

A global analysis of the minimal MFV SMEFT

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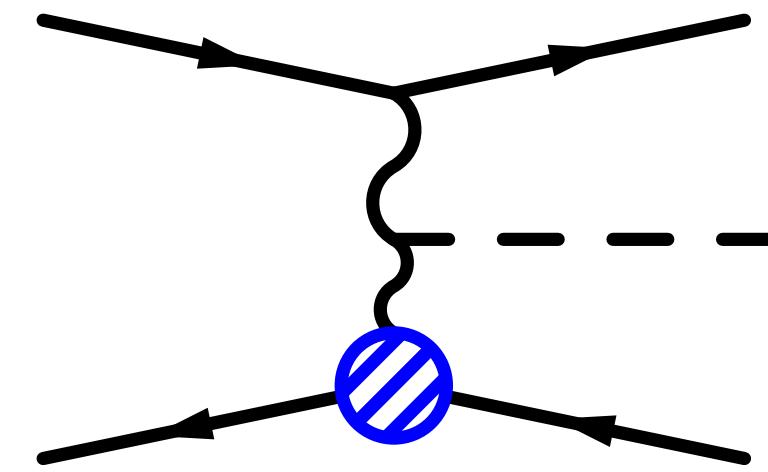
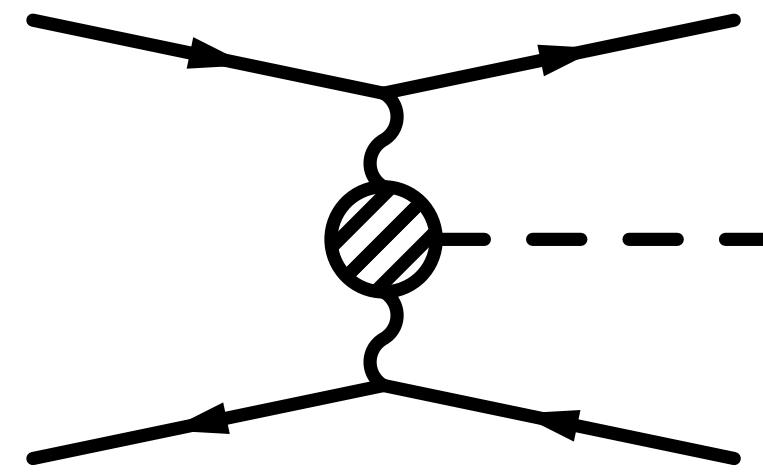


Based on [2311.04963](#) with
Riccardo Bartocci and Tobias Hurth

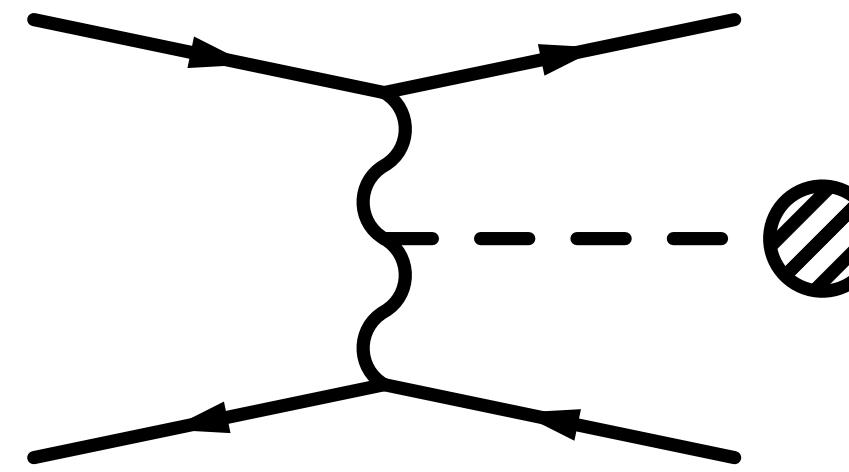
LCHP Boston - June 7, 2024

Why global fits?

One observable can be influenced by many operators

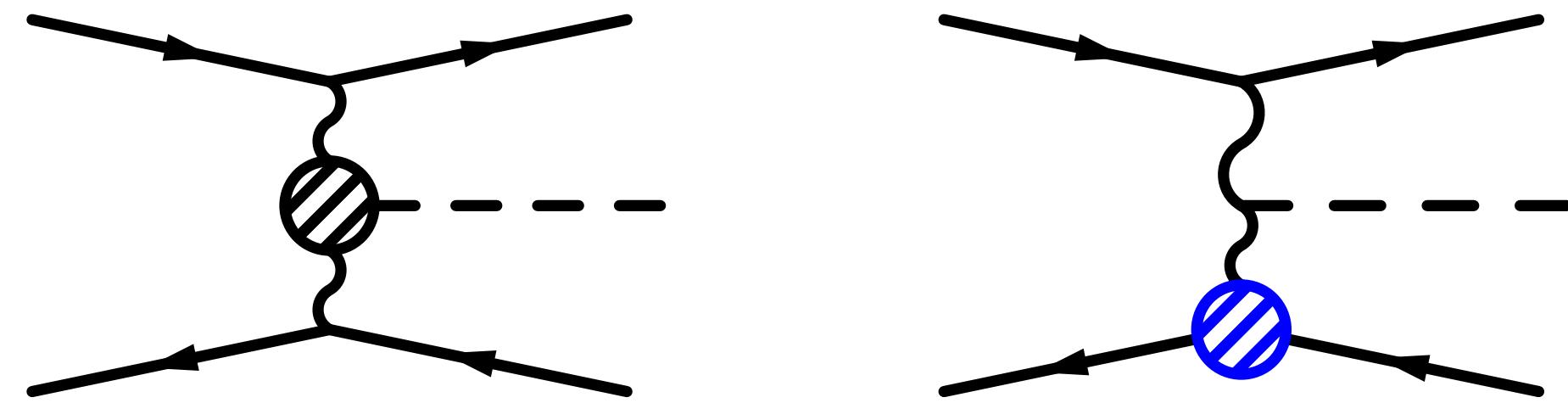


Higgs decay

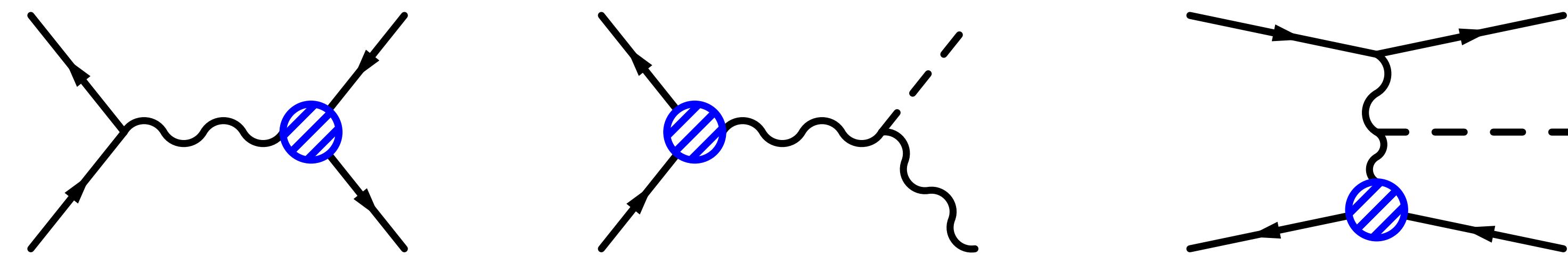


Why global fits?

One observable can be influenced by many operators



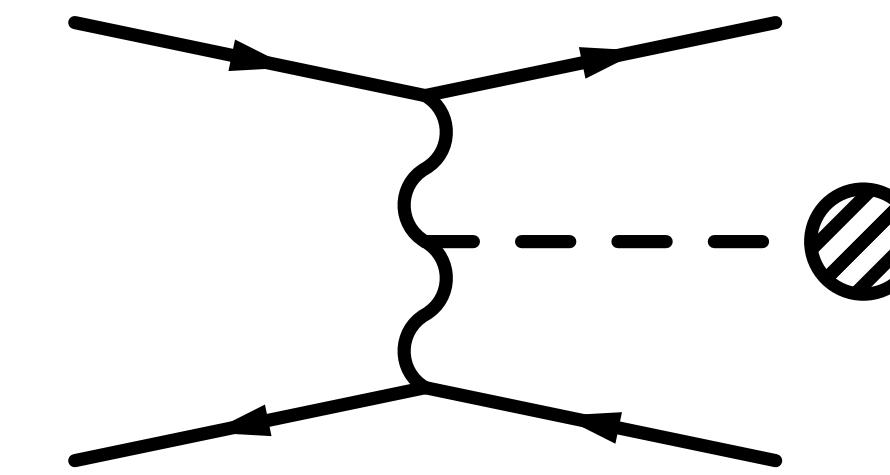
One operator can contribute to many different observables



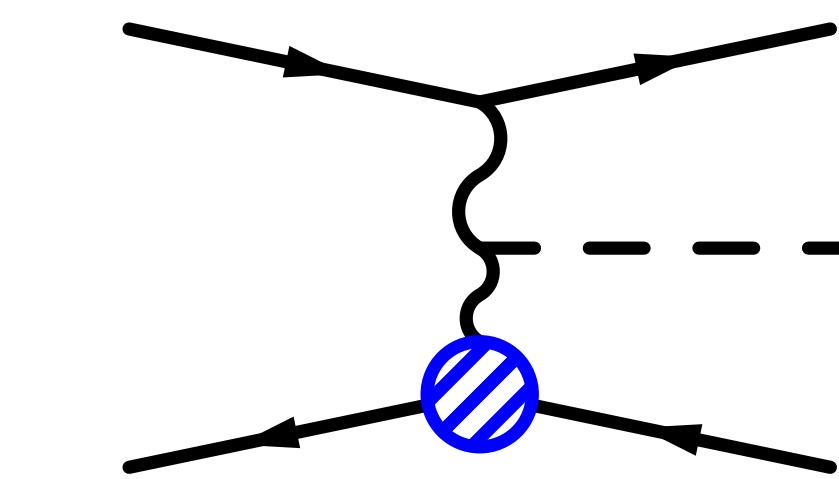
$$e^+ e^- \rightarrow f\bar{f}$$

Zh production

Higgs decay



Need a global fit to take into account the possible crosstalk of observables and operators



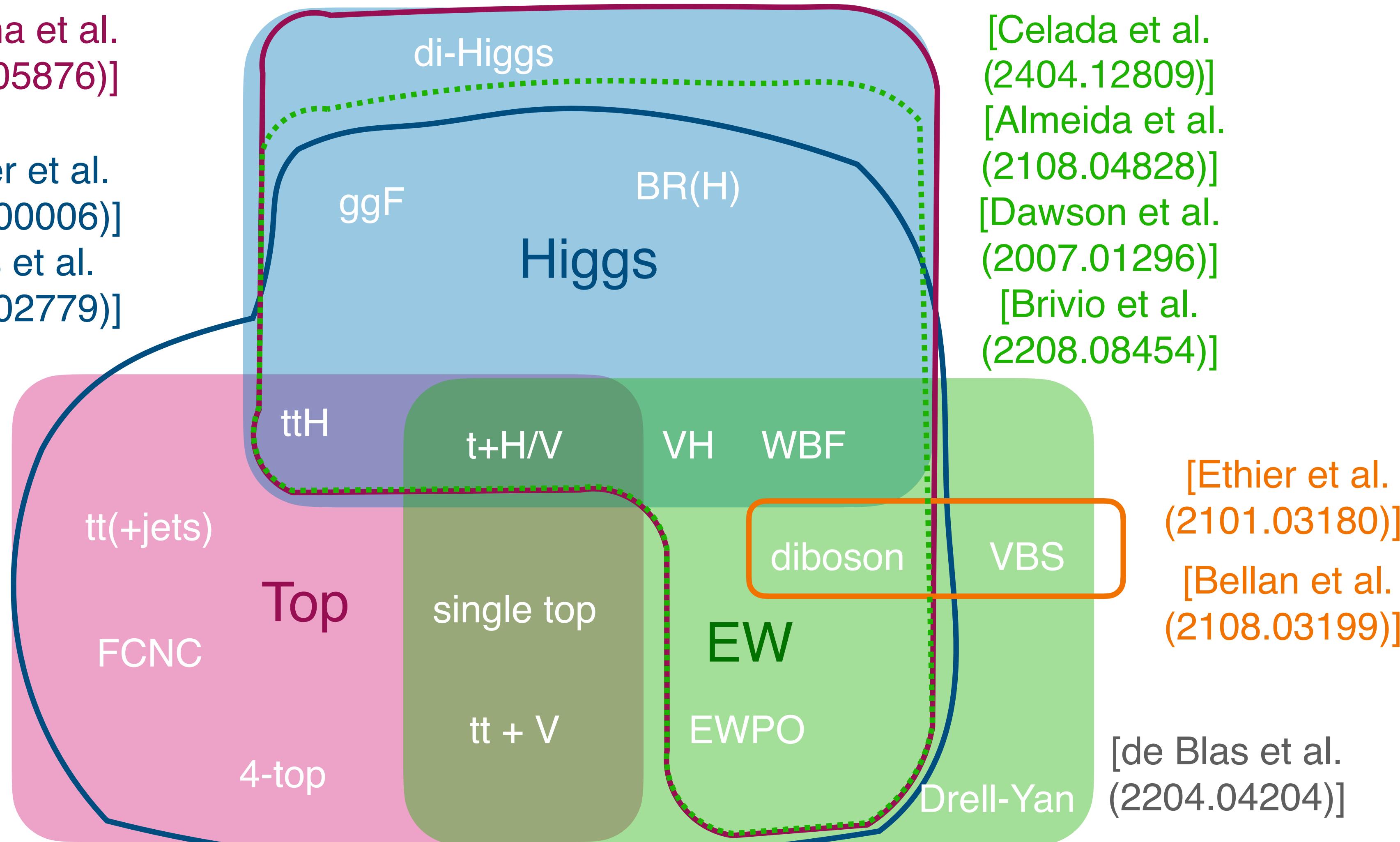
Weak boson fusion
Higgs production

Confronting the SMEFT with data

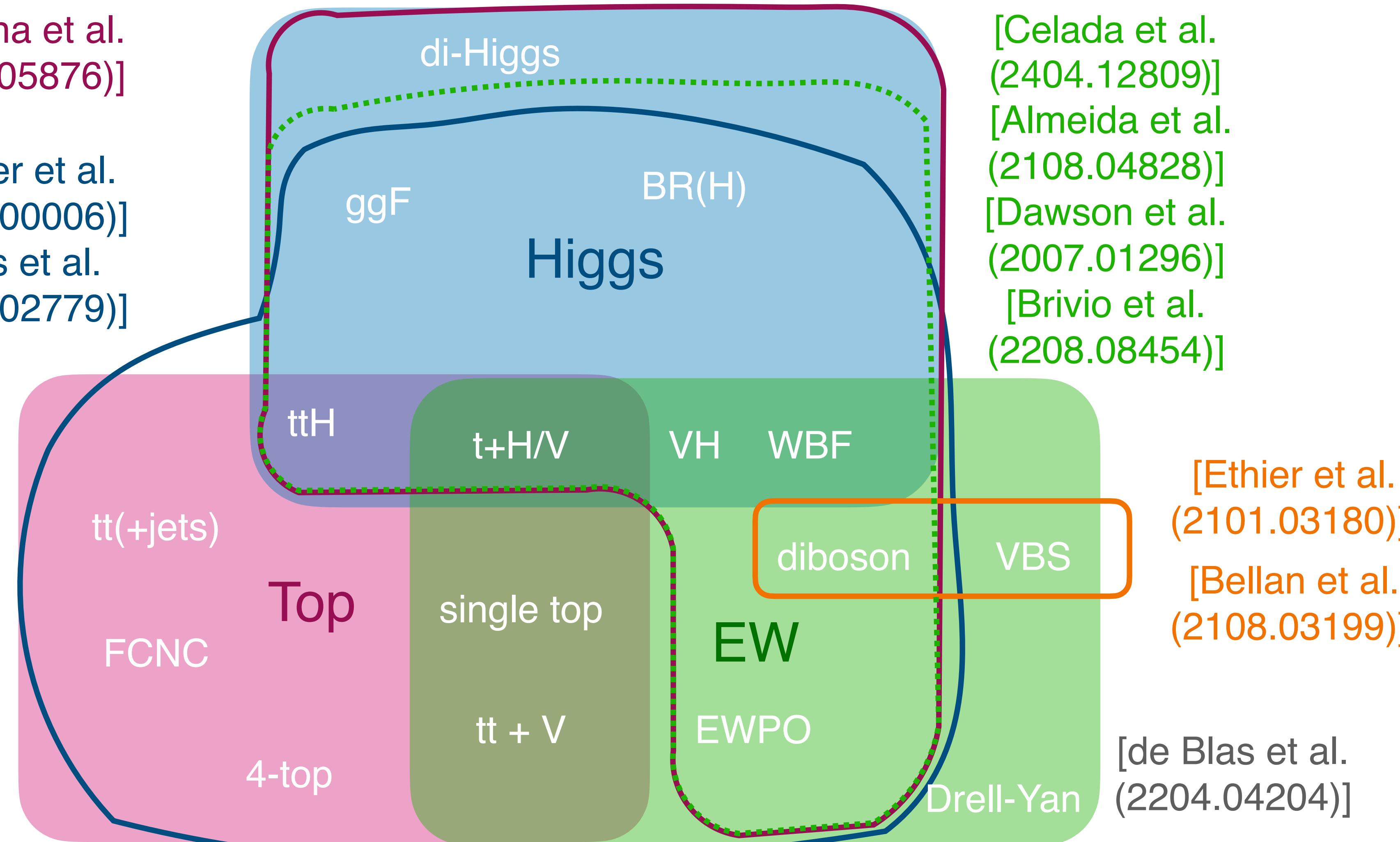
[Anisha et al.
(2111.05876)]

[Ethier et al.
(2105.00006)]

[Ellis et al.
(2012.02779)]



Confronting the SMEFT with data



Outline

Are we ready for a global fit starting from theory assumptions?

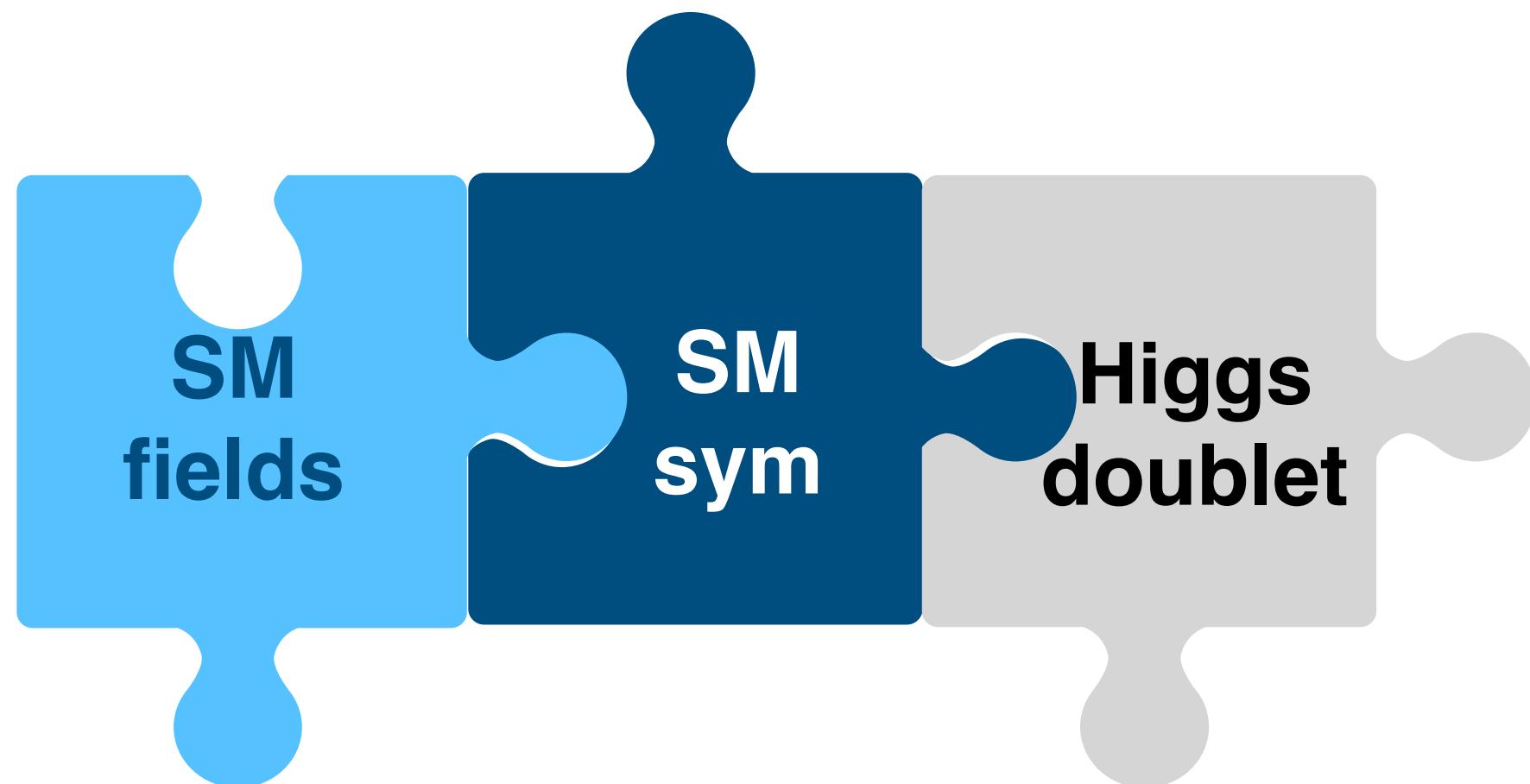
SMEFT@NLO: Curse or blessing?

Based on [2311.04963](#) with
Riccardo Bartocci and Tobias Hurth



Standard Model Effective Field Theory (SMEFT)

[review: Brivio, Trott ([1706.08945](#))]



$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \dots$$

Odd dimensions violate lepton or baryon number

2499 operators at D6

Many of these are different flavor combinations
of the same structure

Reduce number of dof with flavor assumptions

$$\mathcal{O}_{d_H}^{ij} = (H^\dagger H)(\bar{q}_i H d_j)$$

$$3 \times 3 + \text{h.c.} = 18 \text{ Flav. combinations}$$

SMEFT flavor assumptions

More flavor symmetries:

[Faroughy, Isidori, Wilsch, Yamamoto ([2005.05366](#))]

[Greljo, Palavric, Thomsen ([2203.09561](#))]

Assume an **exact** $U(3)^5$ symmetry

$$U(3)^5 = U(3)_\ell \times U(3)_q \times U(3)_e \times U(3)_u \times U(3)_d \quad + \text{no CP odd interactions}$$

Same couplings for top, charm, up quark.

$$\mathcal{O}_{dH} = (H^\dagger H)(\bar{q}_i H d_j) \quad \text{Operator is forbidden under } U(3)^5 \text{ symmetry}$$

Left with **41** operators

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$$\mathcal{O}_{dH} = \sum_{ij} (H^\dagger H) (\bar{q}_i H d_j) \quad \text{Operator is forbidden under } U(3)^5 \text{ symmetry}$$

Left with **41** operators

Minimal flavor violation (MFV)

In the SM, the Yukawas Y_e , Y_u , Y_d are the only sources of the breaking of this symmetry

[Gerard ([1983](#))]

[Chivukula, Georgi ([1987](#))]

[D'Ambrosio, Giudice, Isidori, Strumia ([hep-ph/0207036](#))]

$$\begin{aligned} \mathcal{O}_{dH} &= \sum_{ij} (H^\dagger H) (\bar{q}_i H \textcolor{red}{Y_d} d_j) \\ &\rightarrow (H^\dagger H) ((\bar{q} \Omega_q^\dagger)_i H (\textcolor{red}{\Omega_d Y_q \Omega_d^\dagger}) (\Omega_d d)_j) \end{aligned}$$

Warsaw basis

[Grzadkowski et al. (1008.4884)]

1 : X^3		2 : H^6		3 : $H^4 D^2$		5 : $\psi^2 H^3 + \text{h.c.}$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\square}$	$(H^\dagger H) \square (H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$			Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$						
4 : $X^2 H^2$		6 : $\psi^2 X H + \text{h.c.}$		7 : $\psi^2 H^2 D$			
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$		
$Q_{H\tilde{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$		
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$		
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$		
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$		
$Q_{H\tilde{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$		
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$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$		
8 : $(\bar{L}L)(\bar{L}L)$		Plus more four-fermion operators					
$Q_{\ell\ell}$			$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$				

Warsaw basis under $U(3)^5$

[Grzadkowski et al. (1008.4884)]

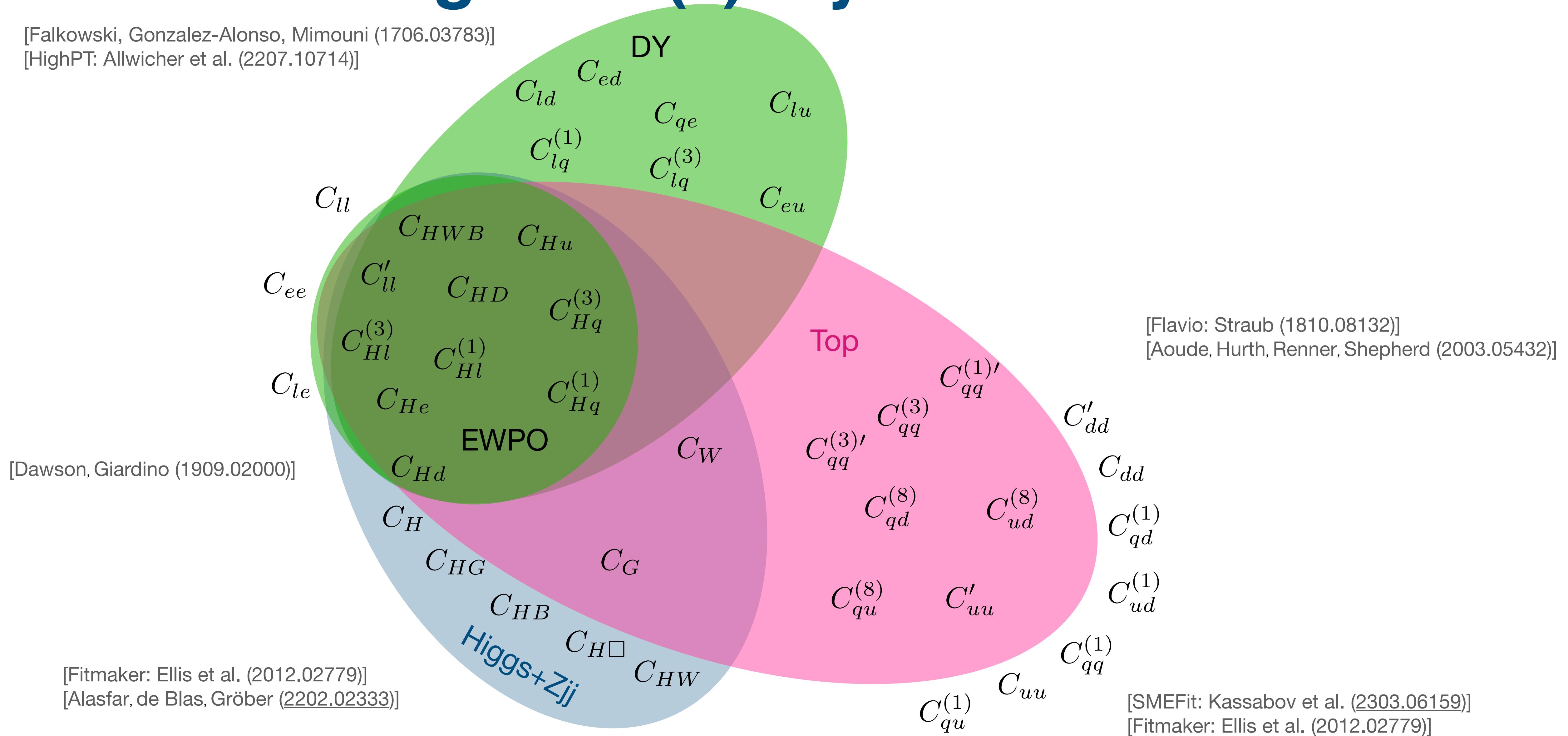
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41 parameters in total

Constraining the $U(3)^5$ symmetric SMEFT

[Falkowski, Gonzalez-Alonso, Mimouni (1706.03783)]

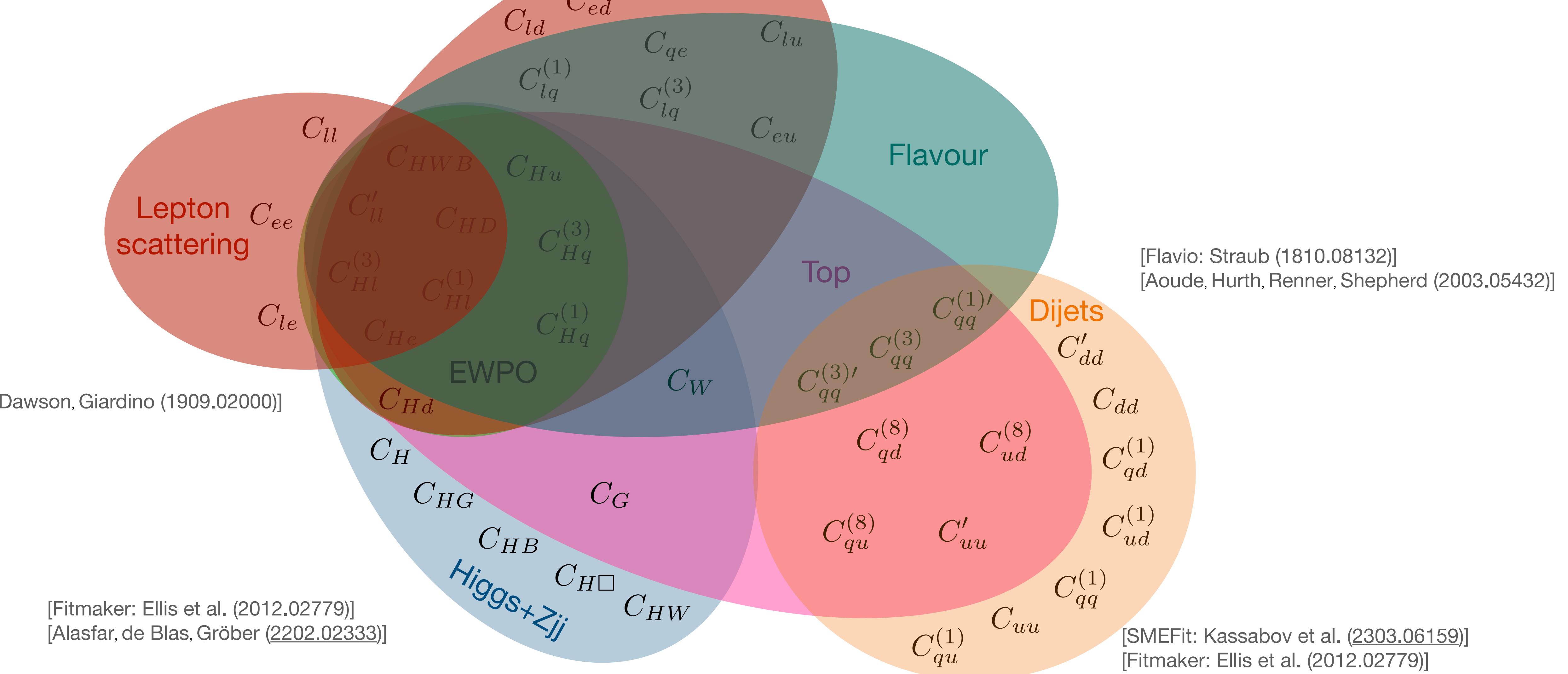
[HighPT: Allwicher et al. (2207.10714)]



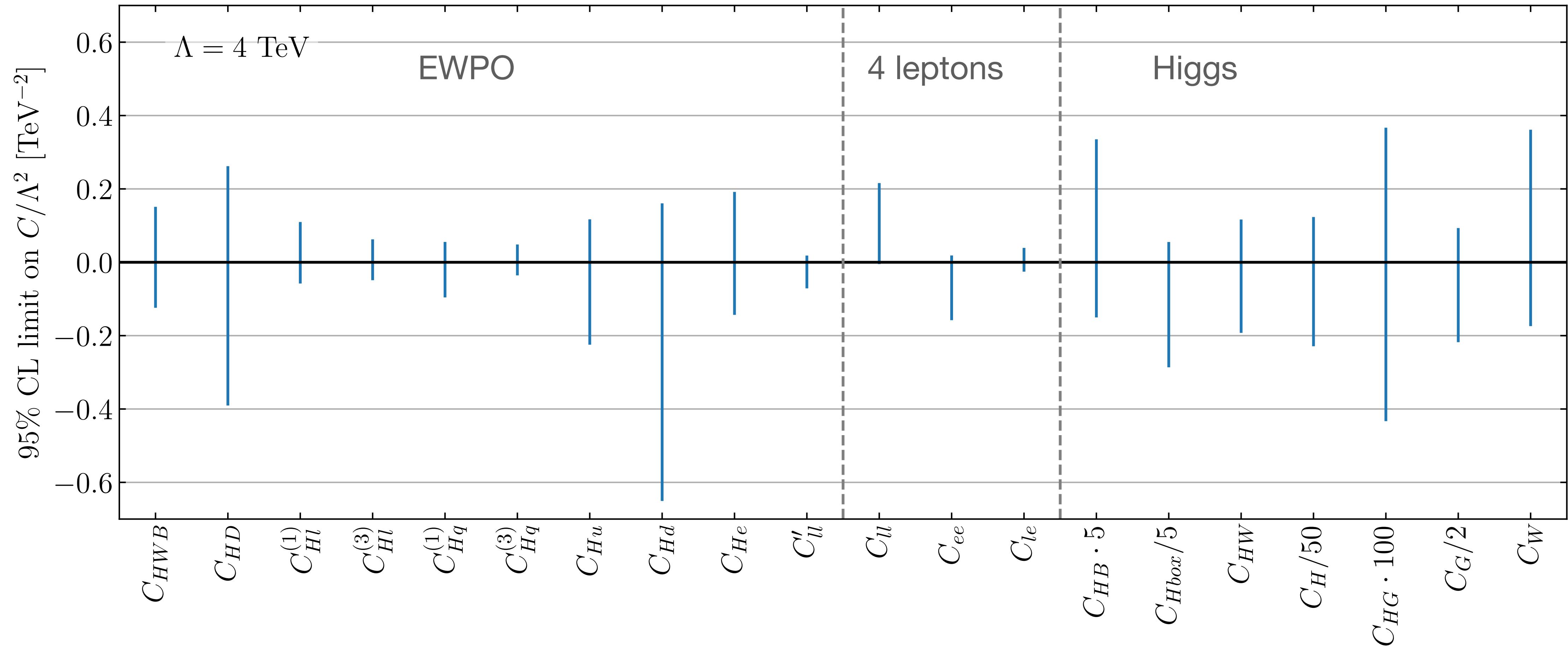
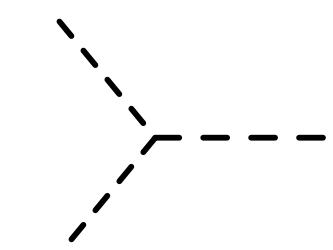
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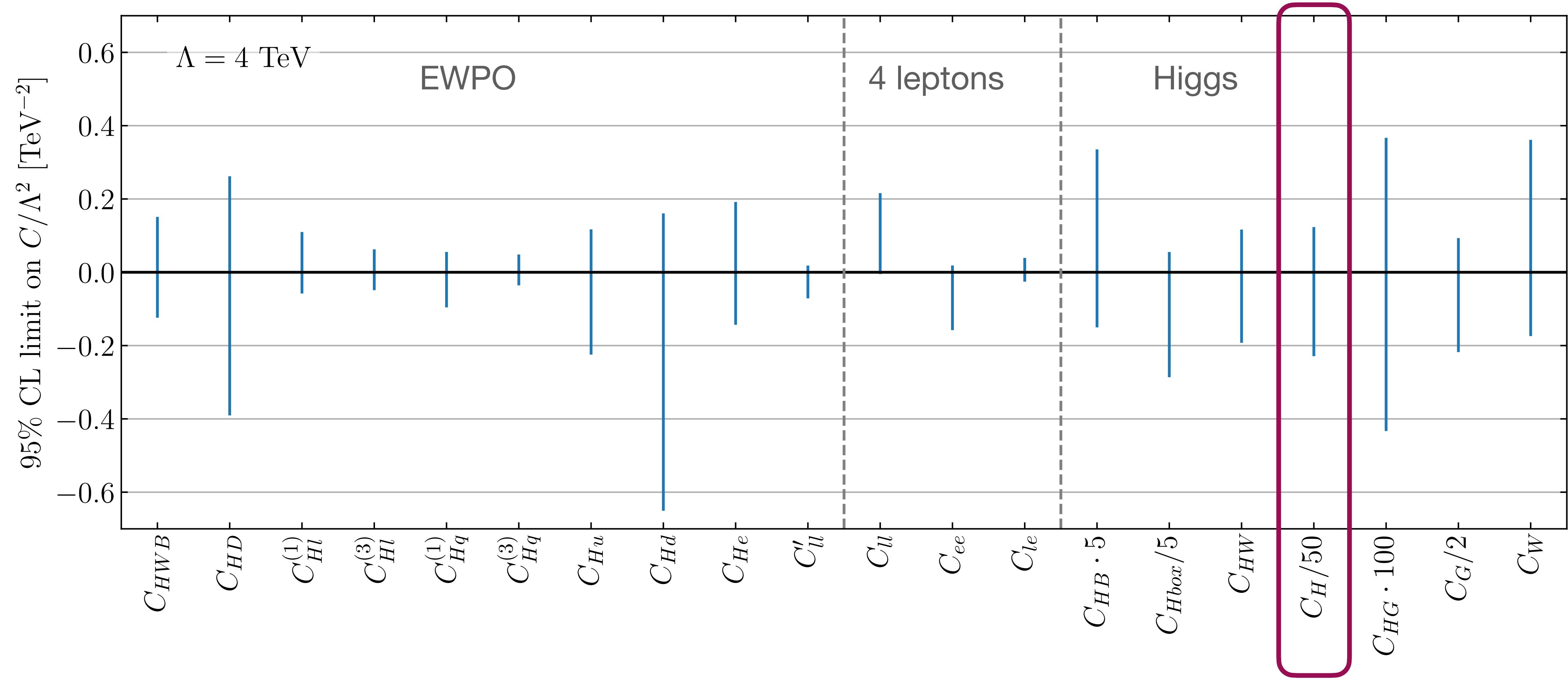
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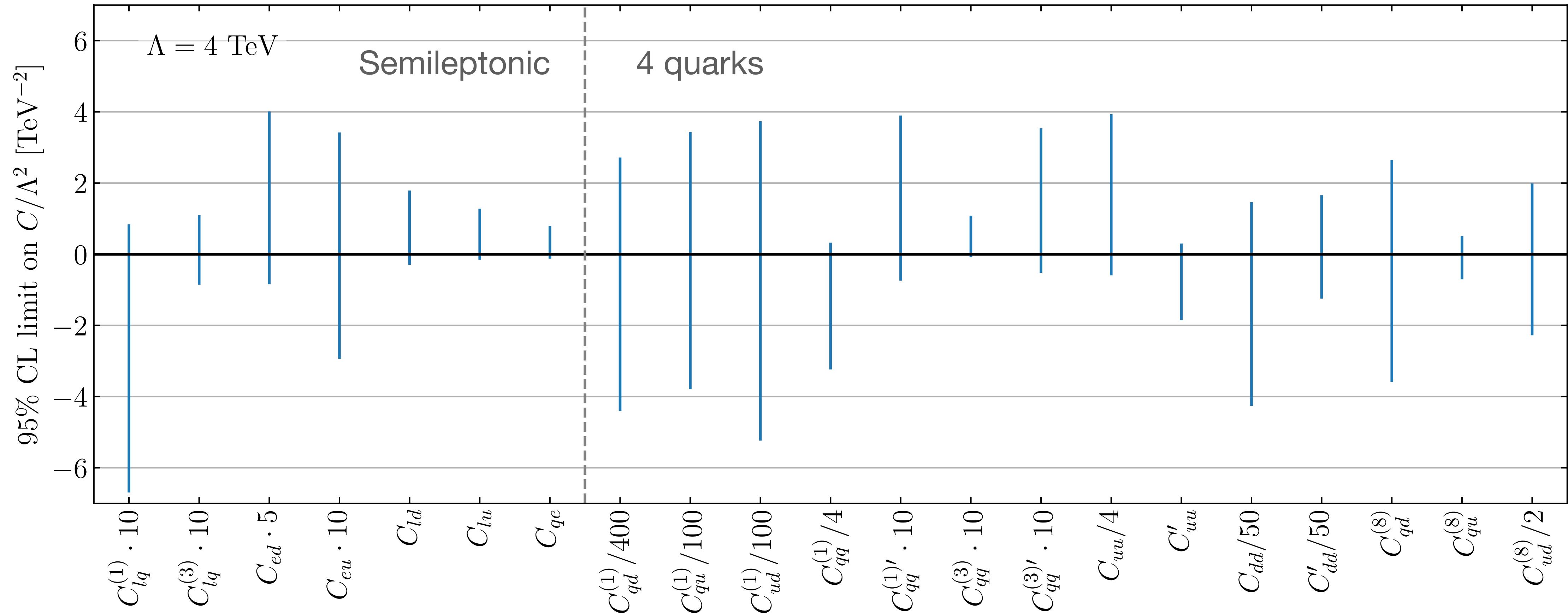
LO fit - Higgs/gauge interactions



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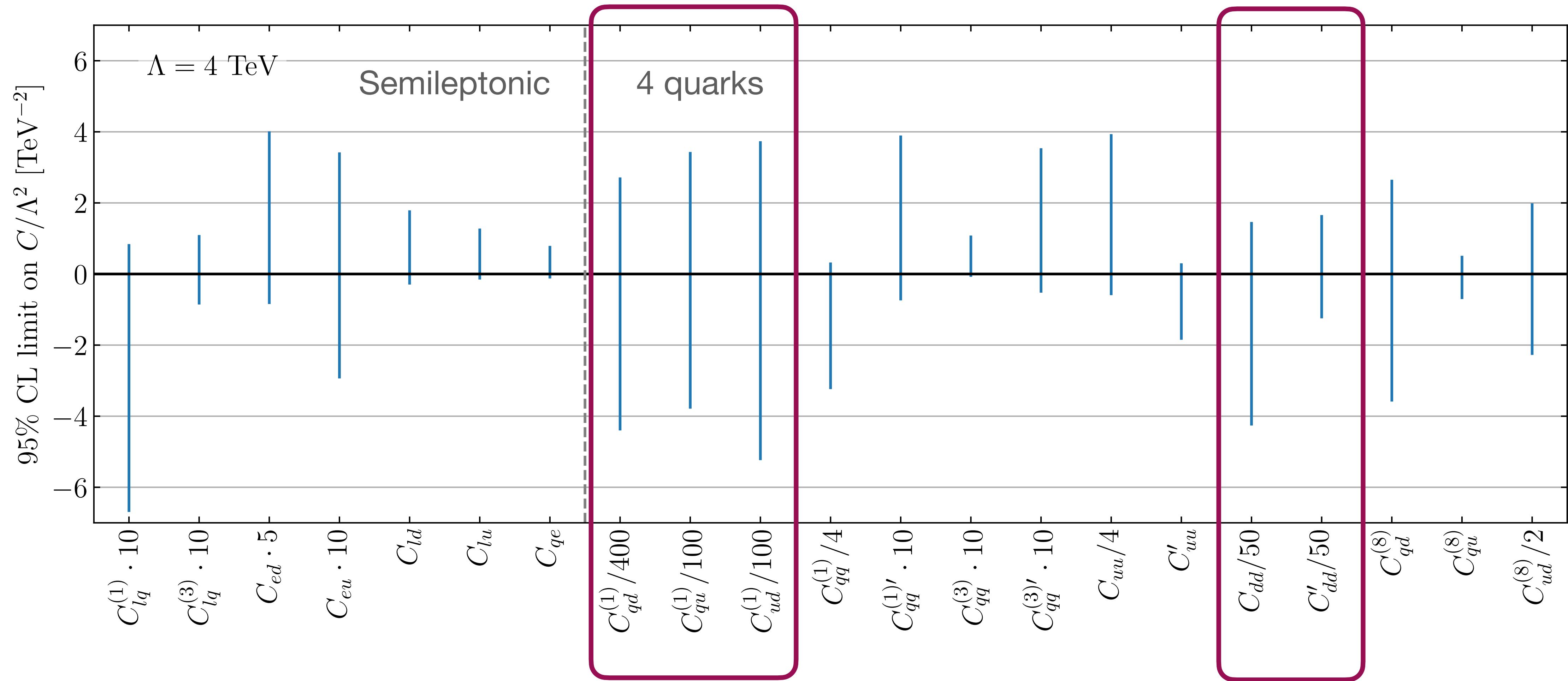


LO fit - 2



LO fit - 2

Do not interfere with
dominant SM diagram in
dijet(+photon) production



NLO to the rescue?

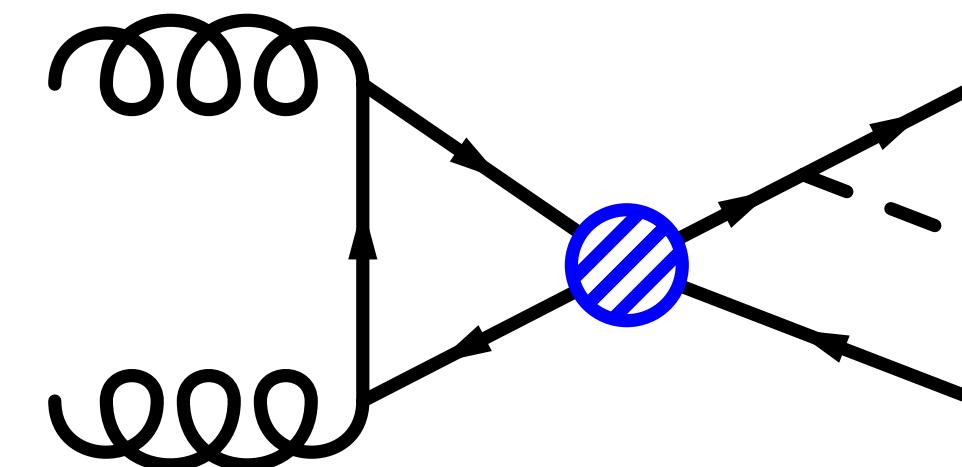
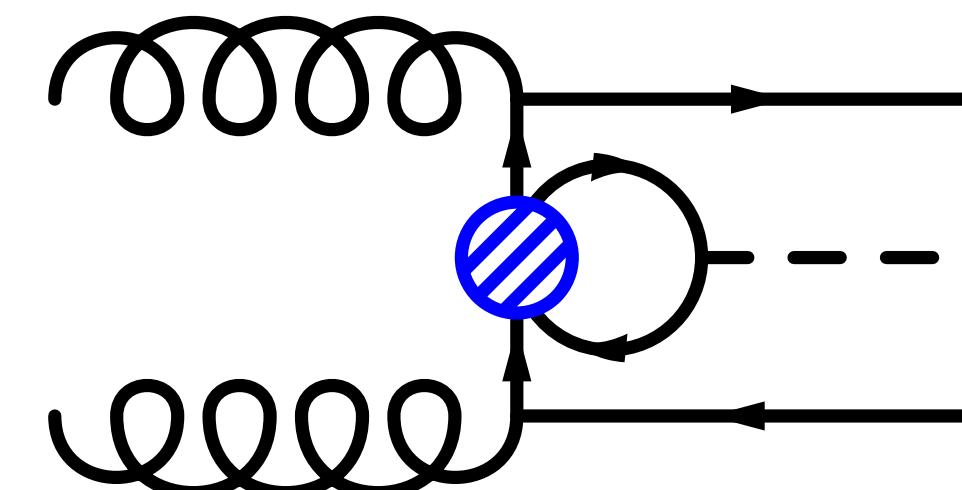
Additional sensitivity from next-to-leading-order (NLO) SMEFT effects

Higgs: $C_{qu}^{(1)}$

Top: $C_{qd}^{(1)}, C_{ud}^{(1)}$

$t\bar{t}h$

$\bar{t}t$

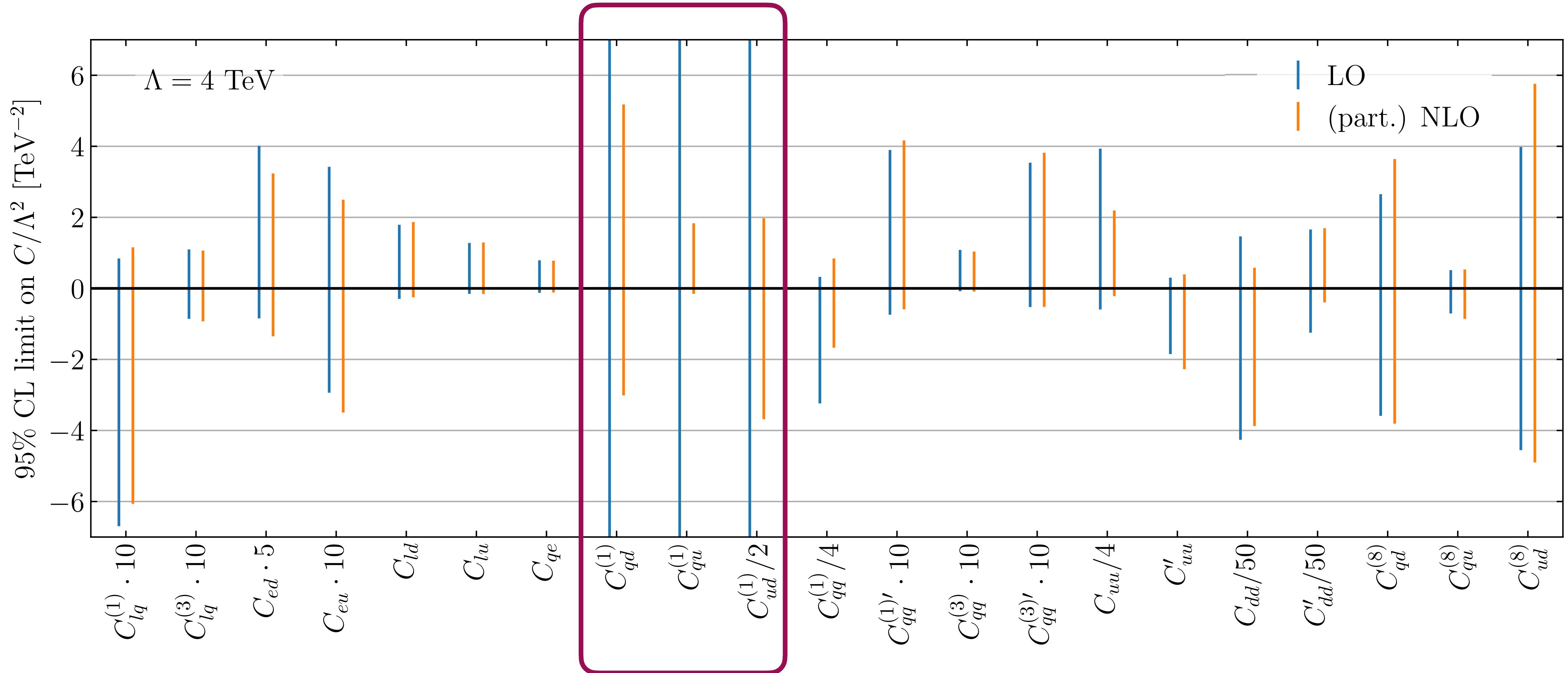


[Alasfar, de Blas, Gröber ([2202.02333](#))]

[SMEFit: Kassabov et al. ([2303.06159](#))]

(Partial) NLO fit

Partial: Not all observables are available at NLO



SMEFT@NLO: Blessing & curse

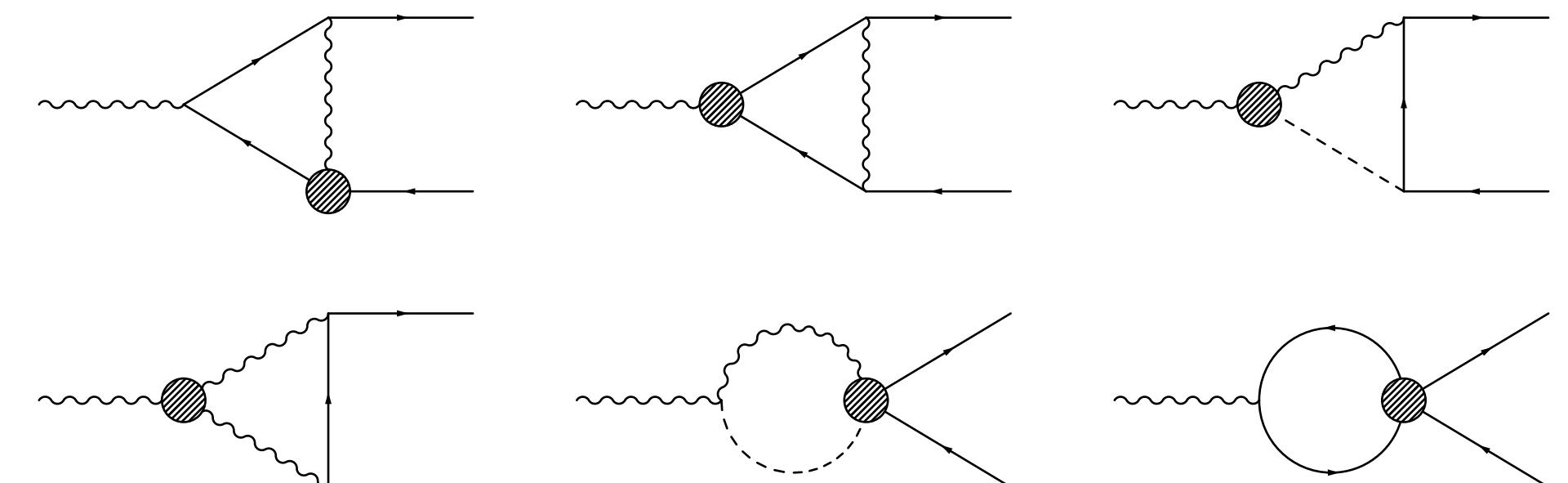
EWPO: 10 ops @LO, 32 ops @NLO ($U(3)^5$ sym)

[Dawson, Giardino (1909.02000)], [AB, Pecjak, Scott, Smith (2305.03763)]

$$\delta\Gamma(Z \rightarrow l^+l^-)^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.1408\mathcal{C}_{\phi e} + 0.191\mathcal{C}_{\phi l}^{(1)} - 0.037\mathcal{C}_{\phi l}^{(3)} + 0.114\mathcal{C}_{ll} - 0.057\mathcal{C}_{\phi D} - 0.0713\mathcal{C}_{\phi WB} \right\} \text{GeV}$$

$$\begin{aligned} \delta\Gamma(Z \rightarrow l^+l^-)^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.1596\mathcal{C}_{\phi e} + 0.1834\mathcal{C}_{\phi l}^{(1)} - 0.0221\mathcal{C}_{\phi l}^{(3)} + 0.0985\mathcal{C}_{ll} - 0.0508\mathcal{C}_{\phi D} - 0.0349\mathcal{C}_{\phi WB} - 0.0001\mathcal{C}_{\phi W} - 0.0002\mathcal{C}_{ed} - 0.0005\mathcal{C}_{ee} + 0.0035\mathcal{C}_{eu} - 0.0002\mathcal{C}_{\phi d} - 0.0042\mathcal{C}_{\phi q}^{(1)} + 0.0032\mathcal{C}_{\phi q}^{(3)} + 0.0049\mathcal{C}_{\phi u} + 0.0002\mathcal{C}_{ld} + 0.0001\mathcal{C}_{le} + 0.0034\mathcal{C}_{lq}^{(1)} - 0.0031\mathcal{C}_{lq}^{(3)} - 0.0045\mathcal{C}_{lu} - 0.0001\mathcal{C}_{\phi \square} - 0.0027\mathcal{C}_{qe} - 0.0007\mathcal{C}_{uB} - 0.0007\mathcal{C}_{uW} - 0.0001\mathcal{C}_W \right\} \text{GeV} \end{aligned}$$

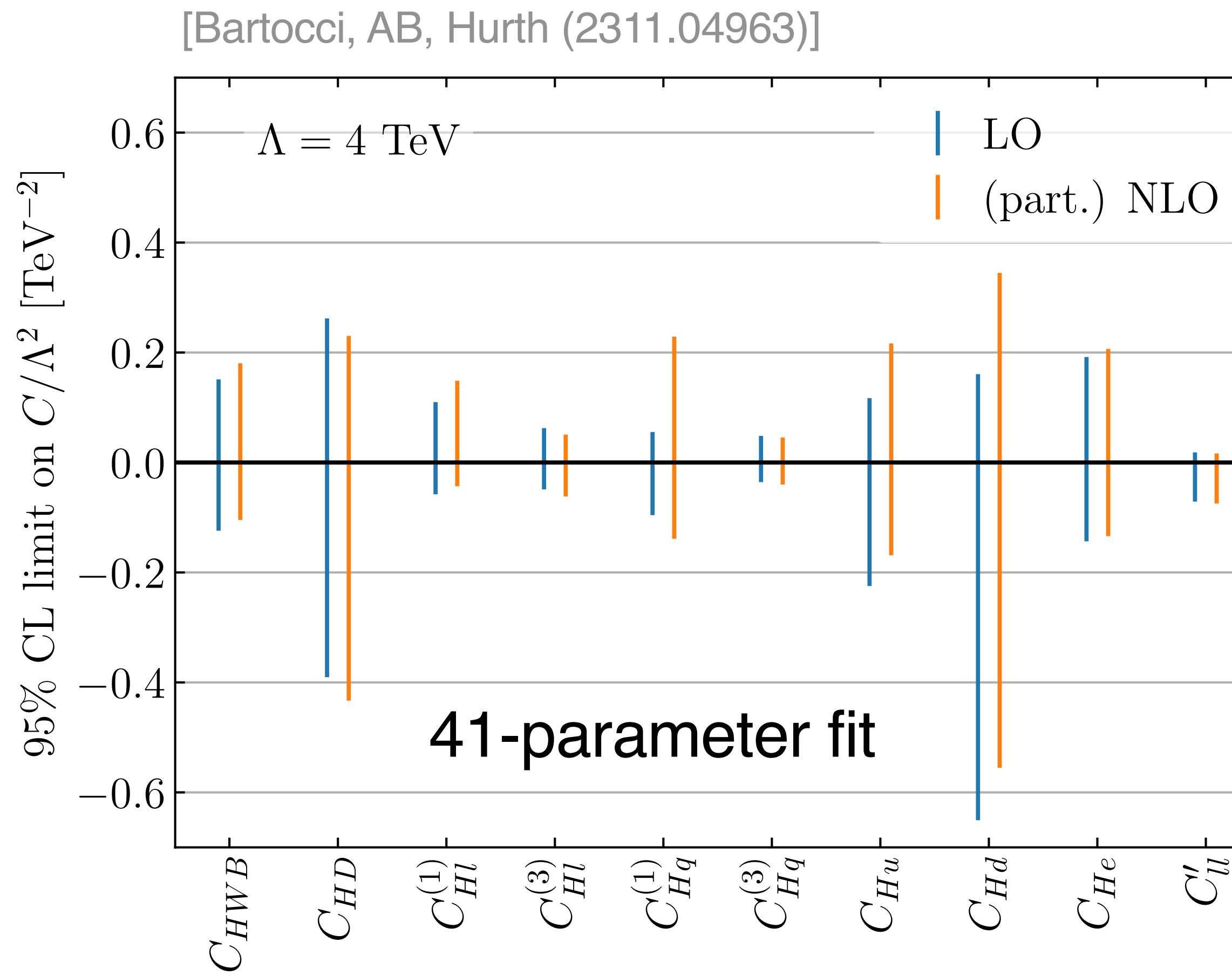
More degrees of freedom contribute to each observable at NLO



precision & degeneracies

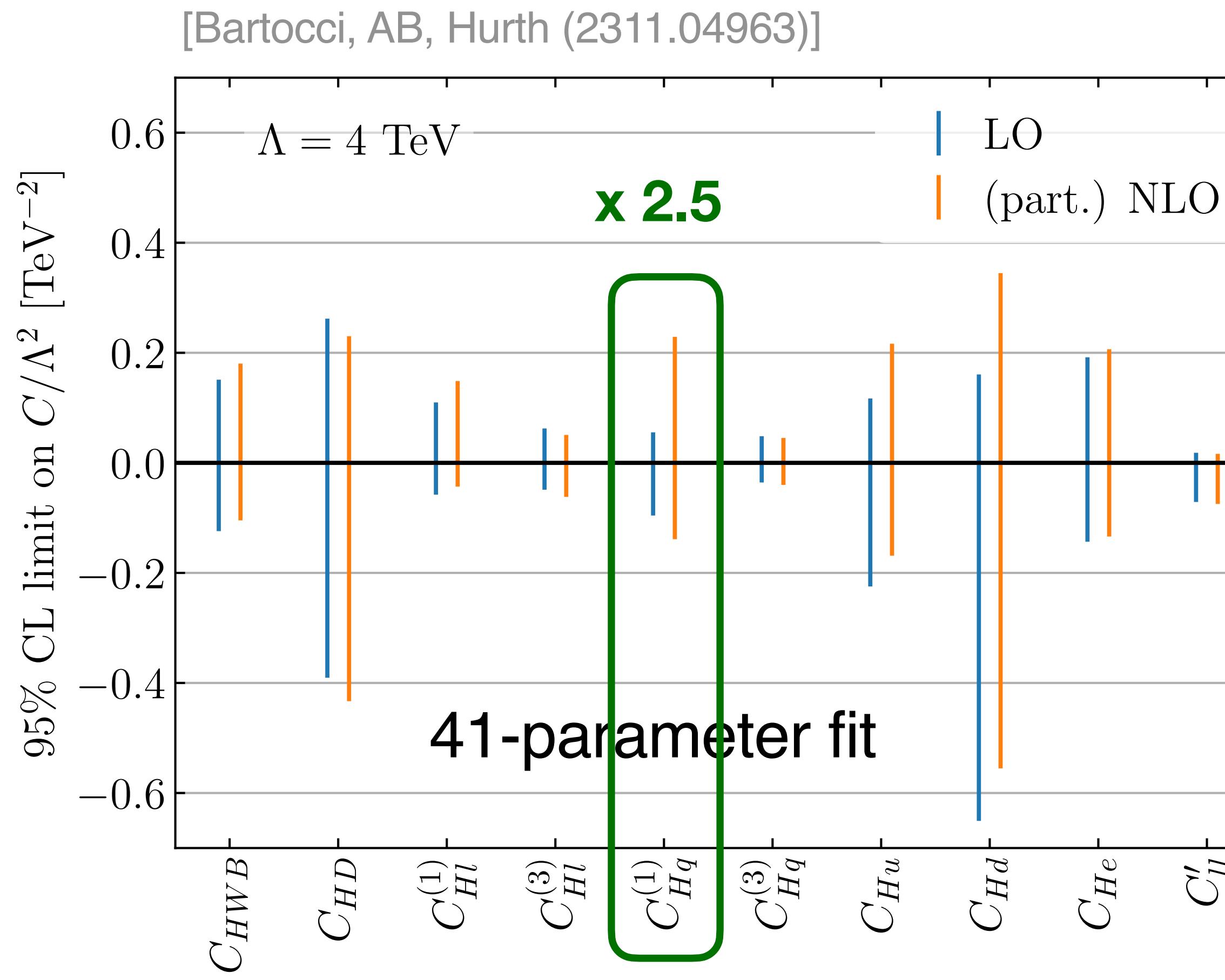
NLO degeneracies

Higgs self coupling [Alasfar, de Blas, Gröber (2202.02333)]



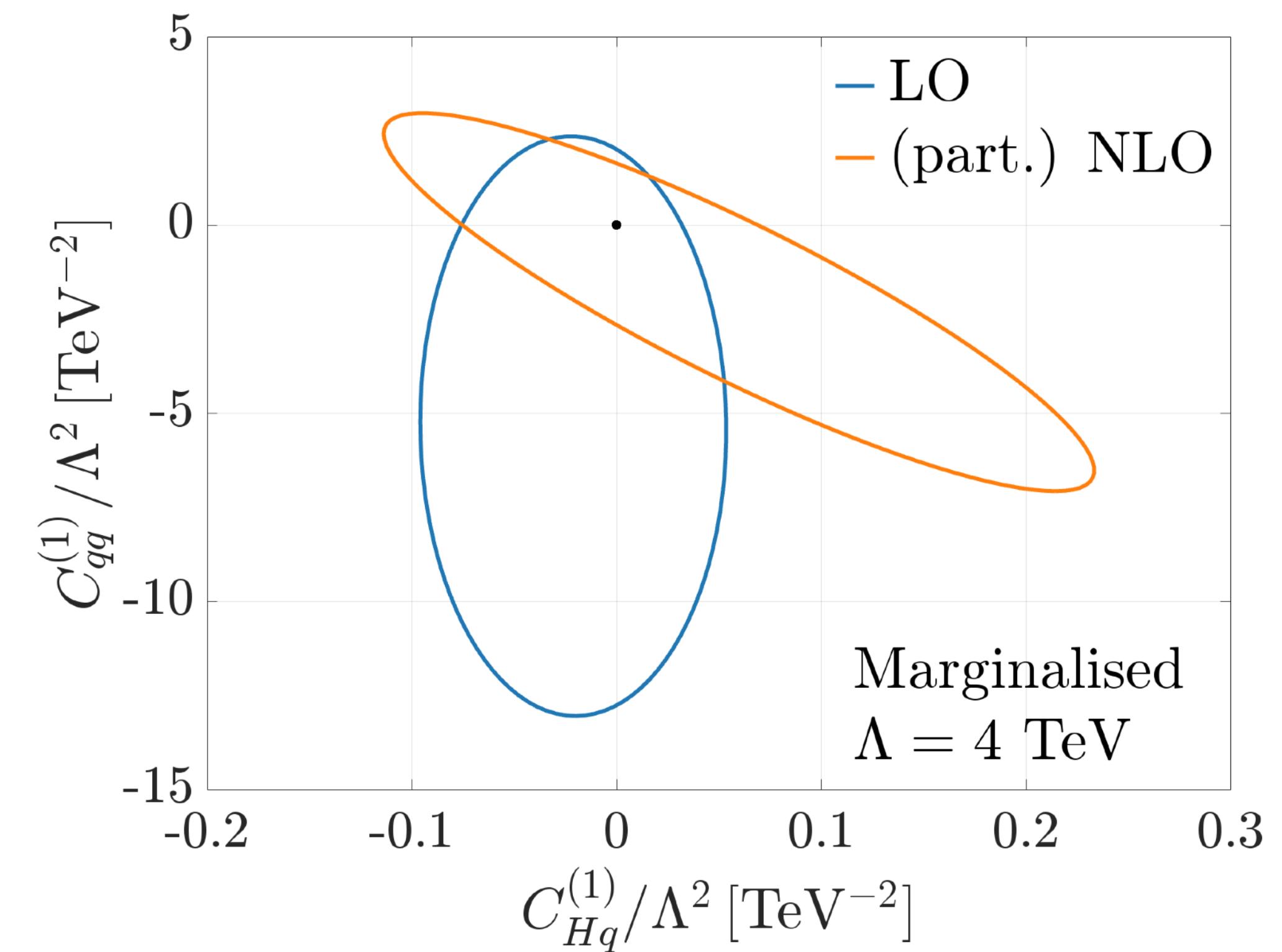
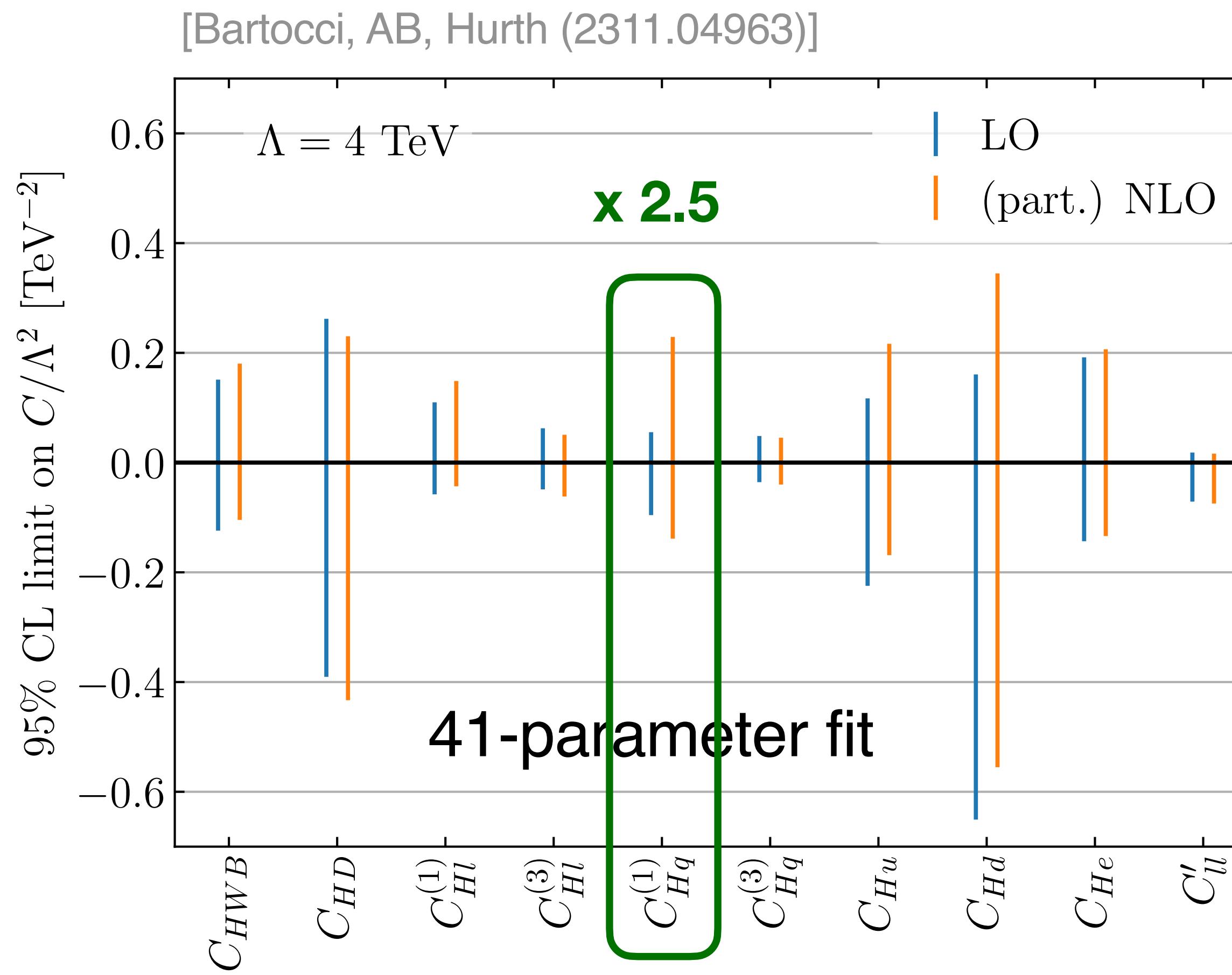
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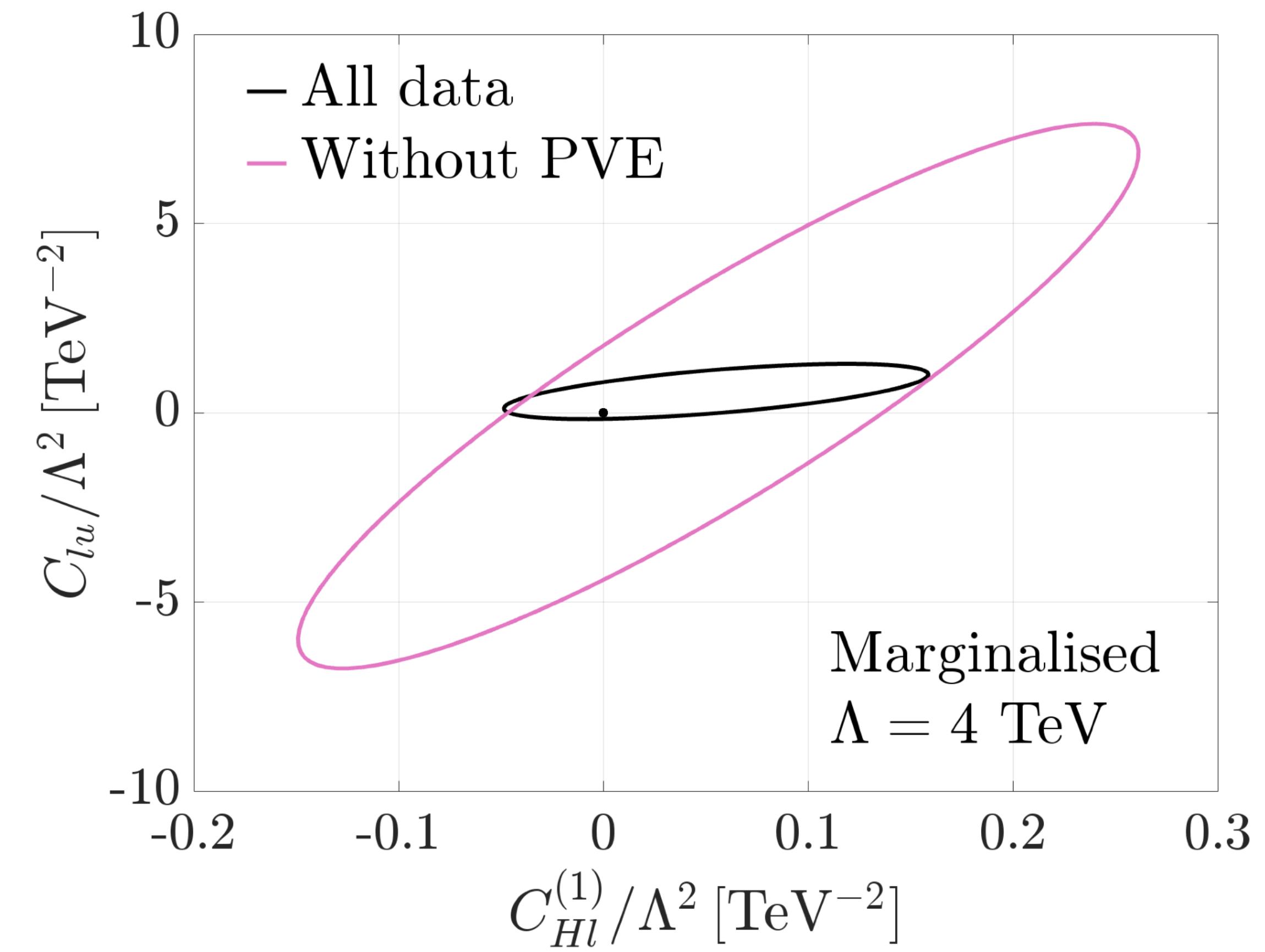
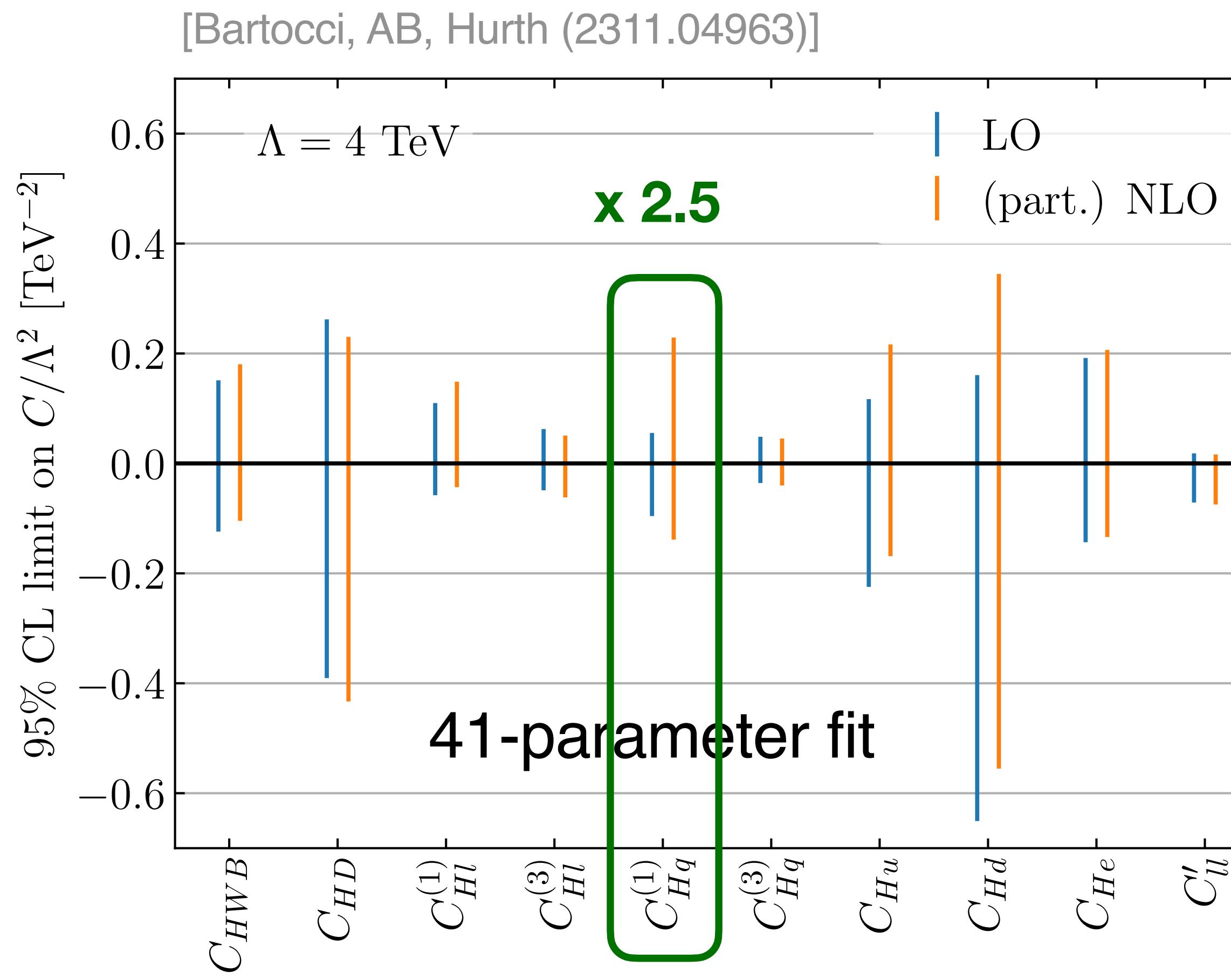
NLO degeneracies

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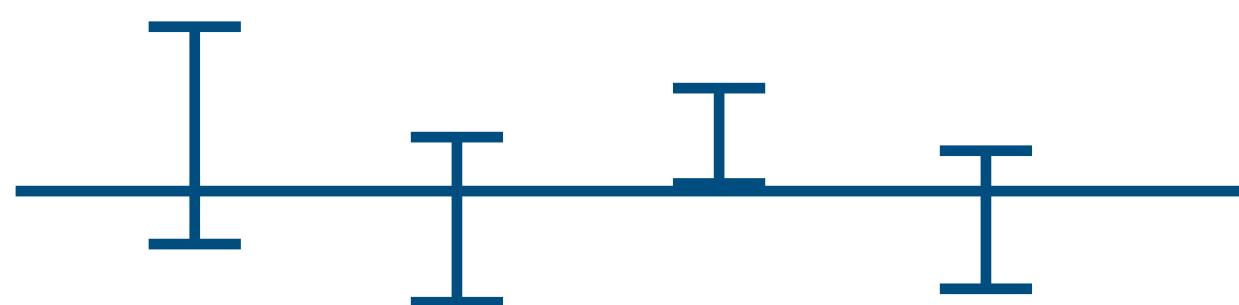
NLO degeneracies

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Conclusions

- Fitting theory-motivated operator sets is becoming a reality
- Degeneracies in NLO SMEFT predictions are manageable with current data
- Combination of many observables needed (different scales)



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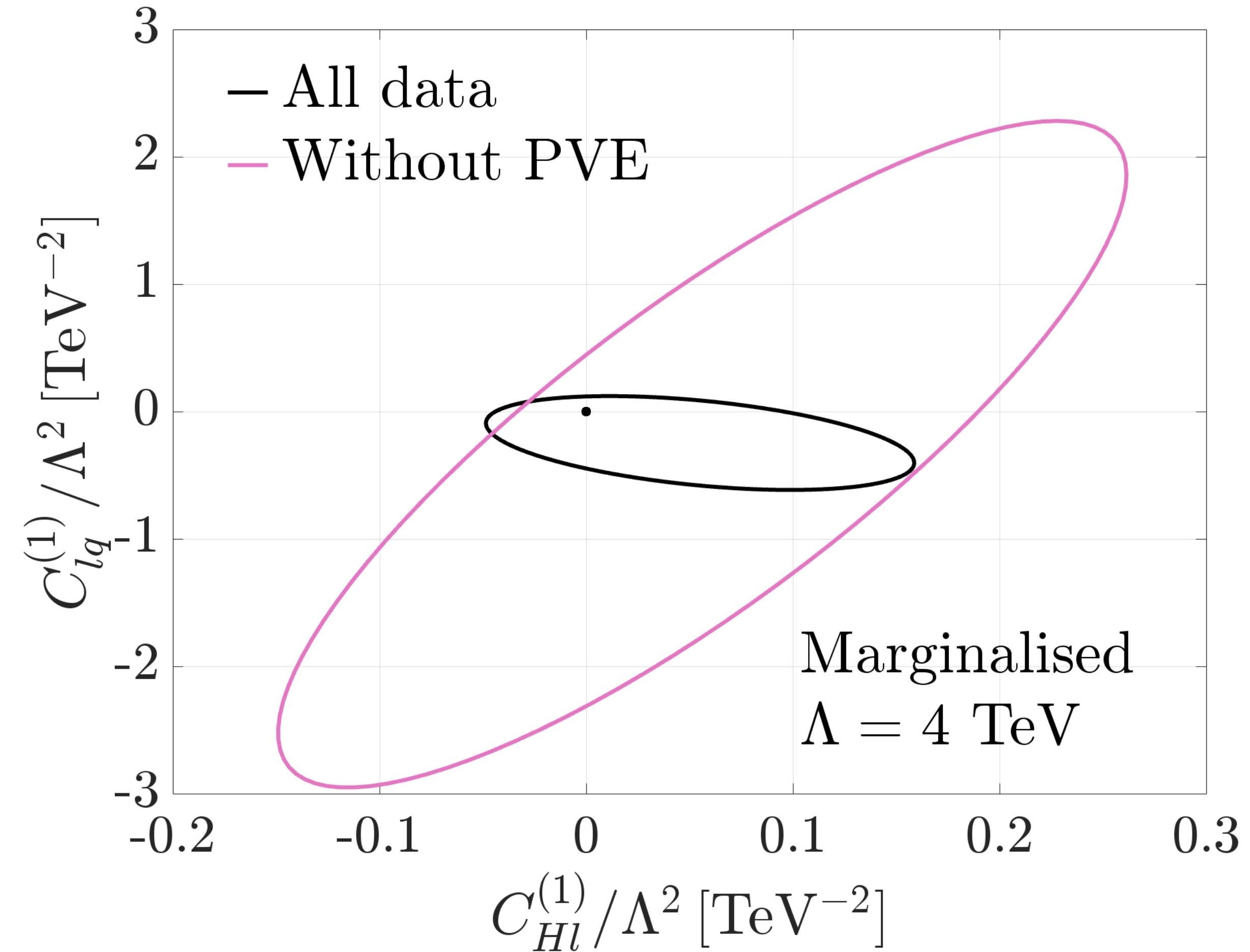
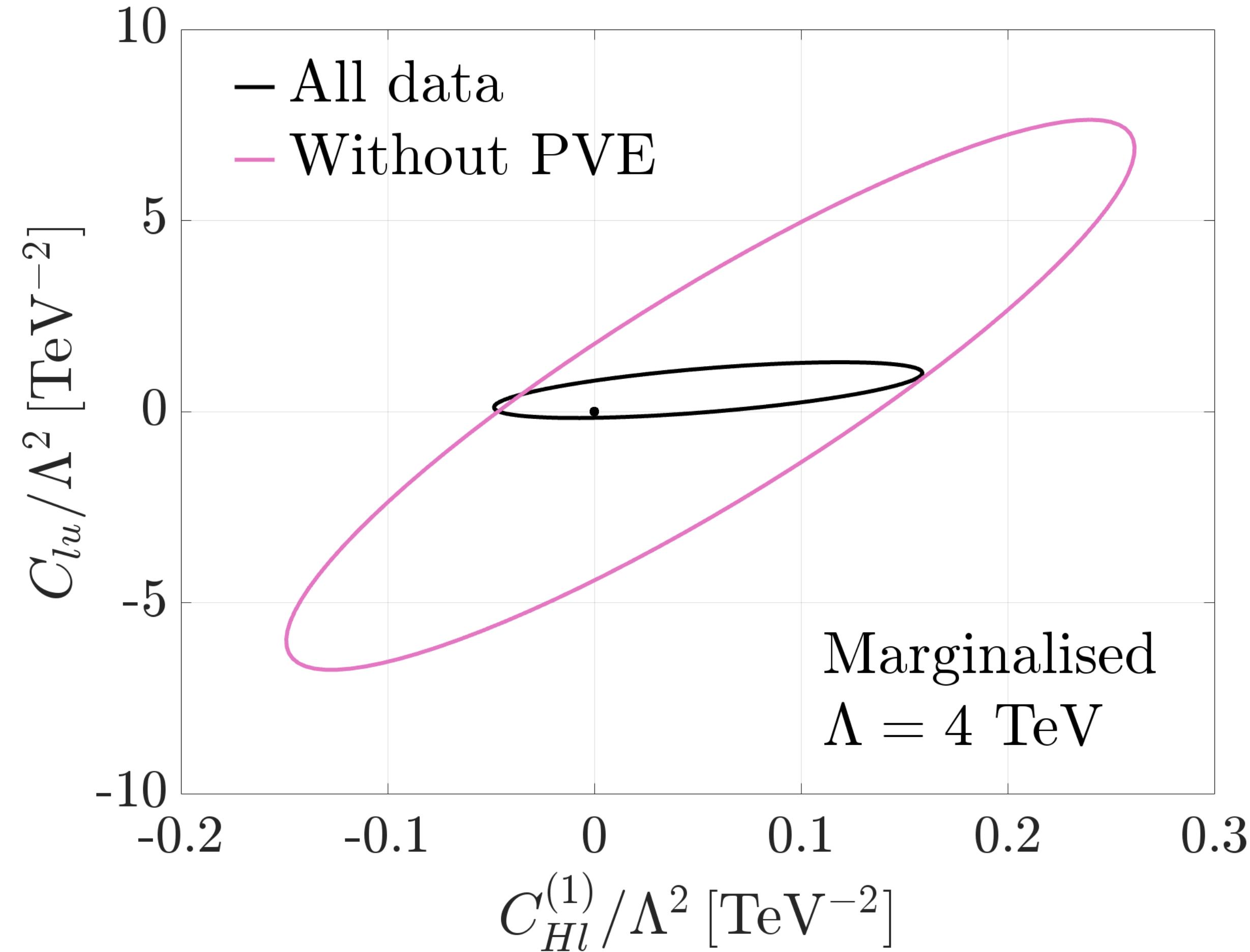
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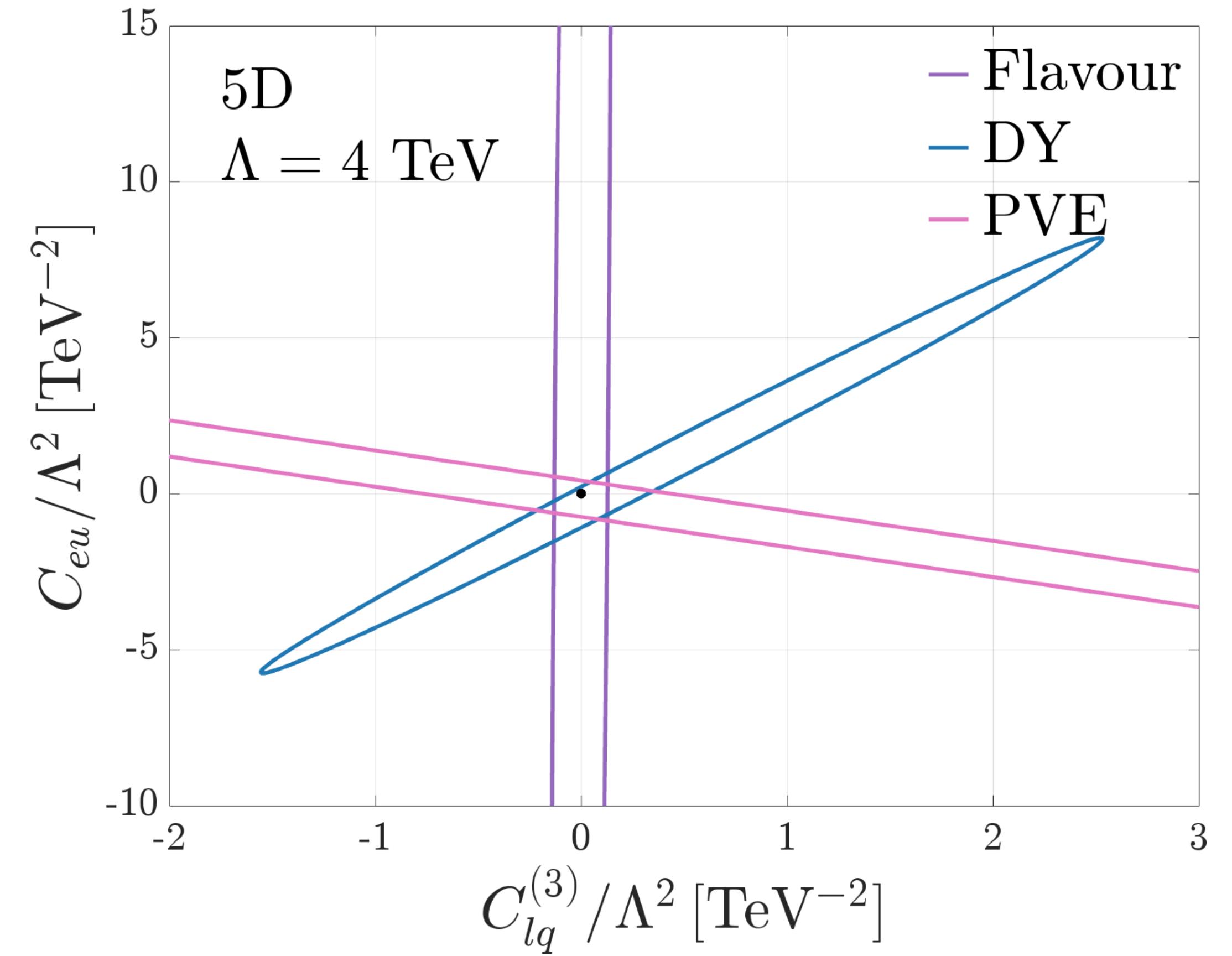
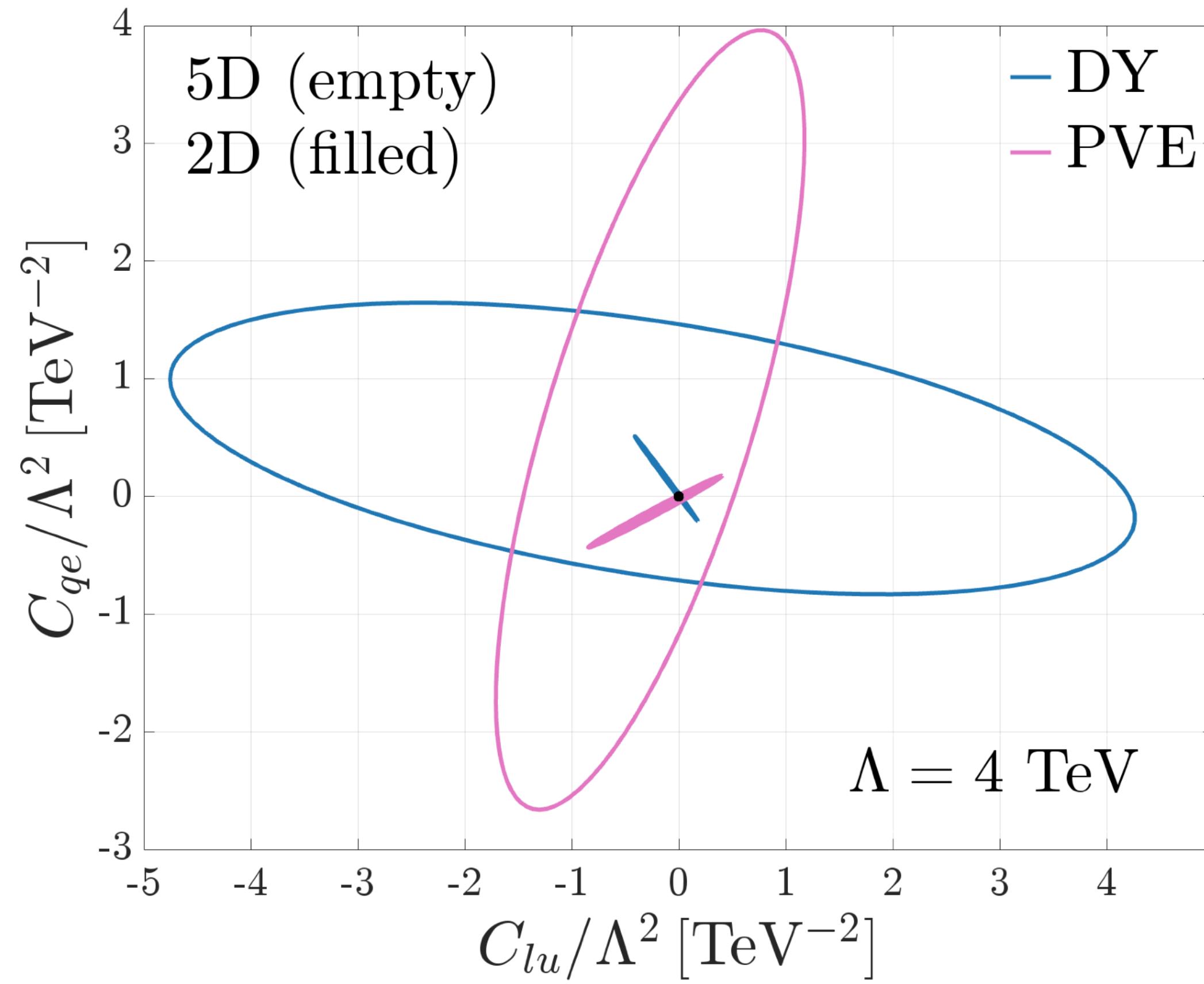
Thank you for your attention!

Backup

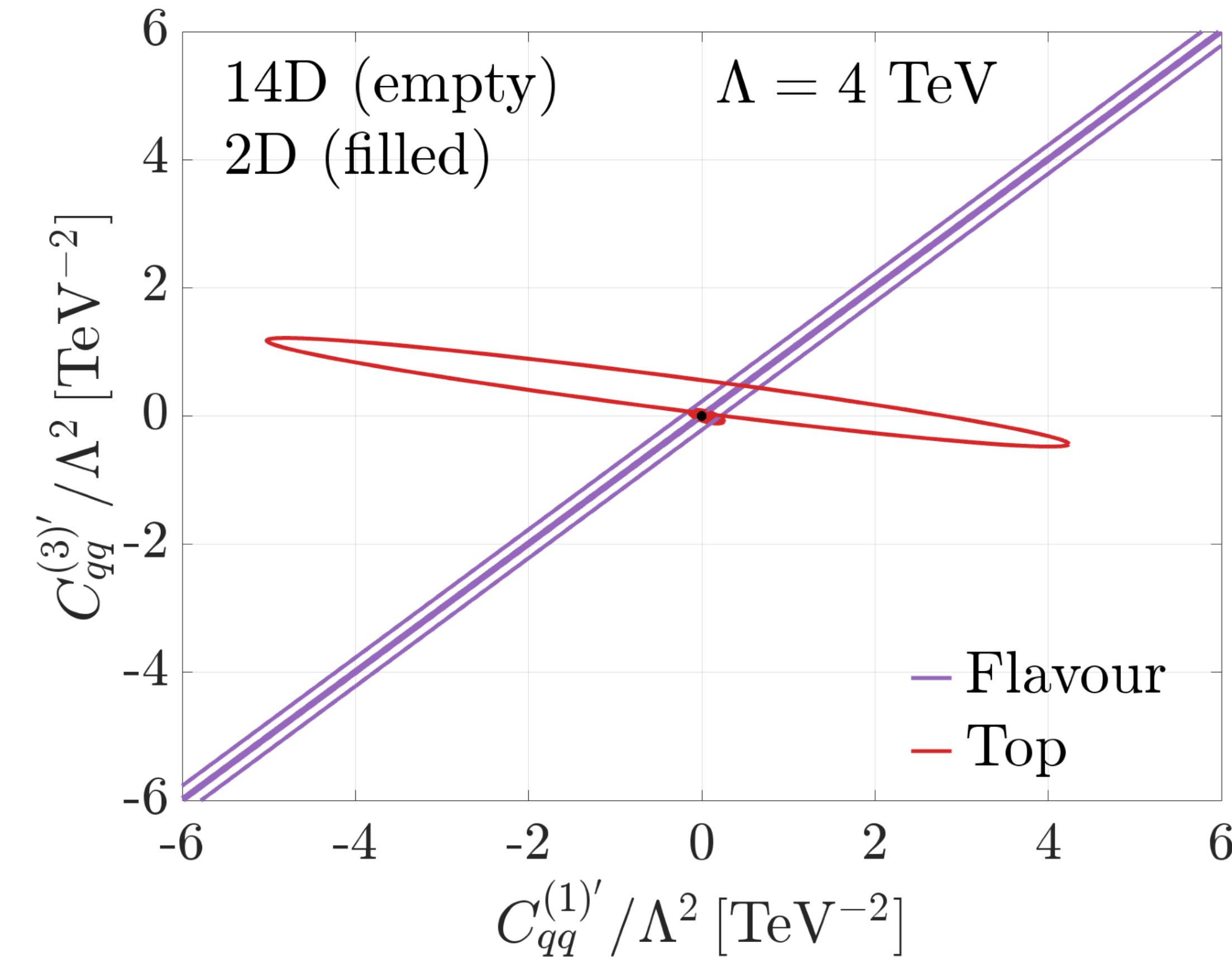
Correlations - PVE and EWPO



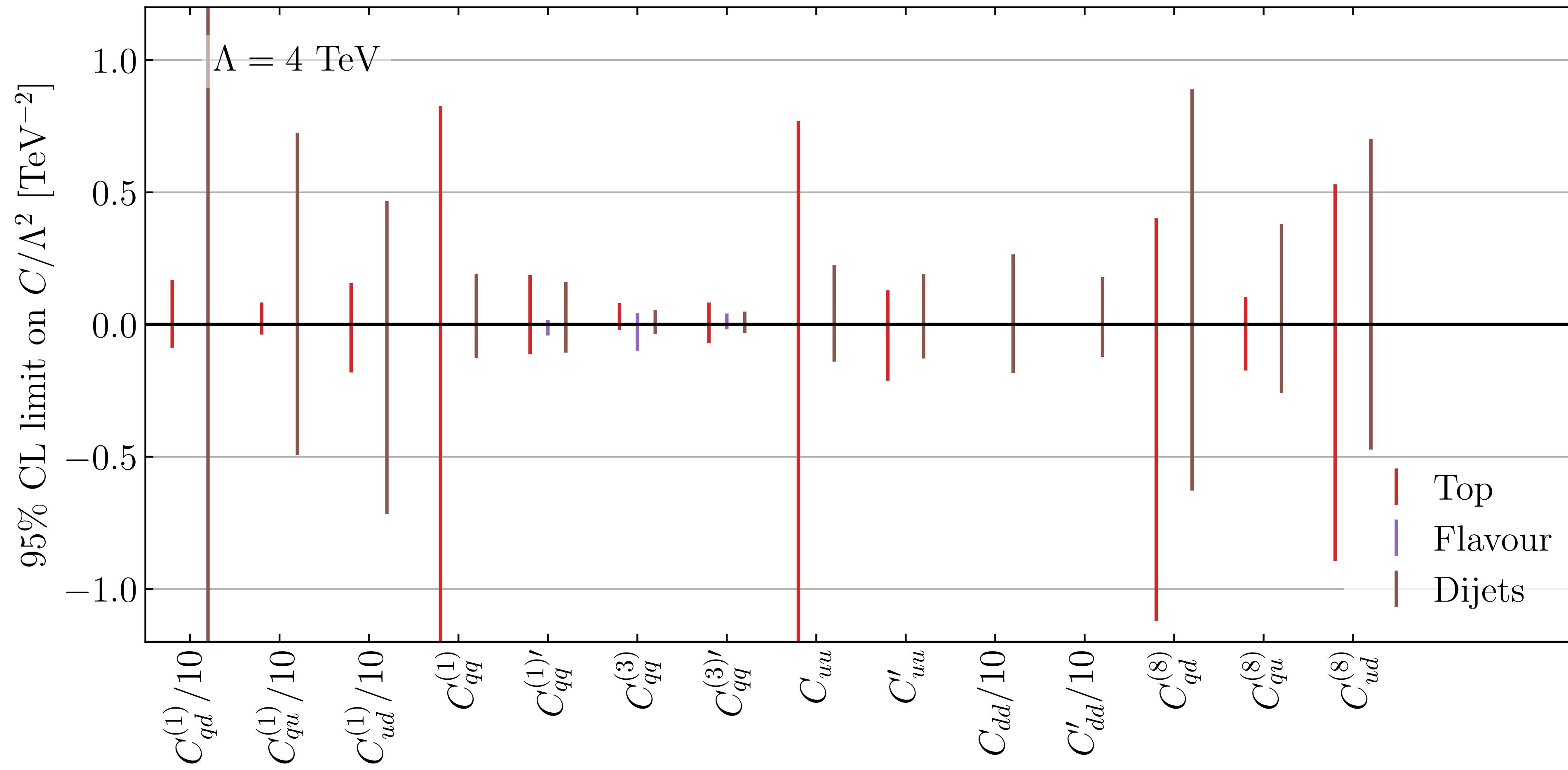
Correlations - PVE and DY



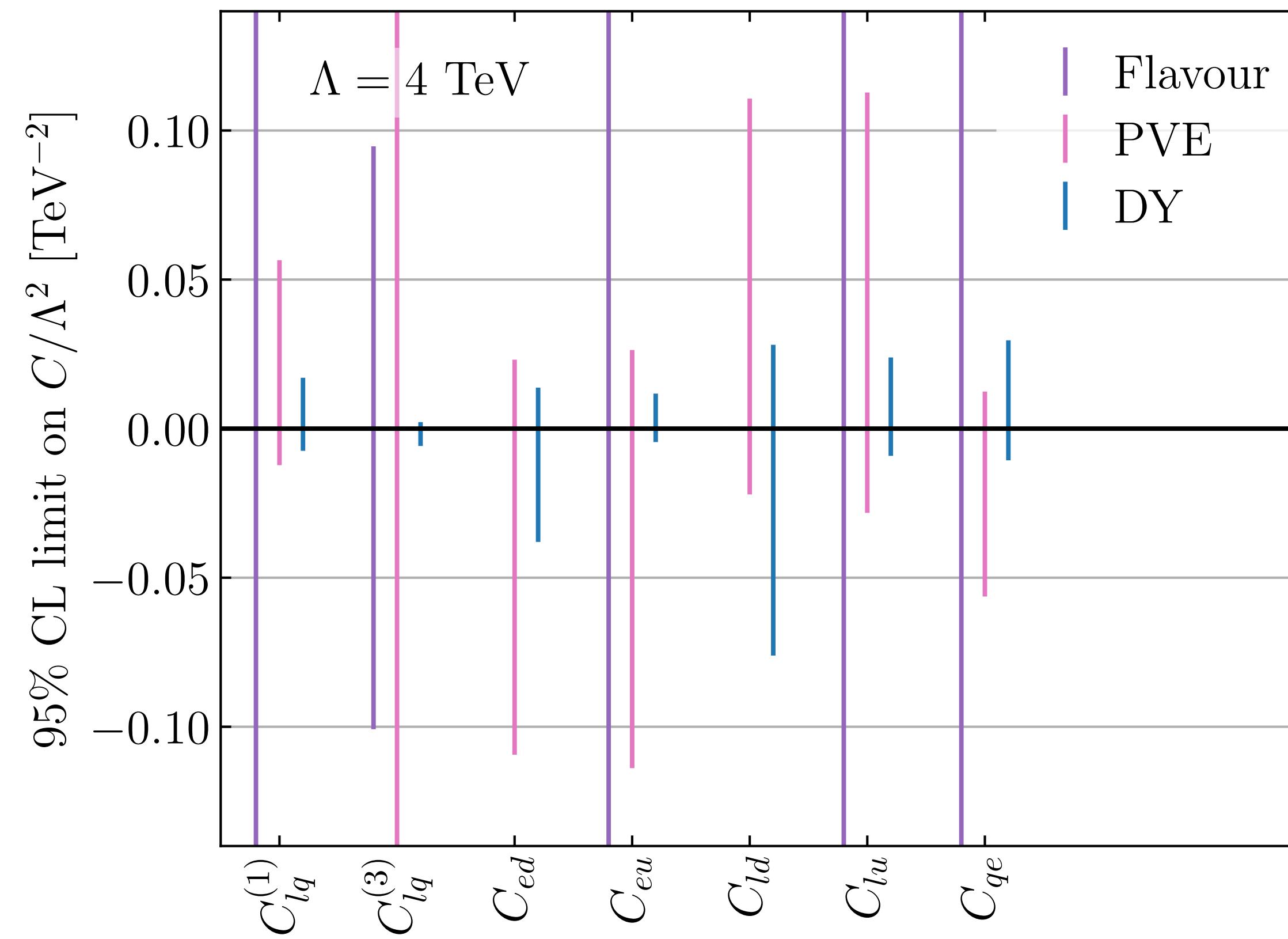
Correlations - top and flavor



1D fits - four-quark operators



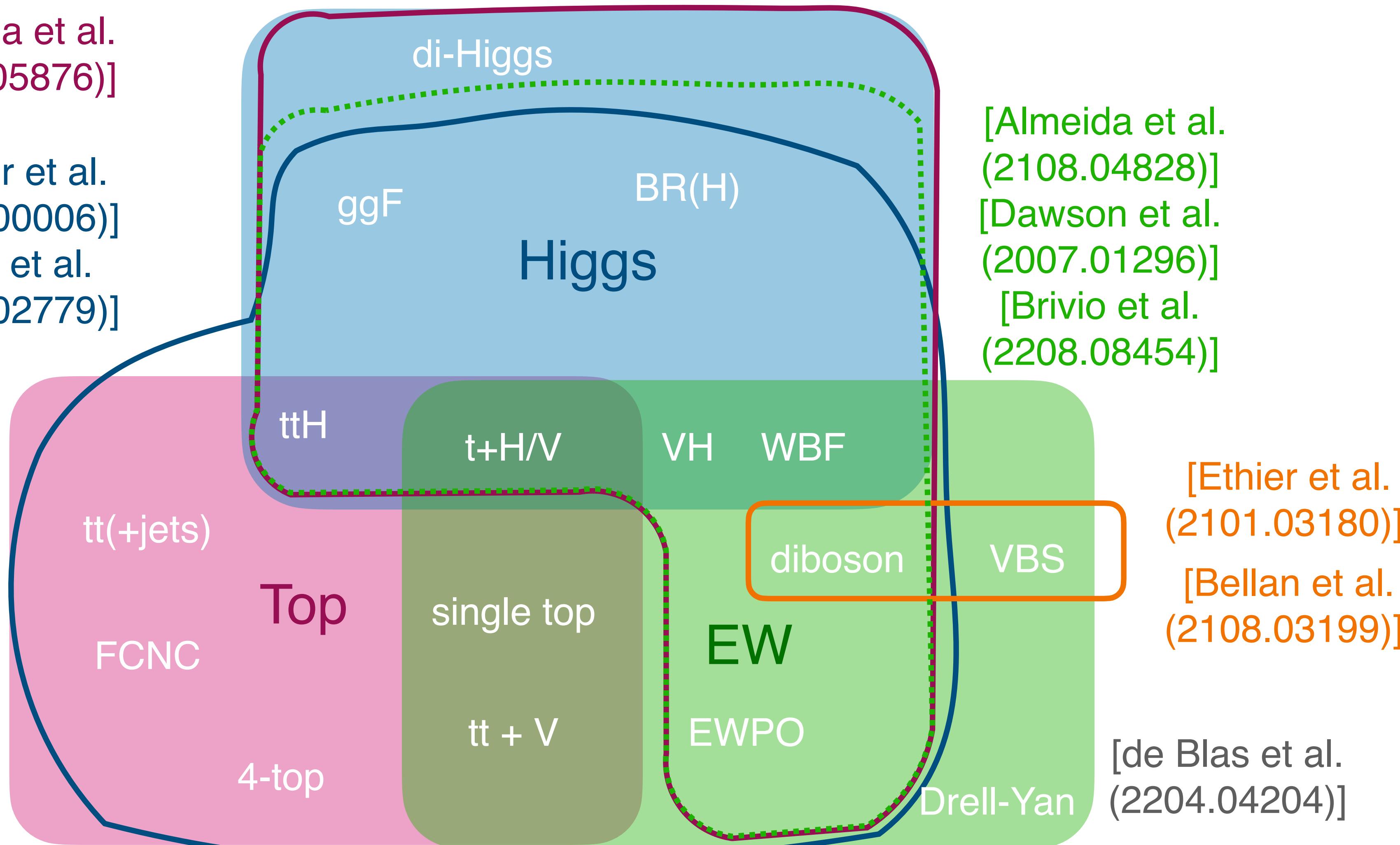
1D fits - semileptonic operators



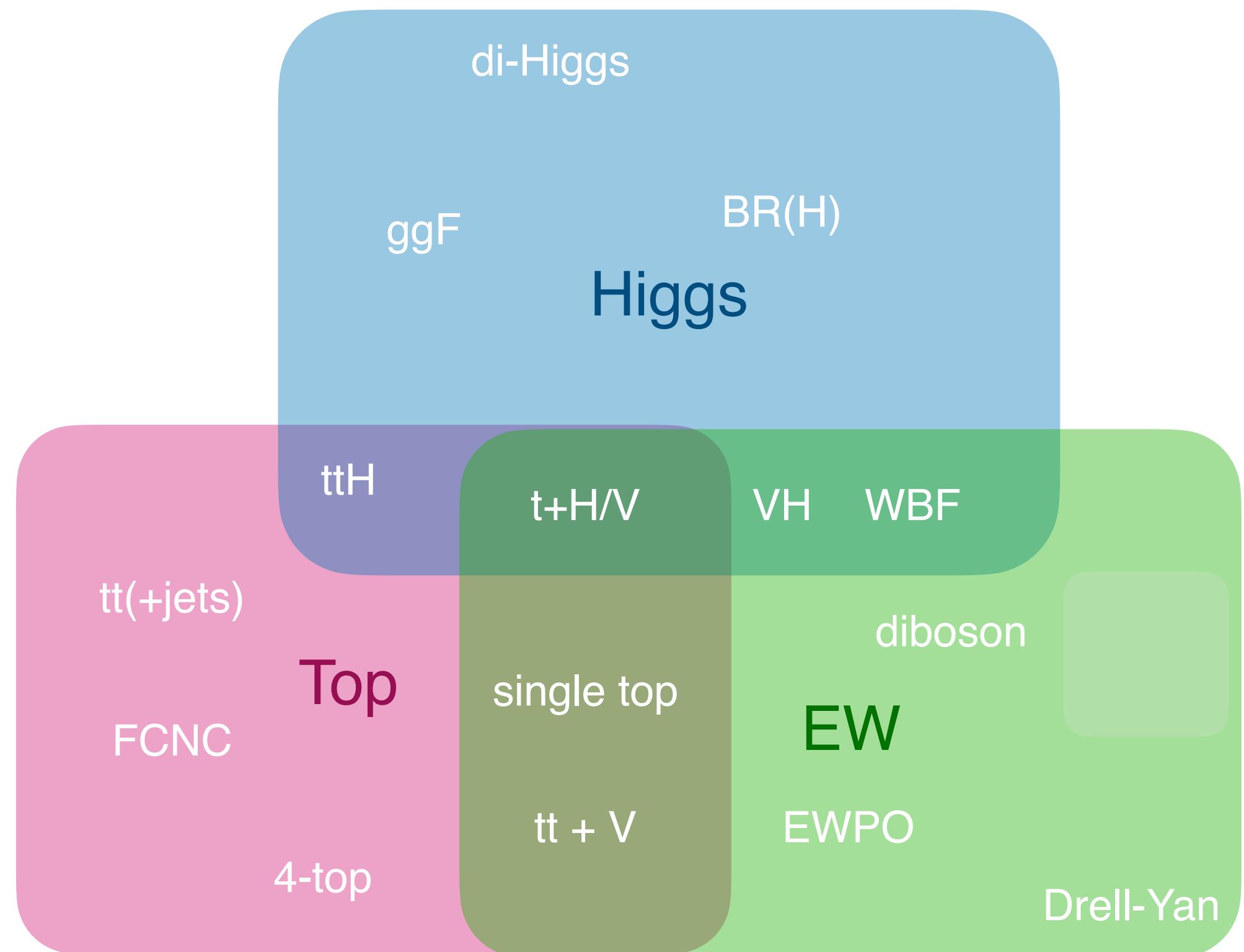
Confronting the SMEFT with data

[Anisha et al.
(2111.05876)]

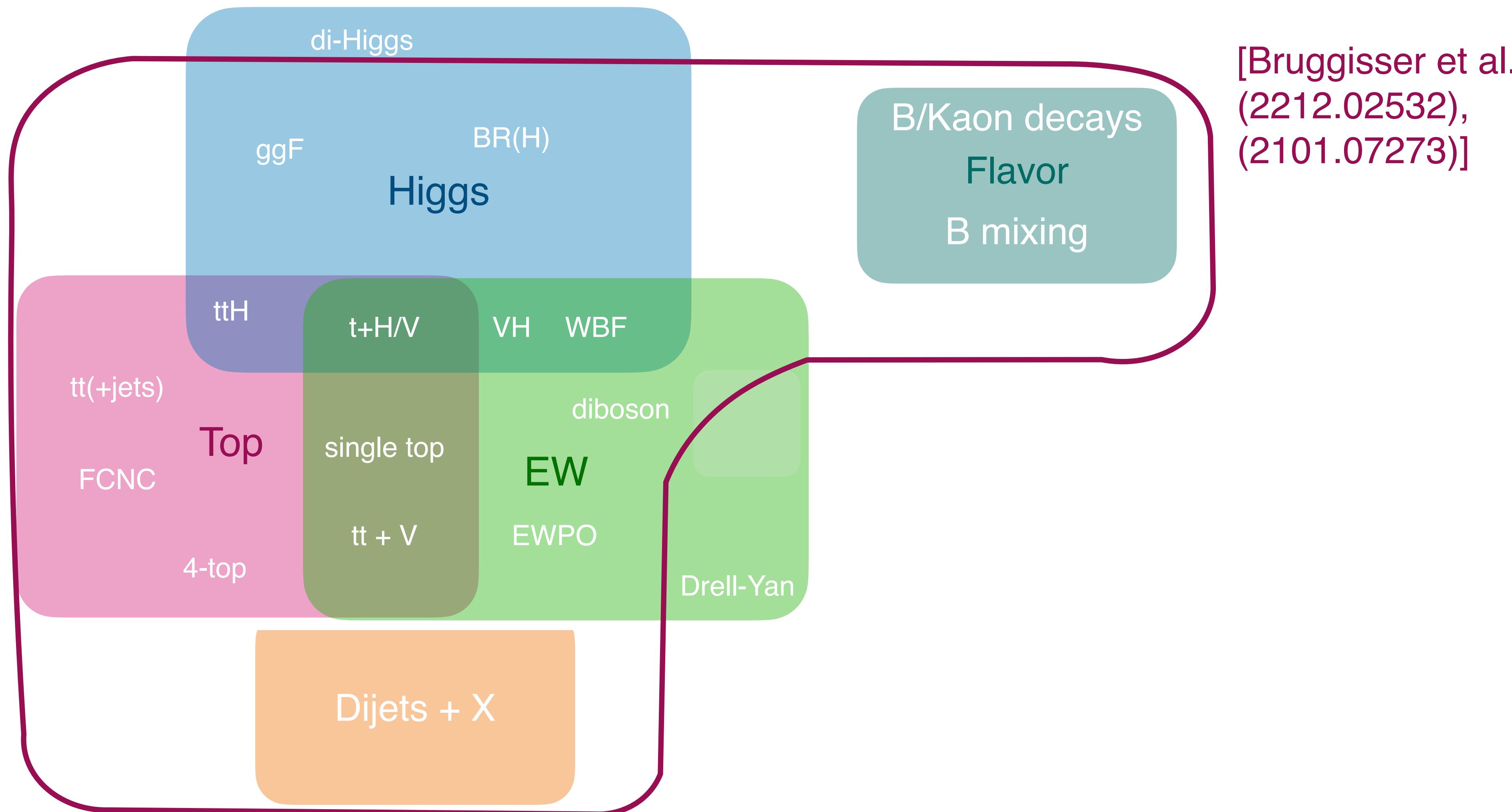
[Ethier et al.
(2105.00006)]
[Ellis et al.
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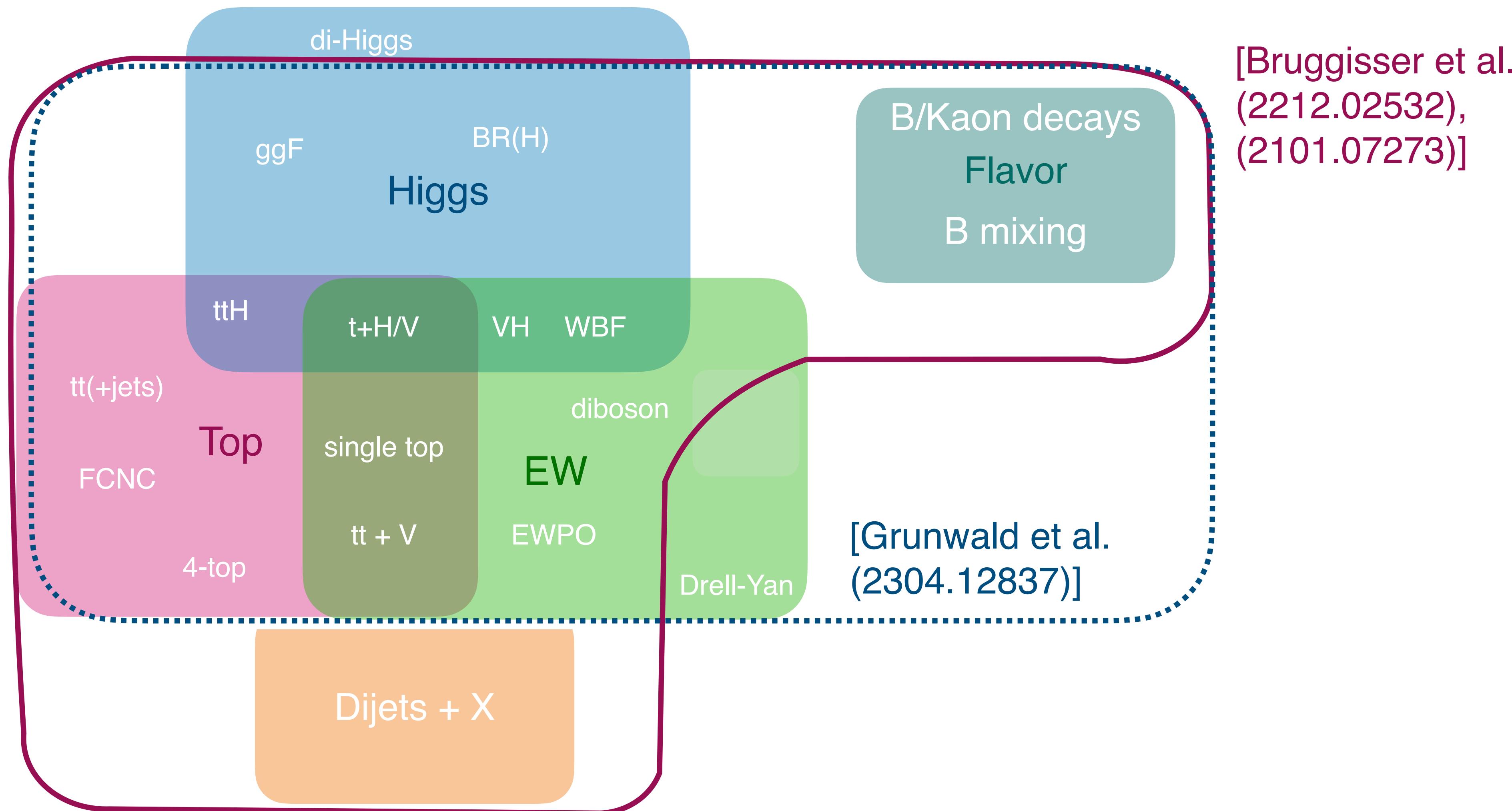
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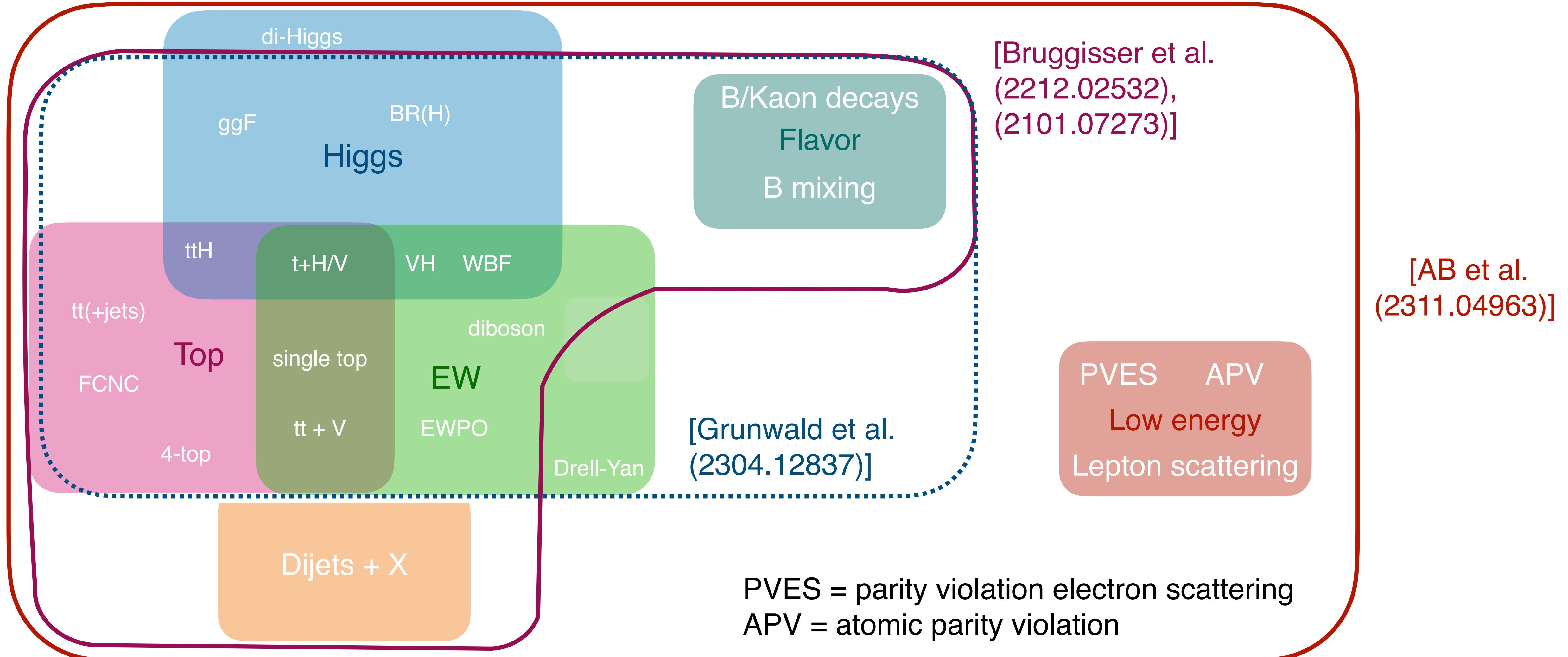
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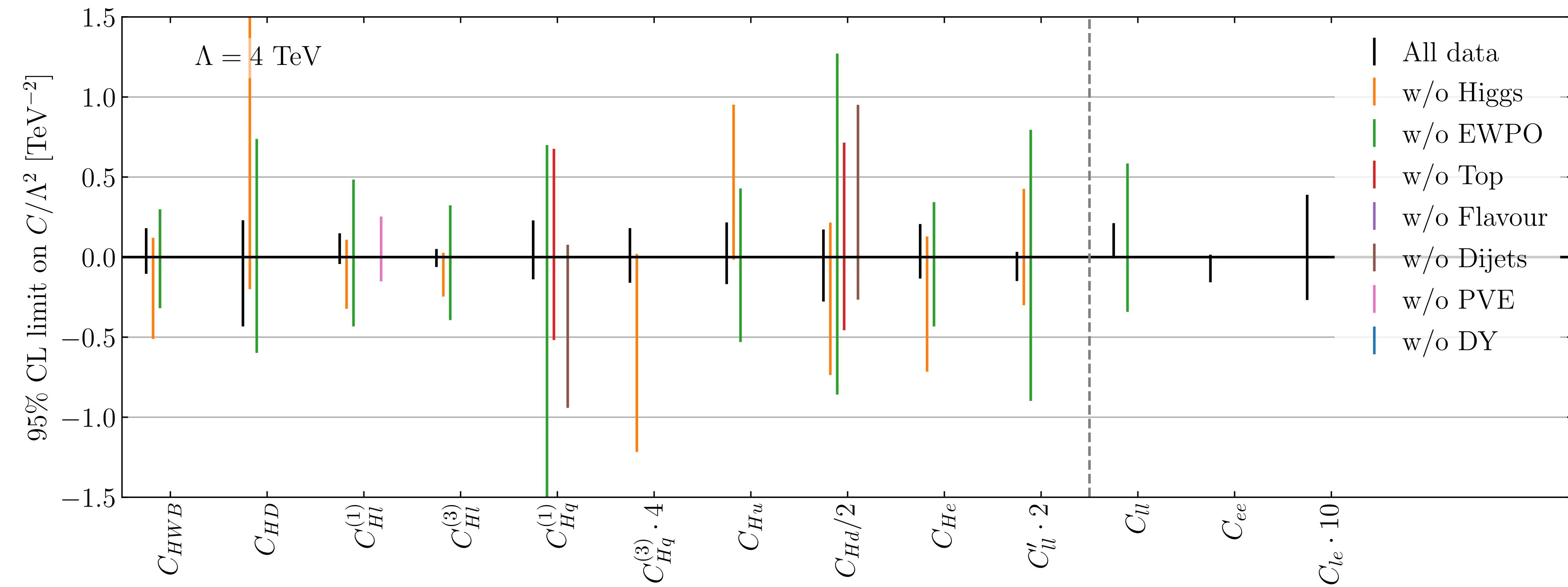
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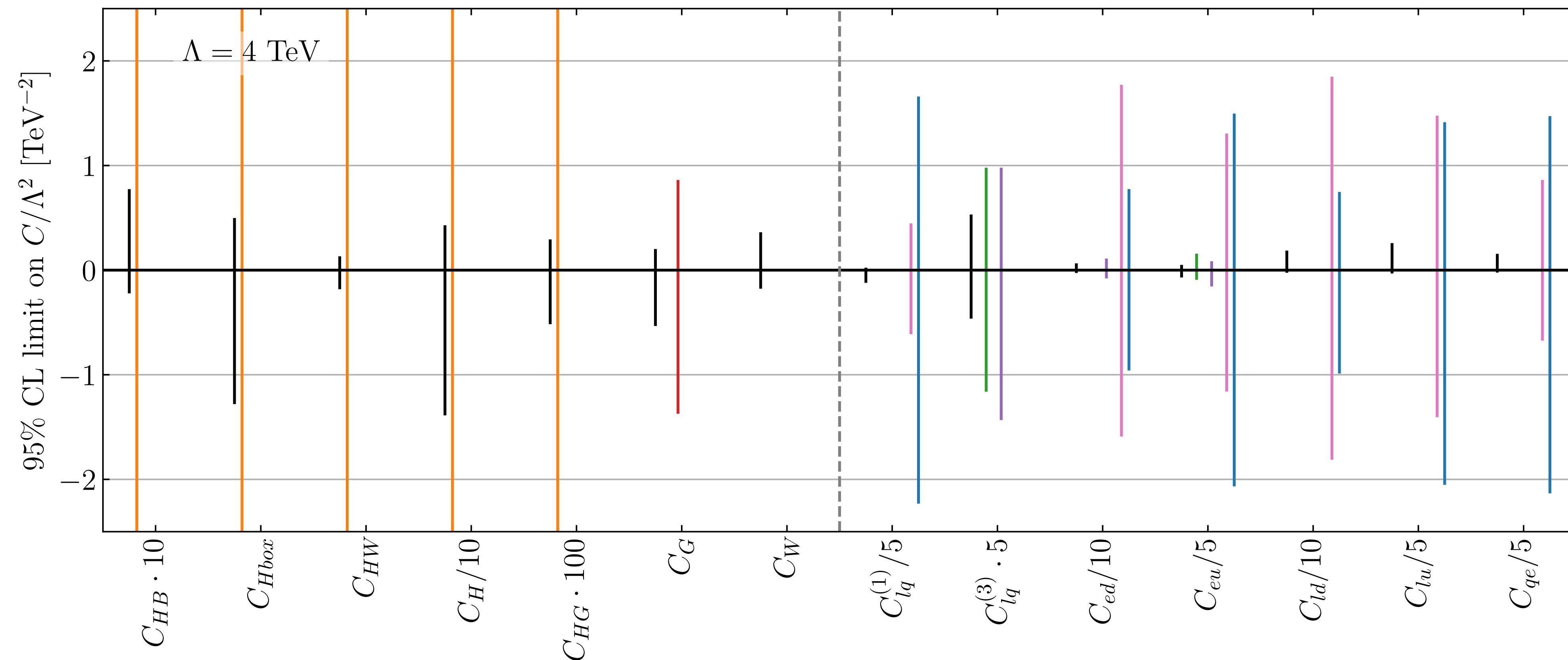
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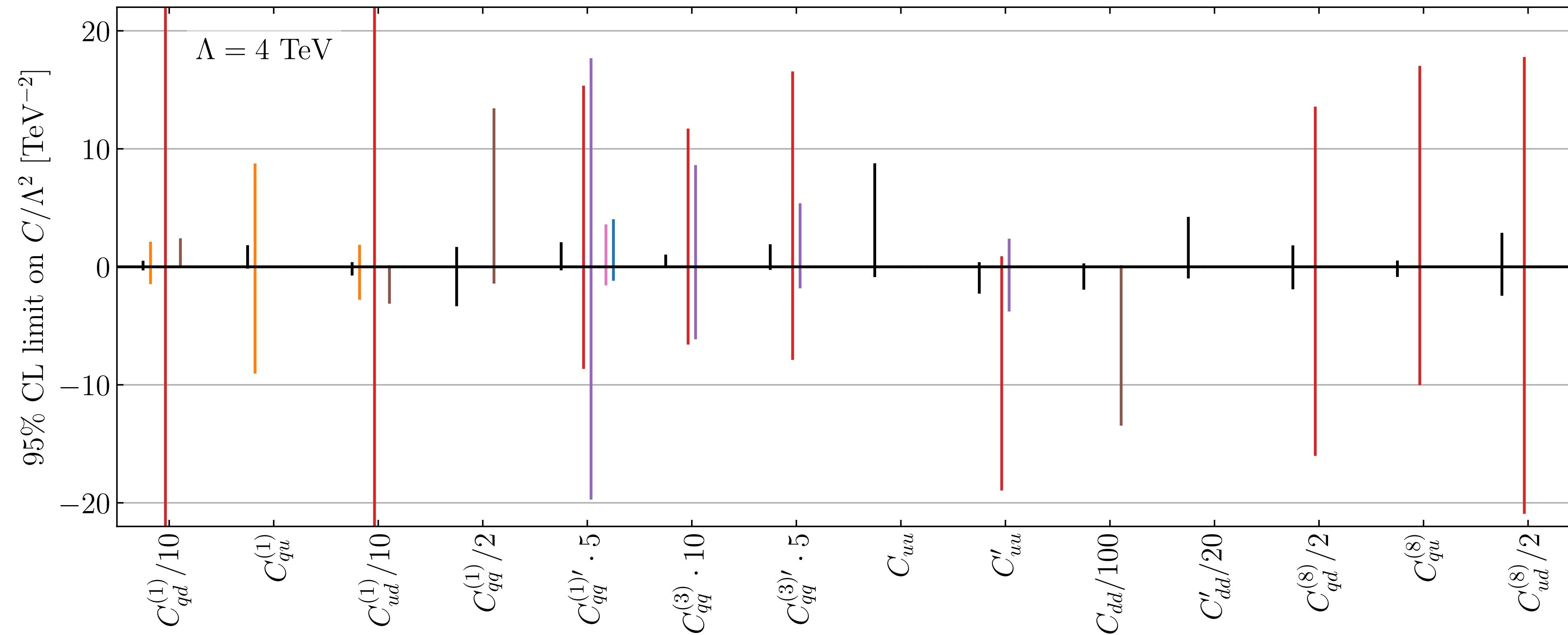
Removing operator sets



Removing operator sets - 2



Removing operator sets - 3



SMEFT fits - a global effort!

Filled to my best knowledge

	Eboli, Gonzalez-Garcia et al	Fitmaker	SFitter	TopFitter	HEPfit	SMEFit	Dawson et al.	Chakrabortty et al.
Input	EWPD+Higgs+VV, DY +VV	EWPD+Higgs+VV + top	EWPD+Higgs+VV, top	top	EWPD+Higgs+VV Flavor	EWPD+Higgs+VV, VBS + diboson, top	EWPD+Higgs+VV	EWPD + Higgs
Linear/quadratic	Both	Linear	Both	Linear	Linear	Both	Linear	Linear
Basis	HISZ	Warsaw	HISZ (Higgs) Warsaw (top)	Warsaw	Warsaw	Warsaw	Warsaw	Warsaw
EW scheme	Alpha	Alpha	Alpha	-	Alpha	mW	mW	Alpha
Flavor assumptions	$SU(3)^5$	$SU(3)^5$ $SU(2)^2 \times SU(3)^3$	$SU(3)^5$ $SU(2)^2 \times SU(3)^3$	$SU(3)^5$	$SU(3)^5$ general	$SU(2)^2 \times SU(3)^3$	$SU(2)^2 \times SU(3)^3$	$SU(3)^5$
NLO QCD included	LO	Top only	Top only	LO	LO	Top only	Vh, diboson, EWPO	EWPO only
Fitting procedure	Chi2	Bayesian	Toy MC, Chi2, Bayesian	Chi2	Bayesian	Toy MC	Chi2	Bayesian
Uncertainties	Gaussian, theory correlated	Gauss	Gauss, Poisson, flat	Gauss	(Asymmetric) Gauss, flat	Gauss	Gauss, uncorrelated	Gauss
UV complete model fits	✗	✓	✓	✓	✓	✗	✓	✓
Specialties	VV + DY	Higgs + EWPO + top + diboson	Correlation of uncertainty classes	Top	Projections	CP odd operators VBS	NLO for VV and Vh	UV complete models
References	1211.4580, 1509.01585, 1805.11108, 1812.01009, 2108.04828	1404.3667, 1803.03252, 2012.02779	1308.1979, 1505.05516, 1604.03105, 1812.07587, 1910.03606	1506.08845, 1512.03360, 1901.03164	1710.0540, 1905.03764, 1907.04311, 1910.14012	1901.05965, 1906.05296, 2101.03180	2007.01296	2009.13394, 2010.04088, 2012.03839, 2111.05876

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Linear/quadratic	Both	Linear	Both	Linear	Linear	Both	Linear	Linear
Basis	HISZ	Warsaw	HISZ (Higgs) Warsaw (top)	Warsaw	Warsaw	Warsaw	Warsaw	Warsaw
EW scheme	Alpha	Alpha	Alpha		Alpha	mW	mW	Alpha
Uncertainties	Correlated, fully correlated	Gauss	Correlated, mostly flat	Gauss	(mostly) Gauss, flat	Gauss	Correlated, uncorrelated	Gauss
UV complete model fits	✗	✓	✓	✓	✓	✗	✓	✓
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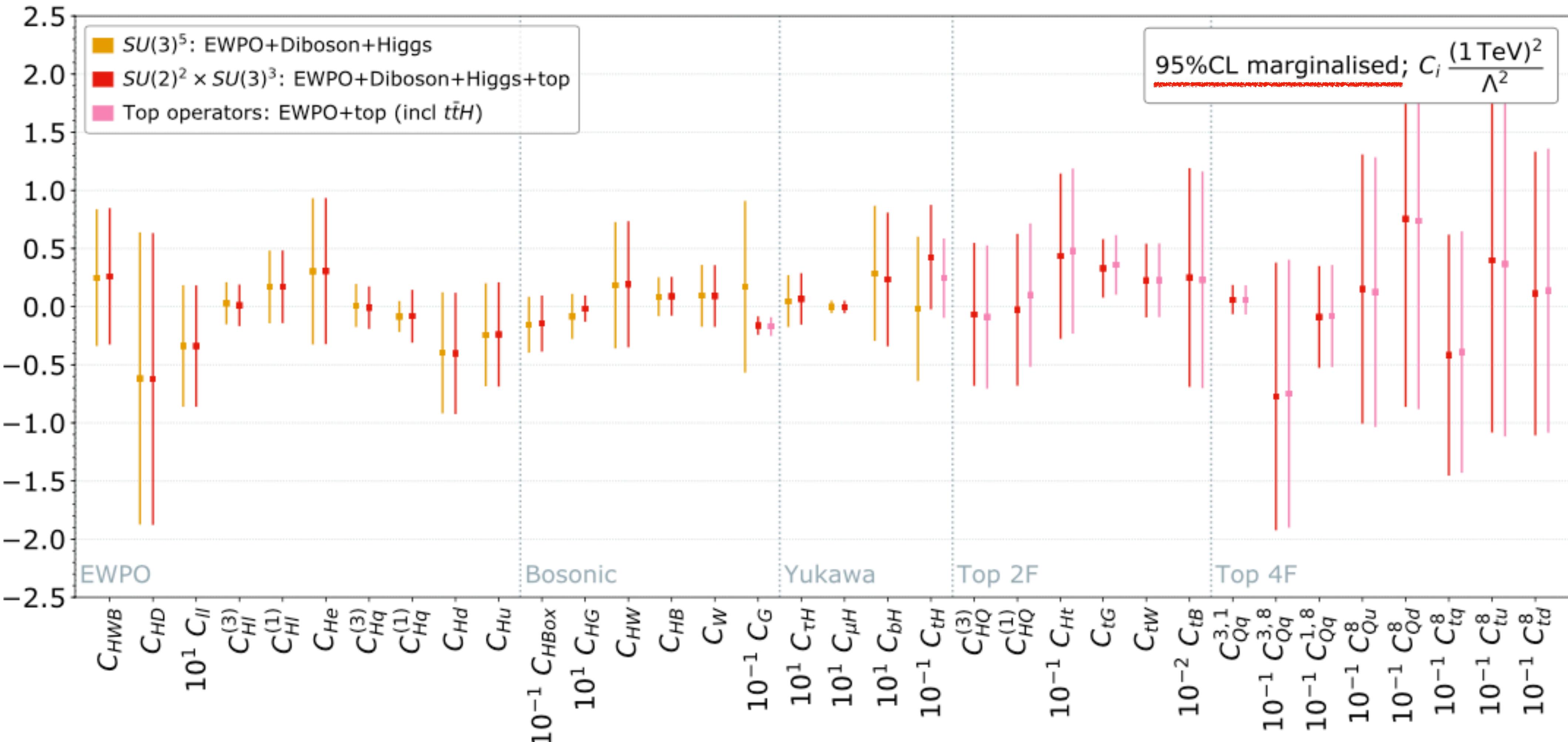
Many groups contribute to the field.
Each of them has their own strength.

Making the top quark stand out

Combining different sectors, e.g. top + Higgs

[Ellis, Madigan, Mimasu, Sanz, You (2012.02779)]

[Ethier et al. (2105.00006)]

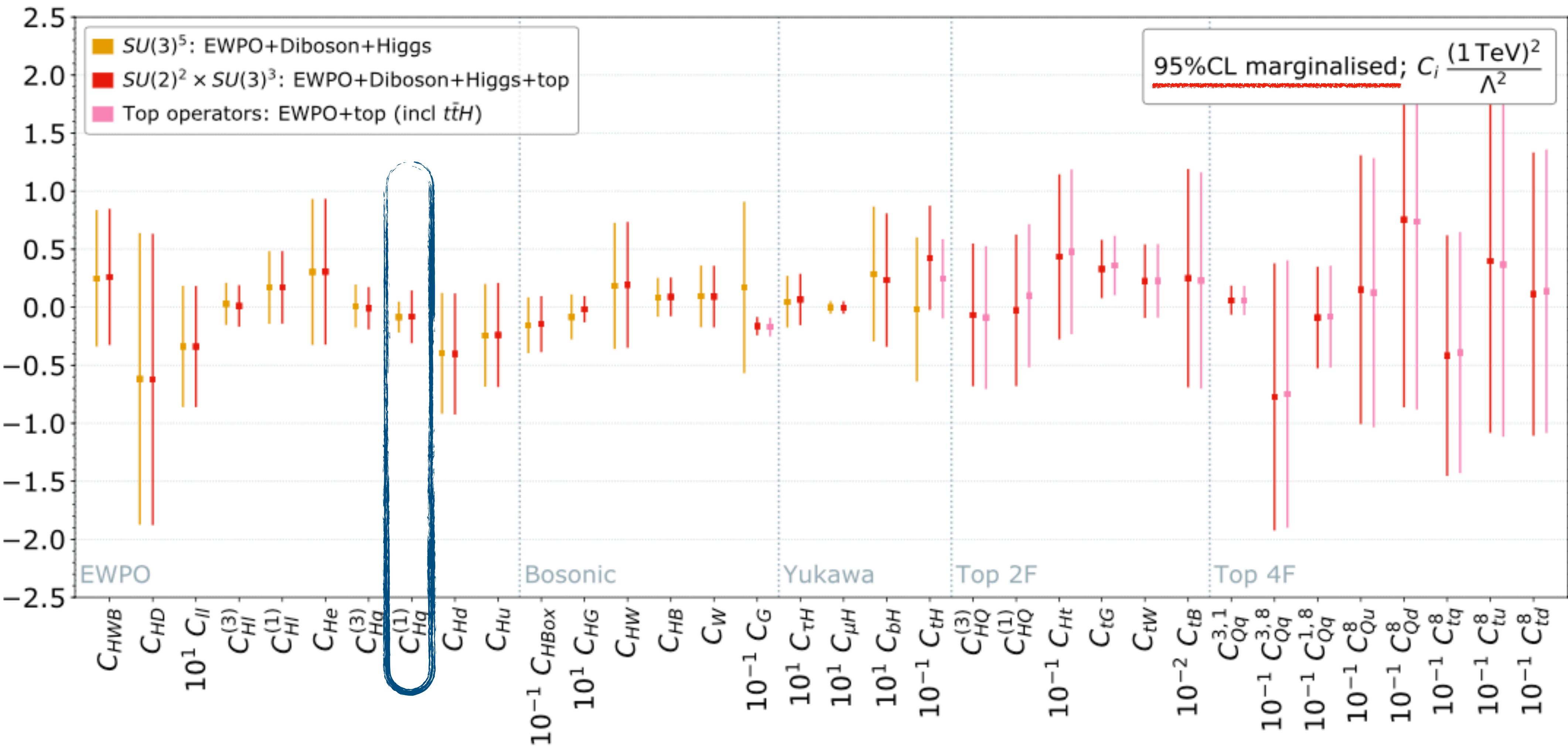


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- Generate a random "toy"-measurement according to the uncertainties
- Construct the Likelihood (or χ^2 equivalently)
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- Report result in Histogram

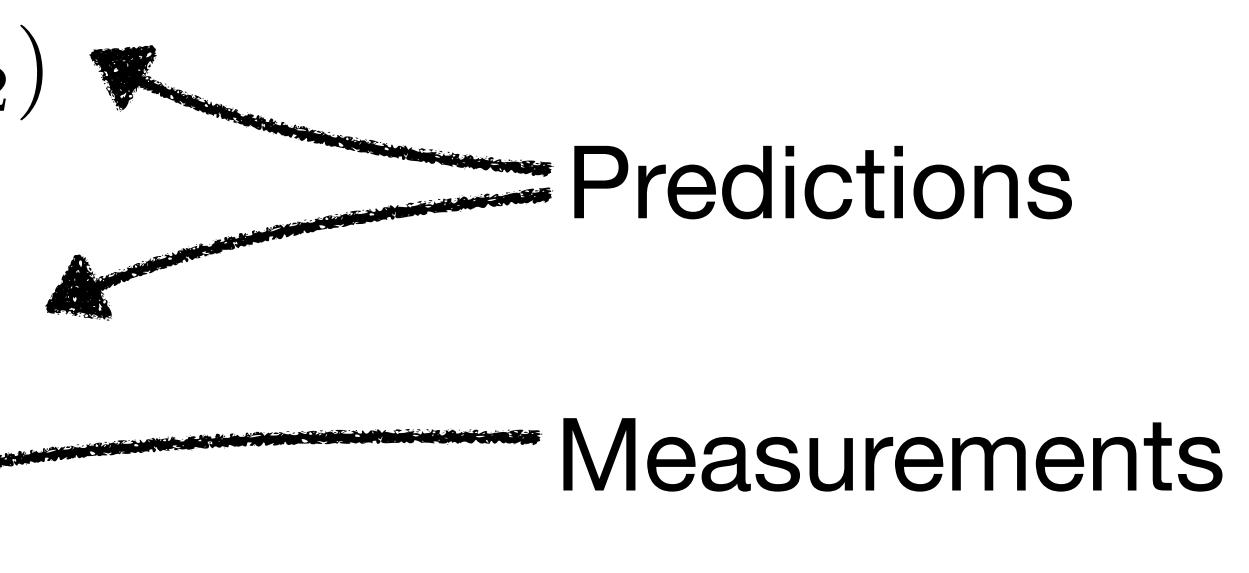
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$$\begin{aligned}\mu_1 &= 105 - 5(c_1 + c_2) \\ \mu_2 &= 93 + 3(c_1 - c_2) \\ \mu_{1/2}^{exp} &= 100 \pm 10\end{aligned}$$

Predictions

Measurements

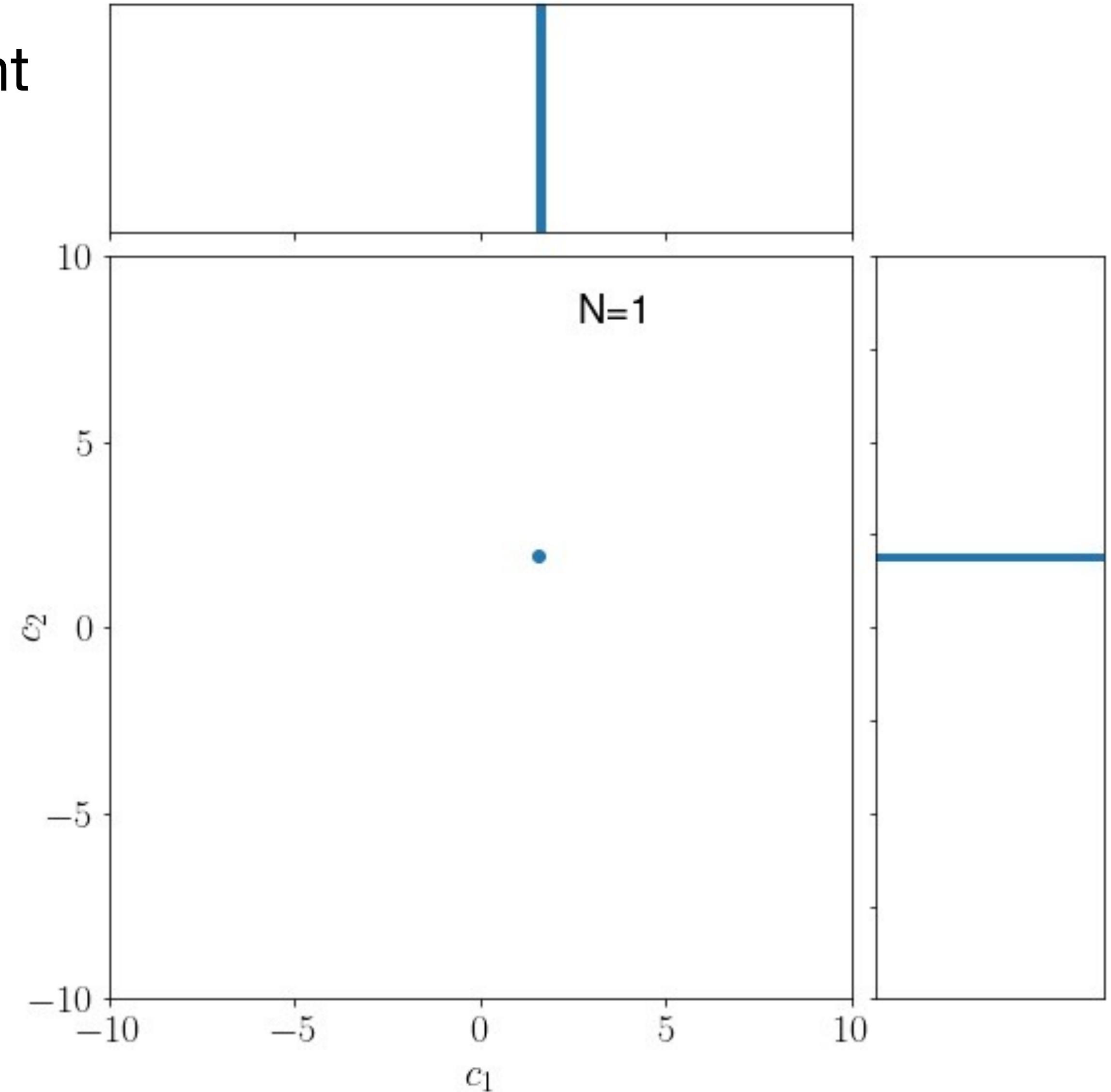


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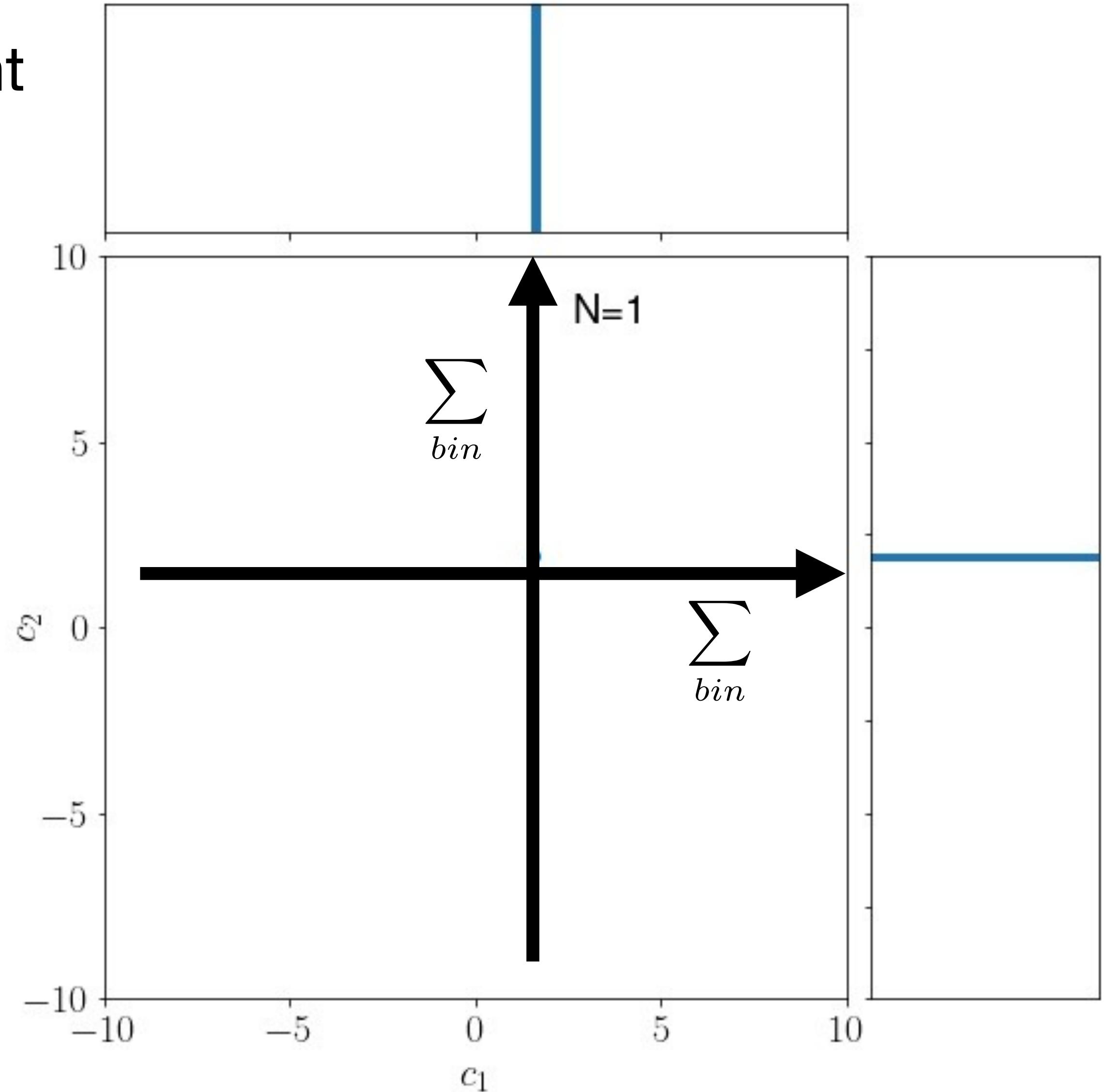
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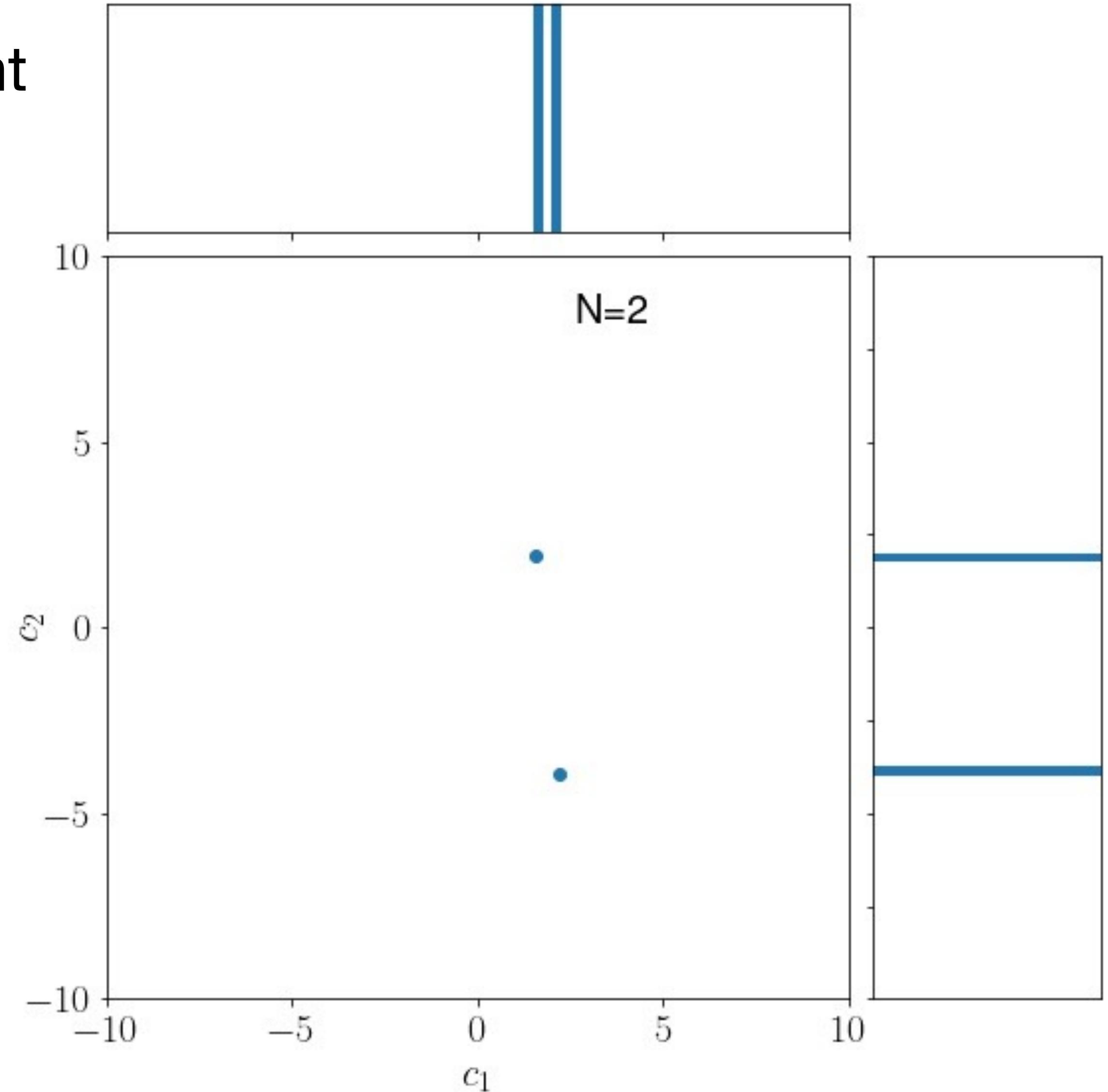


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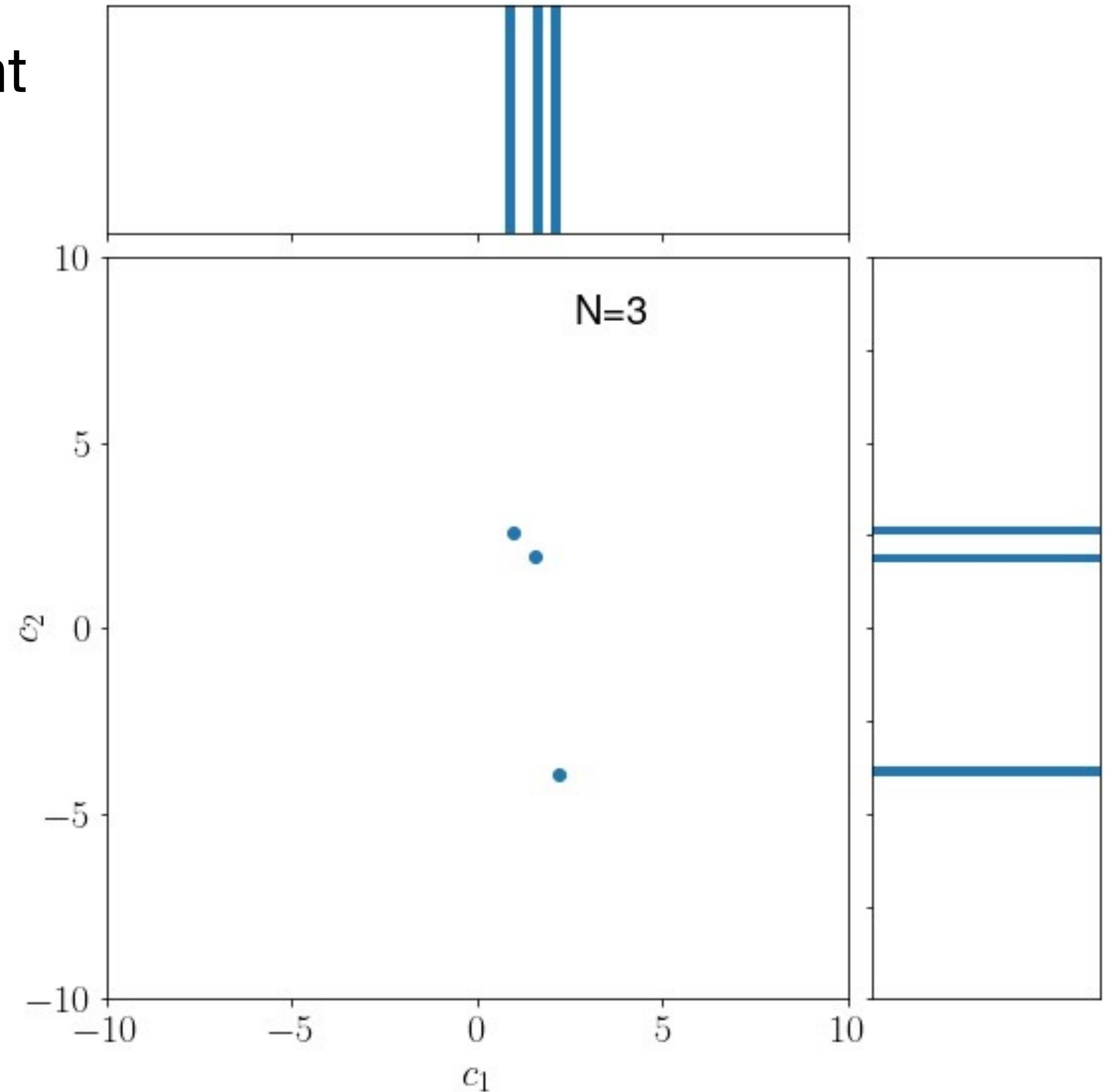


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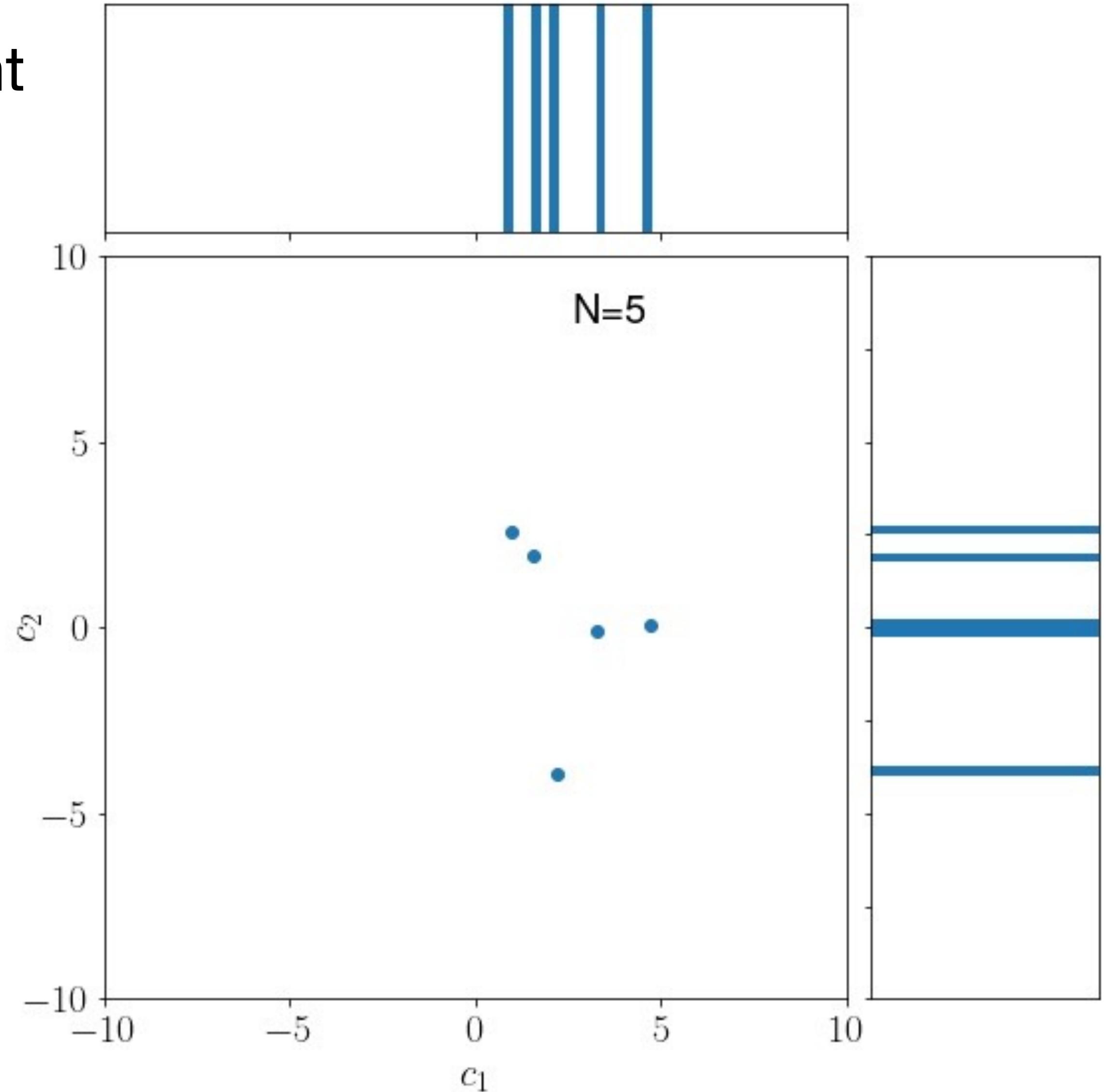


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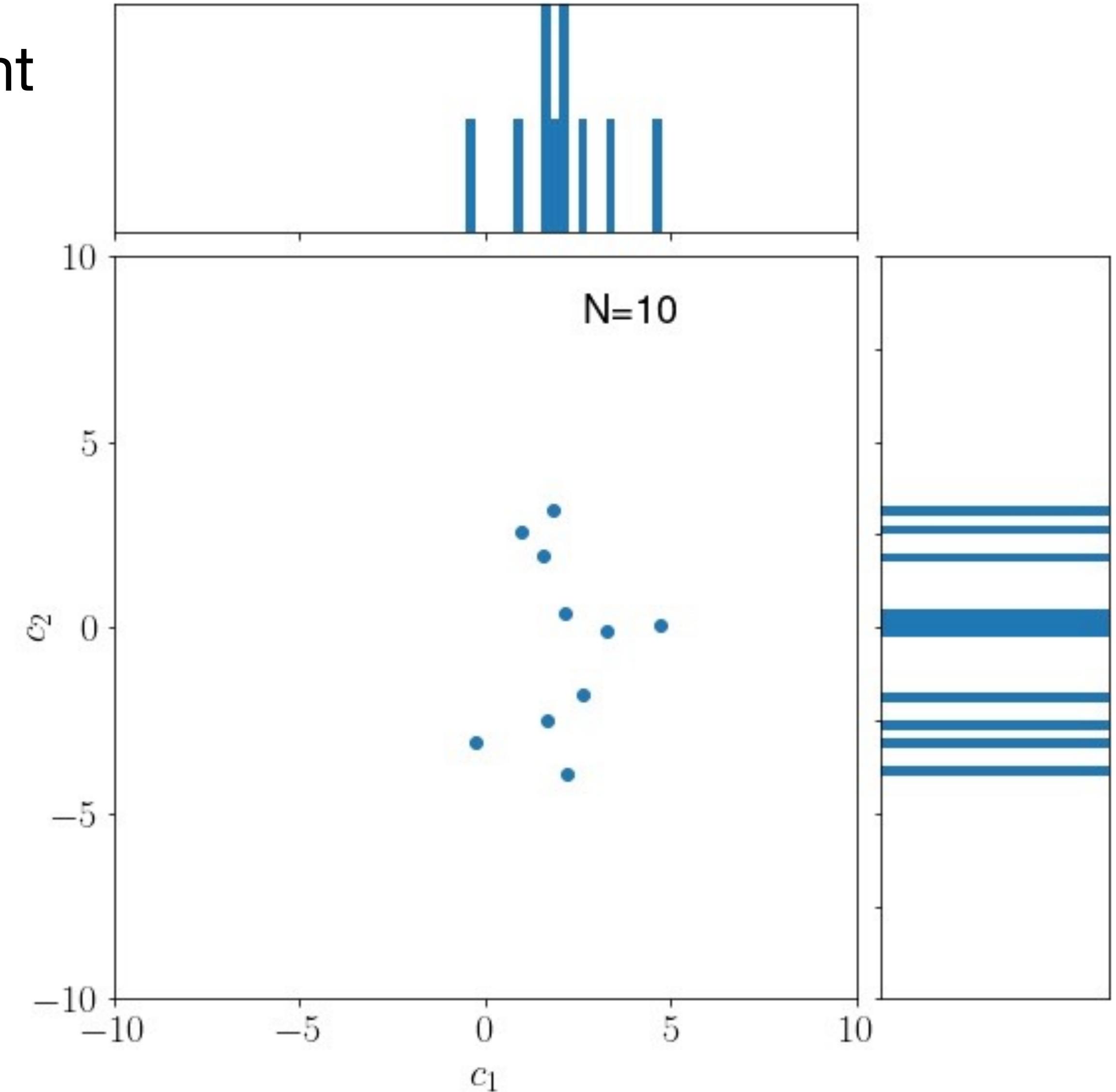


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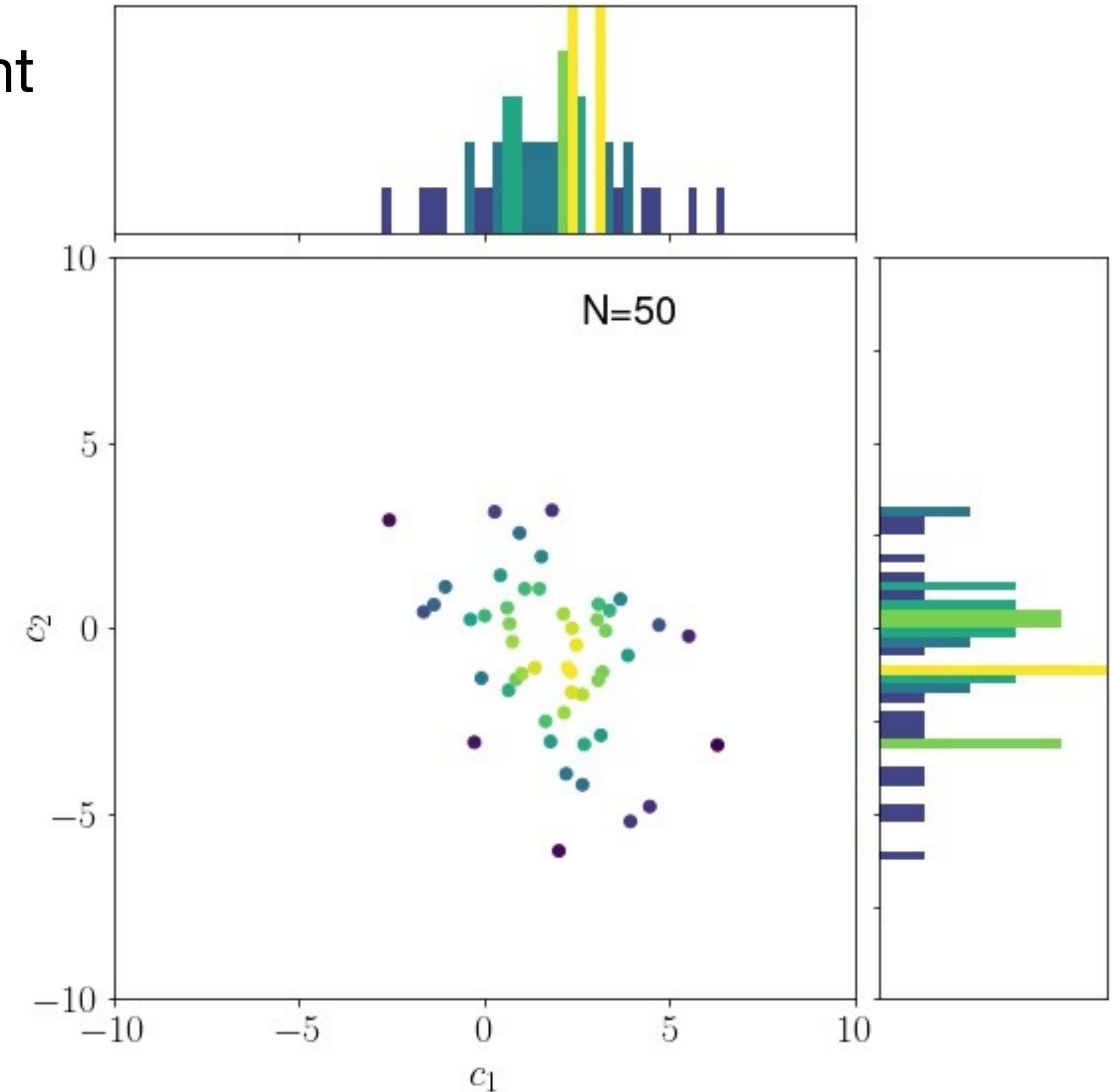


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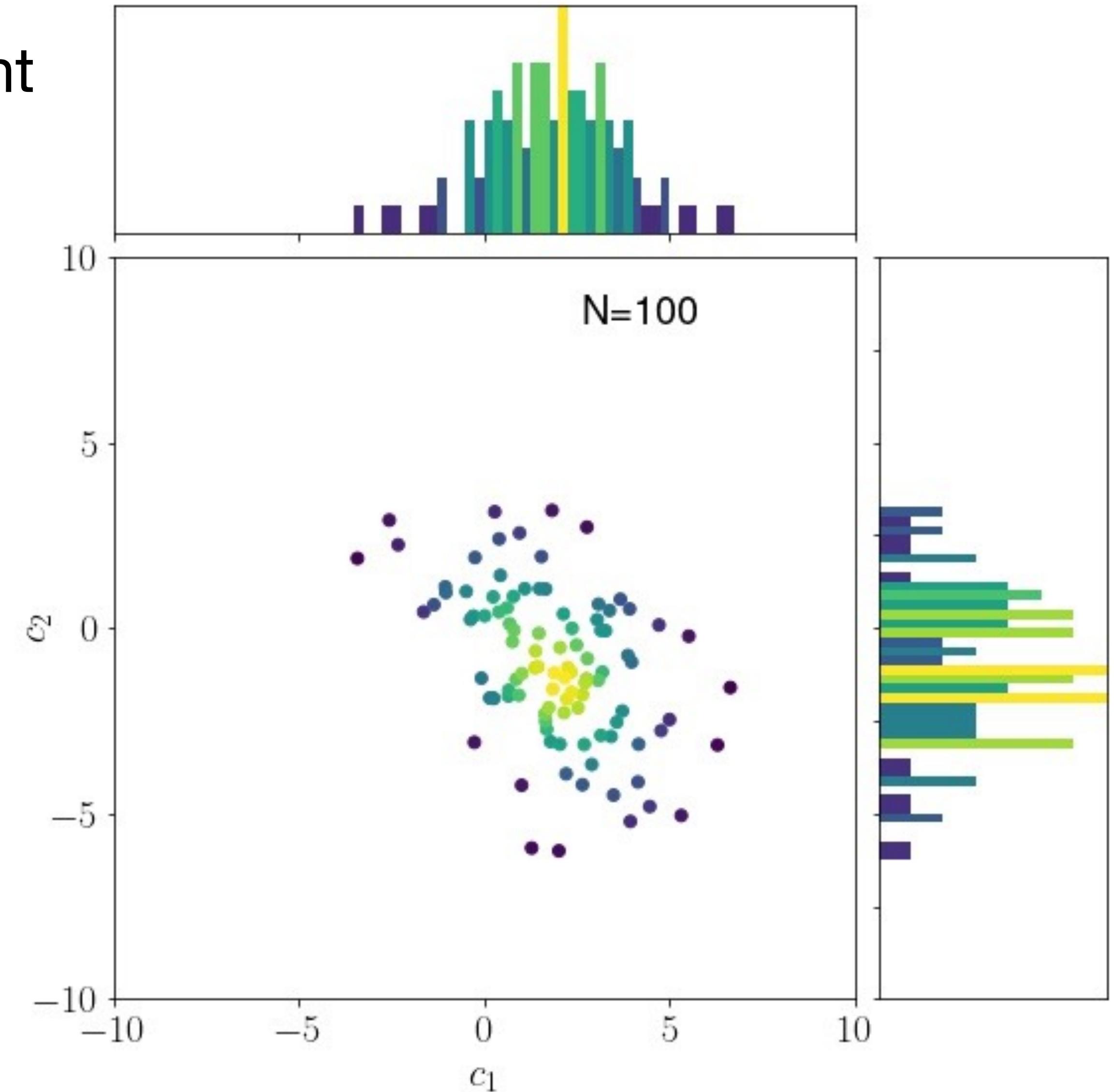


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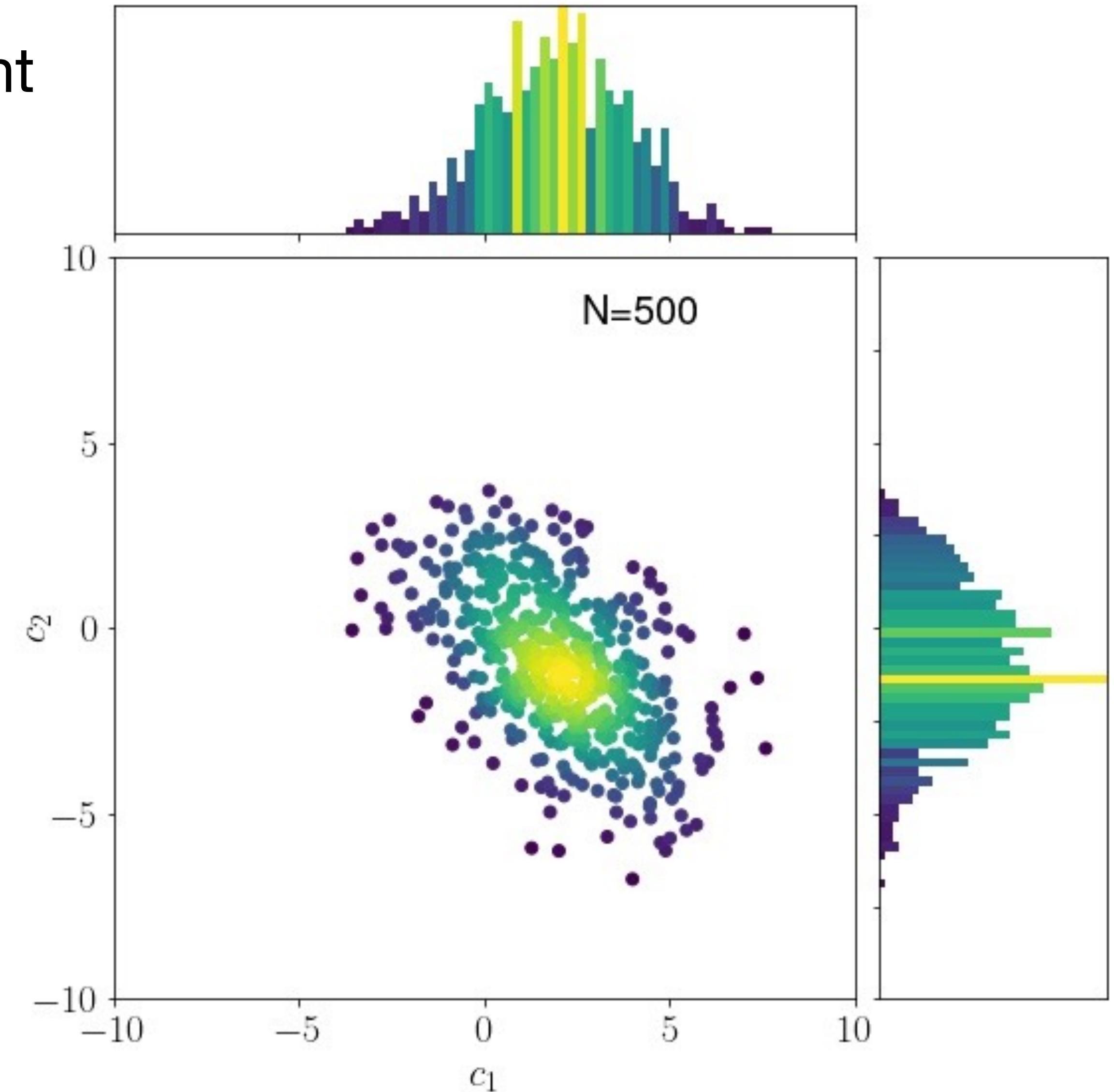


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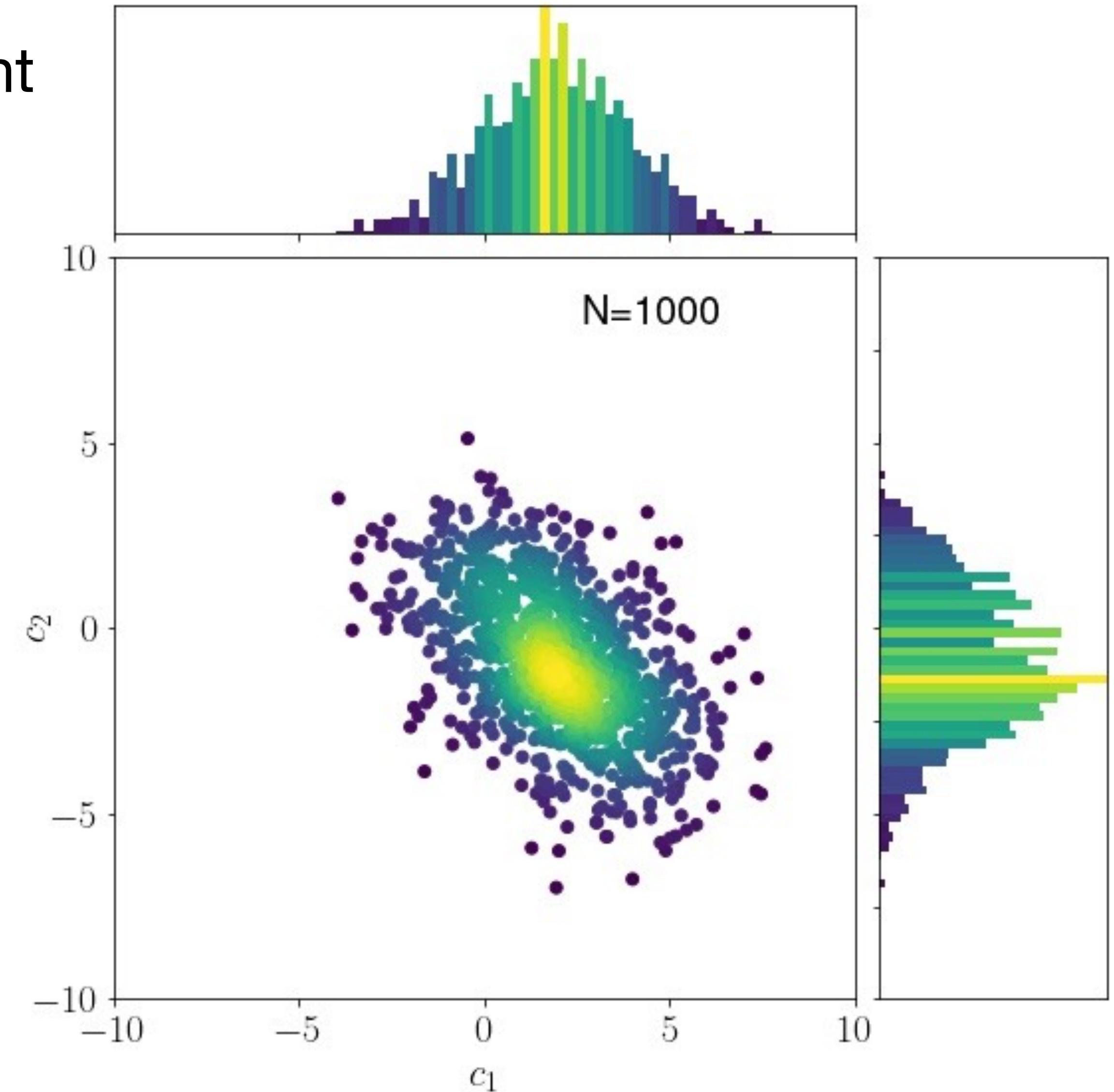
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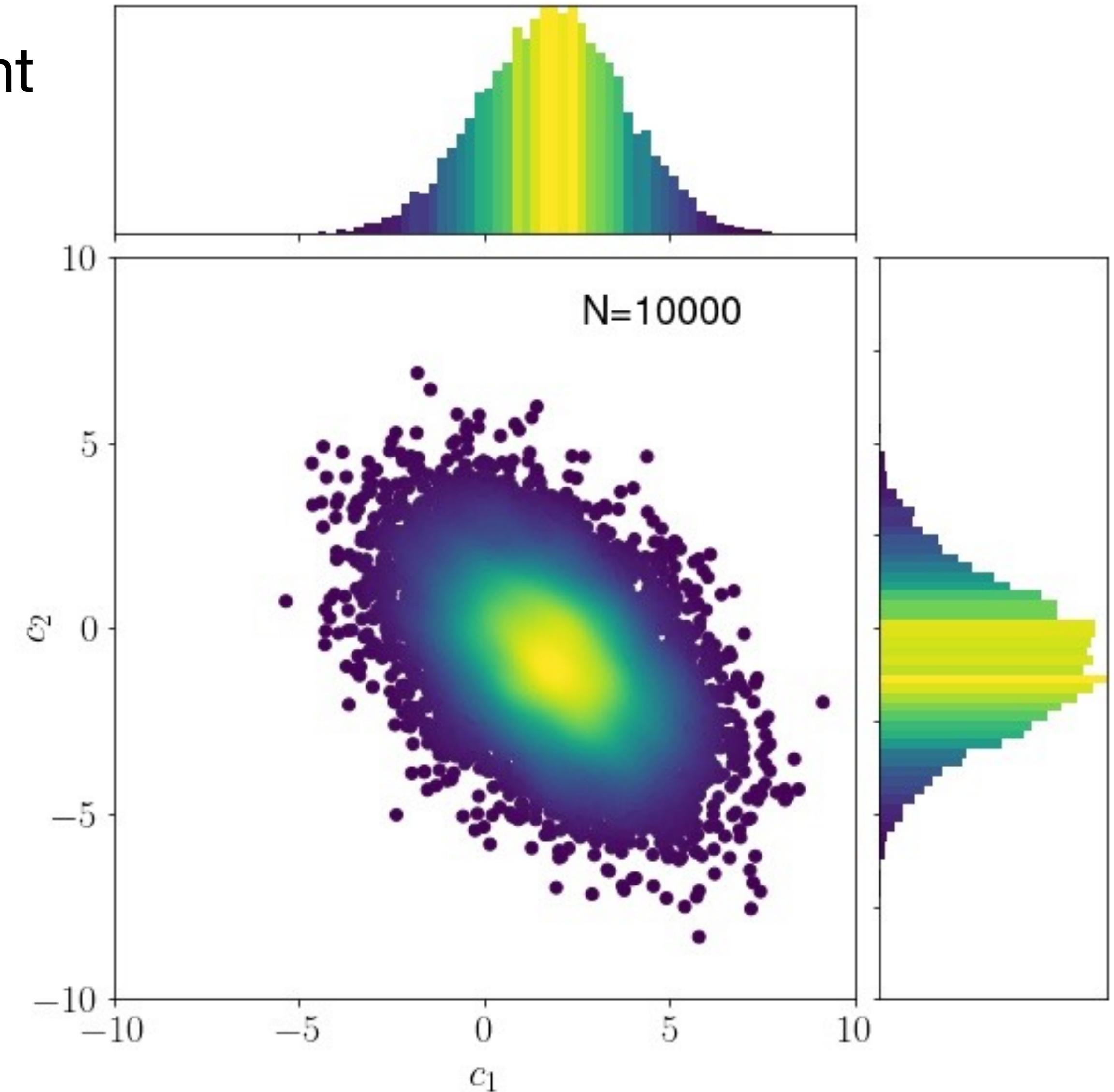
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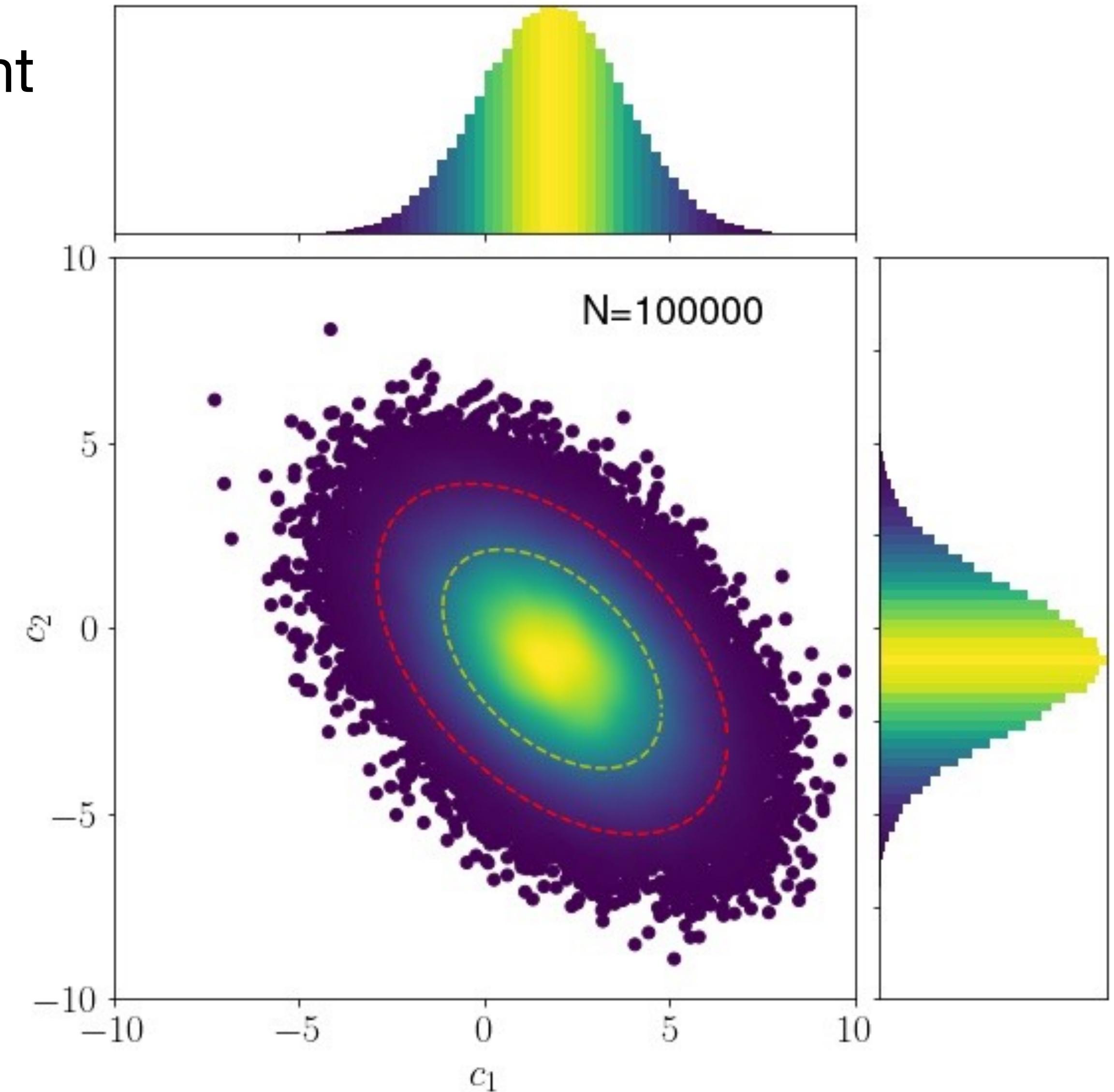


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Markov Chains

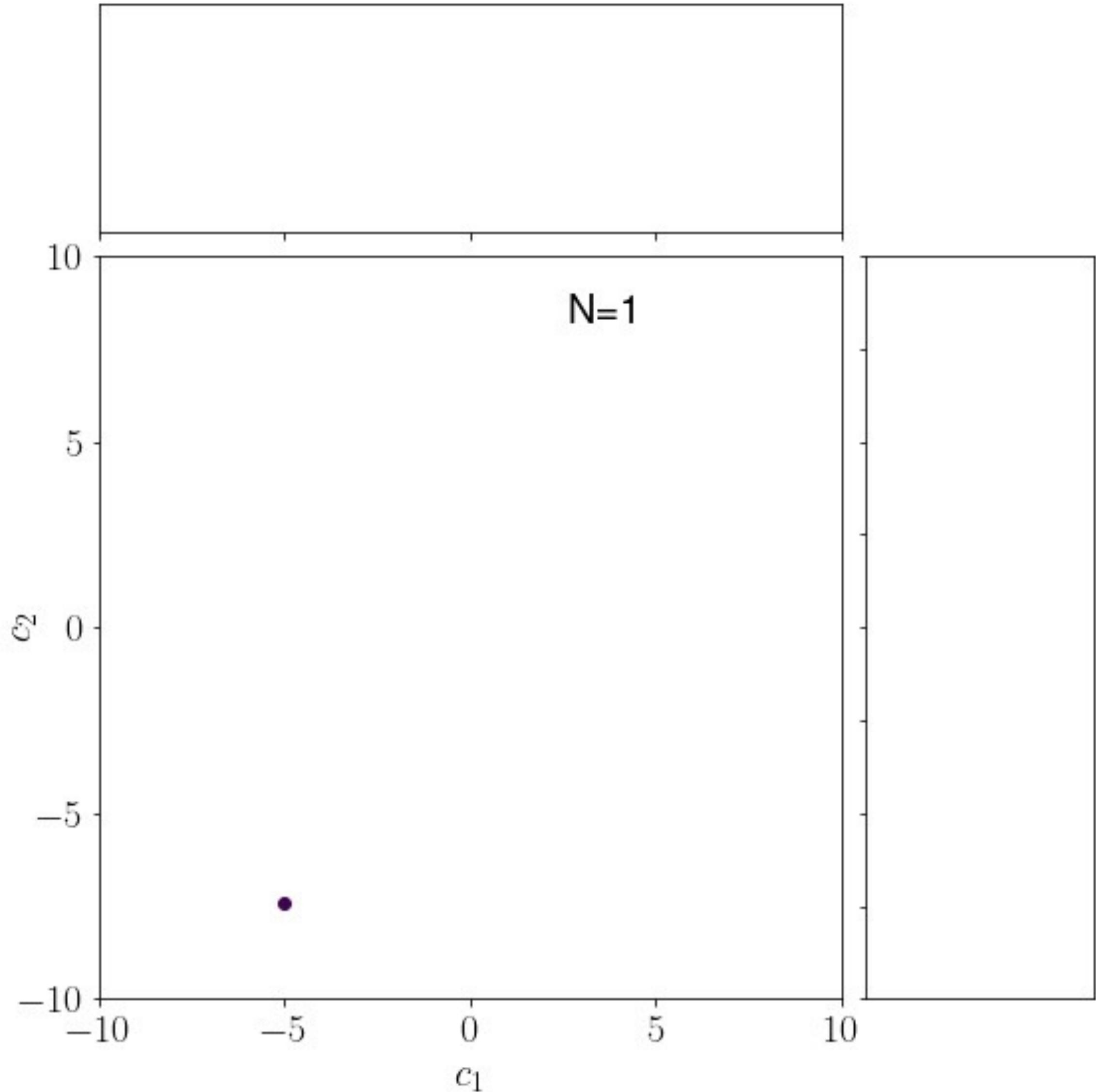
Metropolis-Hastings

- Start at random point x_i
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- Generate random number u in $(0,1)$
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Markov Chains

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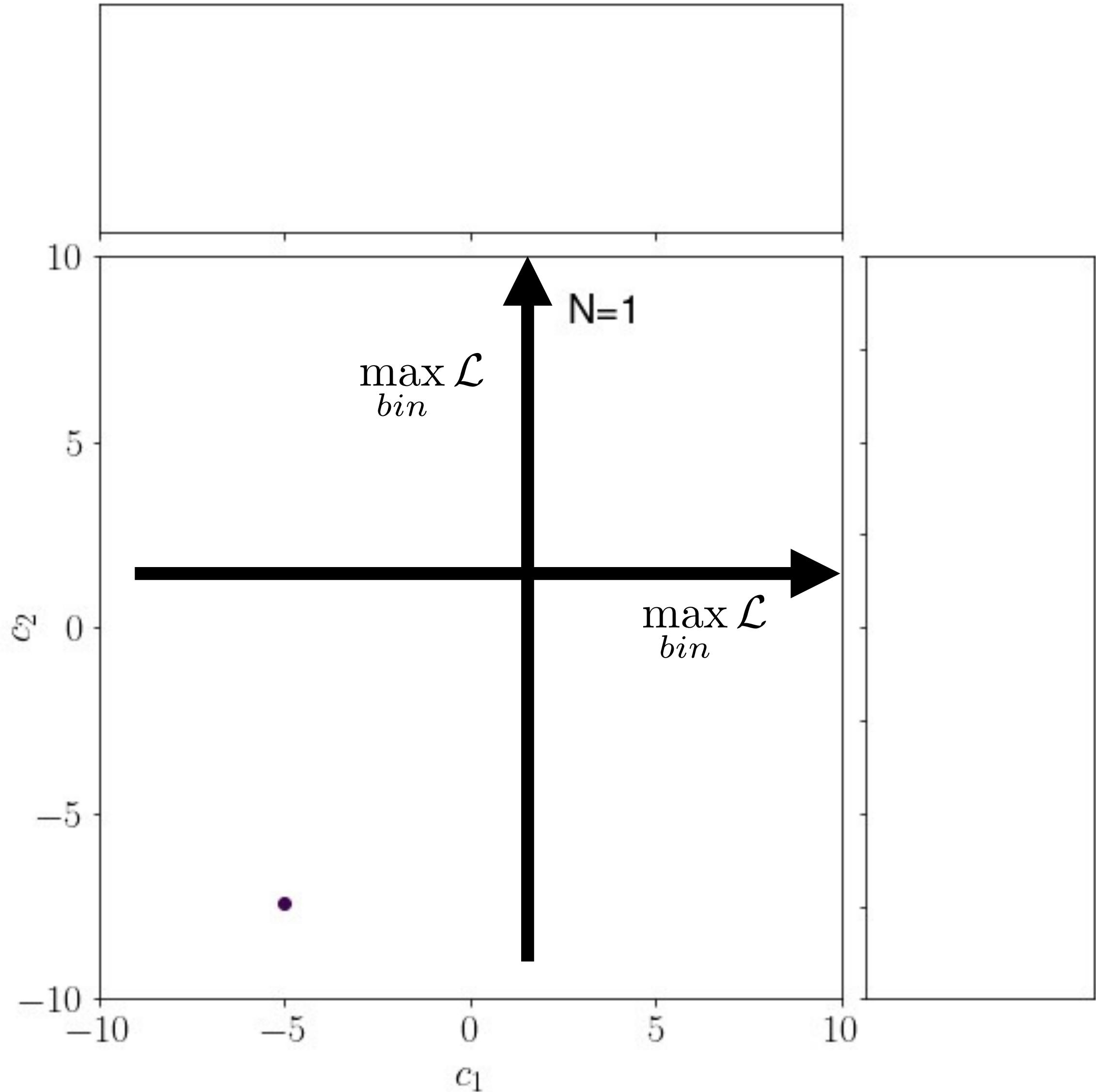
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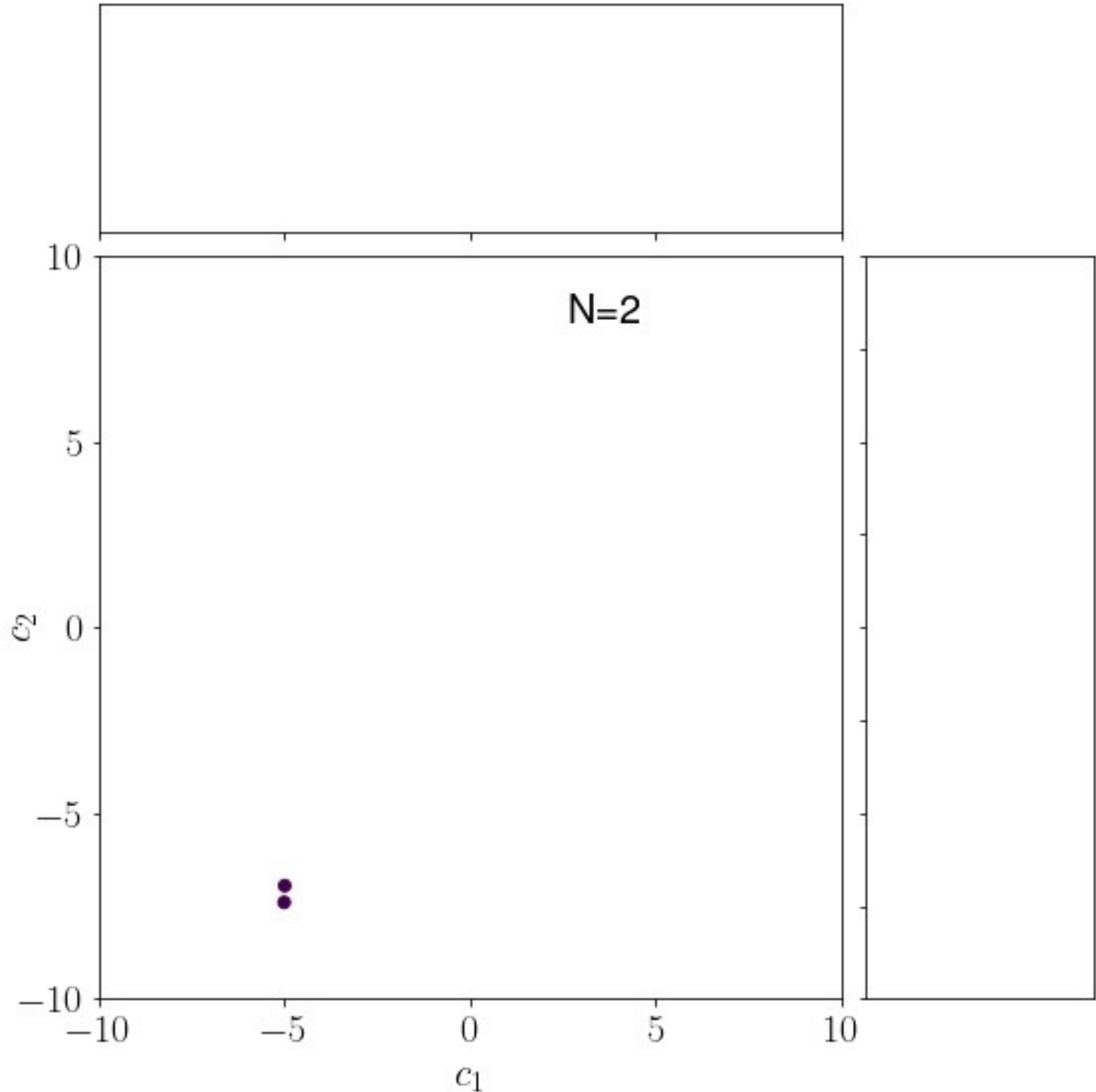
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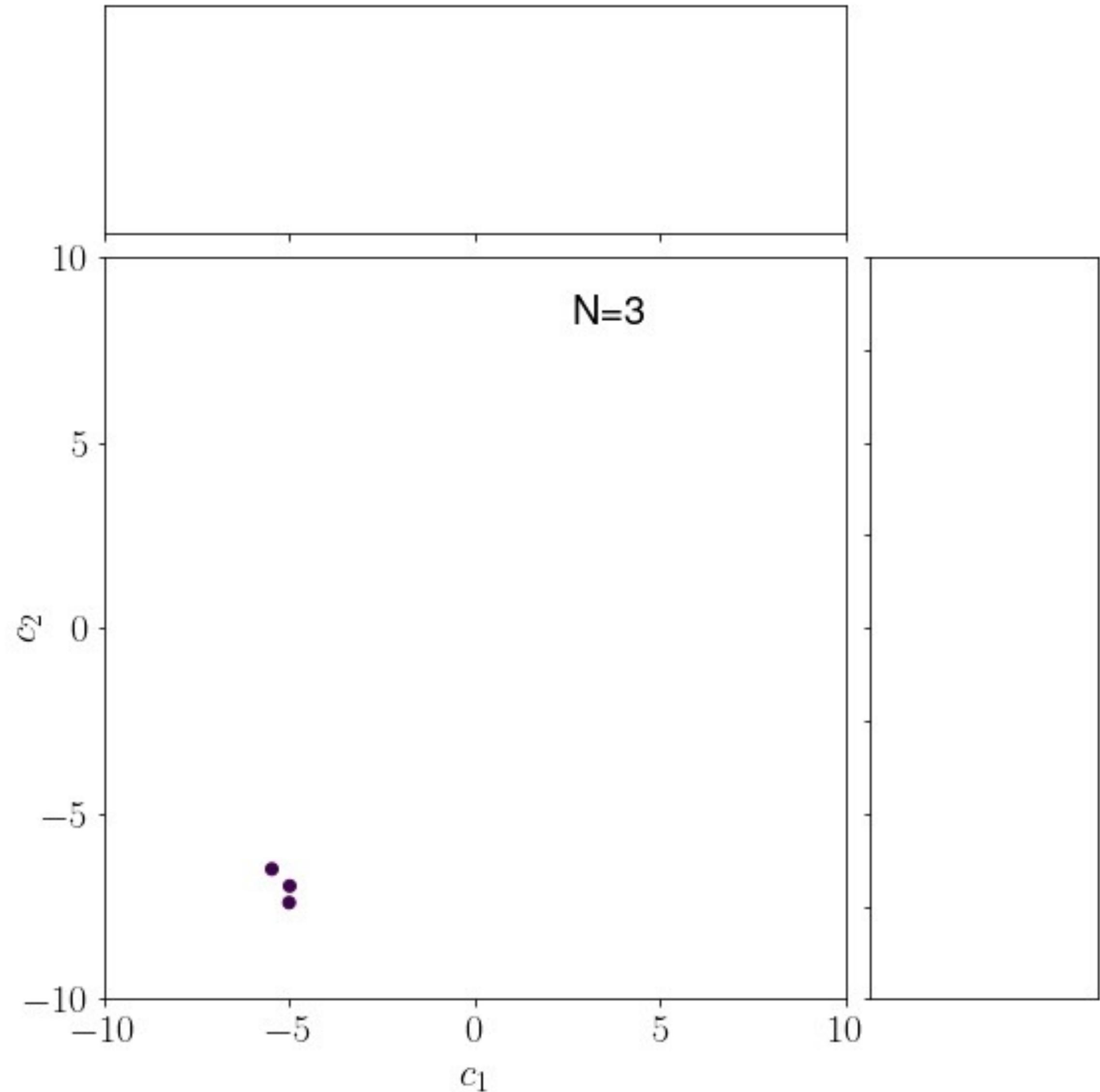
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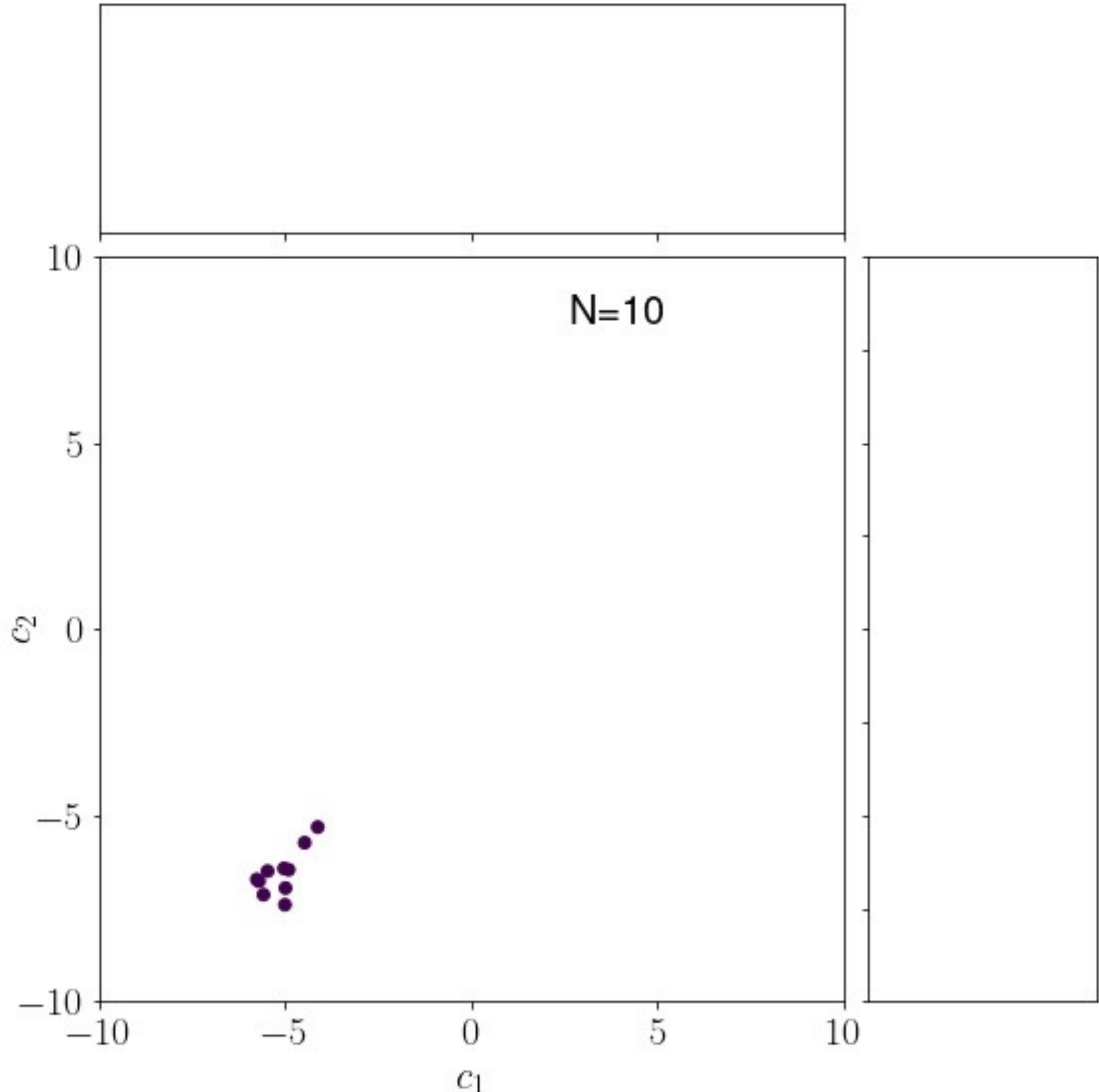
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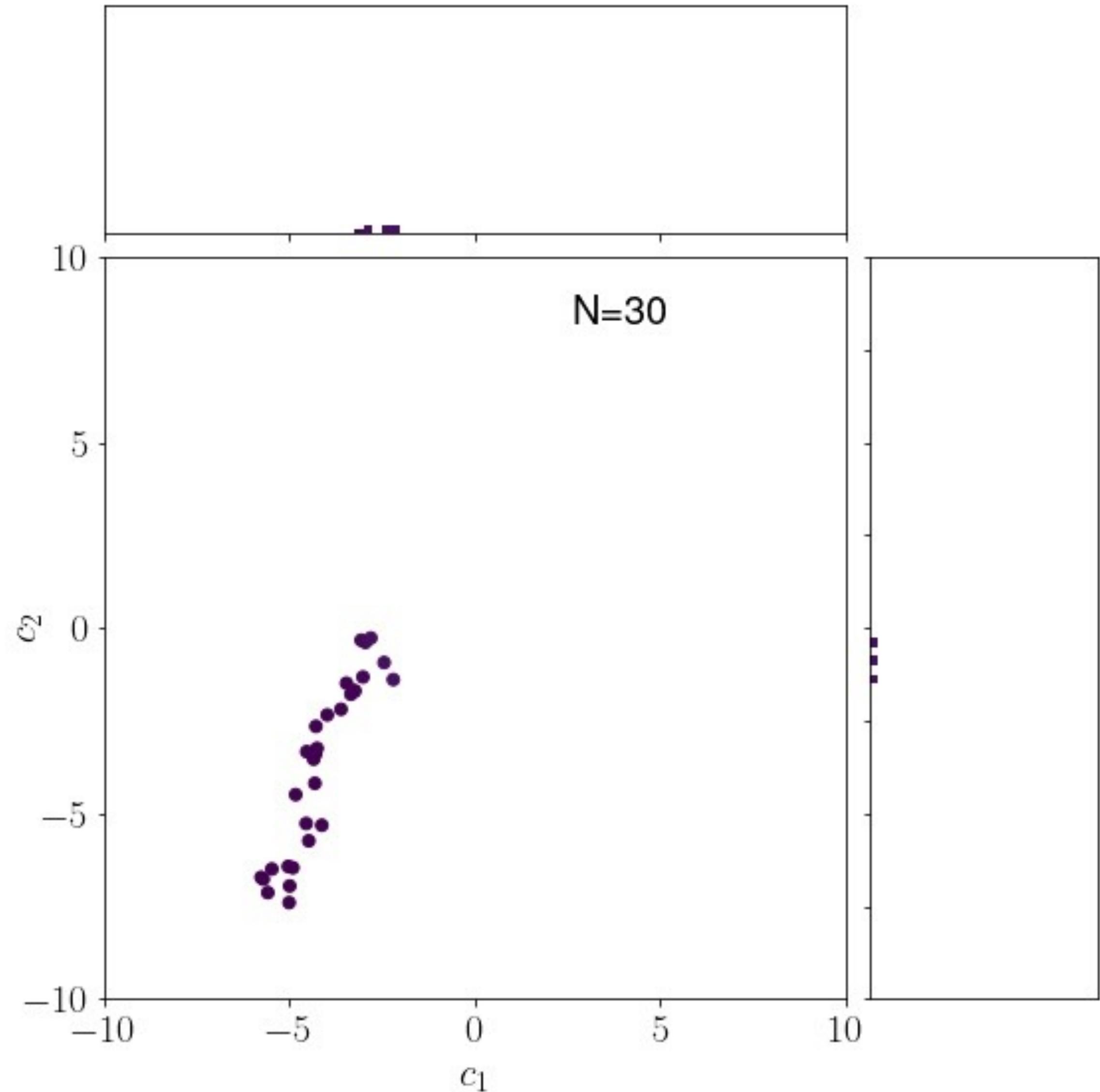
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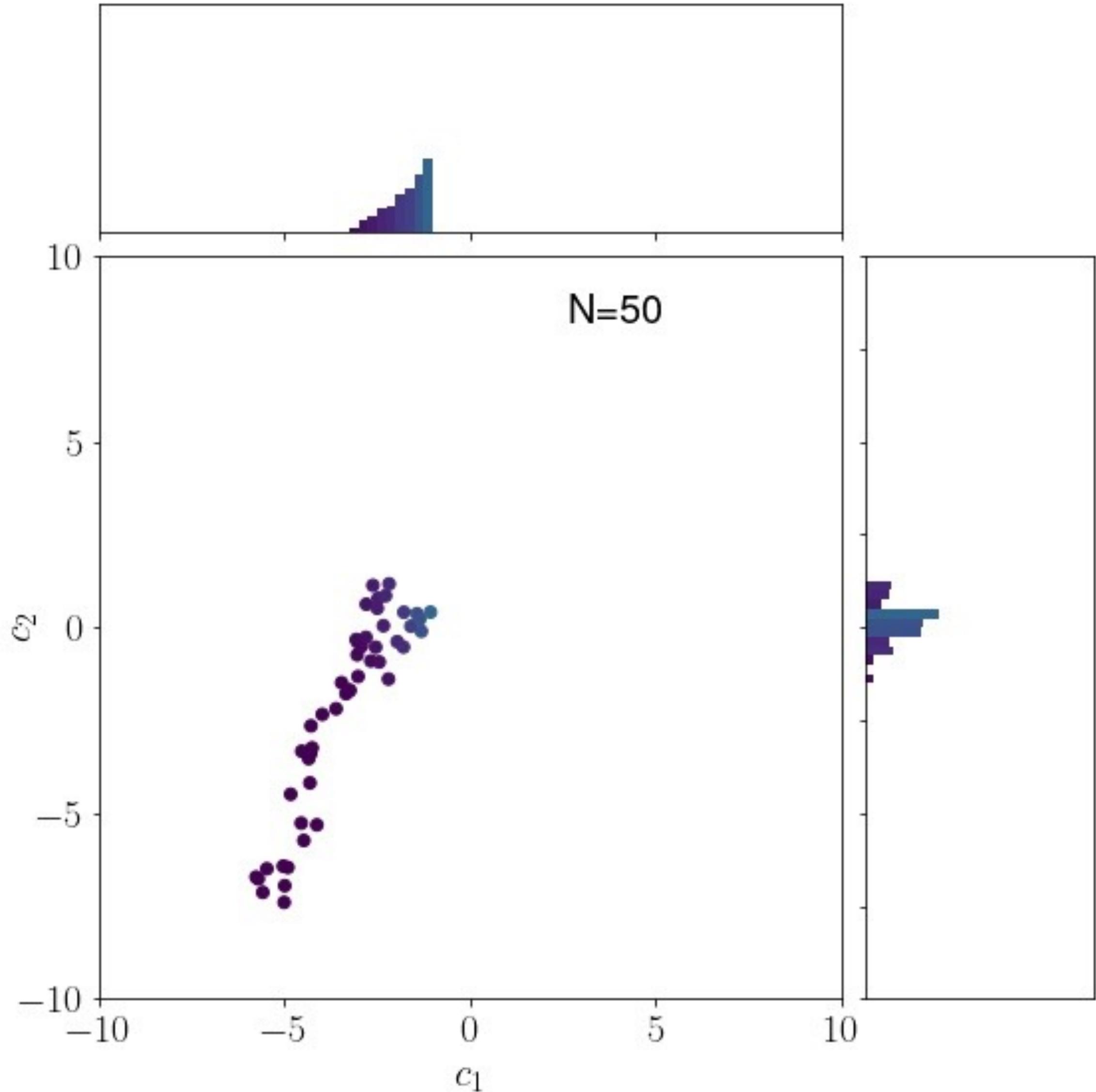
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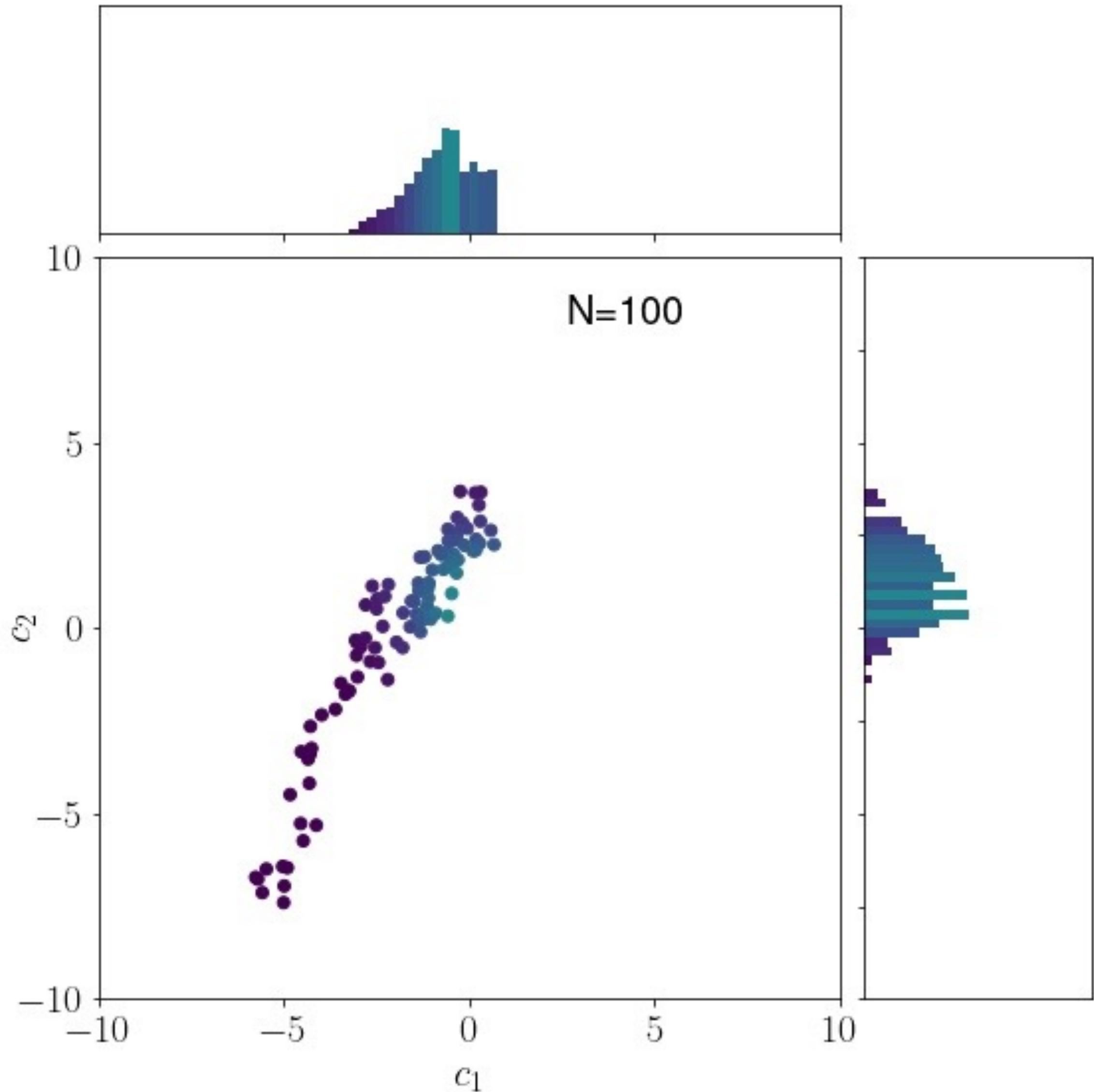
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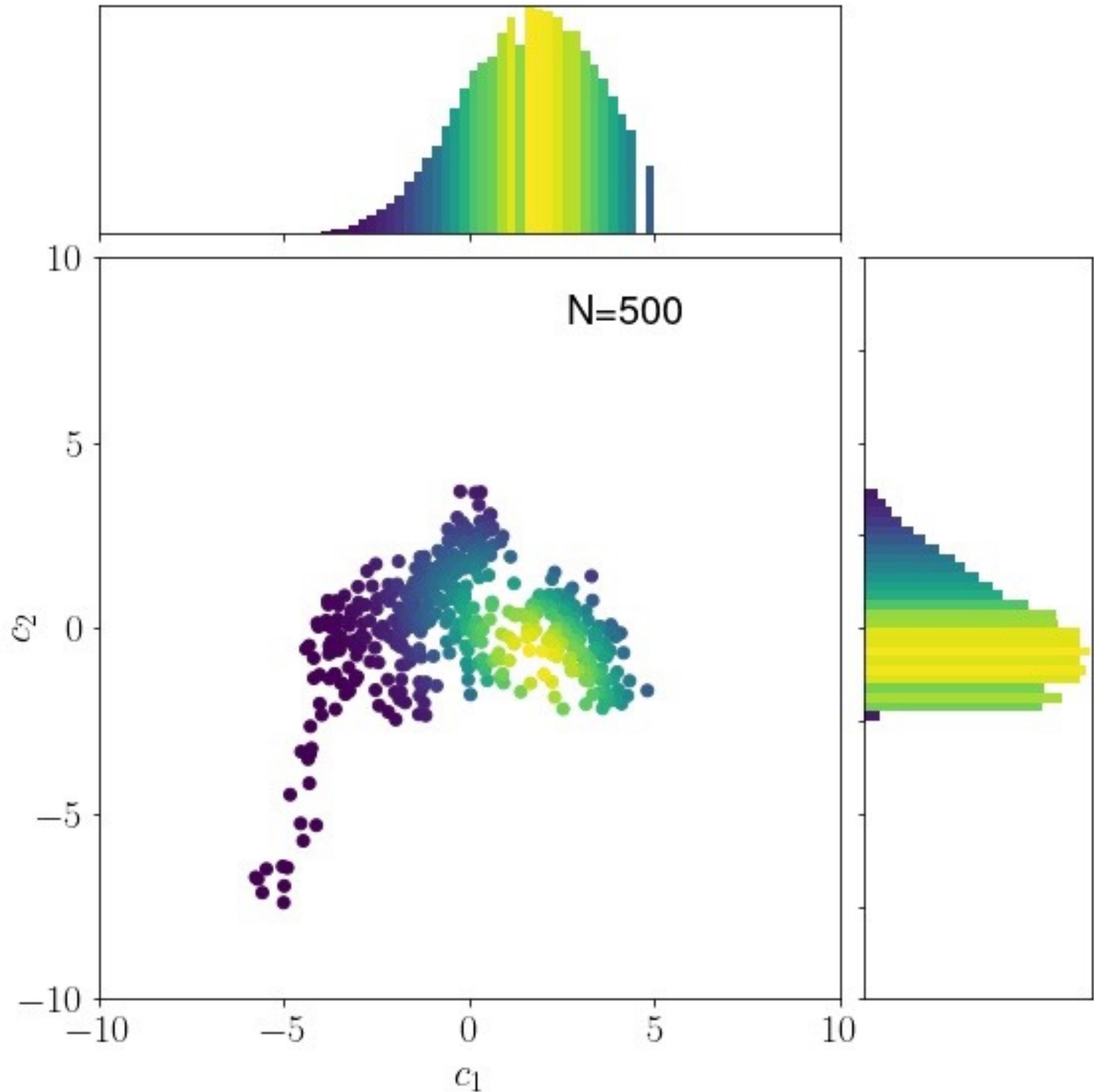
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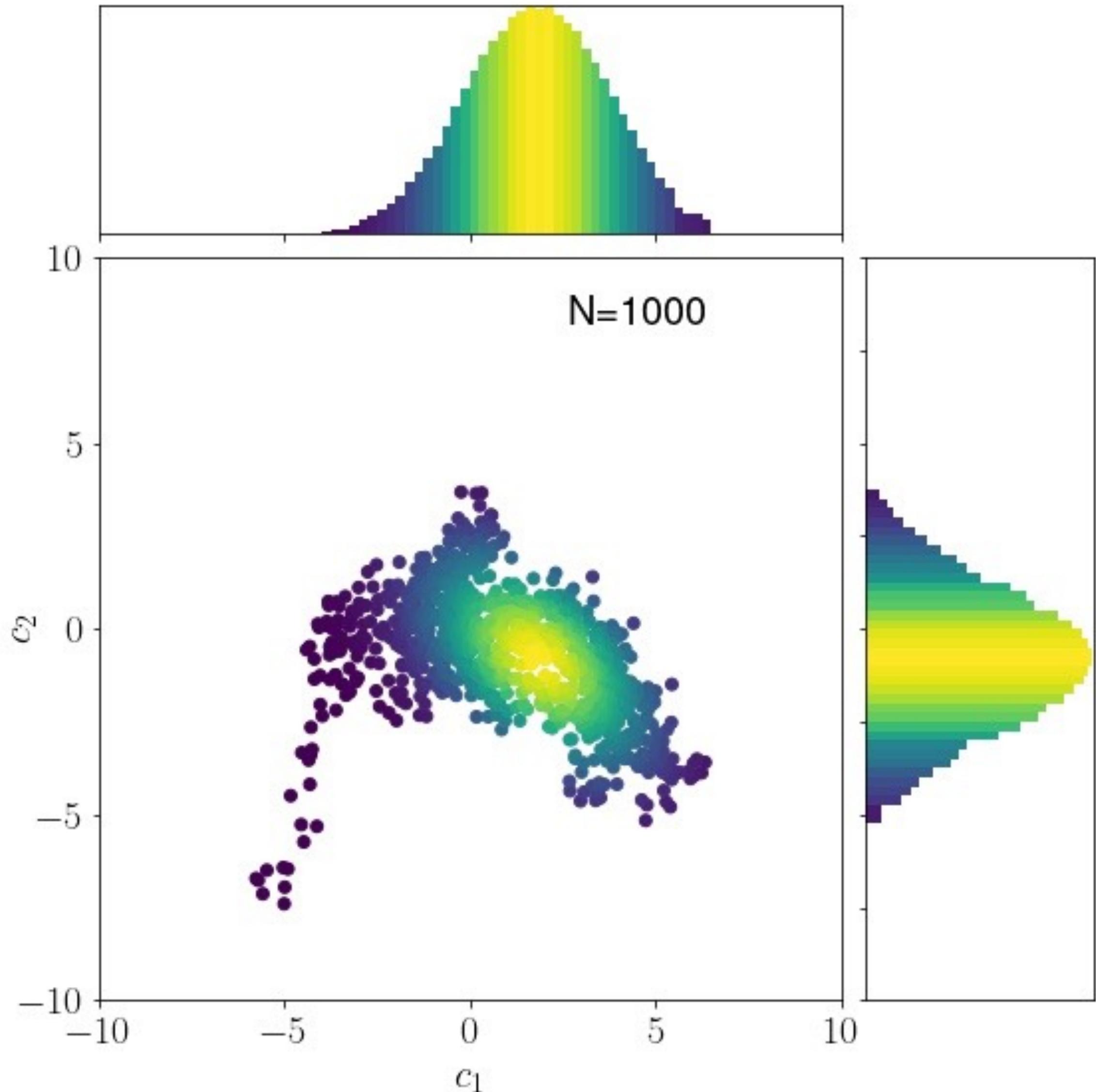
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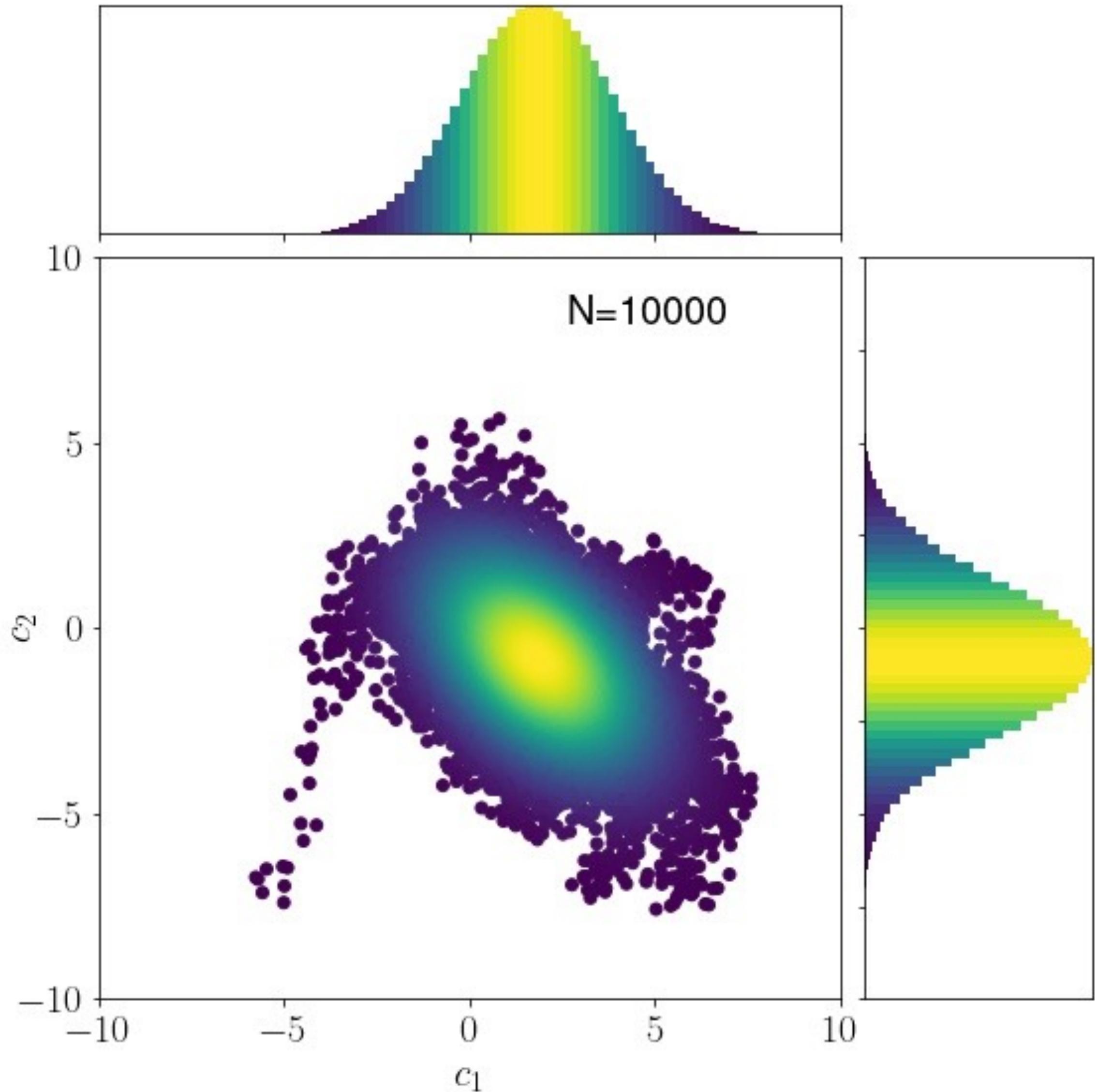
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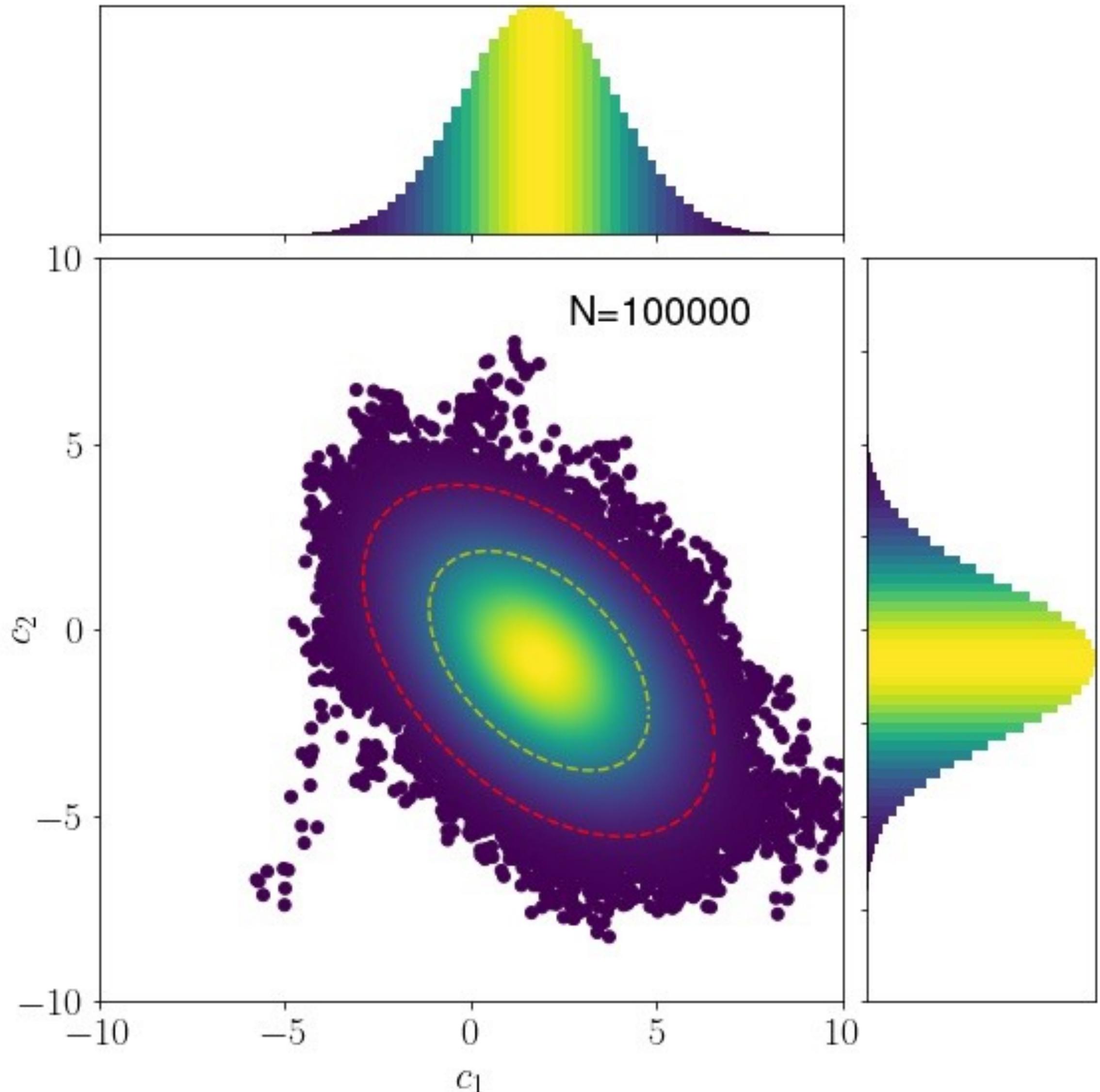
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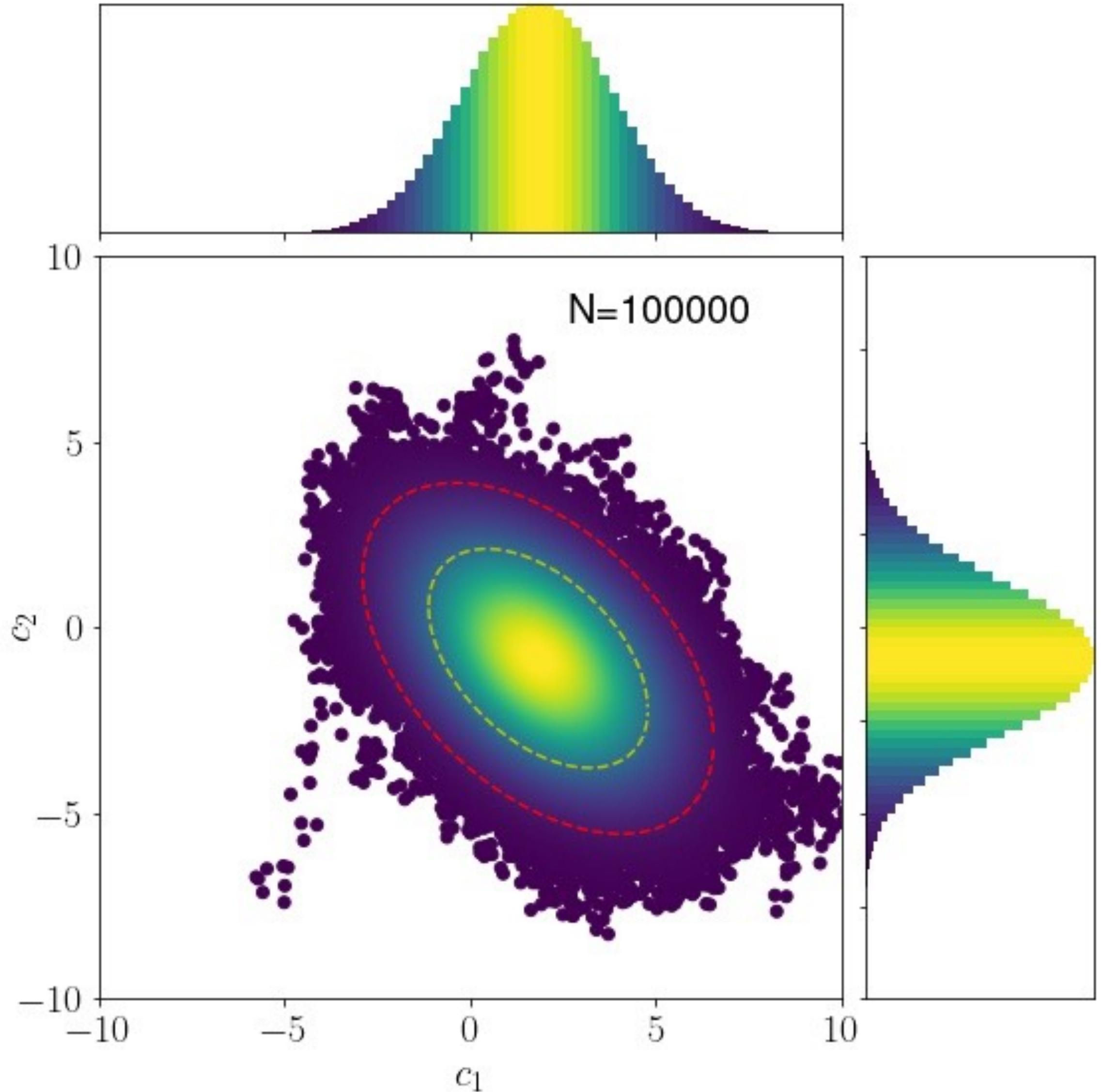
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MC vs. Toys

Markov Chain
Toy Monte Carlo



MC vs. Toys

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