NNLO QCD CORRECTIONS TO $t\bar{t}W$ PRODUCTION AT THE LHC

LHCP Boston 2024

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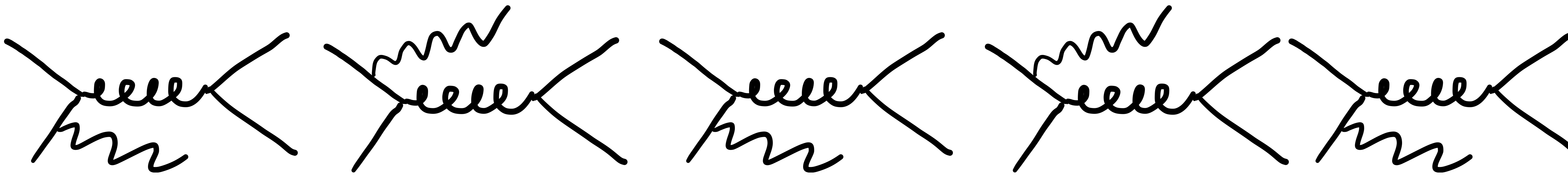
FACULTY OF SCIENCES

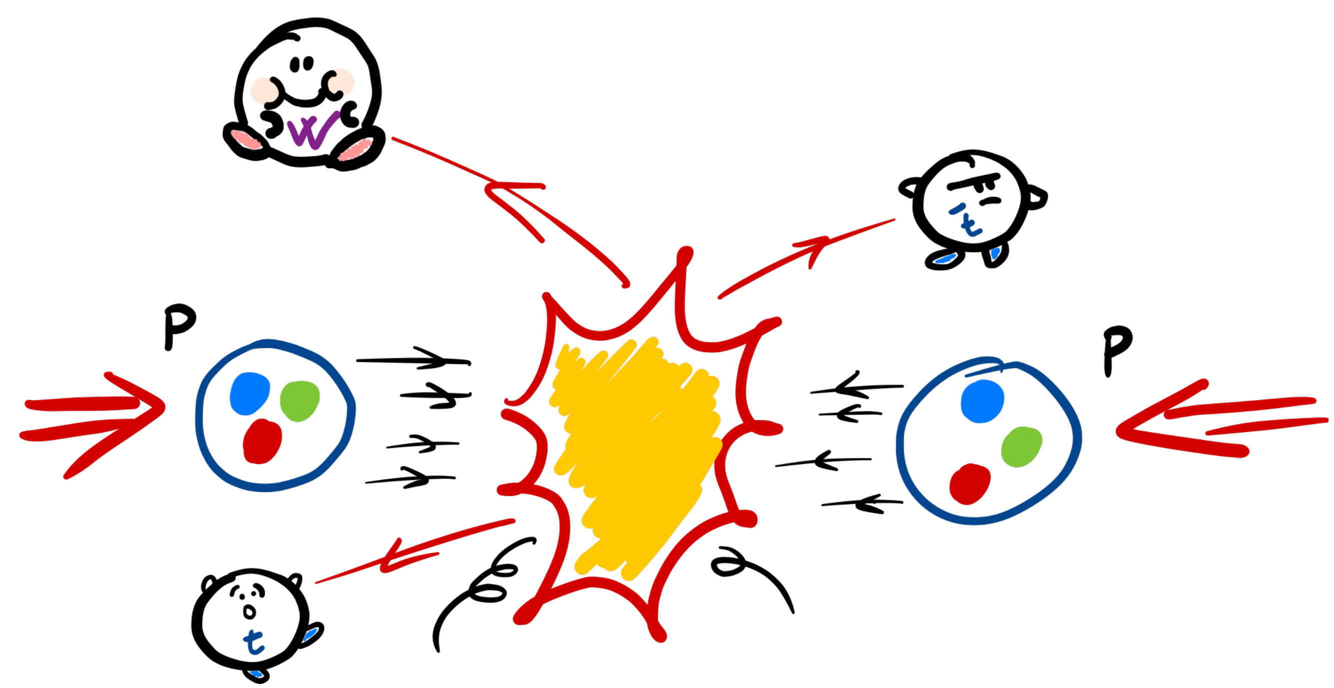
European Research Council
Established by the European Commission

In collaboration with:
L. Buonocore, M. Grazzini,
S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini



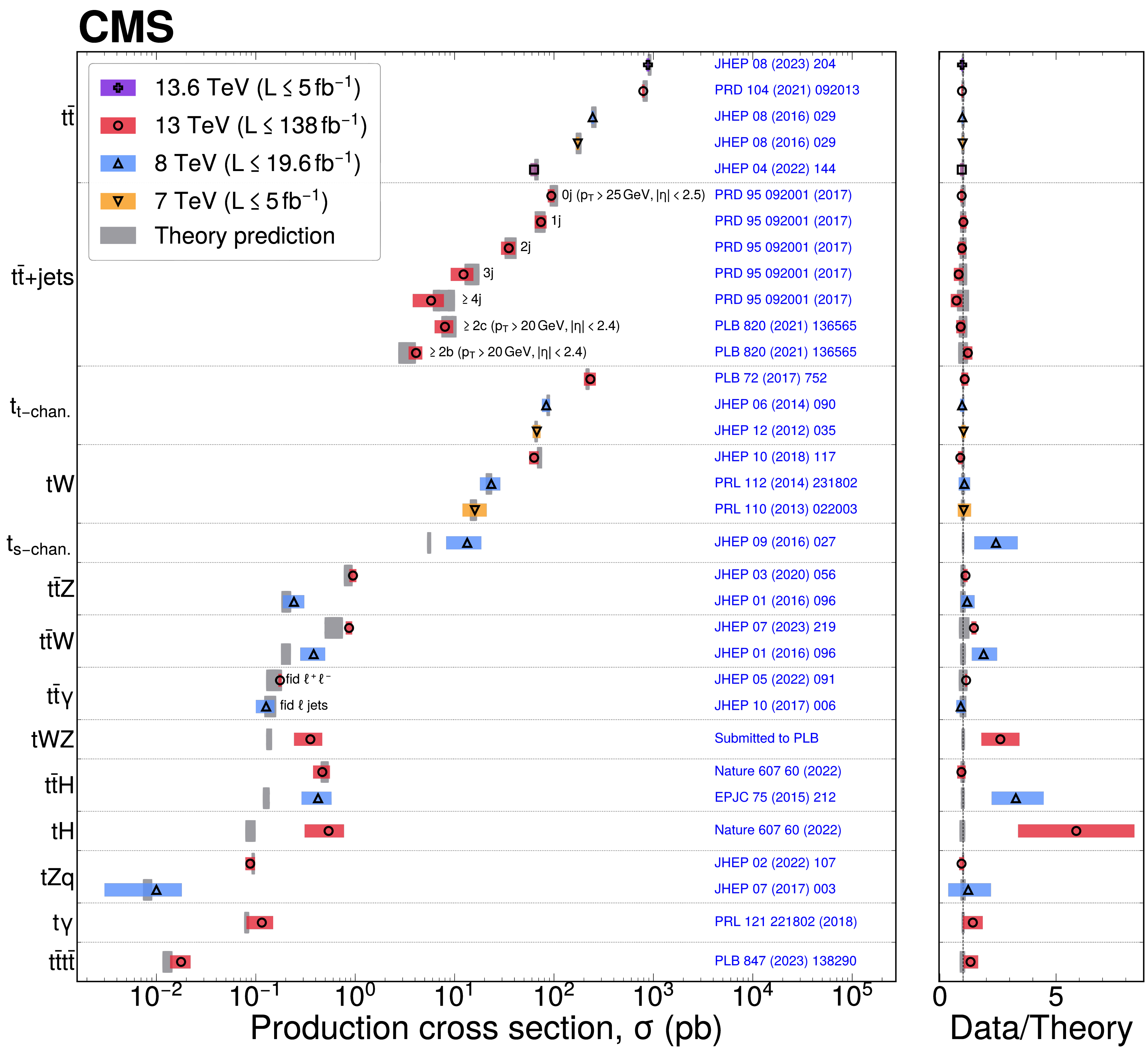
CONTENTS

- **Motivations**;
- Theory bottlenecks:
  - subtraction;
  - two-loop amplitudes;
- $t\bar{t}W$ @ **NNLO**;
- **Conclusions**.



TOP PHYSICS

- See plenary talk by Stefan-



[CMS collaboration -2405.18661]

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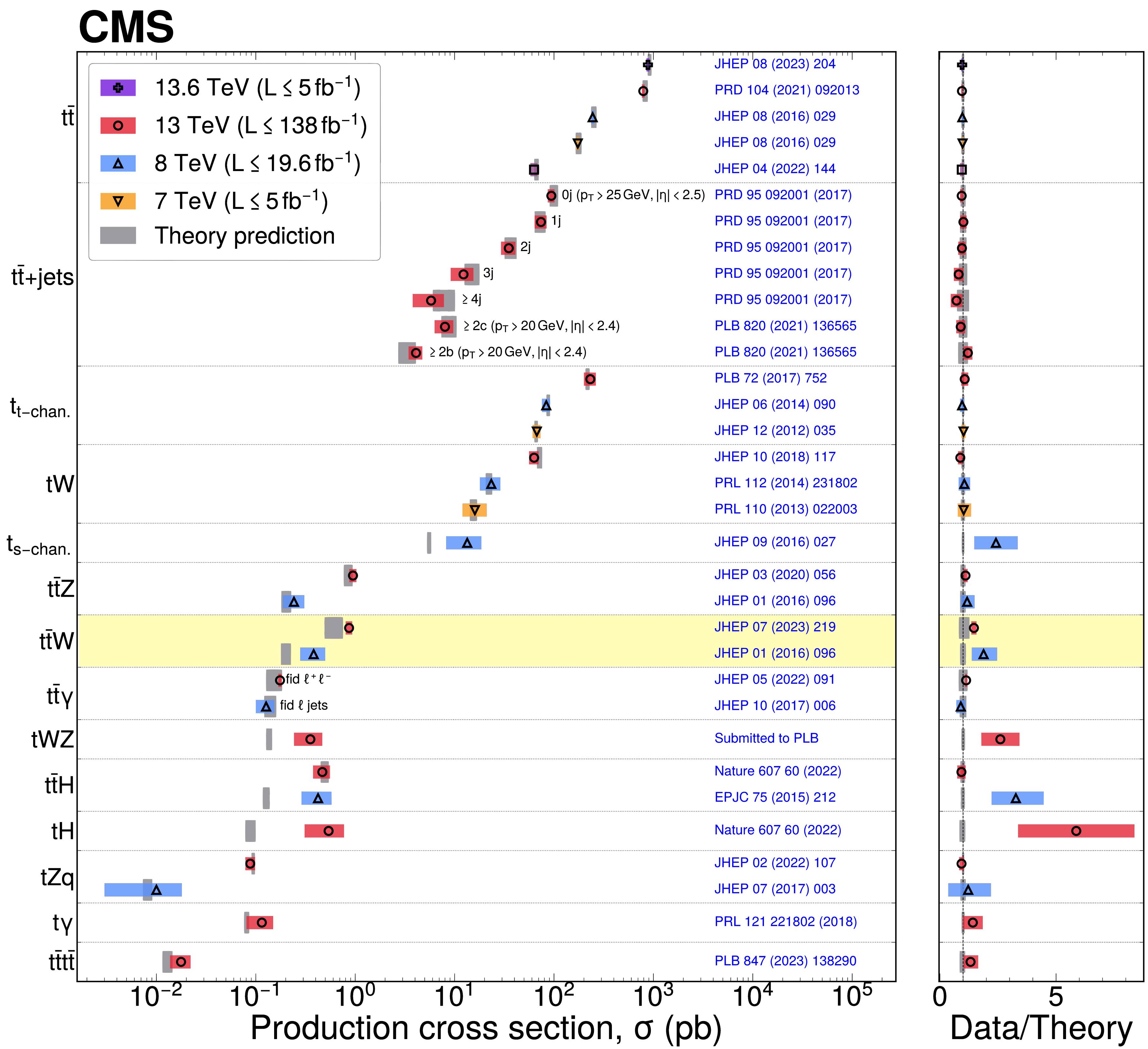
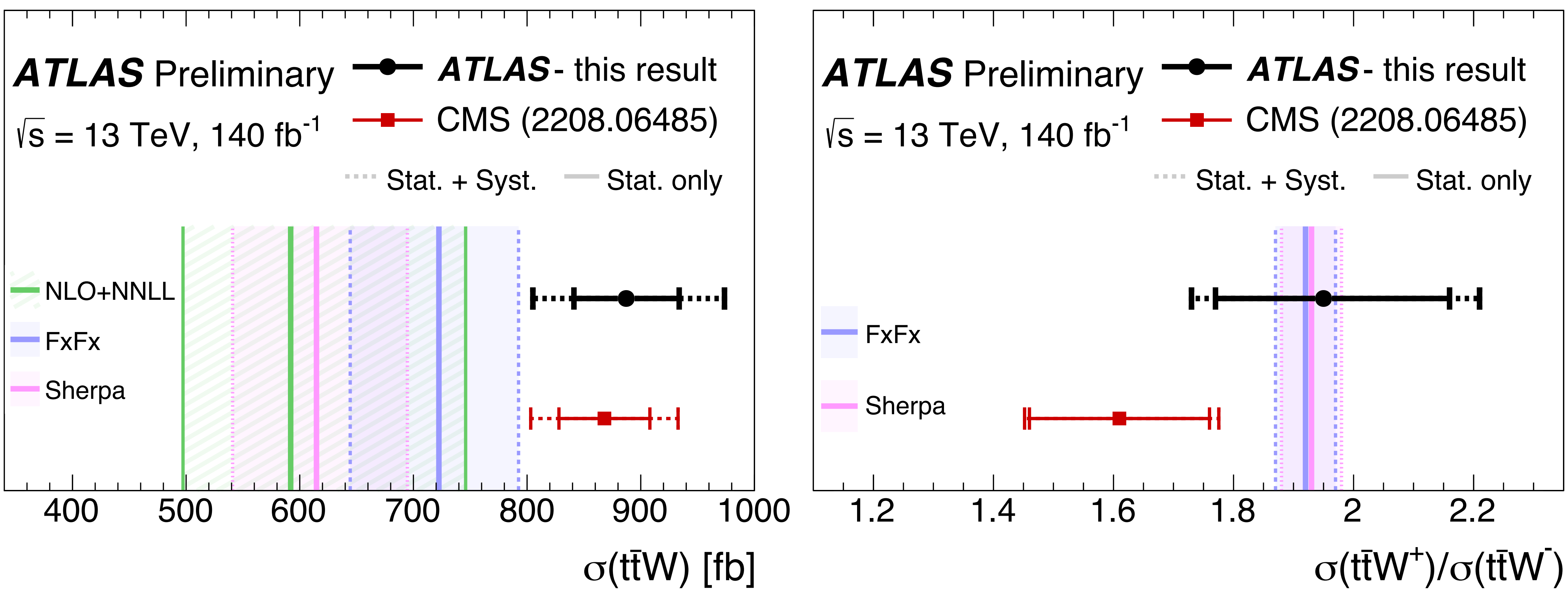


[CMS collaboration -2405.18661]

MOTIVATIONS: $t\bar{t}W$

- Together with $t\bar{t}H$ production, one of the **most massive** Standard Model (SM) signatures accessible at the LHC;
- Relevant as a $t\bar{t}H$ **background**;
- Measurements carried out by the ATLAS and CMS collaborations lead to rates consistently **higher than the SM predictions**;
- Most recent measurements confirm excess at the **2σ level**.



[ATLAS-CONF-2023-019]

STATUS OVERVIEW

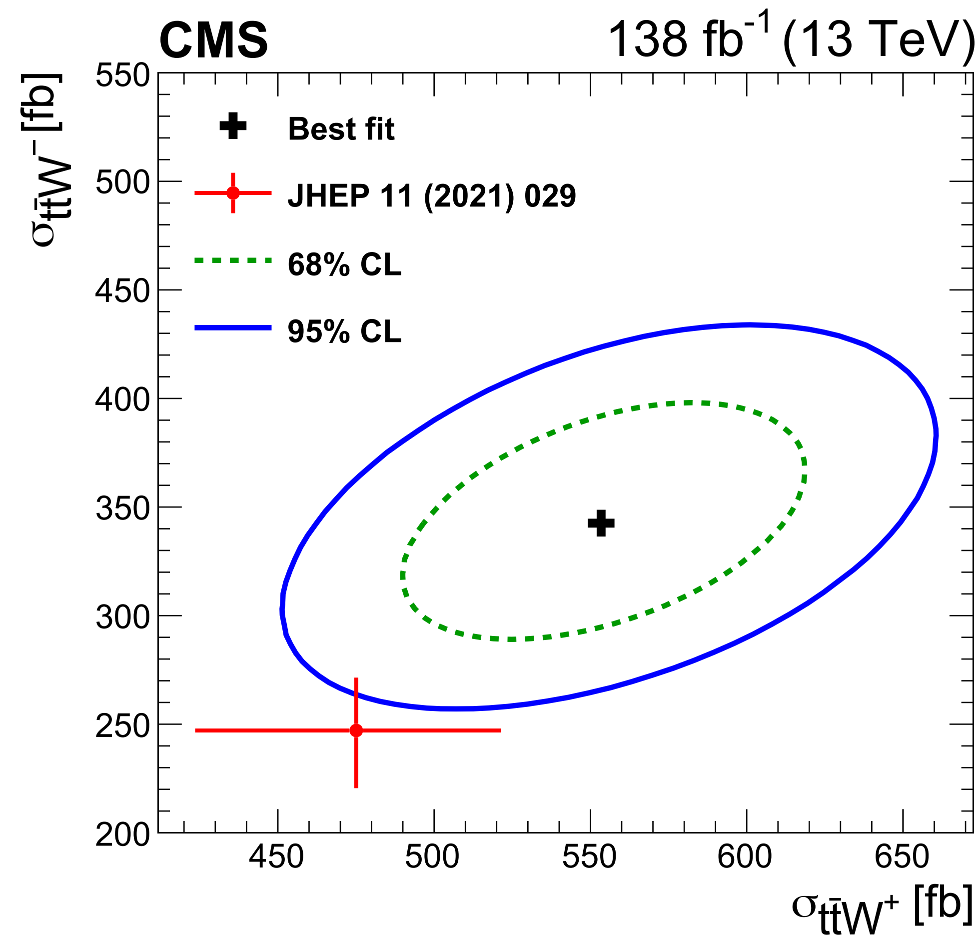
THEORY

- NLO QCD:
  [S. Badger, J. M. Campbell, R. K. Ellis, 1011.6647], [J. M. Campbell, R. K. Ellis, 1204.5678], [A. Denner, G. Pelliccioli, 2102.03264];
- NLO QCD with light jet:
  [G. Bevilacqua, H. Y. Bi, F. Febres Cordero, H. B. Hartanto, M. Kraus, J. Nasufi, L. Reina, and M. Worek , 2109.1581, 2305.03802]
- NLO QCD + EW:
  [S. Frixione, V. Hirschi, D. Pagani, H. S. Shao, M. Zaro, 1504.03446], [R. Frederix, D. Pagani, M. Zaro, 1711.02116], [Denner, Pelliccioli, 2020]
- Resummation of soft gluons:
  [H. T. Li, C. S. Li, S. A. Li, 1409.1460] [A. Broggio, G. Ferroglia, G. Ossola, B. D. Pecjak, 1607.05303], [A. Kulesza, L. Motyka, D. Schwartlaender, T. Stebel, V. Theeuwes, 1812.08662]
- NLO QCD + EW (on-shell) predictions supplemented with multi-jet merging as la FxFx: *[R. Frederix, S. Frixione, 1209.6215] [R. Frederix, I. Tsinikos, 2108.07862]*

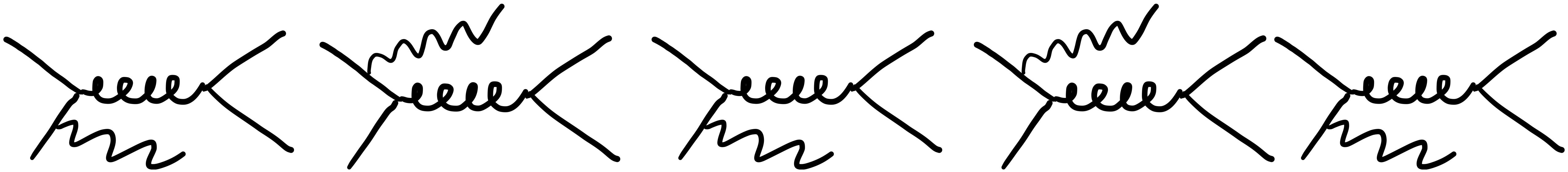
Current theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

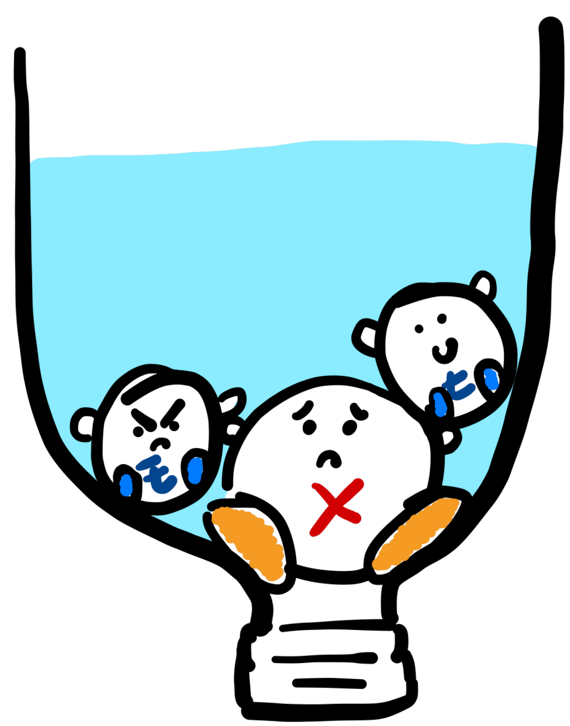
- ATLAS collaboration: *[2401.05299];*
- CMS collaboration: *[2208.06485].*



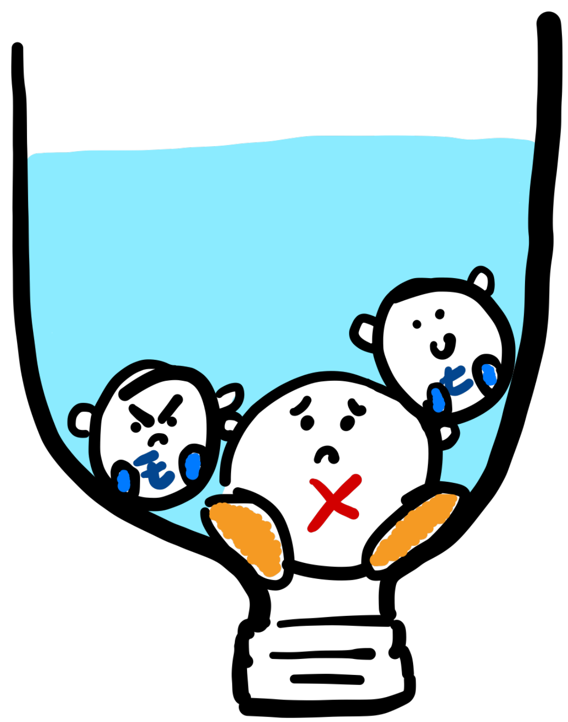
Theory-experiment tension at 2σ level:
Explained by higher order corrections?

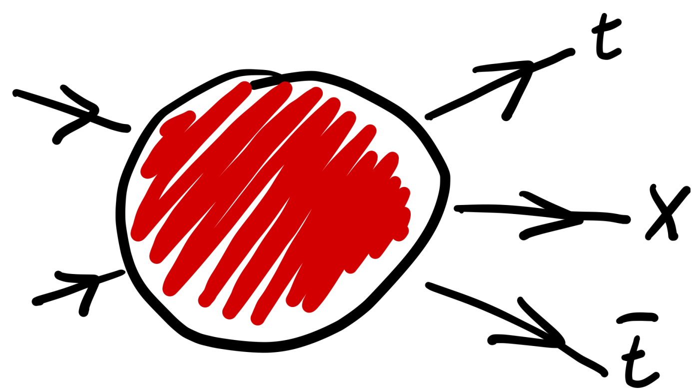


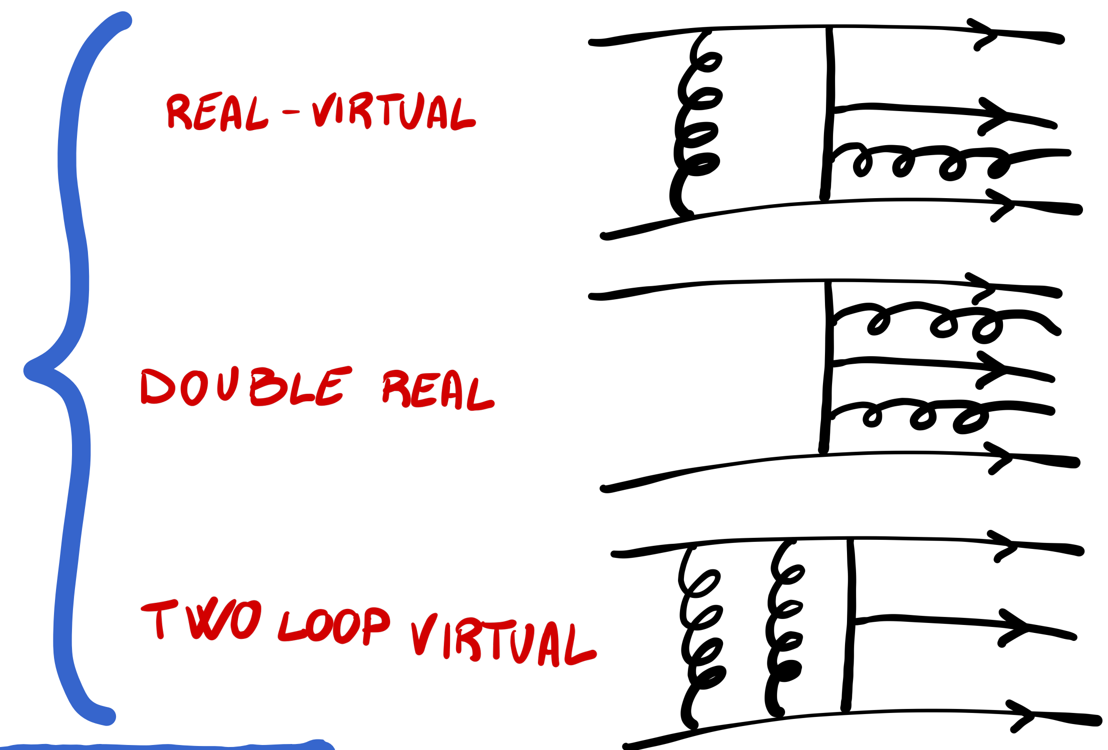
THEORETICAL CHALLENGES



THEORY BOTTLENECKS







What makes NNLO challenging?

Subtraction procedure

- We use q_T-subtraction;
- We generalised the method to this class of processes.

Two loop amplitudes

- Not known: current frontier!

[F. Febres Cordero, G. Figueiredo, M. Kraus, B. Page, L. Reina, 2312.08131],[B. Agarwal, G. Heinrich, S. P. Jones, M. Kerner, S. Y. Klein, J. Lang, V. Magerya, A. Olsson, 2402.03301]

- We developed approximations.

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini Phys.Rev.Lett. 98 (2007)]

$$d\sigma^F_{NNLO} = d\sigma^F_{NNLO}\Big|_{q_T=0} + d\sigma^F_{NNLO}\Big|_{q_T\neq 0}$$

$$d\sigma^{F+jets}_{NLO}$$

$$d\sigma^F_{NNLO} = \mathcal{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{NLO} - d\sigma^{CT}_{NLO}\right]$$

HARD COLLINEAR COEFFICIENT

Contains information on virtual corrections to the process.

$$\mathcal{H}^F_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}^{(2)}$$

Contains the genuine **2-loop contribution**:

$$H^{(2)} = \frac{2\,Re\left(\mathcal{M}^{(2)}(\mu_{IR},\mu_R)\mathcal{M}^{(0)}\right)}{|\mathcal{M}^{(0)}|^2}$$

- APPROXIMATED -

Includes:

- one-loop squared contribution;
- **soft parton contribution**.

- EXACT -

SOFT PARTON CONTRIBUTION

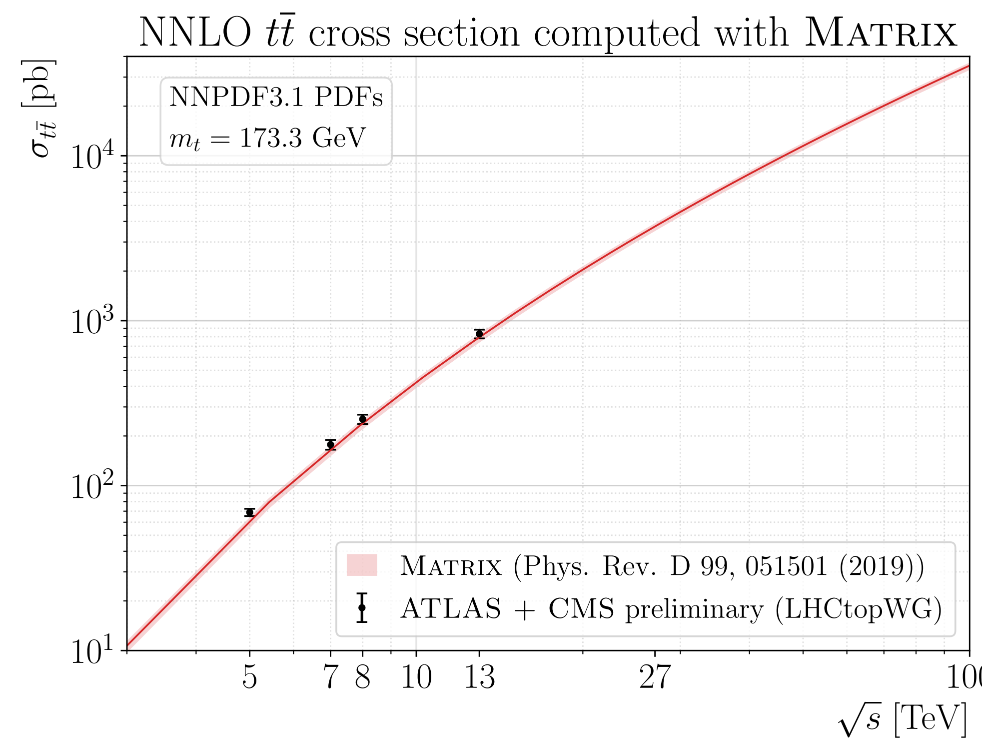
[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786
SD, J. Mazzitelli, In preparation]

The soft contribution from a massive final state was a key ingredient to extend q_T subtraction to a massive coloured final state.

Soft contributions to heavy-quark (Q) production

[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786]

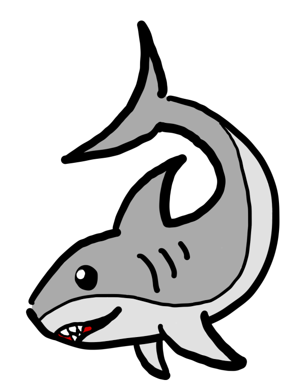
- Applied to top pair and bottom pair production: *[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019, 2020]*;
- Mostly **analytic** expressions;
- Assumption of $Q\bar{Q}$ **back-to-back** at LO.



NEW: generalisation to $Q\bar{Q}F$ kinematics

[SD, J. Mazzitelli, IN PREPARATION]

- removed the back-to-back assumption;
- Extra contribution computed **numerically**;
- On-the-fly numerical integration implemented in a **library**: **SHARK**
  Soft function for **H**eavy quark production in **AR**bitrary **K**inematics



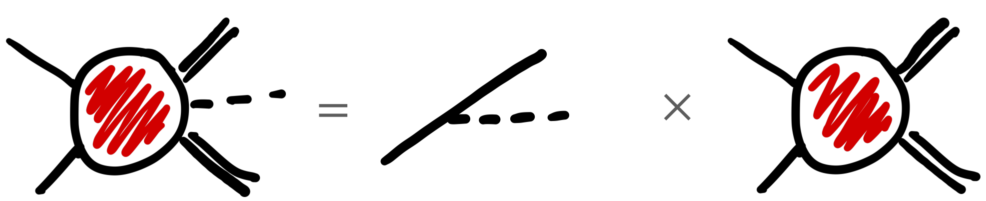
2-LOOP CONTRIBUTION

$$H^{(2)}_{t\bar{t}X} = \frac{2\,Re\left(\mathscr{M}^{(2)}_{t\bar{t}X}(\mu_{IR},\mu_R)\mathscr{M}^{(0)}_{t\bar{t}X}\right)_{appr.}}{|\mathscr{M}^{(0)}_{t\bar{t}X}|^2_{appr.}}$$

- We need to find an **approximation** of the 2-loop **virtual amplitude**;
- Because of the approximation, leftover dependence on the (unphysical) **subtraction scale** μ_{IR}.

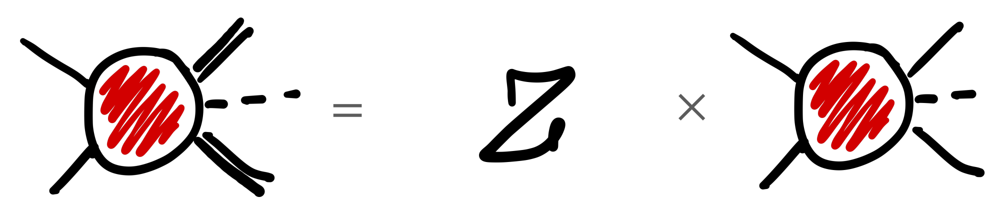
Two independent approximations

Soft approximation



- Captures the leading behavior when the energy and **mass of the associated boson** are smaller than the other relevant scales

Massification procedure



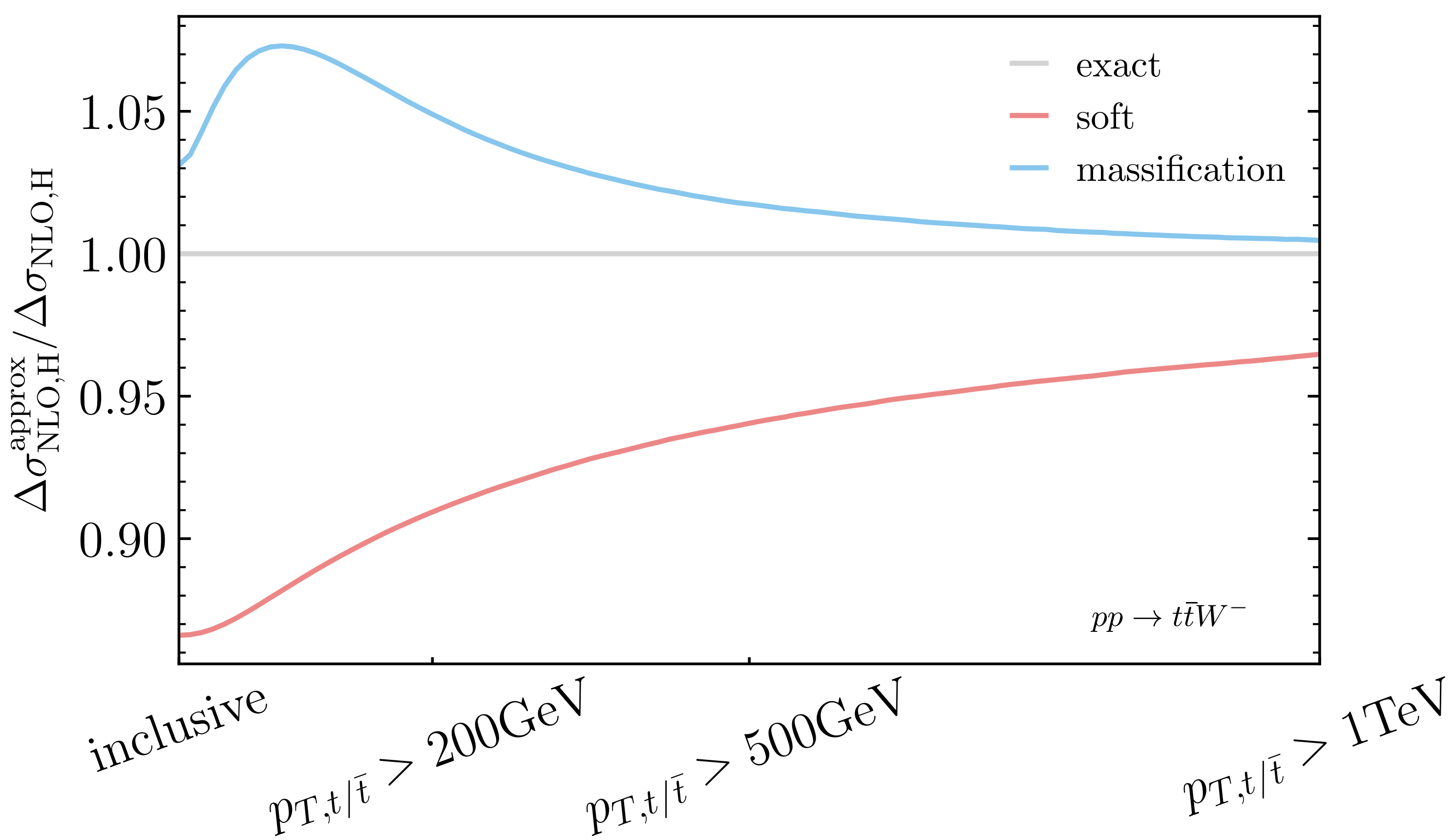
- Captures the leading behavior when the **mass of the top pair** are smaller than the other relevant scales

TESTING THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

To validate our procedure: test the approximations at NLO!

- Both approximations provide a **good estimation** also at the inclusive level;
- We observe a **pattern**: **soft approximation undershoots** the exact result, while the **massification procedure overshoots**;
- As expected, both approximations get closer to the exact result when a **harder cut** is imposed

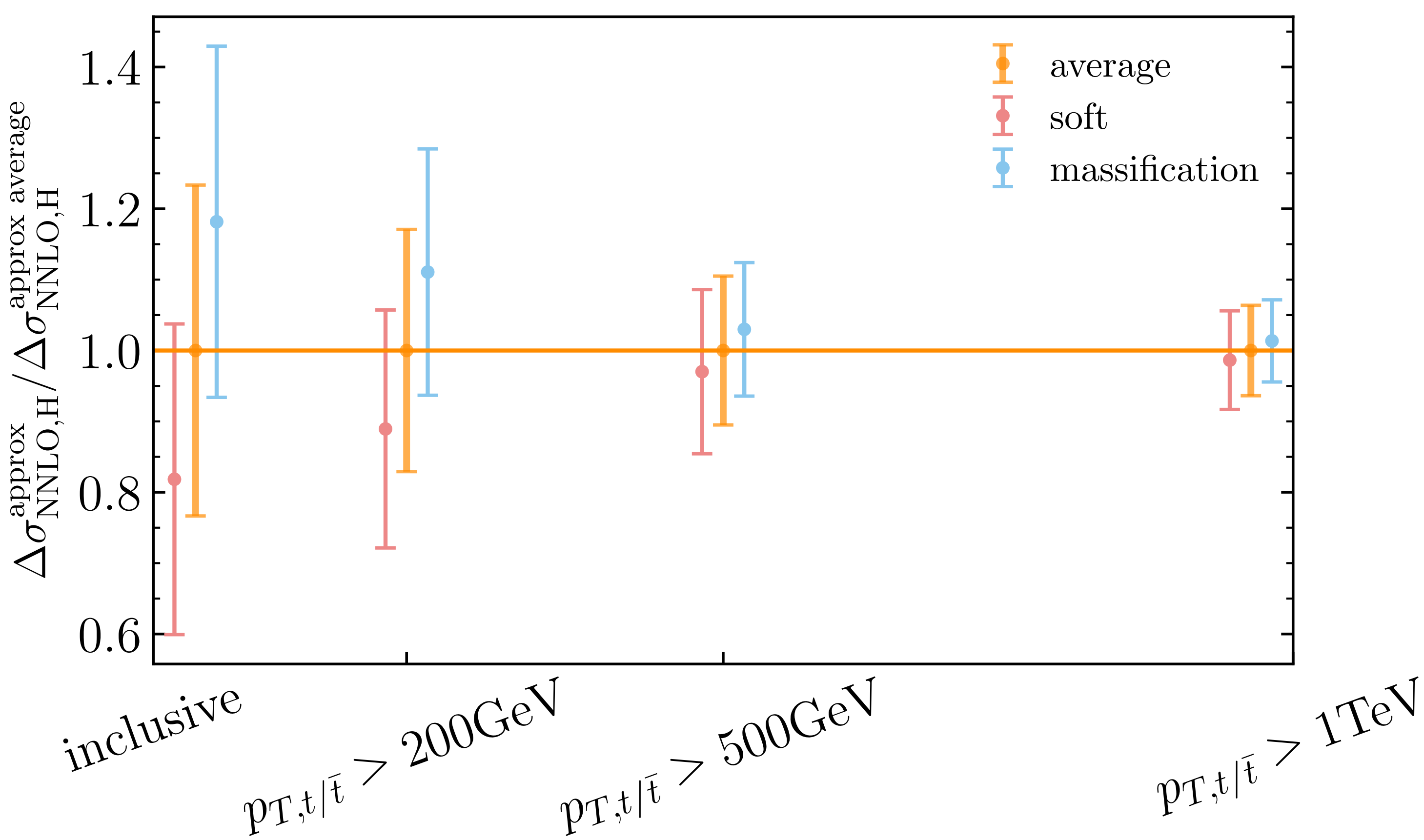


UNCERTAINTIES ESTIMATION

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

How to estimate the NNLO uncertainties of each approximation?

- **Method 1**: we take the difference between exact and approximated result at NLO and we multiply by a **tolerance factor** of **2**;
- **Method 2**: we consider the effect of using a **different subtraction scales** $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$;
- The uncertainty is defined as the **maximum between these two estimates**.



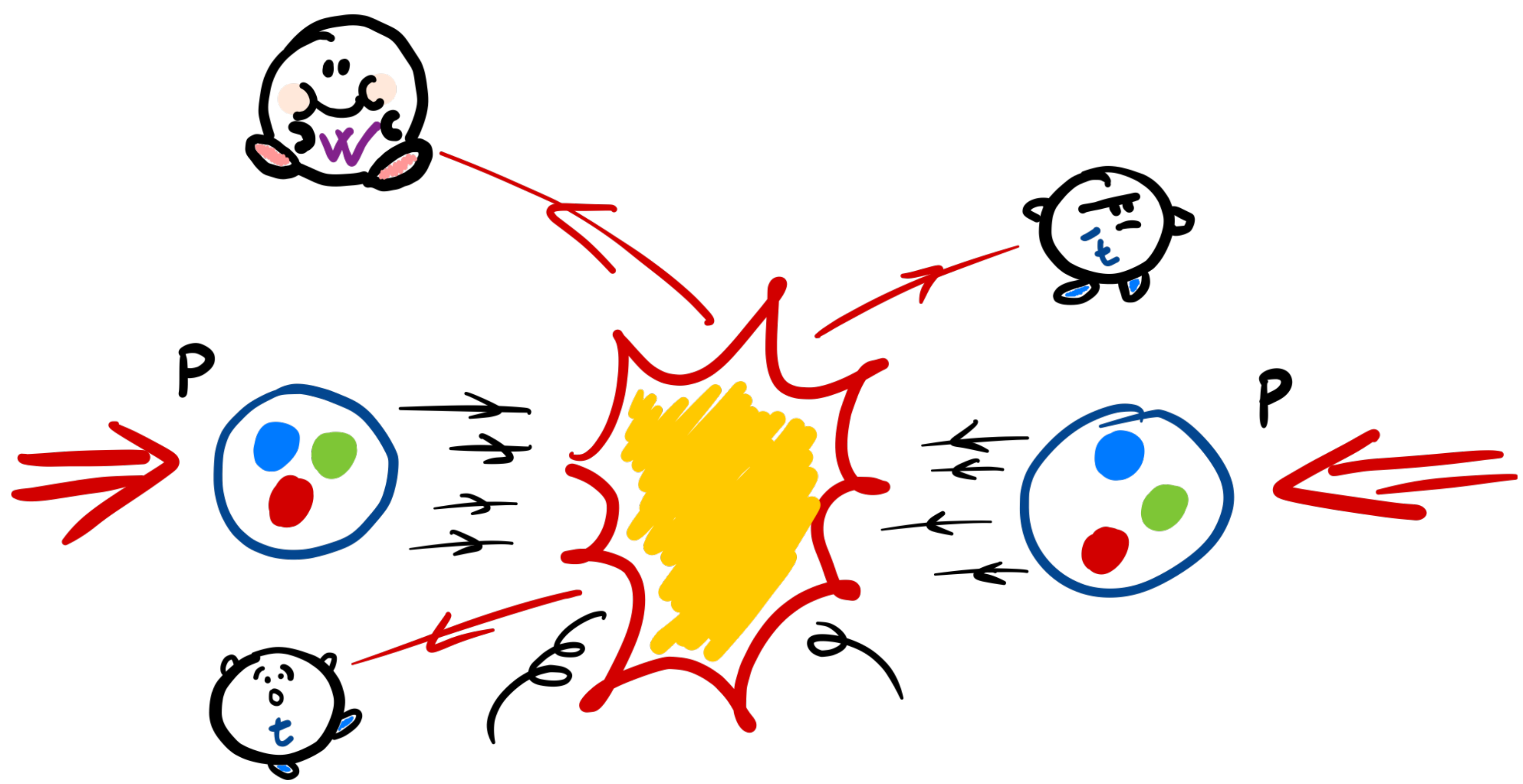
- The two approximations are **fully consistent**;
- Our best prediction is obtained by taking their **average** and **linearly combing** the uncertainties.

Final uncertainty:

- $\pm 25\,\%$ on $\Delta\sigma_{\mathrm{NNLO,H}}$
- $\pm 2\,\%$ on σ_{NNLO}

PHENOMENOLOGY

[ArXiv:2306.16311]



RESULTS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

LHC@13TeV	$\sigma_{t\bar{t}W^+}$ $[fb]$	$\sigma_{t\bar{t}W^-}$ $[fb]$	$\sigma_{t\bar{t}W}$ $[fb]$	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
LO$_{QCD}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.0^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
NLO$_{QCD}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
NNLO$_{QCD}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\,\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\,\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\,\%$	$2.018^{+1.6\%}_{-1.2\%}$
NNLO$_{QCD}$+NLO$_{EW}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\,\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\,\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\,\%$	$2.007^{+2.1\%}_{-2.1\%}$

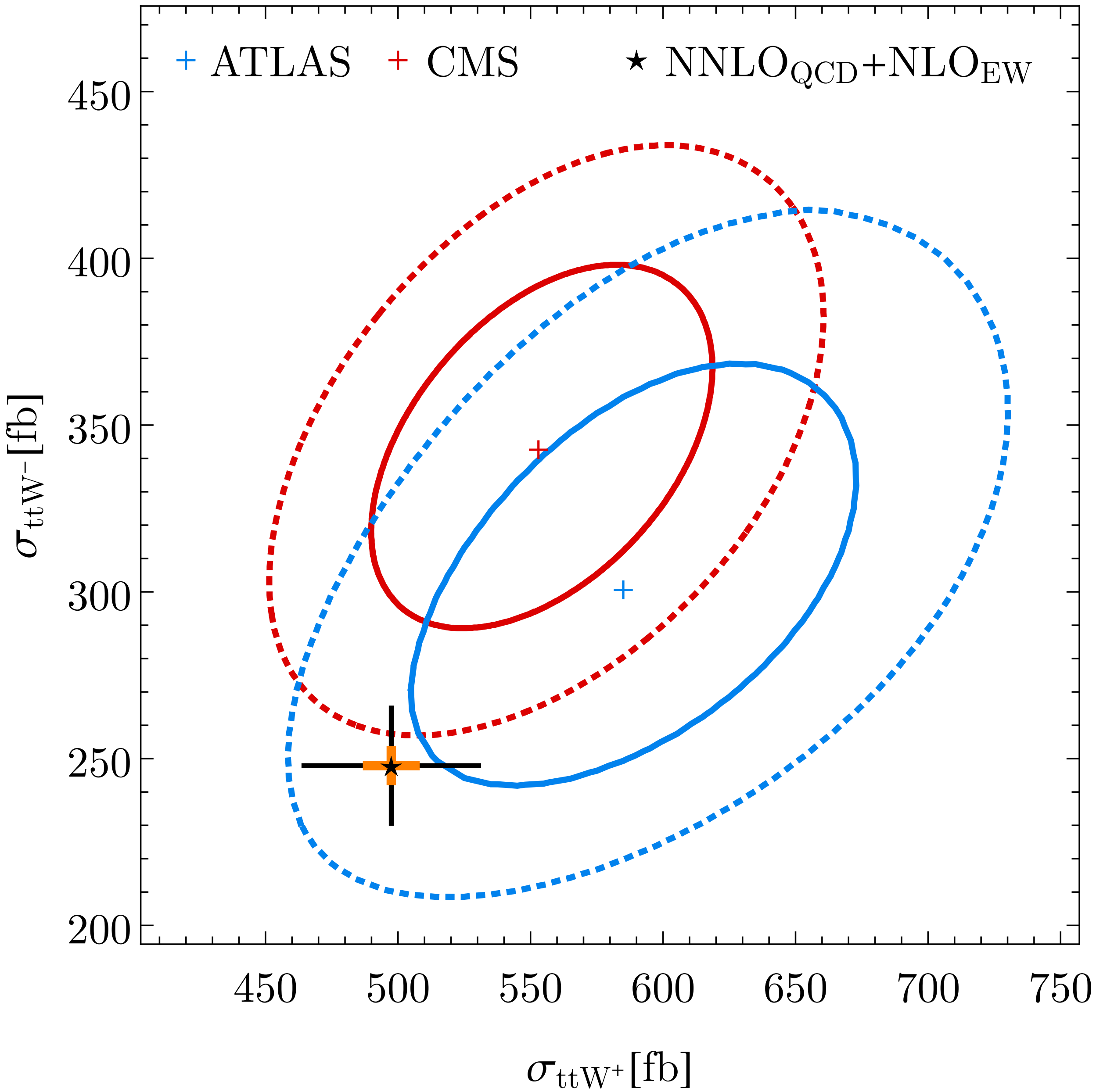
Scale uncertainties

Uncertainties from 2 loop amplitudes

- We choose $\mu_0 = M/2$;
- NNLO predictions show first sign of **perturbative convergence**;
- ratio $\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ have a **very stable** perturbative behavior;
- **PDF uncertainties** $\pm 1.8\,\%$ (computed with MATRIX + PINEAPPL interface *[SD, T. Ježo, S. Kallweit, C. Schwan, in preparation]*)
- α_S **uncertainties** $\pm 1.8\,\%$;
- by combining with EW corrections, we get our **best prediction**;
- to be conservative, scale uncertainties for NNLO$_{QCD}$+NLO$_{EW}$ are **symmetrized**.

RESULTS

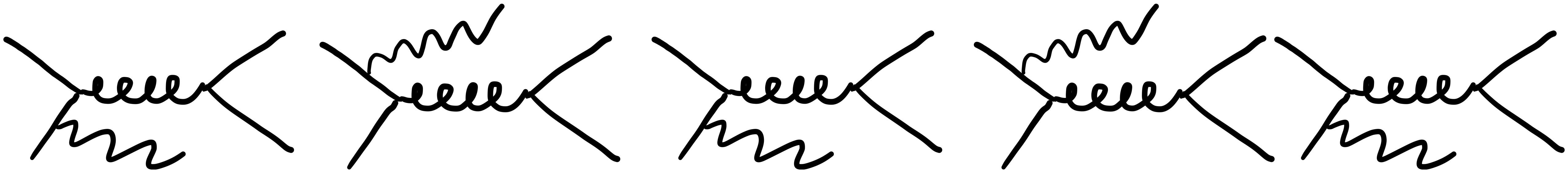
[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]



- We compare our best prediction to **ATLAS and CMS measurements**;
- With respect to the **FxFx prediction**, the current theory reference, higher rate and smaller uncertainties;

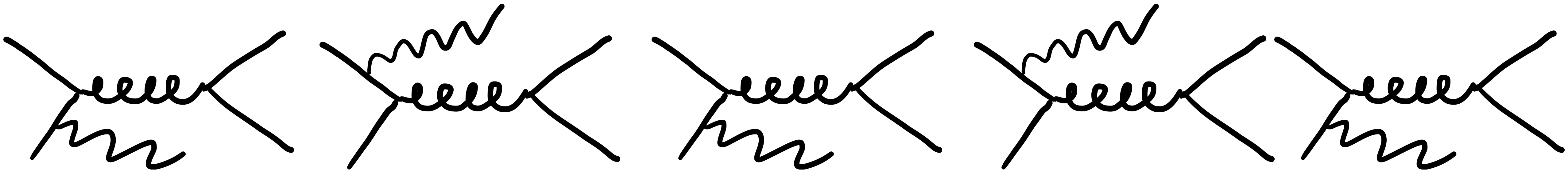
$$\sigma_{t\bar{t}W}^{NNLO_{QCD}+NLO_{EW}} = 745.3^{+6.7\%}_{-6.7\%}$$
$$\sigma_{t\bar{t}W}^{FxFx} = 722.3^{+9.7\%}_{-10.8\%}$$

- Tension remains at the $1\sigma - 2\sigma$ **level**.



SUMMARY

- We computed within q_T subtraction formalism the **NNLO QCD corrections** to $t\bar{t}W$ production;
- The **missing ingredients** we needed for the computation are:
  - **NNLO soft contribution** in arbitrary kinematics;
  - **two-loop amplitudes**: **approximated** with **massification** and **soft approximation**.
- NNLO prediction confirm the observed $1\sigma - 2\sigma$ **tension** with the experimental measurement.



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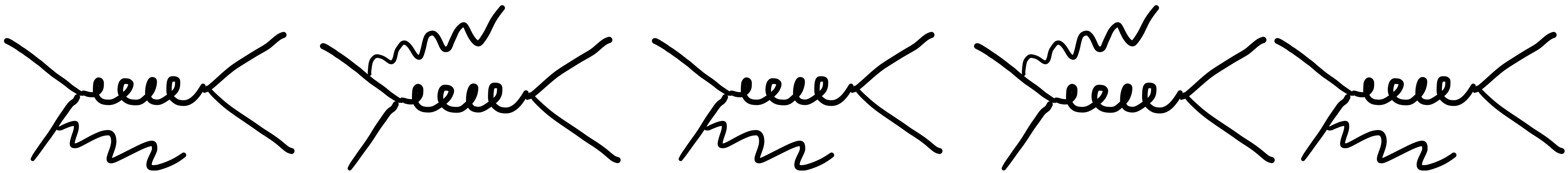
THANKS!



BACKUP SLIDES

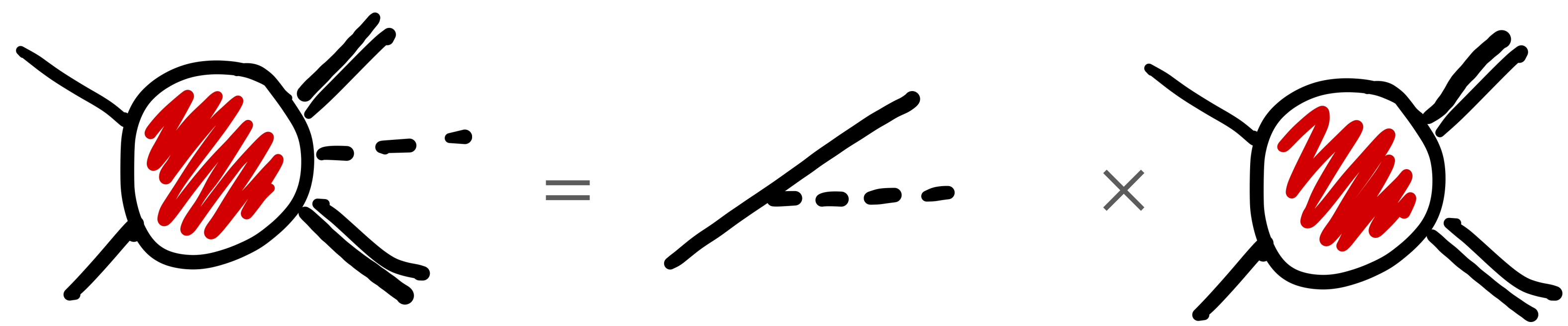
SOFT APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Soft approximation:

$k \to 0, \quad m_W \ll m_t$



$$\mathscr{M}_{q\bar{q}' \to t\bar{t}W}\big(\{p_i\}, k\big) \simeq \frac{g}{\sqrt{2}} \left(\frac{p_2 \cdot \varepsilon^*(k)}{p_2 \cdot k} - \frac{p_1 \cdot \varepsilon^*(k)}{p_1 \cdot k} \right) \mathscr{M}_{q_L \bar{q}'_R \to t\bar{t}}\big(\{p_i\}\big)$$

- The formula captures the leading behavior in the **soft limit** $k \to 0$: the emission from highly **off-shell top propagators** is **not captured**.
- To use the approximation, we need a **recoil prescription** to map the $t\bar{t}X$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}W} \to Q_{t\bar{t}}$);

MASSIFICATION PROCEDURE

[A. A. Penin: 0508127]
[A. Mitov, S. Moch: 0612149]
[T. Becher and K. Melnikov: 0704:3582]

Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + X(k)$

Massification procedure:

$m_t \ll Q$

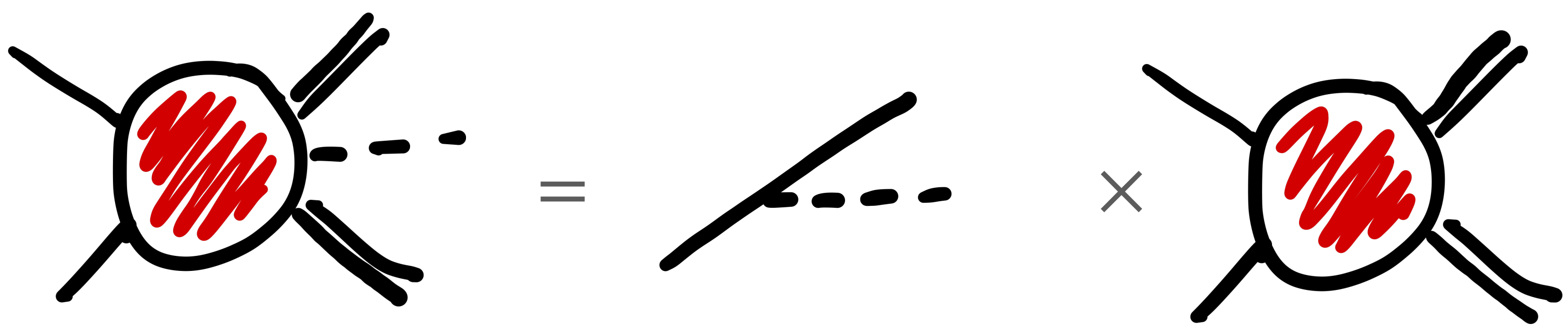
$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right) \mathcal{M}_{q\bar{q}'\to q\bar{q}W}(\{p_i\}, k; \mu_R, \epsilon)$$

- The perturbative factor $Z_{[q]}^{(m_t|0)}$ was computed in *[A. Mitov, S. Moch: 0612149]*;
- The procedure retrieves the correct **mass logarithms**;
- The contribution from **massive top loops** is **not captured**;
- Massification of the amplitudes implemented in a **C++ library**, **WQQAmp** *[L. Buonocore, L. Rottoli, C. Savoini, https://gitlab.com/lrottoli/WQQAmp]*;
- Successfully applied to $b\bar{b}W$ production *[L. Buonocore, SD, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2212.04954]*.

CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the process $c\bar{c} \to t\bar{t}$ available *[P. Bärnreuther, M. Czakon, P. Fiedler: 1312.6279]*: **we can use the soft approximation**.



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}W}\left(\{p_i\},k\right) \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k} - \frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}\left(\{p_i\}\right)$$

- The soft emission of a W selects the **helicity configuration** $\mathcal{M}_{q_L\bar{q}'_R\to t\bar{t}}$;
- In contrast with the $t\bar{t}H$ case, the soft W is emitted by the **initial-state partons**;
- To map the $t\bar{t}W$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}W} \to Q_{t\bar{t}}$), we use use a **prescription symmetrised** with respect to the one employed for $t\bar{t}H$ case:
  - We reabsorb the W momentum equally in the top-quark momenta;
  - We leave unchanged the initial-state parton momenta.

CHOICE OF THE APPROXIMATIONS

[L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: 2306.16311]

- Amplitudes for the massless process $c\bar{c} \to q\bar{q}W$ available *[S. Abreu, F. Febres Cordero, H. Ita, M. Klinkert, B. Page, V. Sotnikov: 2110.07541]*: **we can use the massification procedure**;

$$\mathcal{M}(\{p_i\}, k; \mu_R, \epsilon) \sim Z_{[q]}^{(m_t|0)}\left(\alpha_S(\mu_R), \frac{m_t}{\mu_R}, \epsilon\right) \mathcal{M}^{m_t=0}(\{p_i\}, k; \mu_R, \epsilon)$$

- Massification of the amplitudes implemented in a **C++ library**, **WQQAmp** *[L. Buonocore, L. Rottoli, C. Savoini, https://gitlab.com/lrottoli/WQQAmp]*;
- We need to map the massless kinematics into a massive one: we do it by preserving the momentum of the $t\bar{t}$ pair.

IR SUBTRACTION METHODS (NLO)

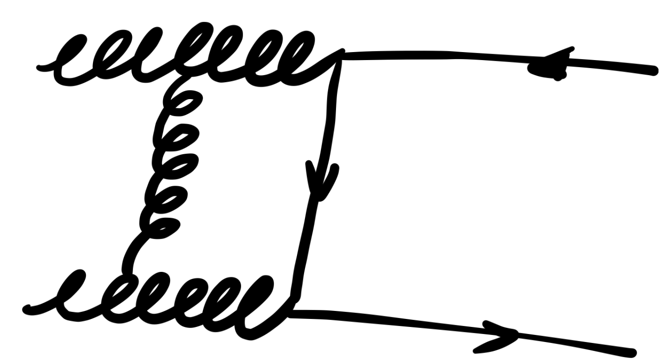
$$\Delta\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V}$$

Divergent — Divergent

REAL

- ➤ No explicit ϵ poles;
- ➤ Singular in unresolved limit.

VIRTUAL



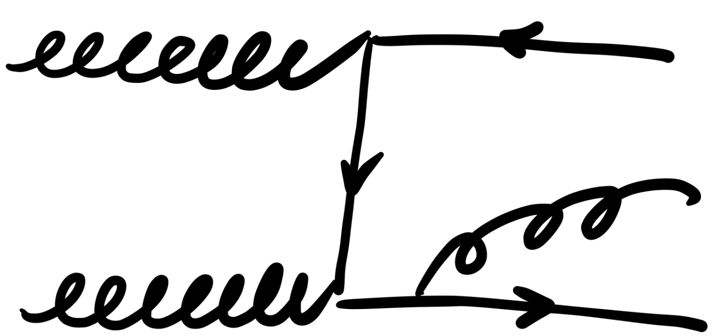
- ➤ Explicit poles up to order $1/\epsilon^2$;
- ➤ No additional PS singularity

IR SUBTRACTION METHODS (NLO)

$$\Delta\sigma^{NLO} = \int d\sigma^{NLO} = \int_{m+1} \left[d\sigma^{R} - d\sigma^{CT} \right] + \left[\int_{m} d\sigma^{V} + \int_{1} d\sigma^{CT} \right]$$

~~Divergent~~ CONVERGENT! ~~Divergent~~ CONVERGENT!

REAL



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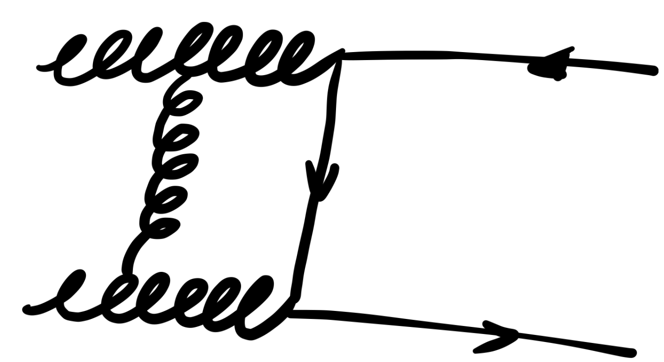
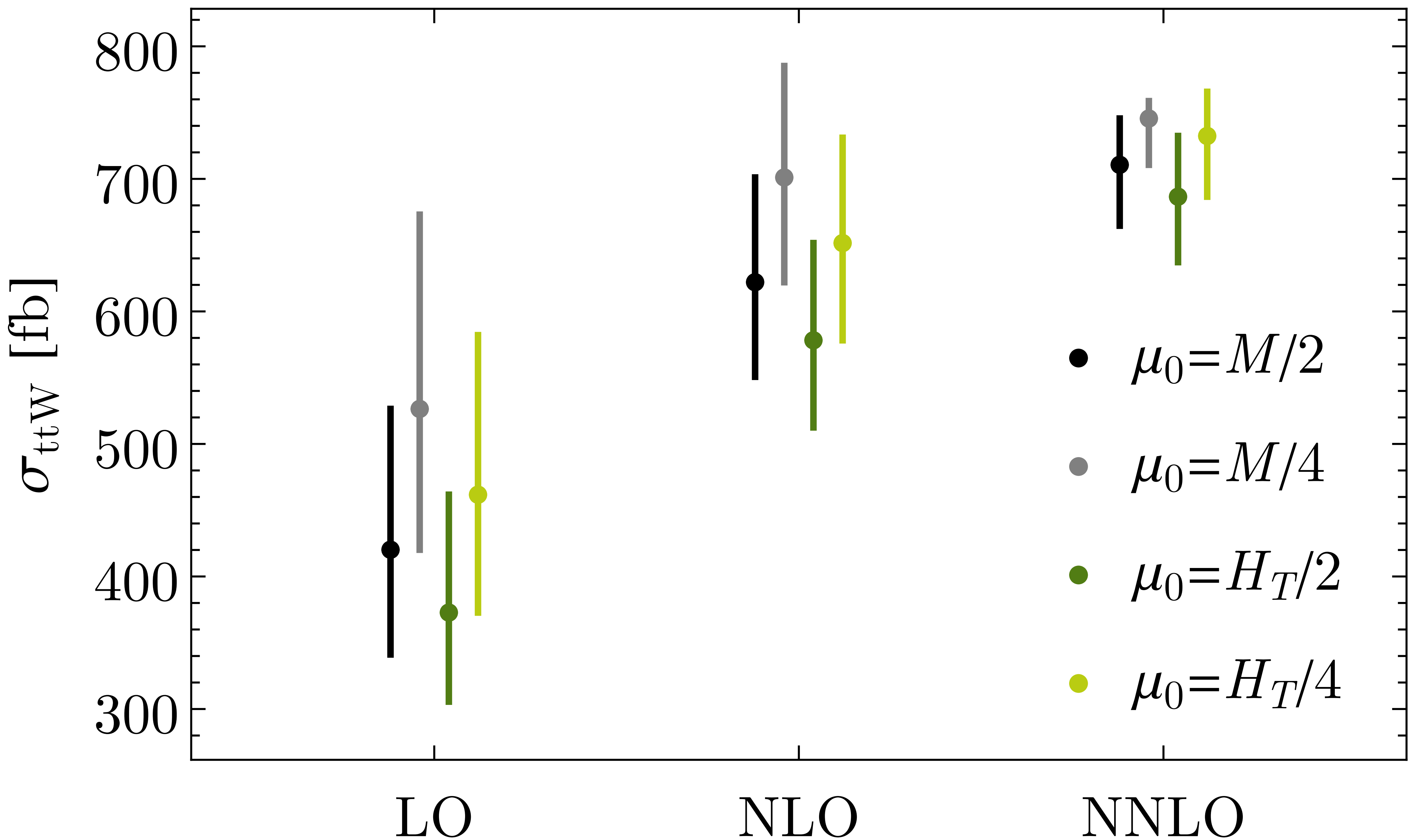
VIRTUAL



➤ *Explicit poles up to order $1/\epsilon^2$;*

➤ *No additional PS singularity*

$t\bar{t}W$: DIFFERENT SCALE CHOICES



THE SLICING

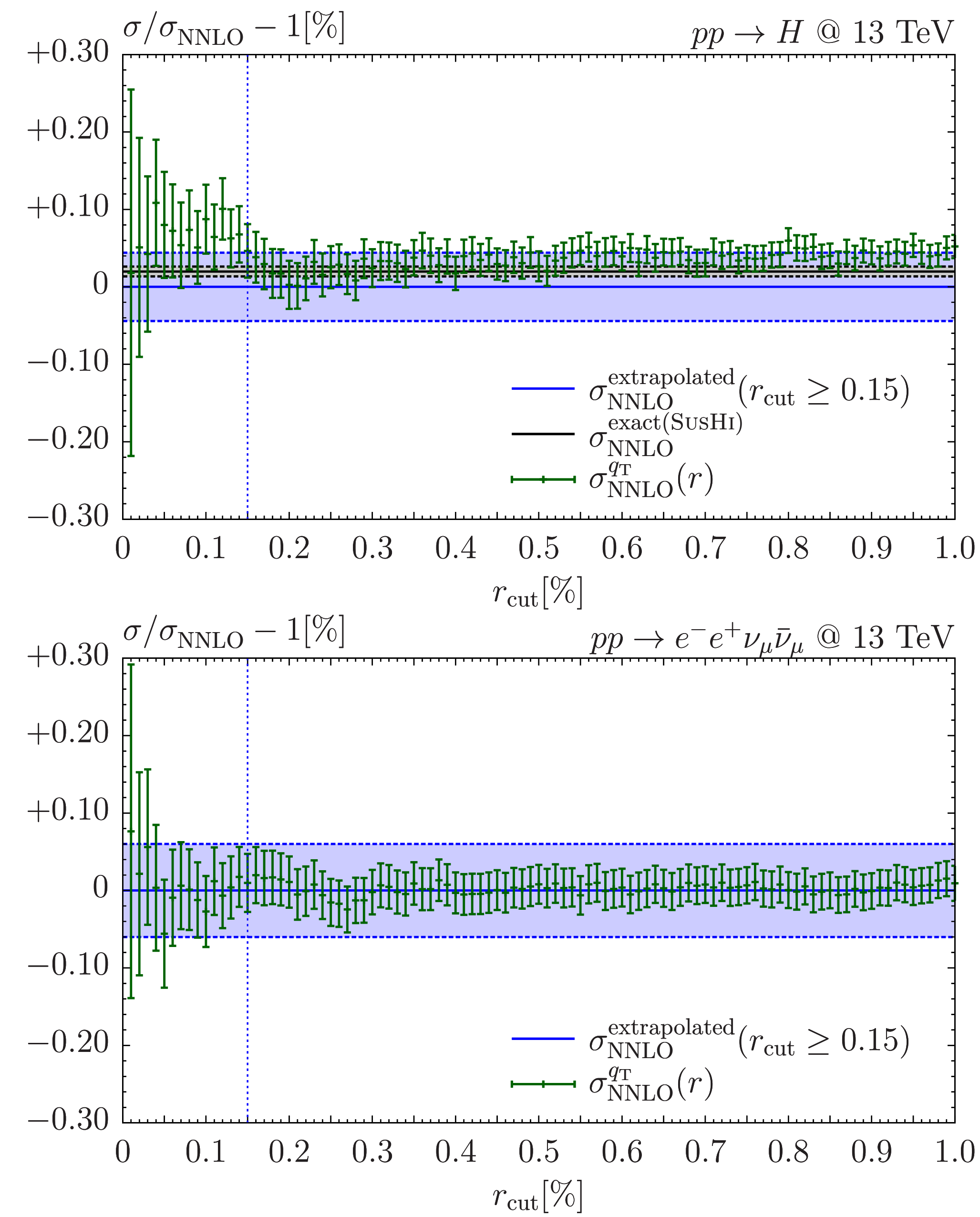
$$d\sigma^{F}_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

$d\sigma^{F+jets}_{(N)LO}$ and $d\sigma^{CT}_{(N)LO}$ are separately divergent.

In practice, q_T subtraction is implemented as a slicing method:

- introducing a cutoff $r_{cut} = Q/M$;
- performing the limit $r_{cut} \to 0$.

Quality of the $q_T \to 0$ extrapolation can be understood looking at the r_{cut} dependence



[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

r_{CUT} DEPENDENCE