Quantum simulation of jet evolution

--- based on arXivs:2002.09757, 2107.02225, 2208.06750, 2305.12490, 2307.01792 and ongoing works

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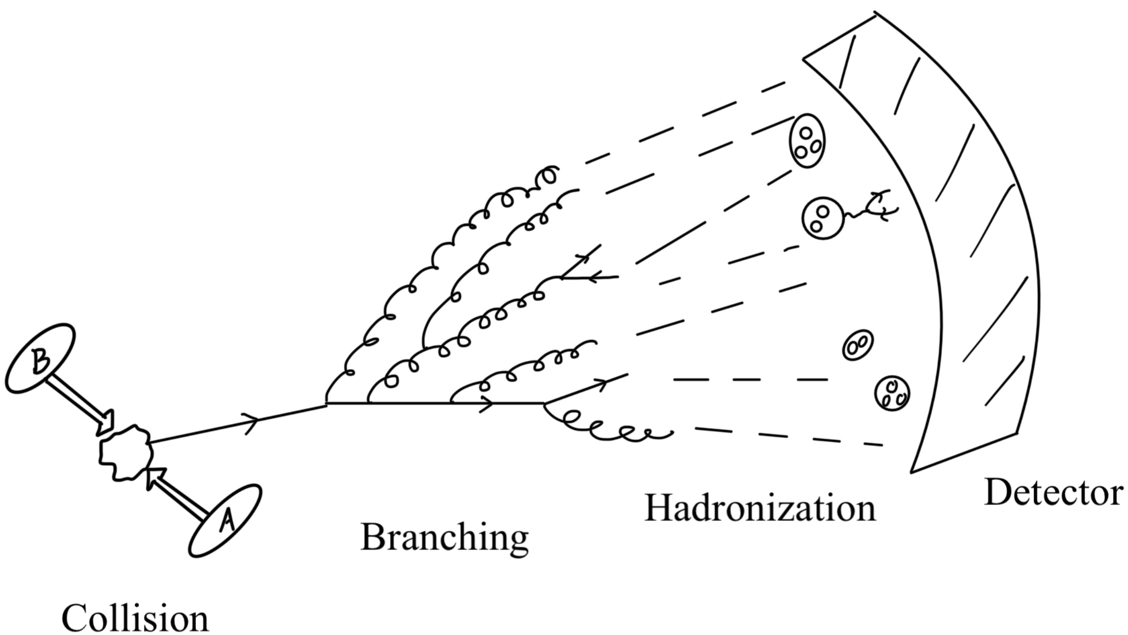
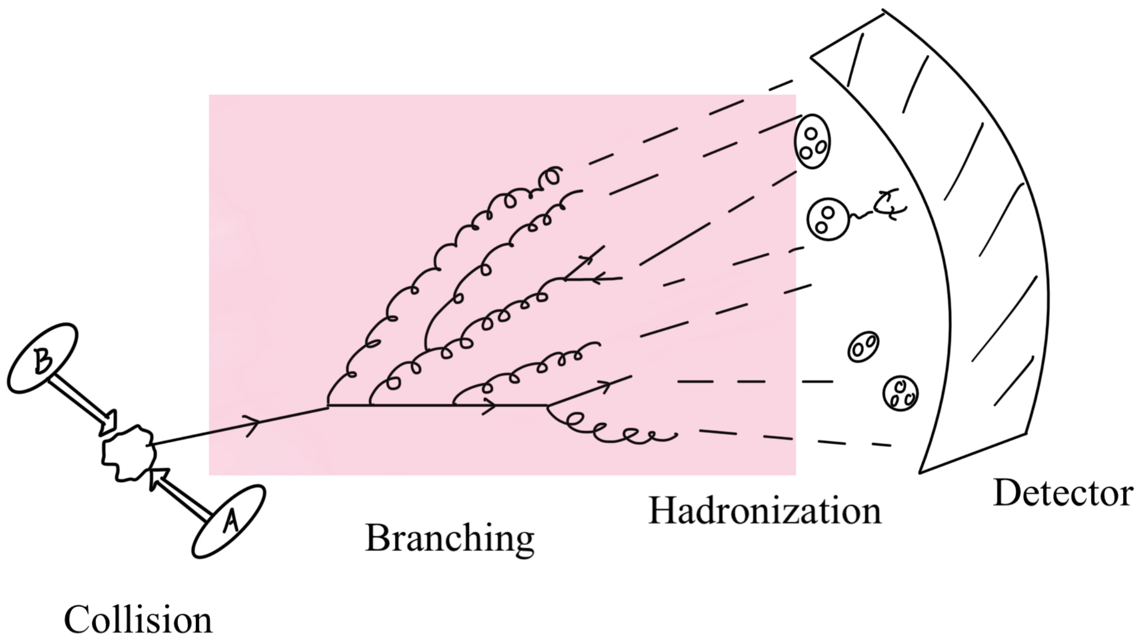
What is a jet?

- In high-energy collisions, a jet is a collimated beam of particles produced by the splitting of a common ancestor (quark or gluon).



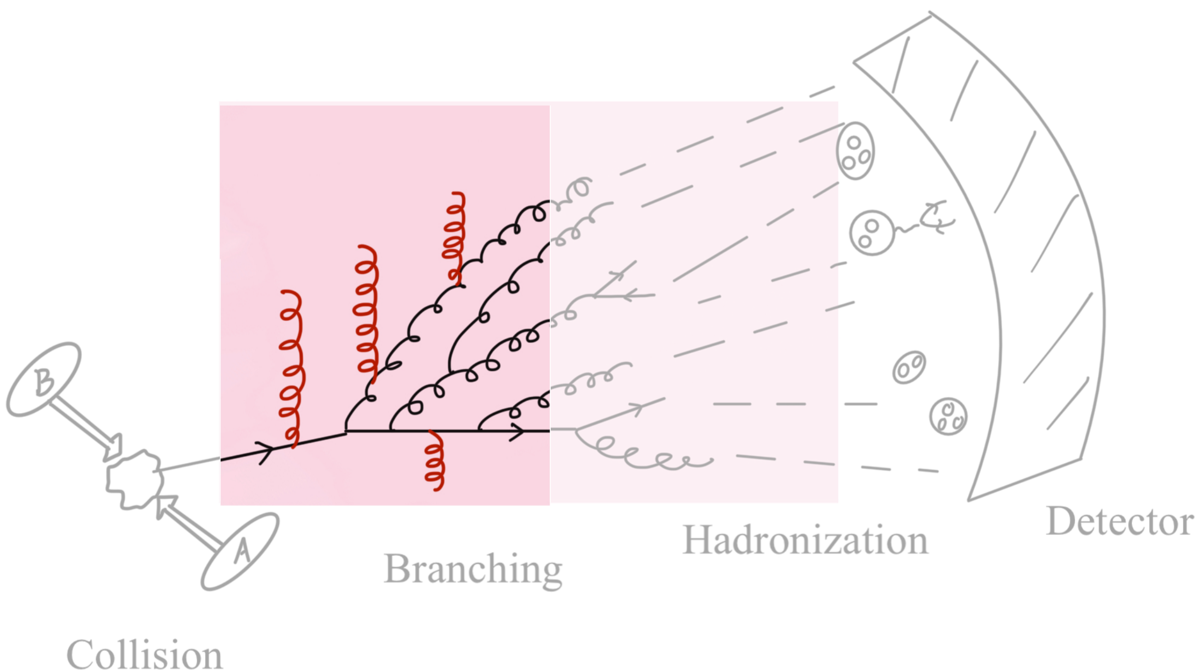
What is a jet?

- A probe of matter, a tool to understand interaction.



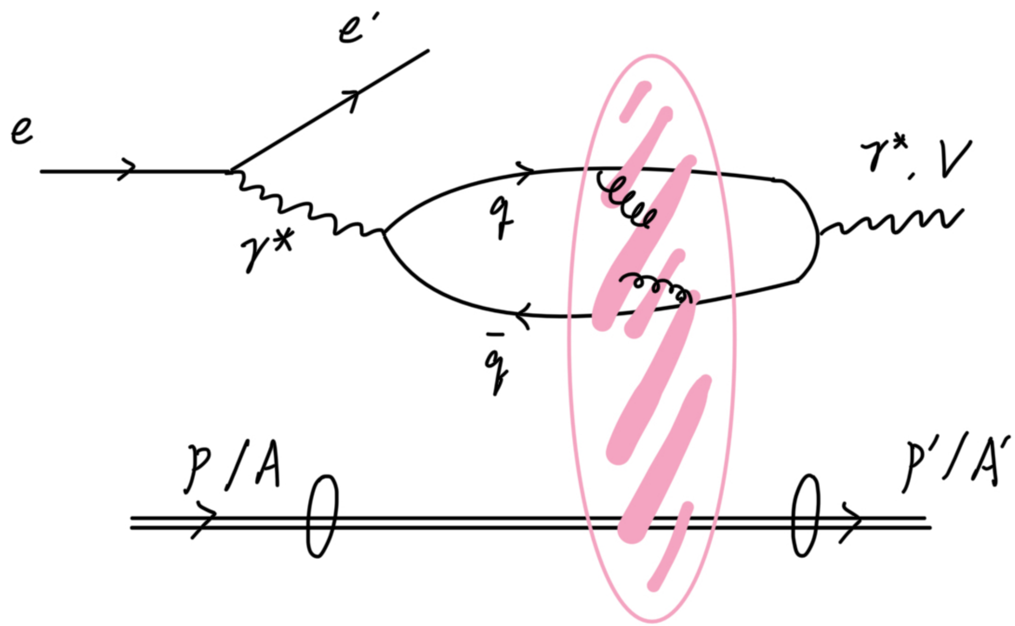
What is a jet?

- An energetic QCD state that evolves and interacts.



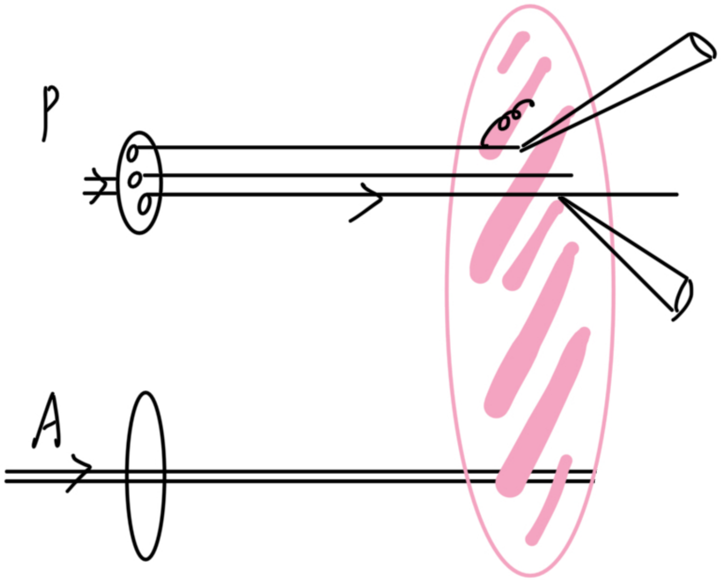
Jet evolution in medium

Deep inelastic scattering e+p/A



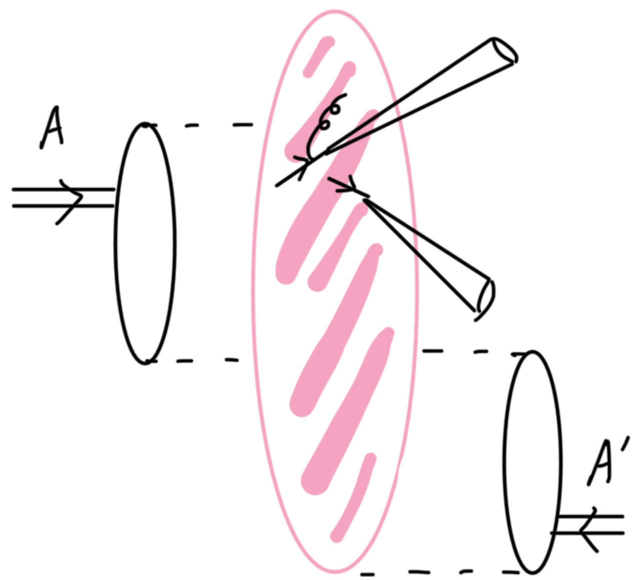
Jet evolution in medium

Proton nucleus scattering p+A



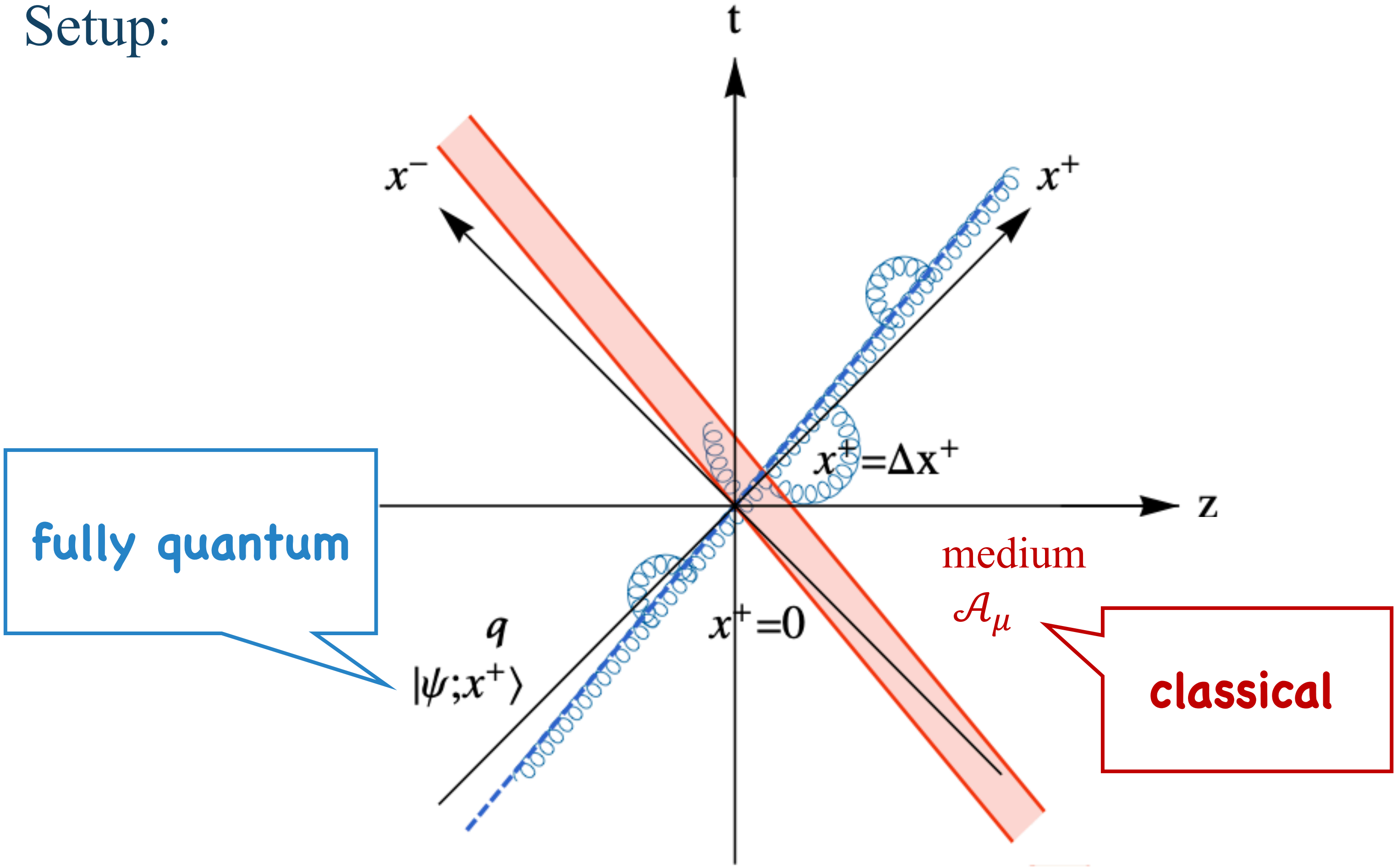
Jet evolution in medium

Heavy ion collisions A+A



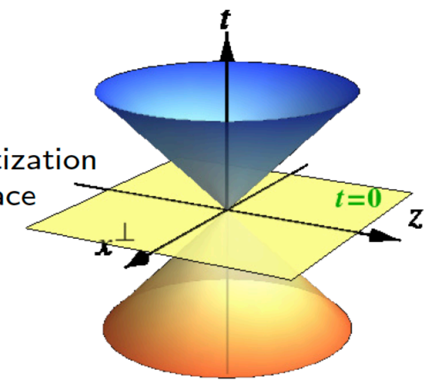
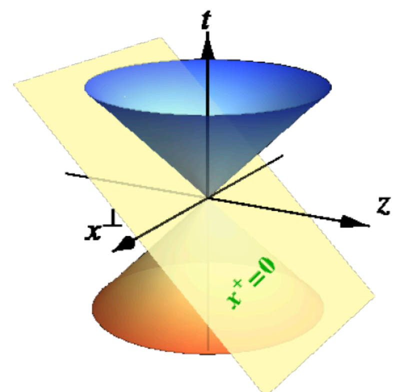
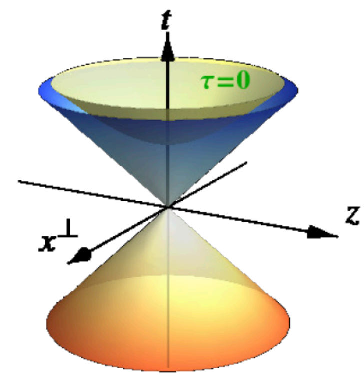
Jet evolution in medium

- Setup:



Time-dependent Basis Light-Front Quantization (tBLFQ)

- **Light-front dynamics**

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$

Time-dependent Basis Light-Front Quantization (tBLFQ)

- **Hamiltonian formalism**

  The state obeys the time-evolution equation

$$\frac{1}{2}P^{-}(x^{+})|\psi(x^{+})\rangle = i\frac{\partial}{\partial x^{+}}|\psi(x^{+})\rangle$$

A nonperturbative treatment:

$$|\psi(x^{+})\rangle = \mathcal{T}_{+}\exp\left[-\frac{\mathrm{i}}{2}\int_{0}^{x^{+}} dz^{+}P^{-}(z^{+})\right]|\psi(0)\rangle$$

$$= \lim_{n\to\infty}\prod_{k=1}^{n}\mathcal{T}_{+}\exp\left[-\frac{\mathrm{i}}{2}\int_{x_{k-1}^{+}}^{x_{k}^{+}} dz^{+}P^{-}(z^{+})\right]|\psi(0)\rangle$$

$$\delta x^{+} = x^{+}/n,\ x_{k}^{+} = k\delta x^{+}(k = 0,1,2,\ldots,n)$$

Time-dependent Basis Light-Front Quantization (tBLFQ)

- **Basis representation**

$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle$$

Operators

$$\begin{pmatrix} \langle 1|U|1\rangle & \langle 1|U|2\rangle & \dots & \langle 1|U|n\rangle \\ \langle 2|U|1\rangle & \langle 2|U|2\rangle & \dots & \langle 2|U|n\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n|U|1\rangle & \langle n|U|2\rangle & \dots & \langle n|U|n\rangle \end{pmatrix}$$

State

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

Jet evolution by tBLFQ

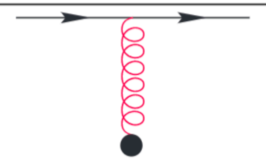
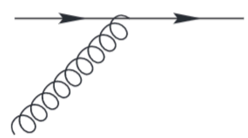
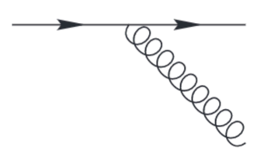
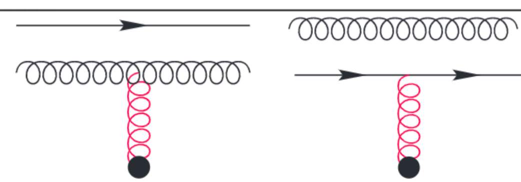
- **The light-front Hamiltonian**

$$\mathcal{L} = -\frac{1}{4}\mathrm{F}_{\mathrm{a}}^{\mu\nu} F_{\mu\nu}^{a} + \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\psi$$

with $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$.

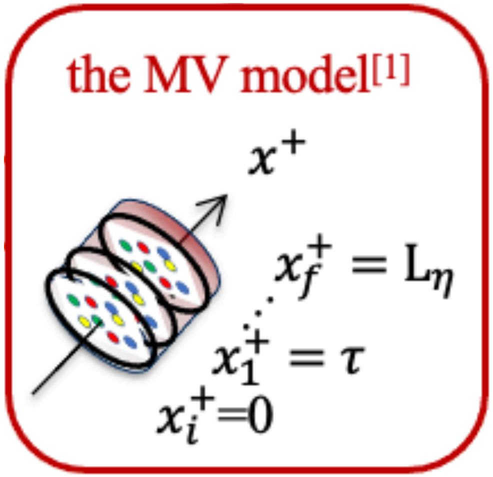
$$\Downarrow$$

In the $|q\rangle + |qg\rangle$ space, $\mathrm{P}^-(x^+) = \mathrm{P}^-_{KE} + V_{qg} + V_{\mathcal{A}}(x^+)$

Fock sector	$\lvert q\rangle$	$\lvert qg\rangle$
$\langle q\rvert$		
$\langle qg\rvert$		

Jet evolution by tBLFQ

- **The background field**, $\mathcal{A}(x^+, \vec{x}_\perp)$, is a classical gluon field described by the color glass condensate[1]



- Color charges are stochastic variables

$$< \rho_a(x^+, \vec{x}_\perp) \rho_b(y^+, \vec{y}_\perp) >$$
$$= g^2 \tilde{\mu}^2 \delta_{ab} \delta^2(\vec{x}_\perp - \vec{y}_\perp) \delta(x^+ - y^+)$$

- The color field is solved from

$$(m_g^2 - \nabla_\perp^2) \mathcal{A}_a^-(x^+, \vec{x}_\perp) = \rho_a\,(x^+, \vec{x}_\perp)$$

where m_g is a chosen infrared regulator.

- Saturation scale: $Q_s^2 = C_F\,(g^2 \tilde{\mu})^2 L_\eta / (2\pi)$

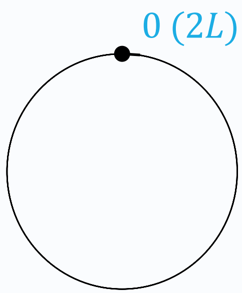
[1]L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

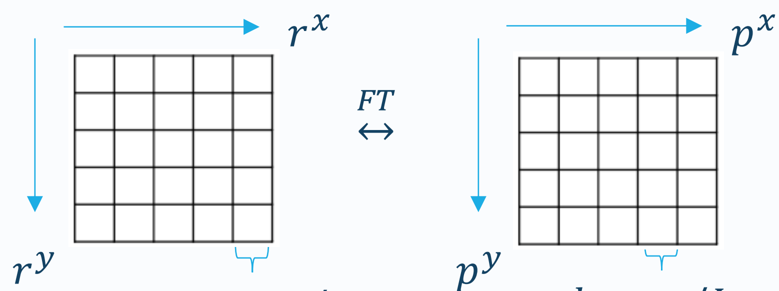
Jet evolution by tBLFQ

- **Basis representation:** discrete momentum states

$$P^-_{\text{KE}}|\beta\rangle = P^-_{\beta}|\beta\rangle, \beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$$

$$|q\rangle: |\beta_q\rangle; \quad |qg\rangle: |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$$





$a_\perp = \mathrm{L}_\perp / N_\perp$ $\quad d_p = \pi / L_\perp$

- $x^- = [0, 2L]$
- $p_l^+ = \frac{2\pi}{L} k_l^+$

  $k_q^+ = \frac{1}{2}, \frac{3}{2}, \dots, K$

  $k_g^+ = K - \frac{1}{2}, K - \frac{3}{2}, \dots, 1$

- $r_l^\perp = [-N_\perp, \dots, N_\perp - 1]\, a_\perp$
- $p_l^\perp = \frac{2\pi}{2L_\perp} k_l^\perp$

  $k_l^\perp = -N_\perp, \dots, N_\perp - 1$

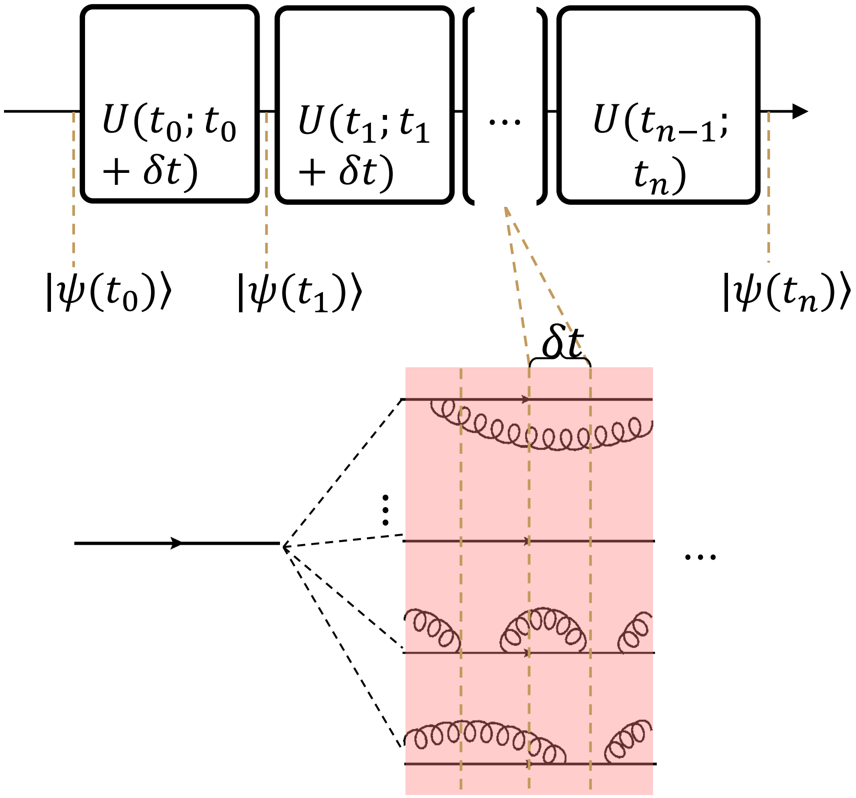
Basis size: $N_{tot} = (2N_\perp)^2 \times 2 \times 3 + \lfloor K \rfloor \times (2N_\perp)^4 \times 4 \times 24$

Jet evolution by tBLFQ

- **Solve the time-evolution equation**

$$|\psi; x^+\rangle \quad = \mathrm{U}(0;\mathrm{x}^+)|\psi; 0\,\rangle$$

$$\mathrm{U}(0;\mathrm{x}^+) =$$
$$\mathcal{T}_+ \exp\{-\tfrac{\mathrm{i}}{2}\int_0^{x^+} dz^+ P^-(z^+)\}$$
$$= \prod_{k=1}^{n} \mathcal{T}_+ \exp\left\{-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right\}$$



Classical→Quantum simulation

Classical simulation

- 1st tBLFQ: $|e\rangle + |e\gamma\rangle$

  Phys.Rev.D 88 (**2013**) 065014, X. Zhao, A. Ilderton, P. Maris, J. P. Vary

  ⋮

- **1st tBLFQ in QCD: $|q\rangle$**

  Phys.Rev.D 101(**2020**)7, 076016, ML, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin and J. P. Vary

- $|q\rangle + |qg\rangle$

  Phys.Rev.D 104 (**2021**) 5, 056014, ML, T. Lappi and X. Zhao;

  Phys.Rev.D 108 (**2023**) 3, 3, ML, T. Lappi, X. Zhao and C. A. Salgado

Quantum simulation

- Quantum strategy

  Eur.Phys.J.C 81 (**2021**) 10, 862, J. Barata and C. A. Salgado

- Nuclear scattering

  Phys.Rev.A 104 (**2021**) 1, 012611, W. Du, J. P. Vary, X. Zhao and W. Zuo

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- $|q\rangle$

  arXiv:**24**04.00819, S. Wu, W. Du, X. Zhao and J. P. Vary

Classical→Quantum simulation

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- $|q\rangle$
  arXiv:**24**04.00819, S. Wu, W. Du, X. Zhao and J. P. Vary

Quantum simulation algorithm

1. Define problem Hamiltonian
2. Basis encoding
3. Prepare initial states
4. Time evolution
5. Measurement

Quantum simulation algorithm

1. Define problem Hamiltonian ✓

   Simplifications in QS:
   - Non-abelian SU(2) color
   - Spin-non-flip transition $q(\uparrow) \leftrightarrow q(\uparrow) + g(\uparrow)$
2. Basis encoding
3. Prepare initial states
4. Time evolution
5. Measurement

Qubit encoding of basis states

- **Binary representation**

basis state **qubit state**

$$|\beta_\psi\rangle \rightarrow |\zeta\rangle \otimes \underbrace{\left(|p_g^x\rangle\, |p_g^y\rangle\, |c_g\rangle \right)}_{|g\rangle} \otimes \underbrace{\left(|p_q^x\rangle\, |p_q^y\rangle\, |c_q\rangle \right)}_{|q\rangle}$$

Discrete longitudinal modes: $\#[K]$

$$\begin{aligned}
&|\mathrm{q}\rangle, k_q^+ = K && \mapsto |\zeta\rangle = |0\rangle \\
&|\mathrm{qg}\rangle, k_q^+ = K-1, k_g^+ = 1 && \mapsto |\zeta\rangle = |1\rangle \\
&|\mathrm{qg}\rangle, k_q^+ = K-2, k_g^+ = 2 && \mapsto |\zeta\rangle = |2\rangle \\
&\qquad\vdots \\
&|\mathrm{qg}\rangle, k_q^+ = \tfrac{1}{2}, k_g^+ = K-1/2 && \mapsto |\zeta\rangle = |K-1/2\rangle
\end{aligned}$$

Qubit encoding of basis states

- **Binary representation**

basis state **qubit state**

$$|\beta_\psi\rangle \rightarrow |\zeta\rangle \otimes \underbrace{\left(|p_g^x\rangle\, |p_g^y\rangle\, |c_g\rangle \right)}_{|g\rangle} \otimes \underbrace{\left(|p_q^x\rangle\, |p_q^y\rangle\, |c_q\rangle \right)}_{|q\rangle}$$

Discrete transverse modes: $\#(2N_\perp)^4$

$$\begin{aligned}
k_l^x &= 0 && \mapsto |p_l^x\rangle = |0\ldots00\rangle \\
k_l^x &= 1 && \mapsto |p_l^x\rangle = |0\ldots01\rangle \\
k_l^x &= 2 && \mapsto |p_l^x\rangle = |0\ldots10\rangle \\
&\vdots \\
k_l^x &= 2N_\perp - 1 && \mapsto |p_l^x\rangle = |1\ldots11\rangle
\end{aligned}$$

Qubit encoding of basis states

- **Binary representation**

basis state **qubit state**

$$|\beta_\psi\rangle \rightarrow |\zeta\rangle \otimes \underbrace{\left(|p_g^x\rangle \, |p_g^y\rangle \, |c_g\rangle \right)}_{|g\rangle} \otimes \underbrace{\left(|p_q^x\rangle \, |p_q^y\rangle \, |c_q\rangle \right)}_{|q\rangle}$$

Color modes: #2×3

$c_q = 0 \quad \mapsto |c_q\rangle = |0\rangle$

$c_q = 1 \quad \mapsto |c_q\rangle = |1\rangle$

$c_g = 0 \quad \mapsto |c_g\rangle = |00\rangle$

$c_g = 1 \quad \mapsto |c_g\rangle = |01\rangle$

$c_g = 2 \quad \mapsto |c_g\rangle = |10\rangle$

Qubit encoding of basis states

- **Binary representation**

basis state **qubit state**

$$|\beta_\psi\rangle \to |\zeta\rangle \otimes \underbrace{\left(|p_g^x\rangle\,|p_g^y\rangle\,|c_g\rangle \right)}_{|g\rangle} \otimes \underbrace{\left(|p_q^x\rangle\,|p_q^y\rangle\,|c_q\rangle \right)}_{|q\rangle}$$

$$\sim \lceil K \rceil (2N_\perp)^4 \left\{ \begin{array}{c} 0 \\ 1 \\ 2 \\ \vdots \\ N_{tot}-1 \end{array} \right. \quad \mapsto \quad \underbrace{\begin{array}{c} 00 \quad \ldots\ 00 \\ 00 \quad \ldots\ 01 \\ 00 \quad \ldots\ 10 \\ \vdots \\ 11 \quad \ldots\ 11 \end{array}}_{n_Q \sim \mathbf{log}\,\lceil K \rceil + \mathbf{4}\,\mathbf{log}(2N_\perp)}$$

Quantum simulation algorithm

1. Define problem Hamiltonian ✓
2. Basis encoding ✓
   Binary representation: $n_Q \sim \log N_{tot}$
3. Prepare initial states
4. Time evolution
5. Measurement

Quantum simulation algorithm

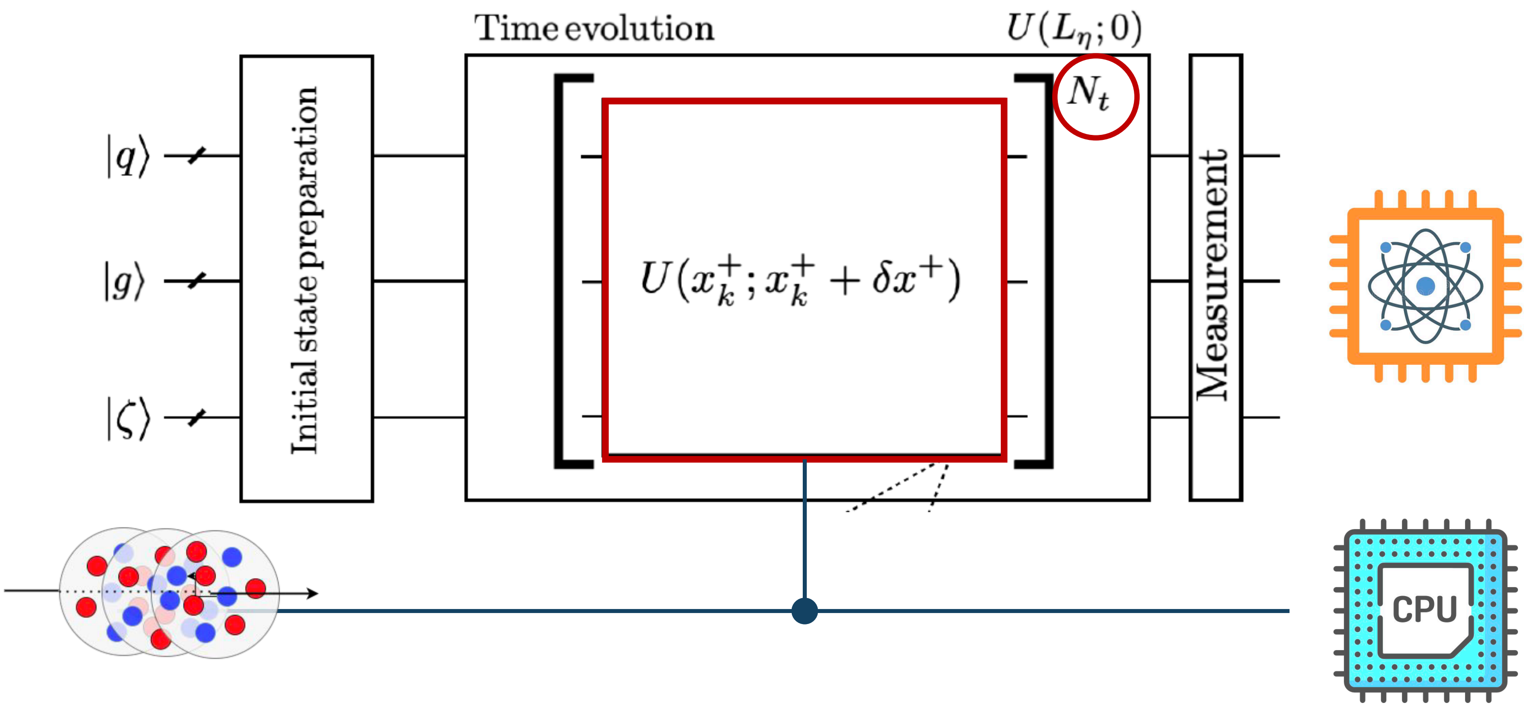
1. Define problem Hamiltonian ✓
2. Basis encoding ✓
3. Prepare initial states ✓

   $|q\rangle$: $\{p_q^x, p_q^y\} = \{0,0\}$, $\quad |c_q\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

4. Time evolution
5. Measurement

Time evolution

Level I. Trotterization in time



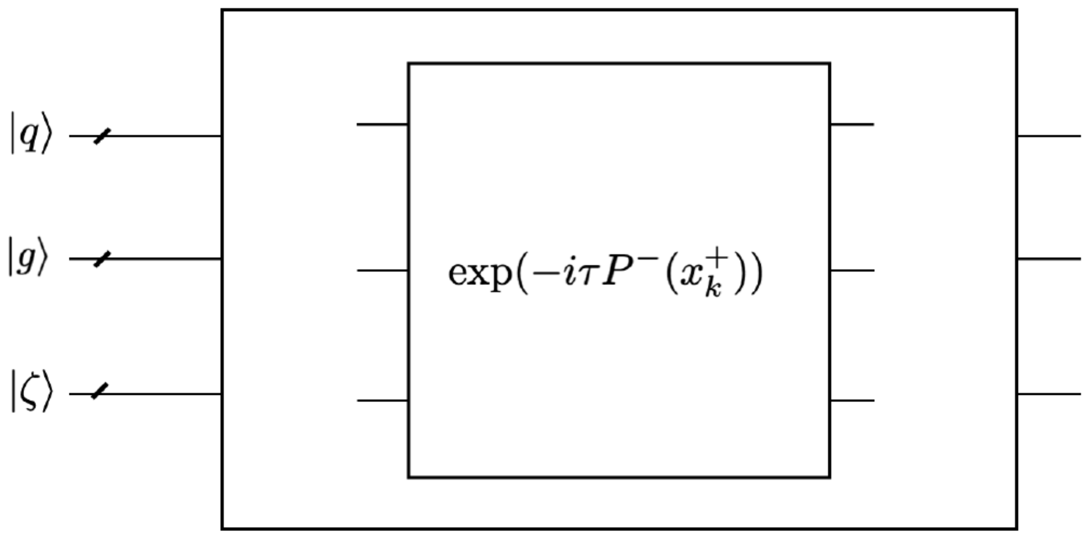
✓ tBLFQ, non-perturbative

Time evolution

Level II. Algorithm within each timestep

Option i. Direct exponentiation

dense H → Pauli strings → Quantum Gates



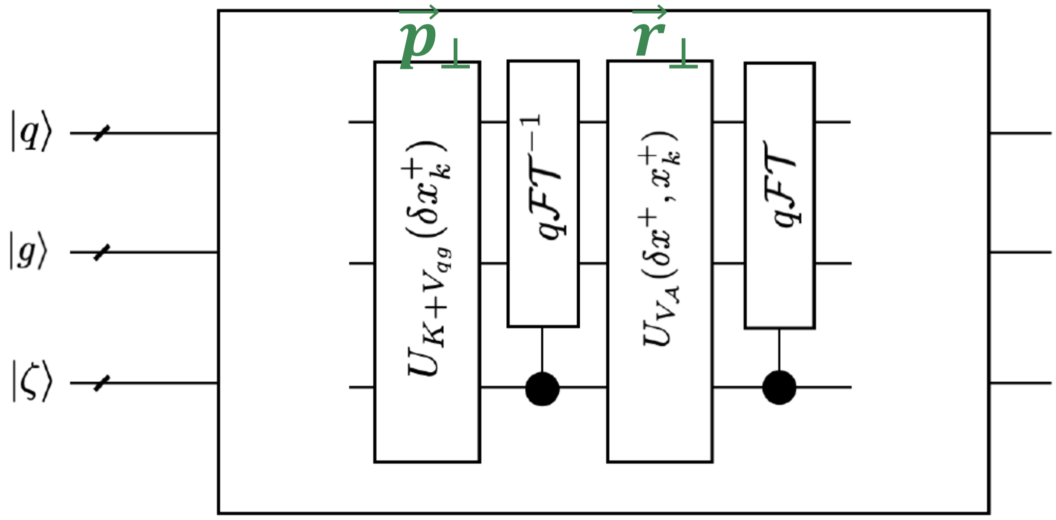
- feasible only for small-sized problem

Time evolution

Level II. Algorithm within each timestep

Option ii. Alternating exponentiation

$$\textit{sparser } H \xrightarrow{\textit{faster}} \textit{Pauli strings} \rightarrow \textit{Quantum Gates}$$



✓ more efficient

Quantum simulation algorithm

1. Define problem Hamiltonian ✓
2. Basis encoding ✓
3. Prepare initial states ✓
4. Time evolution ✓

   Trotterization + alternating mixed space algorithm
5. Measurement

Measurement

- **State collapsing**

$$|\psi; x^{+}\rangle = \sum_{\beta} c_{\beta}(x^{+})|\beta\rangle$$

Quantum state

Amplitude

$$\begin{pmatrix} c_1 \\ \vdots \\ c_i \\ \vdots \\ c_n \end{pmatrix}$$

Classical state

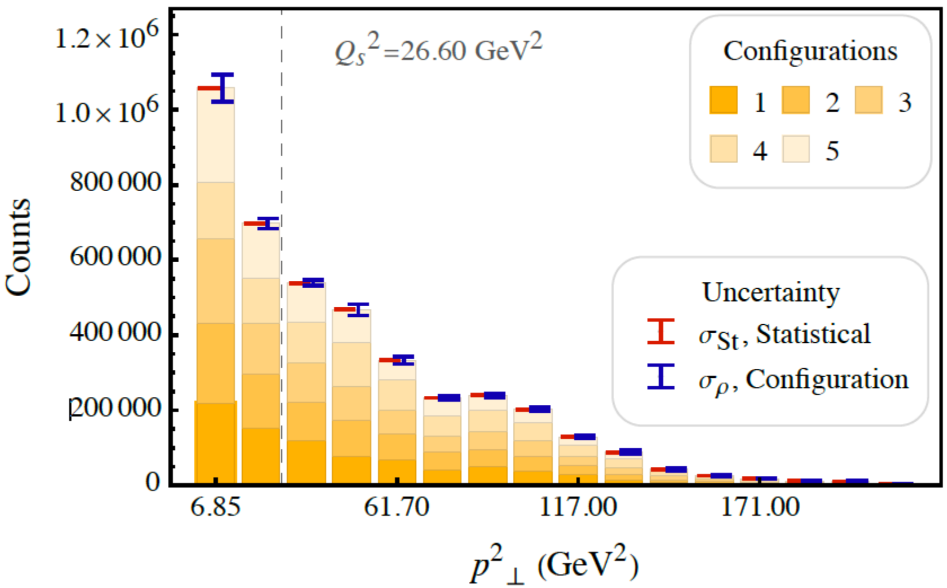
$$P_i = |c_i|^2$$

$$\begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

Measurement

- **State collapsing**

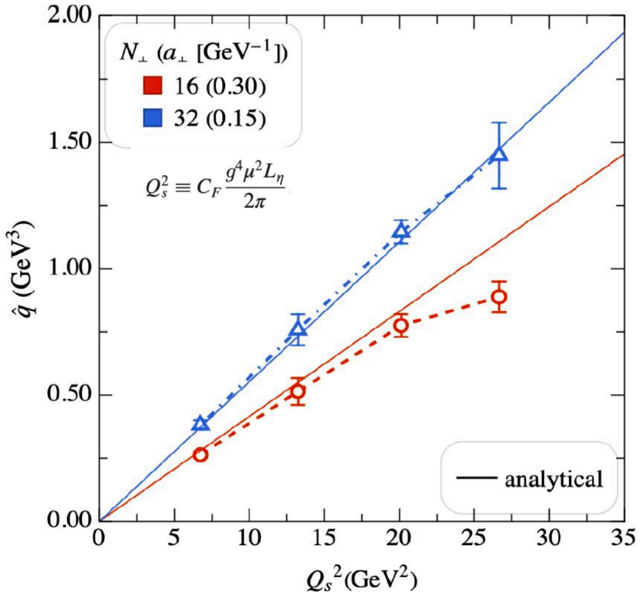
  Multiple shots, histogram

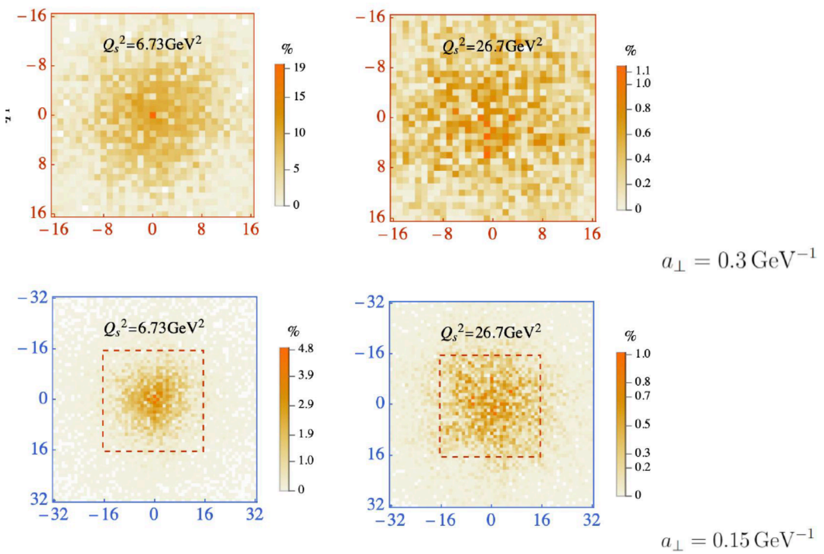


Simulation result

- **Fock space $|q\rangle$**

Jet quenching parameter: $\hat{q} = \dfrac{\Delta\langle p_\perp^2(x^+)\rangle}{\Delta x^+} = \langle \boldsymbol{p}_\perp^2\rangle / L_\eta$ Eikonal analytical: $\sim Q_s^2/L_\eta$





$p_\perp = [-\pi/a_\perp, \pi/a_\perp]$

$n_Q = 10{,}12$

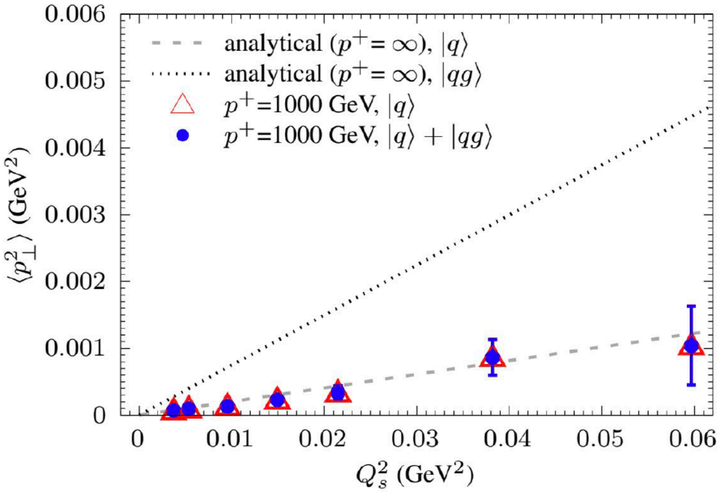
Simulation result

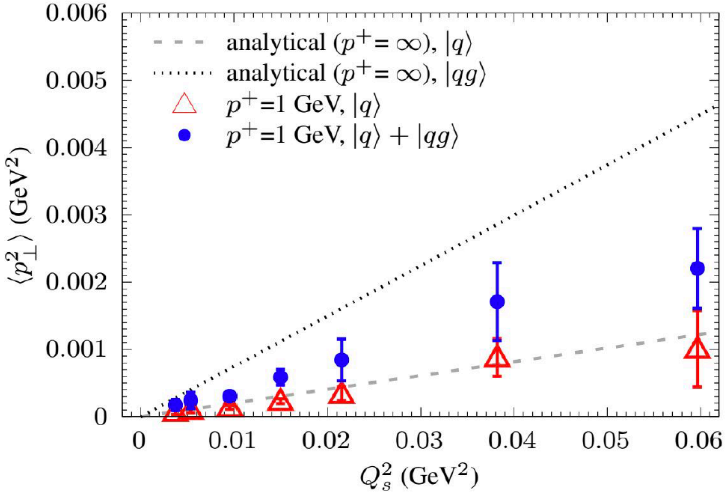
- **Fock space $|q\rangle + |qg\rangle$**

Momentum broadening:

$$\langle p_\perp^2 \rangle = \mathcal{P}_{|q\rangle} \langle p_\perp^2 \rangle_{|q\rangle} + \mathcal{P}_{|qg\rangle} \langle p_\perp^2 \rangle_{|qg\rangle}$$

$$\hat{q}_{Eik}(x = a_\perp m_g/\pi, N_\perp = 1)\big|_{\text{on lattice}} = C_F g^4 \tilde{\mu}^2 \frac{1}{(2\pi)^2}\left[\frac{2}{(x^2+1)^2} + \frac{2}{(x^2+2)^2}\right] \sim Q_s^2/L_\eta$$



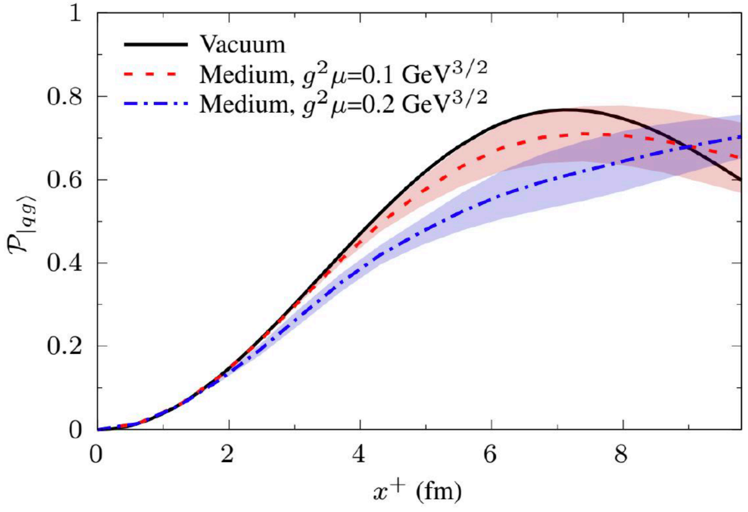


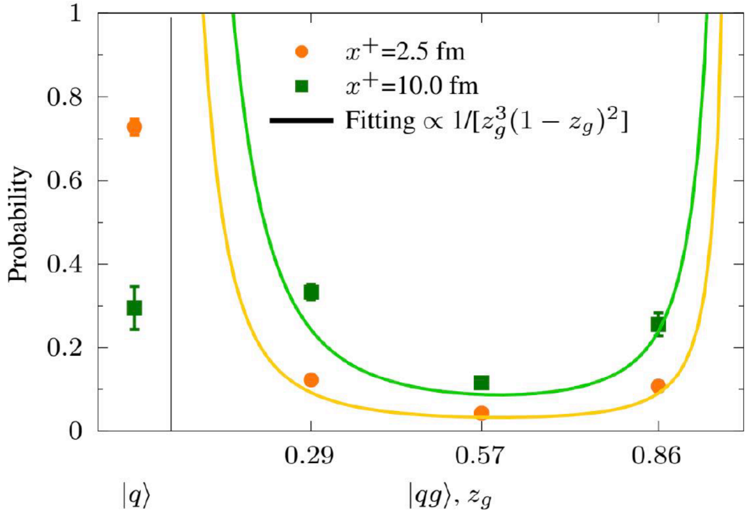
$n_Q = 9$

Simulation result

- **Fock space $|q\rangle + |qg\rangle$**

Dynamical evolution of the jet involving quantum interference (error from stochastic medium)



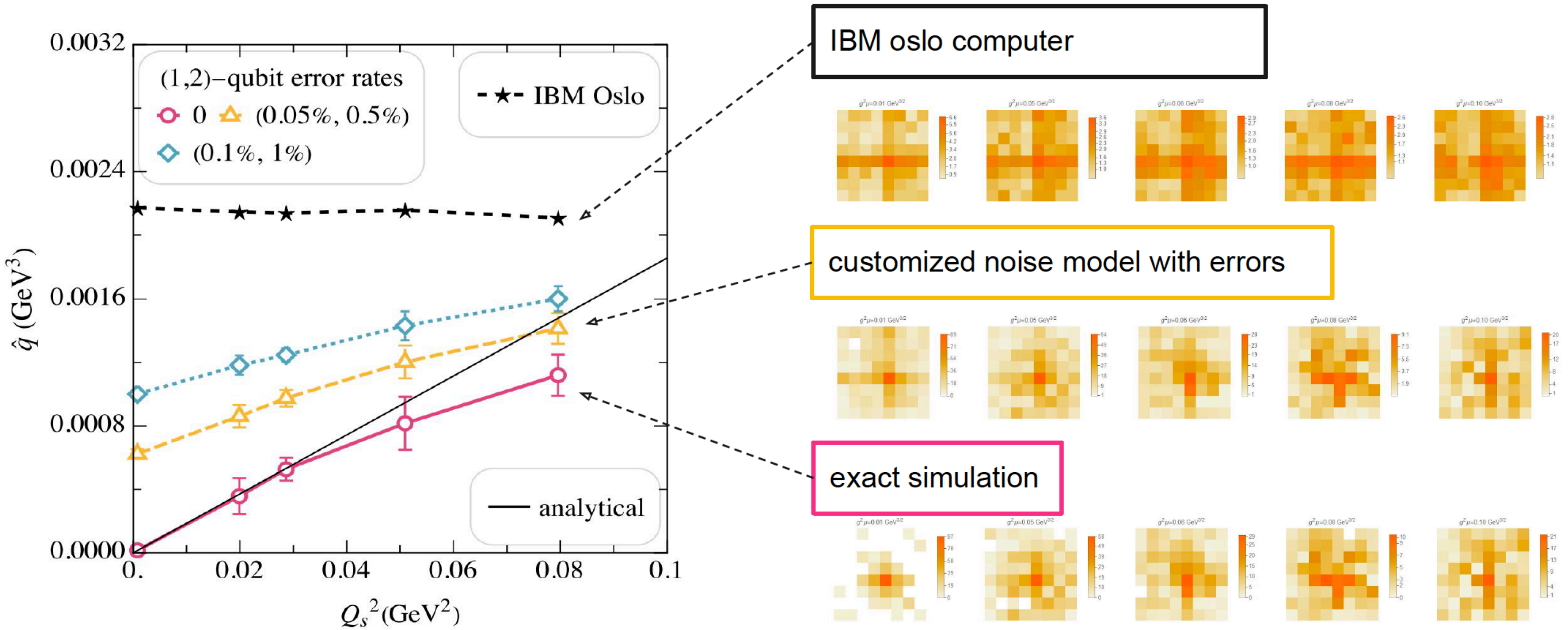


$n_Q = 9$

Real quantum devices

- **Quantum noise**

For a minimized problem (Fock |q>, 64 states, 4 config avg)

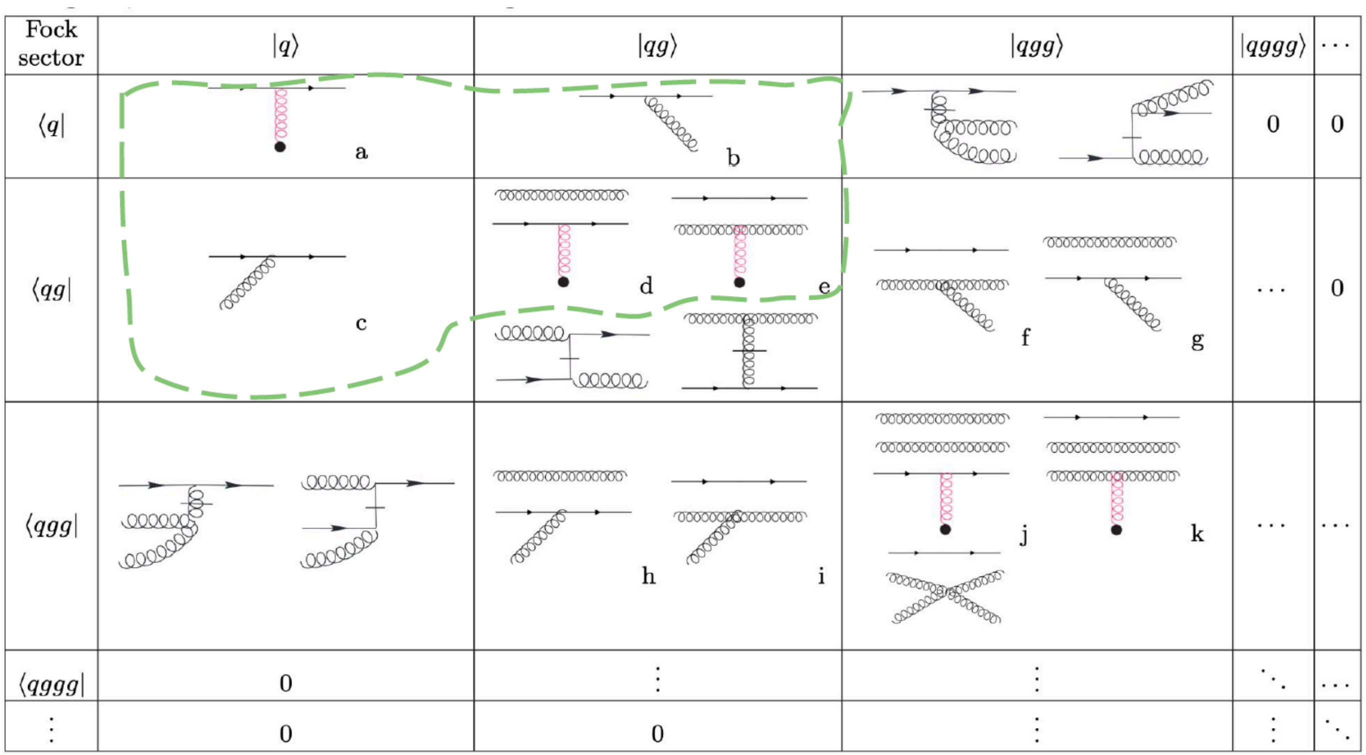


$n_Q = 6$

Ongoing works

- **Towards quantum advantages**

Many-body quantum advantage for |q + x g> Fock sector



Summary

- **Non-perturbative and real-time**: we studied multi-particle jet evolution in a medium using tBLFQ approach on quantum simulator
- **Towards quantum advantages**: problem complexity reduced to linear in particle number and logarithmic in momentum modes
- **Physically meaningful:** despite a small model space, we can study jet evolution with quantum simulation algorithm

Thank you!