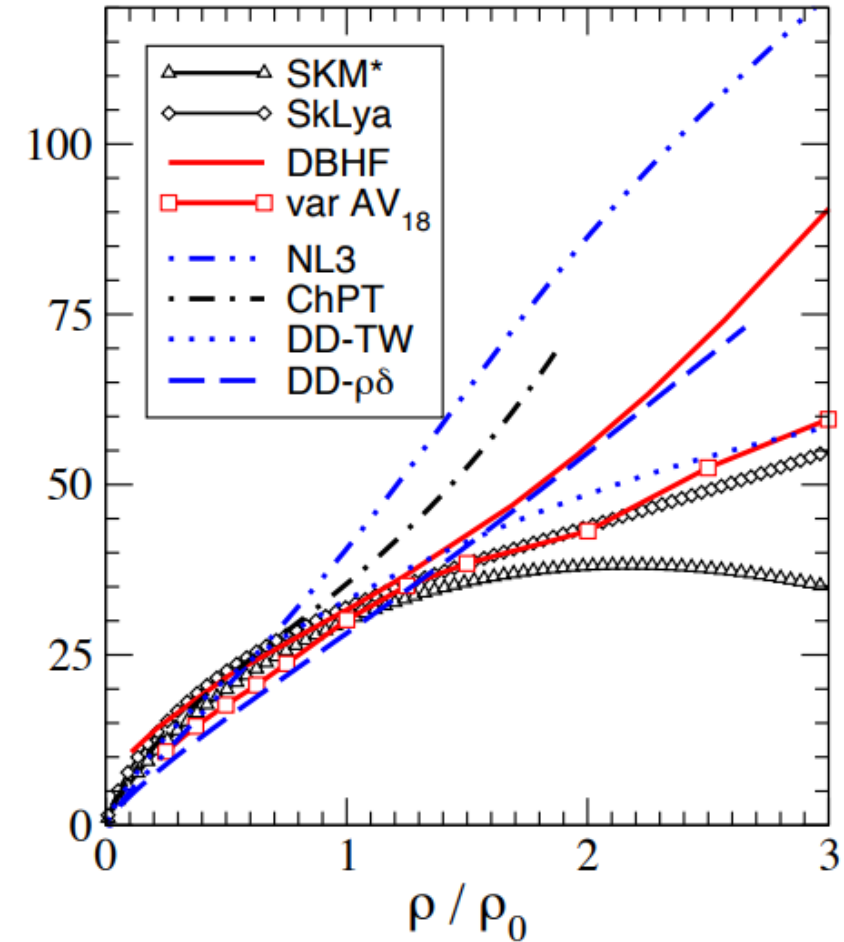
Nuclear Equation of state (EoS)

different models \rightarrow different predictions

nuclear EoS



symmetry energy

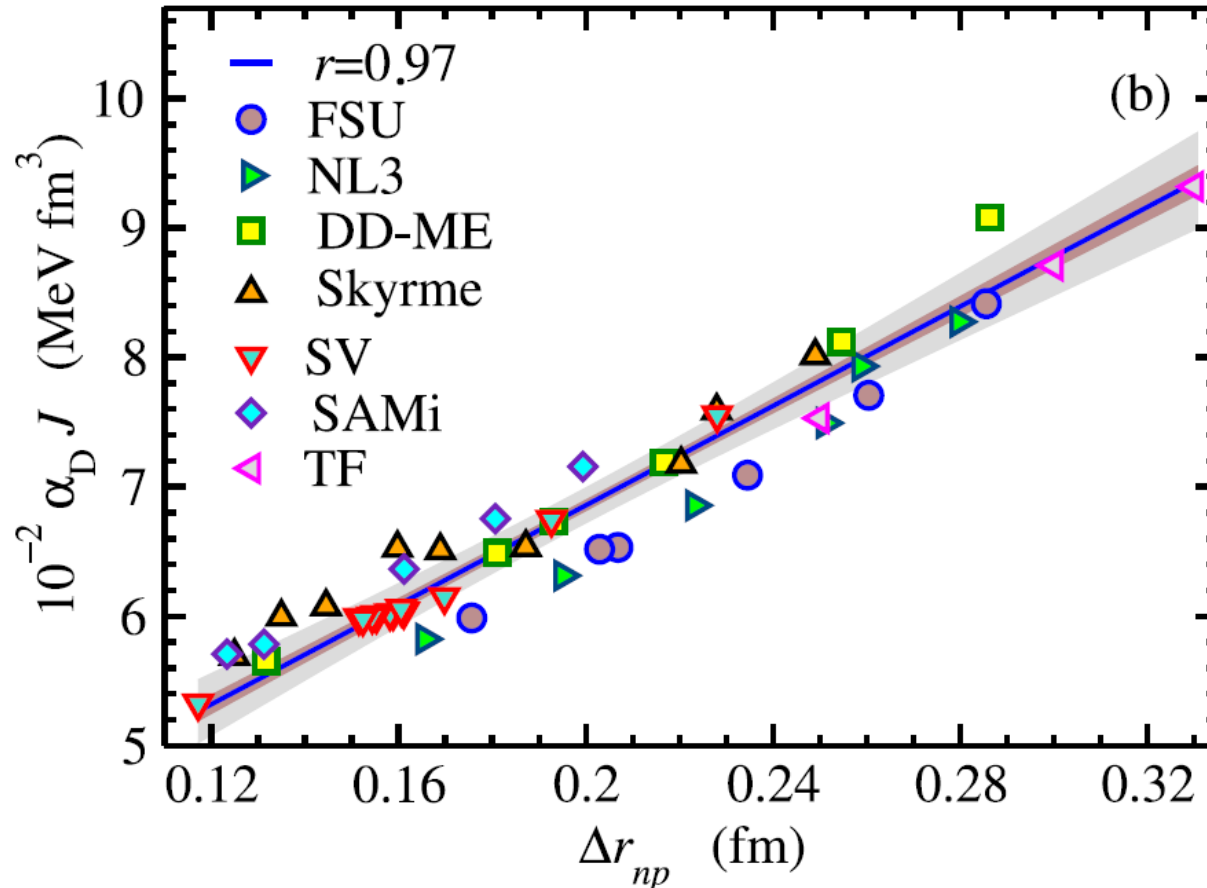


Symmetry energy parameters in EoS

How to determine symmetry energy parameter L ? (nuclear masses are insensitive to L)

Energy density functional (EDF) nuclear models (mean-field calculations) „predictions“

→ strong correlation between **dipole polarizability α_D** and **symmetry energy slope parameter L**



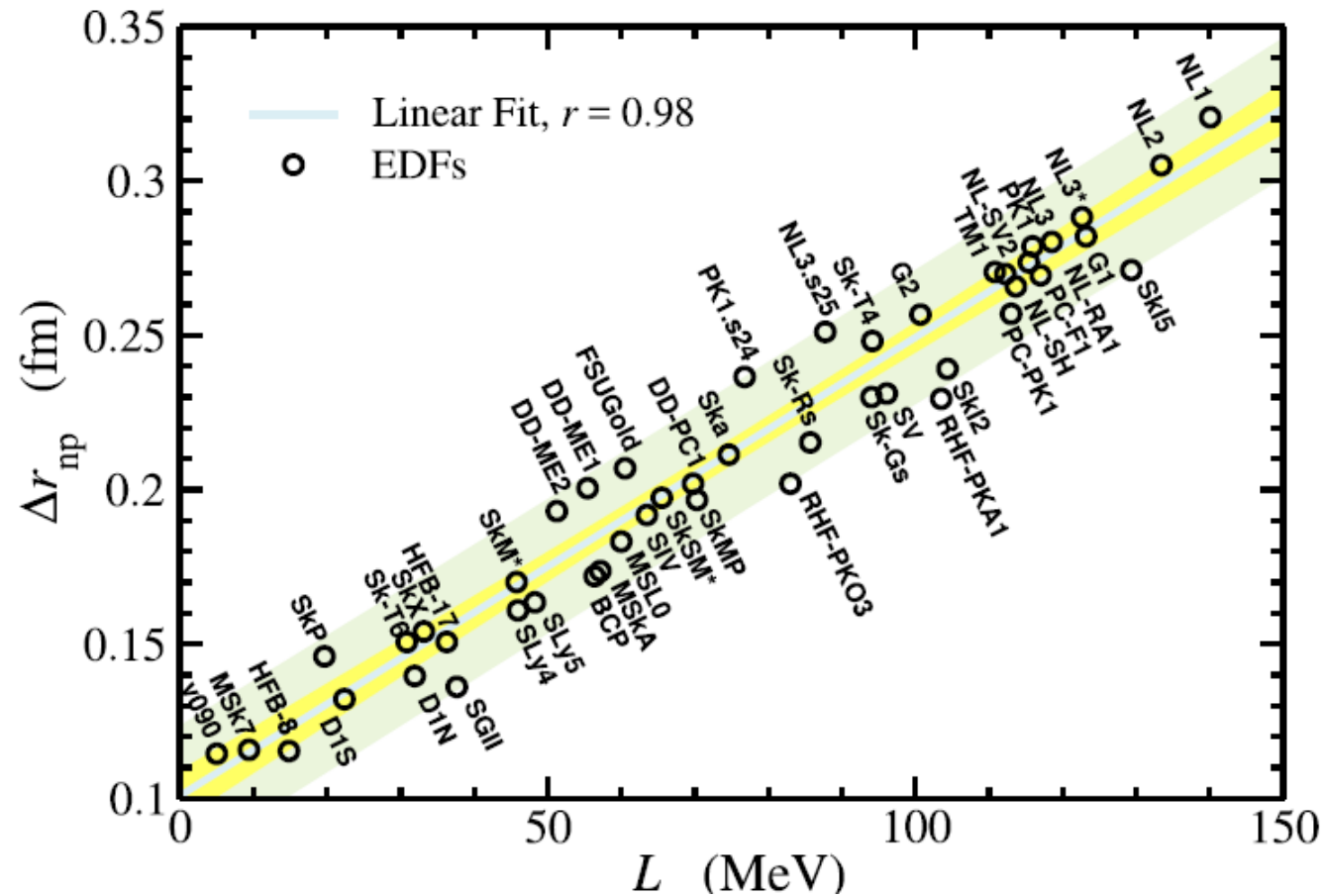
^{208}Pb

EDFs : set of microscopic models developed for precise description of bulk properties across the chart

$$\alpha_D \sim \int \frac{\sigma_{\gamma}^{abs,E1}(\omega)}{\omega^2} d\omega$$

Symmetry energy parameters in EoS

... and strong correlation between **neutron skin thickness** Δr_{np} , and **symmetry energy slope parameter** L

 ^{208}Pb

How to determine (measure) neutron skin thickness?

Pb Radius EXperiment and Ca Radius EXperiment @JLAB

- goal: determination of neutron skin thickness with minimal model-dependence

electroweak parity violating measurements of elastic scattering of longitudinally polarized electrons from ^{208}Pb (PREX) and ^{48}Ca (CREX)

weak coupling of proton to $Z^0 \rightarrow$ sensitive to neutron distribution

\rightarrow neutral weak formfactor extraction from parity violation asymmetry

\rightarrow „neutron“ radius

$$A_{\text{PV}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx \frac{G_F Q^2 |Q_W| F_W(Q^2)}{4\sqrt{2}\pi\alpha Z F_{\text{ch}}(Q^2)} \approx 10^{-7}$$

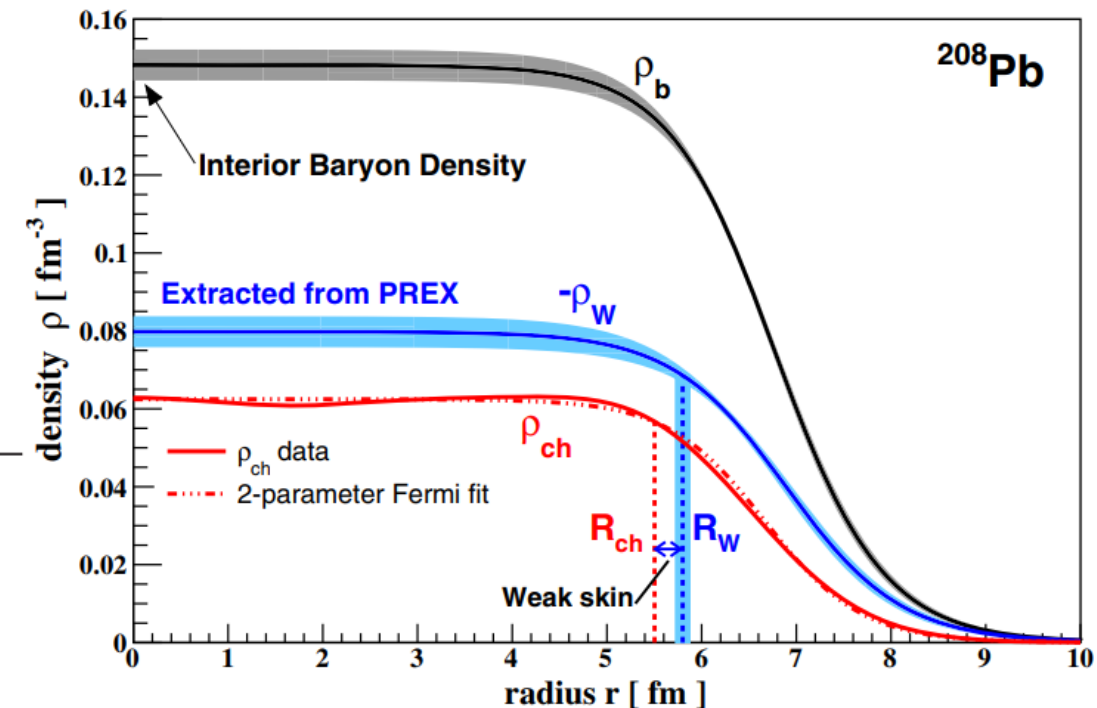
$$R_{\text{skin}} = (0.283 \pm 0.071) \text{ fm}$$

PHYSICAL REVIEW LETTERS **126**, 172502 (2021)

Editors' Suggestion

Featured in Physics

Accurate Determination of the Neutron Skin Thickness of ^{208}Pb
through Parity-Violation in Electron Scattering



Pb Radius EXperiment and Ca Radius EXperiment @JLAB

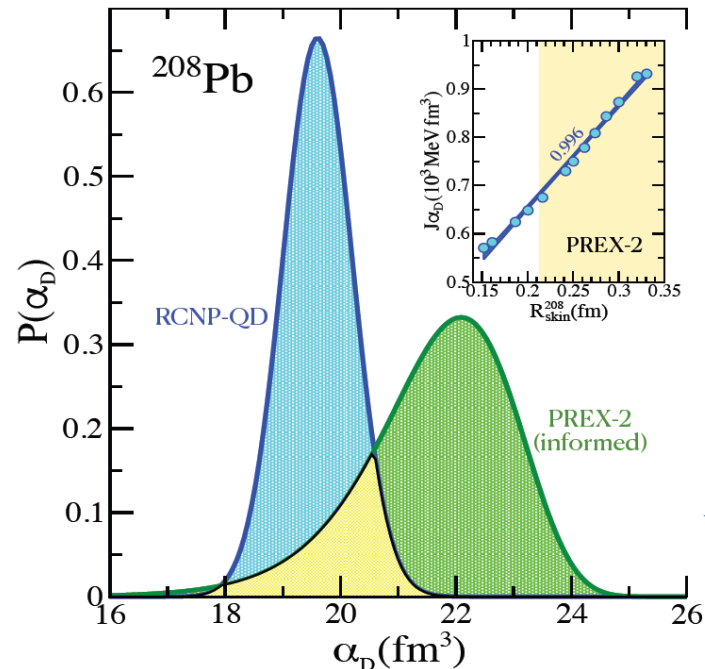
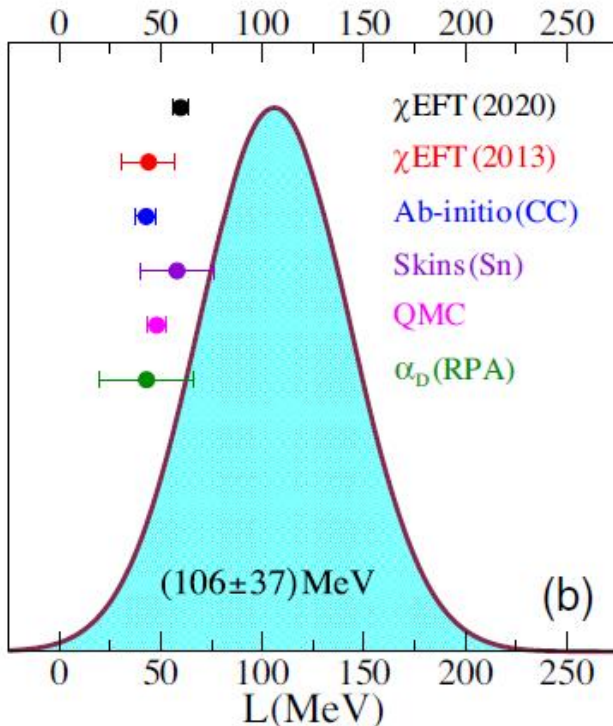
“some tension” between nuclear models and PREX-II data...
CREX → smaller value of neutron skin in ^{48}Ca .

PREX-II data → stiff symmetry energy

dipole polarizability, ab-initio, EFT
→ more soft

$$J = (38.1 \pm 4.7) \text{ MeV},$$

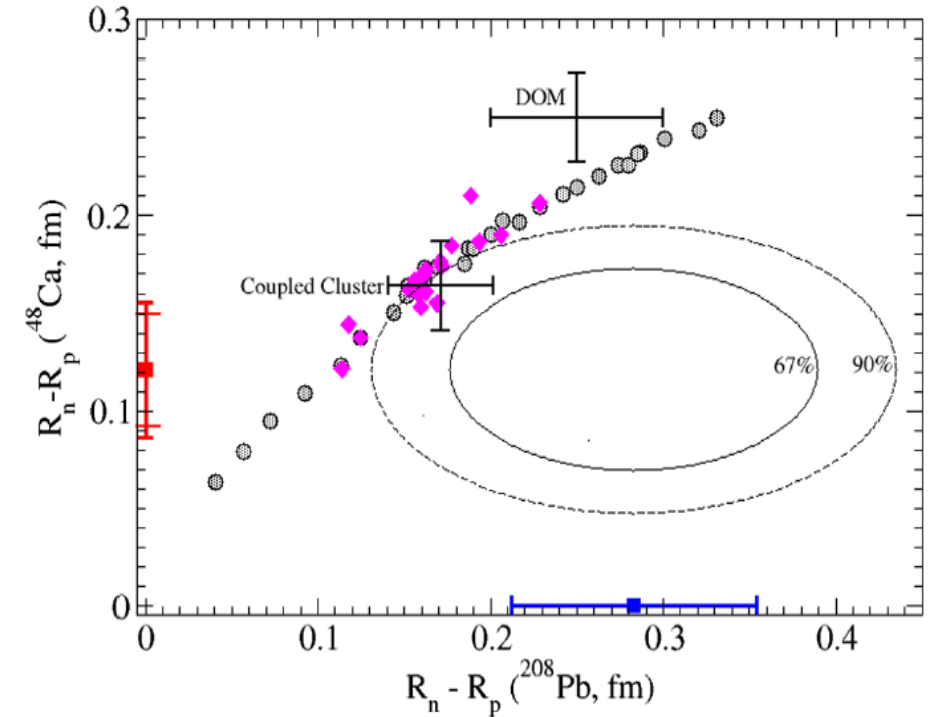
$$L = (106 \pm 37) \text{ MeV}.$$



PHYSICAL REVIEW LETTERS **129**, 042501 (2022)

Editors' Suggestion

Precision Determination of the Neutral Weak Form Factor of ^{48}Ca



PHYSICAL REVIEW C **104**, 024329 (2021)

Editors' Suggestion

Implications of PREX-2 on the electric dipole polarizability of neutron-rich nuclei

J. Piekarewicz

Summary

- laboratory experiments + theory useful for a determination of some fundamental nuclear properties

dipole polarizability, neutron skin thickness

→ constraints on parameters of EoS → neutron matter properties

- nuclear dipole excitations unique determination of dipole polarizability
- measuring of neutron skin → parity-violating electron scattering
- linking neutron stars with properties of nuclei