Importance of LQCD for Precision Theory

L Del Debbio

Higgs Centre for Theoretical Physics
The University of Edinburgh

Selected topics

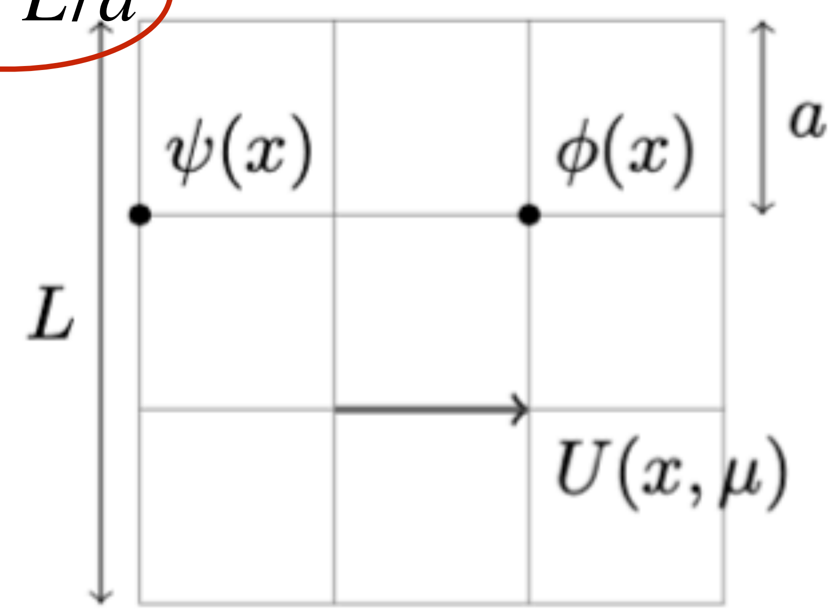
- Very lively time in LQCD, driven by theory/hardware
- Comprehensive reviews at the Lattice conferences (Liverpool UK, 7/24, Fermilab 7/23), new FLAG release later this year
- Focus on just a few topics today
- Lattice Field Theory toolkit - understanding the challenges
- Update on g-2
- Lattice determination of the strong coupling

QFT on a lattice

$N = L/a$

UV cutoff scale

action



$$\boxed{Z = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\phi \; e^{-S[U,\psi,\bar{\psi},\phi]}}$$

- only input parameters: g, am_i — adjusted non perturbatively, hadronic input
- $n_f = 2 + 1$, requires two hadronic masses to fix (am_i)
- compute correlators: $\langle \Phi(x_1)\ldots\Phi(x_n)\rangle = \dfrac{1}{Z}\displaystyle\int \mathcal{D}\Phi \, e^{-S(\Phi)}\, \Phi(x_1)\ldots\Phi(x_n)$
- importance sampling

Road to precision in LQCD

- We know the theory: no **validation**, no modelling, **first-principles** results, multiple discretizations, check universality…

- Computational challenge: cost increases as $a \to 0\,, m \to m^{\mathrm{phys}}$

- **Statistical** errors scale like $1/\sqrt{N_{\mathrm{conf}}}$, improve with computing power

- … if we have independent configurations in the Markov chain — **autocorrelations** in Monte Carlo generation of configurations are a bottleneck

- **Systematic** errors: $a\mu \ll 1 \ll \mu L\,, \quad m \to m_{\mathrm{phys}}$, EFTs describe the extrapolations

Hadronic inputs for phenomenology

- Decay constants, bag parameters, form factors [Cornella's & Smith's talk - this morning], hadron scattering [Gandini's talk & parallel session - yesterday]

- Heavy quarks - large artefacts $(am_b)^n$, multiple discretizations, test of universality. [Cornella's & Smith's talk - this morning]

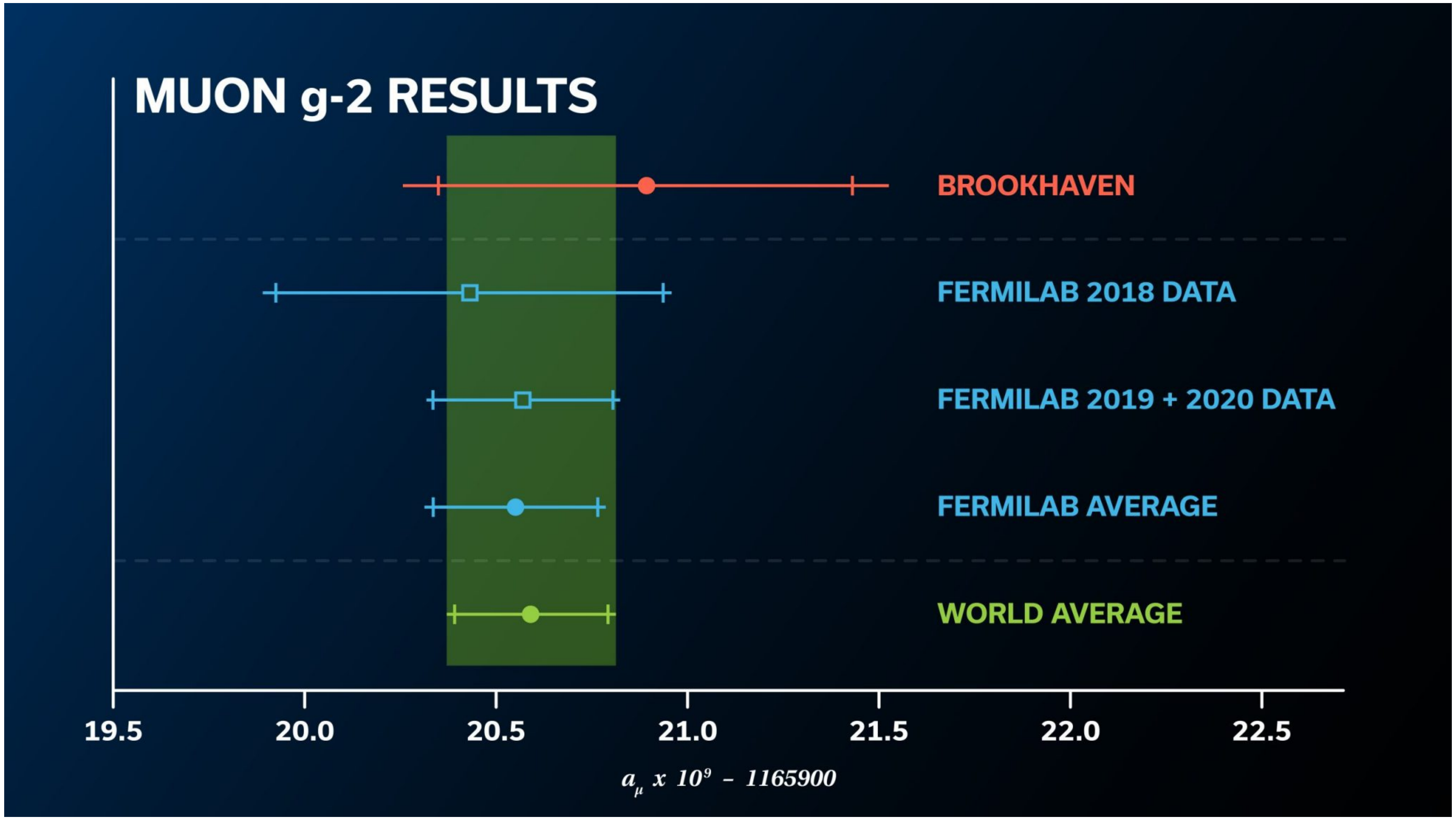
- Precision $\sim 1\,\%$, QED and isospin breaking are now taken into account

- Baryon physics, input for neutrino experiments

- PDFs: *relatively* new direction for LQCD, interesting results

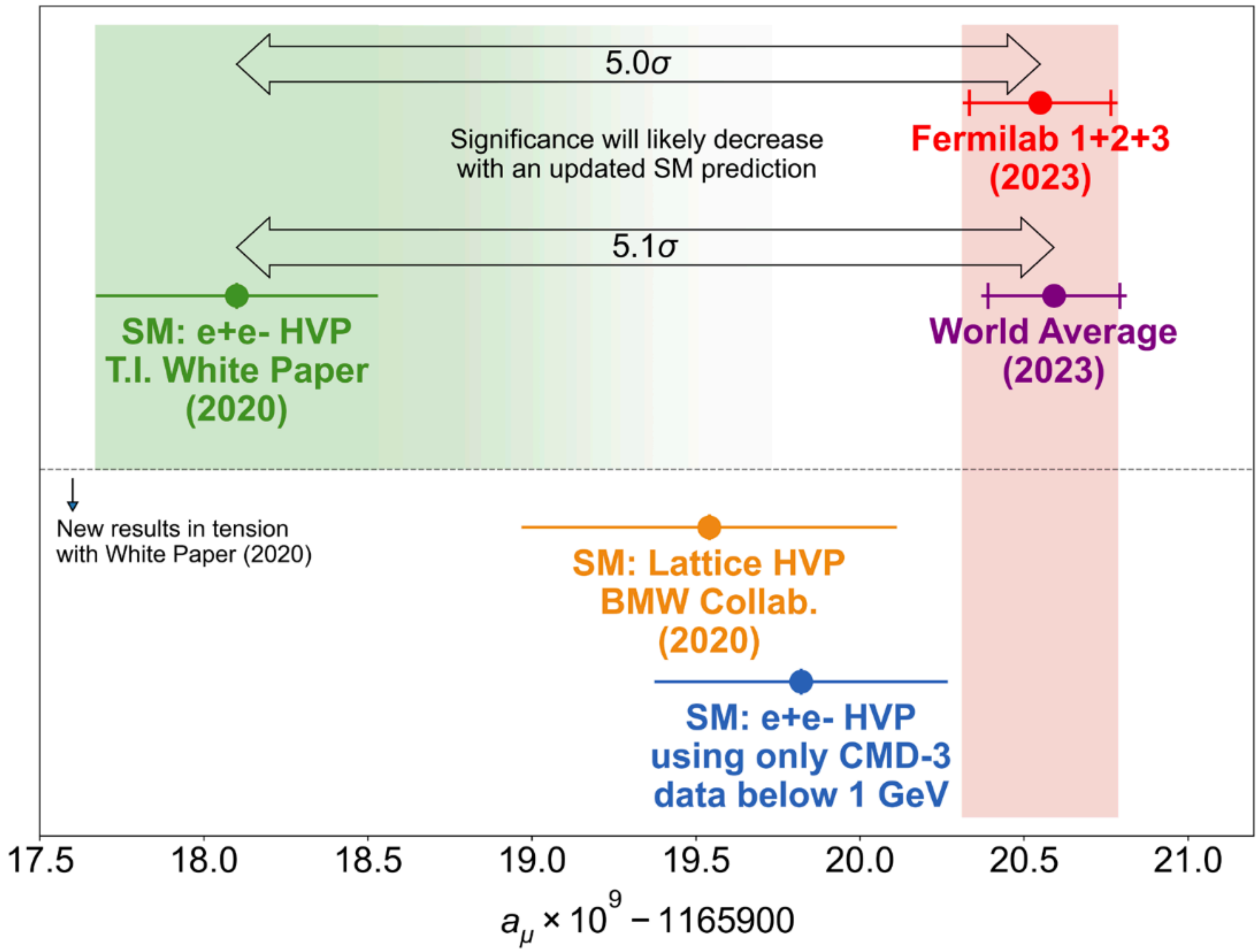
Update on g-2

- Since LHPC23, new result released by Fermilab



[Muon g-2 Collaboration]

A new challenge for theory

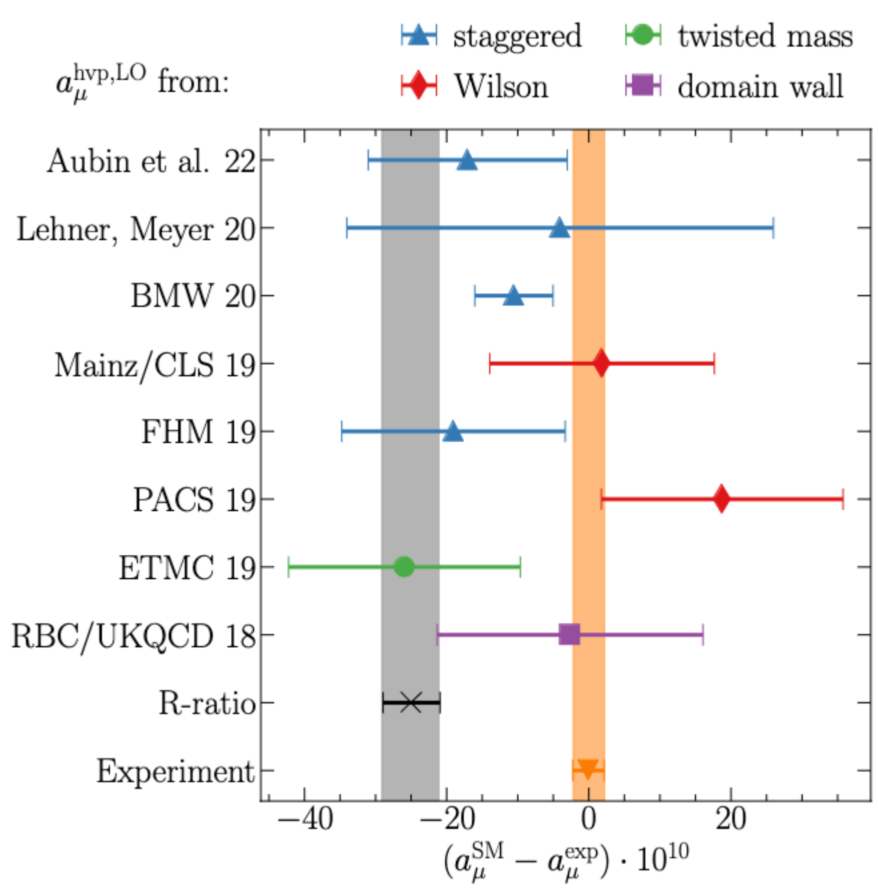


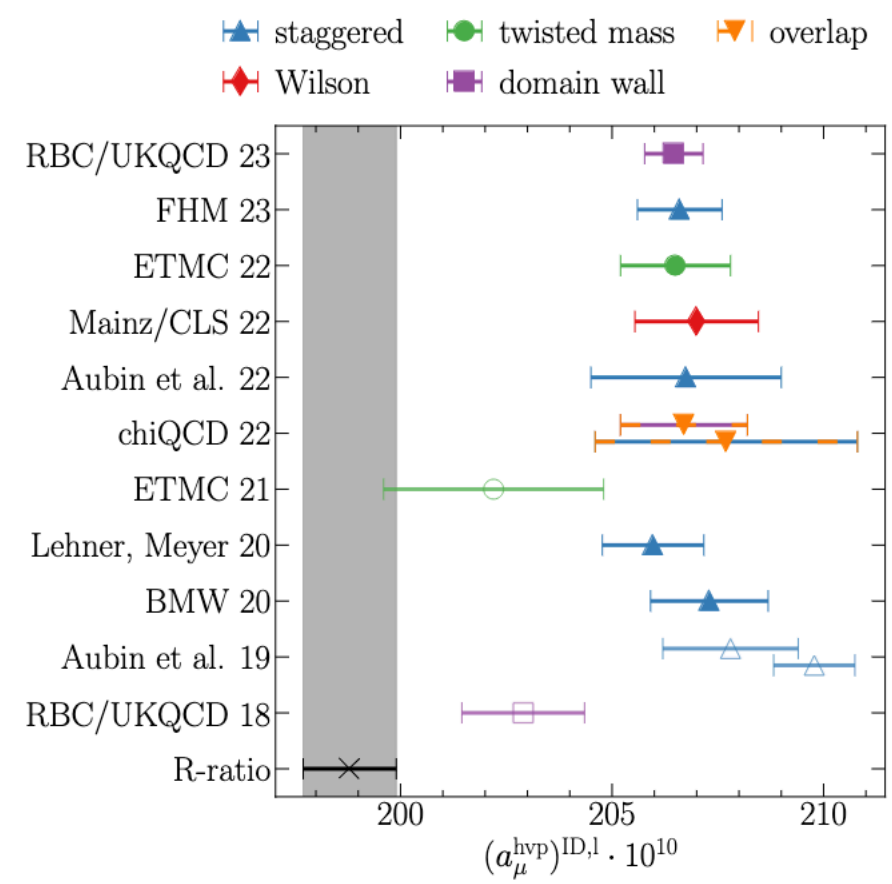
[Mott 08/2023]

Understand the discrepancy between lattice and R-ratio determinations of HVP

Growing consensus between lattice results

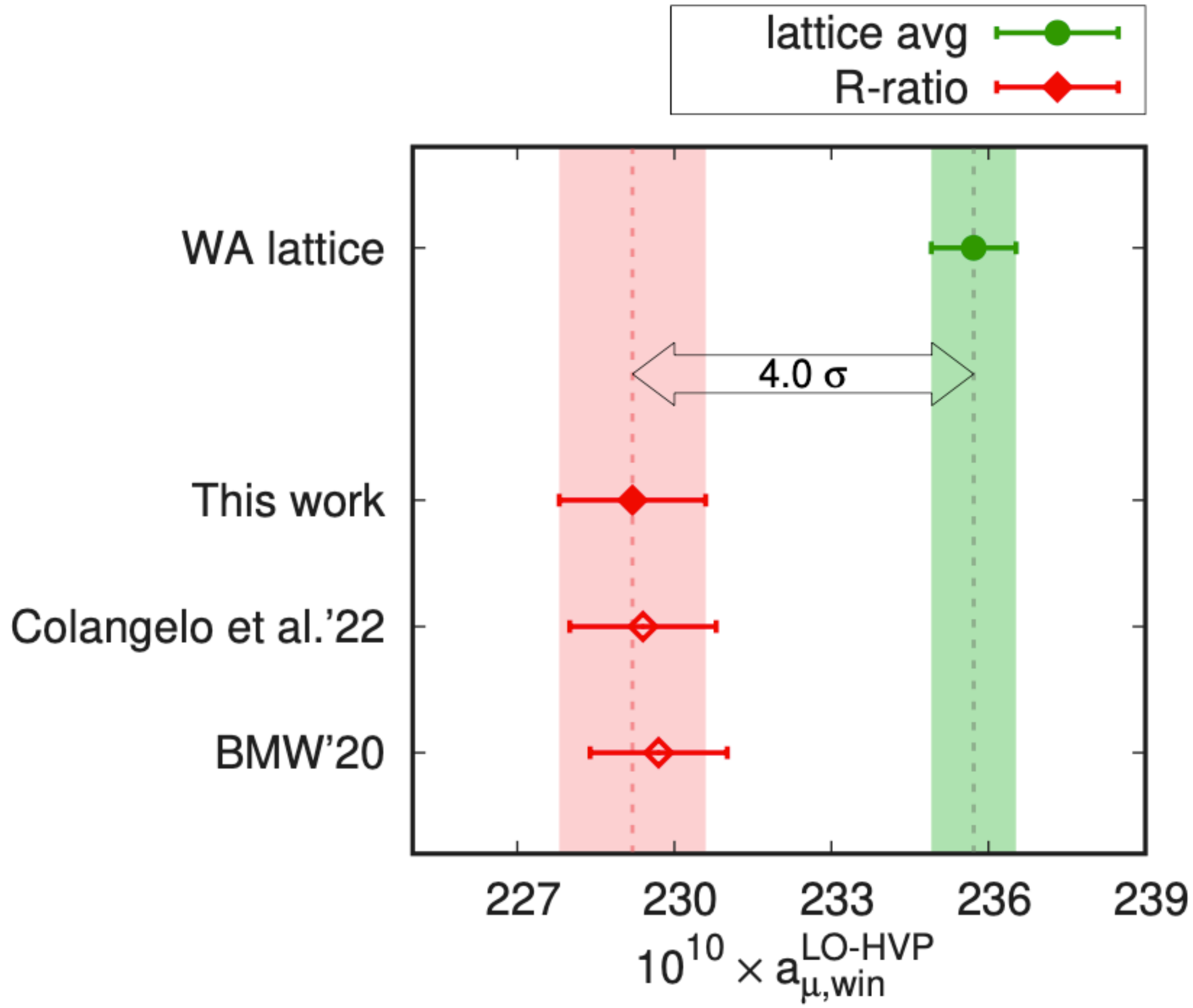
Focus on *window quantities* for precision comparisons (for now!)





[Kuberski 23]

Quantitative tools to understand the discrepancy



[Davier et al 2308.04221]

g-2: inputs & observables

Lattice correlators $$C(t) = \frac{1}{3e^2} \sum_{i=1,2,3} \int d^3x \, \langle J_i(t, \mathbf{x}) J_i(0) \rangle$$

R-ratio $$R(s) = \frac{\sigma\left(e^+e^-(s) \to \mathrm{hadr}\right)}{4\pi\alpha(s)^2/(3s)}$$

Observables $$a_I = \int dt \, K_I(t) \, C(t) = \int ds \, \hat{K}_I(s) \, R(s)$$

a_I^{lat} a_I^{R}

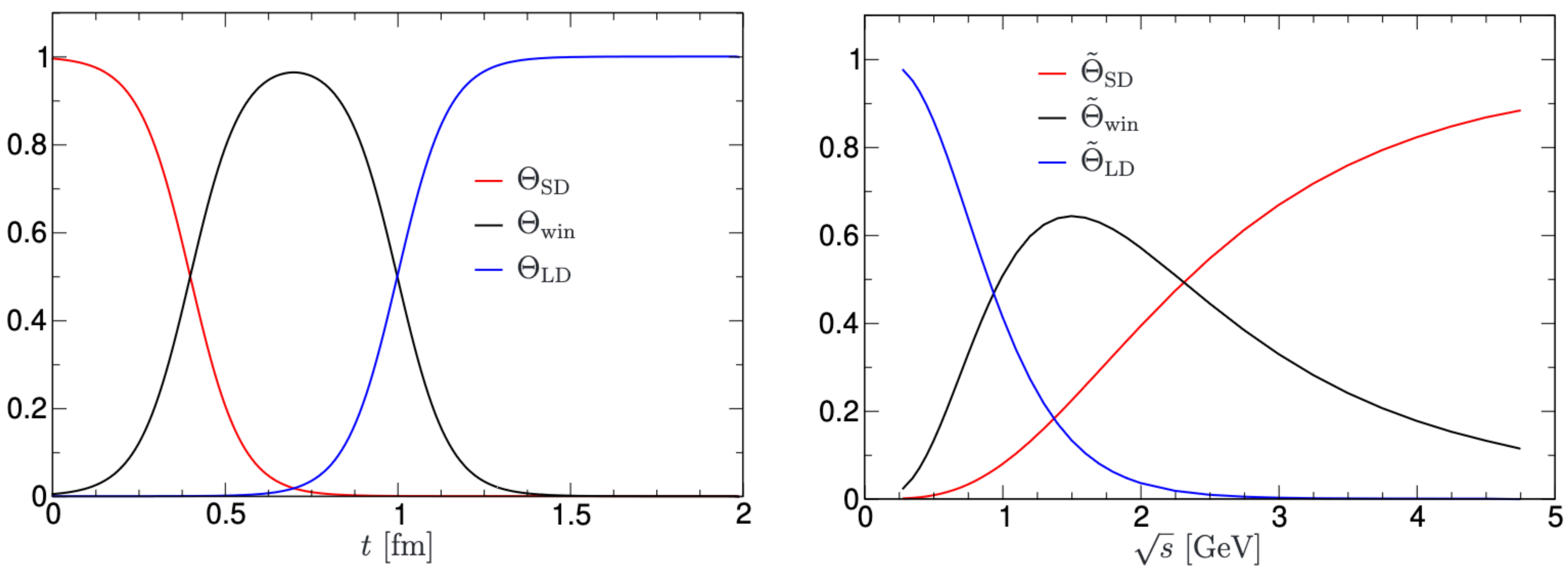
Window quantities & other constraints

Windows introduced by RBC/UKQCD 18, developed by many authors
[Lehner et al 20, Colangelo et al 22, Davier et al 23]

Add weight functions to the integrands

$$a_{I,\mathrm{win}} = \int dt\, W(t)\, K_I(t) C(t)$$

Allows to focus on discrepancies in different energy regions/channels
Systematic errors are easier to control

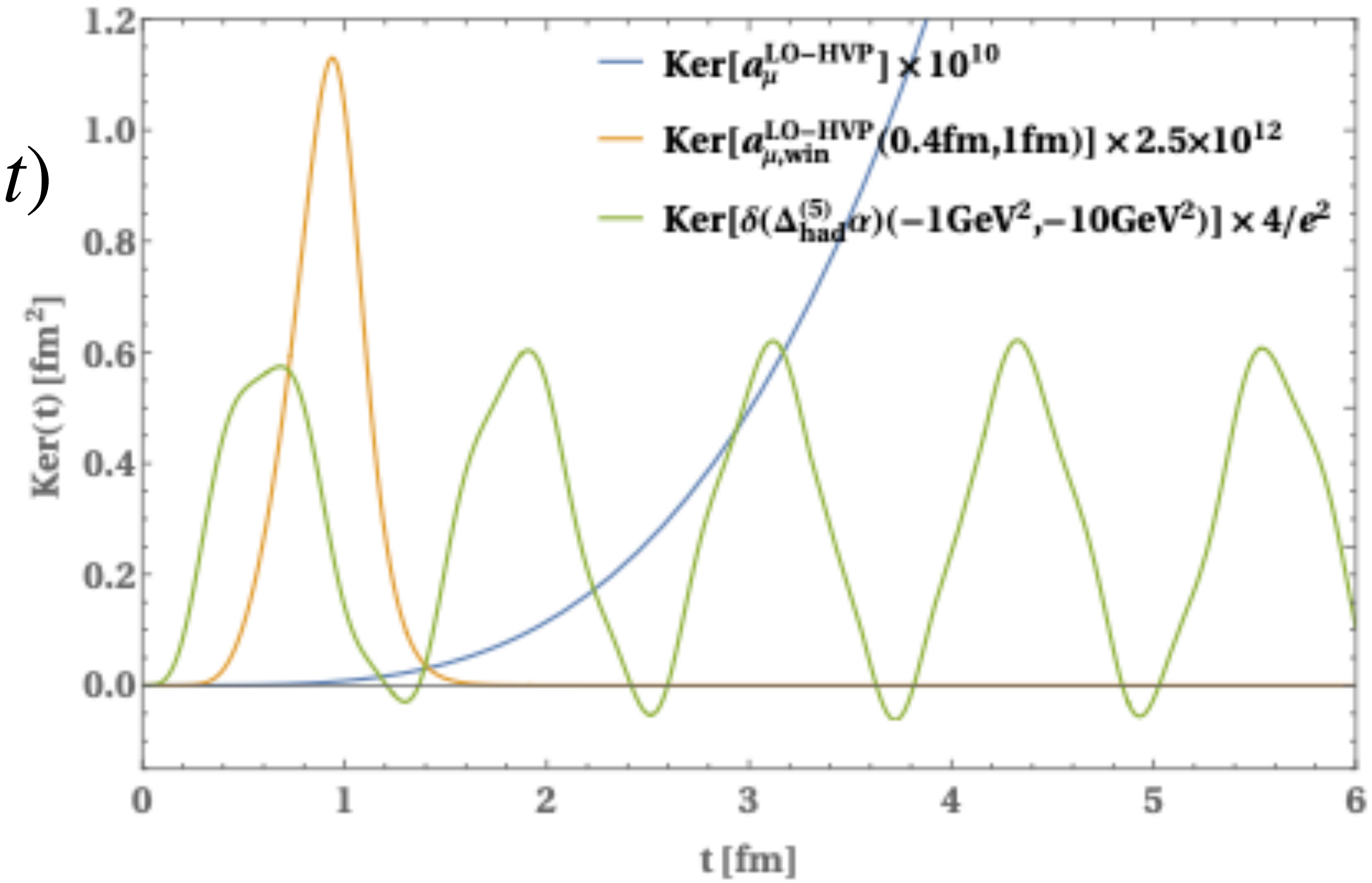


Following Davier et al, 2308.04221, consider the observables

$$a_\mu^{\mathrm{HVP,lat}} = \int dt\, K_{\mu,\mathrm{HVP}}(t)\, C(t)$$

$$a_{\mu,\mathrm{win}}^{\mathrm{HVP,lat}} = \int dt\, W(t; t_i, t_f)\, K_{\mu,\mathrm{HVP}}(t)\, C(t)$$

$$\Delta_{\mathrm{had}}^{(5),\mathrm{lat}} = \int dt\, K_\Delta(t)\, C(t)$$



Observable	lattice [6]	data-driven	diff.	% diff.	σ	p-value [%]
$a_\mu^{\mathrm{LO\text{-}HVP}} \times 10^{10}$	707.5(5.5)	694.0(4.0)	13.5(6.8)	1.9(1.0)	2.0	4.7
$a_{\mu,\mathrm{win}}^{\mathrm{LO\text{-}HVP}} \times 10^{10}$	236.7(1.4)	229.2(1.4)	7.5(2.0)	3.2(0.8)	3.8	0.01
$\left[\Delta_{\mathrm{had}}^{(5)}\alpha(-10\,\mathrm{GeV}^2) - \Delta_{\mathrm{had}}^{(5)}\alpha(-1\,\mathrm{GeV}^2)\right] \times 10^4$	48.67(0.32)[a]	48.02(0.32)	0.65(0.45)	1.3(0.9)	1.4	15.

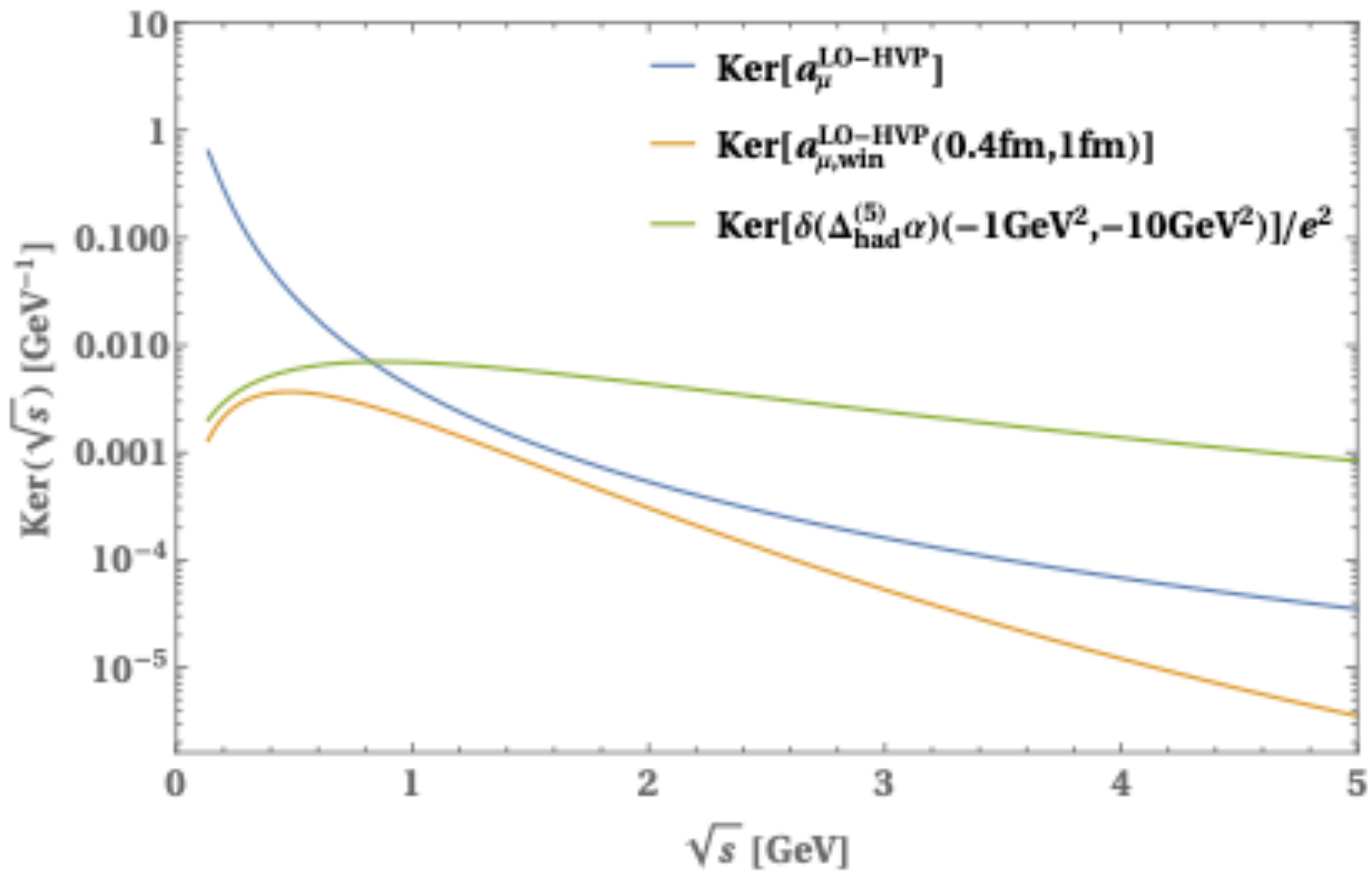
covariance matrices matter!

Reweighting the R ratio contributions

$$a_\mu^{\mathrm{HVP,R}} = \int ds\, \hat{K}_{\mu,\mathrm{HVP}}(s)\, R(s)$$

$$a_{\mu,\mathrm{win}}^{\mathrm{HVP,R}} = \int ds\, \hat{K}_{\mu,\mathrm{HVP,win}}(s)\, R(s)$$

$$\Delta_{\mathrm{had}}^{(5),\mathrm{R}} = \int ds\, \hat{K}_{\Delta}(s)\, R(s)$$



All observables are of the form $\quad a_I^{\mathrm{R}} = \sum_b a_{Ib}^{\mathrm{R}}$

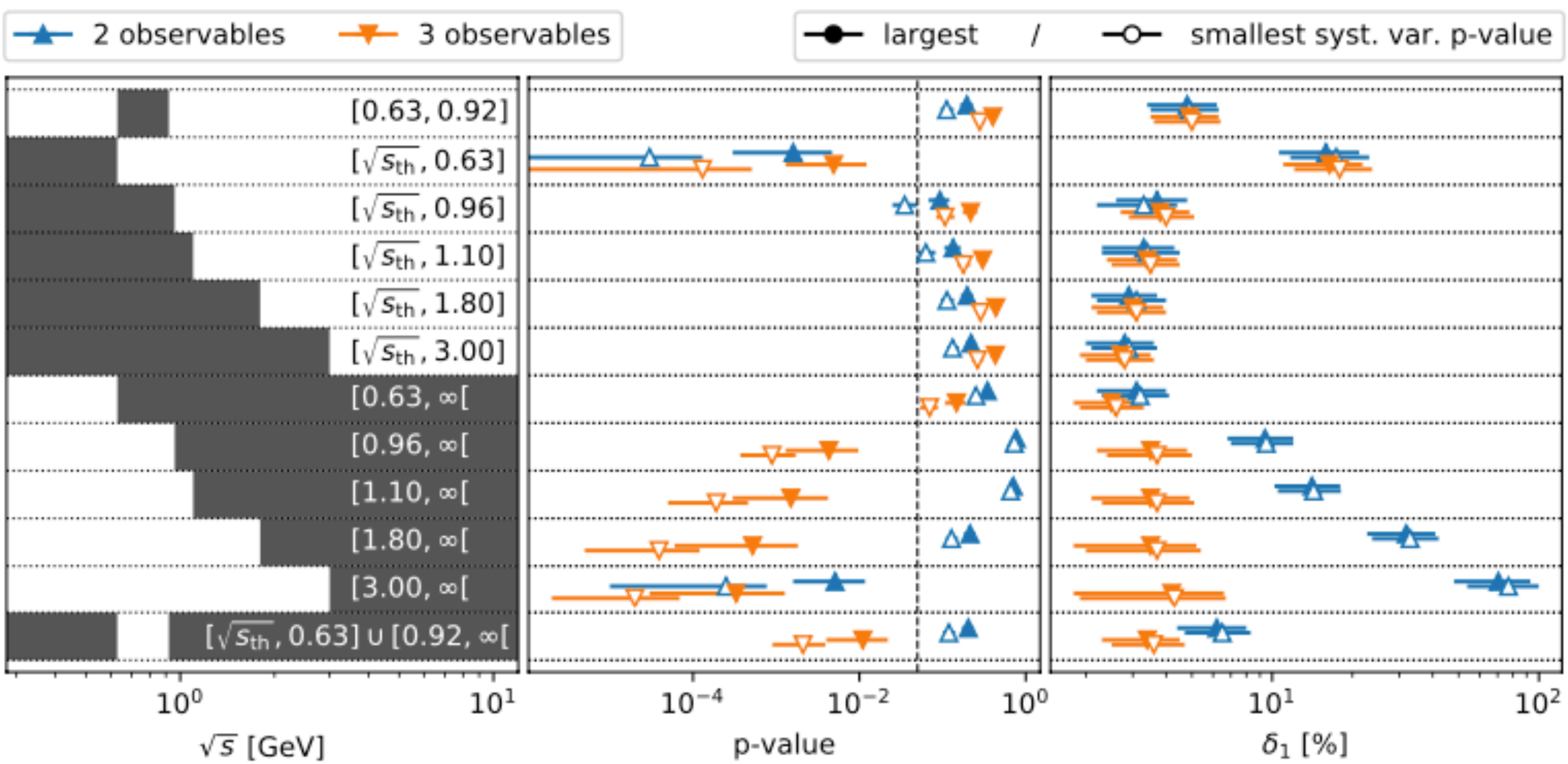
Rescaling in some energy intervals $\quad a_I^{\mathrm{lat}} = \sum_{b\in A} \gamma a_{Ib}^{\mathrm{R}} + \sum_{b\in B} a_{Ib}^{\mathrm{B}}$

yielding $\quad \gamma = \tilde{\gamma}_I = \dfrac{a_I^{\mathrm{lat}} - \sum_{b\in B} a_{Ib}^{\mathrm{R}}}{\sum_{b\in A} a_{Ib}^{\mathrm{R}}}$

Combine multiple observables and fit a common value of γ

$$\chi^2 = \sum_{I,J} \left(\gamma - \tilde{\gamma}_I\right) \left[C^{\tilde{\gamma}}_{\mathrm{lat}} + C^{\tilde{\gamma}}_{\mathrm{R}}\right]^{-1} \left(\gamma - \tilde{\gamma}_J\right)$$

importance of the covariances!



Current state of the analysis

1. Quantitative tools to understand discrepancies

2. Pin down the current tension to a given range of energies in R(s)?

3. Importance of a proper estimation of the covariances!

> *Whether such a strategy could disentangle contributions from different channels in realistic lattice-QCD simulations depends on the covariance matrices, which thus provides further motivation to make both the window quantities and their correlations available.*
>
> *[Colangelo et al 22]*

4. Analyses can be extended/refined

5. Stay tuned

Strong coupling constant & finite volume schemes

Λ parameter/strong coupling

$$\Lambda_S = \mu \left[b_0 \bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\bar{g}^2(\mu)}\right) \exp\left\{ \int_0^{\boxed{\bar{g}(\mu)}} dx \left[\boxed{\frac{1}{\beta(x)}} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right] \right\}$$

Λ_S is a scheme-dependent, non perturbative quantity

Conversion between schemes is exact at one loop

Current FLAG21 results

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1184(8)\,, \quad \Lambda^{(5)}_{\overline{\mathrm{MS}}} = 214(10)\ \mathrm{MeV}$$

Uncertainties are better understood by looking at Λ_S

Precision = understanding systematic errors

1. Extraction of $\bar{g}(\mu)$ - perturbative order of the (lattice) observable O

$$O(Q) = \sum_{k=0}^{n} c_k(s)\alpha(\mu)^k + \mathcal{O}(\alpha(\mu)^{n+1}) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right), \quad s = \mu/Q \sim 1$$

Determination is affected by truncation errors, estimated by scale variations

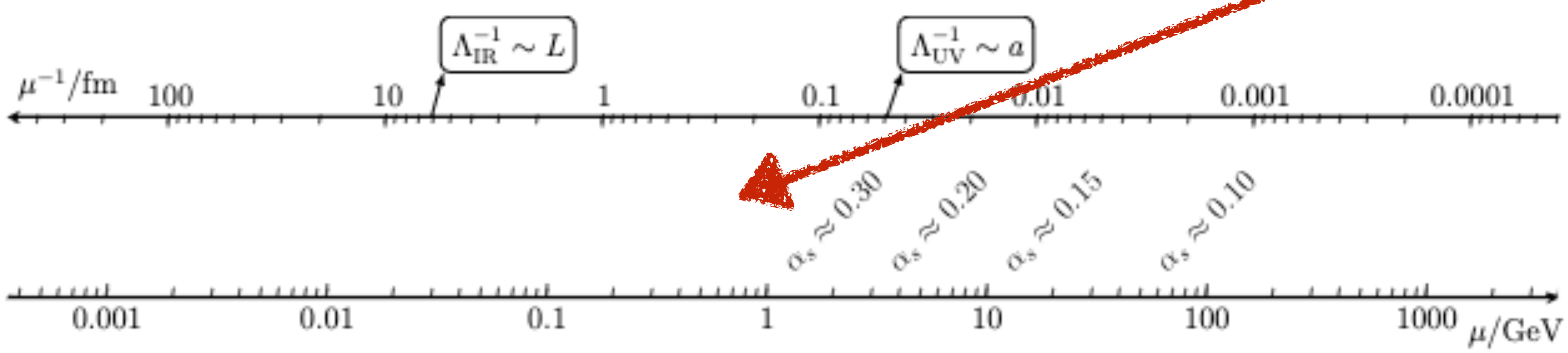
2. Matching to perturbation theory - perturbative order of the beta function

$$\int_0^{\bar{g}(\mu_{\mathrm{PT}})} dx \left[\frac{1}{\beta(x)} - \frac{1}{\beta^{(\ell)}(x)}\right] = \mathcal{O}\left(\bar{g}(\mu_{\mathrm{PT}})^{2(\ell-1)}\right)$$

The Λ parameter is the result of extrapolating to $\bar{g} \rightarrow 0$

Traditional lattice simulations are faced with a multi-scale problem

$a\mu \ll 1 \ll L\mu$



Scale variations to estimate the MHOU

Observable	loops	Q [GeV]	FLAG error [%]	$\delta^\star_{(4)}$ [%]	$\delta_{(2)}$ [%]	$\delta^\star_{(2)}$ [%]
		1.5	1.4		2.6	2.7
Potential	4	2.5		0.9	1.5	1.5
		5.0		0.4	0.8	0.8
HQ r_4		m_c	1.3		2.7	2.8
HQ r_4	3	$2m_c$		1.2	1.5	1.6
HQ r_6		$2m_c$			2.3	1.2
HQ r_8		$2m_c$			2.8	4.8
$-\log W_{11}$	3	4.4	1.0	2.8	3.3	2.5
$-\log W_{12}/u_0^6$		4.4		3.5	3.2	3.1
FSS	3	80		0.1	0.2	0.2

[Idd & Ramos 21]

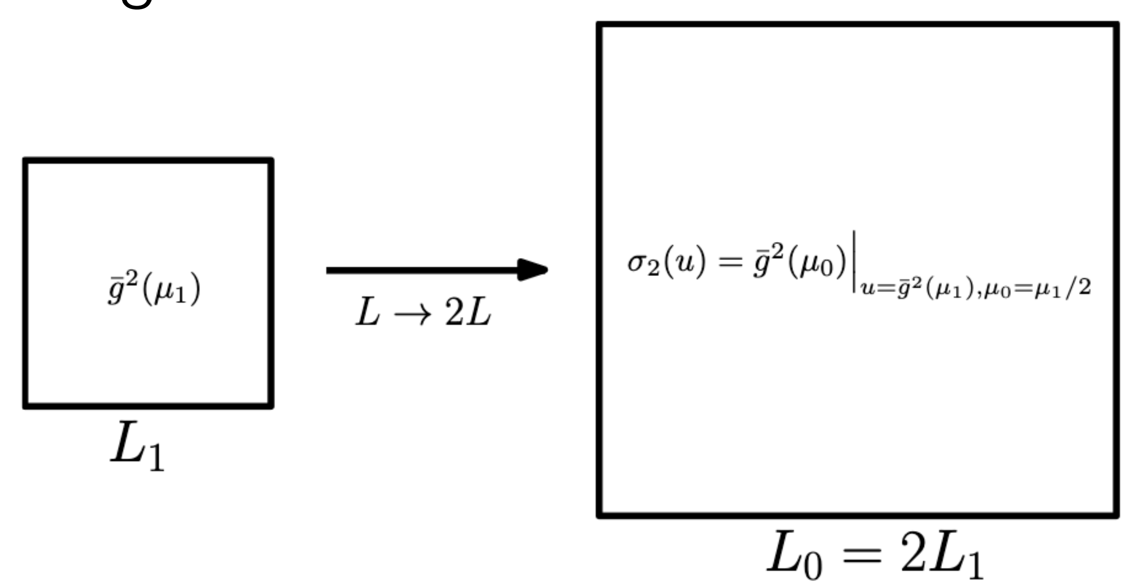
Finite Size Scaling schemes

1. $\mu = 1/L$

2. Observable: $O(Q) = \bar{g}^2_{\mathrm{SF/GF}}(1/L) = \sum_k c_k(s)\alpha_{\overline{\mathrm{MS}}}(\mu)^k$

3. iterative procedure: determine the step scaling function

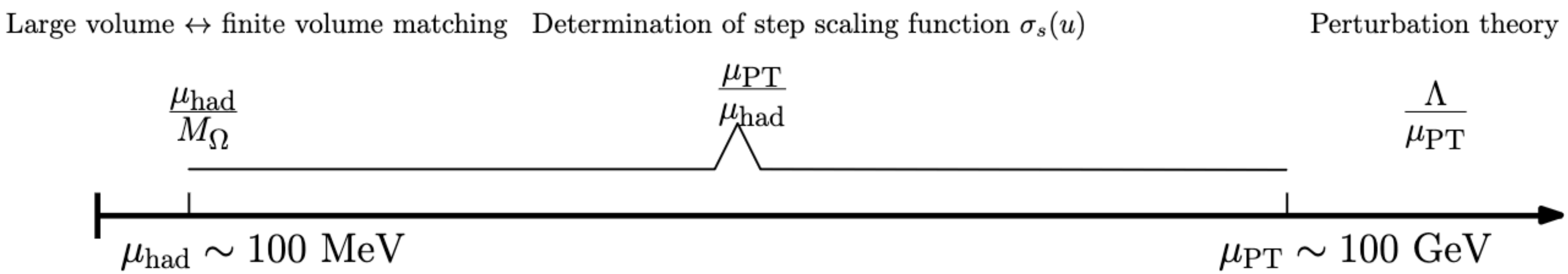
$$\sigma_2(u) = \bar{g}^2_{\mathrm{SF/GF}}(\mu_0)\Big|_{u=\bar{g}^2(2\mu_0)}$$



4. at each step, the scale changes by a factor 2

5. run to very high scales/increasingly small physical volumes

6. match to perturbation theory at high scale



$$\Lambda = M_\Omega^{\rm exp} \times \frac{\mu_{\rm had}}{M_\Omega} \times \frac{\mu_{\rm PT}}{\mu_{\rm had}} \times \frac{\Lambda}{\mu_{\rm PT}}$$

Determination of Λ requires the input of a single (low-energy) scale $M_\Omega^{\rm exp}$!

Lattice QCD provides the theoretical framework to connect $M_\Omega^{\rm exp}$ to Λ

Systematics under control, statistics is needed!

More details in the FLAG release coming soon

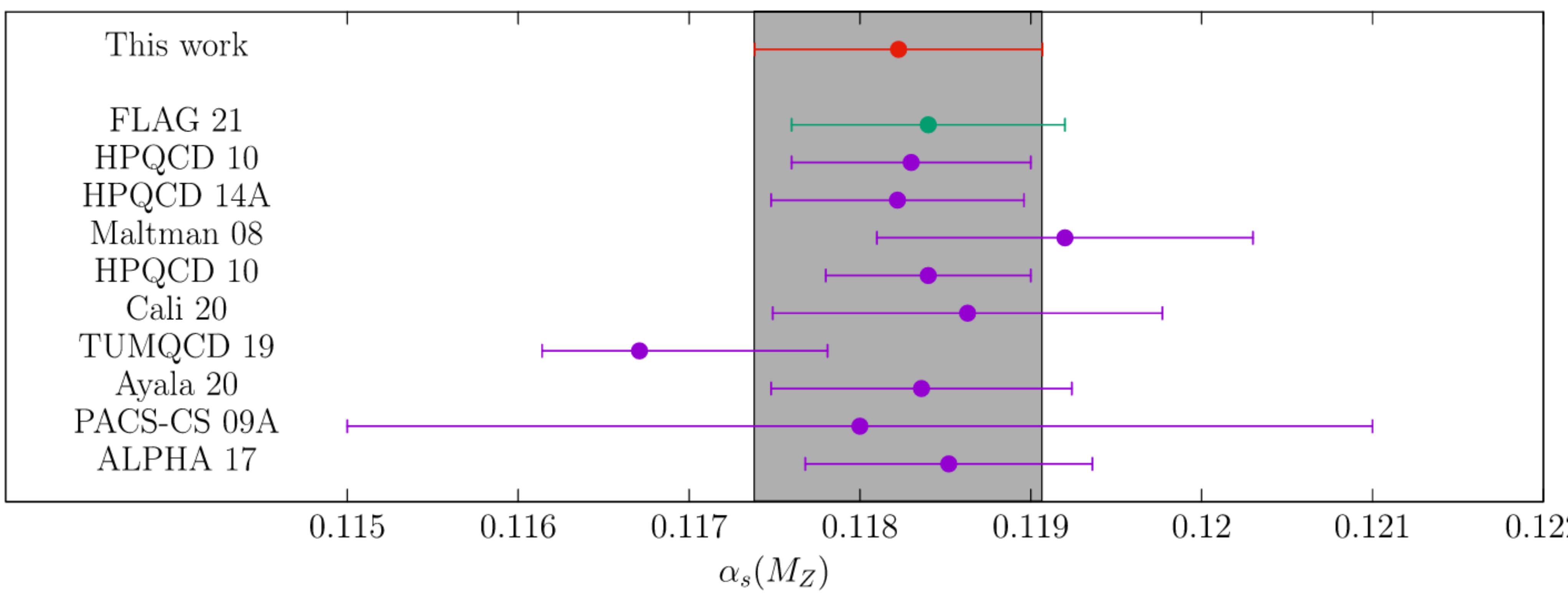
Decoupling for α_s

- Decoupling allows to relate $n_f = 3$ QCD to $n_f = 0$ QCD

- Step scaling methods in $n_f = 0$ QCD allow high-precision results

- Result dominated by statistical errors

[Dalla Brida 22]



Summary for α_s

The lattice determination dominates the PDG — theoretically clean

Step scaling methods allows to connect hadronic and perturbative scales

Decoupling provides a strong confirmation of the robustness of the lattice result

Determinations based on data are limited by physical scales of processes and need to consider carefully the experimental covariances

Determinations at hadron colliders need to perform a simultaneous fit of PDFs and α_s

[Forte & Kassabov 20]

Outlook

- Focus in this talk on g-2: tools to understand the current discrepancy
- Importance of understanding covariances in detail!
- … and on α_s: consolidated finite size results, $< 1\,\%$ precision
- Decay constants, quark masses, form factors, baryon, hadron spectroscopy, PDFs, see FLAG forthcoming new review, lattice conference proceedings
- Exascale computing will have a big impact on LQCD precision
- Algorithmic improvements, new ideas from AI/ML