Heavy Quarkonium Propagation in the QGP

LHC Physics 2024

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5 June 2024

Introduction

Motivation: to probe the medium formed in heavy ion collisions

- the medium formed in current heavy ion collision experiments provides an unprecedented window into the Universe in the first moments after the Big Bang
- energy densities are possibly sufficiently high to create a deconfined quark gluon plasma (QGP) in which quarks and gluons propagate freely
- due to their large mass and short formation times, heavy quarks, and specifically their bound states, provide ideal probes of the QGP
- Matsui and Satz[1] postulated heavy quarkonium suppression (a reduction in the yields of heavy quark-antiquark states states in heavy ion collisions relative to proton-proton collisions) as a signature that such a deconfined medium is indeed created

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[1] Phys. Lett. B178, 416 (1986)

Theory Approaches

Plethora of Models and Approaches

- Duke-MIT approach, Munich-KSU approach, Nantes model, parton-hadron string dynamics, Saclay model, Santiago comover interaction model, statistical hadronization model, Texas A&M model, Tsingua model, cold nuclear matter effects

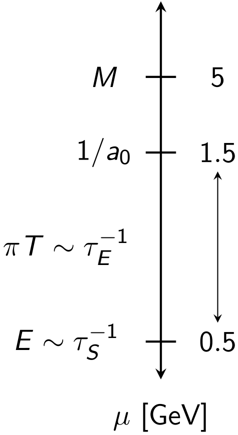
Different Evolution Equations

- Lindblad equation, Langevin equation, Boltzmann equation

Theory Status summarized in recent publication

- Eur. Phys. J. A 60 (2024) 4, 88 (Andronic, et. al.)

Physical Setup



- hierarchically ordered scales of the problem are heavy quark mass M, (inverse) Bohr radius a_0 of bound state, (π times) medium temperature T and binding energy E of state
- dimensionful quantities of combined system define the system intrinsic time scale τ_S, the environment correlation timescale τ_E and relaxation time $\tau_R \sim a_0^{-2} (\pi T)^{-3}$

Method

- theory tools:[2]
  - Open Quantum Systems: allows for the rigorous treatment of a quantum system of interest coupled to and evolving out of equilibrium with an environment
  - Effective Field Theories: allow to systematically exploit hierarchies of scale in physical systems to isolate contributions from the relevant scales
- use these tools to derive an evolution equation describing the in-medium evolution of heavy quarkonium
- use computational tools to solve the resulting equations and extract observables of interest for comparison against experiment including the nuclear modification factor R_{AA} and the elliptic flow v_2

[2] see references in Eur. Phys. J. A 60 (2024) 4, 88 (Andronic, et. al.)

Hierarchies and Simplifying Assumptions

quantum Brownian motion

for

$$\tau_R,\, \tau_S \gg \tau_E,^{3}$$

where τ_R, τ_S, and τ_E are the relaxation, system intrinsic, and environment correlation time scales, respectively, the system realizes **quantum Brownian motion**

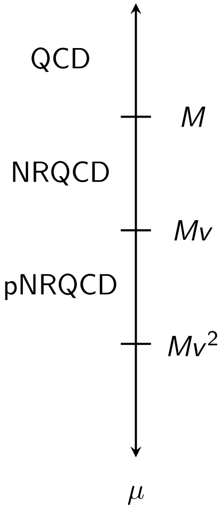
Simplifying Approximations

hierarchy of scales allows for two simplifying approximations:

- **Born approximation**: quarkonium has little effect on the medium at time scales of interest; density matrix factorizes
- **Markov approximation**: only the state of the quarkonium at time t is necessary to describe its evolution at time t

[3]in lower temperature regime, $\tau_R \gg \tau_S, \tau_E$ is realized and system is in quantum optic limit

Nonrelativistic EFTs of QCD



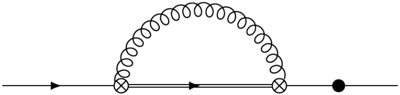
- due to heavy quark mass M, the relative velocity in a heavy-heavy bound state is small: i.e., $v \ll 1$ and the system is nonrelativistic
- integrating out the hard scale M gives rise to NRQCD
- integrating out the soft scale Mv gives rise to pNRQCD; degrees of freedom are color singlet (heavy quarkonium) bound states and color octet (unbound, open flavor) scattering states interacting with gluons at the ultrasoft scale Mv^2

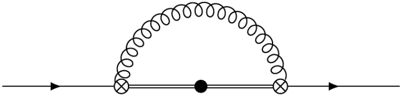
Diagrammatic Evolution of $\rho_s(t)$[4]

singlet evolution given by

$$\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = -i\,[h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t))$$

where

$\Sigma_s \rho_s(t) \sim$ 

$\Xi_{so}(\rho_o(t)) \sim$ 

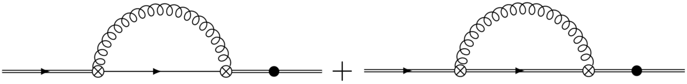
[4] Phys. Rev. D 97 (2018) 7, 074009 (Brambilla, Escobedo, Soto, Vairo)

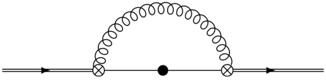
Diagramatic Evolution of $\rho_o(t)$[5]

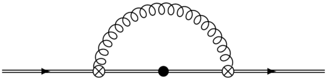
octet evolution given by

$$\frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} = -i\,[h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$

where

$\Sigma_o \rho_o(t) \sim$ 

$\Xi_{os}(\rho_s(t)) \sim$ 

$\Xi_{oo}(\rho_o(t)) \sim$ 

[5] Phys. Rev. D 97 (2018) 7, 074009

Evolution as Lindblad Equation[6]

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left[C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\{ C_i^{n\dagger} C_i^n, \rho(t)\} \right],$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \left(\frac{r^2}{2}\gamma + \frac{\kappa}{4MT}\{r_i, p_i\} \right) \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}} \left(r^i + \frac{ip_i}{2MT} + \frac{\Delta V_{os}}{4T} r_i \right) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$+ \sqrt{\kappa} \left(r_i + \frac{ip_i}{2MT} + \frac{\Delta V_{os}}{4T} r_i \right) \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}} \left(r_i + \frac{ip_i}{2MT} \right) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

free parameters are κ and γ which we extract from lattice data

[6] Phys.Rev.D 108 (2023) 1, L011502 (Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, PVG)

QTraj Implementation[7]

1. initialize wave function $|\psi(t_0)\rangle$
2. generate random number $0 < r_1 < 1$, evolve with H_{eff} until

$$|| \, e^{-i\int_{t_0}^{t} dt' H_{\text{eff}}(t')} |\psi(t_0)\rangle \, ||^2 \leq r_1,$$

   and initiate a quantum jump
3. quantum jump
   3.1 if singlet, jump to octet; if octet, generate random number $0 < r_2 < 1$ and jump to singlet if r_2 less than the branching fraction to singlet; otherwise, remain in octet
   3.2 generate random number $0 < r_3 < 1$; if $r_3 < l/(2l+1)$, $l \to l-1$; otherwise, $l \to l+1$.
   3.3 multiply wavefunction by r and normalize
4. Continue from step 2.

[7] Comput. Phys. Commun. 273 (2022) 108266 (Ba Omar, et. al.)

Medium Interaction

- medium evolution implemented using a $3+1$D dissipative relativistic hydrodynamics code using a realistic equation of state fit to lattice QCD measurements
- approximately $1-2\times10^5$ physical trajectories
  - production point sampled in transverse plane using nuclear binary collision overlap profile $N_{AA}^{\text{bin}}(x,y,b)$, initial p_T from an E_T^{-4} spectrum, and ϕ uniformly in $[0,2\pi)$
  - $\sim$30 quantum trajectories per physical trajectory
  - allows for computing observables as a function of transverse momentum p_T

Nuclear Modification Factor R_{AA}[8]

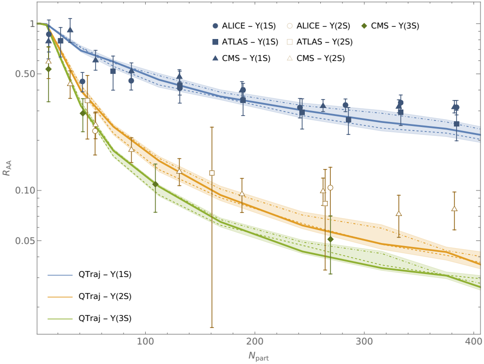


Figure: R_{AA} of the $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ as a function of N_{part}; experimental data from the ALICE, ATLAS and CMS collaborations.

- state of the art extraction of R_{AA} using 3-loop potentials with excellent agreement with vacuum spectrum
- LO in $E/(\pi T)$ expansion $\rightarrow$ coupling to medium terminated at $T_F = 250$ MeV

[8] 2403.15545 (Brambilla, Magorsch, Strickland, Vairo, PVG) (to appear in Phys. Rev. D)

Nuclear Modification Factor R_{AA}[9]

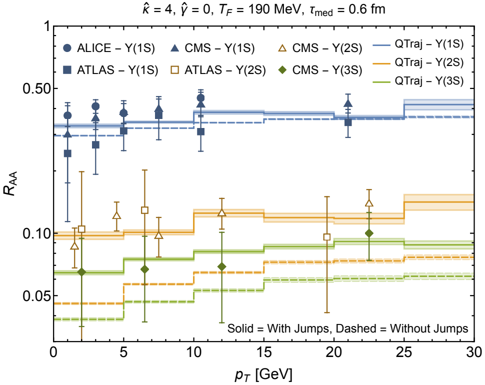


Figure: R_{AA} of the $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ as a function of p_T; experimental data from the ALICE, ATLAS and CMS collaborations.

- extraction of R_{AA} using Coulomb potentials
- NLO in $E/(\pi T)$ expansion allowing coupling to medium to $T_F = 190$ MeV
- regeneration necessary to match experimental data

[9] Phys.Rev.D 108 (2023) 1, L011502 (Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, PVG)

Double Ratio $2S/1S$[10]

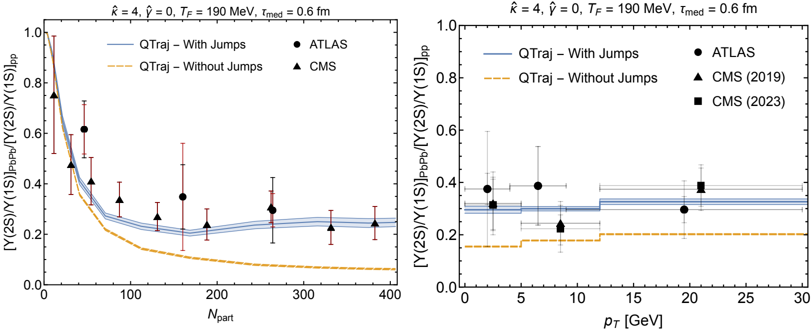


Figure: Double ratio of $R_{AA}(2S)$ to $R_{AA}(1S)$ as a function of N_{part} (left) and p_T (right); experimental data from the ATLAS and CMS collaborations.

[10] Phys.Rev.D 108 (2023) 1, L011502

Double Ratio $3S/2S$[11]

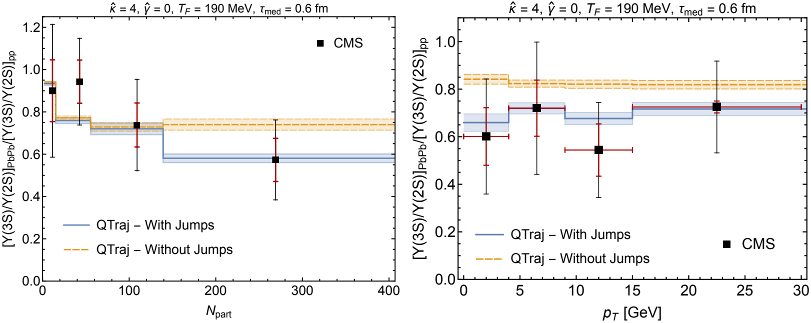


Figure: Double ratio of $R_{AA}(3S)$ to $R_{AA}(2S)$ as a function of N_{part} (left) and p_T (right); experimental data from the CMS collaborations.

[11] Phys.Rev.D 108 (2023) 1, L011502

Elliptic Flow v_2[12]

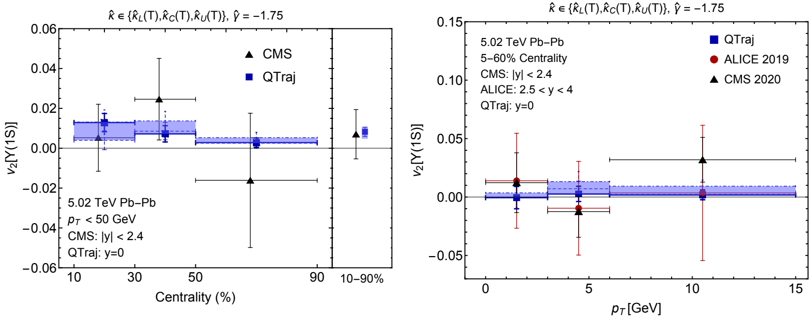


Figure: Elliptic flow v_2 of the $\Upsilon(1S)$ as a function of N_{part} (left) and p_T (right); experimental data from the CMS and Alice collaborations. Older result with Coulomb potential at LO in $E/(\pi T)$ with $T_f = 250$ MeV.

[12] Phys.Rev.D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

Conclusions

- due to hierarchies of scale, system of in-medium bottomonium ideally described using EFT methods, specifically pNRQCD, and the OQS formalism
- evolution equation takes the form of a Lindblad equation
- computational methods necessary to solve the Lindblad equation and extract the nuclear modification factor R_{AA}
- use `QTraj` code to solve the Lindblad equation and extract R_{AA} and v_2 of ground and excited states as functions of N_{part} and p_T
- quantum recombination necessary to match experimental data
- method and results are fully quantum, non abelian, and heavy quark number conserving; take into account dissociation and recombination; and depend only on the transport coefficients κ and γ

Thank you!

Backup Slides

Elliptic Flow v_2 of Excited States[13]

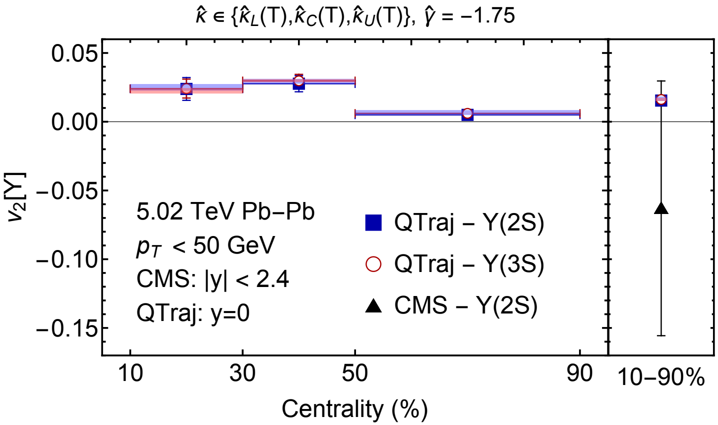


Figure: Elliptic flow v_2 of the $\Upsilon(2S)$ and $\Upsilon(3S)$ as a function of $N_{\rm part}$; experimental data from the CMS collaboration. Coulomb potential at LO in $E/(\pi T)$ with $T_f = 250$ MeV.

[13] Phys.Rev.D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

pNRQCD Lagrangian

$$\mathcal{L}_{\text{pNRQCD}} = \text{Tr}\Big[S^\dagger(i\partial_0 - h_s)S + O^\dagger(iD_0 - h_o)O + O^\dagger \mathbf{r}\cdot g\,\mathbf{E}\,S$$
$$+S^\dagger \mathbf{r}\cdot g\,\mathbf{E}\,O + \frac{1}{2}O^\dagger\left\{\mathbf{r}\cdot g\,\mathbf{E}\,,\;O\right\}\Big]$$

- singlet and octet field S and O interacting via chromo-electric dipole vertices
- $h_{s,o} = \frac{\mathbf{p}^2}{M} + V_{s,o}$: singlet, octet Hamiltonian
  - $V_s = -\frac{C_f\alpha_s(1/a0)}{r}$: attractive singlet potential
  - $V_o = \frac{\alpha_s(1/a0)}{2N_c r}$: repulsive octet potential

Quantum Trajectories Algorithm

- Monte Carlo method to solve the Lindblad equation
- less memory intensive due to use of wave function $|\psi\rangle$ rather than density matrix ρ
- absorb quantum number conserving diagonal evolution terms of Lindblad equation into a non-Hermitian effective Hamiltonian

$$H_{eff} = H - \frac{i}{2}\sum_n C_n^\dagger C_n$$

  Lindblad equation becomes

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i\left(H_{eff}\rho(t) - \rho(t)H_{eff}^\dagger\right) + \sum_n C_i^n \rho(t) C_i^{n\dagger}$$

- H_{eff} term reduces trace of ρ and preserves quantum numbers of state
- C_n term changes quantum numbers of state and ensure overall evolution is trace preserving

H_{eff} Evolution

- evolve wavefunction with H_{eff}

$$|\psi(t+\delta t)\rangle = (1 - iH_{eff}\delta t)|\psi(t)\rangle$$

- H_{eff} evolution preserves quantum numbers of the state and decreases its norm

$$\begin{aligned}\langle\psi(t+\delta t)|\psi(t+\delta t)\rangle &\approx 1 - i\langle\psi(t)|(H_{eff} - H_{eff}^{\dagger})|\psi(t)\rangle\delta t \\ &= 1 - \delta p\end{aligned}$$

where

$$\delta p = \sum_n \langle\psi(t)|C_n^{\dagger}C_n|\psi(t)\rangle\delta t = \sum_n \delta p_n$$

- decrease in norm related to probability a change of quantum numbers, implemented by $C_n|\psi(t)\rangle$, occurs

Monte Carlo

(normalized) evolution of state

$$|\tilde{\psi}(t+\delta t)\rangle = \begin{cases} \frac{|\psi(t+\delta t)\rangle}{\sqrt{1-\delta p}} \text{ with probability } 1-\delta p \\ \frac{C_n|\psi(t)\rangle}{\sqrt{\delta p_n/\delta t}} \text{ with probability } \delta p \end{cases}$$

i.e., with probability $1-\delta p$, the state evolves as governed by H_{eff}, and with probability δp, is acted on by the collapse operator C_n

simulation

- generate a random number $0 < r_1 < 1$
- evolve state with H_{eff} until norm squared $< r_1$
- generate additional random number(s) to determine which collapse operator C_n to apply

Equivalence of Evolution and Convergence

equivalence of evolution

$$\begin{aligned}\rho(t+\delta t) =& (1-\delta p)\frac{|\psi(t+\delta t)\rangle}{\sqrt{1-\delta p}}\frac{\langle\psi(t+\delta t)|}{\sqrt{1-\delta p}} \\ &+ \delta p \sum_n \frac{\delta p_n}{\delta p}\frac{C_n|\psi(t)\rangle}{\sqrt{\delta p_n/\delta t}}\frac{\langle\psi(t)|C_n^\dagger}{\sqrt{\delta p_n/\delta t}} \\ =& \rho(t) - i[H_{eff}\rho(t) - \rho(t)H_{eff}^\dagger]\delta t + \sum_n C_n\rho(t)C_n^\dagger\delta t,\end{aligned}$$

as given by Lindblad equation

convergence

- calculate expectation values using evolved state
- evolve many states and average to converge to result of directly solving the Lindblad equation

T_f Variation[14]

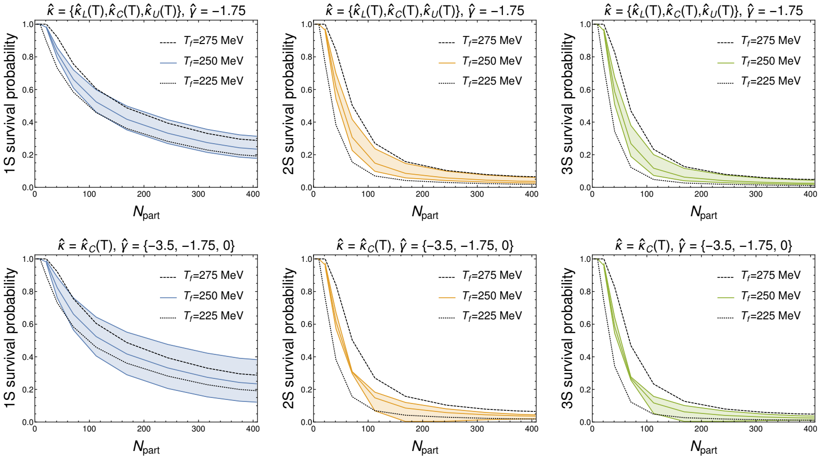


Figure: Variation in the survival probability of the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ varying T_f by $\pm 10\%$.

[14] JHEP 05 (2021) 136 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)