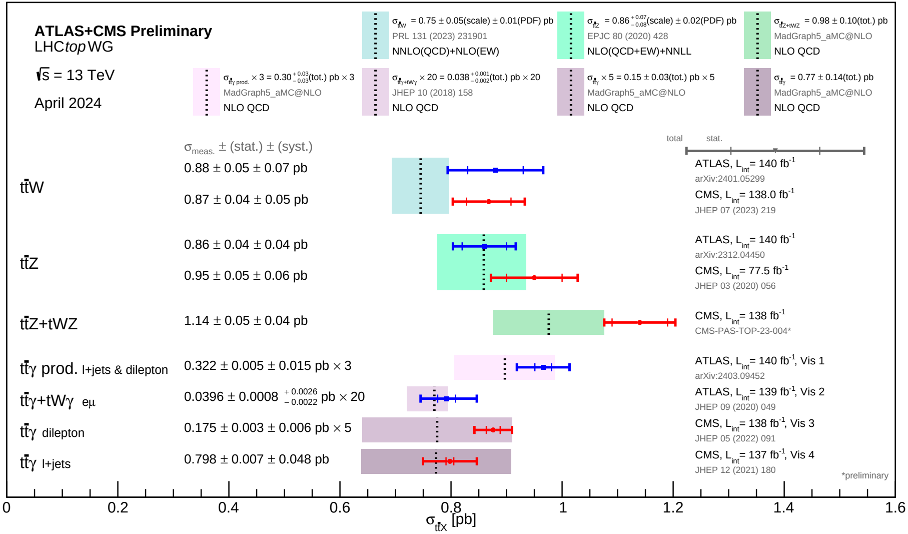
Precision top quark physics

Stefan Kallweit

12th Edition of the Large Hadron Collider Physics Conference
Boston, USA, June 3–7, 2024

Associated top-quark pair production at the LHC



Disclaimer

subjective selection of results, without the attempt to cover the full field of *precision top quark physics*

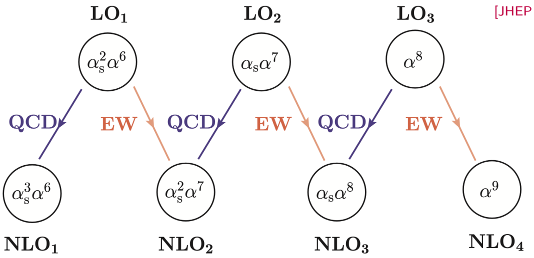
➥ **focus on recent fixed-order predictions for associated top-quark pair production**

Outline

1. Introduction
2. Ingredients for precision calculations on (associated) top-pair production
   - NLO corrections in full Standard Model
   - subtraction of IR singularities at NNLO QCD
   - towards two-loop amplitudes for massive $2 \to 3$ processes
3. Recent results on (associated) top-pair production
   - $t\bar{t}\gamma(\gamma)$ in dilepton decay channel using NWA in full NLO SM
   - $t\bar{t}Z$ in dilepton decay channel with full off-shell decays in full NLO SM
   - $t\bar{t}H$ at NNLO QCD (approximated 2-loop amplitudes)
   - $t\bar{t}W^{\pm}$ at NNLO QCD (approximated 2-loop amplitudes)
   - towards $t\bar{t}$ off-shell at NNLO QCD (approximated 2-loop amplitudes)
4. Conclusions & Outlook

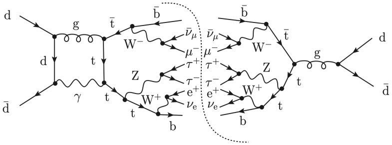
Full NLO Standard Model (SM) corrections in associated heavy-quark pair production

[JHEP 09 (2023) 072]



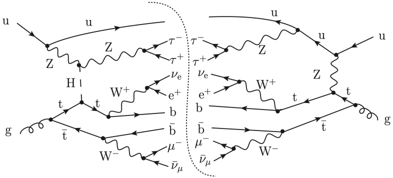
- Partonic channels enter at different orders:
  - gg at **LO$_1$**, $g\gamma$ at **LO$_2$**, $\gamma\gamma$ at **LO$_3$** (only neutral FS)
  - $q\bar{q}$ at **LO$_1$** (QCD), **LO$_3$** (EW), and **LO$_2$** (interference)
- **NLO$_1$** and **NLO$_4$** genuine QCD and EW corrections, respectively
- **NLO$_2$** and **NLO$_3$** in general not unique EW or QCD corrections
- **NLO$_3$** enhancements due to opening of new real topologies

- sample **NLO$_2$** virtual correction



  ➥ EW to **LO$_1$** and QCD to **LO$_2$**

- sample **NLO$_3$** real correction



  ➥ enhanced tZ-scattering topologies

q_T subtraction method for associated heavy-quark pair production at NNLO QCD

Extension of q_T subtraction method to production of heavy coloured particles ($F = Q\bar{Q},\ Q\bar{Q}X$, etc.)

$$d\sigma_F^{\mathrm{NNLO}} = \mathcal{H}_F^{\mathrm{NNLO}} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma_{F+\mathrm{jet}}^{\mathrm{NLO}} - d\sigma_{F,\mathrm{CT}}^{\mathrm{NNLO}}\right]_{\mathrm{cut}_{q_T}\to 0}$$

- basic idea: $\mathrm{d}\sigma_{\mathrm{F}}^{\mathrm{NNLO}}\Big|_{q_T\neq 0} = \mathrm{d}\sigma_{\mathrm{F+jet}}^{\mathrm{NLO}}$ with known singular $q_T \to 0$ behaviour (from q_T resummation), which is used to construct a non-local counterterm [Catani, Grazzini (2007)]
- built upon subtraction method for colourless F, extended to deal with soft final-state singularities [Bozzi, Catani, de Florian, Grazzini (2006), Catani, Grazzini (2007), Catani, Cieri, de Florian, Ferrera, Grazzini (2013)]
  - counterterm $d\sigma_{F,\mathrm{CT}}^{\mathrm{NNLO}}$ accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglia, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
  - massive NLO subtraction required for real-emission part $\mathrm{d}\sigma_{\mathrm{F+jet}}^{\mathrm{NLO}}$, e.g. massive dipole subtraction [Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]
  - $\mathcal{H}_{\mathrm{NNLO}}^F$ contains, besides 2-loop amplitudes and compensation for subtraction of counterterm, remainder of integrated final-state soft singularities
    - known for heavy-quark pairs [Catani, Devoto, Grazzini, Mazzitelli (2023), Angeles-Martinez, Czakon, Sapeta (2018)]
    - more involved kinematics for associated heavy-quark pair production [Devoto, Mazzitelli (to appear)]

➥ talk by S. Devoto

Amplitudes for heavy coloured particle production at NNLO QCD

2-loop amplitudes for on-shell $t\bar{t}$ production

- numerically: $q\bar{q}$ [Czakon (2008)] and gg [Bärnreuther, Czakon, Fiedler (2014)] channels, also polarized [Chen, Czakon, Poncelet (2017)]
- analytically in leading colour: $q\bar{q}$ [Bonciani, Ferroglia, Gehrmann, Studerus (2009)] and gg [Badger, Chaubey, Hartanto, Marzucca (2021)]
- analytically, in full colour: $q\bar{q}$ [Mandal, Mastrolia, Ronca, Bobadilla Torres (2022)]

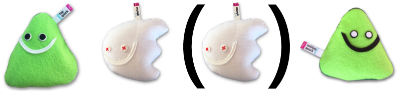
2-loop amplitudes for 2 $\longrightarrow$ 3 production processes with massless quarks

- analytically, in leading colour: $b\bar{b}H$ [Badger, Hartanto, Kryś, Zoia (2021)]
- analytically, in leading colour: $b\bar{b}W$ [Badger, Hartanto, Zoia (2021), Abreu, Cordero, Ita, Klinkert, Page, Sotnikov (2022)]
- analytically, in leading colour: $b\bar{b}Z$ [Sotnikov, Mazzitelli, Wiesemann ('24)]

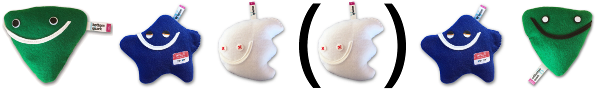
Recent publications/preprints on developments towards $t\bar{t}H$ 2-loop amplitudes ➥ hot topic!

- *Two-Loop Master Integrals for Leading-Color $pp \to t\bar{t}H$ Amplitudes with a Light-Quark Loop* [Cordero, Figueiredo, Kraus, Page, Reina ('23)]
- *One loop QCD corrections to $gg \to t\bar{t}H$ at $\mathcal{O}(\epsilon^2)$* [Buccioni, Kreer, Liu, Tancredi (2024)]
- *On the high-energy behavior of massive QCD amplitudes* [Wang, Xia, Yang, Ye (2024)]
- *Two-loop amplitudes for $t\bar{t}H$ production: the quark-initiated N_f-part* [Agarwal, Heinrich, Jones, Kerner, Klein, Lang, Magerya, Olsson (2024)]
- *Two-loop QCD amplitudes for $t\bar{t}H$ production from boosted limit* [Wang, Xia, Yang, Ye ('24)]

Theory status of $pp \to t\bar{t}\gamma(\gamma) + X$



- NLO QCD for on-shell production $(t\bar{t}\gamma)$ [Duan, Guo, Han, Ma, Wang, Zhang (2009 & 2011), Maltoni, Pagani, Tsinikos (2016)]
- NLO EW for on-shell production $(t\bar{t}\gamma)$ [Duan, Li, Song, Wang, Zhang (2017)]
- NLO SM for on-shell production $(t\bar{t}\gamma(\gamma))$ [Pagani, Shao, Tsinikos, Zaro (2021)]
- NLO+NNLL for on-shell production $(t\bar{t}\gamma)$ [Kidonakis, Tonero (2023)]
- NLO+PS for on-shell production $(t\bar{t}\gamma)$ [Kardos, Trócsányi (2015)]
  $(t\bar{t}\gamma\gamma)$ [Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer,Torrielli, Zaro (2014), Kardos, Trócsányi (2015), van Deurzen, Frederix, Hirschi, Luisoni, Mastrolia, Ossola (2016)]



- NLO QCD for NWA production with leptonic decays $(t\bar{t}\gamma)$ [Melnikov, Schulze, Scharf (2011)]
  with leptonic and lepton+jet decays $(t\bar{t}\gamma(\gamma))$ [Stremmer, Worek (2023)]
- NLO QCD for off-shell production with leptonic decays $(t\bar{t}\gamma)$ [Bevilacqua, Hartanto, Kraus, Weber, Worek (2018 & 2020)]
- NLO SM for NWA on-shell production with leptonic decays $(t\bar{t}\gamma(\gamma))$ [Stremmer, Worek ('24)]

$pp \to t\bar{t}\gamma(\gamma) + X$ production in NWA with leptonic decays at NLO SM

Calculational details

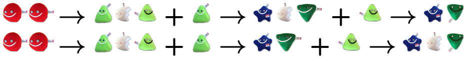
➥ talk by D. Stremmer

- Implementation in HELACDIPOLES MC program [Czakon, Papadopoulos, Worek (2009)]
- phase space integration with PARNI [van Hameren (2009)] and KALEU [van Hameren ('10)]
- amplitudes generated with RECOLA [Actis, Denner, Hofer, Scharf, Uccirati (2013), Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)], tensor reduction and scalar integrals from COLLIER [Denner, Dittmaier, Hofer (2017)]; qp rescue system based on CUTTOOLS [Ossola, Papadopoulos, Pittau (2007 & 2008)] with ONELOOP scalar integrals [van Hameren (2011)]
- Nagy-Soper subtraction for QCD and QED singularities [Bevilacqua, Czakon, Kubocz, Worek (2013)] extended to NWA [Bevilacqua, Lupattelli, Stremmer, Worek (2023)]
- phase space restrictions on subtraction terms [Nagy, Trócsányi (1999), Nagy (2003), Bevilacqua, Czakon, Papadopoulos, Pittau, Worek (2009), Czakon, Hartanto, Kraus, Worek (2015)]

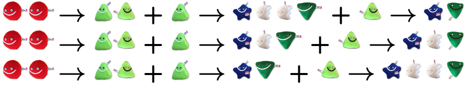
Composition of photon emissions ($t\bar{t}\gamma\gamma$)

- Prod.

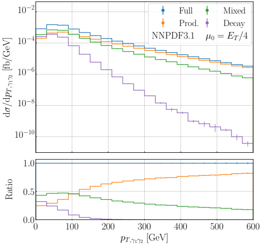
- Mixed

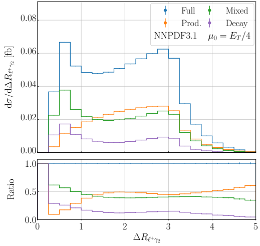


- Decay



$p_{T,\gamma_1\gamma_2}$ **$\Delta R_{\ell^+\gamma_2}$** [JHEP 08 (2023) 179]



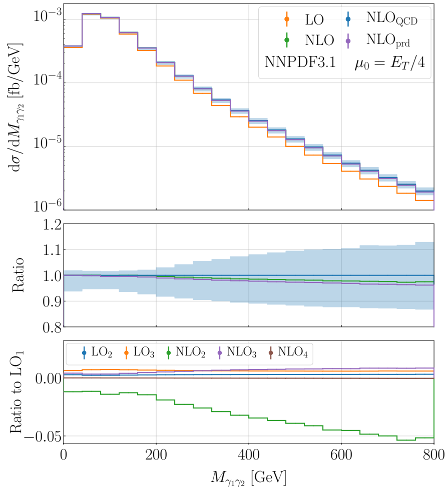


Breakdown of subleading NLO SM contributions to $pp \to t\bar{t}\gamma\gamma + X$

		σ_i [fb]	Ratio to LO$_1$
LO$_1$	$\mathcal{O}(\alpha_s^2\alpha^6)$	$0.15928(3)^{+31.3\%}_{-22.1\%}$	1.00
LO$_2$	$\mathcal{O}(\alpha_s^1\alpha^7)$	$0.0003798(2)^{+25.8\%}_{-19.2\%}$	+0.24%
LO$_3$	$\mathcal{O}(\alpha_s^0\alpha^8)$	$0.0010991(2)^{+10.6\%}_{-13.1\%}$	+0.69%
NLO$_1$	$\mathcal{O}(\alpha_s^3\alpha^6)$	+0.0110(2)	+6.89%
NLO$_2$	$\mathcal{O}(\alpha_s^2\alpha^7)$	−0.00233(2)	−1.46%
NLO$_3$	$\mathcal{O}(\alpha_s^1\alpha^8)$	+0.000619(1)	+0.39%
NLO$_4$	$\mathcal{O}(\alpha_s^0\alpha^9)$	−0.0000166(2)	−0.01%
LO		$0.16076(3)^{+30.9\%}_{-21.9\%}$	1.0093
NLO$_{\rm QCD}$		$0.1703(2)^{+1.9\%}_{-6.2\%}$	1.0690
NLO$_{\rm prd}$		$0.1694(2)^{+1.7\%}_{-5.9\%}$	1.0637
NLO		$0.1700(2)^{+1.8\%}_{-6.0\%}$	1.0674

- **NLO$_1$** inevitable (scale dependence)
- **NLO$_4$** phenomenologically irrelevant

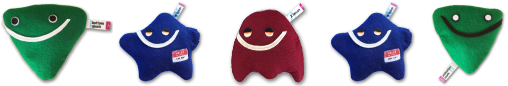
$m_{\gamma_1\gamma_2}$ [arXiv:2403.03796]



- **NLO$_2$** dominant among subleading NLO terms inclusively; EW Sudakov logarithms in high-energy tails of distributions
- partial (accidental) cancellations between **NLO$_2$** and **NLO$_3$** (less pronounced for $t\bar{t}\gamma\gamma$ than for $t\bar{t}\gamma$)
- **NLO$_{\rm prd}$** (subleading NLO corrections only to decay stage) provides very good approximation of full **NLO** result

Theory status of $pp \to t\bar{t}Z + X$

- NLO QCD for on-shell production [Lazopoulos, McElmurry, Melnikov, Petriello (2008), Kardos, Trócsányi, Papadopoulos (2012)]
- NLO SM for on-shell production [Frixione, Hirschi, Pagani, Shao, Zaro (2015), Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2021)]
- NLO+PS for on-shell production [Maltoni, Pagani, Tsinikos (2016), Garzelli, Kardos, Papadopoulos, Trócsányi (2012 & 2012)]
- NLO+NNLL for on-shell production [Broggio, Ferroglia, Ossola, Pecjak, Sameshima (2017), Kulesza, Motyka, Schwartländer, Stebel, Theeuwes (2019 & 2020)]
- NLO+NNLL plus NLO SM for on-shell production [Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos (2019)]



- NLO QCD for NWA production with leptonic decays [Röntsch, Schulze (2014 & 2015)]
- NLO+PS for NWA production with leptonic decays [Ghezzi, Jäger, Chavez, Reina, Wackeroth (2022)]
- NLO QCD for off-shell production with leptonic decays $(2\ell 2\nu)$ [Bevilacqua et al. (2019)] (4ℓ) [Bevilacqua et al. (2022)]
- NLO SM for off-shell production with leptonic decays [Denner, Lombardi, Pellicciol (2023)]

Off-shell $pp \to t\bar{t}Z + X$ production with leptonic decays at NLO SM

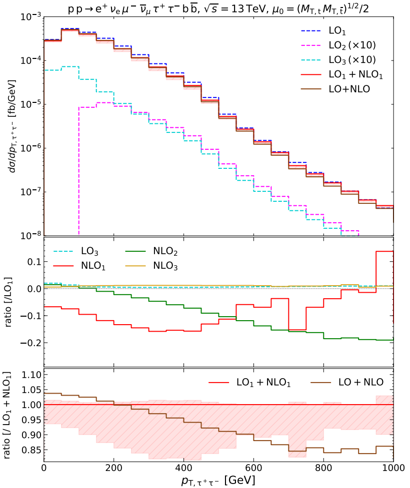
Calculational details

- phase space integration with MoCaNLO (private code)
- amplitudes generated with Recola [Actis, Denner, Hofer, Scharf, Uccirati (2013), Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)], tensor reduction and scalar integrals from Collier [Denner, Dittmaier, Hofer (2017), Denner, Dittmaier (2003 & 2006 & 2012)]
- dipole subtraction for QCD and QED singularities [Catani, Seymour (1997), Dittmaier (2000), Catani, Dittmaier, Seymour, Trócsányi (2002), Dittmaier, Kabelschacht, Kasprzik (2008)]
- resonance treatment through complex-mass scheme [Denner, Dittmaier, Roth, Wackeroth (1999), Denner, Dittmaier, Roth, Wieders (2005 & 2012), Denner, Dittmaier (2006, 2020)]

Pattern of typical transverse-momentum distributions

- **NLO_1** necessary to stabilize scale dependence; size depends on scale choice, which mostly affects **LO_1**
- **NLO_2** shows typical negative corrections due to Sudakov logarithms in the tail: predominantly an NLO EW correction
- **subleading orders** (LO_2, LO_3, NLO_3) at the percent level

$p_{T,\tau^+\tau^-} = p_{T,Z}$ [JHEP 09 (2023) 072]

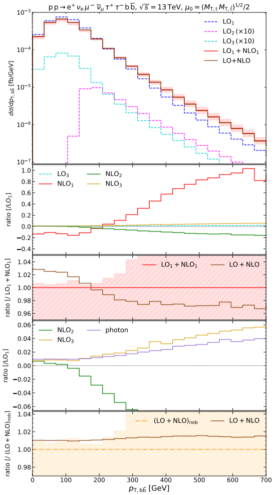


Breakdown of contributions for off-shell $pp \to t\bar{t}Z + X$ in NLO SM

perturbative order	σ_{nob} [ab]	$\frac{\sigma_{\text{nob}}}{\sigma_{\text{nob, LO}_1}}$	σ_{b} [ab]	$\frac{\sigma_{\text{b}}}{\sigma_{\text{nob, LO}_1}}$	σ [ab]	$\frac{\sigma}{\sigma_{\text{LO}_1}}$
LO_1	$107.246(5)^{+35.0\%}_{-24.0\%}$	1.0000	0.31378(9)	+0.0029	$107.560(5)^{+34.9\%}_{-23.9\%}$	1.0000
LO_2	$0.7522(2)^{+11.1\%}_{-9.0\%}$	+0.0070	−0.6305(2)	−0.0059	0.1217(3)	+0.0011
LO_3	$0.2862(1)^{+3.4\%}_{-3.4\%}$	+0.0027	0.7879(2)	+0.0073	$1.0742(3)^{+12.1\%}_{-14.9\%}$	+0.0100
NLO_1	−11.4(1)	−0.1072	0.518(3)	+0.0048	−10.9(1)	−0.1016
NLO_2	−0.89(1)	−0.0083	0.109(3)	+0.0010	−0.78(1)	−0.0072
NLO_3	1.126(4)	+0.0105	−0.089(4)	−0.0008	1.037(6)	+0.0096
NLO_4	−0.0340(9)	−0.0003	−0.0180(9)	−0.0002	−0.052(1)	−0.0005
LO_1+NLO_1	$95.8(1)^{+0.4\%}_{-11.2\%}$	+0.8933	0.832(3)	+0.0078	$96.6(1)^{+0.4\%}_{-10.7\%}$	+0.8984
LO	$108.285(5)^{+34.7\%}_{-23.8\%}$	+1.0097	0.4713(3)	+0.0044	$108.756(5)^{+34.5\%}_{-23.7\%}$	+1.0111
LO+NLO	$97.0(1)^{+0.5\%}_{-11.2\%}$	+0.9052	0.991(6)	+0.0092	$98.0(1)^{+0.4\%}_{-10.6\%}$	+0.9114

- **LO_2** small due to cancellation of $g\gamma$ channel and $b\bar{b}$-induced interference; at large p_T, $g\gamma$ dominates, exceeding even **LO_3** contribution
- **NLO_1** with large K-factors and LO-like scale uncertainties in $\boldsymbol{p_{T,b\bar{b}}}$ tail
- partial cancellation between **NLO_2** and **NLO_3**; tiny **NLO_4** effect

$\boldsymbol{p_{T,b\bar{b}}}$ [JHEP 09 (2023) 072]



Theory status of $pp \to t\bar{t}H + X$

- NLO QCD for on-shell production [Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas (2001 & 2003), Reina, Dawson (2001), Reina, Dawson, Wackeroth (2002), Reina, Dawson, Wackeroth, Jackson, Orr (2001 & 2003)]
- NLO SM for on-shell production [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- NLO+PS for on-shell production [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli (2011), Garzelli, Kardos, Papadopoulos, Trócsányi (2011), Hartanto, Jäger, Reina and Wackeroth (2015)]
- NLO+NNLL for on-shell production [Kulesza, Motyka, Stebel, Theeuwes (2016), Broggio, Ferroglia, Pecjak, Signer, Yang (2016), Broggio, Ferroglia, Pecjak, Yang (2017)]
- NNLO QCD for on-shell production [Catani, Fabre, Grazzini, SK (2021), Catani, Devoto, Grazzini, SK, Mazzitelli, Savoini (2023)]

- NLO EW for NWA production with leptonic decays [Chen, Guo, Ma, Zhang, Zhang (2014)]
- NLO QCD for off-shell production with leptonic decays [Denner, Feger (2015), Stremmer, Worek (2022)]
- NLO EW for off-shell production with leptonic decays [Denner, Lang, Pellen, Uccirati (2017)]

On-shell $pp \to t\bar{t}H + X$ production at NNLO QCD

Calculational details

- Implementation in the MATRIX framework [Grazzini, SK, Wiesemann (2018)]
- amplitudes from OPENLOOPS [Cascioli, Maierhöfer, Pozzorini (2012), Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)] using on-the-fly tensor reduction [Buccioni, Pozzorini, Zoller (2018)] and scalar integrals from COLLIER [Denner, Dittmaier, Hofer (2017)]
- q_T slicing combined with dipole subtraction
- full calculation exact, apart from 2-loop amplitudes

➥ **Soft Higgs boson approximation** $(k \to 0, m_H \ll m_t)$

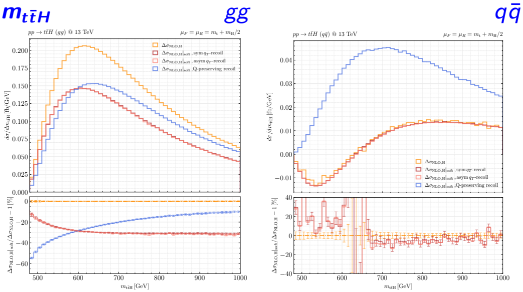
$$\mathcal{M}_{t\bar{t}H}(\{p_i\}, k) \simeq F(\alpha_s(\mu_R); \tfrac{m_t}{\mu_R})\, \mathcal{J}_0(k)\, \mathcal{M}_{t\bar{t}}(\{\tilde{p}_i\})$$

- F soft limit of scalar heavy-quark form factor [Bernreuther et al. (2005), Blümlein et al. (2017)]
- $\mathcal{J}_0(k) = \frac{m_t}{v}\left(\frac{m_t}{p_t \cdot k} + \frac{m_t}{p_{\bar{t}} \cdot k}\right)$ eikonal factor
- $\mathcal{M}_{t\bar{t}}$ $gg/q\bar{q} \to t\bar{t}$ amplitude [Bärnreuther et al. (2014)]

➥ reweighting with full Born amplitude applied

Validation of soft-Higgs approximation at NLO

[Devoto, Grazzini, SK, Mazzitelli, Savoini (prel.)]



- Ambiguities in soft approximation approach
  - projection from $t\bar{t}H$ to $t\bar{t}$ phase space
  - choice of approximation scale

➥ performance at 1-loop level and ambiguities used to derive a conservative error estimate

NNLO QCD results for on-shell $pp \to t\bar{t}H + X$ production

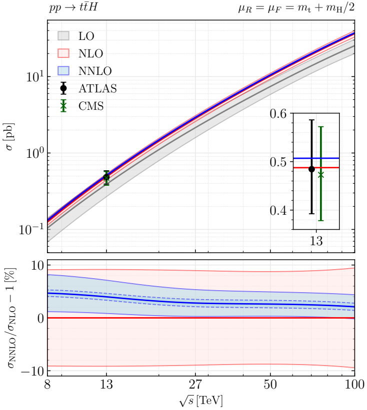
[PRL 130, 111902 (2023)]

σ (pb)	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

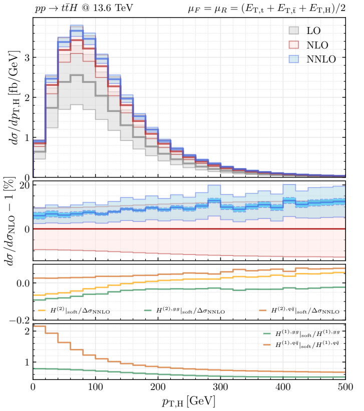
	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ (fb)	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO},H}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO},H}\vert_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO},H}\vert_{\text{soft}}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

- significant reduction of perturbative uncertainties (max. of 7-point scale variation, assigned symmetrically)
- error estimate due to soft approximation much smaller
  - ➥ only ±0.6% of σ_{NNLO}

[PRL 130, 111902 (2023)]



$p_{T,H}$ [Devoto, Grazzini, SK, Mazzitelli, Savoini (prel.)]



- same error estimate applied differentially looks reasonable as well
- ➥ further studies ongoing, including alternative approximations ...

Theory status of $pp \to t\bar{t}W^{\pm} + X$

- NLO QCD for on-shell production [Badger, Campbell, Ellis (2011)]
- NLO SM for on-shell production [Frixione, Hirschi, Pagani, Shao, Zaro (2015), Frederix, Pagani, Zaro (2018)]
- NLO+NNLL for on-shell production [Li, Li, Li (2014), Broggio, Ferroglia, Ossola, Pecjak (2016), Kulesza, Motyka, Schwartländer, Stebel, Theeuwes (2019)]
- NLO+NNLL plus NLO SM [Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos (2019)] +N^{3}LL [Kidonakis, Foster (2024)]
- NLO SM for on-shell production with multi-jet merging [Frederix, Tsinikos (2021)]
- NNLO QCD plus NLO SM for on-shell production [Buonocore, Devoto, Grazzini, SK, Mazzitelli, Rottoli, Savoini (2023)]

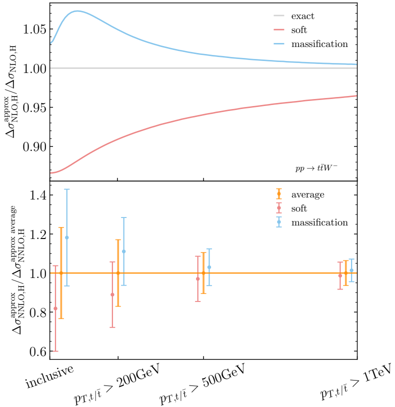
- NLO QCD for NWA production with leptonic decays [Campbell, Ellis (2012)]
- NLO QCD for off-shell production with leptonic decays [Bevilacqua, Bi, Hartanto, Kraus, Worek (2020), Denner, Pellicciolí (2020), Bevilacqua, Bi, Hartanto, Nasufi, Kraus, Worek (2021)]
- NLO QCD+EW for off-shell production with leptonic decays [Denner, Pellicciolí (2021)]

On-shell $pp \to t\bar{t}W^{\pm} + X$ production at NNLO QCD

Calculational details

➥ talk by S. Devoto

[PRL 131, 231901 (2023)]



- MATRIX implementation, as for $t\bar{t}H$ at NNLO QCD
- full calculation exact, apart from 2-loop amplitudes
  - ➥ use two complementary approximations

1. **Soft W boson approximation** ($k \to 0, m_W \ll m_t$)
   - ➥ analogous approach as for $t\bar{t}H$
2. **Massification approach** ($m_t \ll Q_{t\bar{t}W}$) [Mitov, Moch (2007)]
   - use massless $Wq\bar{q}$ amplitudes [Badger et al. (2021), Abreu et al. (2022)] and reconstruct m_q dependence (up to power corrections)
     - ➥ successfully applied in MATRIX for $Wb\bar{b}$ production [Buonocore, Devoto, SK, Mazzitelli, Rottoli, Savoini (2023)]
   - ➥ reweighting with full Born amplitude applied
   - error estimate derived similarly to soft approximation

➥ Validate the two approximations in a phase space region where both should work: **large $p_{T,t/\bar{t}}$**
NLO: both converge against exact result **NNLO:** the two approximations agree well within errors

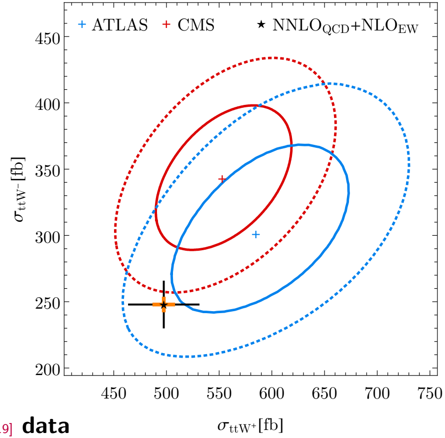
➥ average with linearly combined errors as **best prediction** (improvements under investigation)

NNLO QCD results for on-shell $pp \to t\bar{t}W^{\pm} + X$ production

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%}_{-5.8\%}{}^{+8.0\%}_{-7.5\%}$	$301^{+9.3\%}_{-9.0\%}{}^{+11.6\%}_{-10.3\%}$	$890^{+5.6\%}_{-5.6\%}{}^{+7.9\%}_{-7.9\%}$	$1.95^{+10.8\%}_{-9.2\%}{}^{+8.2\%}_{-6.7\%}$
CMS [10]	$553^{+5.4\%}_{-5.4\%}{}^{+5.4\%}_{-5.4\%}$	$343^{+7.6\%}_{-7.6\%}{}^{+7.3\%}_{-7.3\%}$	$868^{+4.6\%}_{-4.6\%}{}^{+5.9\%}_{-5.9\%}$	$1.61^{+9.3\%}_{-9.3\%}{}^{+4.3\%}_{-3.1\%}$

- positive NNLO corrections: $\approx +15\%$ of σ_{NLO}
- sizeable impact of 2-loop contribution: $\approx 6 - 7\%$ of σ_{NNLO}
- approximation error estimate: $\approx 1.8\%$ of σ_{NNLO}
  - ➥ smaller than perturbative uncertainties at NNLO: $\approx 6.7\%$
- compatible with FxFx result: $\sigma_{\mathrm{FxFx}} = 722.4\,\mathrm{fb}^{+9.7\%}_{-10.8\%}$
  [Frederix, Tsinikos (2021)]

[PRL 131, 231901 (2023)]



Comparison against ATLAS [ATLAS-CONF-2023-019] **and CMS** [JHEP 07 (2023) 219] **data**

➥ Agreement remains at the 1σ (ATLAS) and 2σ (CMS) level, respectively.

(ATLAS result superseeded by [JHEP 05 (2024) 131] : $\sigma_{t\bar{t}W} = 880 \pm 50(\mathrm{stat.}) \pm 70(\mathrm{syst.})\,\mathrm{fb}$)

Theory status of $pp \to t\bar{t} + X$

- NNLO QCD for on-shell production [Czakon, Fiedler, Mitov (2013), Czakon, Heymes, Mitov (2016 & 2016 & 2017), Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2019), Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019 & 2020)]
- NNLO QCD+NLO SM for on-shell production [Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro (2017 & 2018)]
- NNLO+NNLL' for on-shell production [Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, Yang (2018)] +N^{3}LL [Kidonakis, Guzzi, Tonero (2023)]
- NNLO+PS for on-shell production [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi (2021 & 2022)]

- NNLO QCD for NWA production with leptonic decays [Czakon, Mitov, Poncelet (2021)]
- NLO QCD for off-shell production with leptonic decays [Denner, Dittmaier, SK, Pozzorini (2011 & 2012), Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek (2011)], with $m_b \neq 0$ [Frederix (2014), Cascioli, SK, Maierhöfer, Pozzorini (2014)], $+1jet$ [Bevilacqua, Hartanto, Kraus, Worek (2016 & 2016)] and with semileptonic decays [Denner, Pellen (2018)]
- NLO QCD+EW for off-shell production with leptonic decays [Denner, Pellen (2016)]
- NLO+PS for off-shell production with (semi-)leptonic decays [Ježo, Lindert, Nason, Oleari, Pozzorini (2016), Ježo, Lindert, Pozzorini (2023)]
- towards NNLO QCD for off-shell production [Buonocore, Devoto, Grazzini, SK, Lindert, Mazzitelli, Savoini (in progress)]

Off-shell $pp \to t\bar{t} + X$, i.e. $pp \to W^+W^-b\bar{b} + X$, at NNLO QCD

Calculational details

- MATRIX implementation, as for any $Q\bar{Q}X$ process at NNLO QCD, but numerically challenging . . .
- full calculation exact, apart from 2-loop amplitudes
  ➥ **use a double-pole approximation (DPA)**

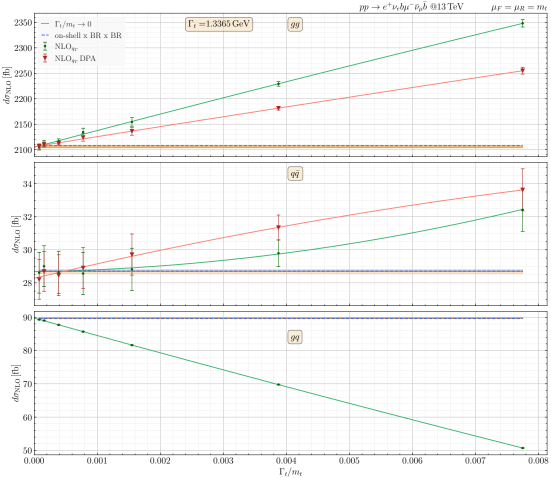
➥ validation against the $t\bar{t}$ cross section in NWA, by numerically taking the limit $\Gamma_t \to 0$

NLO results: exact and full DPA available

- perfect agreement of results achieved with
  - exact 1-loop amplitude
  - full DPA (fact. and non-fact. corrections)

  with $t\bar{t}$ result in NWA in the limit $\Gamma_t \to 0$
- full DPA reproduces exact result very well at physical Γ_t: only $\approx 4\%$ difference on $\Delta\sigma_{\rm NLO}$
  ➥ apply DPA also at NNLO QCD

[Buonocore, Devoto, Grazzini, SK, Lindert, Mazzitelli, Savoini (preliminary)]



Contributions from off-diagonal channels for off-shell $pp \to t\bar{t} + X$ at NNLO QCD

NNLO results: off-diagonal channels

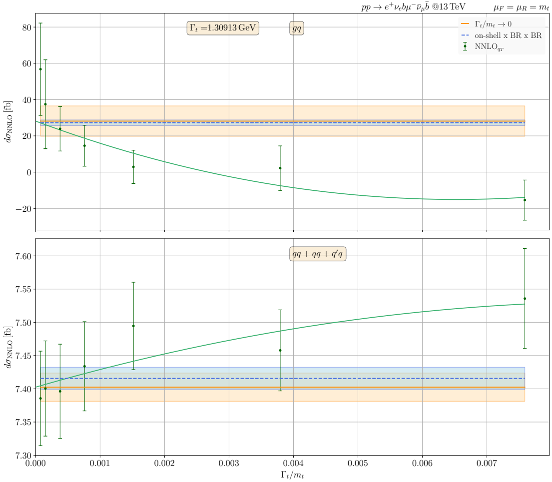
- full calculation exact, no approximation required
- non-trivial double extrapolation to get result for on-shell limit: $\mathrm{cut}_{q_T} \to 0$ and $\Gamma_t \to 0$
- validation through agreement with on-shell result
  - ➥ numerical complexity of relevant contributions comparable to diagonal channels

NNLO results: diagonal channels

- full calculation exact, apart from 2-loop amplitudes
- $\mathrm{cut}_{q_T} \to 0$ behaviour under good control
  - ➥ crucial check of our slicing method
- 2-loop amplitudes approximated in DPA
  - factorizable corrections widely validated
  - non-factorizable corrections not available yet

➥ not ready to present results yet . . .

[Buonocore, Devoto, Grazzini, SK, Lindert, Mazzitelli, Savoini (preliminary)]



Conclusions & Outlook

NLO SM corrections to associated top-quark production with decays

- off-shell calculations available (or at least feasible) for up to $2 \to 8$ scattering processes
  - typically dominated by NLO QCD (LO_1+NLO_1) prediction and the leading EW corrections (NLO_2)
  - enhancement of subleading contributions possible, e.g. due to opening of new topologies

➥ calculations also important in order to judge if subleading terms can eventually be neglected

NNLO QCD corrections to associated top-quark production

- first inclusive results available for $t\bar{t}H$ and $t\bar{t}W^{\pm}$ on-shell production with approximated 2-loop amplitudes
  - ➥ more detailed studies, also on differential level
  - ➥ similar strategies might be applicable also for other processes of this class
- 2-loop amplitudes for $2 \to 3$ processes are a very active field
  - ➥ first results with exact (at least leading-colour) amplitudes presumably not in the too far future (?)

New developments on top-quark pair production

- off-shell $t\bar{t}$ production at NNLO QCD with 2-loop amplitudes in DPA looks promising . . .
- recent progress towards $t\bar{t}j$ amplitudes [Badger et al. ('24)]: maybe NNLO QCD for $t\bar{t}j$ feasible soon (?)