Renormalisation group running effects in $pp \to t\bar{t}h$ in the SMEFT


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Introduction

- The **Standard Model (SM)**: a great success, but it must be extended (baryon asymmetry, dark matter...)

- Many **New Physics (NP)** theories have been proposed, but no clear experimental indication.

- **Effective Field Theories (EFTs)** offer a powerful and pragmatic approach to the search for NP with minimal UV assumptions.

- This talk focuses on **Standard Model Effective Field Theory (SMEFT)**.
The SMEFT

- Parametrise heavy NP effects with a tower of gauge/Lorentz invariant operators:

\[ L_{\text{SMEFT}} = L_{\text{SM}} + \sum_{D_i>4} \frac{C_i}{\Lambda D_i - 4} O_i, \]

\[ O_i \sim \partial n_d^i \phi n_\phi^i A n_A^i \psi n_\psi^i, \quad D_i = n_d^i + n_\phi^i + n_A^i + \frac{3}{2} n_\psi^i. \]
The SMEFT

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[\[ O_i \sim \partial n^i_\phi \phi n^i_\phi A{n^i_A} \psi n^i_\psi, \]

\[ \mathcal{D}_i = n^i_d + n^i_\phi + n^i_A + \frac{3}{2} n^i_\psi. \]

- \( \phi, A, \psi \): SM fields.
- Gauge group: \( \text{SU}(3)_C \otimes \text{SU}(2)_W \otimes \text{U}(1)_Y \).
The SMEFT

- Parametrise heavy NP effects with a tower of gauge/Lorentz invariant operators:

\[ \mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_{D_i > 4} \frac{C_i}{\Lambda^{D_i - 4}} O_i, \]

\[ O_i \sim \partial^{n_d} \phi^{n_\phi} A^{n_A} \psi^{n_\psi}, \quad \mathcal{D}_i = n_d + n_\phi + n_A + \frac{3}{2} n_\psi. \]

- \( \phi, A, \psi \): SM fields.
- Gauge group: \( \text{SU}(3)_C \otimes \text{SU}(2)_W \otimes \text{U}(1)_Y \).

- Dominant effect in collider physics at \( D = 6 \) (Warsaw basis, 2499 operators, [Grzadkowski, Iskrzynski, Misiak, Rosiek, '10]).
SMEFT: how should we use it?

Figure: Courtesy of P. Azzi.
SMEFT: how should we use it?

- Information on UV mediator from IR dynamics: see [Altmannshofer, Gori, Lehmann, Zuo, '23]

Figure: courtesy of L. Alasfar
Running effects

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- Crucial ingredient to connect different energy scales (e.g.: matching scale $\Lambda$, experiment $\sim \nu \ll \Lambda$).
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- Renormalization procedure induces energy-dependent parameters.

- Crucial ingredient to connect different energy scales (e.g.: matching scale \( \Lambda \), experiment \( \sim v \ll \Lambda \)).

- The scale dependence of the coefficients is encoded in the Renormalization Group Equations (RGEs) (1-loop):

\[
\mu \frac{dC_i(\mu)}{d\mu} = \frac{1}{16\pi^2} \Gamma_{ij}(\mu) C_j(\mu).
\]

- \( \Gamma_{ij}(\mu) \): Anomalous Dimension Matrix (ADM).
Structure of ADM in the SMEFT

• $\Gamma_{ij}(\mu)$: known at 1-loop [(Alonso), Jenkins, Manohar, Trott, ’13].

• $\Gamma_{ij}(\mu)$ depends on $\mu$ through the couplings:

$$\Gamma_{ij}(\mu) = g_1^2(\mu)\Gamma_{ij}^{(g_1^2)} + g_2^2(\mu)\Gamma_{ij}^{(g_2^2)} + \ldots$$

• Exactly solvable with only one coupling (typically $g_2^s$, $\Gamma_{ij}(\mu) = g_2^s(\mu)\Gamma_{ij}^{(g_2^s)}$, [Maltoni, Vryonidou, Zhang, ’16], [Battaglia, Grazzini, Spira, Wiesemann, ’21]).

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  $$\Gamma_{ij}(\mu) = g^2_1(\mu)\Gamma_{ij}^{(g^2_1)} + g^2_2(\mu)\Gamma_{ij}^{(g^2_2)} + \ldots$$

- Exactly solvable with only one coupling (typically $g^2_s$, $\Gamma_{ij}(\mu) = g^2_s(\mu)\Gamma_{ij}^{(g^2_s)}$), [Maltoni, Vryonidou, Zhang, '16], [Battaglia, Grazzini, Spira, Wiesemann '21]).

- $\Gamma(g_i)$ do not commute: **analytical solution is impossible.**
Solving the RGEs I

1. **Approximate solution** (first leading log):

\[
C_i(\mu_F) = C_i(\mu_I) + \Gamma_{ij}(\mu_I)C_j(\mu_I) \frac{\log (\mu_F/\mu_I)}{16\pi^2}.
\]

- Neglects dependence on \( \mu \) of \( \Gamma \).
- Ok only if \( \Gamma_{ij}(\mu_I)C_j(\mu_I) \frac{\log (\mu_F/\mu_I)}{16\pi^2} \ll 1 \)
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2. **Numeric solution**:

   - More precise.
   - Slow! Problem for extensive phenomenological analyses.
Solving the RGEs II

• Example: a Monte Carlo analysis in an hadronic collider:

  - \( \hat{s} = x_1 x_2 E_{\text{collider}}^2 \): different for each event.
  - \( C_i(\mu = \Lambda) \xrightarrow{\text{RG flow}} C_i(\mu \sim \sqrt{\hat{s}}) \) for each event.
  - Can be optimized (grid).
Solving the RGEs II

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- State of the art:
  - DsixTools ( [Fuentes-Martin, Ruiz-Femenia, Vicente, Virto, '20], Mathematica).
  - wilson ( [Aebischer, Kumar, Straub, '18], python) SMEFT running ported from DsixTools.
RGESolver [S.D.N., Silvestrini, '22]

- A C++ library that performs RG evolution of SMEFT coefficients.
- General flavour structure (assuming $L$, $B$ conservation, 2499 operators).
- Tested against DsixTools.
- High time efficiency: (numeric running: $\mathcal{O}(0.1 \text{s})$ vs $\mathcal{O}(10 \text{s})$ (DsixTools)).
- Flavour back-rotation implemented.
- Inclusion in HEPfit ([De Blas et. al., '19]) in progress!
$pp \rightarrow \bar{t}th@LHC$ (SM)

- Tree-level in the SM:

(a) $gg \rightarrow t \bar{t}h$

(b) $gg \rightarrow t \bar{t}$

(c) $q \bar{q} \rightarrow t \bar{t}h$

(d) $q \bar{q} \rightarrow t \bar{t}$
Some operators induce a rescaling of the SM vertices.

- \( O_{t\phi} = (\phi^\dagger \phi) \bar{Q}_L \phi t_R \) rescales the top Yukawa coupling:

\[
O_{t\phi} \xrightarrow{\text{S.S.B.}} \frac{v^2}{2} \frac{h}{\sqrt{2}} \bar{t}t + \ldots, \quad g_{t\bar{t}h} = -i \frac{m_t}{v} + i \frac{v^2}{\sqrt{2}} C_{t\phi}. 
\]
$pp \rightarrow t\bar{t}h@LHC$ (SMEFT) I

- Some operators induce a rescaling of the SM vertices.
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- Some operators induce new interaction vertices!

$\mathcal{O}_{\phi G} = (\phi^\dagger \phi) G^A_{\mu\nu} G^{A\mu\nu}$

- New couplings:

$ggh, gghh, gggh, \ldots.$
\[ \mathcal{O}_{uu}^{1133} = (\bar{q} \gamma^\mu \slashed{P} \gamma^\nu \slashed{P} q) (t \gamma_\mu \slashed{P} t) \]

- New coupling: \( \bar{q} q \bar{t} t \).
$pp \rightarrow \bar{t}t h \oplus $LHC (SMEFT) II

\[ \mathcal{O}_{uu}^{1133} = (\bar{q} \gamma^\mu P_R q) (t \gamma_\mu P_R t) \]

- New coupling: $\bar{q}qt\bar{t}t$.

\[ \mathcal{O}_{tG} = (\bar{Q}_L \sigma^{\mu\nu} T^A t_R) \tilde{\phi} G^{A}_{\mu\nu} \]

- New couplings: $\bar{t}tgg, \bar{t}tgh, \bar{t}tgggh$.

- Modifies $\bar{t}t g$. 
Dynamical vs Fixed scale I

- RGEs connect different energy scales: \( \Lambda = O(\text{TeV}) \rightarrow \mu_R \).
- How to choose \( \mu_R \)?
Dynamical vs Fixed scale I

- RGEs connect different energy scales: \( \Lambda = \mathcal{O}(\text{TeV}) \to \mu_R \).
- How to choose \( \mu_R \)?

- We set some Wilson coefficients at the scale \( \Lambda = 2 \text{TeV} \) (inside the bounds in [Ethier et al.,'21]) and test their impact on differential distributions.
- New bounds in [Celada et al.,'24].
- We compare two different choices:
  - Fixed scale: \( \mu_R = m_t \) (same for all the events).
  - Dynamical scale: \( \mu_R = (p_{T,h} + p_{T,t} + p_{T,\bar{t}})/2 \) (changes event by event).
Dynamical vs Fixed scale II

- Sizeable/important effect for large coefficients \( \sim 1 \text{ TeV}^{-2} / \sim 100 \text{ TeV}^{-2} \).
- Numeric running in both cases!

Conservative scenario: \( C_{4t} \sim 1 \text{ TeV}^{-2} \).
Extreme scenario: \( C_{4t} \sim 100 \text{ TeV}^{-2} \).
Numeric vs. 1LL

- Small ($< 5\%$, not shown) / sizeable effect for large coefficients ($\sim 1\, \text{TeV}^{-2} / \sim 100\, \text{TeV}^{-2}$).
- Numeric running: $\simeq 20\,\text{mins}$, 1LL running: $\simeq 40\,\text{s}$,
  $\mu_R = \mu_F = (p_{T,h} + p_{T,t} + p_{T,\bar{t}})/2$ (dyn. ren. scale).
\[ g_{htt} = \frac{m_t}{v} \left( 1 - \frac{v^2}{\sqrt{2}} C_{t\phi} \right) \] is the effective Higgs-top coupling.

\[ \beta_{t\phi} \propto y_t^3 \left( C_{Qt}^{(1)} + \frac{4}{3} C_{Qt}^{(8)} \right), \]

\[ O_{Qt}^{(1,8)} = (\bar{Q}_L \gamma^\mu (T^A) Q_L) (\bar{t}_R \gamma_\mu (T^A) t_R). \]
$g_s$ vs $y_t$

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  \]

- $C_{Qt}^{(8)}$ contributes via penguin diagrams to the running of operators (such as $O_{uu}^{33ii}$, $i = 1, 2$) entering at tree-level.

- $C_{Qt}^{(1)}$ **does not**! We can compare $g_s$ vs $y_t$ running effects.
$g_s \text{ vs } y_t$

- $g_{htt} = \frac{m_t}{v} \left(1 - \frac{v^2}{\sqrt{2}} C_{t\phi}\right)$ is the effective Higgs-top coupling.
  $$\beta_{t\phi} \propto y_t^3 \left(C_{Q_t}^{(1)} + \frac{4}{3} C_{Q_t}^{(8)}\right),$$
  $$O_{Q_t}^{(1,8)} = (\bar{Q}_L \gamma^\mu (T^A) Q_L) (\bar{t}_R \gamma_\mu (T^A) t_R).$$
- $C_{Q_t}^{(8)}$ contributes via penguin diagrams to the running of operators (such as $O_{UU}^{33ii}$, $i = 1, 2$) entering at tree-level.
- $C_{Q_t}^{(1)}$ does not! We can compare $g_s$ vs $y_t$ running effects

We set $C_{Q_t}^{(1,8)} \neq 0$ individually in such a way they contribute (almost) in the same way to the Yukawa-induced running.
$g_s$ vs $y_t$

\[ C_{Qt}^{(1)}(\Lambda) = \frac{4}{3} \times 20 \text{ / TeV}^2. \]

\[ \beta_{t\phi} \propto y_t^3 \left( C_{Qt}^{(1)} + \frac{4}{3} C_{Qt}^{(8)} \right) \]

- Same behaviour in $g_{h\bar{t}t}$ for both operators!
\[ C_{Qt}^{(1)}(\Lambda) = \frac{4}{3} \times 20 \text{ / TeV}^2. \]

\[ C_{Qt}^{(8)}(\Lambda) = 20 \text{ / TeV}^2. \]

\[ \beta_{t\phi} \propto y_t^3 \left( C_{Qt}^{(1)} + \frac{4}{3} C_{Qt}^{(8)} \right) \]
$g_s$ vs $y_t$ III

\[ C^{(1)}_{Qt}(\Lambda) = \frac{4}{3} \times 20 / \text{TeV}^2. \]

\[ C^{(8)}_{Qt}(\Lambda) = 20 / \text{TeV}^2. \]

\[ \beta_{t\phi} \propto y_t^3 \left( C^{(1)}_{Qt} + \frac{4}{3} C^{(8)}_{Qt} \right) \]

- The difference between fixed and dynamical scale shows the importance of running effects.
- $\Delta \neq 0$ for $C^{(1)}_{Qt} \rightarrow$ Top Yukawa contributions are important!
Conclusions

- Running effects are a crucial ingredient for precision physics in the next future.

- Sizable differences can arise when employing a dynamical renormalisation scale vs a fixed renormalization scale.

- In presence of large Wilson coefficients, the leading-log solution of the RGEs shows sizeable differences w.r.t. the numeric integration.

- Yukawa contributions can be as important as strong ones in some cases.
Conclusions

- Running effects are a crucial ingredient for precision physics in the next future.

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- Yukawa contributions can be as important as strong ones in some cases.

Thank you for your attention!
Impact of SM running of $g_s$

- $g_s = g_s(m_t)$ in the fixed case scenario (instead of $g_s = g_s(\mu_F) = g_s((\not{p}_T,h + \not{p}_T,t + \not{p}_T,\bar{t})/2)$ in the SM@1 loop).

\[ C_{Qt}^{(1)} = 20 \text{ TeV}^{-2}. \]
$p_{T,h} \text{ vs } \mu_F$

$C_{Q_t}^{(1)} = 20 \text{ TeV}^{-2}.$
Back-rotation effects

**Figure:** from ([Aebischer,Kumar,'20])
Running, logs, divergent parts I

\[ iM \propto \frac{g_s^2}{16\pi^2} \left( A + B \left( \frac{1}{\Lambda} + \log \frac{\mu_R^2}{m_t^2} \right) \right) + C_{2t2q}(\mu_R) - \delta_{2t2q}. \]
Running, logs, divergent parts I

\[ i\mathcal{M} \propto \sim C_{4t}(\mu_R) \frac{g_s^2}{16\pi^2} \left( A + B \left( \frac{1}{\Lambda} + \log \frac{\mu^2}{m_t^2} \right) \right) + C_{2t2q}(\mu_R) - \delta_{2t2q}. \]

- Divergences determine the ADM: \( \delta_{2t2q} = -\frac{\Gamma_{2t2q,4t}}{32\pi^2 c} C_{4t} \)
Running, logs, divergent parts I

\[ iM \propto \sim \mathcal{C}_{4t}(\mu_R) \frac{g_s^2}{16\pi^2} \left( A + B \left( \frac{\mu_R^2}{m_t^2} + \log \frac{\mu_R^2}{m_t^2} \right) \right) + \mathcal{C}_{2t2q}(\mu_R) - \delta_{2t2q}. \]

- Divergences determine the ADM: \[ \delta_{2t2q} = -\frac{\Gamma_{2t2q,4t}}{32\pi^2\epsilon} C_{4t}. \]

\[ C_{2t2q}(\mu_R) = C_{2t2q}(\Lambda) + C_{4t}(\mu_R) \frac{g_s^2}{16\pi^2} \log \frac{\Lambda^2}{\mu_R^2} \quad (1LL) \]

\[ C_{4t}(\mu_R) \approx C_{4t}(\Lambda) + \ldots \]
If we neglect the finite term ($A$) and we assume that $\mu_R \sim v \sim m_t$, we can include loop effects without any computation!

\[
i\mathcal{M} \propto C_{4t}(\Lambda) \frac{g_s^2}{16\pi^2} \left( A + B \left( \log \frac{m_t^2}{\mu_R^2} + \log \frac{\mu_R^2}{\Lambda^2} \right) \right) + C_{2t2q}(\Lambda) \\
\simeq C_{4t}(\Lambda) \frac{g_s^2}{16\pi^2} B \log \frac{\mu_R^2}{\Lambda^2} + C_{2t2q}(\Lambda)
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\]

\[
\simeq C_4t(\Lambda) \frac{g_s^2}{16\pi^2} B \log \frac{\mu_R^2}{\Lambda^2} + C_{2t2q}(\Lambda)
\]

In some cases, finite terms can be phenomenologically relevant! \([\text{Alasfar, de Blas, Gröber, '22}]\).

Not interesting in our case: **the focus is on running effects!**
Is the SMEFT general enough?

- The SMEFT assumes a SM-like Higgs boson:
  \[ (\tilde{\varphi}, \varphi) = \frac{v + h}{\sqrt{2}} \cdot U, \quad U = \exp \left( i \frac{\pi^I \tau^I}{v} \right). \]

- The Higgs EFT (HEFT) instead assumes a more general scenario: \( U \) and \( h \) are treated separately.

- \( \text{SM} \subset \text{SMEFT} \subset \text{HEFT} \).

- Less correlations between coefficients in HEFT: (e.g., in SMEFT \( g_{5h} = v g_{6h} \) but not in HEFT).

- Measure correlation \( \rightarrow \) insights about EW SSB.

- More about this topic in [Brivio, Trott, '17].
Sum over external polarizations in QCD

- Ex: 2 external gauge bosons, $\mathcal{M} = \epsilon^{\mu_1}(p_1)\epsilon^{\mu_2}(p_2)\mathcal{M}_{\mu_1\mu_2}$.
- **Ward identity**: $p_1^{\mu_1}\mathcal{M}_{\mu_1\mu_2} = p_2^{\mu_2}\mathcal{M}_{\mu_1\mu_2} = 0$ ("each photon is independent").
- In QED (abelian), we can use:

$$\sum_{\text{Pol}} \epsilon_{\mu}(p)\epsilon_{\nu}(p)^* = -g_{\mu\nu} + \frac{n_{\mu}p_{\nu}}{(n \cdot p)} + \frac{n_{\nu}p_{\mu}}{(n \cdot p)} - \frac{p_{\mu}p_{\nu}}{(n \cdot p)^2}.$$
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  \]

- **Ward identity \rightarrow Slavnov-Taylor identity:**
  \( p_1^{\mu_1}\epsilon^{\mu_2}(p_2)\mathcal{M}_{\mu_1\mu_2} = \epsilon^{\mu_1}(p_1)p_2^{\mu_2}\mathcal{M}_{\mu_1\mu_2} = 0 \).
- **Weaker than QED:** all the particles must be on-shell.
- **Terms \( \propto p_1, p_2 \) cannot be dropped.**
- **If we do, we must compensate subtracting (incoherently) MEs with external ghosts ([Malmquist, Kachelrieß, '21]).**