EIC synergies: nuclear imaging at the EIC/UPCs

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**Diffraction in optics**

- **Diffraction**: in momentum-space the positions of the minima and maxima of diffraction pattern are determined solely by the target size \( R \).

Yuri V. Kovchegov, QUANTUM CHROMODYNAMICS AT HIGH ENERGY.
Diffraction in the High Energy Collisions
Diffractive vector meson production

High energy factorization:

1. $\gamma^* \rightarrow q\bar{q}$ splitting, wave function $\psi^\gamma(r, Q^2, z)$
2. $q\bar{q}$ dipole scatters elastically
3. $q\bar{q} \rightarrow J/\Psi$, wave function $\psi^V(r, Q^2, z)$

Diffractive scattering amplitude ($\sim$ Fourier transform of the spatial structure of target).

$$A^{\gamma^*p \rightarrow Vp} \sim \int d^2 b dz d^2 r \psi^\gamma \psi^V(r, z, Q^2) e^{-ib \cdot \Delta} N(r, x, b)$$

Impact parameter, $b$, is the Fourier conjugate of the momentum transfer, $\Delta \approx \sqrt{-t}$

**IP-Sat:** $N(r_T, b_T, x) = 1 - \exp (-r_T^2 F(r_T, x) T_p(b_T))$, accesses the spatial structure ($T_{p/A}$)

$$F(r_T, x) = \frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2)$$

$g(x, \mu^2)$, gluon density at $x$ and scale $\mu^2$ ($\mu^2 \sim \mu_0^2 + 1/r_T^2$).

In this work, we describe the dipole target interactions by CGC model. (See details in back-up)

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; Mäntysaari, Schenke, 1603.04349; Mäntysaari, 2001.10705
Coherent and incoherent processes

• **Coherent** \[ \sigma_{\text{coherent}} \sim |\langle \mathcal{A} \rangle_\Omega|^2 \]
  Target stays intact, (|initial state\rangle = |final state\rangle)
  Probes the average shape of the target.

• **Incoherent** \[ \sigma_{\text{incoherent}} \sim \langle |\mathcal{A}|^2 \rangle_\Omega - |\langle \mathcal{A} \rangle_\Omega|^2 \]
  Target breaks apart, (|initial state\rangle \neq |final state\rangle)
  Probes the variance of event-by-event initial state fluctuations in target structure.

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; Mäntysaari, Schenke, 1603.04349; Mäntysaari, 2001.10705
Accessing nuclear deformation at small $x$
Different deformation parameters control the geometric deformation at different length scale.

• Probe the nuclear geometric deformation by the diffractive process.

Generalized Woods-Saxon profile

$$
\rho(r, \Theta, \Phi) \propto \frac{1}{1 + \exp \left( \frac{[r - R(\Theta, \Phi)]/a}{1} \right)} , 
R(\Theta, \Phi) = R_0 \left[ 1 + \beta_2 \left( \cos \gamma Y_{20}(\Theta) + \sin \gamma Y_{22}(\Theta, \Phi) \right) + \beta_3 Y_{30}(\Theta) + \beta_4 Y_{40}(\Theta) \right]
$$

Taken from Giuliano’s slide
Large $\beta_2$ enhances the fluctuations of the configurations projected onto x-y plane.

$\beta_2$ enhances incoherent cross section at small $|t|$.

Multi-scale imaging: Nuclear deformations

\[ \beta_2, \beta_3, \beta_4 \] manifest themselves at different \(|t|\) regions (different length scales).

High $|t|$ region of $\gamma^* + A$ incoherent cross section probes sub-nucleon structures.


Illustration by G. Giacalone
Probing Nuclear Deformations in UPCs

Plot is from J. D. Brandenburg’s slide.
Beautiful interference pattern observed in UPCs by STAR people.

The interference effect is sensitive to the nuclear geometry.

STAR Sci. Adv. 9 (2023) no.1, eabq3903.
Double-slit interference in UPCs

\[
\frac{d\sigma_{p\rightarrow p^+p^-}}{d^2P_1dq_1dy_1dy_2} = \frac{1}{2(2\pi)^3 (Q^2 - M_V^2)^2 + M_V^2 \Gamma^2} f_{\rho\pi\pi}^2 \left\{ \int d\phi_{q_1} dB_1 dB_1 \langle M^i(y, q_1, B_1) M^{\dagger j}(y, q_1, B_1) \rangle \Omega P_1 P_1 \Theta(|B_1| - B_{\text{min},\Omega}) \right\}
\]

• The amplitude:

\[
M^i(x_1, x_2, q_1, B_1) = \int d^2 b_1 e^{-i q_1 \cdot b_1} \left[ \Phi_{A_1}(x_1, x_2, b_1 - B_1) + \Phi_{A_2}(x_2, x_1, b_1 - B_1) \right]
\]

• Subscripts \( A_1 \) and \( A_2 \) refer to the colliding nuclei. \( x_1 \) and \( x_2 \): Bjorken \( x \), \( b \): impact parameter of the photon-nucleus collision, \( B \): impact parameter of the nucleus-nucleus collision.

• The function \( F_{A_1} \) is the photon flux.

• Diffractive scattering amplitude

\[
\mathcal{A}_{\gamma^*p\rightarrow Vp} \sim \int d^2 b d\zeta d^2 r \psi_{\gamma^*} \psi^V(r, z, Q^2) e^{-ib \cdot \Delta} N(r, x, b)
\]

Our model nicely reproduces the $\cos(2\Delta\Phi)$ modulation.

In U+U, larger $\beta_2$ leads to slightly more pronounced $\cos(2\Delta\Phi)$ modulation.

STAR Sci. Adv. 9 (2023) no.1, eabq3903.
Interference in Au+Au and U+U

- In U+U, larger $\beta_2$ leads to flatter spectra (smaller radius). Larger $\beta_2$ has larger incoherent at low $q^2_\perp$, leads to the flatter $dN/dq^2_\perp$. Also the initial photon kT is more important.

STAR Sci. Adv. 9 (2023) no.1, eabq3903.
Probing isobar, Ru/Zr

<table>
<thead>
<tr>
<th>system</th>
<th>$R_0$ [fm]</th>
<th>$a_0$ [fm]</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>case1 (Ru+Ru)</td>
<td>5.09</td>
<td>0.46</td>
<td>0.16</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>case2 (Ru+Ru)</td>
<td>5.09</td>
<td>0.46</td>
<td>0.16</td>
<td>0.20</td>
<td>0.0</td>
</tr>
<tr>
<td>case3 (Ru+Ru)</td>
<td>5.09</td>
<td>0.46</td>
<td>0.06</td>
<td>0.20</td>
<td>0.0</td>
</tr>
<tr>
<td>case4 (Ru+Ru)</td>
<td>5.09</td>
<td>0.52</td>
<td>0.06</td>
<td>0.20</td>
<td>0.0</td>
</tr>
<tr>
<td>case5 (Zr+Zr)</td>
<td>5.02</td>
<td>0.52</td>
<td>0.06</td>
<td>0.20</td>
<td>0.0</td>
</tr>
</tbody>
</table>

$$R(\Theta, \Phi) = R_0 \left[ 1 + \beta_2 \left( \cos \gamma Y_{20}(\Theta) + \sin \gamma Y_{22}(\Theta, \Phi) \right) + \beta_3 Y_{30}(\Theta) + \beta_4 Y_{40}(\Theta) \right]$$

- The vector meson production in isobar UPCs is sensitive to the nuclear structures.
- "By eyes", the "full" Ru/Zr (case1/case5) is closest to data.

Spatial imaging of polarized deuterons

- Angular dependence of the coherent cross section in $e + d^1$ of transverse polarizations.
- Highlight the importance of polarized target in future EIC.

Summary

• Diffractive vector meson production can “see” the nuclear shape and fluctuations at different length scales!
• Vector meson productions in UPCs open up opportunities for investigations the nuclear structures.
• Spatial image of polarized target in the future.
Thanks for Your Attentions!
Back Up
Dipole-target scattering amplitude (CGC)

- The dipole amplitude $N$ can be calculated from Wilson line $V(x)$

$$N \left( b = \frac{x + y}{2}, r = x - y, x_p \right) = 1 - \frac{1}{N_c} \text{Tr} \left( V(x) V^\dagger(y) \right) \quad V(x) = P \exp \left( -ig \int dx - \frac{\rho(x^-, x)}{\nabla^2 + m^2} \right)$$

- Using MV model for Gaussian distribution of color charge $\rho$:

$$\langle \rho^a(b_\perp) \rho^b(x_\perp) \rangle = g^2 \mu^2(x, b_\perp) \delta^{ab} \delta^{(2)}(b_\perp - x_\perp)$$

$Q_s$: saturation scale, $Q_s/g^2\mu$ is a free parameter, $Q_s$ is determined from IP-Sat parametrization.

- Or, equivalently, factorize $\mu(x, b_\perp) \sim T(b_\perp)\mu(x)$

$N(r, x, b)$ accesses to the spatial structure of the target ($T_{p/A}$).

Schenke, etc.al. PhysRevLett.108.252301, PhysRevC.86.034908, Mäntysaari, Schenke, 1603.04349;
• Pb+Pb $dN_{ch}/d\eta$ data favors the small Nq case.
• $v_2 - p_T$ correlator in p+Pb is a promising observable.

eU cross section for $\rho$ production

Bmin distribution in UPCs

Proton geometry fluctuations

- Proton's event-by-event fluctuating density profile:
  \[ T_p(\mathbf{b}_⊥) = \frac{1}{N_q} \sum_{i=1}^{N_q} p_i T_q(\mathbf{b}_⊥ - \mathbf{b}_{⊥,i}), \quad P(\ln p_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{\ln^2 p_i}{2\sigma^2} \right]. \]

- The density profile of each spot is:
  \[ T_q(\mathbf{b}) = \frac{1}{2\pi B_q} e^{-b^2/(2B_q)} \]

- The spot positions \( \mathbf{b}_i \) are sampled from:
  \[ P(b_i) = \frac{1}{2\pi B_{qc}} e^{-b_i^2/(2B_{qc})} \]

Schenke, etc. all. PhysRevLett.108.252301, PhysRevC.86.034908, Mäntysaari, Schenke, 1603.04349;
Double-slit interference in UPCs

\[
\frac{d\sigma^{\rho \to \pi^+ \pi^-}}{d^2 P_\perp dq_\perp dy_1 dy_2} = \frac{1}{2(2\pi)^3 (Q^2 - M_V^2)^2 + M_V^2 \Gamma^2} \int \frac{d\phi_{q_\perp} dB_\perp \langle M^i(y, q_\perp, B_\perp) M_{\text{out}}^{\dagger}(y, q_\perp, B_\perp) \rangle \Omega P_\perp P_\perp \Theta(|B_\perp| - B_{\text{min}})}{f_{\rho \pi}}
\]

- The amplitude:

\[
M^i(x_1, x_2, q_\perp, B_\perp) = \int d^2 b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ \tilde{\mathcal{A}}(\mathbf{b}_\perp)_{A_1, x_1} \tilde{\mathcal{F}}_{A_2}^j(x_2, b_\perp - B_\perp) + \tilde{\mathcal{A}}(\mathbf{b}_\perp)_{A_2, x_2} \tilde{\mathcal{F}}_{A_1}^j(x_1, b_\perp + B_\perp) e^{-i\mathbf{q}_\perp \cdot \mathbf{B}_\perp} \right],
\]

\[
A^{\gamma* p \to V p} \sim \int d^2 b dz d^2 r \tilde{\mathcal{F}}_{A}^j(r, z, Q^2) e^{-i\mathbf{b} \cdot \Delta} N(r, x, \mathbf{b})
\]

Probing $^{20}\text{Ne}$ and $^{16}\text{O}$

$^{20}\text{Ne}$

$^{16}\text{O}$

Nucleon density distribution is taken from G. Giacalone.

- Incoherent cross section at small $|t|$ captures the deformation of the $^{20}\text{Ne}$.
- Significant difference between $^{20}\text{Ne}$ and $^{16}\text{O}$ diffractive cross sections is observed.

JIMWLK evolution to smaller $xp$

JIMWLK evolution: absorb quantum fluctuations at intermediate $x$ range as the color sources of smaller $x$.

- JIMWLK evolution doesn't wash out this effects.

H. Mantysaari, B. Schenke PRD, 98, 034013.
Yuri V. Kovchegov, QUANTUM CHROMODYNAMICS AT HIGH ENERGY
Probing protons at different resolutions

- The $\rho$ mesons probe proton fluctuations at large length scales.
- Large differences observed for $\rho$ productions between $Nq=3$ and $Nq=9$ MAPs.

Model parameters and the Exp. Data ($\gamma^* + p \rightarrow J/\psi + p^*$)

Parameterize proton shape ($T_p$)

- Number of hot spots $N_q$
- Proton size $B_{qc}$
- Hot spot size $B_q$
- Hot spot density fluctuations $\sigma$
- Min. distance between hot spots $d_{q,min}$
- Overall color charge density: $Qs(x)/g^2\mu$
- Infrared regulator $m$

- 7D parameter space; generated 1000 training points for the model emulator

• Incoherent-to-coherent ratio effectively suppresses model uncertainties from wave functions.

• At smaller $x_p$, nucleon is smoother, reduces the fluctuations, decreases Incoherent-to-coherent ratio.

• JIMWLK evolution doesn’t wash out difference between different $\beta_2$ ($\beta_2$ controls overall shape).

H.Mantysaari, B.Schenke, C. Shen and W. Zhao, [arXiv:2303.04866].
H.Mantysaari, B.Schenke  PRD, 98, 034013.
Degeneracy in the number of hot spots

- The likelihood of number of hot spots $N_q$ increases monotonously.
- Large $N_q$ partially compensated by large $Q_s$ fluctuations, $\sigma \propto \sqrt{N_q}$, "number of effective hot spots" $< N_q$
- Proton’s event-by-event fluctuating density profile:

$$T_p(b_{\perp}) = \frac{1}{N_q} \sum_{i=1}^{N_q} p_i T_q(b_{\perp} - b_{\perp,i}) , \quad P(\ln p_i) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{\ln^2 p_i}{2\sigma^2} \right].$$

MAP of fixed $N_q=3$ and $N_q=9$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>$N_q = 9$</th>
<th>$N_q = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ [GeV]</td>
<td>Infrared regulator</td>
<td>0.780</td>
<td>0.246</td>
</tr>
<tr>
<td>$B_{qc}$ [GeV$^{-2}$]</td>
<td>Proton size</td>
<td>3.98</td>
<td>4.45</td>
</tr>
<tr>
<td>$B_q$ [GeV$^{-2}$]</td>
<td>Hot spot size</td>
<td>0.594</td>
<td>0.346</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Magnitude of $Q_s$ fluctuations</td>
<td>0.932</td>
<td>0.563</td>
</tr>
<tr>
<td>$Q_s/(g^2\mu)$</td>
<td>$Q_s \Rightarrow$ color charge density</td>
<td>0.492</td>
<td>0.747</td>
</tr>
<tr>
<td>$d_{q,\text{Min}}$ [fm]</td>
<td>Min hot spot distance</td>
<td>0.265</td>
<td>0.254</td>
</tr>
<tr>
<td>$N_q$</td>
<td>Number of hot spots</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>$S$</td>
<td>Hydro normalization</td>
<td>0.1135</td>
<td>0.235</td>
</tr>
</tbody>
</table>

- The $N_q=3$ and $N_q=9$ have the different configurations at large length scales.
- “See” them by the different probes.

• Some parameters are well constrained.
• The 2D RMS proton radius $R_{rms} = \sqrt{2(B_{qc} + B_q)} \sim 0.6$ fm, which is consistent with the results in heavy-ion collisions.

Dipole-target scattering amplitude (CGC)

- The dipole amplitude $N$ can be calculated from Wilson line $V(x)$

$$N \left( b = \frac{x + y}{2}, r = x - y, x_p \right) = 1 - \frac{1}{N_c} \text{Tr} \left( V(x) V^\dagger(y) \right) \quad V(x) = P \exp \left( -ig \int dx - \frac{\rho(x^-, x)}{\nabla^2 + m^2} \right)$$

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Impact parameter, $b$, is the Fourier conjugate of the momentum transfer, $\Delta \approx \sqrt{-t}$

$N(r, x, b)$ dipole-target scattering amplitude.

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; Mäntysaari, Schenke, 1603.04349; Mäntysaari, 2001.10705
Dipole-target scattering amplitude (IP-Sat)

\[ N(r_T, b_T, x) = 1 - \exp \left( -r_T^2 F(r_T, x)T_p(b_T) \right) \text{ accesses to the spatial structure } (T_p/A) \]

\[ F(r_T, x) = \frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2). \quad x g(x, \mu^2), \text{ gluon density at } x \text{ and scale } \mu^2 \quad (\mu^2 \sim \mu_0^2 + 1/r_T^2). \]

\[ \mathcal{A}^{\gamma^*p \rightarrow Vp} \sim \int d^2b dz d^2r \psi^{\gamma*}\psi^V(r, z, Q^2) e^{-ib \cdot \Delta} N(r, x, b) \]

• Diffractive scattering amplitude is roughly proportional to Fourier transform of the spatial structure function of target (Tp/A).

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; Mäntysaari, Schenke, 1603.04349; Mäntysaari, 2001.10705
Probing isobar, Ru/Zr

- Differences of incoherent $J/\Psi$ productions cross section between case2 -- case6 are within 5%.
- The difference between case1 and others mainly comes from dmin.

H.Mantysaari, B.Schenke, C. Shen and W. Zhao, in progress.