Top-antitop spin correlation and entanglement

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Spin correlation and entanglement

Polarization, P and spin correlation matrix, C determine the angular distribution of the decay products in the helicity basis as in [1212.4888]

\[ \frac{d\sigma}{d\Omega d\bar{\Omega}} = \sigma_{\text{norm}} (1 + \kappa \vec{P} \cdot \Omega + \bar{\kappa} \vec{P} \cdot \bar{\Omega} - \kappa \bar{\kappa} \Omega \cdot C \cdot \bar{\Omega}) \]

\( \kappa \) - spin analyzing power of top/antitop decay products

\( \Omega \) – unit vector in the direction of the decay product

\( 2 \times 3(P) + 3 \times 3(C) = 15 \) coefficients \( Q_m \)

Alternatively, we can define \( \chi \) - opening angle between the two decay products, then

\[ \frac{d\sigma}{d\cos \chi} = A(1 + D\kappa\bar{\kappa} \cos \chi) \]

where

\[ C_{nn} + C_{rr} + C_{kk} = Tr(C) = -3D \]

and \( \tilde{\chi} \), where the sign of n-component in one of the decay products is inverted

\[ \frac{d\sigma}{d\cos \tilde{\chi}} = A(1 + \tilde{D}\kappa\bar{\kappa} \cos \tilde{\chi}) \]

\[ C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} \]

The system is considered separable if its density matrix can be factored into that of individual states

\[ \rho = \sum_n p_n \rho_n^t \rho_n^t \]

Otherwise, it is considered entangled \( \rightarrow \) Peres-Horodecki criterion [2003.02280]

Entanglement is a result of spin correlation.

Two approaches – both presented

- Use full angular information of two decay products (e.g. charged leptons, or a lepton and a d-type quark) to measure the full matrix C and then construct \( \Delta_E \)

- Use the distribution in \( \chi \) and \( \tilde{\chi} \) to measure D and \( \tilde{D} \)
Spin correlation and entanglement

There are four maximally entangled states

\[ |\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle), \]

\[ |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle). \]

at high \( M_{tt} \) triplet vector state \((\Phi^+ - \Phi^-, \Psi^+, \Phi^+ + \Phi^-)\)

Peres-Horodecki criterion

\[ \Delta_{E} = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1 \]

\[ \tilde{D} > \frac{1}{3} \]

at low \( M_{tt} \) singlet pseudoscalar state \( \Psi^- \)

Peres-Horodecki criterion

\[ \Delta_{E} = Tr(C) = -3D > 1 \]

\[ D < -\frac{1}{3} \]

Plot from Afik, De Nova

EPJP136(2021)9,907

hep-ph:2003.02280

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Dilepton vs l+jets channels

- Dilepton based on PRD 100 (2019) 072002
  - Lower branching ratio
  - $|\kappa|=1$ for charged leptons, which are easy to ID $\rightarrow$ Ideal channel for spin correlation
  - Lower $p_T$ cuts for leading/subleading lepton (25/20 GeV) $\rightarrow$ higher efficiency at the threshold
  - Worse $M_{tt}$ resolution, not ideal for differential measurement
- Best for threshold
  - high entanglement
  - potential for “toponium” observation
  - mostly time-like separated events
- CMS Top-23-001

- Lepton+jets
  - Higher branching ratio
  - $|\kappa|=1$ for down-type quarks, but they are harder to identify – employ AI ($\sim 66\%$)
  - Higher $p_T$ cut for single lepton (30 GeV) and for 4 jets (30 GeV) $\rightarrow$ lower efficiency at the threshold, but OK for high $M_{tt}$
  - Better $M_{tt}$ resolution, good for differential measurement
- Advantage for high $M_{tt}$
  - high entanglement
  - potential for observation of Bell Inequality violation
  - mostly space-like separated events
- CMS Top-23-007
Signal modeling

- NLO POWHEG+Pythia8
  - Dilepton: \( p_T \) reweighting to match the top quark \( p_T \) spectrum from a fixed order ME calculation at NNLO
  - Lepton+jets: NN-based reweighting to match NNLO distributions at reco level
- Add “toponium” (pseudo-scallar color singlet predicted by non-relativistic QCD)
  - \( M(\text{toponium}) \sim 344 \text{ GeV}, \sigma \sim 6.5 \text{ pb} \)
    - Sumino, Fujii, Hagiwara, Murayama & Ng (PRD’93)
    - Jezabek, Kuhn & Teubner (Z.Phys.C’92)
    - B. Fuks et al. (PRD 104 (2021) 034023)
  - affects the invariant mass distribution and the spin correlations at the threshold

F. Maltoni et al. JHEP03(2024)099 06/05/24

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To extract $D$ we measure the distribution in the sensitive variable – $\cos \chi$

\[
\frac{d\sigma}{d \cos \chi} = A(1 + D \kappa \cos \chi)
\]

Optimize $M_{tt}$ cut to maximize sensitivity to entanglement

Determine the effect of acceptance and efficiency by comparing $D_{\text{reco}}(M_{\text{reco}})$ vs $D_{\text{gen}}(M_{\text{gen full phase space}})$

The $t\bar{t}$bar entanglement is observed at $> 5.0 \sigma$ level for $345 < M_{tt} < 400$ GeV, $\beta < 0.9$

$\sim 1.5\sigma$ tension with the expectation if toponium is not included
We pursue both strategies — evaluation of the full correlation matrix $C$ and polarization vectors $P$ as well as $D$ and $\bar{D}$ measurements.

- The measurements are done inclusively and differentially in bins of $M_{tt}$, $\cos \theta$ and top quark $p_T$.
- Event reconstruction (jet-parton assignment) is performed using CNN.
- Remove events with NN score $S_{NN} < 0.1$.
- Divide events into categories based on lepton flavor, number of b-tags, and NN score.

Fraction of $tt$ events with correctly assigned jets to partons including $d$-type quark.
**L+jets**: Example of the fit

- In each \((M_{tt}, \cos \theta)\) bin \(\cos \chi\) distribution is fit to the reco-level templates

![Graph showing the distribution of \(M_{tt}\) and \(\cos \theta\) for different mass bins](image)

- **Pre-fit**
  - \(300-400\) GeV
  - \(400-500\) GeV
  - \(500-600\) GeV
  - \(600-700\) GeV
  - \(700-800\) GeV
  - \(800-900\) GeV
  - \(900-1000\) GeV
  - \(>1000\) GeV

- **Post-fit**
  - \(\theta - 0.4 - 0.7 - 1\)

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- Full measurement of the vectors $P$ and matrix $C$ is performed using templates defined based on the functions of angles of top and antitop decay products

$$\Sigma_m = \sigma_{\text{norm}} \left\{ \kappa \sin \theta_p \cos \phi_p, \ldots - \kappa \bar{\kappa} \cos \theta_p \cos \theta_{\bar{p}} \right\}$$

- The total cross section is a linear combination of these templates with coefficients $Q_m$ that are the components of $P$ and $C$

$$\Sigma_{\text{tot}} = \Sigma_0 + \sum_{m=1}^{15} Q_m \Sigma_m$$

- The templates $T_m$ are defined at the reco level.

To avoid generating events with every possible combination of $Q_m$ the events are reweighted with weights defined at the gen level

$$w_i = \frac{\Sigma_i}{\Sigma_{\text{tot}}}$$

To minimize the bias due to variation of $Q_m$ or $T_m$ within the bin we perform the measurement in finer bins in $M_{tt}$ and $\cos \theta$, then combine
Full measurement of the $P$ and $C$ is performed inclusively and differentially in bins of $M_{tt}$, $\cos \theta$ and top $p_T$.

Full covariance matrix will be provided with the published result.

A good agreement with the SM prediction is observed.
L+jets – entanglement results

- We quantify the entanglement using Peres Horodecki criterion
- Significant entanglement is observed in the high $M_{tt}$ region

Based on full matrix $\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$

<table>
<thead>
<tr>
<th>$m(t\bar{t}) &lt; 400$ GeV</th>
<th>$m(t\bar{t}) &gt; 800$ GeV</th>
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<tr>
<td>2.2 (2.4)$\sigma$</td>
<td>6.7 (5.6)$\sigma$</td>
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Low $M_{tt}$
- $D < -\frac{1}{3}$
- $138$ fb$^{-1}$ (13 TeV)

High $M_{tt}$
- $\tilde{D} > \frac{1}{3}$
- $138$ fb$^{-1}$ (13 TeV)
Excluding classical explanation

- What is the maximum value of $\Delta_E$ that can still be explained by the non-quantum communication ($v\leq c$)?
- In this case only top and antitop decays separated by a time-like interval are entangled
- The rest of the events must be separable
- Since top and antitop decay vertices are not observed, the fraction of space-like events, $f$, can only be determined statistically

\[ \Delta_{E_{critical}} = f(\Delta_E = 1) + (1-f)(\Delta_E = 3) \]

Max($|C_{ii}|$) = 1

- ArXiv:2110.10112v2 – Fig9 – fraction of space-like events

<table>
<thead>
<tr>
<th>CMS Preliminary</th>
<th>138 fb$^{-1}$ (13 TeV)</th>
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Observed $\Delta_E$ exceeds $\Delta_{E_{critical}}$ by $>5\sigma$
excluding classical explanation
Angular distributions of the top and antitop quarks were used to measure their polarization and spin correlation matrix, $C_{ij}$ inclusively and in bins of $M_{tt}$, $\cos\theta$ and top quark $p_T$.

In some regions of phase space top and antitop get entangled, which can be demonstrated using Peres-Horodecki criterion based on their spin correlation matrix.

Maximally entangled states are a singlet produced at the threshold, and a triplet produced at high $M_{tt}$.

Both dilepton and single lepton channels were used for spin correlation studies.

- dilepton channel is more sensitive at the production threshold,
- 1+jets channel is better suited for high $M_{tt}$

Based on $D$ measurement in dilepton channel the entanglement was observed at $>5\sigma$ level at low $M_{tt}$

- $345 < M_{tt} < 400 \text{ GeV}, \beta < 0.9$

Using full matrix measurement the entanglement was observed at $6.7\sigma$ level at high $M_{tt}$

- $M_{tt} > 800 \text{ GeV}, |\cos\theta| < 0.4$

The later result was found to exceed the maximum entanglement achievable by classical communication by $>5\sigma$
Space-like separated events

Spin correlations are evaluated based on the direction of the top quark decay products. Hence, the time of top (antitop) decays $t_1(t_2)$ is considered to be the moment when the measurements is performed. Events are space-like separated if

$$\frac{1-\beta}{1+\beta} t_1 < t_2 < \frac{1+\beta}{1-\beta} t_1$$

Fraction of space-like events

$$f = \frac{1-\beta}{1+\beta}$$

$$\Delta_E \text{critical} = f(\Delta_E = 1) + (1-f)(\Delta_E = 3)$$

$$B_E \text{critical} = f(B_E = \sqrt{2}) + (1-f)(B_E = 2)$$
Systematics

- The analysis is statistics limited
- Theoretical uncertainties
  - Mtop, renormalization/factorization scale, NNLO, EW
- NB. Toponium effect is small for lepton+jets \( \sim 5E-04 \)
- Experimental uncertainties:
  - Jet energy scale, b-tagging efficiency
Impacts

Measurement: $m(\bar{t}t) \text{ vs } |\cos(\theta)|$, $D$

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CMS preliminary

Pull (obs)